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**A taste of cosmology
(in 3 lectures)**

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Institut de Ciències
del Cosmos



IEEC



Cosmology

Program:

- Introduction, Hubble law, the expanding Universe
- Dark matter and large-scale cosmological structure, clustering
- Cosmic microwave background
- Inflation
- Dark energy and outlook for the future

Lectures and additional material will appear at
<http://icc.ub.edu/~liciaverde/cernlectures.html>

Cosmology

Cosmos= Universe, Order, beauty
-logy= study

Greek!

Study of the Universe as a whole

Aim at getting an understanding of:

- its origin
- its structure and composition
(where do galaxies, stars, planets, people come from?)
- its evolution
- its fate

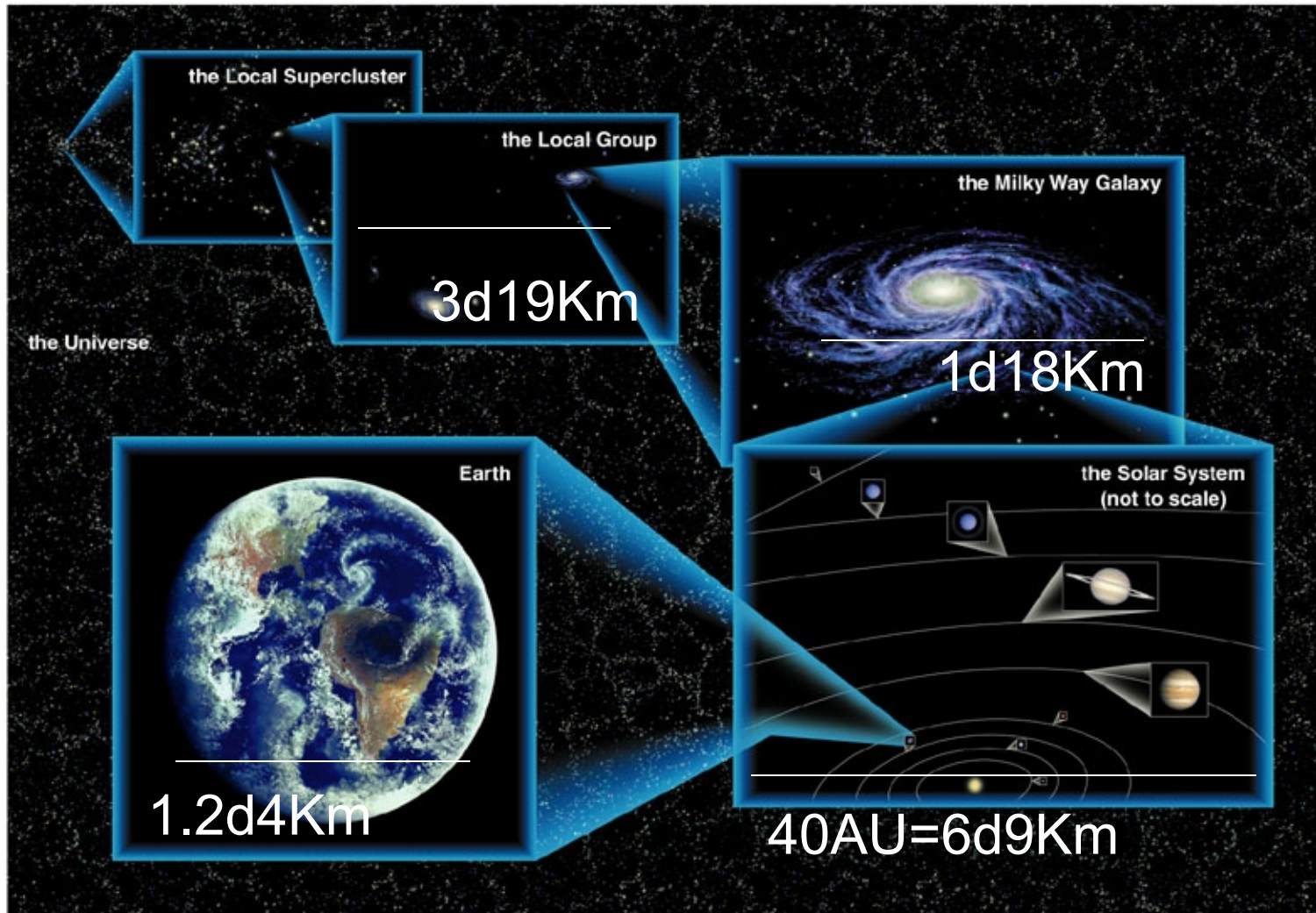
What to expect...

Concepts are mind-bending
but maths are simple

Quick learning curve: a first year graduate student or a good undergraduate can do new work worth of publication

This is a crash course. Can't cover everything, to give you an introduction and a flavor.
If you studied cosmology already may be boring.

Scales involved!



New units of measure

For distance, we use pc, Kpc & Mpc

$$1 \text{ pc} = 3.086 \times 10^{16} \text{ m}$$
$$1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$$

For comparison, mean Earth-Sun distance (Astronomical Unit):

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$
$$1 \text{ pc} = 2.1 \times 10^5 \text{ AU}$$

• Cosmologists often express masses
• in units of the solar mass:

$$1 M_{\odot} = 1.99 \times 10^{30} \text{ kg}$$

Looking far away is looking back in time!



8 minutes ago



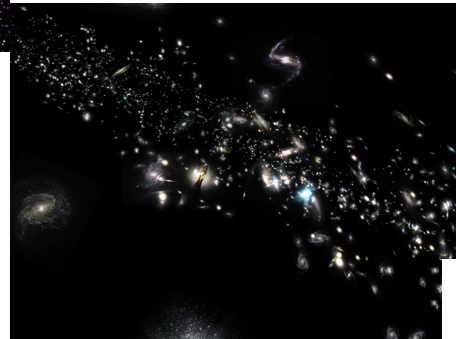
28000 years ago

Cosmic archeology



Andromeda, M31

2.2 million years ago



3 billion
years ago

Looking far away in space= looking back in time

stars



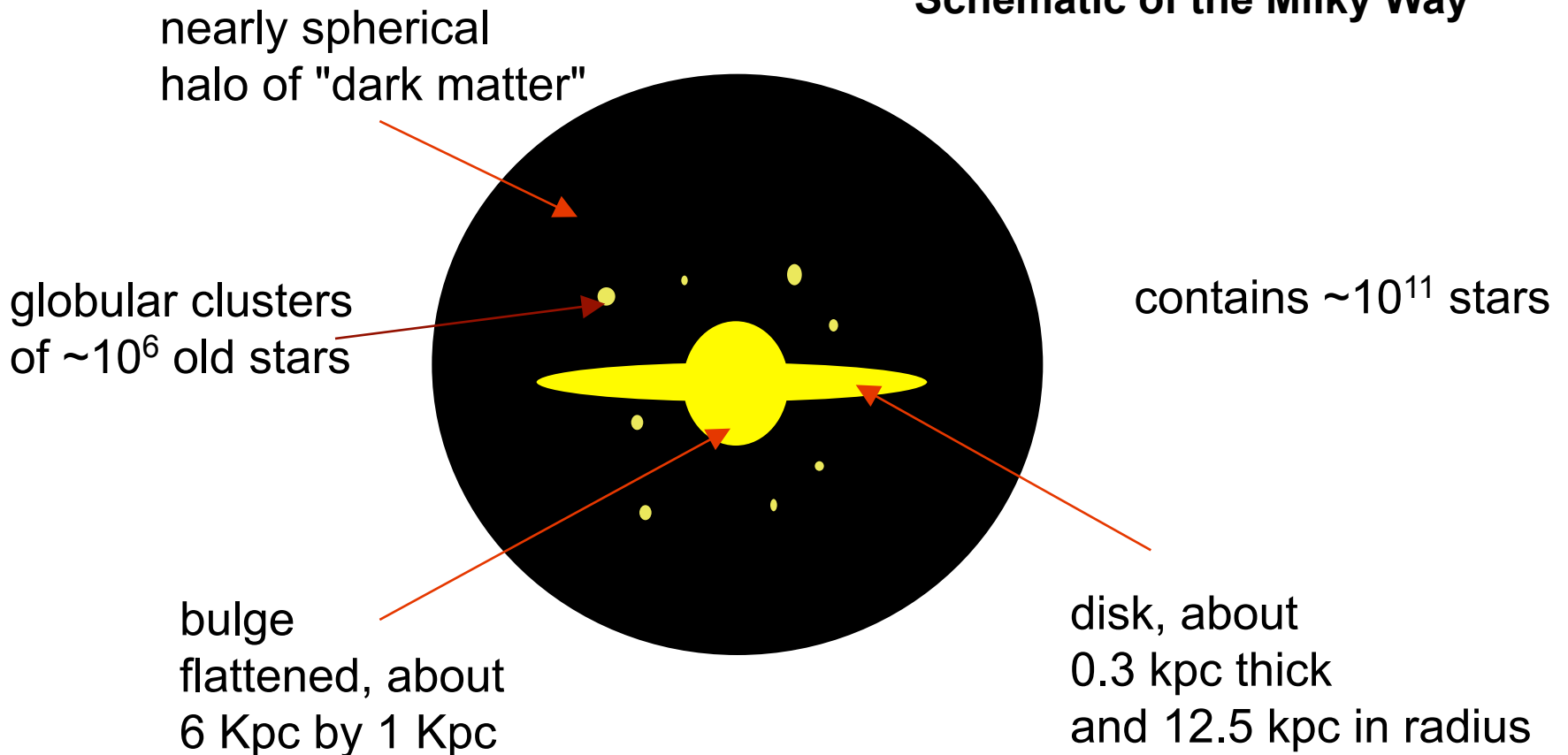
HST image of stars being born,
but it has no direct use
for Cosmology;
exploding stars (supernovae)
are very useful
(we'll see at the very end)

By the way, we are star-dust

galaxies

Collections of $\sim 10^{11} \sim 10^{12}$ Stars

Schematic of the Milky Way



galaxies



galaxies

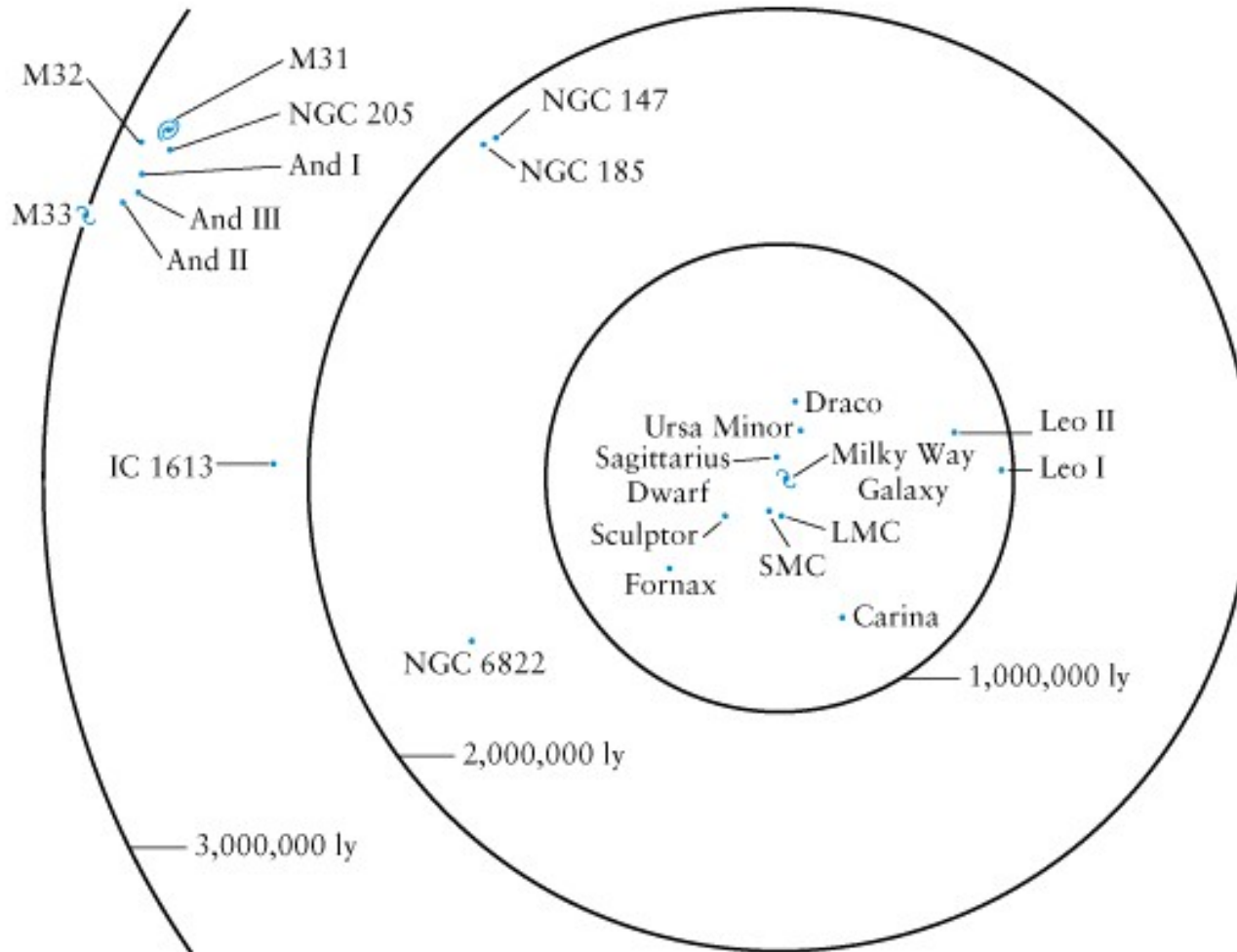


The Tadpole Galaxy → UGC 10214  HUBBLESITE.org

galaxies



The local group



Entering the regime of cosmology....

