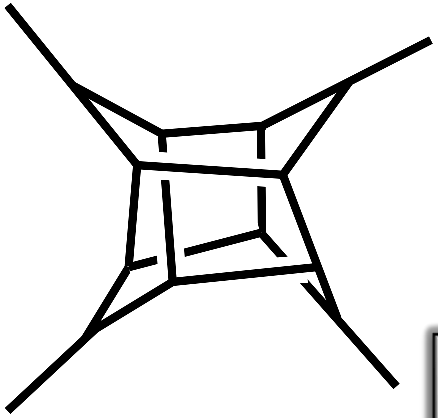
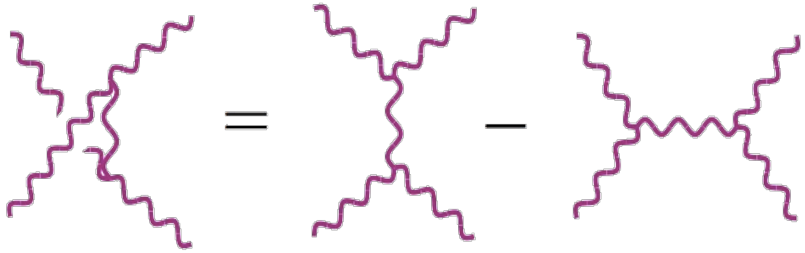
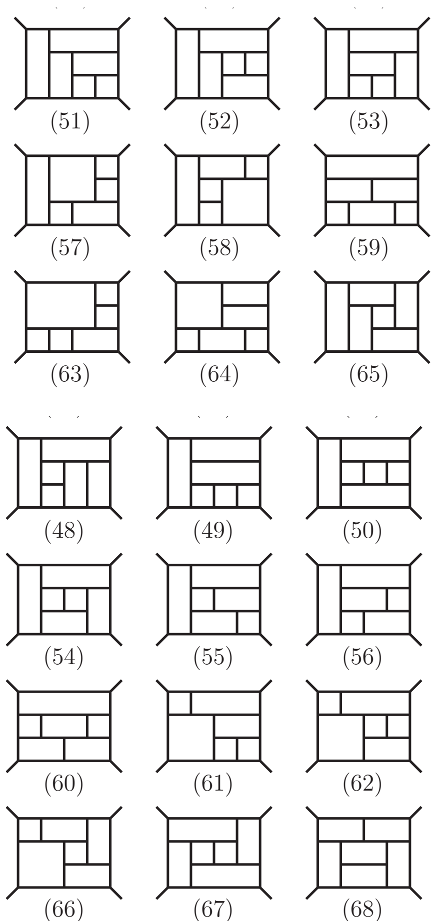
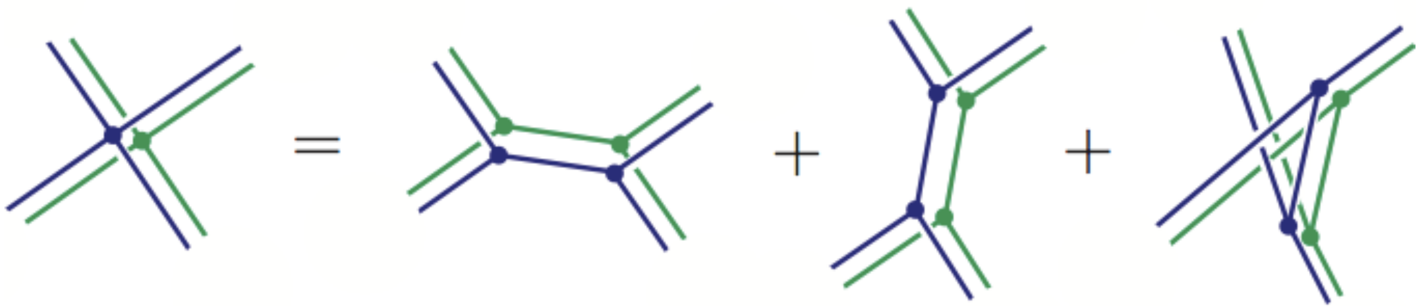






Henrik Johansson

- Scattering amplitudes
- SYM, Supergravity, BLG/ABJM
- Planar & non-planar
- Gravity UV behavior
- Color-kinematics duality
- Towards 2-loop QCD



TH Retreat, Nov 7, 2013

Made it to Hollywood ☺

The Parking Spot Escalation

Sheldon Cooper busy with 6 loops…

This talk:

