Predictive Monte Carlo tools for colliders

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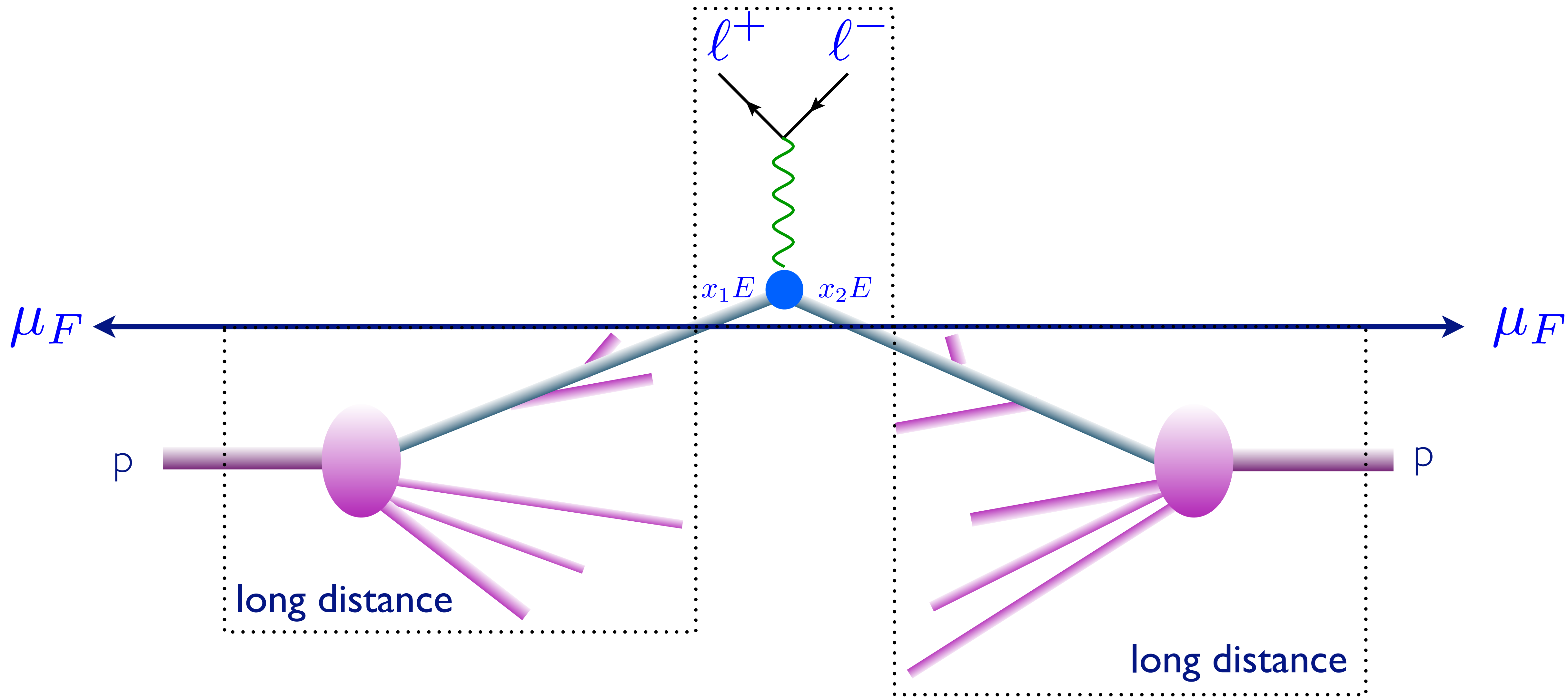
Lecture II

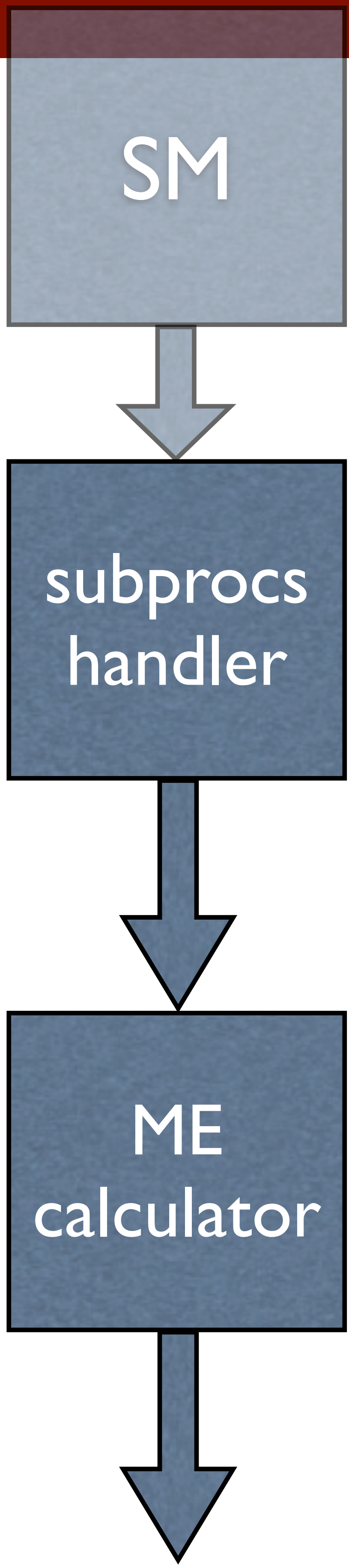
Master formula for the LHC



$$\sum_{a,b}\int dx_1 dx_2 d\Phi_{\rm FS}\; f_a(x_1,\mu_F) f_b(x_2,\mu_F)\; \hat\sigma_{ab\to X}(\hat s,\mu_F,\mu_R)$$

Phase-space integral | Parton density functions | Parton-level cross section

General structure



Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a d d~ u u~ g
d~ d -> a d d~ c c~ g
s~ s -> a d d~ u u~ g
s~ s -> a d d~ c c~ g
...

"Automatically" generates a code to calculate |M|2 for arbitrary processes with many partons in the final state.
Use Feynman diagrams with tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺

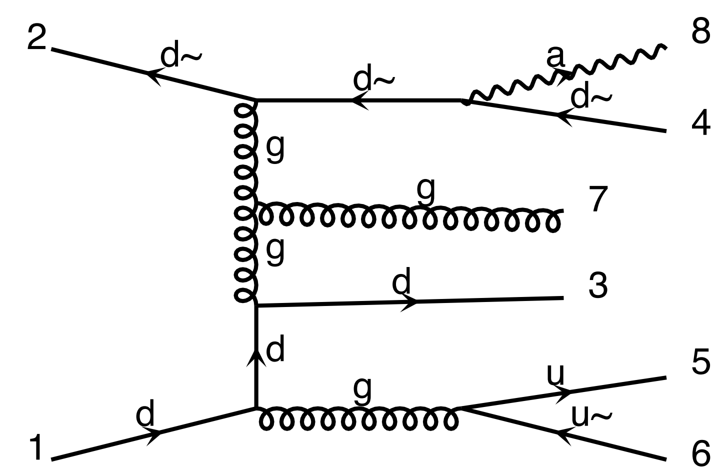
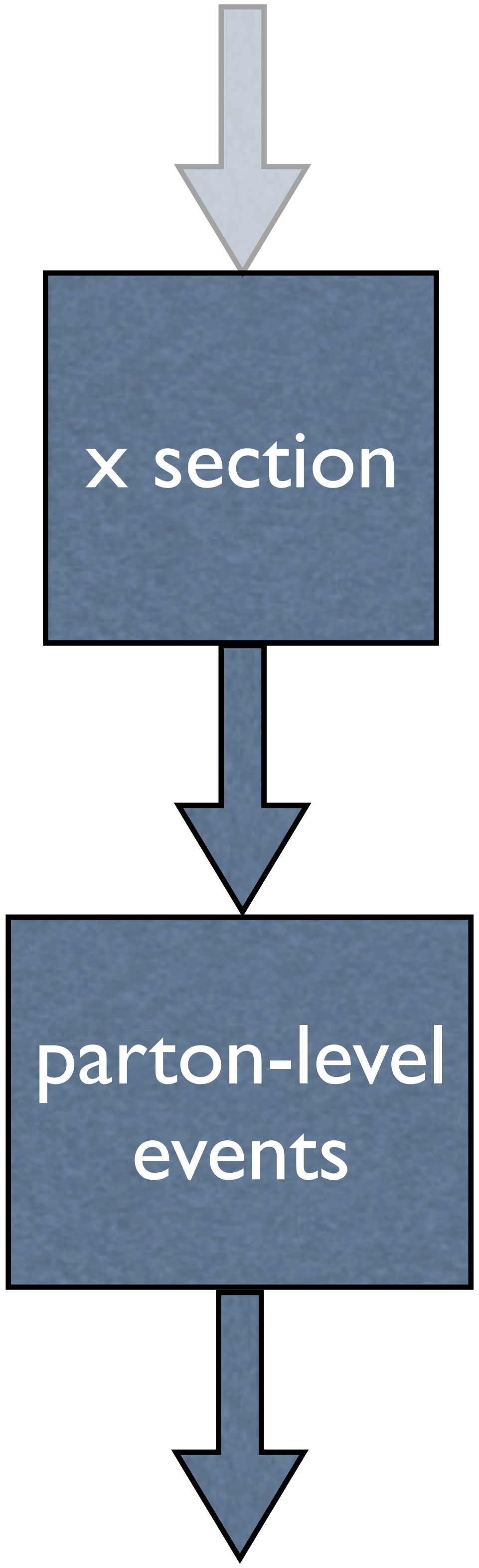
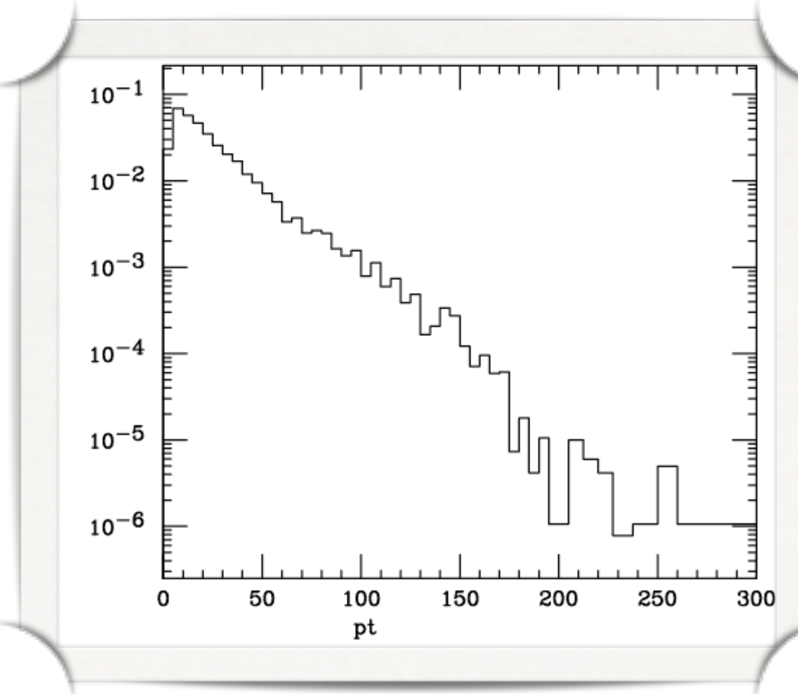


diagram 949 QCD=5, QED=1

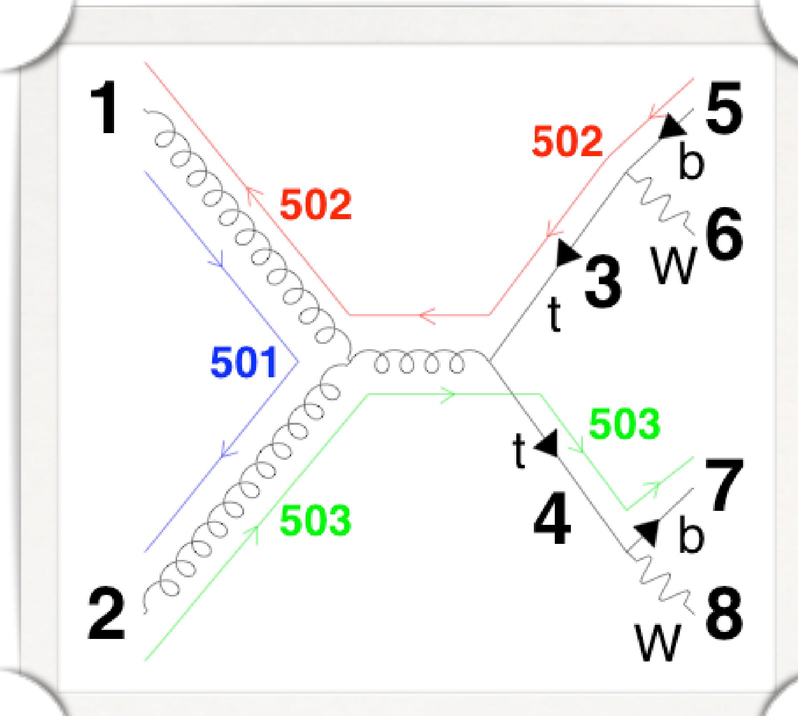
General structure



Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.

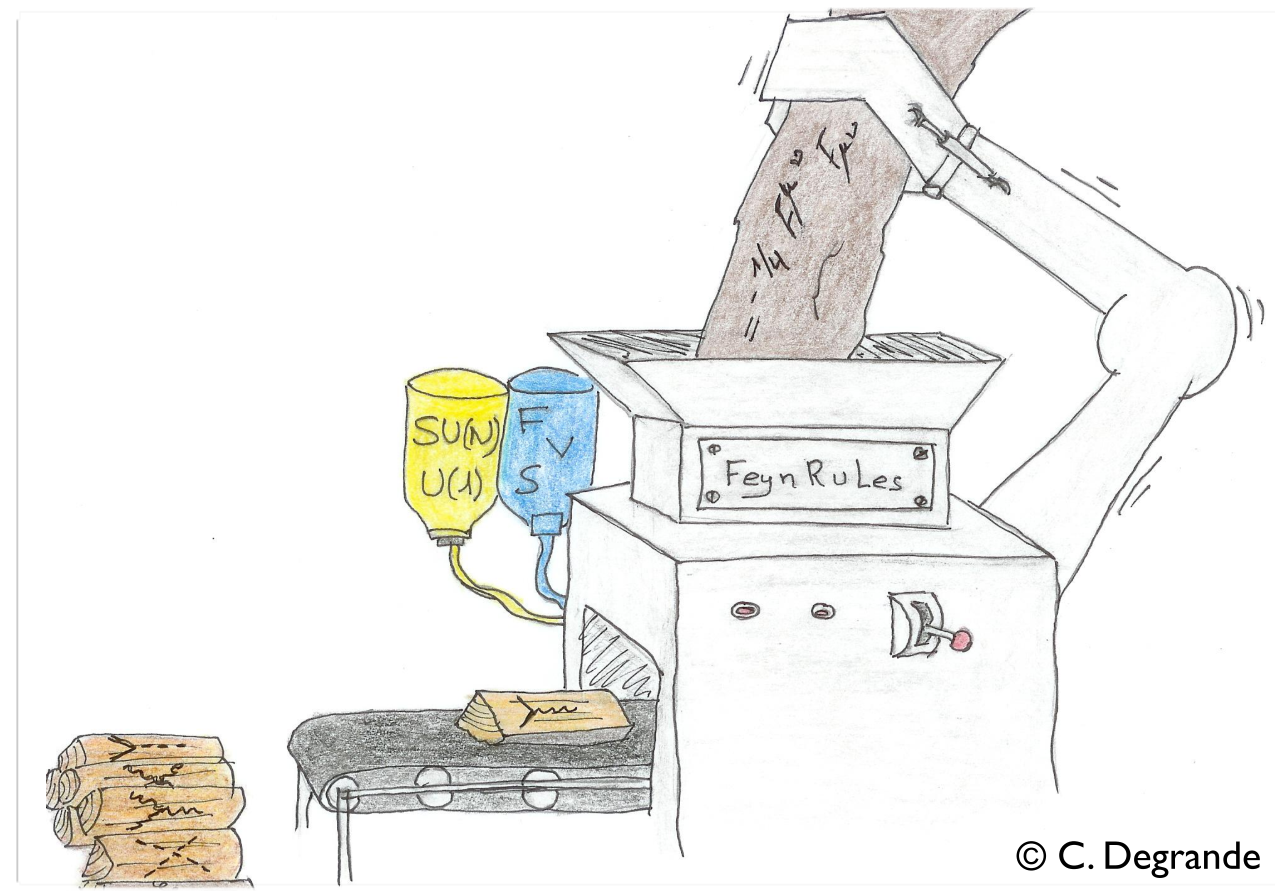


Codes

- Example of tree-level Monte Carlo codes:
  - Alpgen: fast matrix elements due to use of recursion relations. SM only.
  - Comix (Sherpa): fast matrix elements due to use of recursion relations. Some BSM models implemented (however, e.g. no Majorana particles).
  - MadGraph: Feynman diagrams to generate matrix elements which results in high unweighting efficiency. Virtually all BSM models are (or can be) implemented.
- and more: CalcHEP/CompHEP, Whizard, HELAC,...

FeynRules

- FeynRules comes with a set of interfaces, that allow to export the Feynman rules to various matrix element generators.
- Interfaces coming with current public version
  - ➡ CalcHep / CompHep
  - ➡ FeynArts / FormCalc
  - ➡ MadGraph
  - ➡ Sherpa
  - ➡ Whizard / Omega
  - ➡ Universal FeynRules Output



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FEYNRULES

- The input requested form the user is twofold.

- The Model File:
  Definitions of particles and parameters (e.g., a quark)

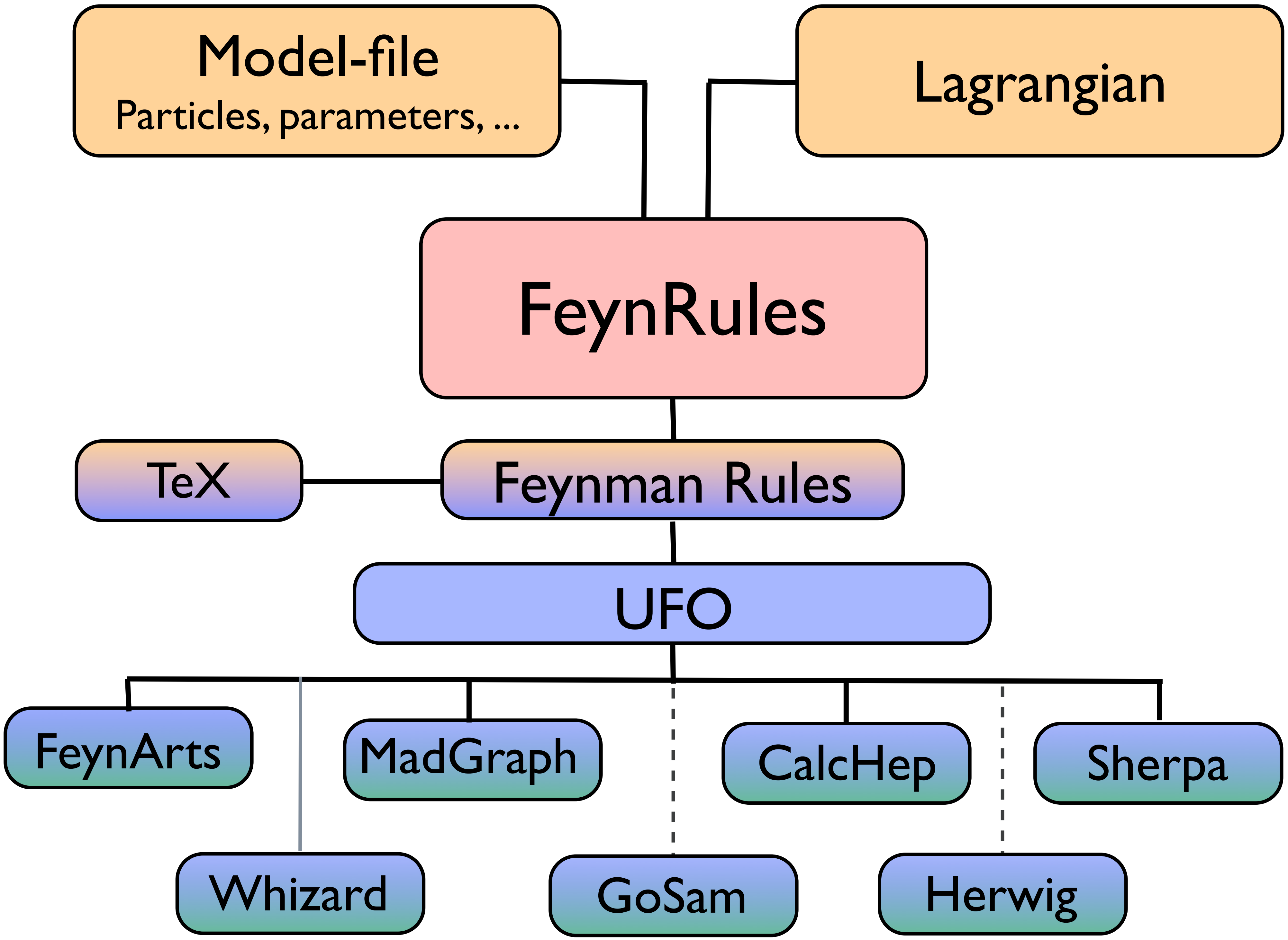
```
F[1] ==
  {ClassName     ->  q,
   SelfConjugate ->  False,
   Indices       ->  {Index[Colour]},
   Mass          ->  {MQ,  200},
   Width         ->  {WQ, 5}   }
```