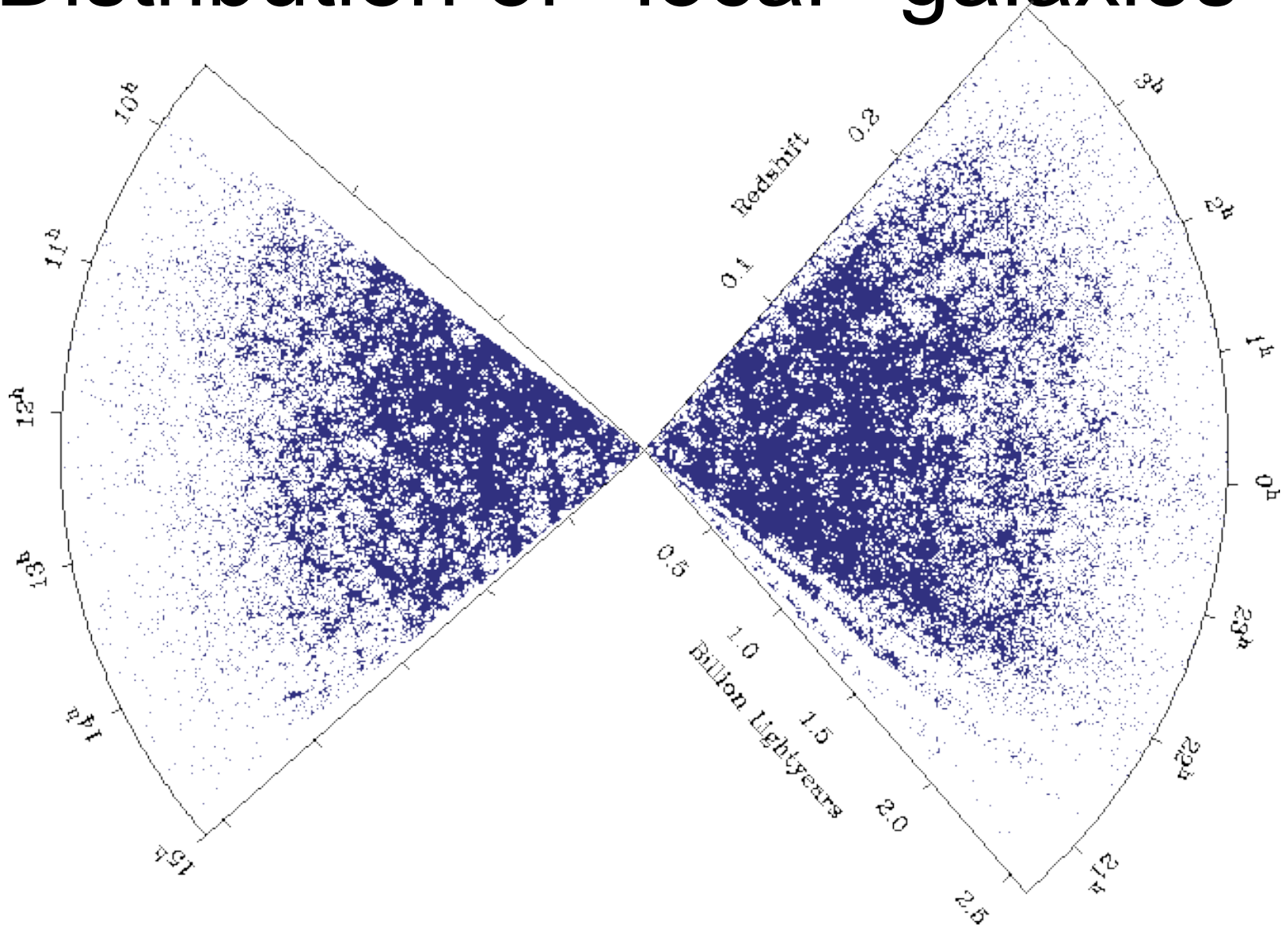
groups



Groups and clusters




Distribution of “local” galaxies



Hubble deep field



The Hubble Deep Field North  HUBBLESITE.org

Not only pretty pictures

Nature is written in the mathematical language (Galileo)

The laws of physics are the same in the entire Universe

The universe is comprehensible (by us)

Need physics and maths (physical cosmology)

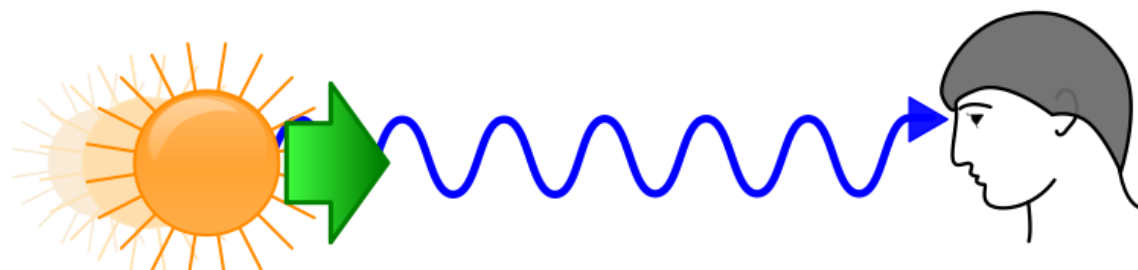
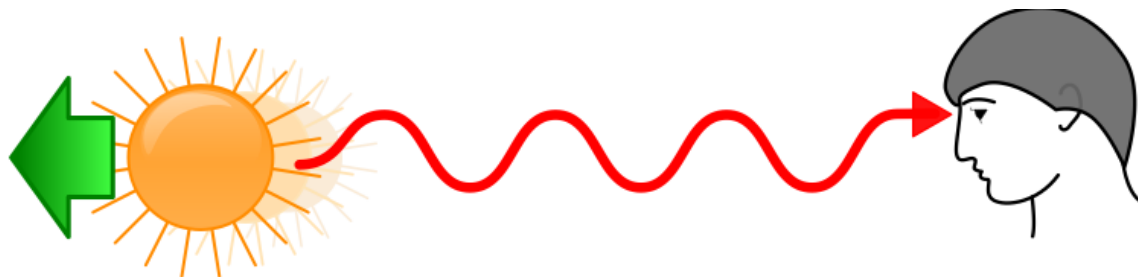
Deep links to fundamental physics

*The same fundamental physical laws that are being studied
for example at CERN with accelerators.*

Distances are difficult:
velocities are “easy”
Thank you Edwin Hubble



REDSHIFT



REDSHIFT

$$z = \frac{\lambda_{\text{obsv}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \quad \text{or} \quad 1 + z = \frac{\lambda_{\text{obsv}}}{\lambda_{\text{emit}}}$$

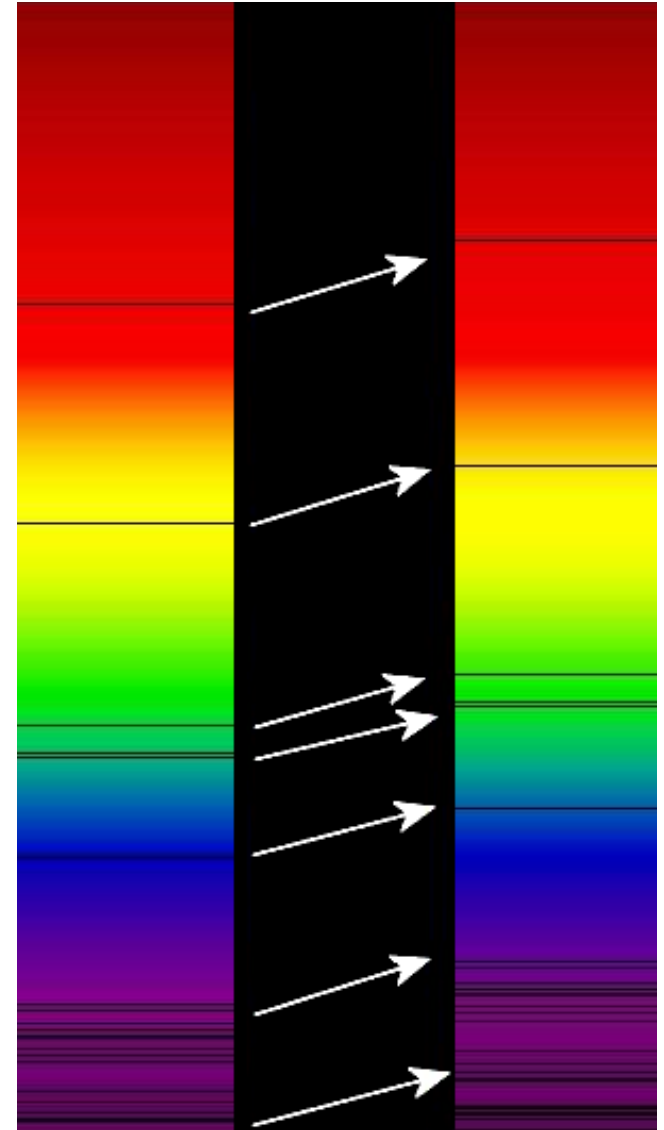
In relativity:

$$1 + z = \gamma \left(1 + \frac{v_{\parallel}}{c} \right)$$

$$z \approx \frac{v_{\parallel}}{c} \quad \text{For small velocity}$$

or

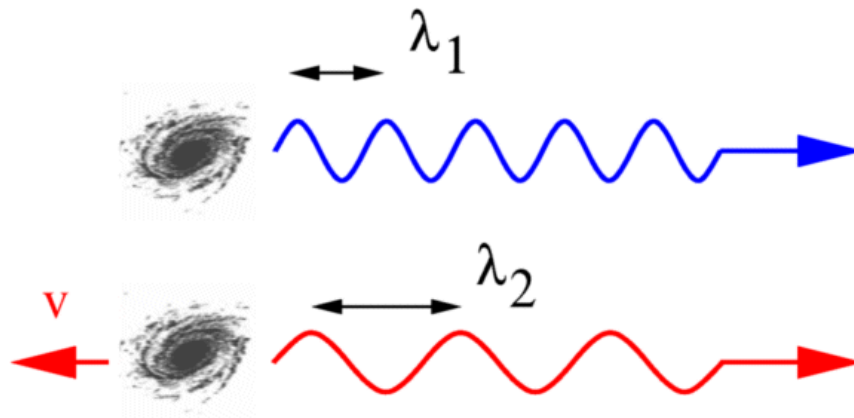
$$1 + z = \frac{1 + v \cos(\theta)/c}{\sqrt{1 - v^2/c^2}}$$



Hubble's Law

1912 - 1920s: Vesto Slipher finds most galaxies are redshifted

nebulae

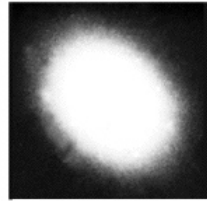


Vesto Slipher (1875 - 1969)

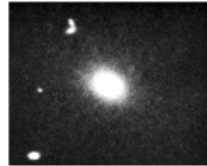
$$1 + z = \frac{\lambda_2}{\lambda_1} \simeq 1 + \frac{v}{c}$$

