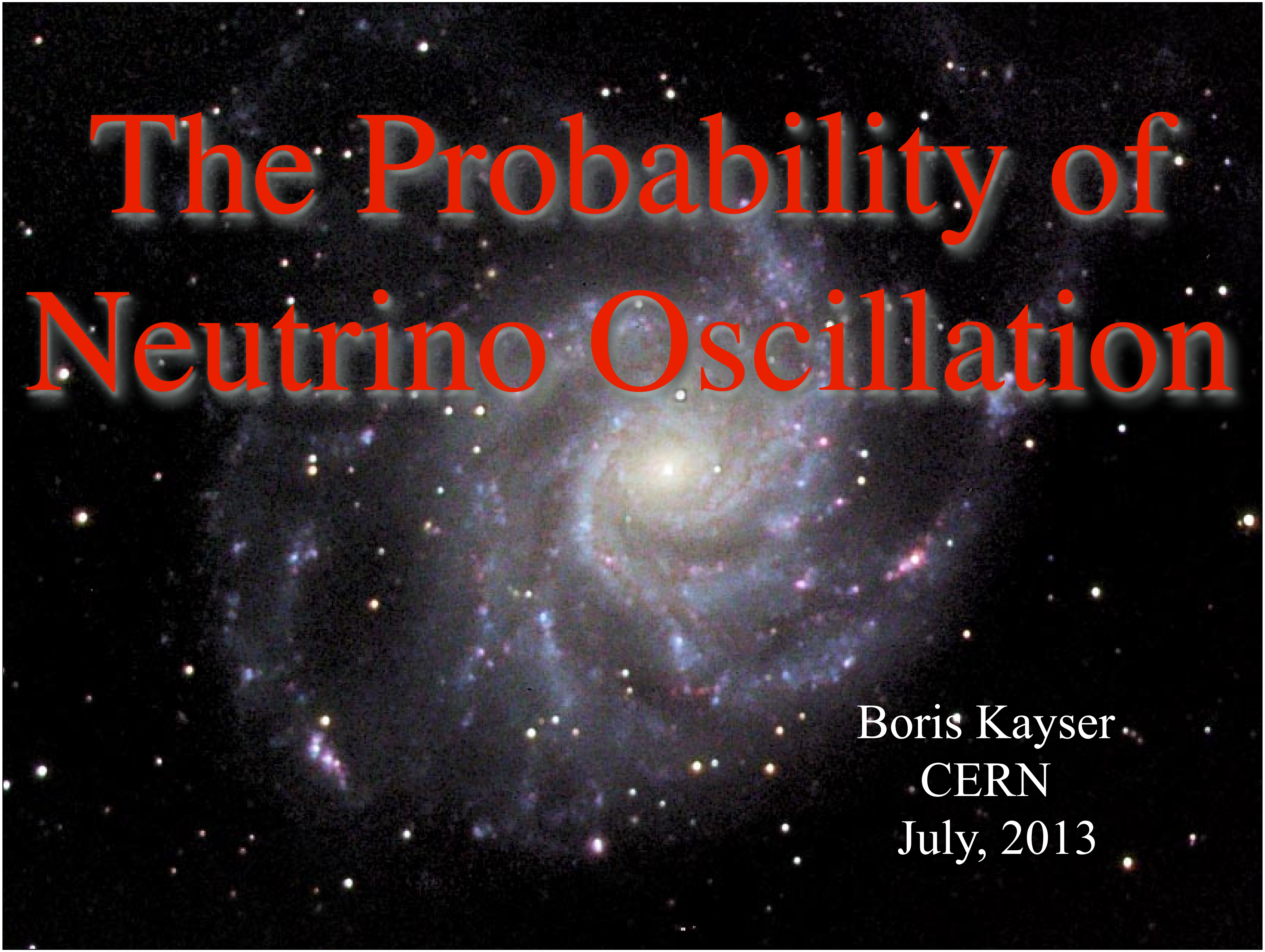

The Probability of Neutrino Oscillation

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Probability for Neutrino Oscillation in Vacuum

$(\hbar = \mathrm{c} = 1)$

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$

For Antineutrinos –

We assume the world is CPT invariant.

C: Replaces particle by antiparticle

P: Reverses helicity

T: Reverses the arrow of time

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \stackrel{CPT}{=} P(\nu_\beta \to \nu_\alpha) = P(\nu_\alpha \to \nu_\beta; U \to U^*)$$

Thus,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+} 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

A complex U would lead to the CP violation

$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \neq P(\nu_\alpha \to \nu_\beta)$.