- The Lagrangian:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu}\, G_a^{\mu\nu} + i\bar{q}\,\gamma^\mu\, D_\mu q - M_q\, \bar{q}\, q$$

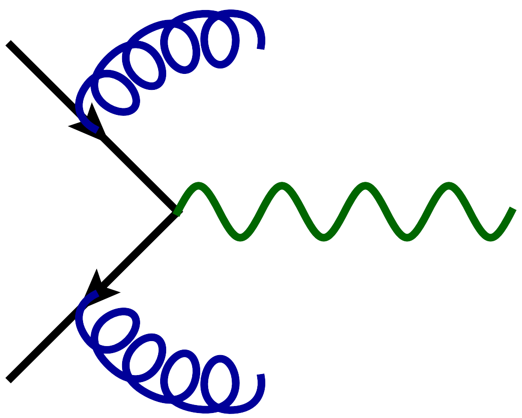
```
L =
-1/4 FS[G,mu,nu,a] FS[G,mu,nu,a]
+ I qbar.Ga[mu].del[q,mu]
- MQ qbar.q
```

FeynRules

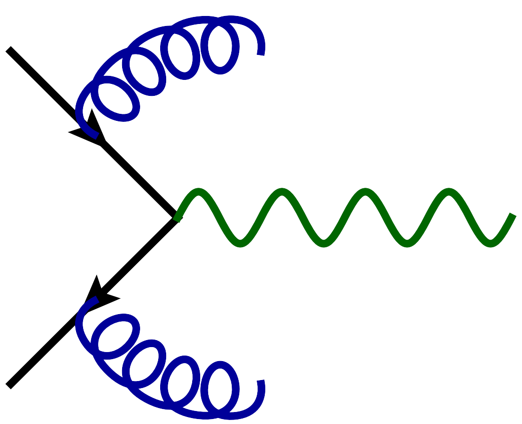

Model-file
Particles, parameters, ...
Lagrangian
FeynRules
TeX
Feynman Rules
UFO
FeynArts
MadGraph
CalcHep
Sherpa
Whizard
GoSam
Herwig

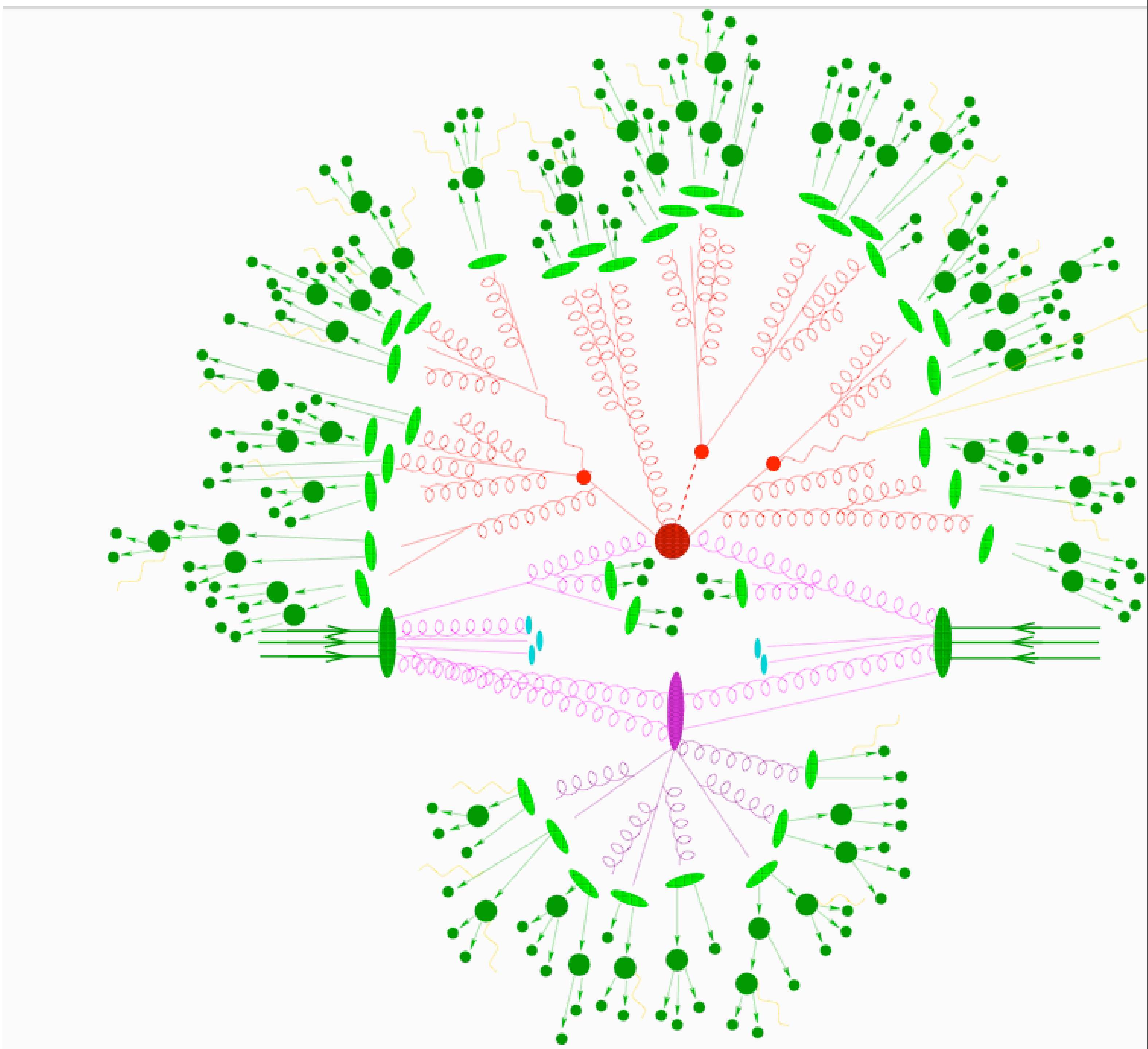
Limits of fixed-order predictions

Limits of fixed-order predictions

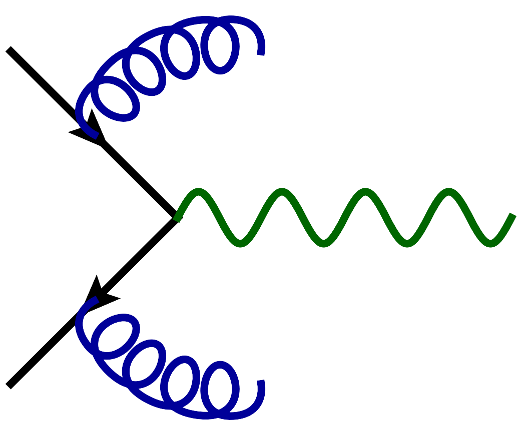


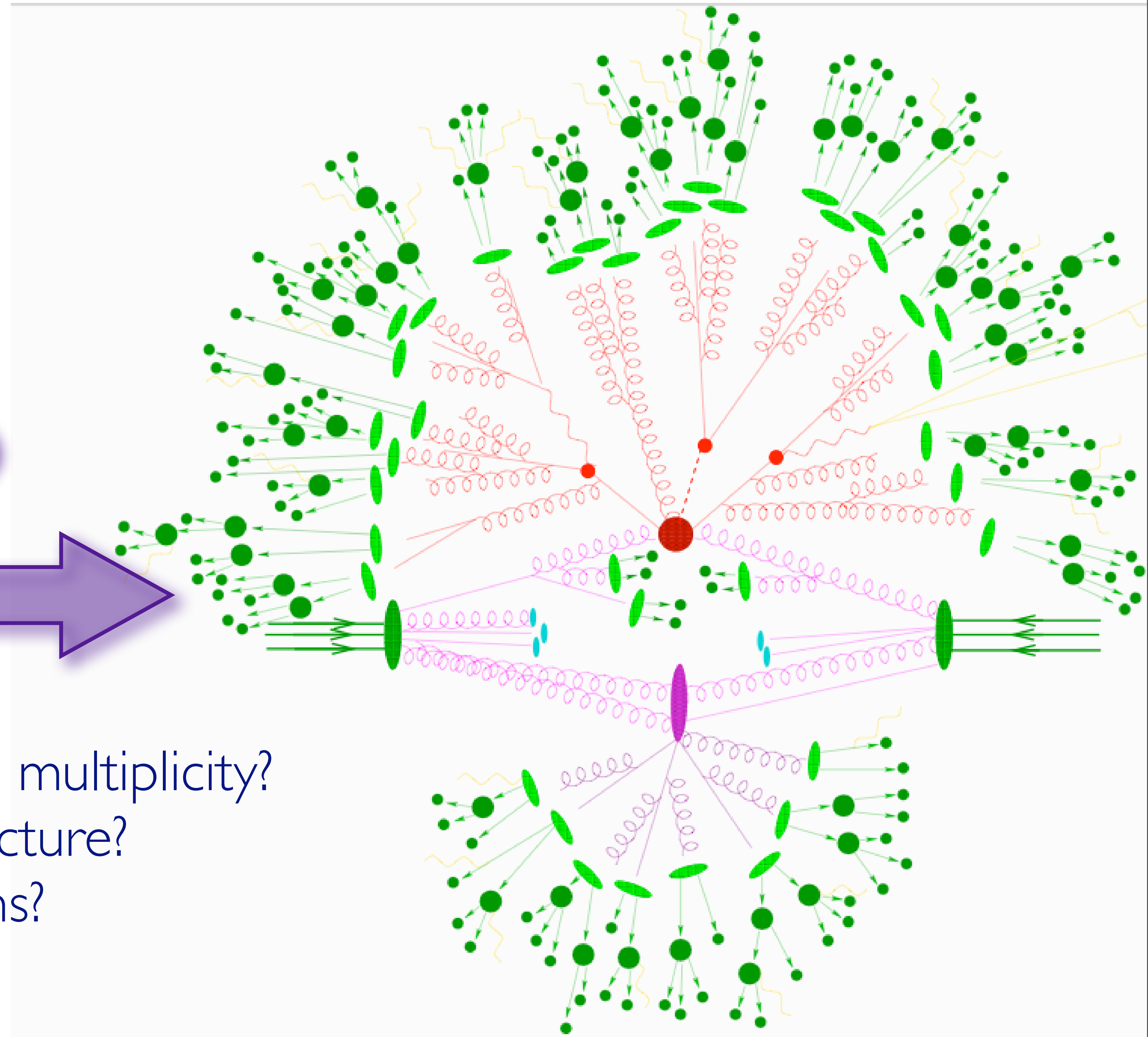
Limits of fixed-order predictions





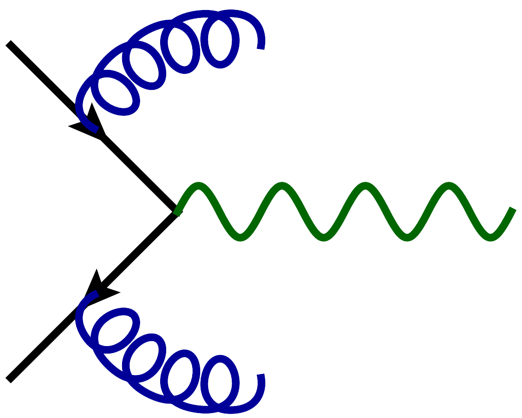
Limits of fixed-order predictions

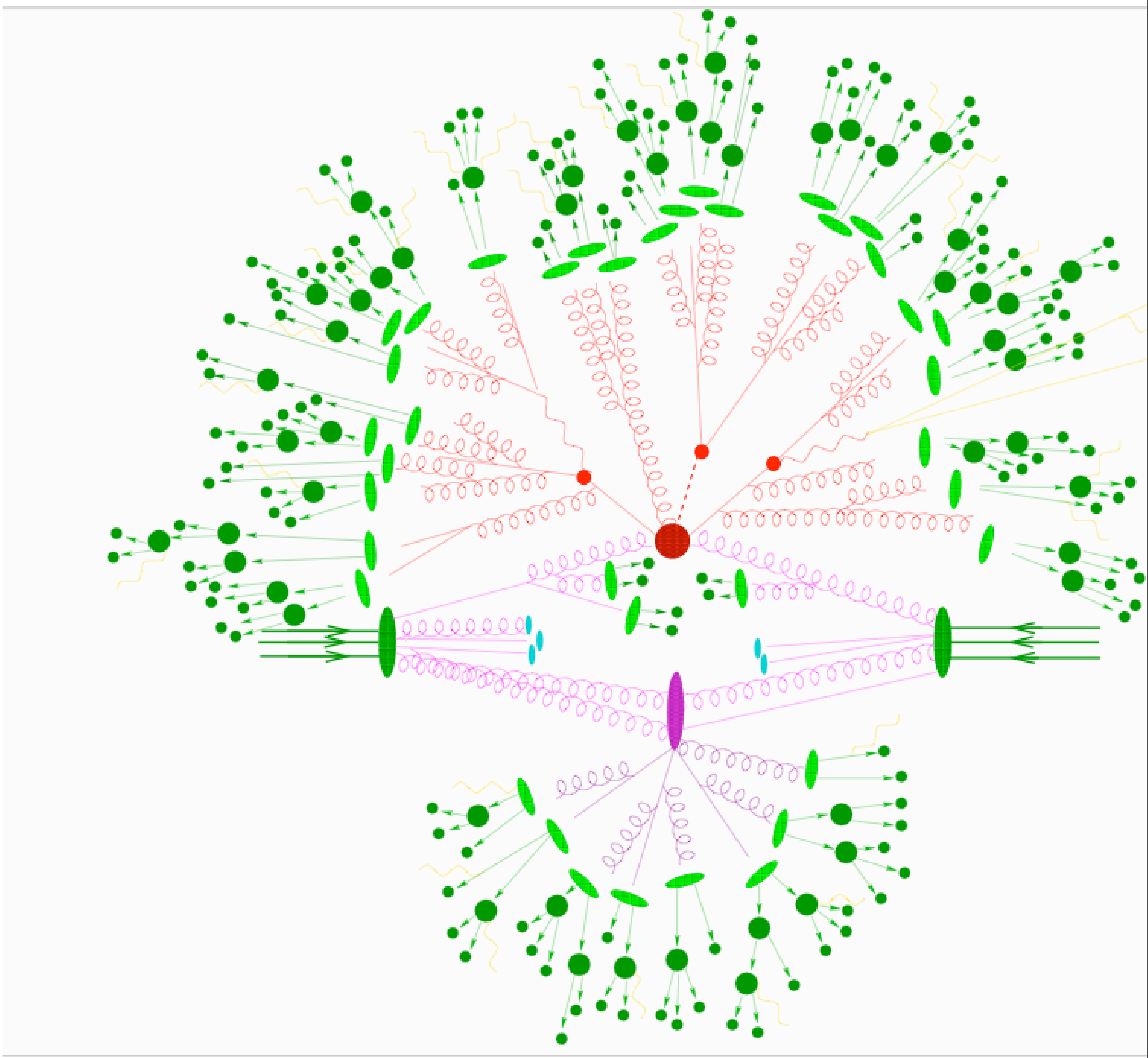




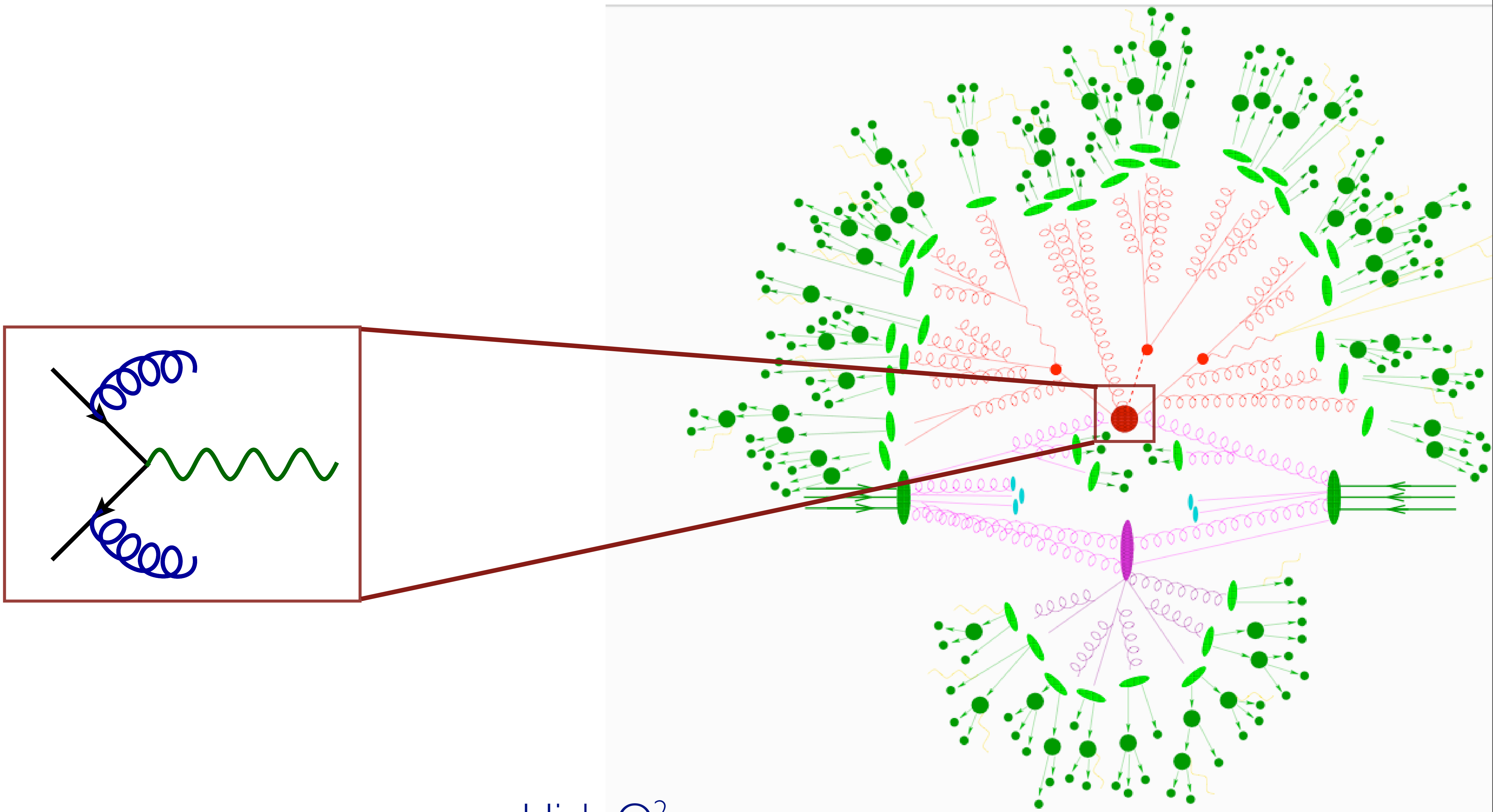
- Particle multiplicity?
- Jet structure?
- Hadrons?