The Doppler (red)shift

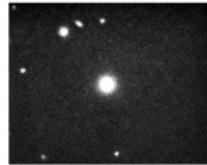
GALAXIES in



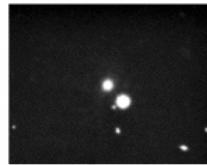
Virgo



Ursa Major



Corona Borealis

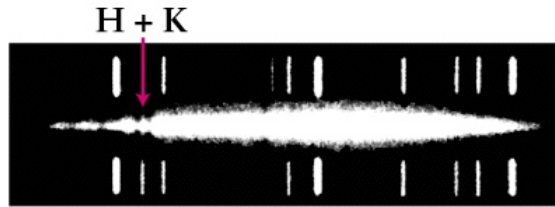


Boötes



Hydra

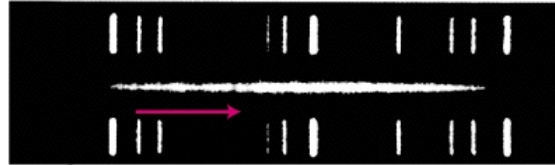
REDSHIFTS



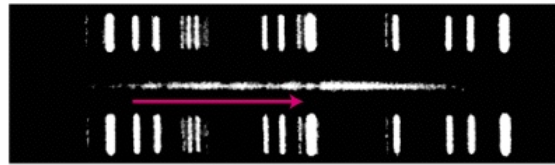
1200 km/s



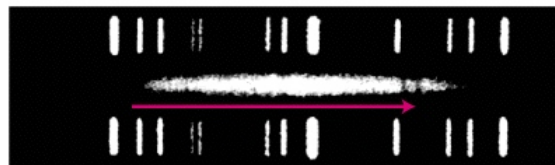
15,000 km/s



22,000 km/s



39,000 km/s

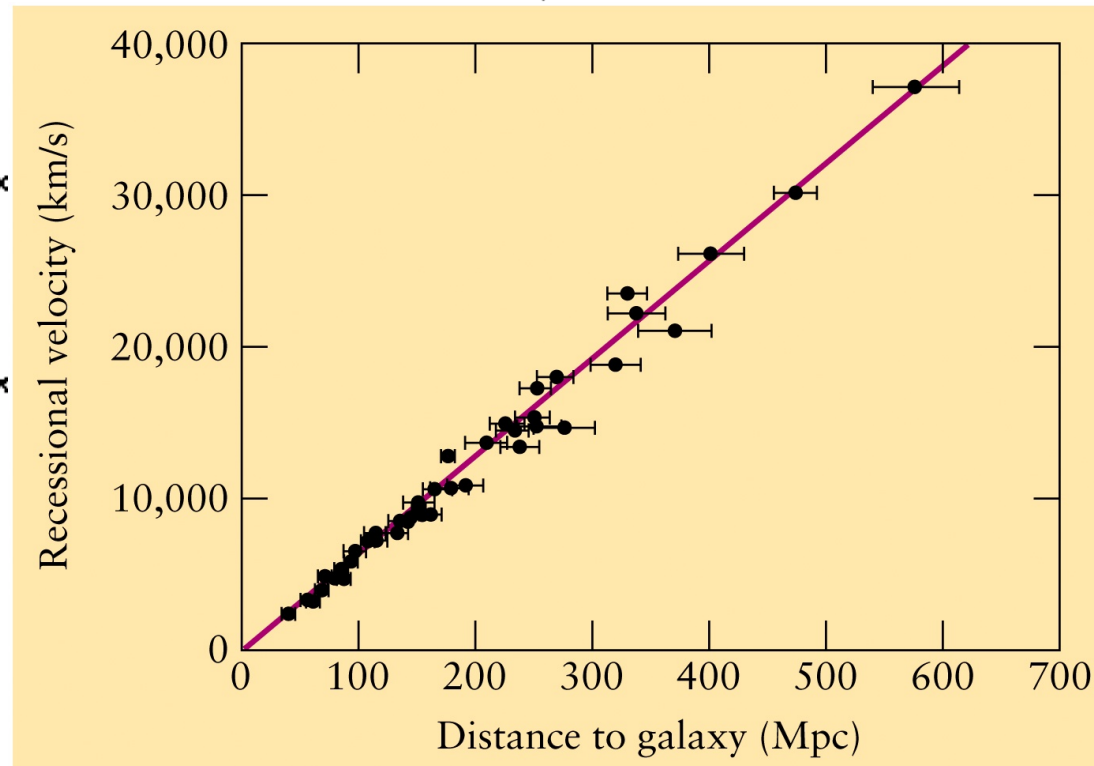


61,000 km/s

Hubble's Law

$$cz = v = H_0 d$$

$$H_0 = 74.2 \pm 3.8 \text{ km/s/Mpc}$$



Hubble 1929
(PNAS vol 15)

Aside: the great debate (1920)

http://antwrp.gsfc.nasa.gov/htmltest/gifcity/cs_nrc.html



Harlow Shapley



Herber Curtis

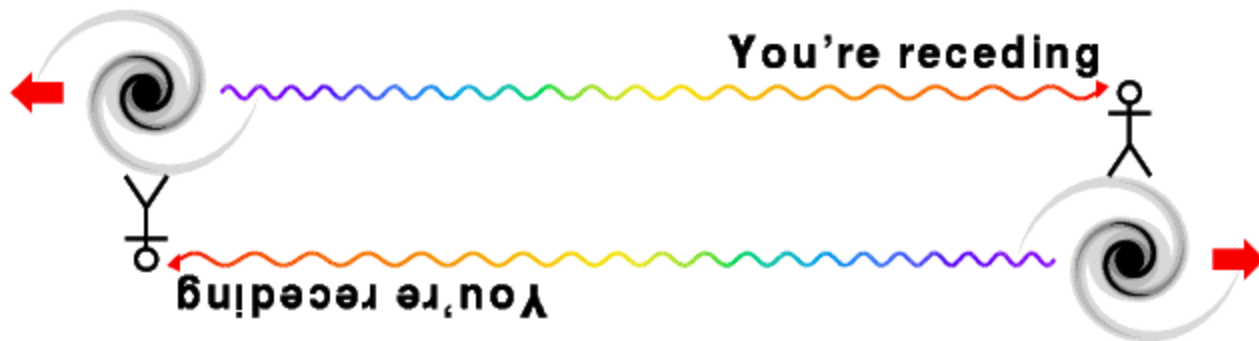
1924: Hubble closes the Shapley-Curtis debate

Galaxy \neq Universe

& Hubble classified galaxies

The many uses of Hubble LAW

- Determine distances (caveats...)
- The universe is expanding
(is it? ...Into what?)



Is the expansion of the Universe surprising?

Einstein's view of Newton's considerations.

How bright would the night sky be if the distribution of stars was infinite?

Olbers' paradox:

(1826 but from 1576)



Olbers' paradox

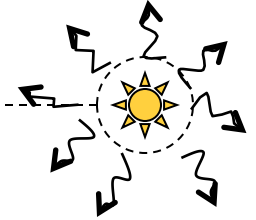
How bright would the night sky be if the distribution of stars was infinite?

Flux from a star

$$f = \frac{L}{4\pi r^2}$$



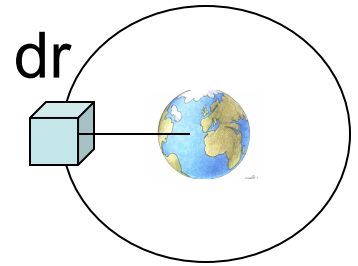
r



Intensity of radiation
from a shell of stars
per steradian

$$dJ = \frac{L}{4\pi r^2} n r^2 dr$$

Density,
for simplicity assume constant



If the Universe is infinite:

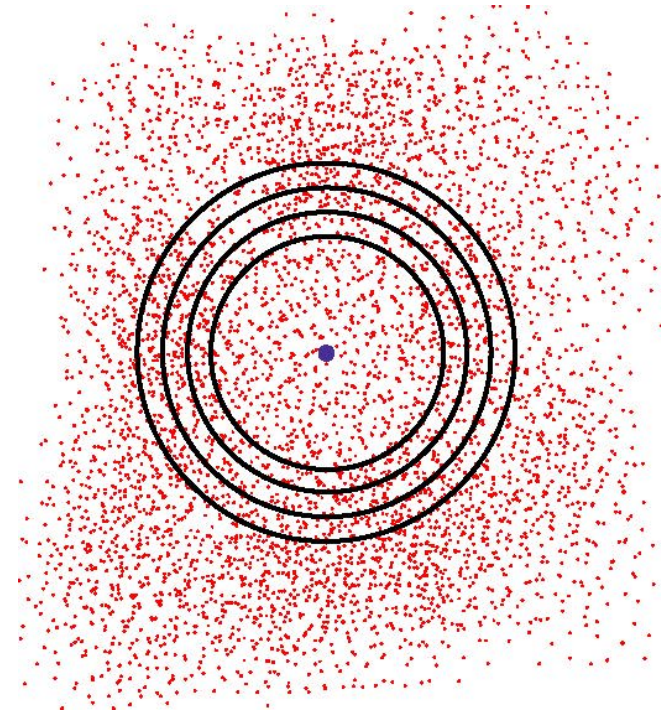
$$J = \int_{r=0}^{r=\infty} dJ = \frac{nL}{4\pi} \int_0^{\infty} = \infty$$

Olbers: “but... the night sky is actually dark!”

Woops!

“Solutions” to Olber’s Paradox

- The brightness of stars goes down as $1/r^2$.
 - BUT... The number of stars goes UP by r^2 !
- Dust clouds obscure the light from distant stars/galaxies.
 - BUT... Those clouds would heat up... and we would see THEM!



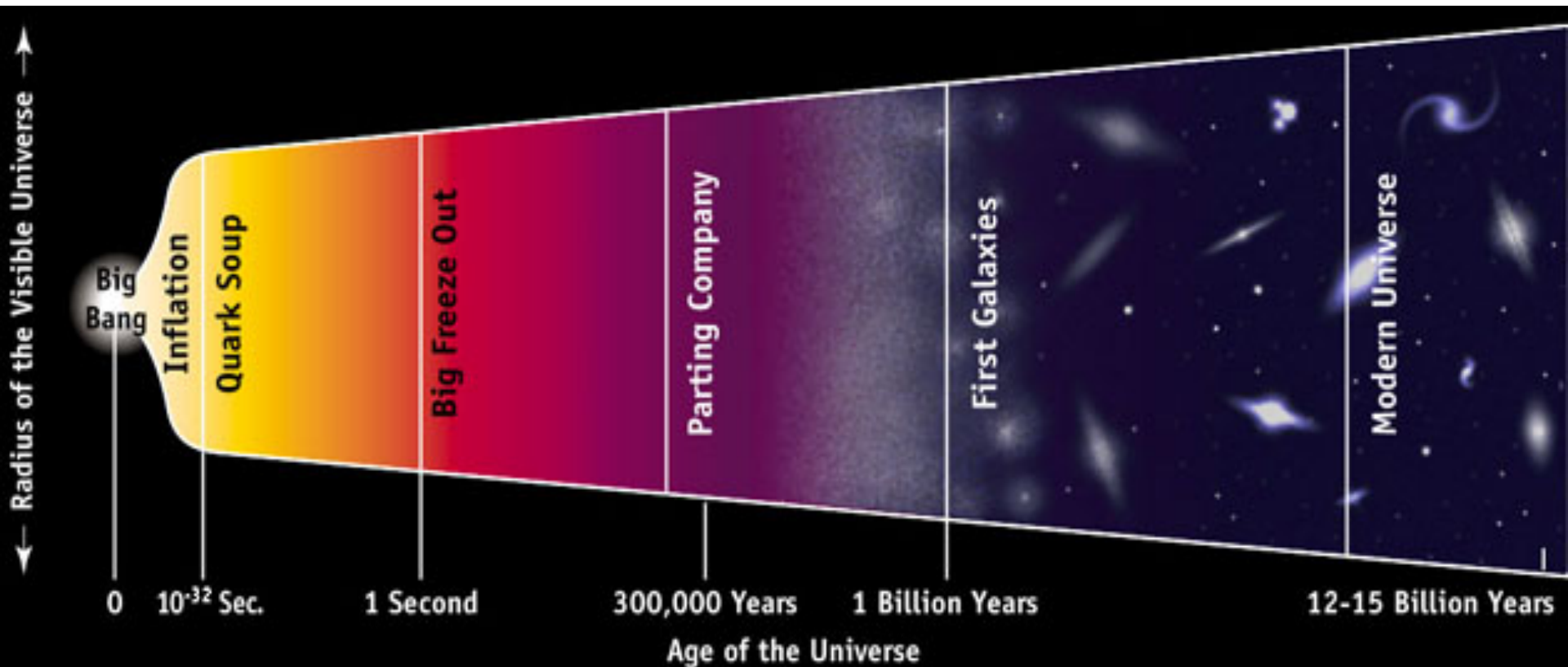
Something has to GIVE: Either the Universe is not INFINITE OR the Universe is not STATIC.

EINSTEIN believed in the STATIC Universe:

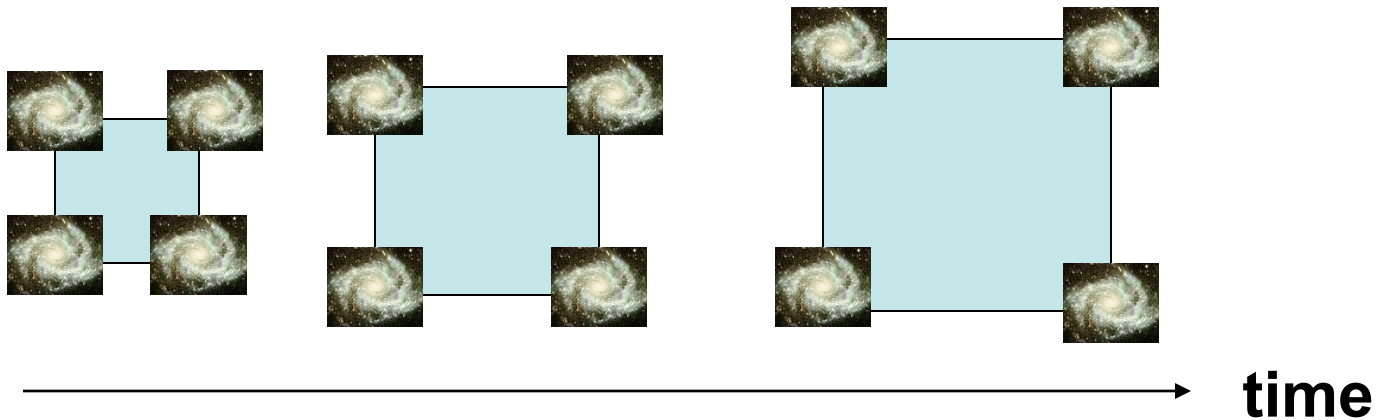
- cosmological constant
- uniform distribution of galaxies
- UNSTABLE

The universe had a beginning!

The extremely successful BIG BANG theory!