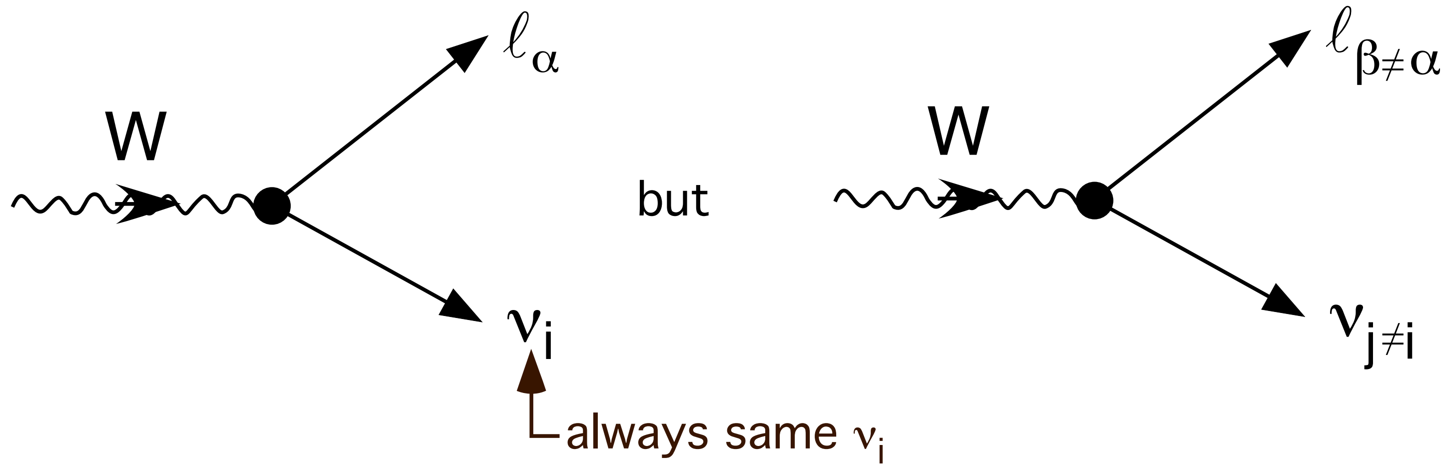
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow$ ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha,i} = 0, \text{ so that } P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* $\Rightarrow$ Mixing

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

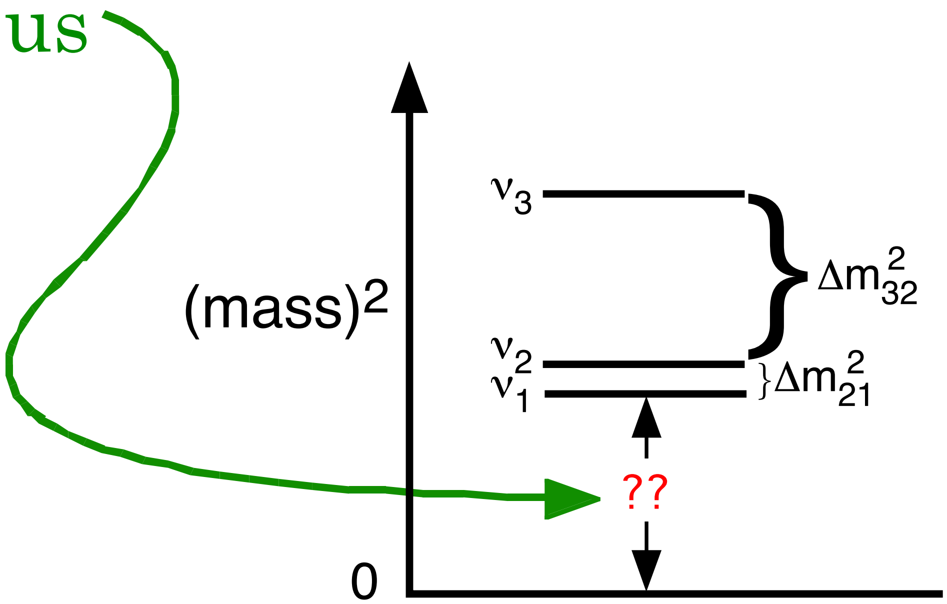
$\sin^2[1.27 \Delta m^2 (\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2 (\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

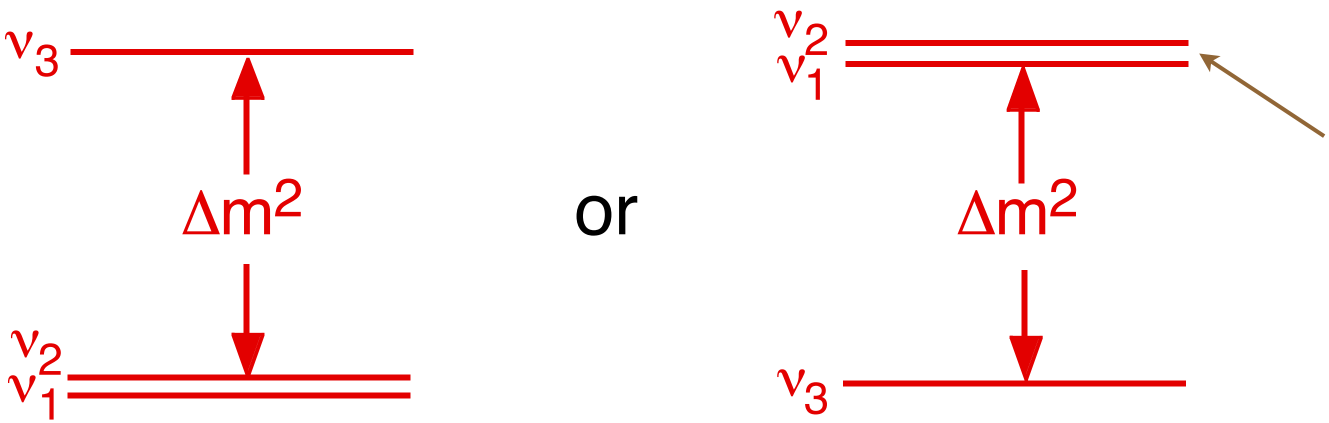
$$\sum_{\text{All }\beta} P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be "*sterile*".

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

When the Spectrum Is—



Invisible if $\Delta m^2 \dfrac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

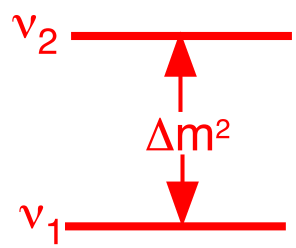
$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3} U_{\beta 3}|^2 \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) \cong 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana ~~CP~~ phase

$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

For $\beta \neq \alpha$, $P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.