Limits of fixed-order predictions

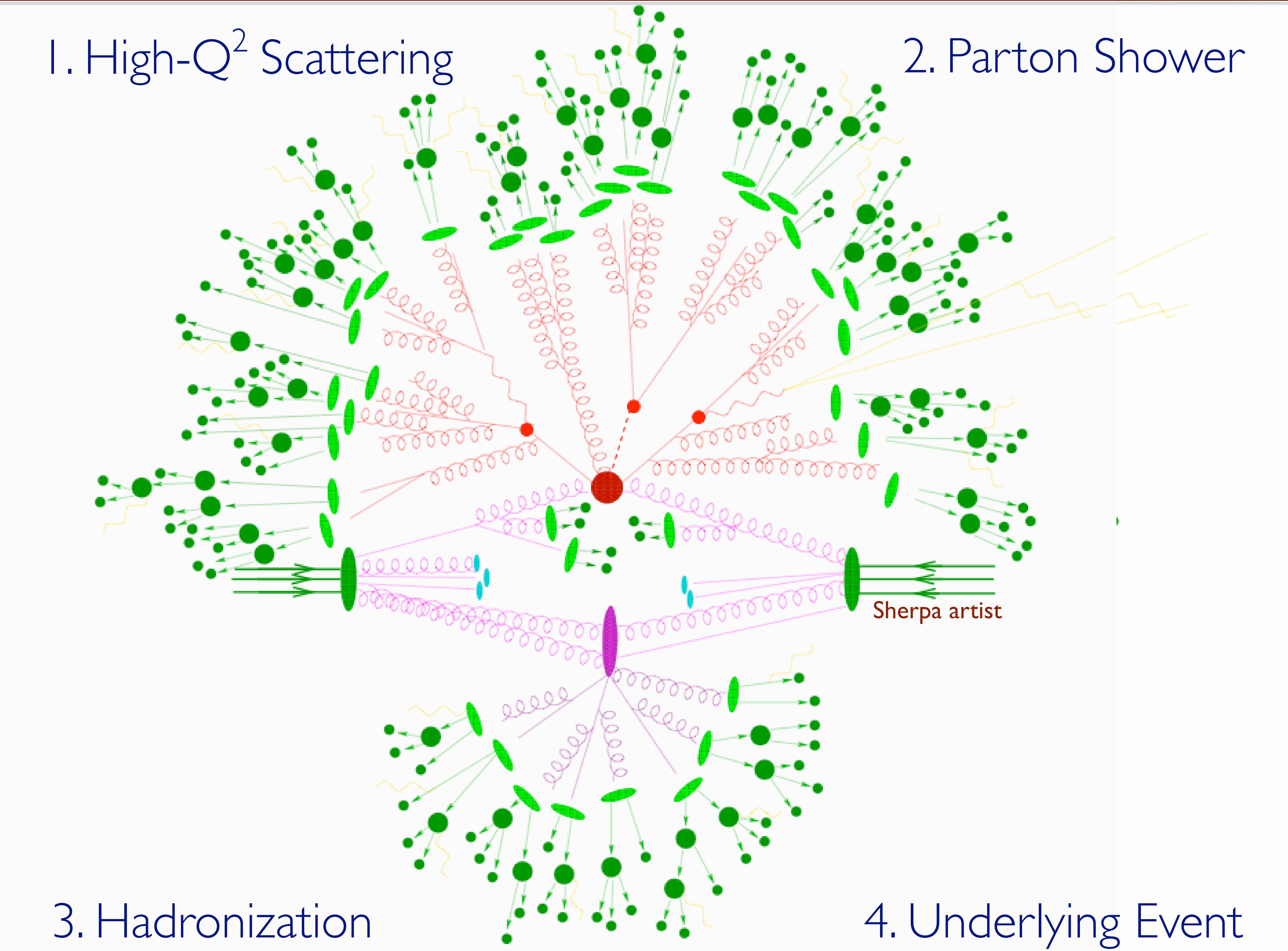




Limits of fixed-order predictions

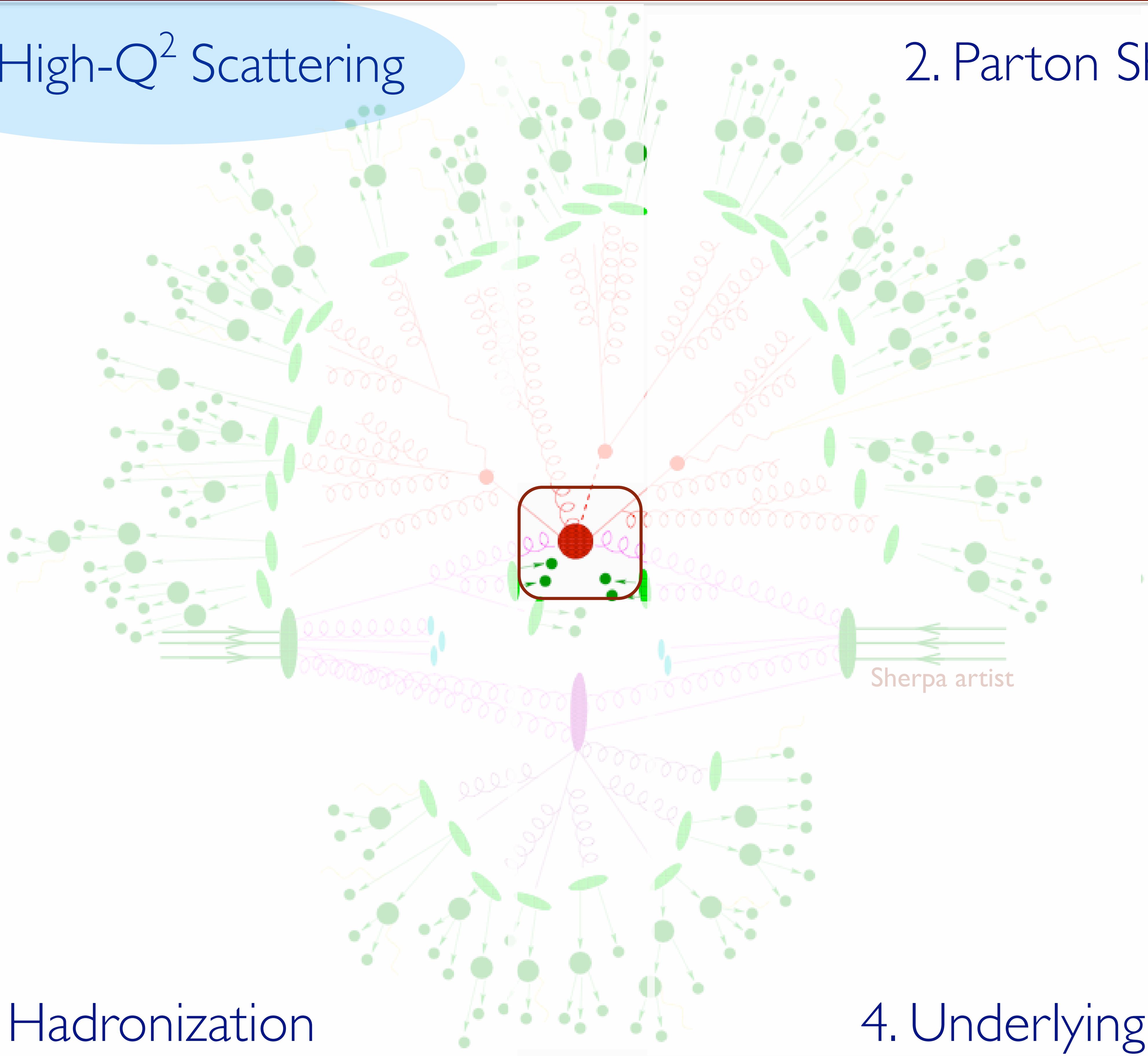


High Q^2


1. High-Q^2 Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event
Sherpa artist

1. High-Q^2 Scattering

2. Parton Shower



3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

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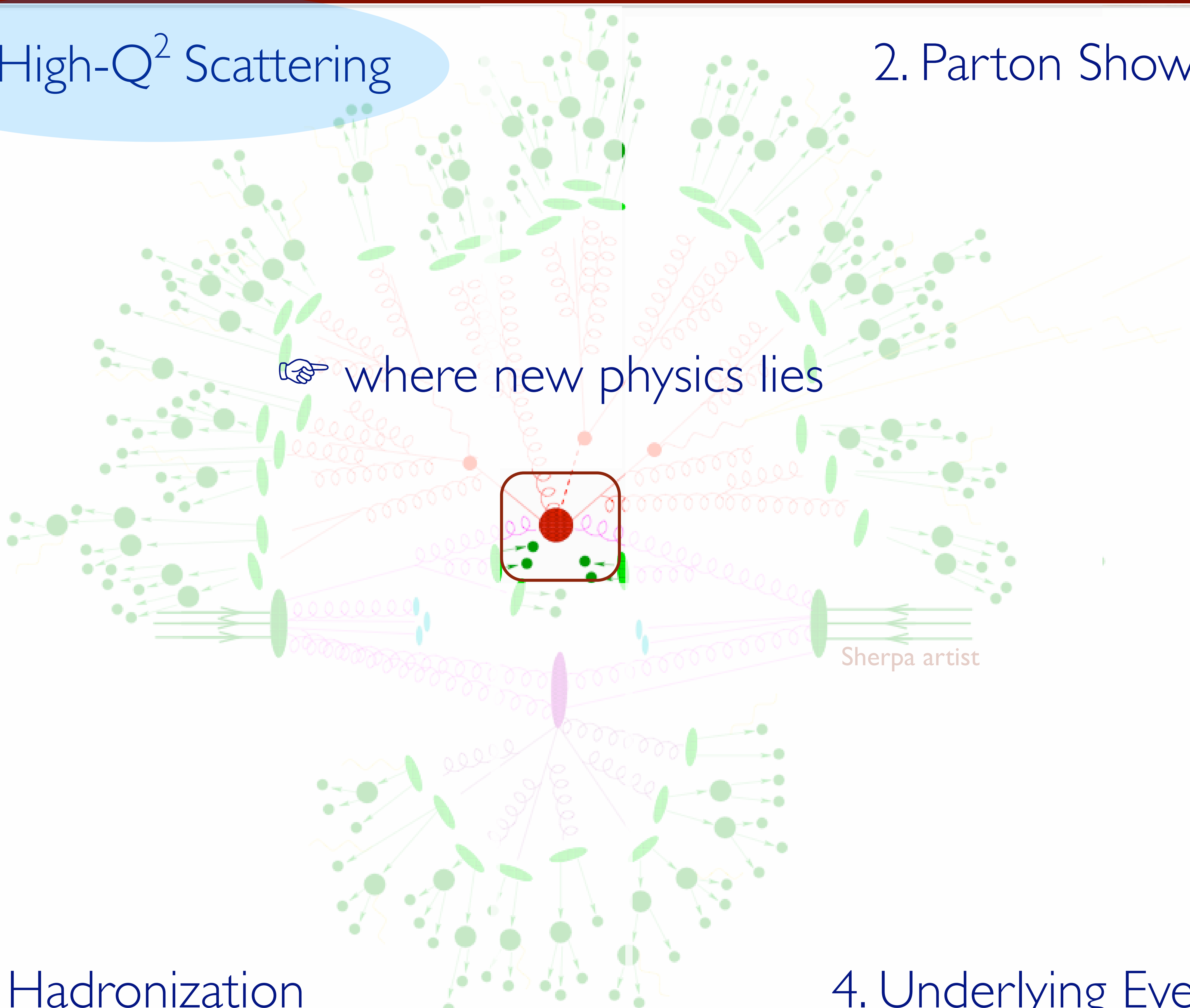
☞ where new physics lies



3. Hadronization

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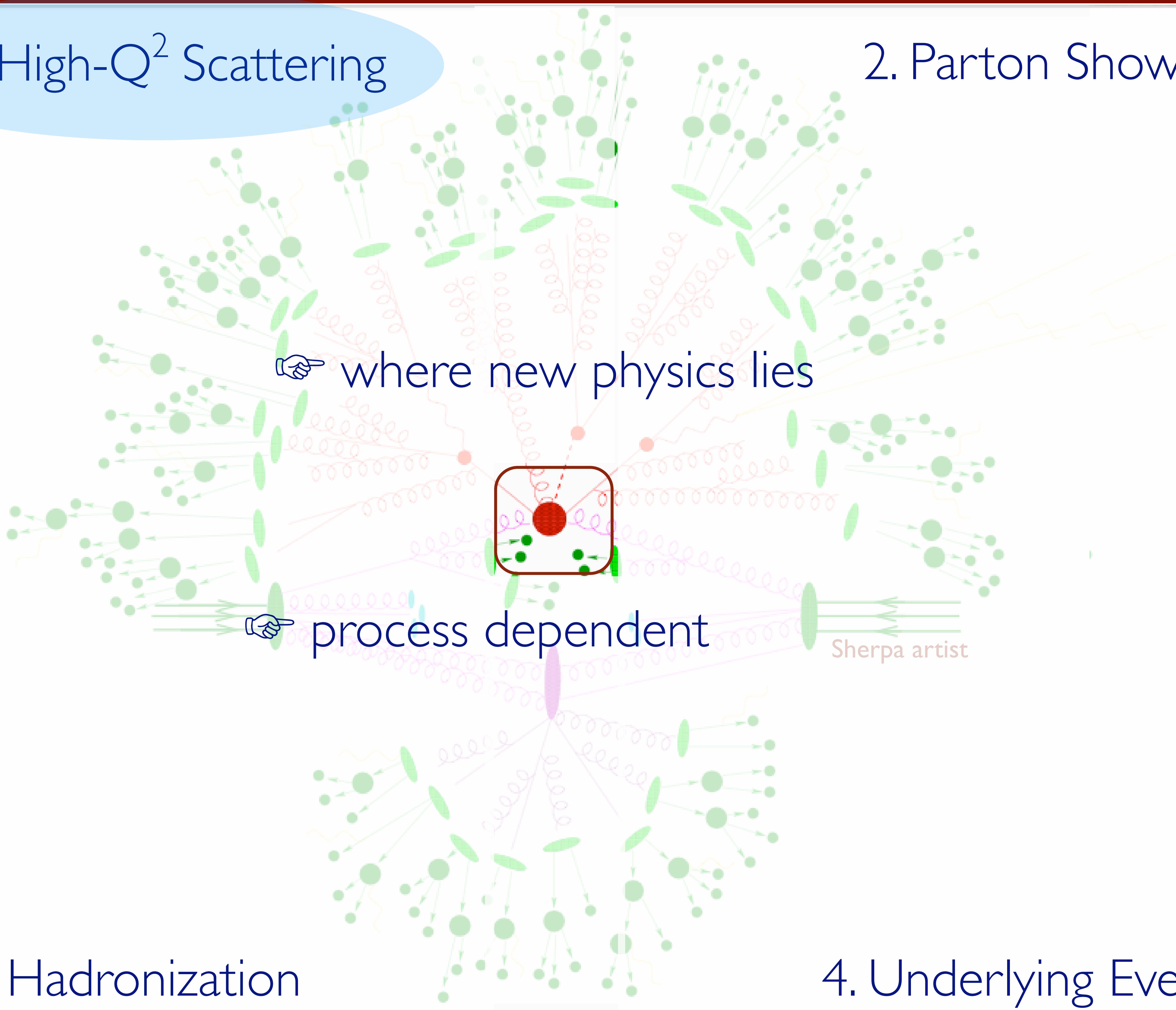

☞ process dependent

3. Hadronization

4. Underlying Event

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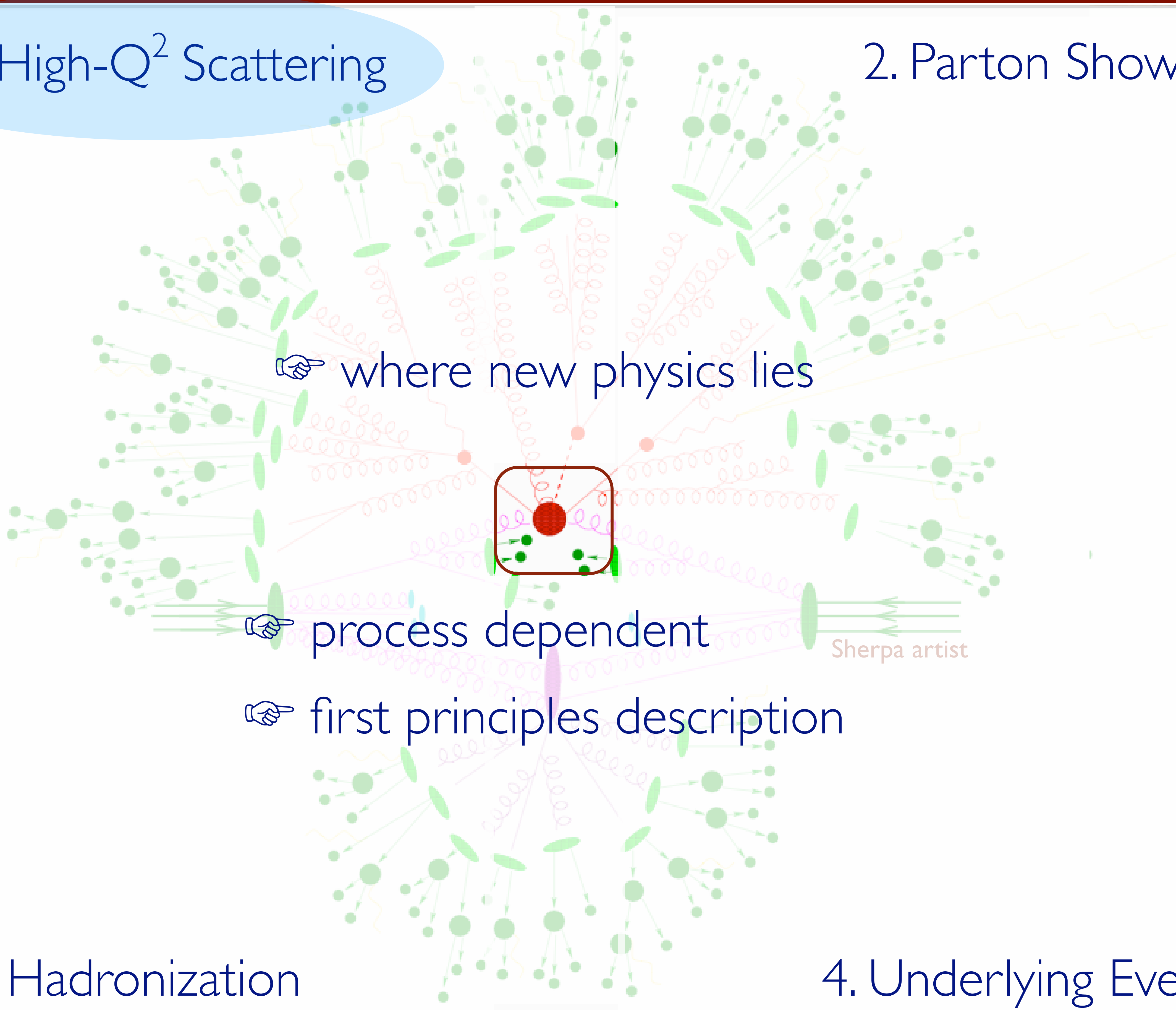

- ☞ where new physics lies
- ☞ process dependent
- ☞ first principles description

3. Hadronization

4. Underlying Event

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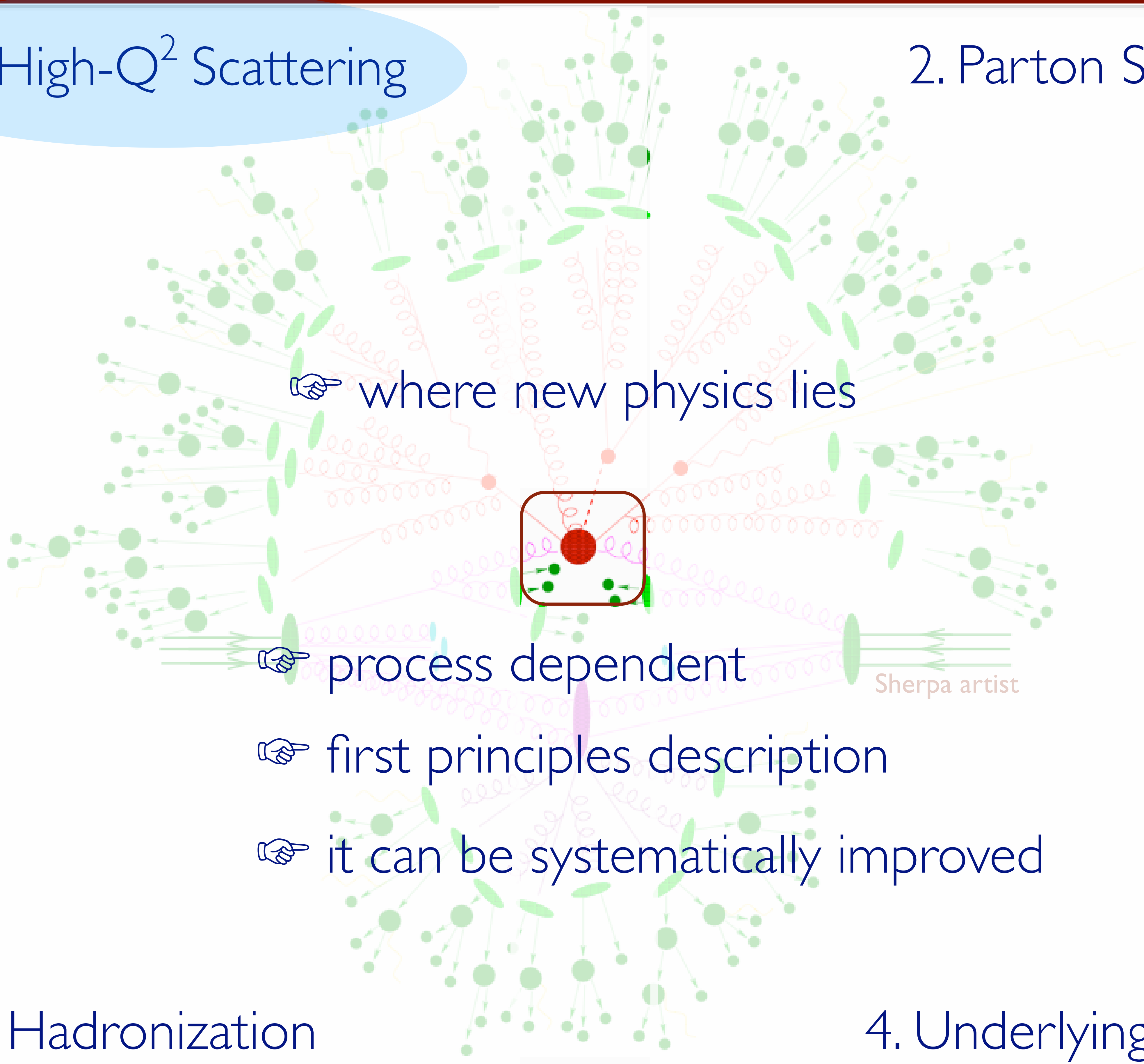
2. Parton Shower