The scale factor a



$$\mathbf{r}(t) = \mathbf{r}(t_0) a(t)$$

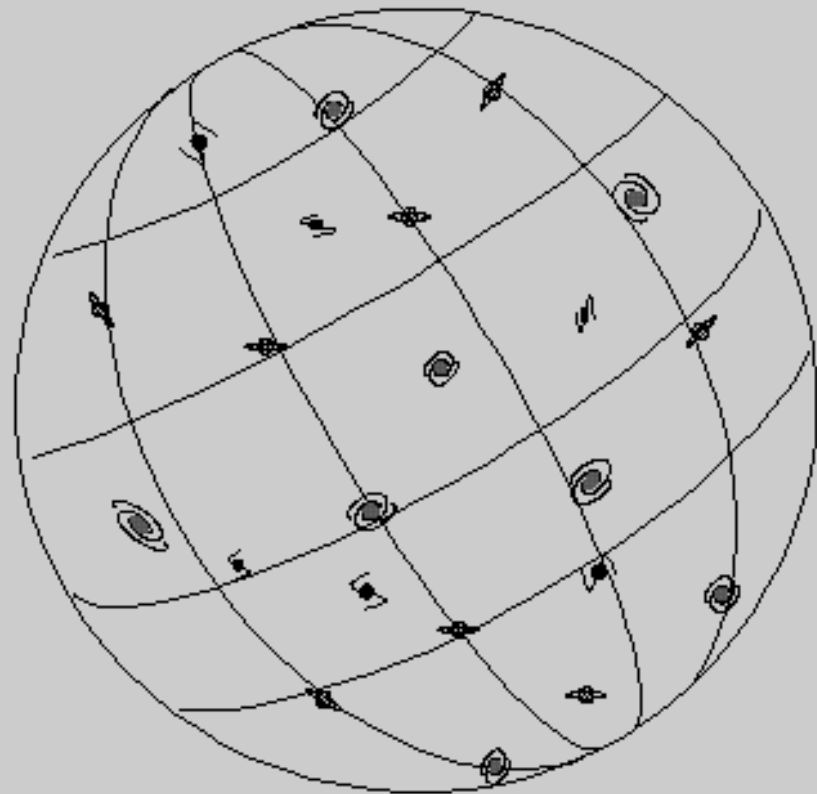
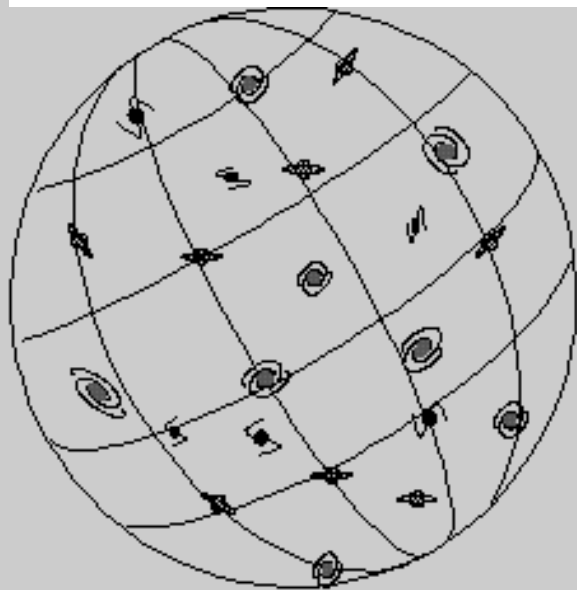
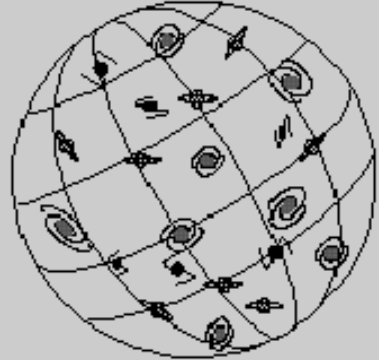
Comoving coordinates!

$$\mathbf{v}_{12} = d\mathbf{r}_{12}/dt = \dot{a} \mathbf{r}_{12}(t_0) = \dot{a}/a \mathbf{r}_{12}(t)$$

Looks like Hubble law

$$H = \frac{\dot{a}}{a}$$

Important!



How old is the Universe?

$$t_0 = r/v = r/(H_0 r) = 1/H_0$$

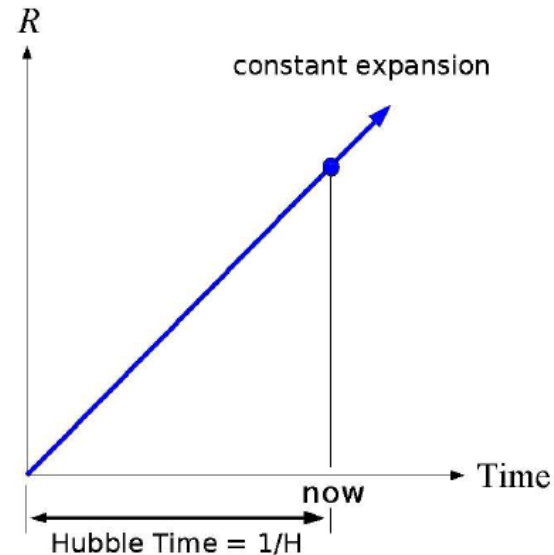
Hubble time

Remember Olbers?

Hubble radius c/H_0
Also called Hubble horizon

The Hubble time is the age of the present day Universe assuming a constant expansion rate.

$$t_H = \frac{1}{H_0} = \frac{D_H}{c}$$



Exercise: compute numerical values.

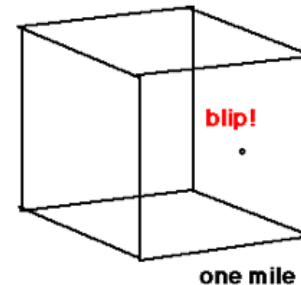
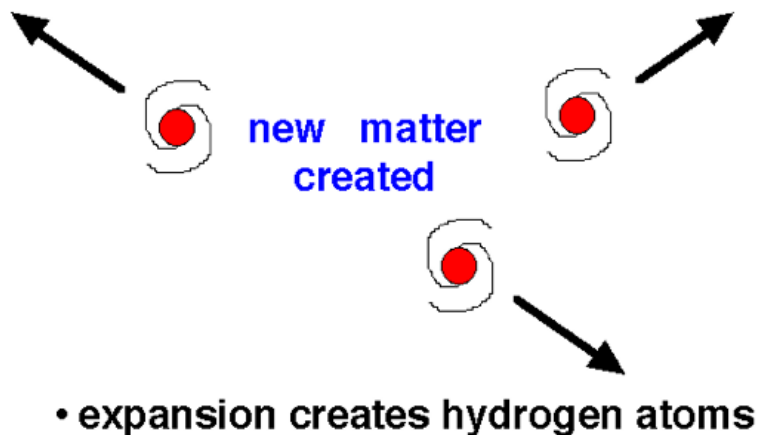
Another interpretation

(historical interest only)

The steady state Universe (Fred Hoyle)

Infinitely old; Infinitely big; Constant density
Expanding (Hubble's Law)

CONTINUOUS MATTER CREATION

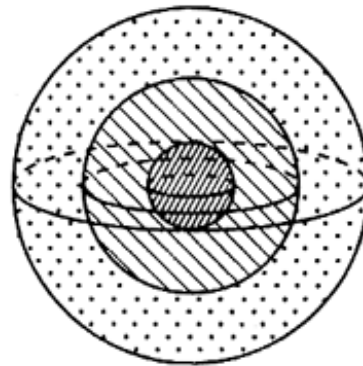
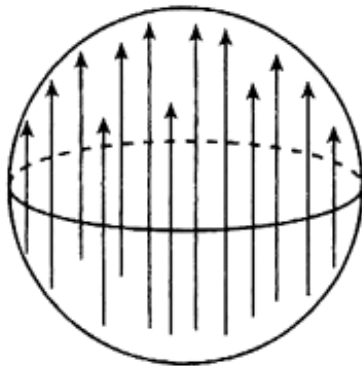
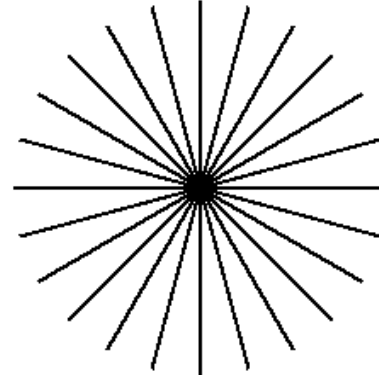
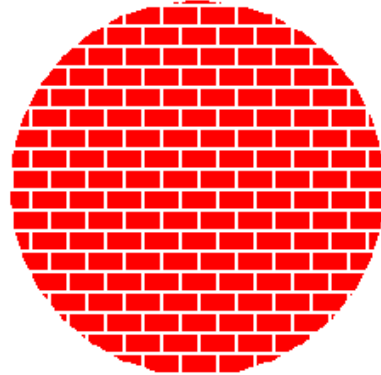


- one hydrogen atom appears every year in a one-cubic mile volume

Exercise: how did I get this number?

Some assumptions

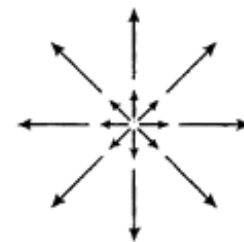
The Universe is homogeneous and isotropic on large scales



Homogeneous
Not isotropic

Isotropic
Not homogeneous

(a)



Homogeneous
Not isotropic

Isotropic
Not homogeneous

(b)

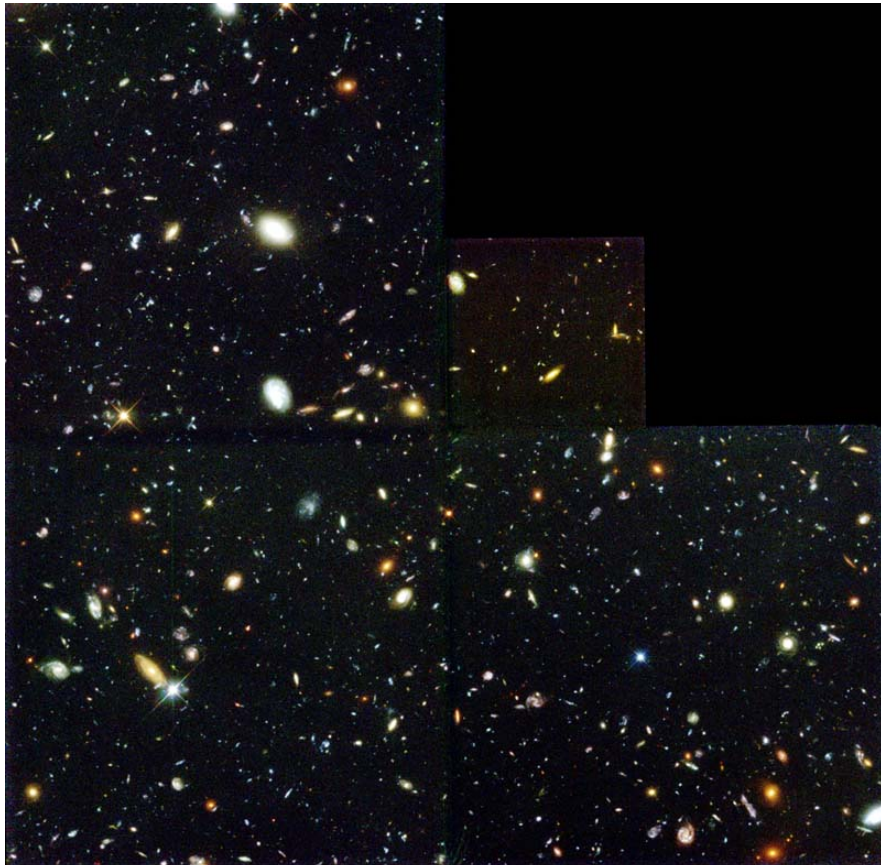
Figure 1.5 Illustrations of how homogeneity and isotropy are not equivalent in (a) three dimensions and (b) two dimensions. In the first example of each, a unique direction is picked out but translation invariance is maintained. In the second example of each, all directions are the same (rotation invariance) but a radial gradient exists.

