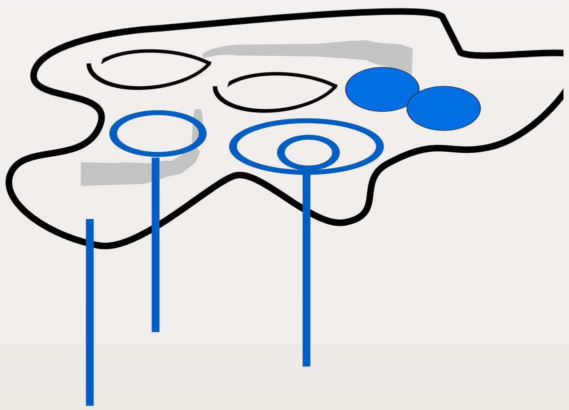
Geometry of Topological Strings & Branes

W.Lerche, TH Retreat 2013

Motivation:

Typical brane + flux configuration on a Calabi-Yau space



closed string (bulk) moduli t

open string (brane location + bundle) moduli u

3+1 dim world volume with effective N=1 SUSY theory

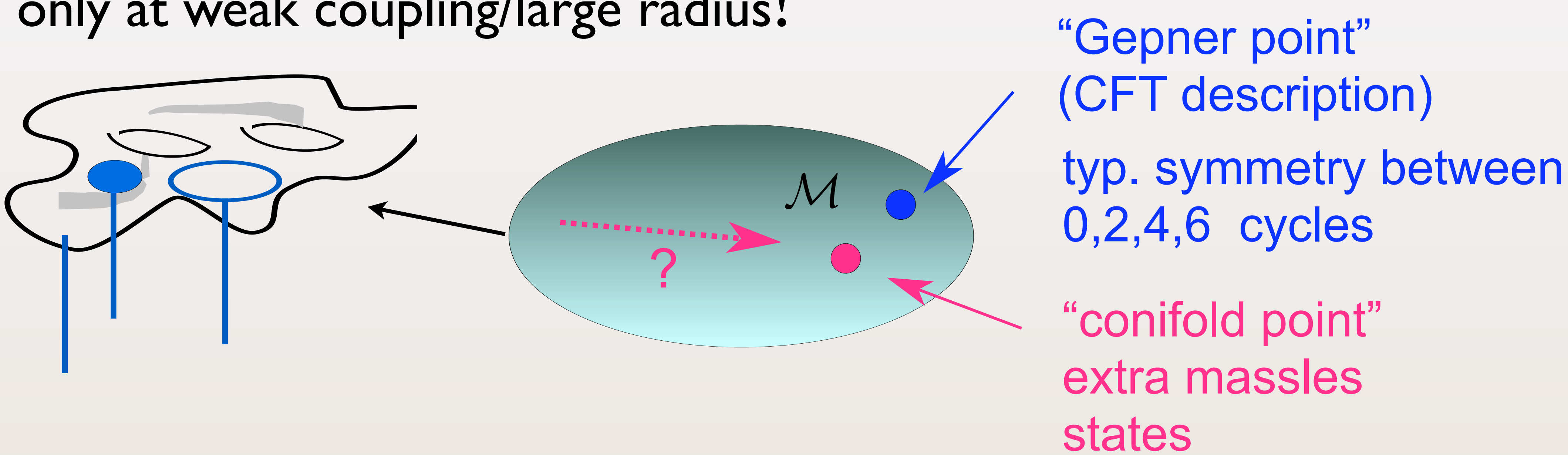
What are the exact effective superpotential, the vacuum states, gauge couplings, etc ? (topological =holomorphic BPS quantities)

$$\mathcal{W}_{\text{eff}}(\Phi, t, u) = ?$$

....well developed geometrical techniques mostly for non-generic brane configurations (non-compact, -intersecting) branes only !
(mirror symmetry, localization, matrix models...)

Classical versus Quantum Geometry

Classical geometry ("branes wrapping p-cycles", gauge bundle configurations on top of them) makes sense only at weak coupling/large radius!



Classical geometry:
cycles, gauge (“bundle”) configurations on them

Quantum corrected geometry:
(instanton) corrections wipe out notions of classical geometry

Most of string phenomenology deals with (semi-)classical regime!

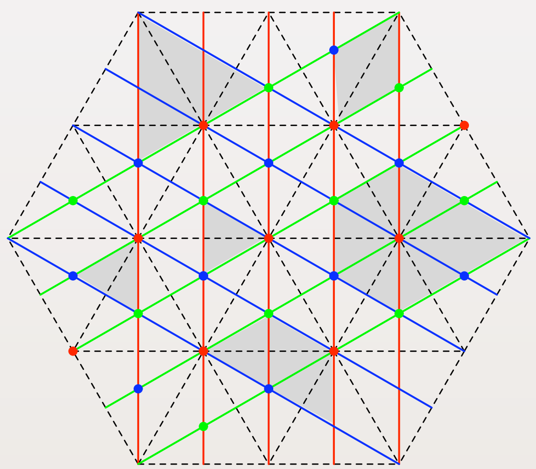
Important: need to develop formalism capable of describing the physics of general D-brane configurations (here: topological D-branes)

Matrix Factorizations and Homological Mirror Symmetry

- Important concept (Kontsevich): derived/Fukaya categories

- Translate math. language to physics:
  homological mirror symmetry <-->
  boundary Landau-Ginzburg theory/matrix factorizations

$$Q(x) \cdot Q(x) \; = \; W_{LG}(x)\,\mathbf{1}$$



- Open string mirror symmetry becomes (really) interesting for intersecting branes; there is a **much** richer diversity of world-sheet instantons and "GW" invariants.

- My task: flatness equations on A∞ category,
  generalized Gauss-Manin connection

- Compute correlation functions on T2, K3 (Quintic?)

$$\mathcal{W}_{eff} = C_{XXY}(t)\,\mathrm{Tr}XXY + C_{XXYXXY}(t)\,\mathrm{Tr}(XXY)^2 + \ldots$$