☞ where new physics lies

☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

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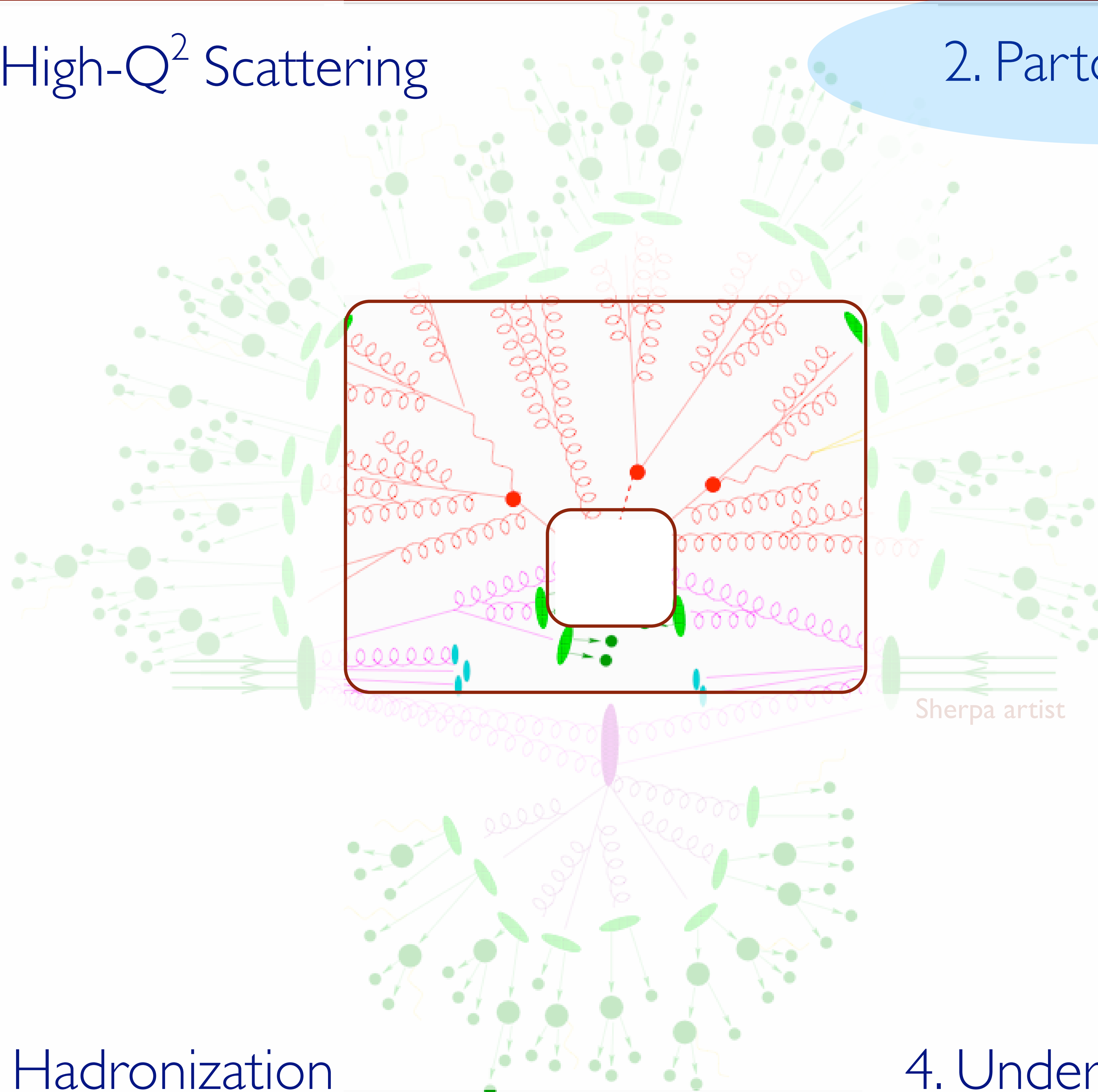
2. Parton Shower



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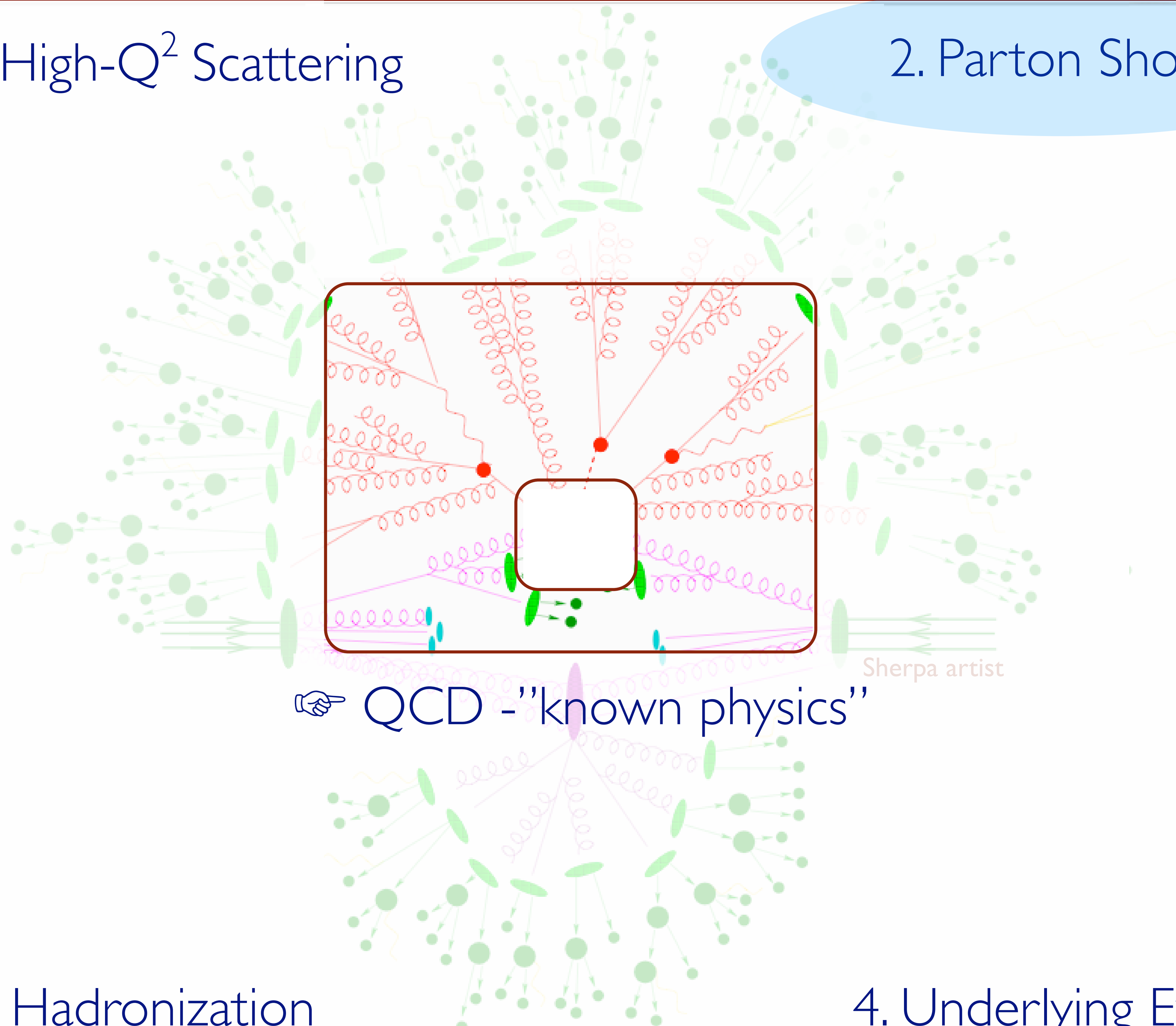

☞ QCD -"known physics"

3. Hadronization

4. Underlying Event

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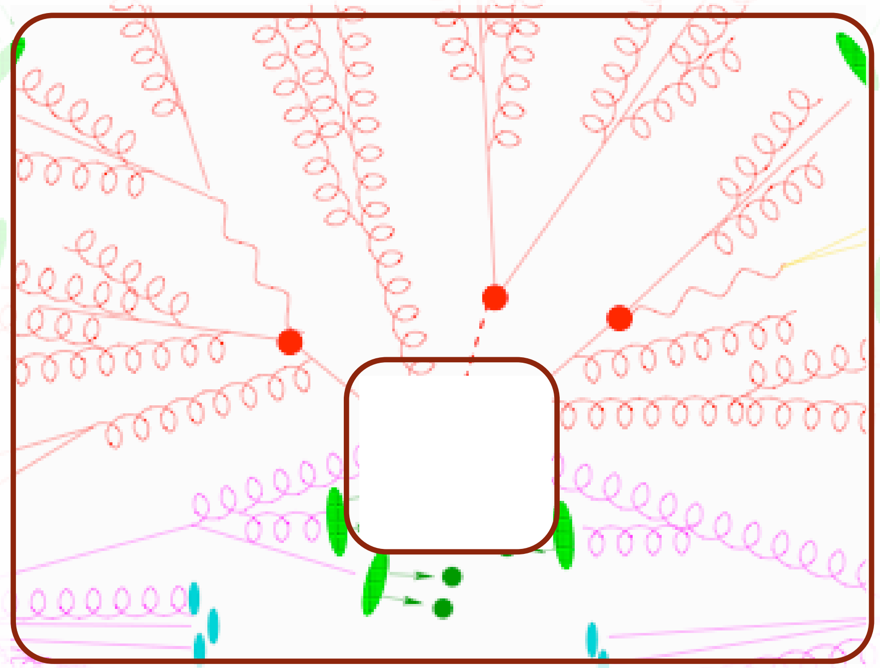

Sherpa artist

☞ QCD -"known physics"

☞ universal/ process independent

3. Hadronization

4. Underlying Event

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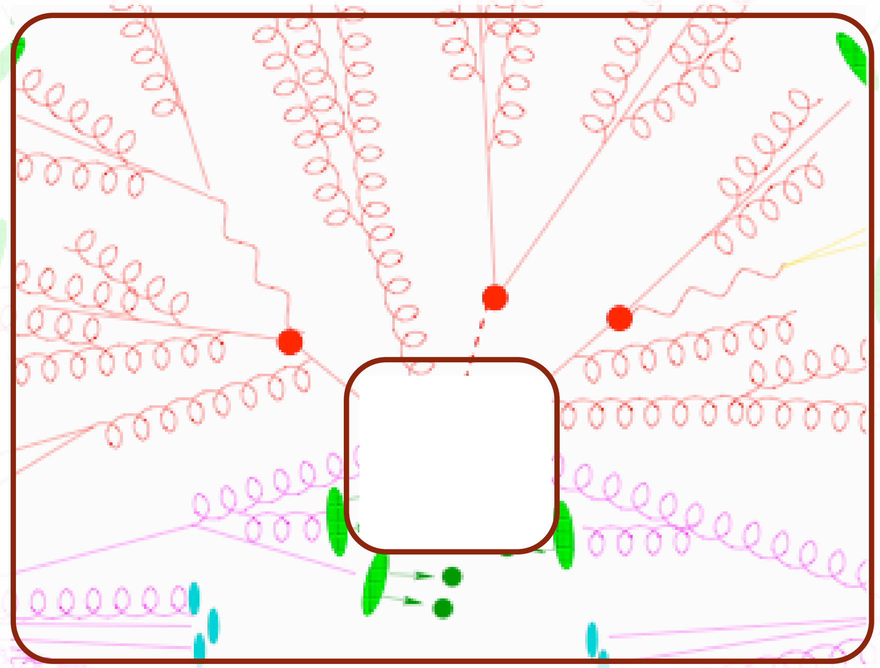

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- ☞ QCD -"known physics"
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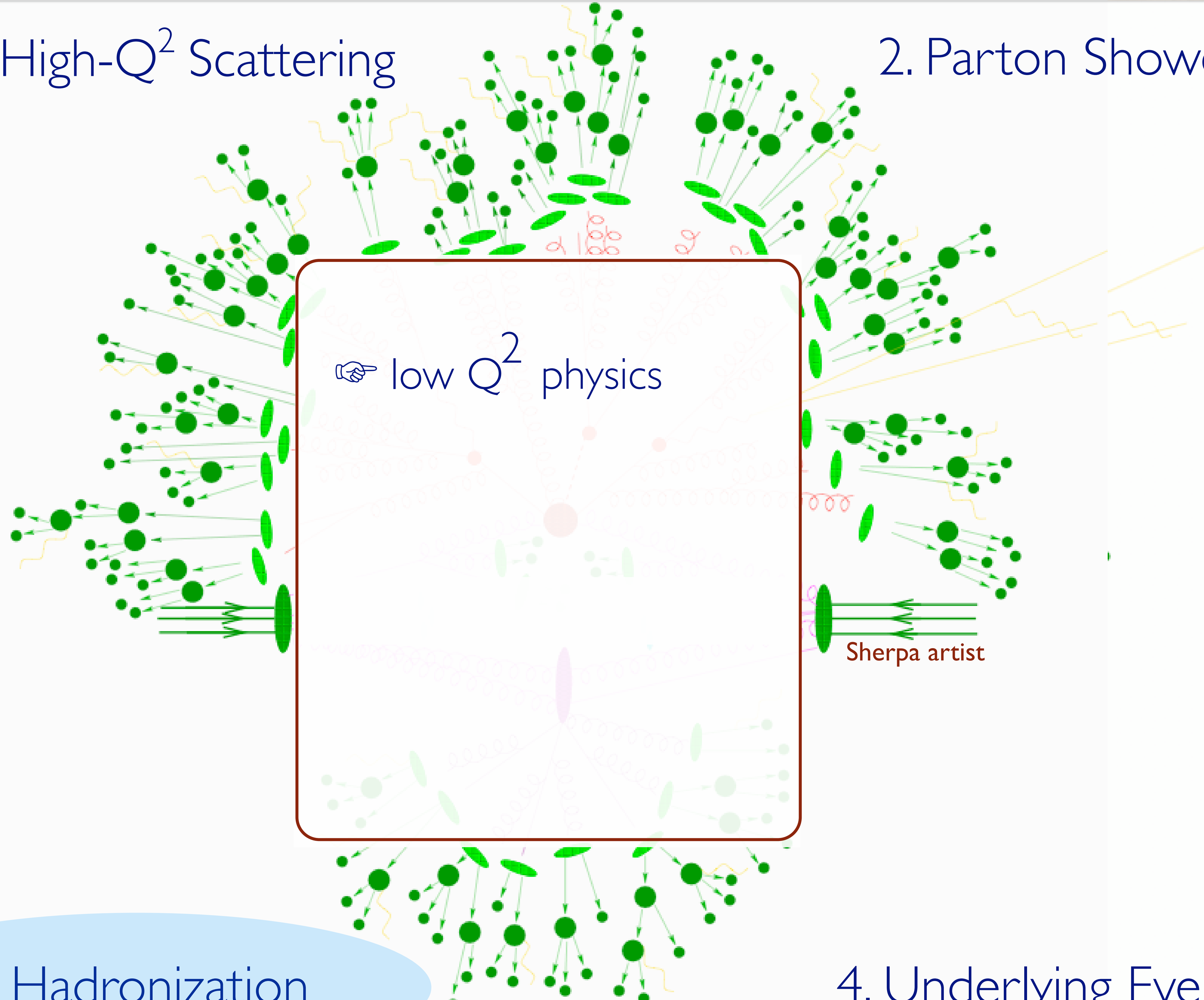

☞ low Q^2 physics

3. Hadronization

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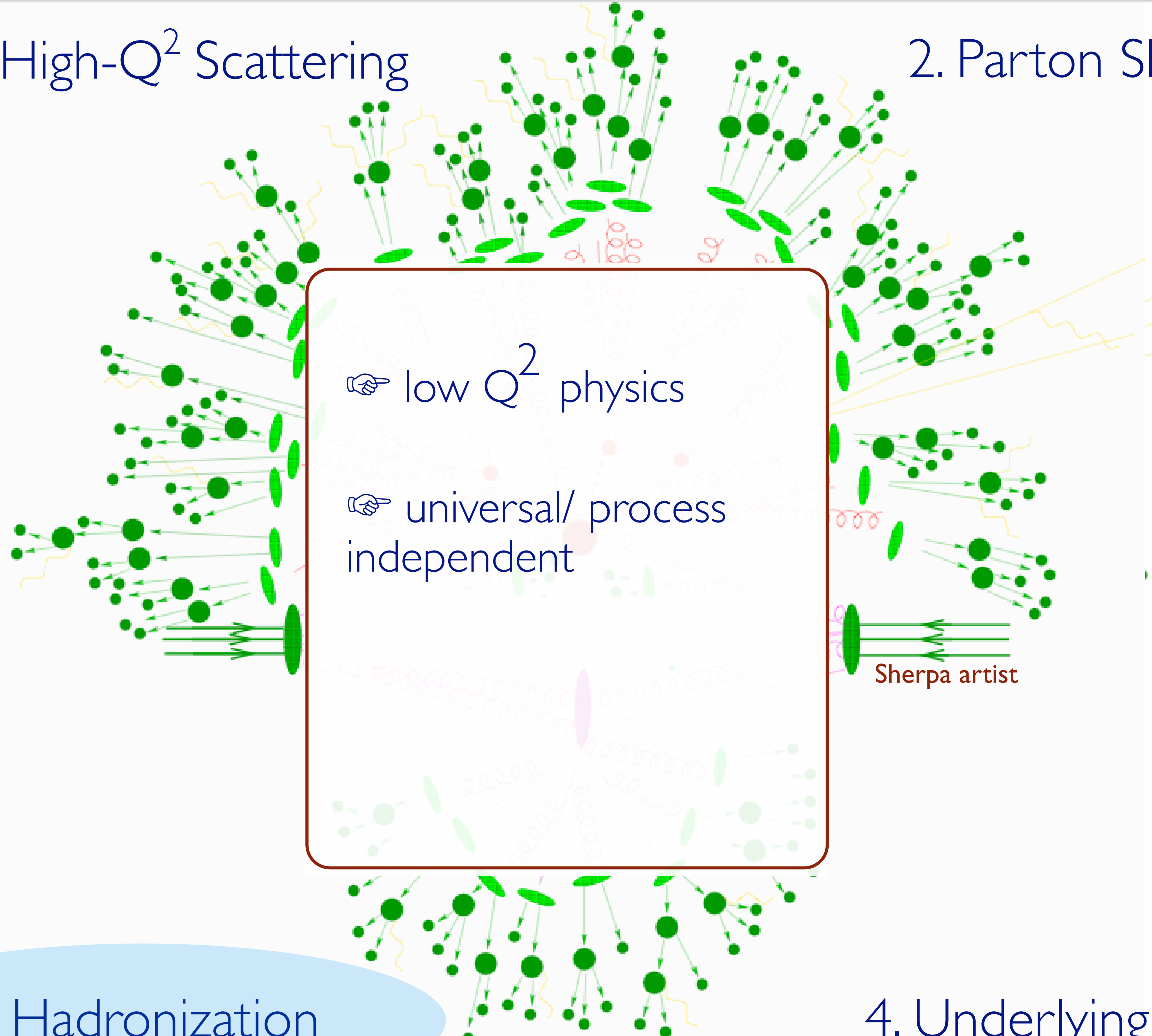

☞ low Q^2 physics

☞ universal/ process independent

3. Hadronization

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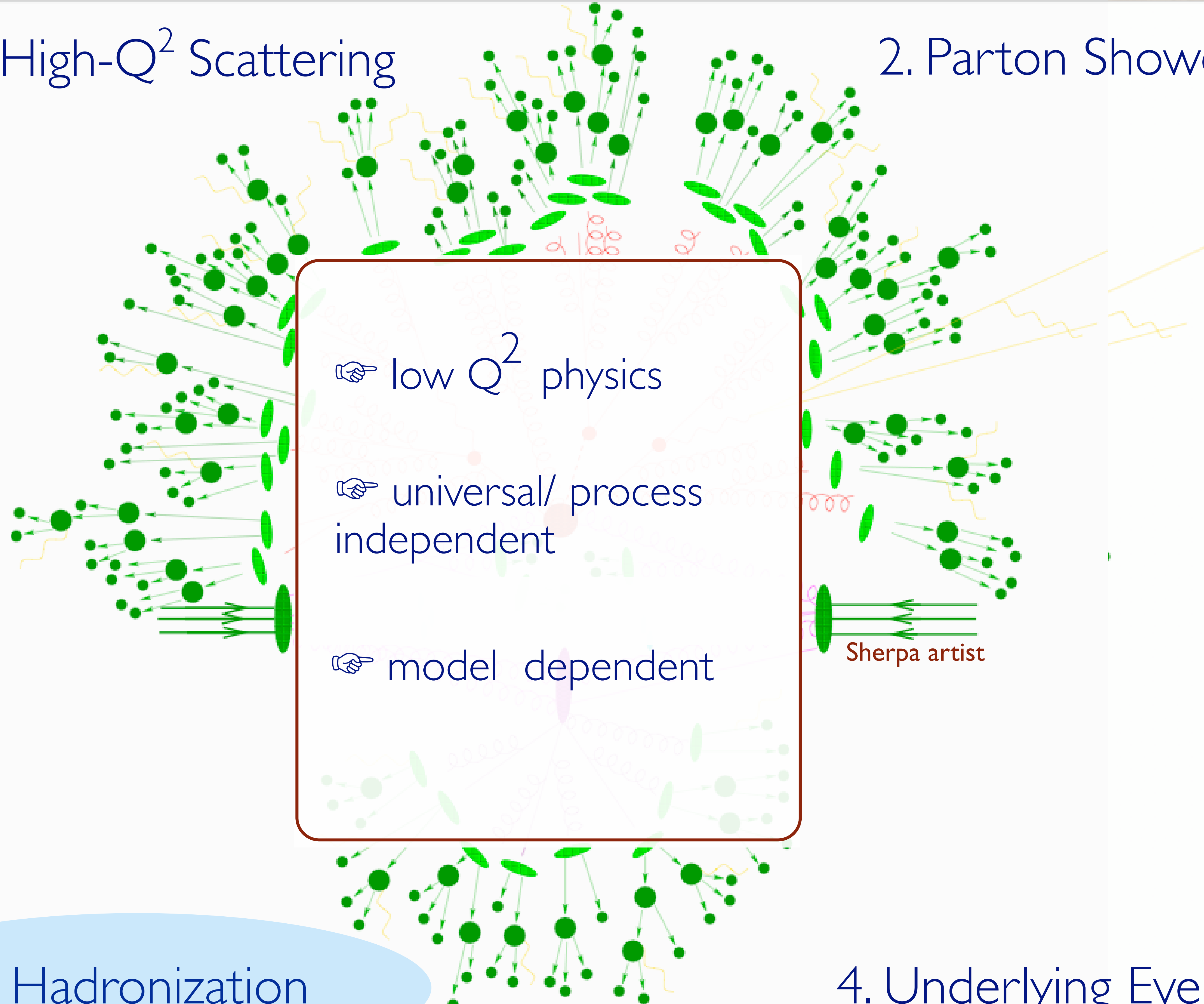