Supported by observations

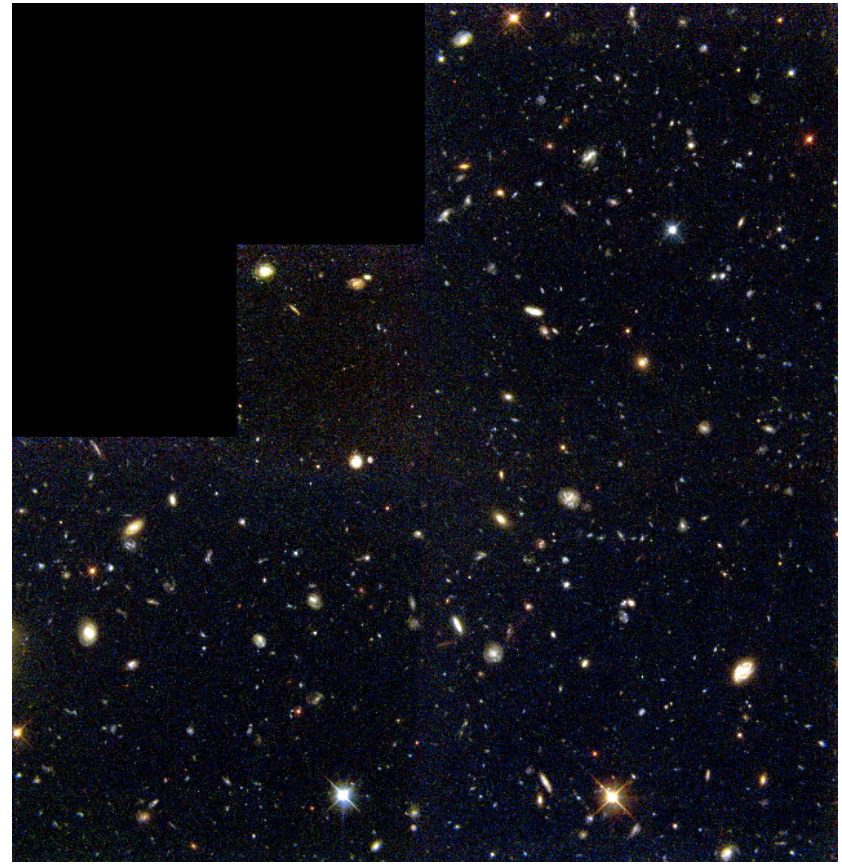
The universe is isotropic—

it looks the same in every direction

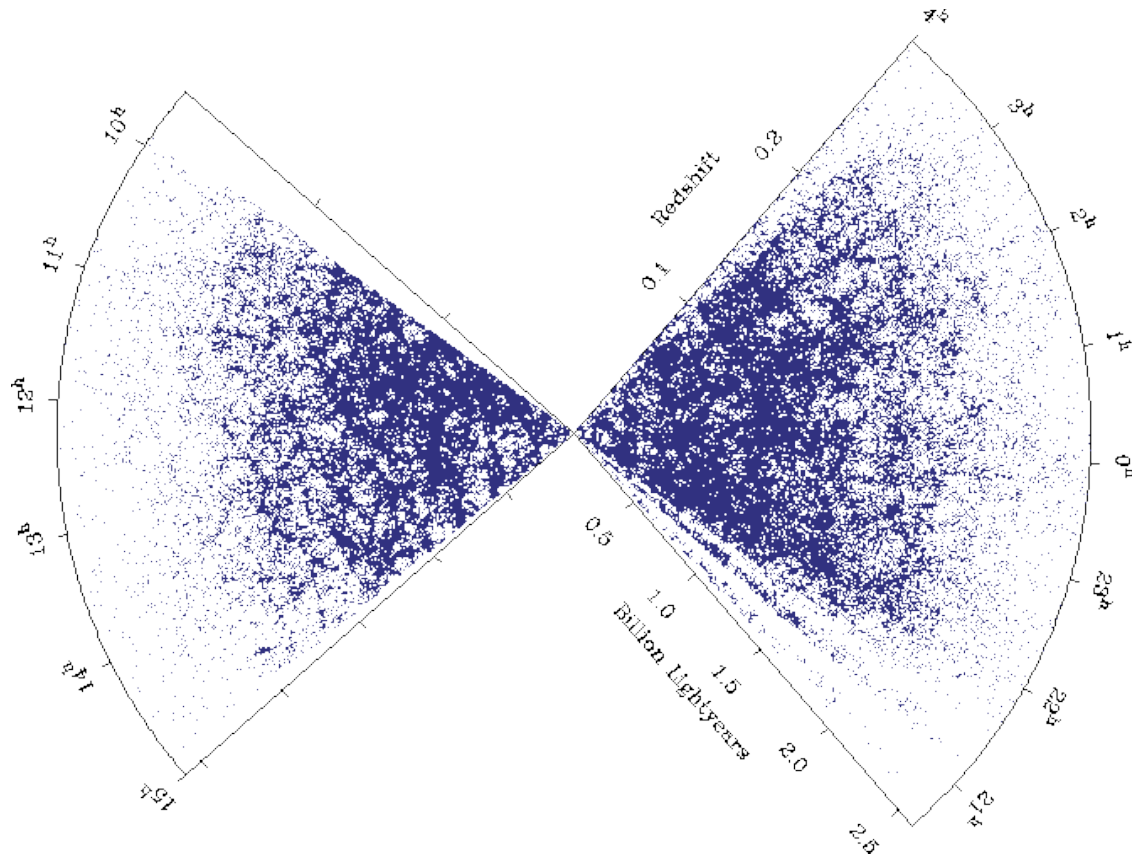
HDF-North



HDF-South



The universe is homogeneous—each volume is about like every other volume
Large volumes of the sky in different directions, 100's of Mpc in size, look about the same.



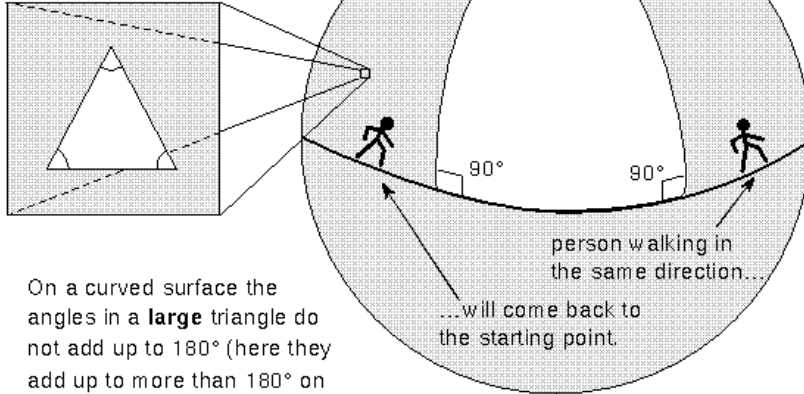
The importance of the Cosmological Principle

The isotropic and homogeneous nature of the universe are often spoken of together as the cosmological principle. Basically, it says that the universe is more or less the same everywhere, and it looks more or less the same from any location.

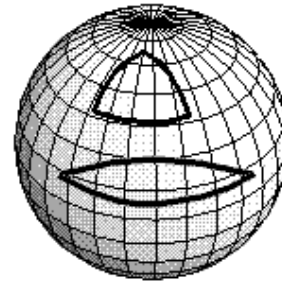
Two consequences: there is no preferred location (i.e., a center) in the universe; and our own Milky Way (and Sun and...) is not in any particularly special place.

Geometry

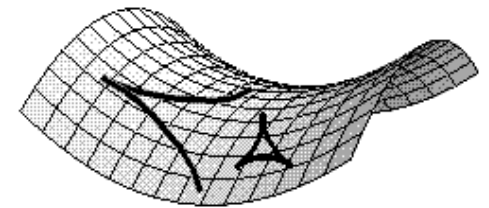
On small scales the surface looks flat. The angles of **small** triangles add up to 180° .



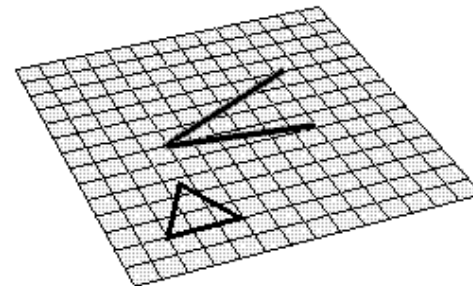
On a curved surface the angles in a **large** triangle do not add up to 180° (here they add up to more than 180° on this type of curved surface).



Universe with *positive* curvature. Diverging lines converge at great distances. Triangle angles add to more than 180° .

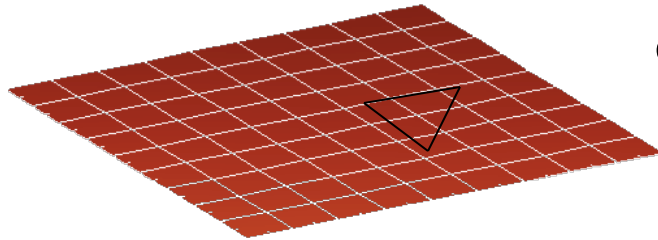


Universe with *negative* curvature. Lines diverge at ever increasing angles. Triangle angles add to less than 180° .



Universe with no curvature. Lines diverge at constant angle. Triangle angles add to 180° .

Geometry & Metrics



$$\alpha + \beta + \gamma = \pi$$

flat

$$ds^2 = dr^2 + r^2 d\theta^2$$

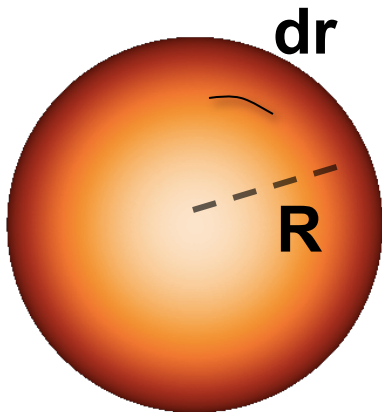
Area of triangle



$$\alpha + \beta + \gamma = \pi - A/R^2$$

Negatively curved

$$ds^2 = dr^2 + R^2 \sinh^2(r/R) d\theta^2$$



$$\alpha + \beta + \gamma = \pi + A/R^2$$

Positively curved

$$ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2$$

Finite area, max separation

In 3D and in general...

$$d\theta^2 \longrightarrow d\theta^2 + \sin^2\theta d\phi^2 = d\Omega^2$$

$$ds^2 = dr^2 + S_k(r)^2 d\Omega^2$$

$$S_k(r) = \begin{cases} \sqrt{k}^{-1} \sin(r\sqrt{k}), & k > 0 \\ r, & k = 0 \\ \sqrt{|k|}^{-1} \sinh(r\sqrt{|k|}), & k < 0. \end{cases}$$

\nearrow **R**

k may be taken to belong to the set $\{-1, 0, +1\}$

Changing coordinate system $x = S_k(r)$:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2/R^2} + x^2 d\Omega^2.$$

Friedman-Robertson Walker metric

In 4 dimensions and introducing back the scale factor

$$ds^2 = \underset{\substack{\uparrow \\ c^2}}{dt^2} - a^2(t) \left[\frac{dx^2}{1 - kx^2} + x^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

c^2

If $k=0$ Minkowski

OR

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_k(r)^2 d\Omega^2 \right]$$

Comoving coords again!

t is COSMIC TIME:

time seen by an observer who sees the universe expanding uniformly

Knowing $a(t)$, k , and R_0 is “all” you need!

Compute distances:

At fixed time spatial geodesic (angles are fixed)

$$ds = a(t) dr$$

Proper distance:

$$d_p = a(t) \int_0^r dr = a(t)r = a(t)r(x) = a(t)S_k(r)^{-1}$$

Hubble law again:

$$\dot{d}_p = \dot{a}r = \frac{\dot{a}}{a}d_p$$

$$H_0 = \left(\frac{\dot{a}}{a} \right)_{t=t_0}$$

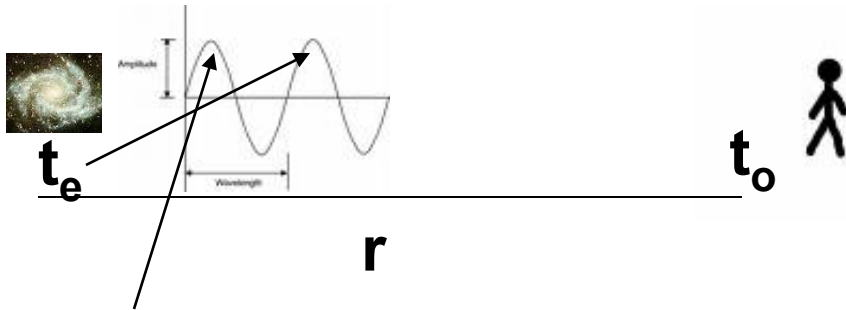
$$v_p(t_0) = H_0 d_p(t_0)$$

Redshift, again

$ds = 0 \longrightarrow c^2 dt^2 = a(t)^2 dr^2$ Photons travel along null geodesics

$$c \frac{dt}{a(t)} = dr$$

t only r only

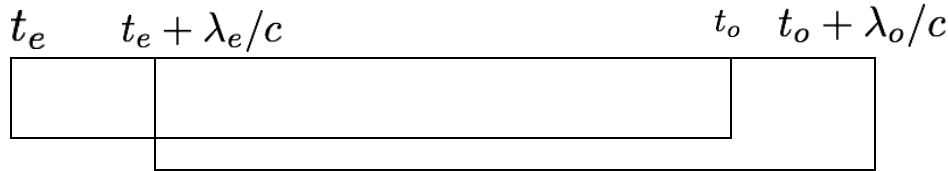


$$c \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_0^r dr = r$$

$$c \int_{t_e + \lambda_e/c}^{t_o + \lambda_o/c} \frac{dt}{a(t)} = \int_0^r dr = r$$

$$c \int_{t_e + \lambda_e/c}^{t_o + \lambda_o/c} \frac{dt}{a(t)} = c \int_{t_e}^{t_o} \frac{dt}{a(t)}$$

The next crest



$$\frac{1}{a(t_e)} \int_{t_e}^{t_e + \lambda_e/c} dt = \frac{1}{a(t_o)} \int_{t_o}^{t_o + \lambda_o/c} dt \longrightarrow \frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_o)}$$

Should remind you of z

$$\frac{\lambda_o}{\lambda_e} = (1 + z) = \frac{a(t_o)}{a(t_e)}$$

If an object has $z=3$, what was the size of the Universe?

How do we measure the geometry then?

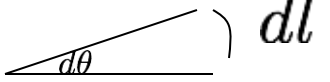
$$\alpha + \beta + \gamma = \pi + kA/R_0$$

$K=+1 \rightarrow$ finite size: circumference $2\pi R_0$ In the past even smaller

If $2\pi R_0 \ll ct_0 \sim c/H_0$??????

Angular size of objects

If I happen to know $dl \dots$

$$d\theta = \frac{dl}{a(t)S_k(r)}$$


AH!

Standard ruler, angular diameter distance

brightness

Standard candle

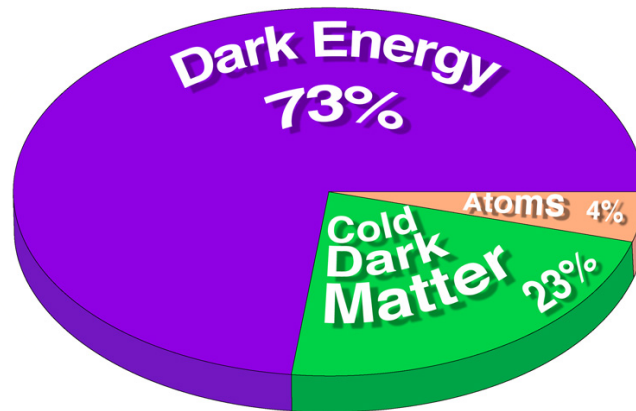
$$F = \frac{L}{4\pi r^2} \longrightarrow F = \frac{L}{4\pi S_k^2(1+z)^2}$$

Ah! Ha!

- In GR space tells mass how to move, mass tells space how to curve.
- Suspicion: $a(t)$ related to content of Universe?
- Really need GR....

**The theory that really describe the Universe at the largest scales must be GR.
Dwell a bit on Einstein vs Newton...**

Composition of the Universe



Radiation...

**TODAY WE KNOW THIS (how we got there ...more in the next lectures)
All the component scale with scale factor in different ways so today the composition is this
(we' ll see how we know this later on) but in the past, was different!**

There is more than meets the eye

In the solar system sun + planets

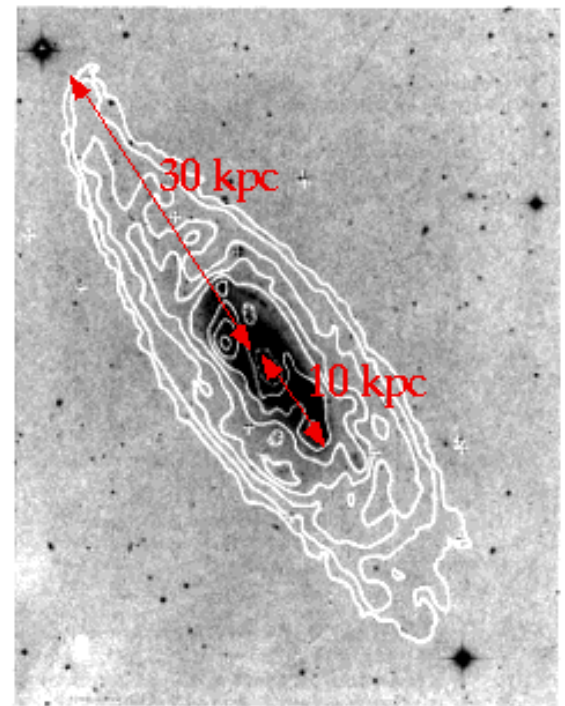
Mass-to-light ratio

Let's consider galaxies



optical light

rotating



Gas (H, 21cm)

Dynamical Estimation of Galaxy Masses

From Kepler's and Newton's laws, the rotation speed of an atom of gas (mass m_{gas}) around a galaxy (mass M) is given by

$$(M + m_{\text{gas}}) P^2 = M P^2 = a^3$$

where P and a are the period and semi-major axis of the orbit, and the atom of gas is much less massive than the galaxy. If the atom's orbit is circular, then a is the radius of the circle (r), and, according to Kepler's 2nd law, the atom's speed is constant. So

$$v = 2 \pi r / P$$

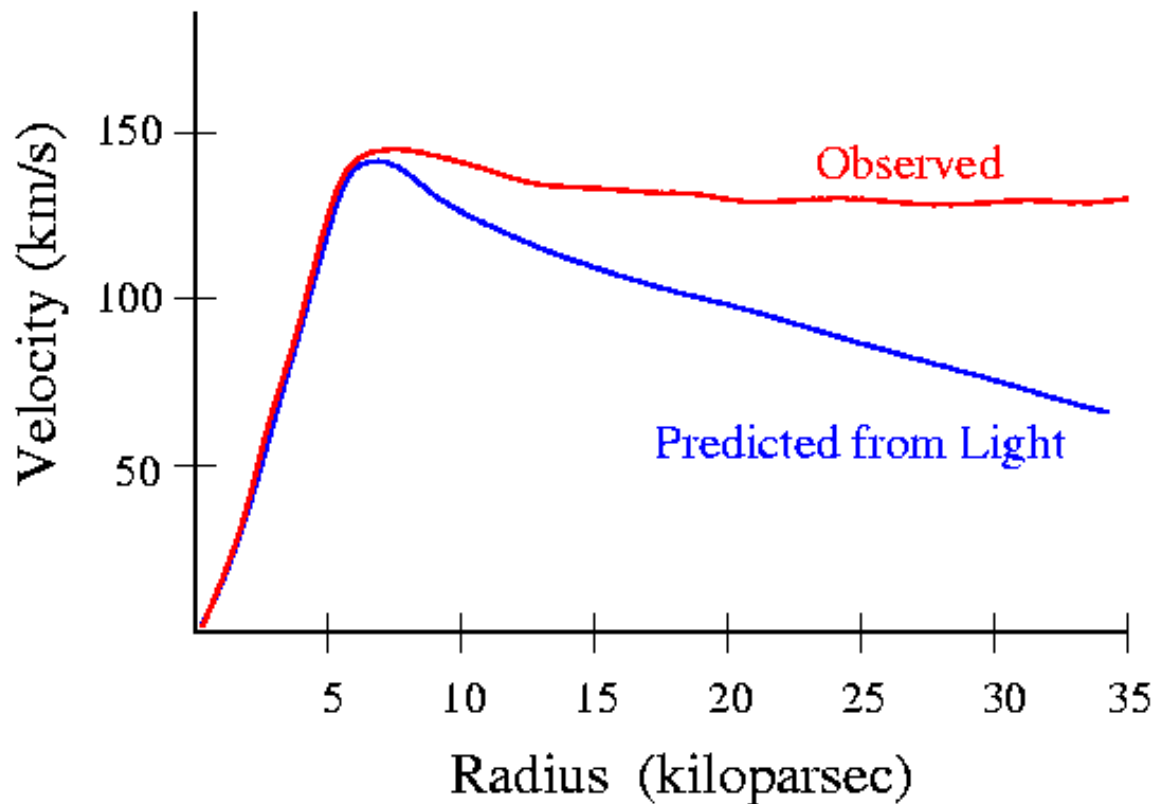
Putting this all together with a bit of math then yields

$$v = 2 \pi \sqrt{M} \cdot \frac{1}{\sqrt{r}} \quad \text{or} \quad M \propto v^2 r$$

Dynamical Estimation of Galaxy Masses

According to Newton's laws, once outside a galaxy, the rotation velocity of gas should decrease with distance. But that's not observed!

$$v = 2\pi\sqrt{M} \cdot \frac{1}{\sqrt{r}}$$

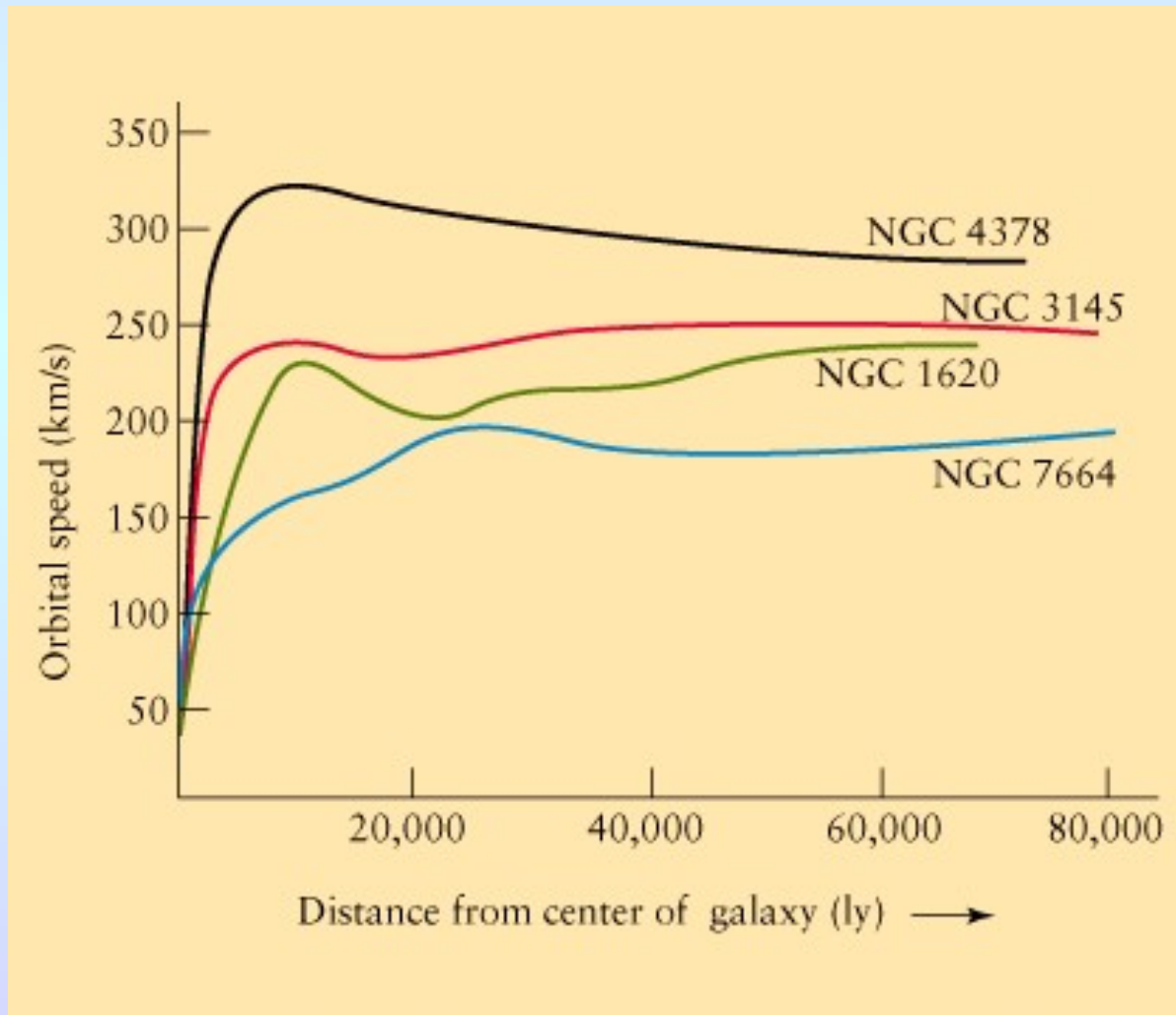


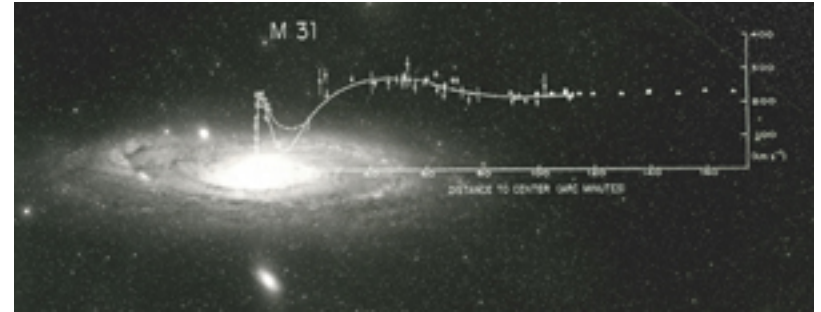
There must be a lot of mass in the outer regions of galaxies that we are not observing!

This is called
DARK MATTER

Dynamical Estimation of Galaxy Masses

Virtually all spiral galaxies have these “flat” **rotation curves**. The outer regions of spirals must be dominated by dark matter.