☞ low Q^2 physics

☞ universal/ process independent

☞ model dependent

3. Hadronization

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1. High-Q^2 Scattering

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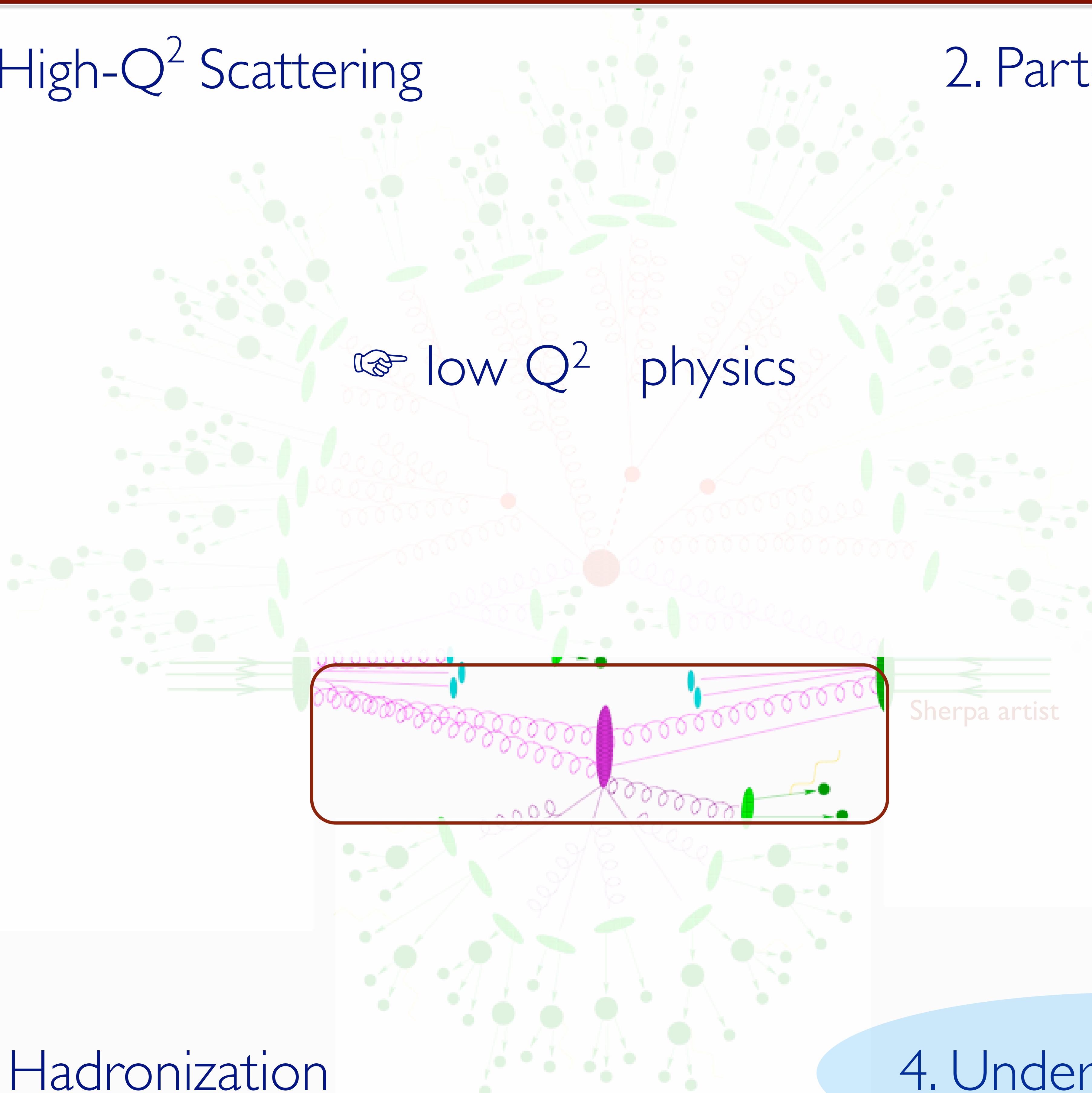
☞ low Q^2 physics

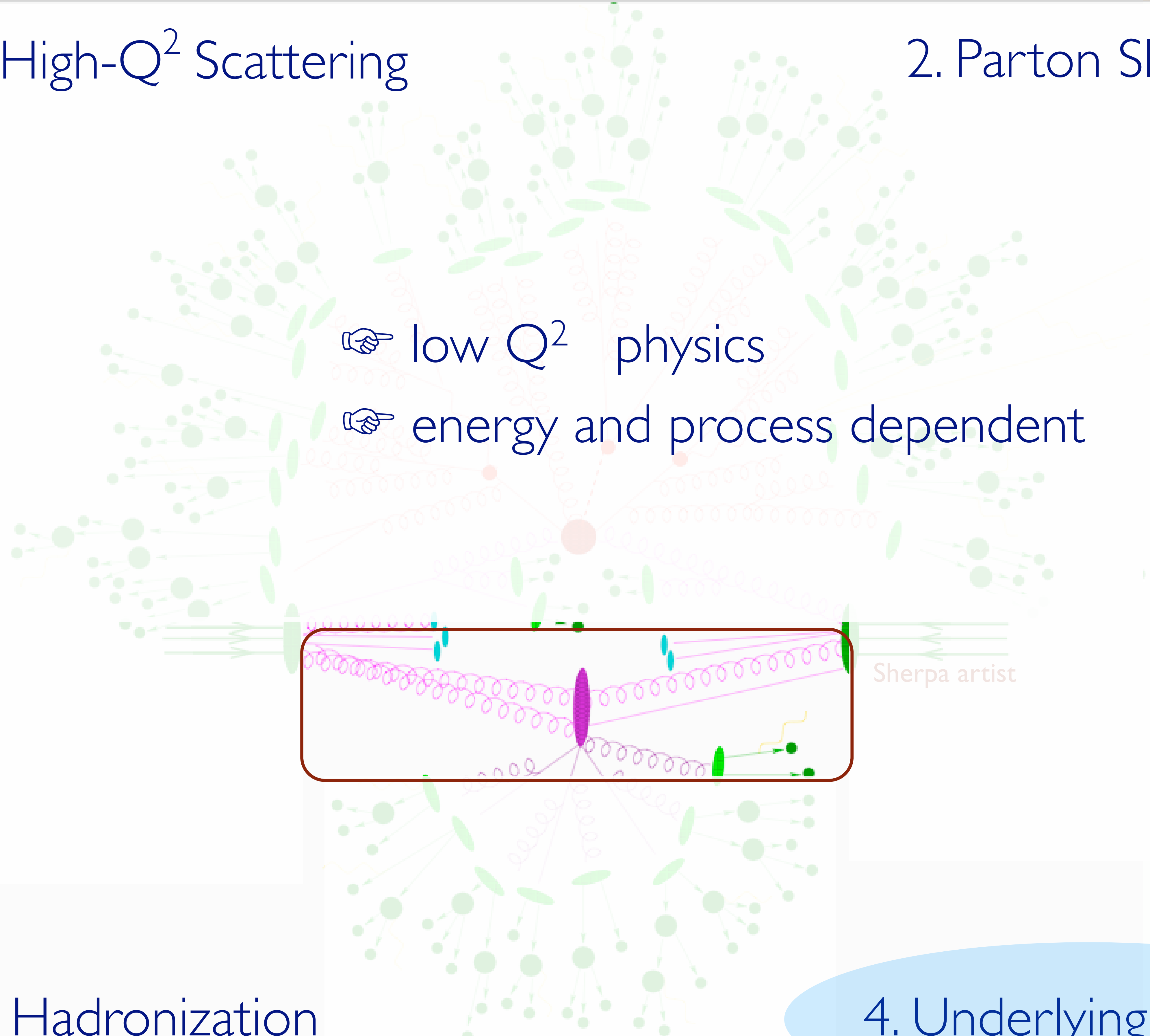


3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



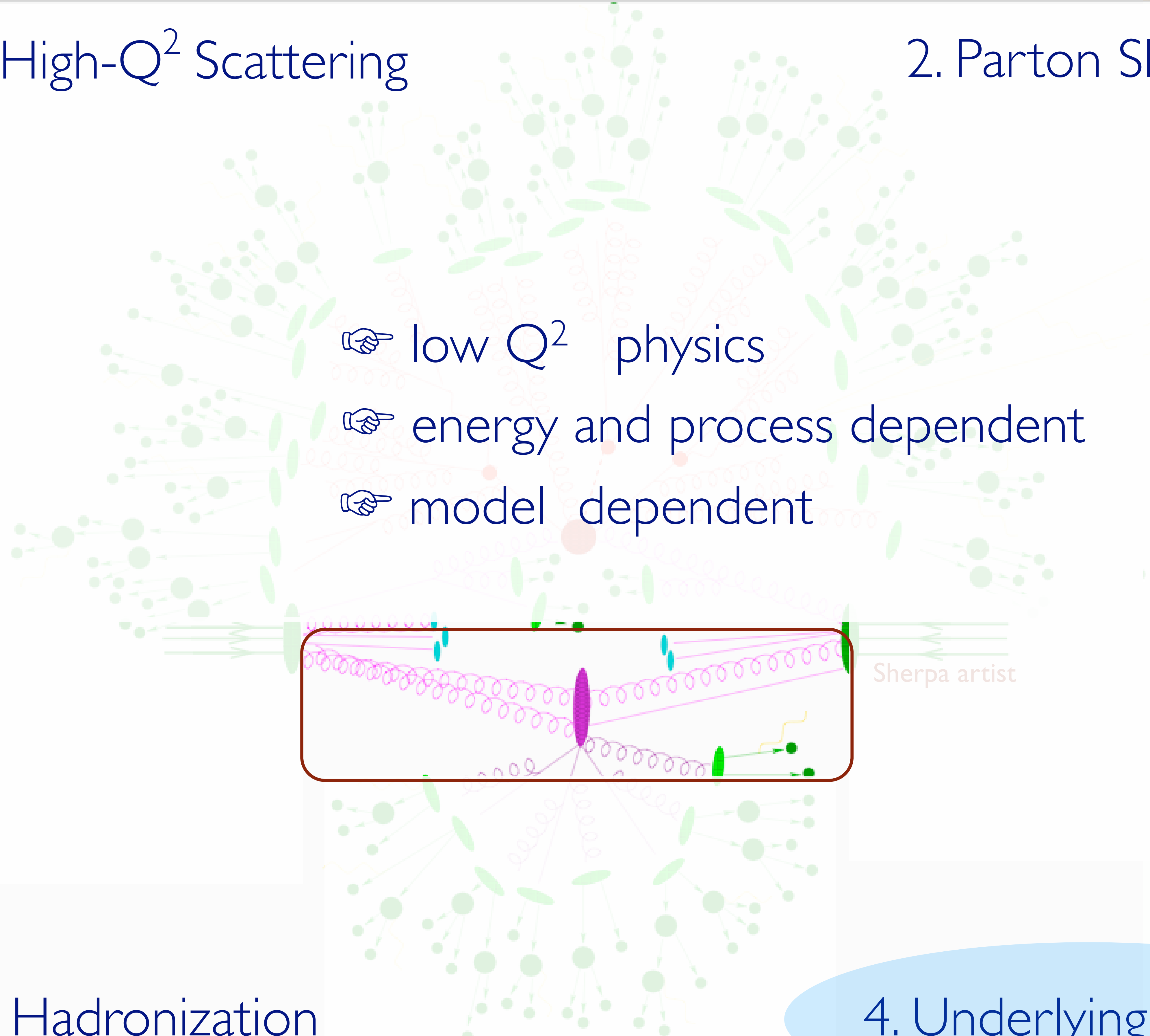
- ☞ low Q^2 physics
- ☞ energy and process dependent

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



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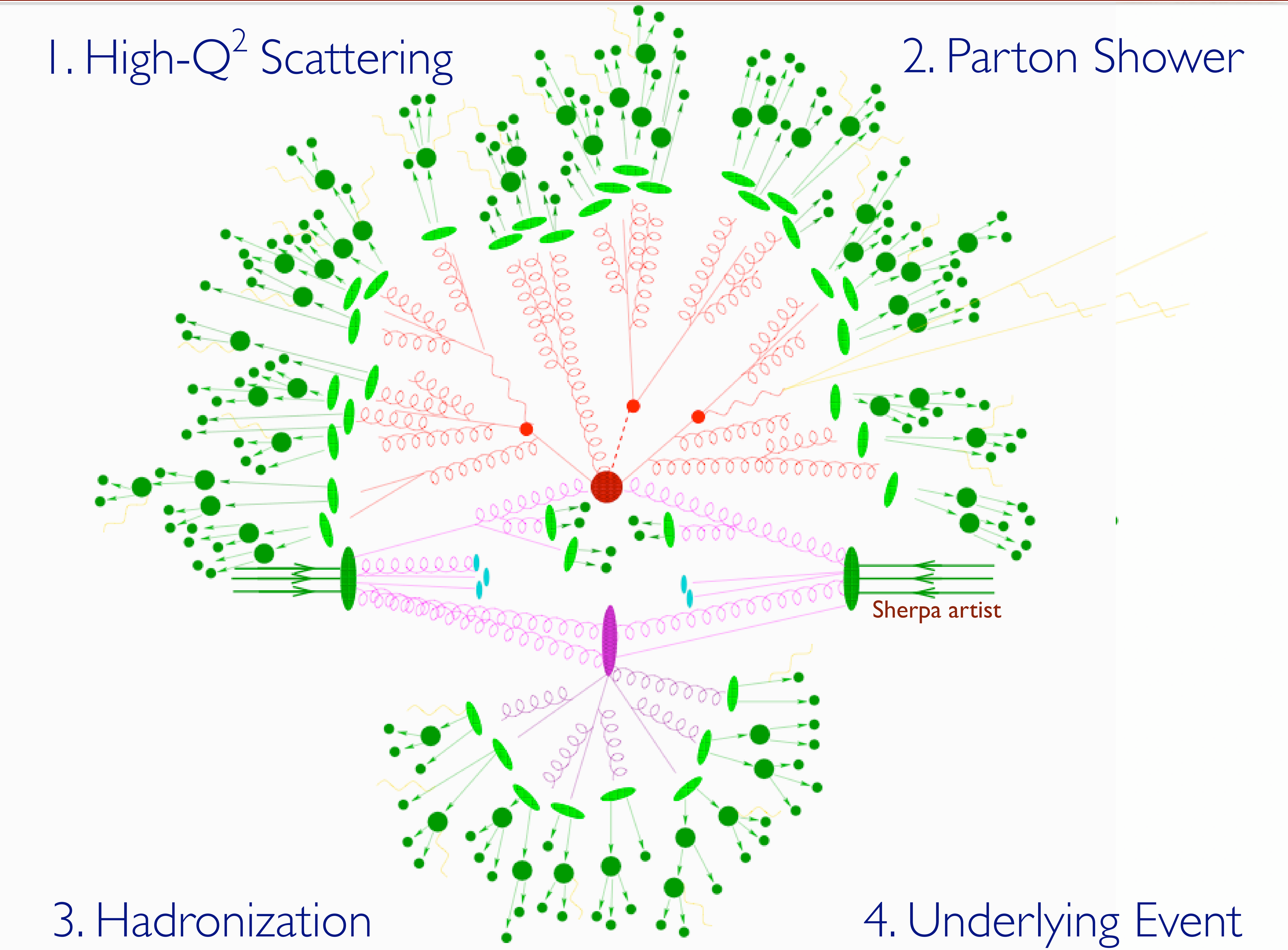
1. High-Q^2 Scattering
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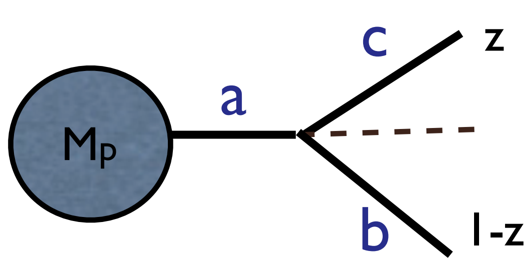
Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be unitary: the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

Parton showers

ME involving q →q g (or g → gg) are strongly enhanced when they are close in the phase space:



$$z = E_b/E_a\,, t = k_a^2$$

$$\theta = \theta_b + \theta_c = \frac{\theta_b}{1-z} = \frac{\theta_c}{z} = \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}}$$

$$\frac{1}{(p_q+p_g)^2} \simeq \frac{1}{2E_qE_g(1-\cos\theta)}$$

$$d\sigma_{N+1} = d\sigma_N \frac{dt}{t}\frac{d\phi}{2\pi}dz\frac{\alpha_S}{2\pi}|K_{ba}(z)|^2$$

$$d\bar{\sigma}_{N+1} = d\bar{\sigma}_N \frac{dt}{t}dz\frac{\alpha_S}{2\pi}P_{ba}(z)$$

In the collinear limit the cross section factorizes. The splitting can be iterated.

Parton showers

It is easy to iterate the branching process:

$$a(t) \longrightarrow b(z) + c\,, \quad b(t') \longrightarrow d(z') + e$$

$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{dt'}{t'} dz' \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ba}(z) P_{db}(z')$$

This is a generalized Markov process (in the continuum), where the probability of the system to change (discontinuosly) to another state, depends only on present state and not how it got there:

$$\tau_1 < \ldots < \tau_n \implies$$

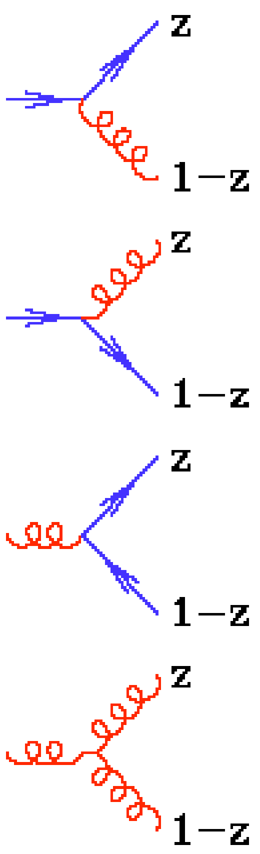
$$P\Big(x(\tau_n) < x_n | x(\tau_{n-1}), \ldots, x(\tau_1)\Big) = P(x(\tau_n) < x_n | x(\tau_{n-1}))$$

No memory!

Parton showers

The spin averaged (unregulated) splitting functions for the various types of branching ar

$$
\begin{aligned}
\hat{P}_{qq}(z) &= C_F\left[\frac{1+z^2}{(1-z)}\right],\\
\hat{P}_{gq}(z) &= C_F\left[\frac{1+(1-z)^2}{z}\right],\\
\hat{P}_{qg}(z) &= T_R\left[z^2+(1-z)^2\right],\\
\hat{P}_{gg}(z) &= C_A\left[\frac{z}{(1-z)}+\frac{1-z}{z}+z(1-z)\right].
\end{aligned}
$$



$$C_F=\tfrac{4}{3}, C_A=3, T_R=\tfrac{1}{2}.$$

Comments:
* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* Pqg has no soft divergences.

PARTON SHOWERS

Following a given line in a branching tree, it is clear that contributions coming from the strongly-ordered region will be leading:

$$Q^2 \gg t_1 \gg t_2 \gg \ldots t_N \gg Q_0^2$$

$$\sigma_N \propto \sigma_0 \alpha_S^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \ldots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_S^N}{N!} \left(\log \frac{Q^2}{Q_0^2} \right)^N$$

Denote by $\Phi_a[E, Q^2]$

the ensemble of parton cascades initiated by a parton a of energy E and emerging from a hard process with scale Q^2 (Generating functional). Also, define

$$\Delta(Q_1^2, Q_2^2)$$

as the probability that a **does not branch** for virtualities $Q_1^2 > t > Q_2^2$

Parton showers

With this, it easy to write a formula that takes into account all the branches associated to a parton a:

$$\Phi_a[E,Q^2] = \Delta_a(Q^2,Q_0^2)\Phi_a[E,Q_0^2] + \int_{Q_0^2}^{Q^2} \frac{dt}{t}\Delta_a(Q^2,t)\sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)\Phi_b[zE,t]\Phi_c[(1-z)E,t]$$

Simple interpretation. First term describes the evolution to Q_0, where no branching has occurred. The second term is the contribution coming from evolving with no branching up to a given t and then branching there. Now conservation of probability imposes that:

$$1 = \Delta_a(Q^2,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dt}{t}\Delta_a(Q^2,t)\sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

Which can be solved to give an explicit expression for **Δ**.

Parton showers

$$\Delta_a(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ab}(z)\right]$$

Which gives an explicit expression for the Sudakov form factor, i.e. la probability that a parton will not branch in going from the virtuality Q^2 to $Q^2{}_0$.

Proof:

derive the conservation of probability equation

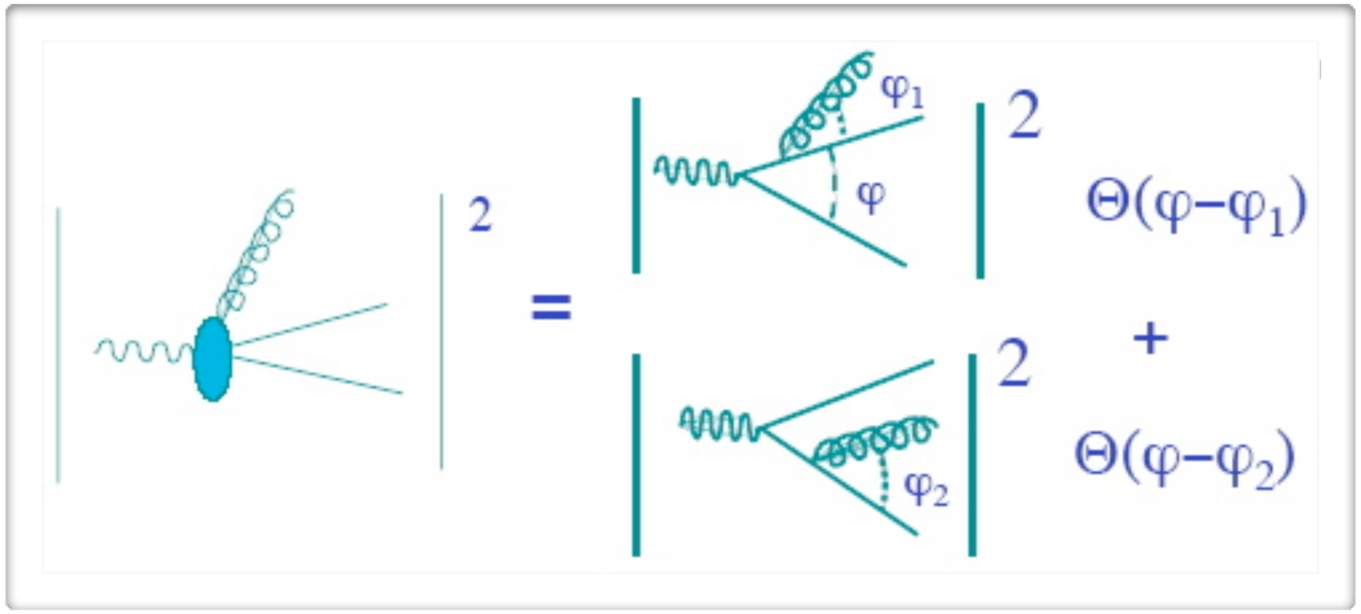
$$0 = \frac{d\Delta_a}{dQ_0^2}(Q^2, Q_0^2) - \frac{\mathcal{P}_a}{Q_0^2}\Delta_a(Q^2, Q_0^2)\,, \qquad \mathcal{P}_a = \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

and impose the initial condition

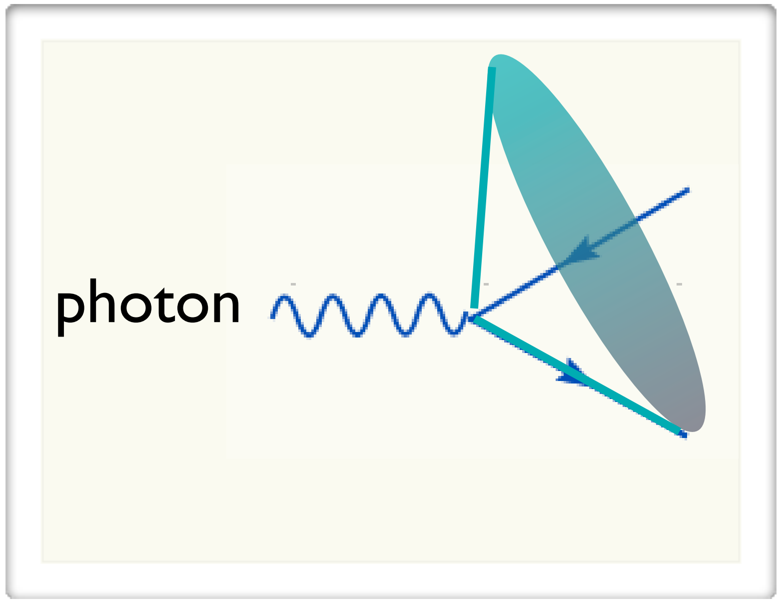
$$\Delta_a(Q^2, Q^2) = 1$$

Note that: $\Delta_a(Q^2, t) = \dfrac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(t, Q_0^2)}$ and therefore sometimes the second argument is not used.

Angular ordering



Radiation inside cones around the orginal partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



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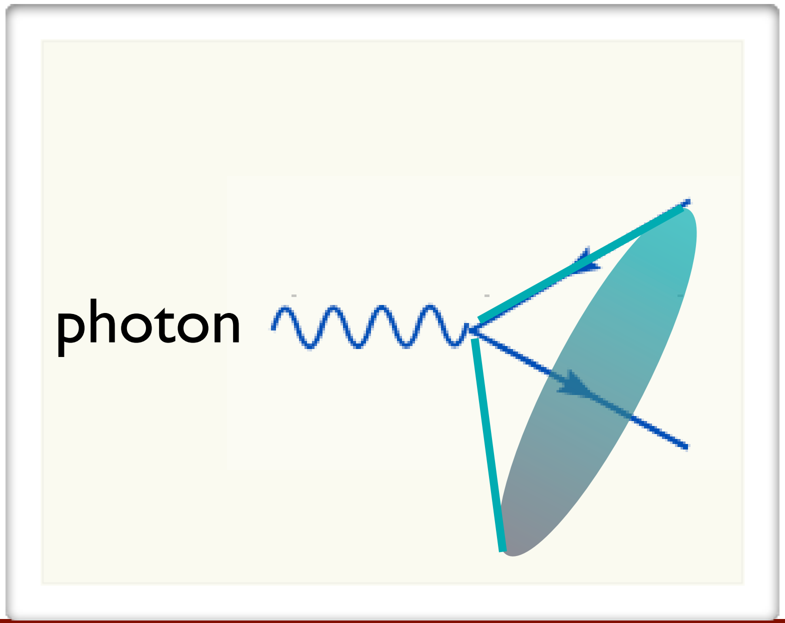
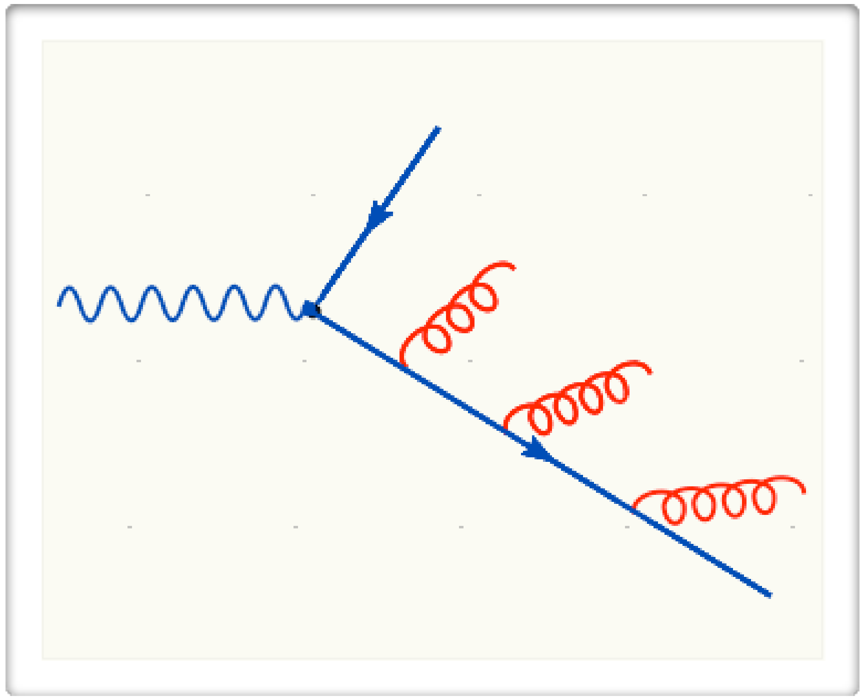
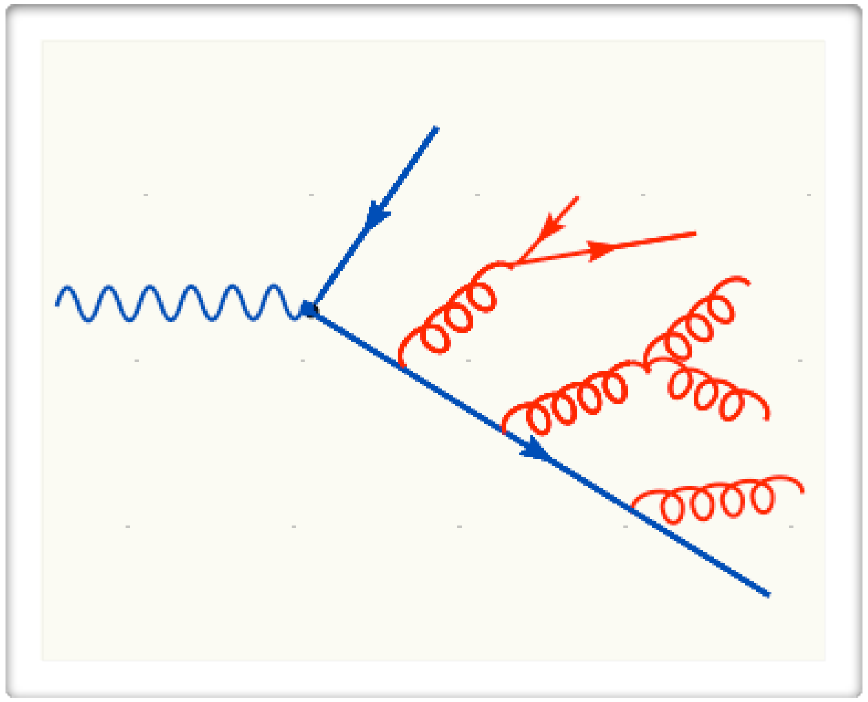


Angular ordering





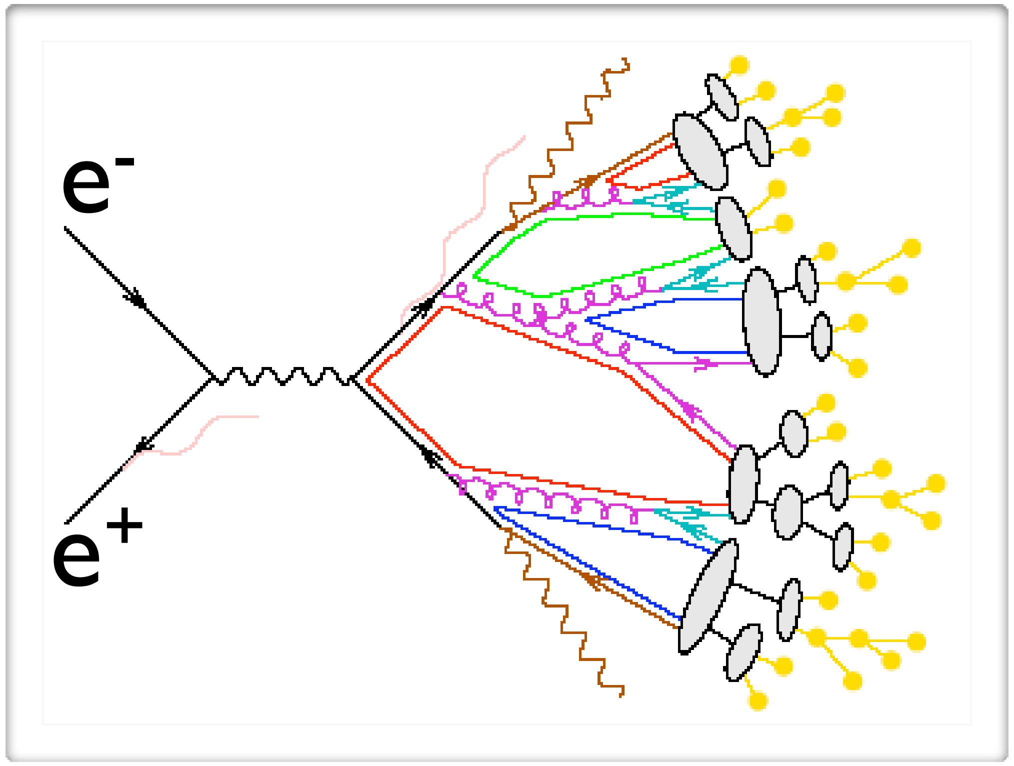
- The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.
- One can generalize it to a generic parton of color charge Q_k splitting into two partons i and j, $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-ordered cones the emission is coherent and can be treated as if it was directly from color charge Q_k.
- KEY POINT FOR THE MC!
- Angular ordering is automatically satisfied in θ ordered showers! (and easy to account for in p_T ordered showers).

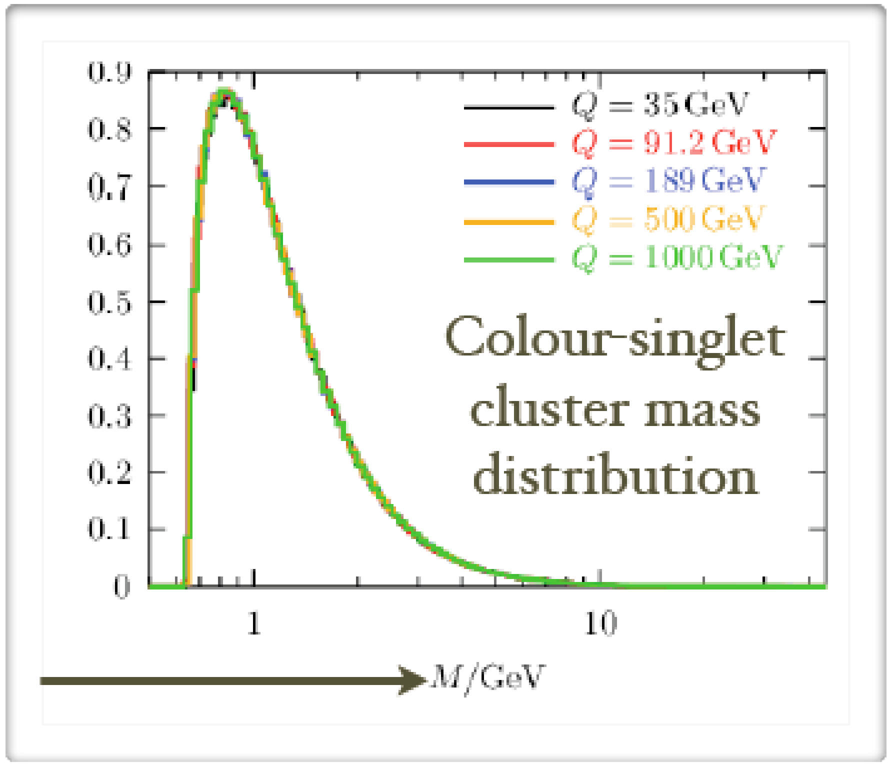
Hadronization

- The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.
- Physically, we observe hadrons, not (colored) partons.
- We need a non-perturbative model in passing from partons to colorless hadrons.
- There are two models (string and cluster), based on physical and phenomenological considerations.

Cluster model

The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





String model

From lattice QCD one sees that the color confinement potential of a quark-antiquark grows linearly with their distance: V(r) ~ kr, with k ~ 0.2 GeV. This is modeled with a string with uniform tension (energy per unit length) k that gets stretched between the qq pair.

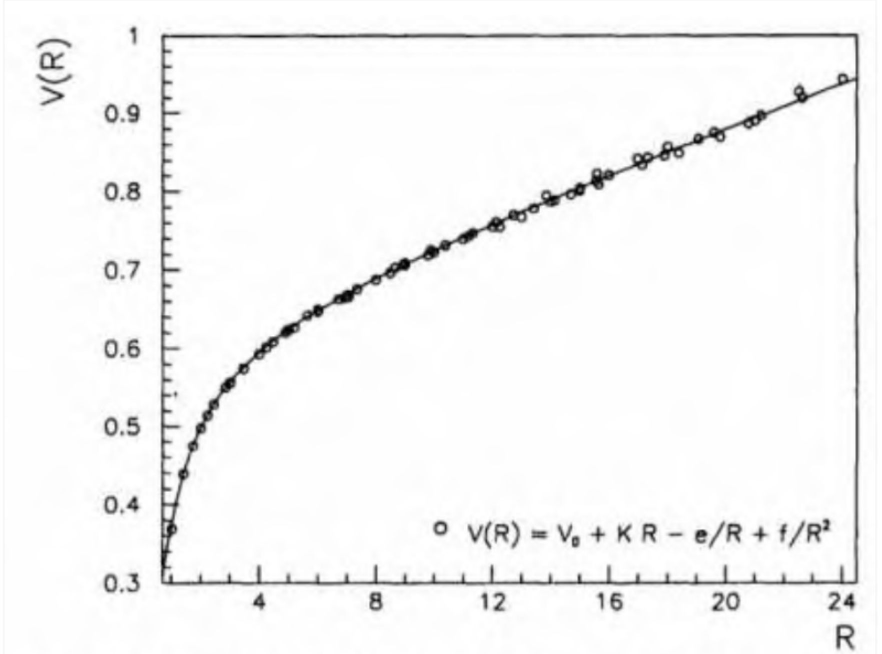
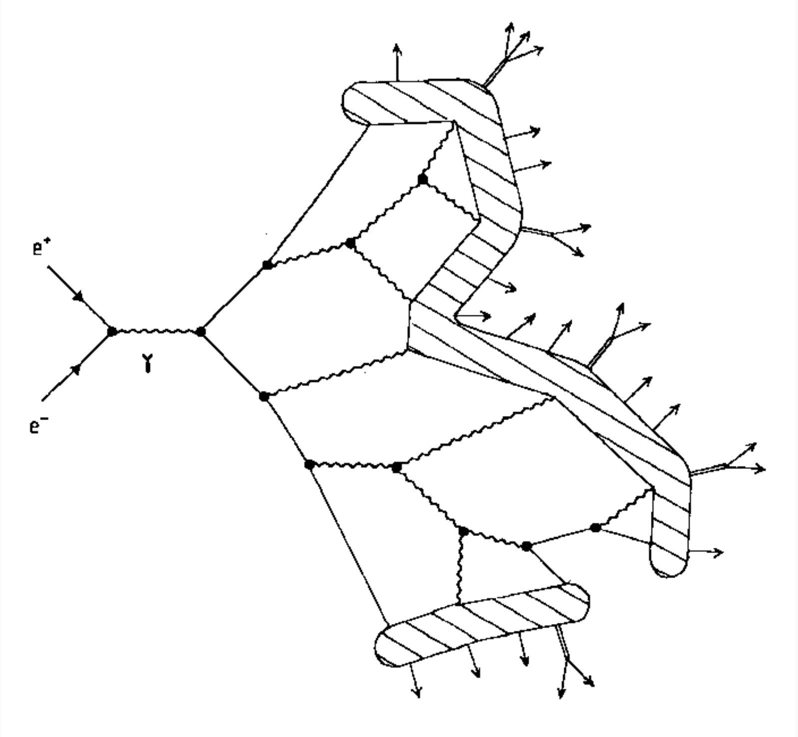
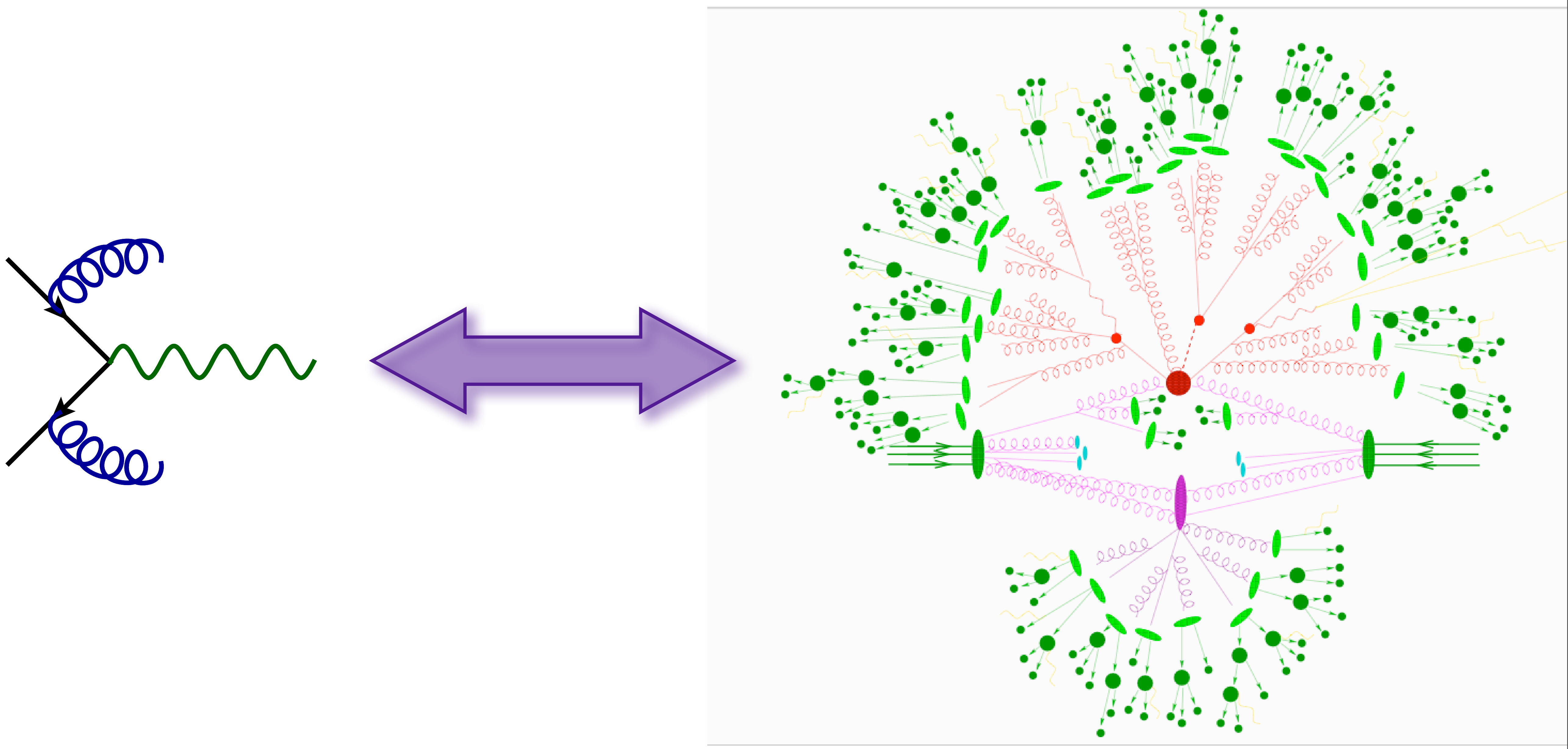


Fig. 2.9. QCD potential *vs.* *R* (in lattice units) from lattice QCD. Figure from ref. [23].