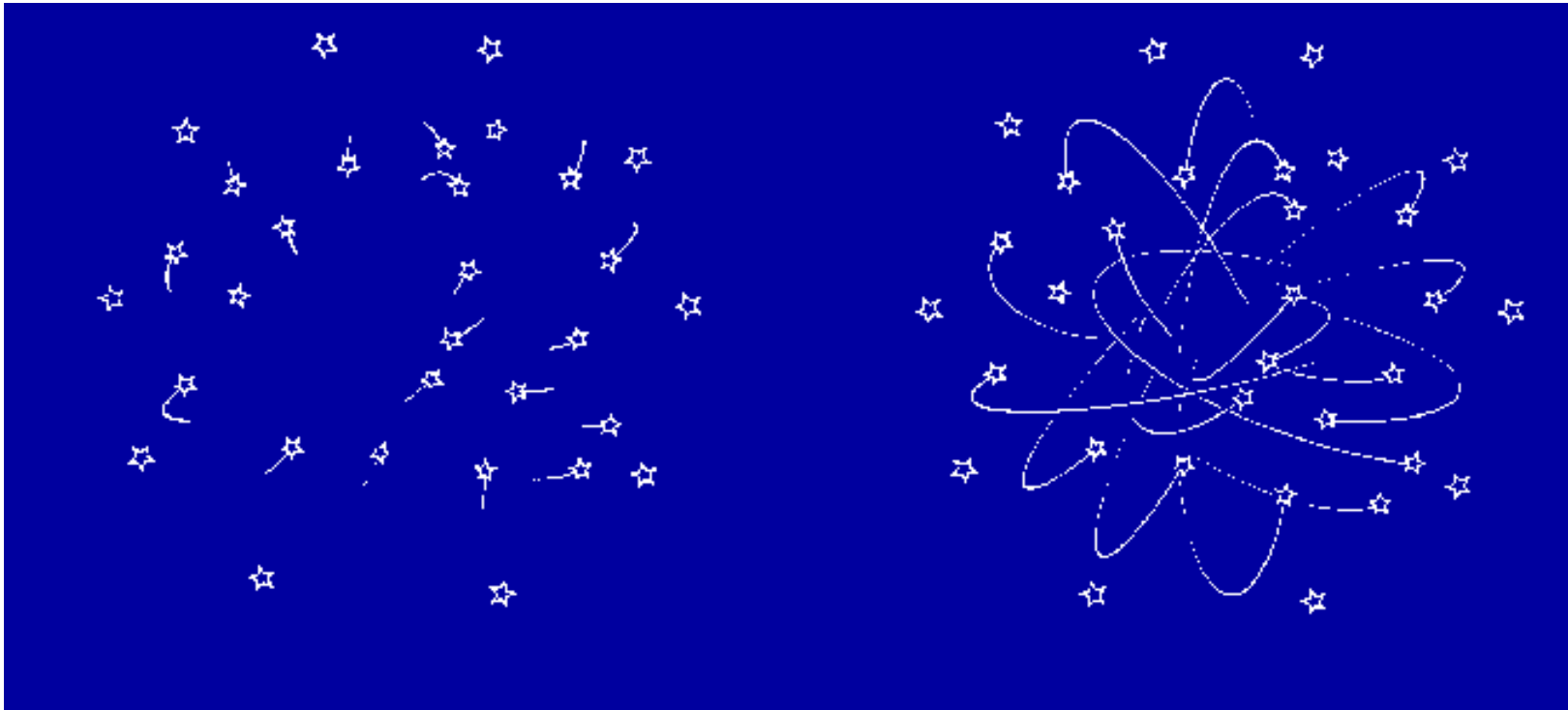


"In a spiral galaxy, the ratio of dark-to-light matter is about a factor of ten. That's probably a good number for the ratio of our ignorance-to-knowledge."

Mass-to-light ratio ~ 10

Mass measurements in galaxy clusters

For a group of objects, there always must be balance between gravity and velocity. Too little velocity, and gravity takes over, making the cluster smaller. Too much velocity, and the objects escape the group's gravity, causing the group to evaporate.



By measuring the Doppler shifts of galaxies in a cluster, you can measure the cluster's gravity. And from gravity, you get mass.

Virial theorem

Kinetic energy = -1/2 potential energy

$$\frac{1}{2}M\langle v^2 \rangle = \frac{\alpha GM^2}{2r_h}$$

Coma cluster

$$\sigma_{v_{los}} \sim 1000 \text{ km/s}$$

$$\sigma_v \sim 3\sigma_{v_{los}}$$

$$r_h \sim 1.5 \text{ Mpc}$$

$$M \sim 2 \times 10^{15} M_{\odot}$$

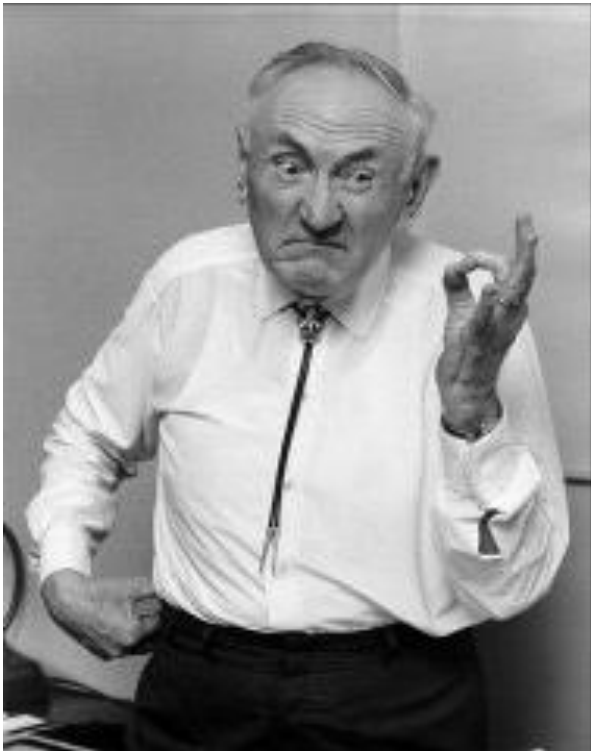
$$M_{\odot} \sim 2 \times 10^{30} \text{ Kg}$$

whoa

Mass-to-light ratio ~100 to 300

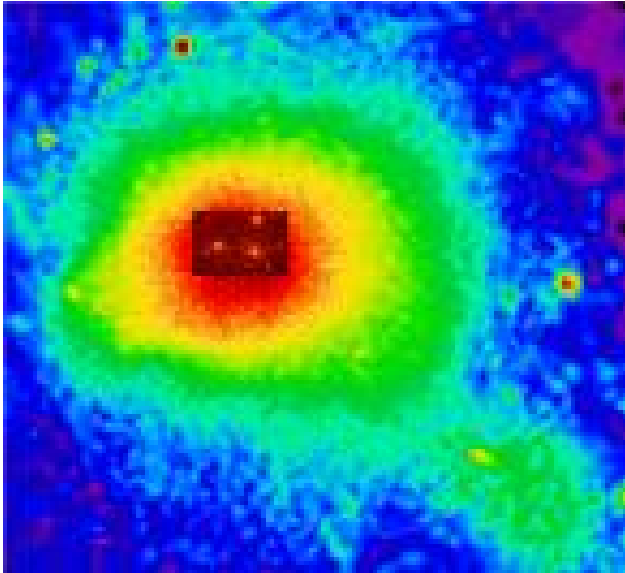
The Mass of Clusters

As Fritz Zwicky noticed in 1933, galaxies in clusters move much too fast for the amount of matter we see. The clusters must contain a lot of unseen matter providing extra gravity.



Once again – **dark matter**

& X-rays

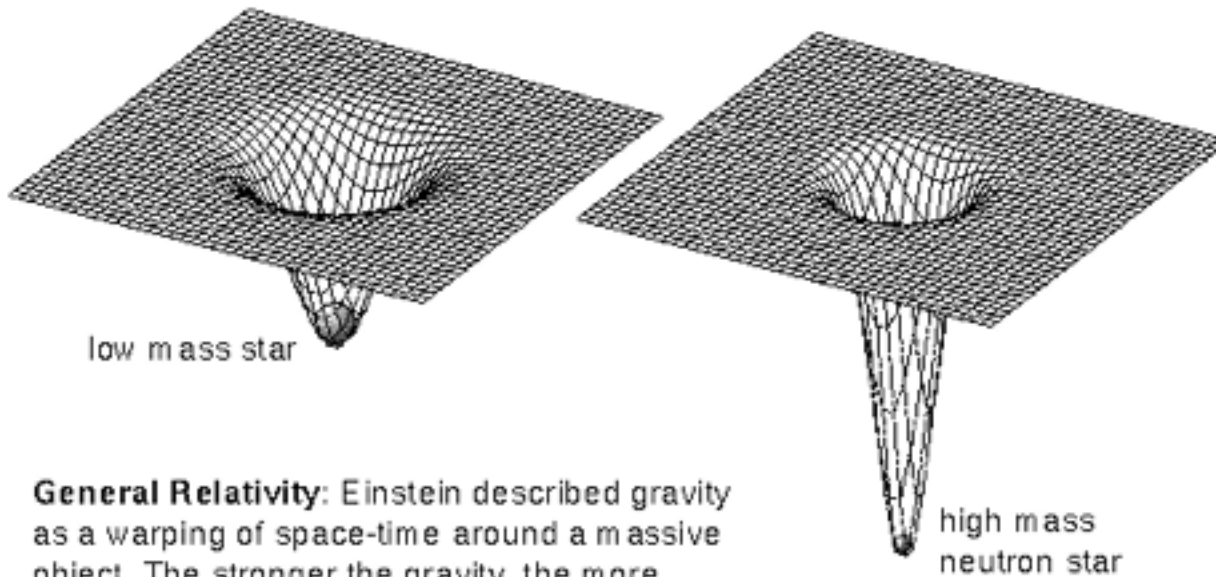


$$K.E. \sim \frac{3}{2}kT = \frac{3}{2}m_H\sigma_v^2$$

$$T \sim 6 \times 10^7 K; \lambda \sim \frac{c}{\nu} = \frac{ch}{KT}$$

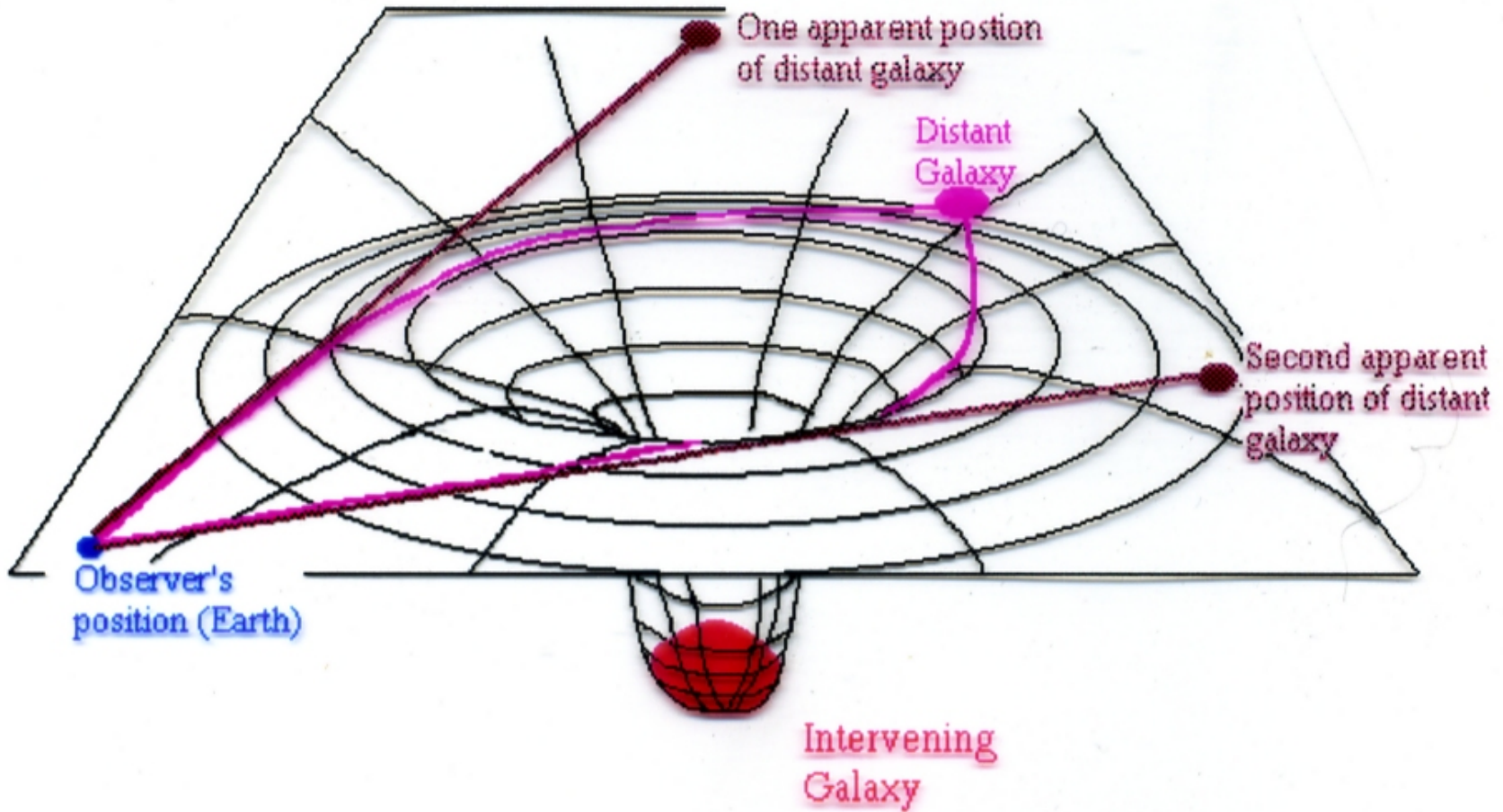
Mass bends space-time

Global geometry \neq local geometry



General Relativity: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.