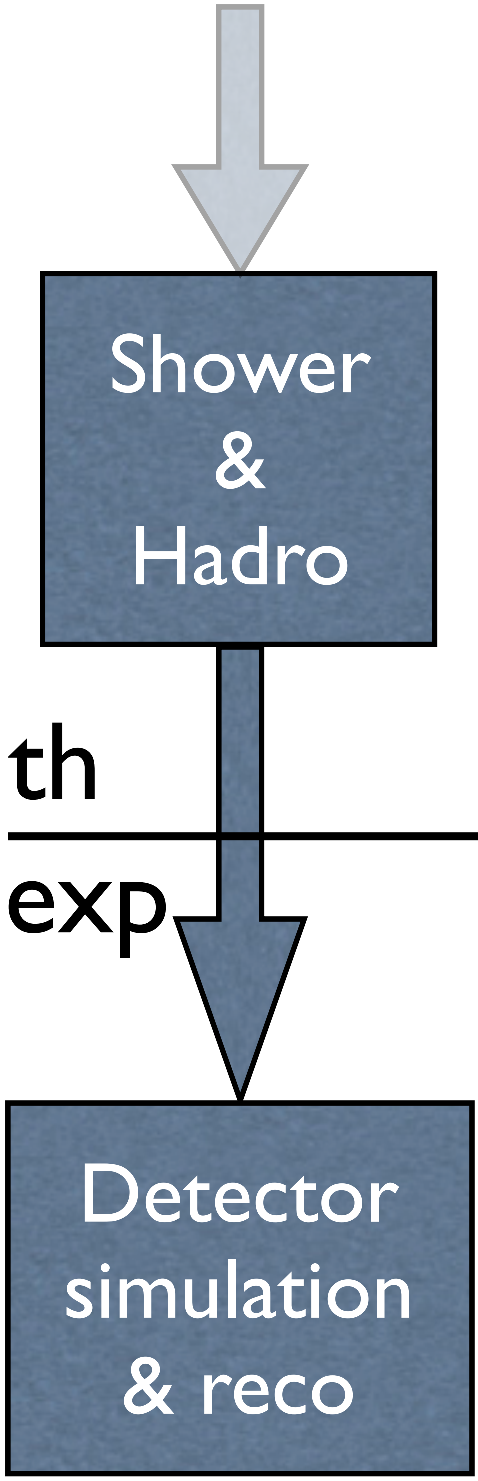
When quark-antiquarks are too far apart, it becomes energetically more favorable to break the string by creating a new qq pair in the middle.

Exclusive observable



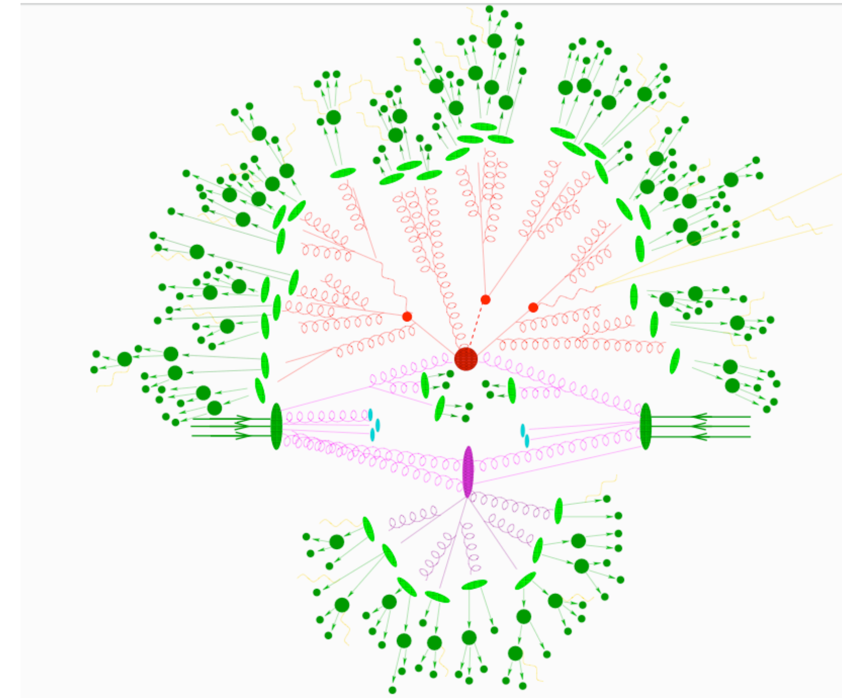
A parton shower program associates one of the possible histories (and pre-histories in case of pp collisions) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

General structure

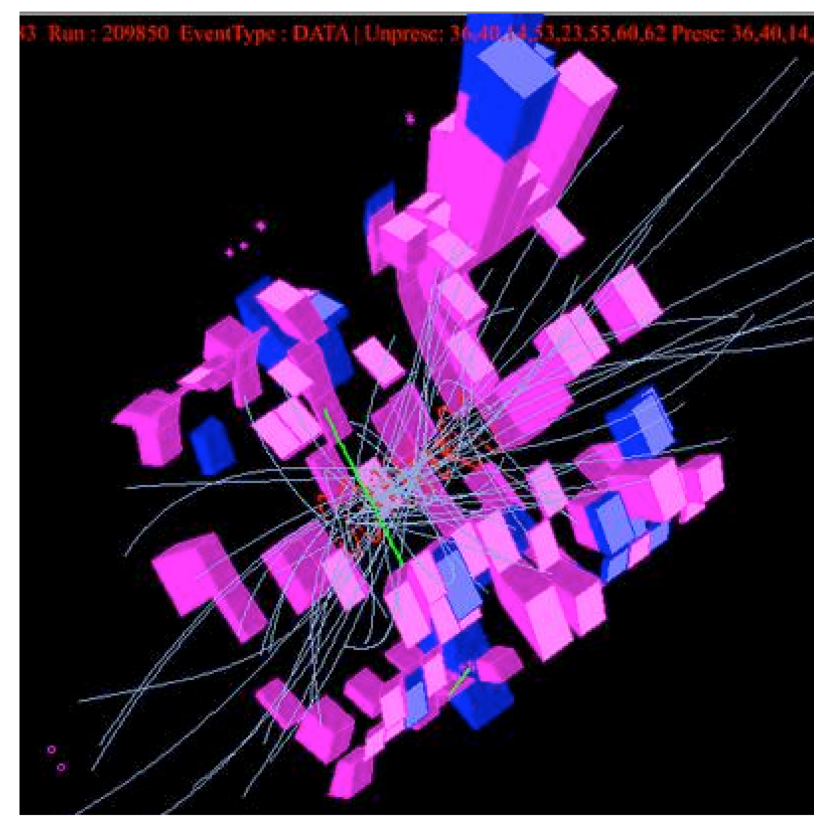


Events in the LH format are passed to the showering and hadronization⇒

high multiplicity hadron-level events



Events in HepMC format are passed through fast or full simulation, and physical objects (leptons, photons, jet, b-jets, taus) are reconstructed.



Parton shower MC event generators

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- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
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Shower MC Generators: PYTHIA, HERWIG, SHERPA

SM Status circa 2002

pp→ n particles

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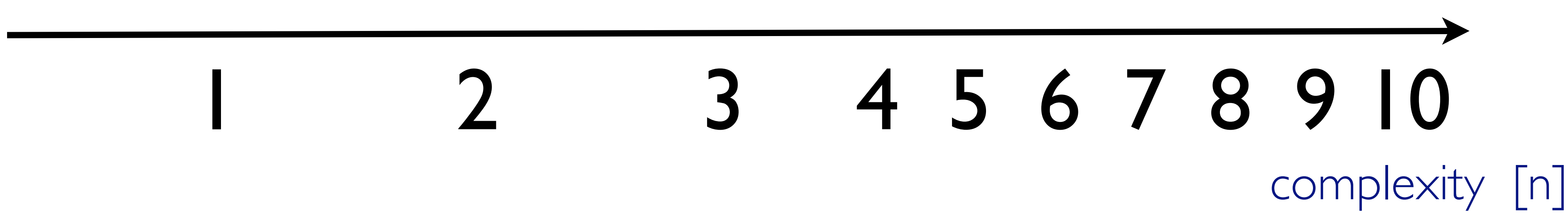

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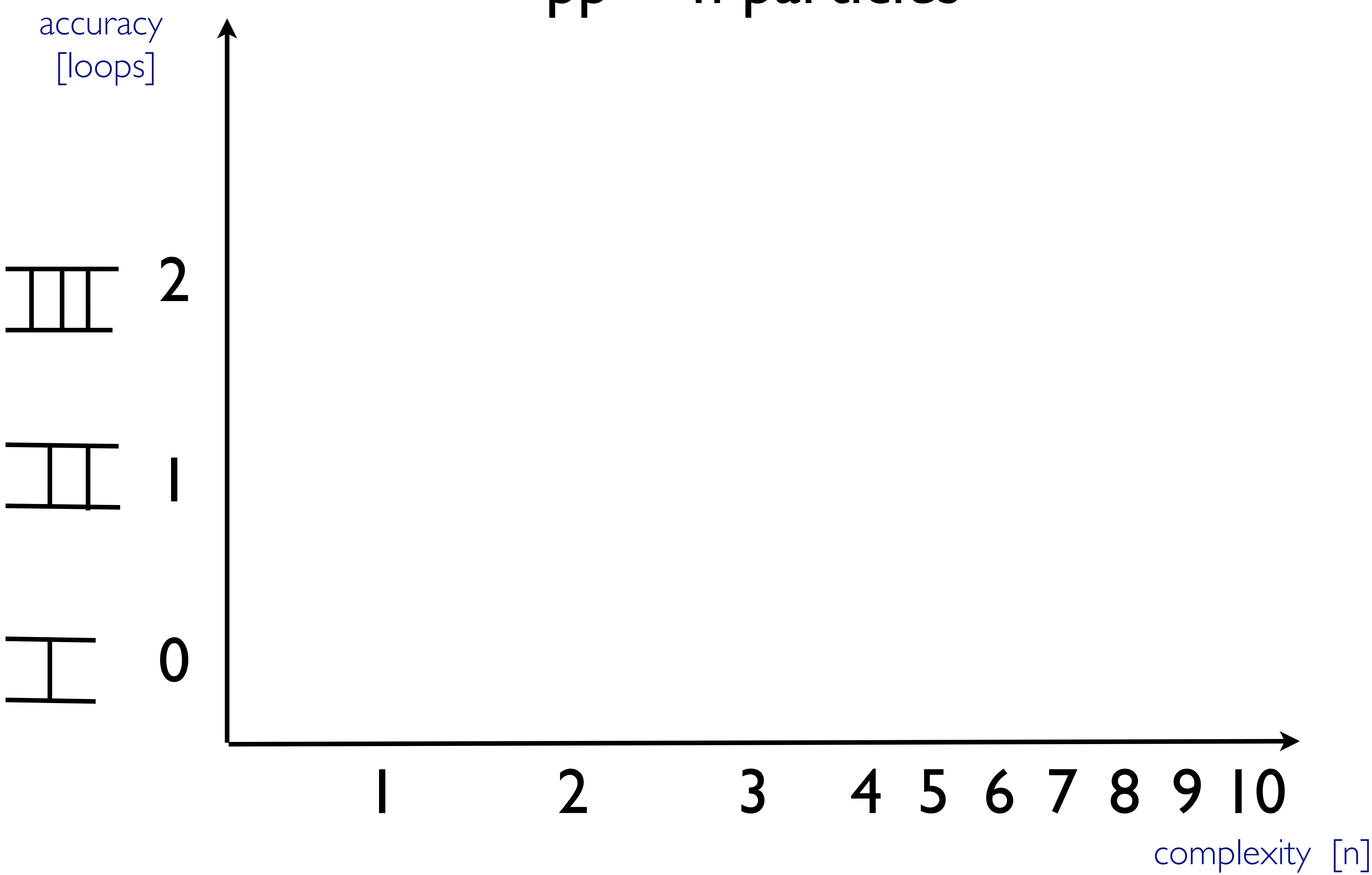

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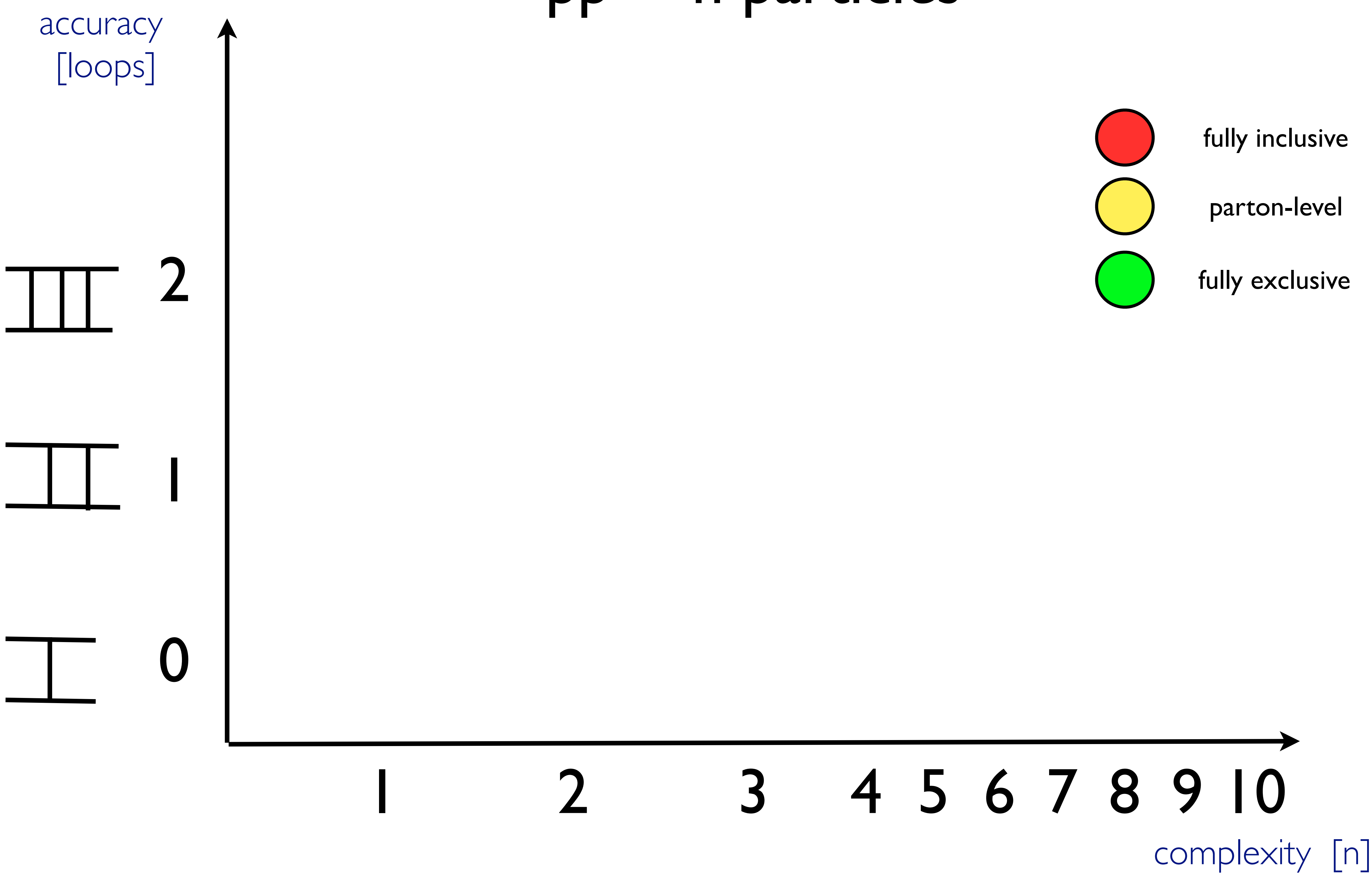

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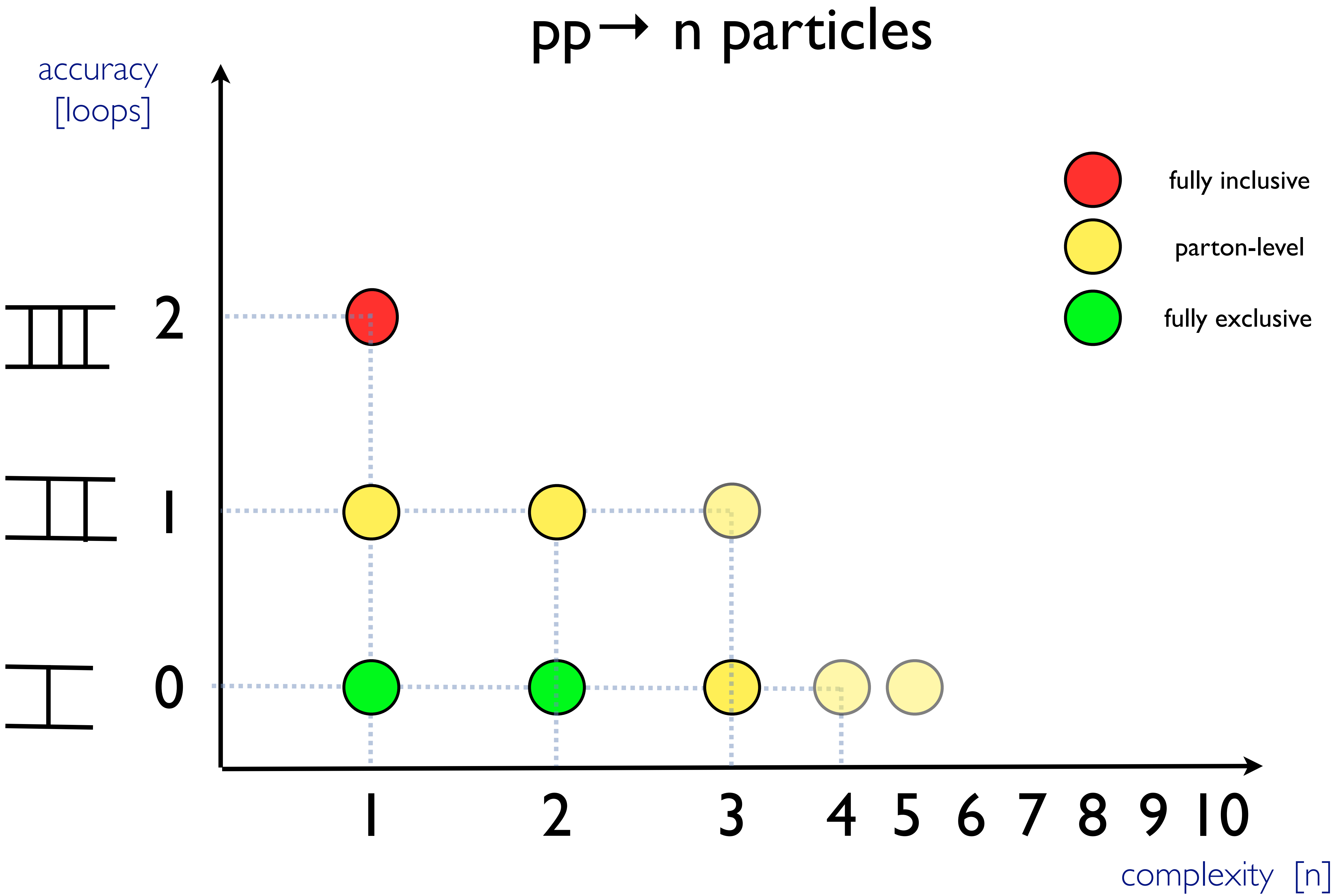