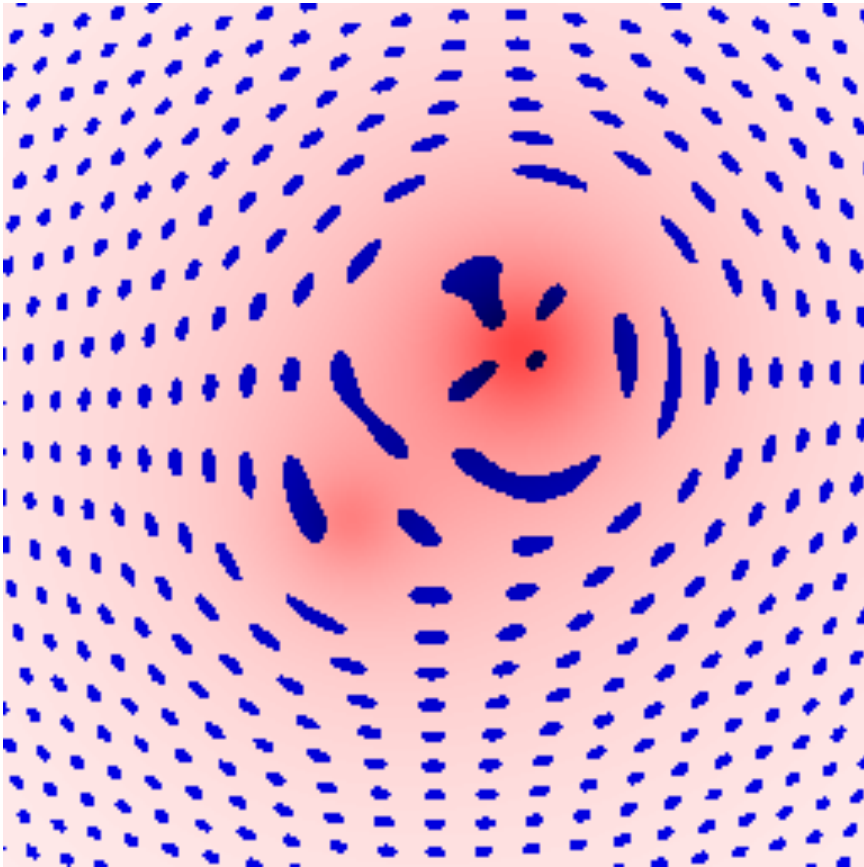
Gravitational lensing



A light ray can be deflected by gravity. The greater the gravity, the greater the bending. As a result, a collection of matter can act as a **gravitational lens**.

Gravitational Lenses

In the case of a background point source, the result might be multiple images of the object. In the case of a larger source (*i.e.*, a galaxy), the result can be arcs and arclets.



In this image, the blue areas show background galaxies, while the red represents foreground mass.

Example of a Gravitational Lens

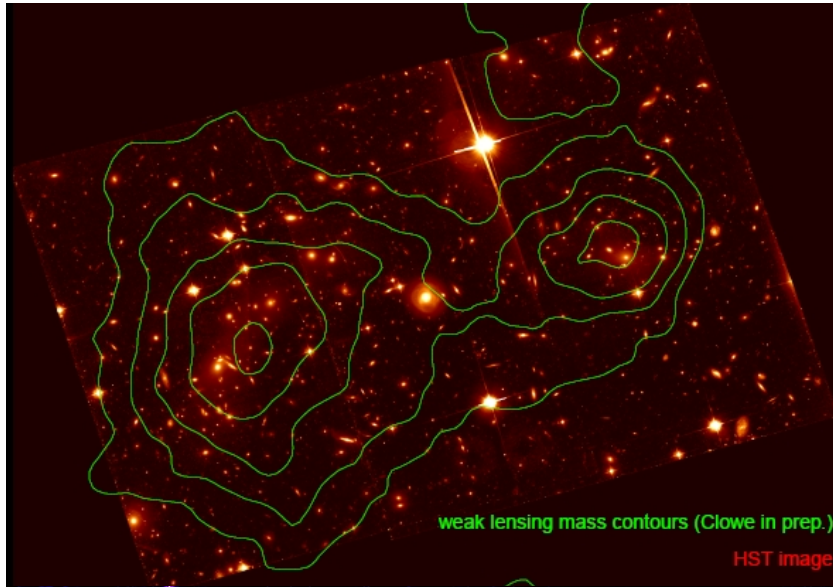


So what is the dark matter?

- (Nucleosynthesis)
- Theories of particle physics
- Structure formation

Must be something unknown to Earth
Weakly interacting!

Some “direct” evidence



1E 0657-56

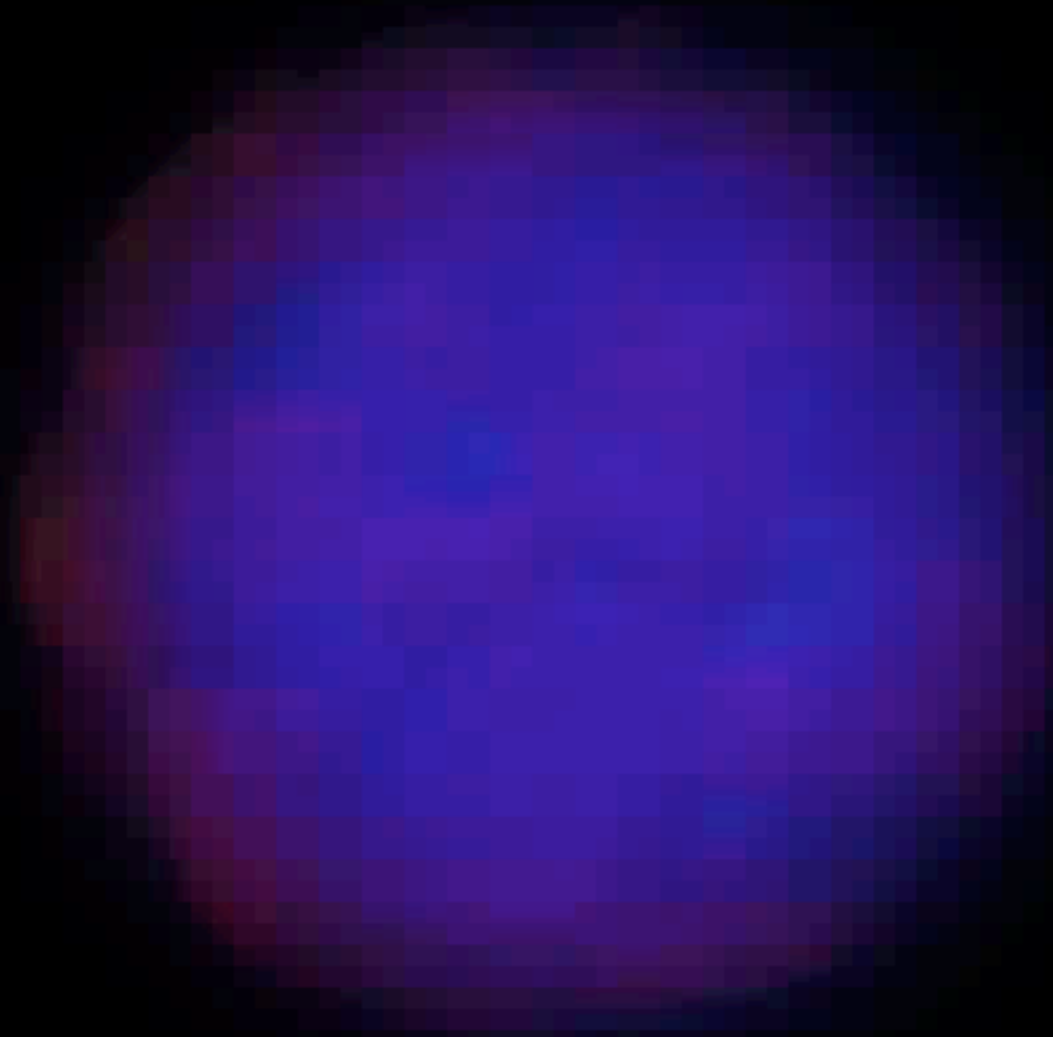
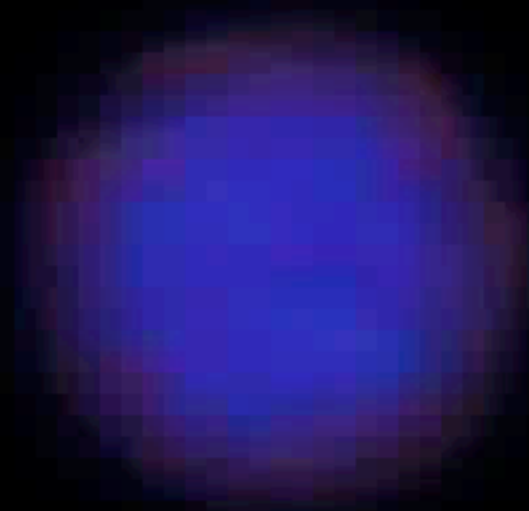


Bullet cluster

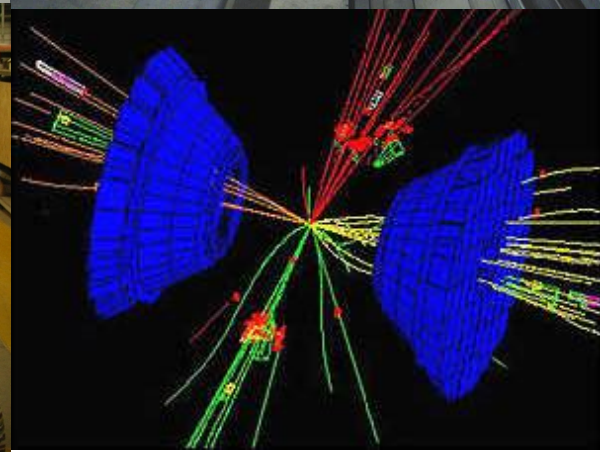
REAL DATA!




Computer-SIMULATION




DIY dark matter

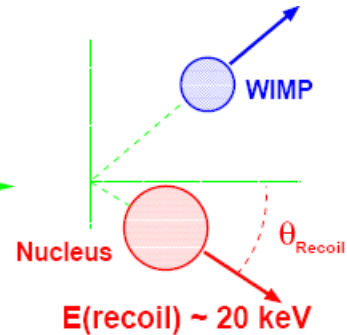


WIMP
From galactic halo

($v \sim 250$ km/s)

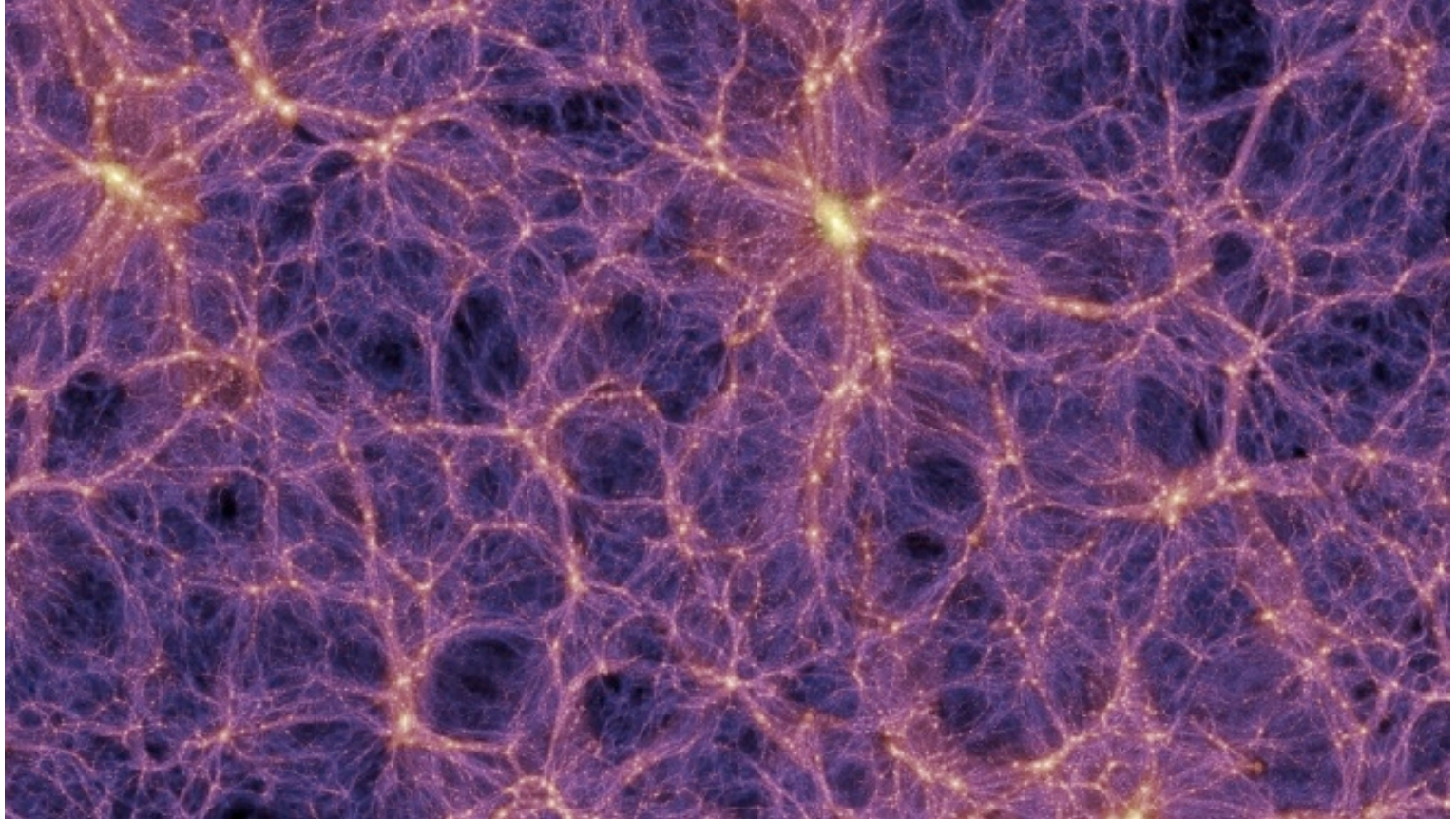
Nucleus
in laboratory

($v = 0$ km/s)

Elastic WIMP
scattering




Computer simulation of dark matter distribution



Key concepts today

The expansion of the Universe

Hubble's Law

Redshift

Olbers' paradox

Geometry

Dark matter