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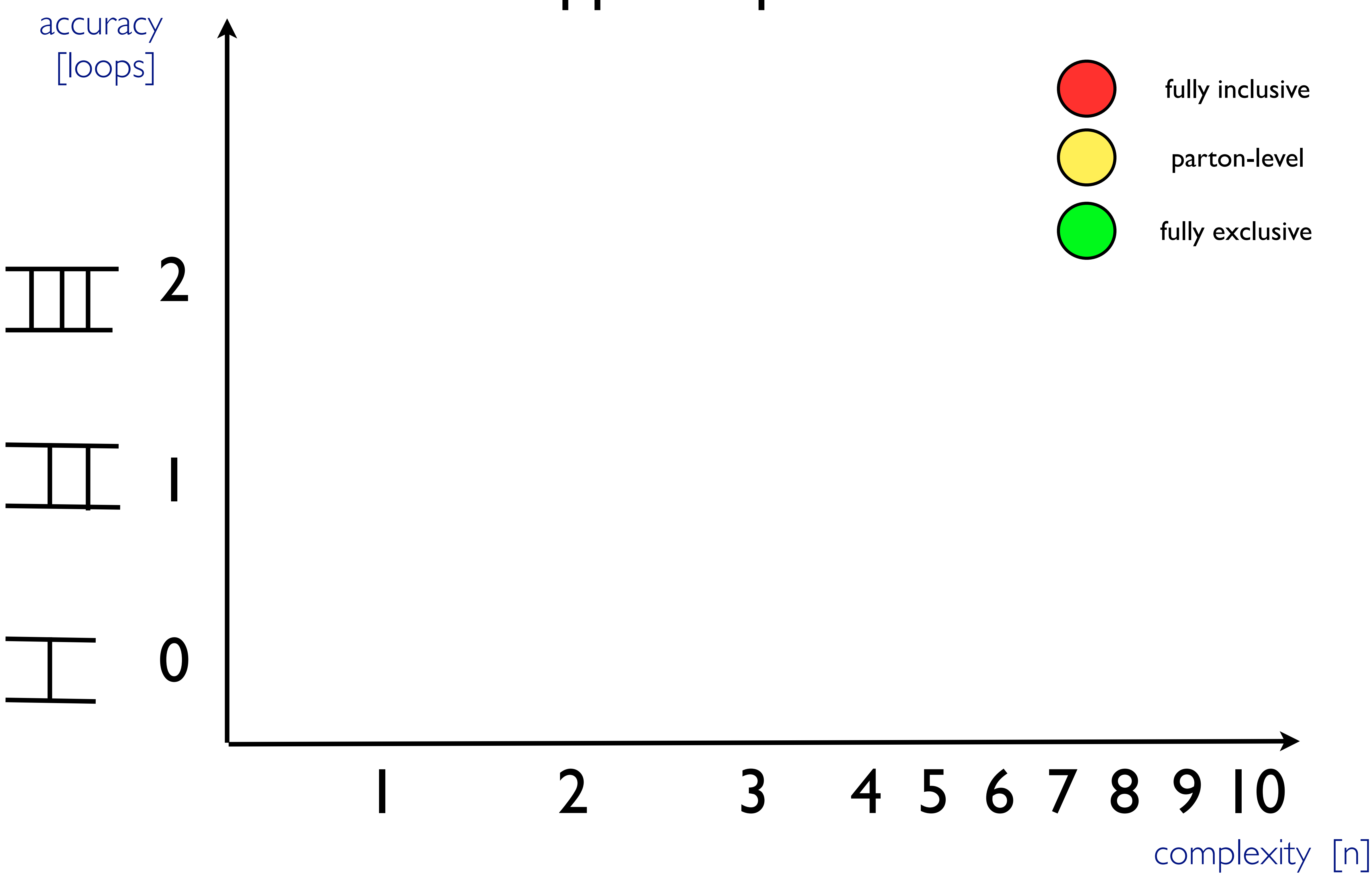
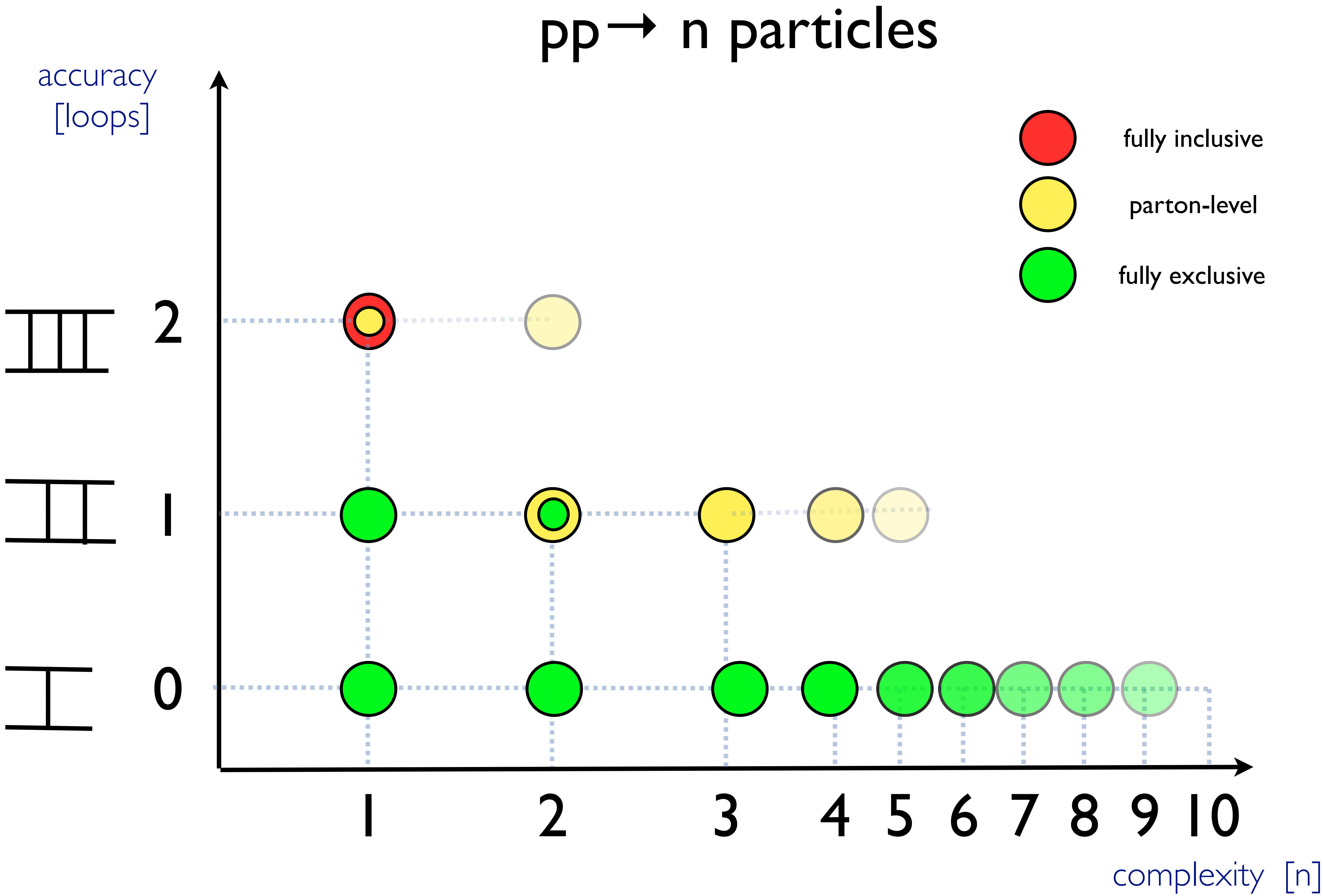
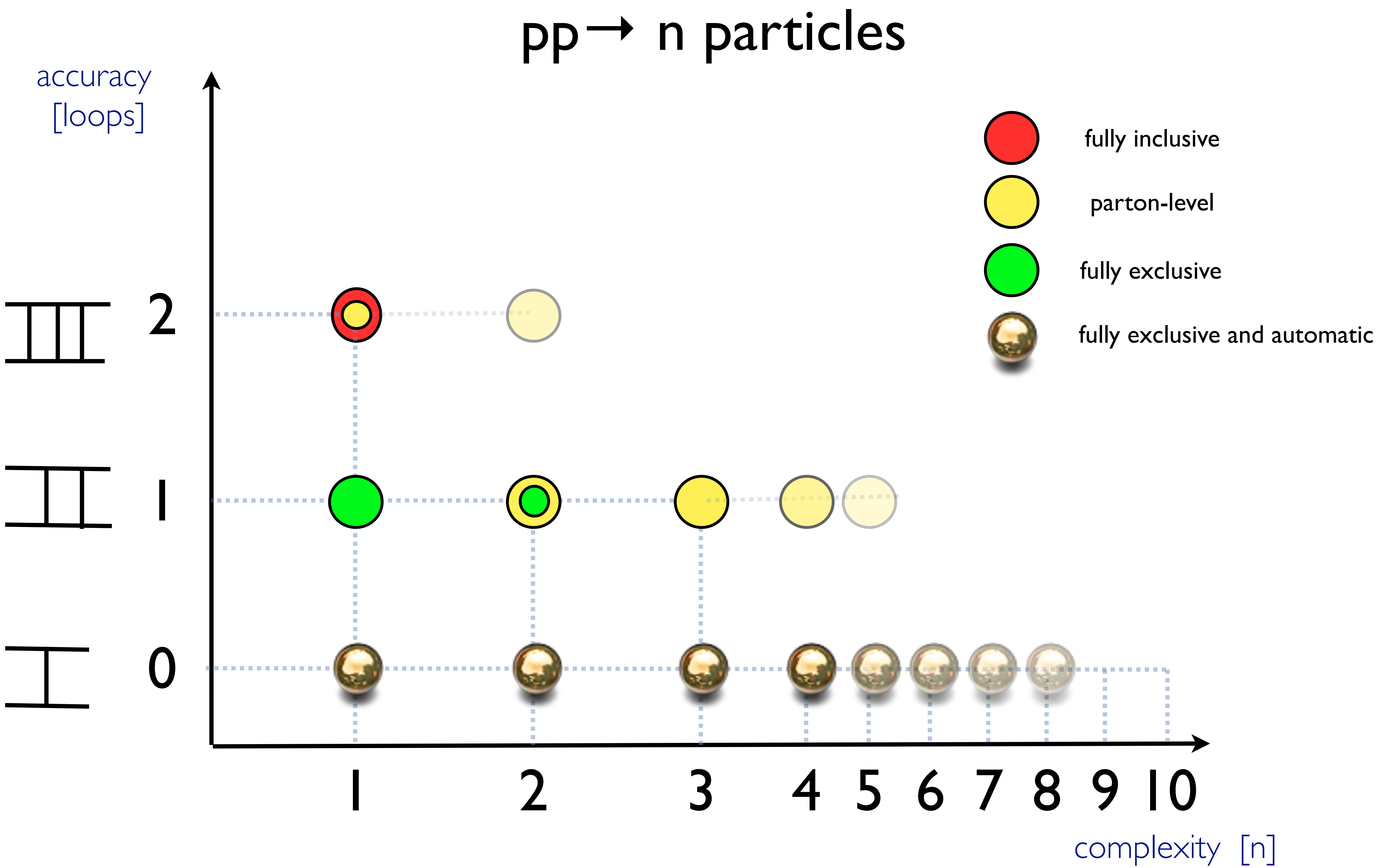


SM STATUS : SINCE 2007

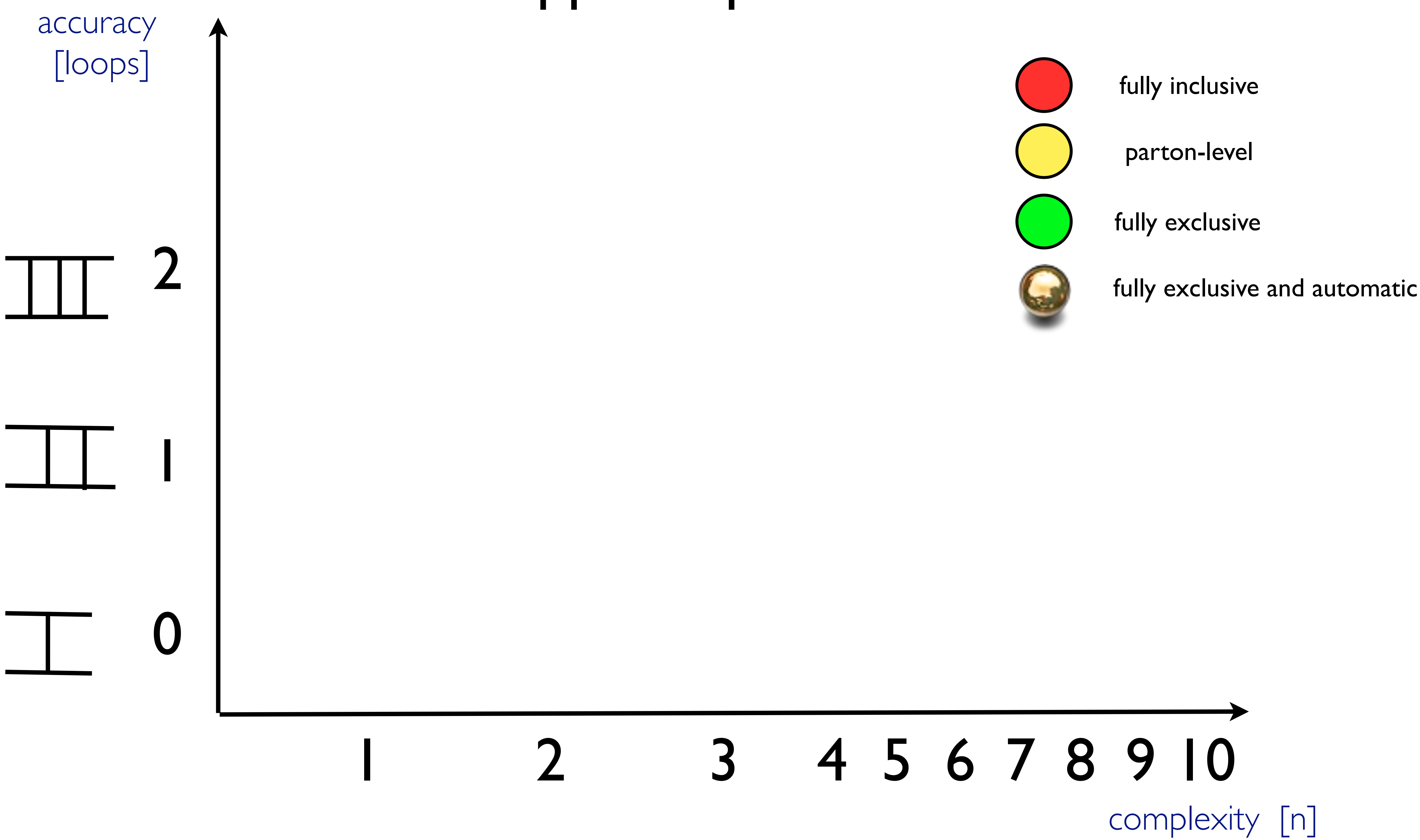


SM STATUS : SINCE 2007



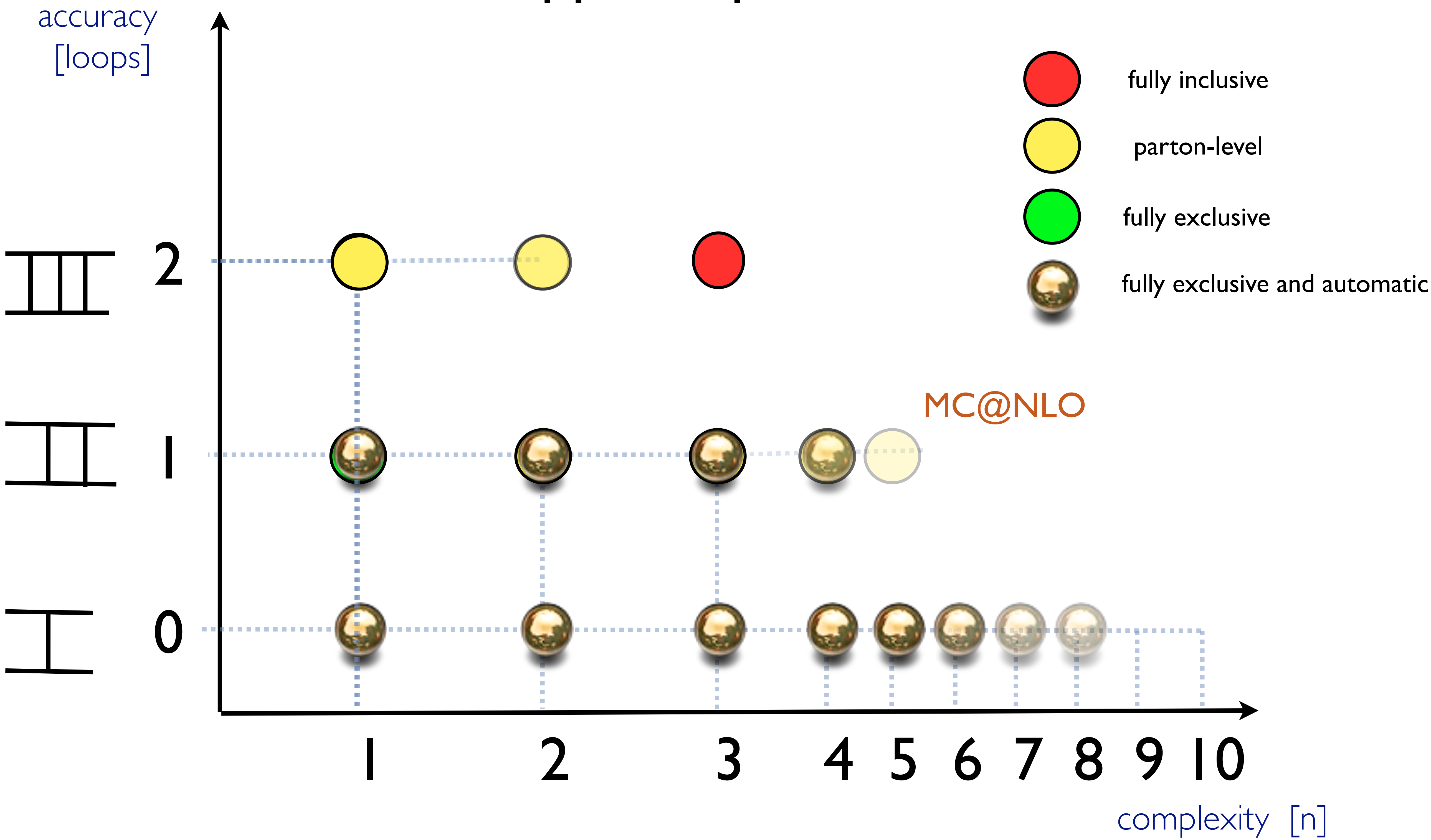
SM Status: now

pp→ n particles

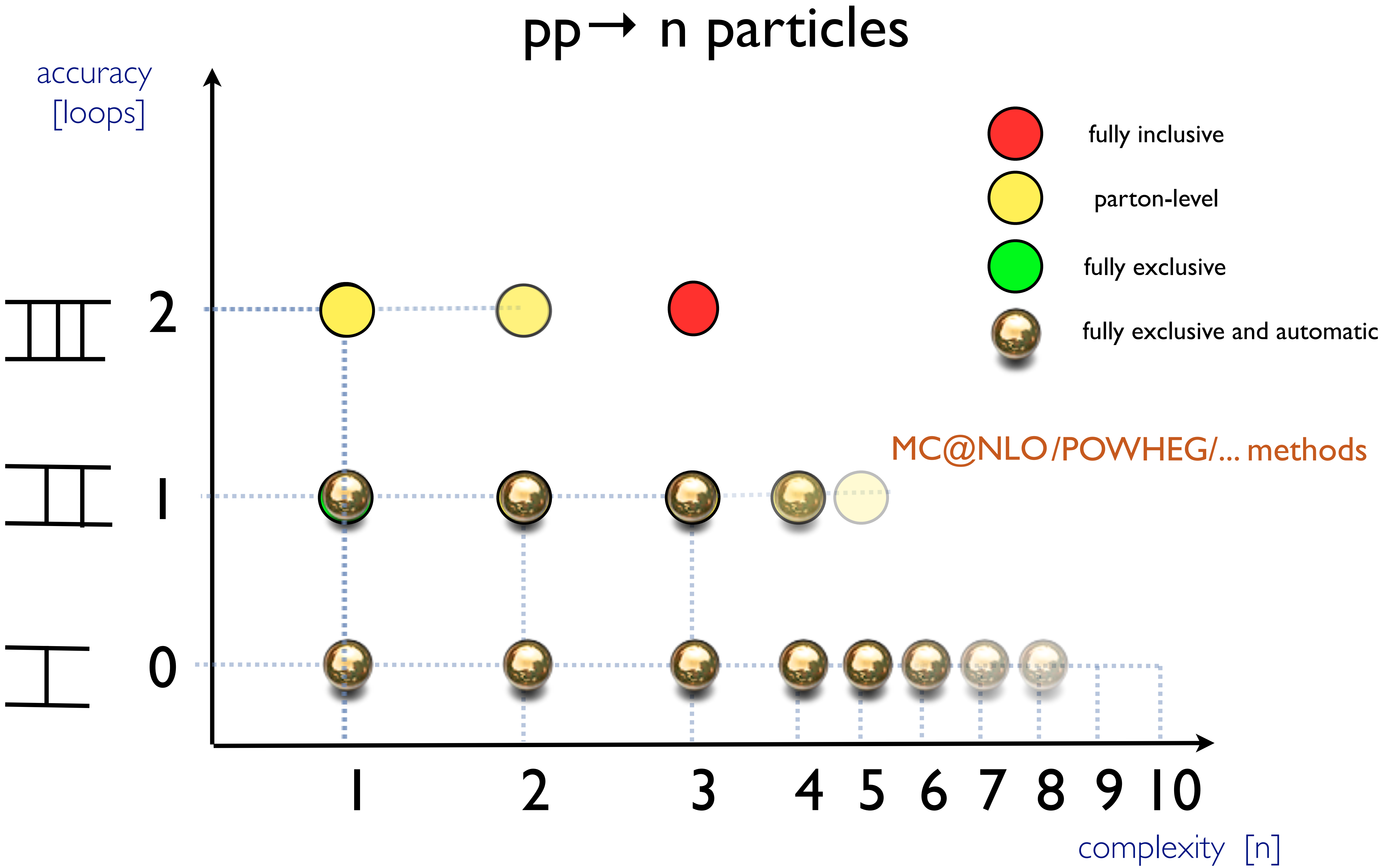


SM Status: now

pp→ n particles



SM Status: now



SUMMARY

- Simulations of HEP events play a central role to compare theory with experiments.
- **Accurate** predictions in the form of NLO calculations have now been automatised and are now available for wide range of applications.
- Turning them into **realistic** predictions is possible through parton shower algorithms that dress partons with radiation and eventually turn partons into hadrons. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive.
- **Predictive and flexible (and free!)** MC's are publicly available to all the HEP community.