1) UV divergences in *D*=4 and higher dimensions:

- Question of SUGRA UV finiteness/divergences

2) Hidden structures in gauge theory and gravity:

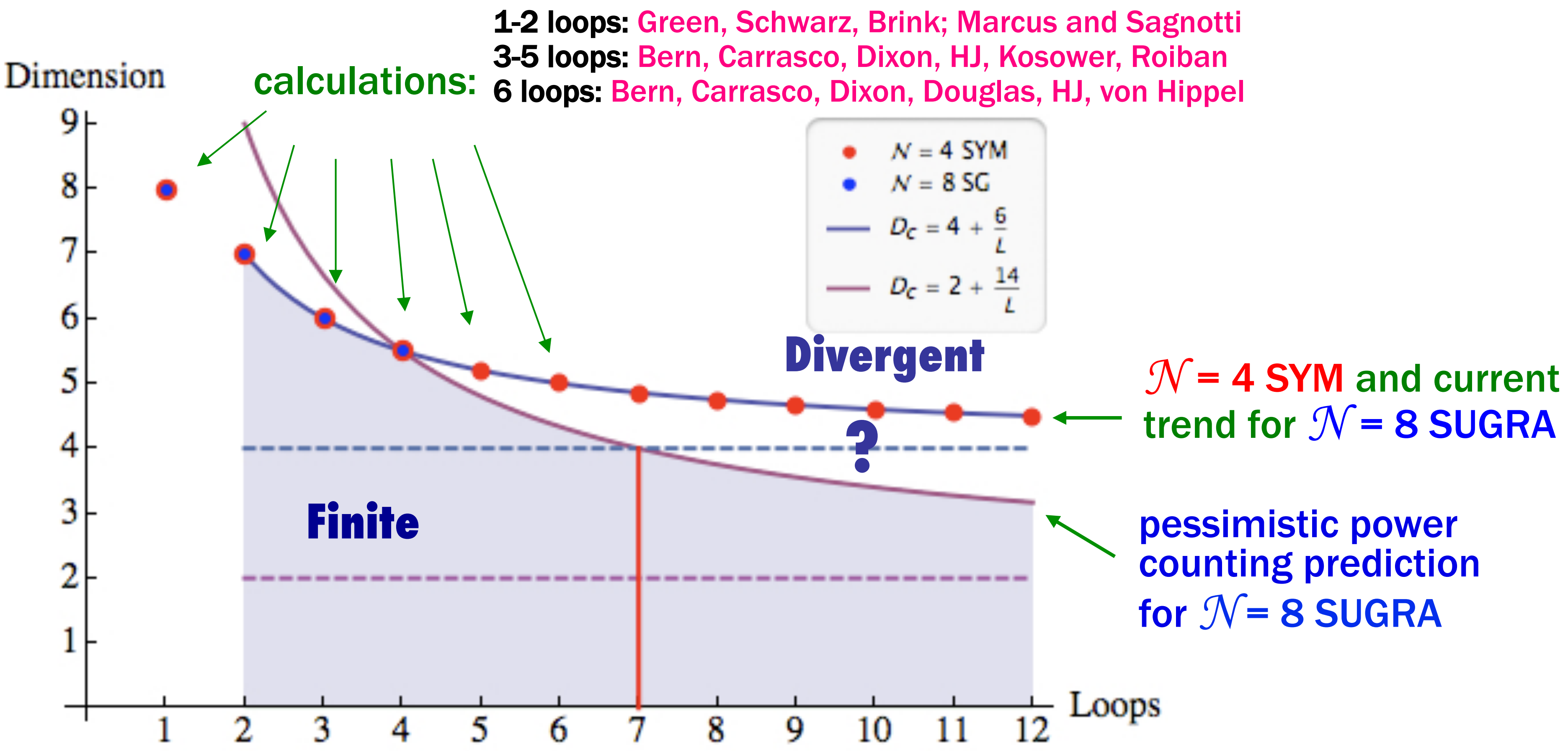
- Duality between color and kinematics
- Gravity as a double copy of gauge theory

3) General multi-loop methods:

- Towards two-loop QCD automation

1) Question of supergravity finiteness...

Parameter space for UV divergences in $\mathcal{N}= 8$ SUGRA and $\mathcal{N}= 4$ SYM



5-loop UV calc. will give strong indication of $\mathcal{N}= 8$ finiteness/divergence

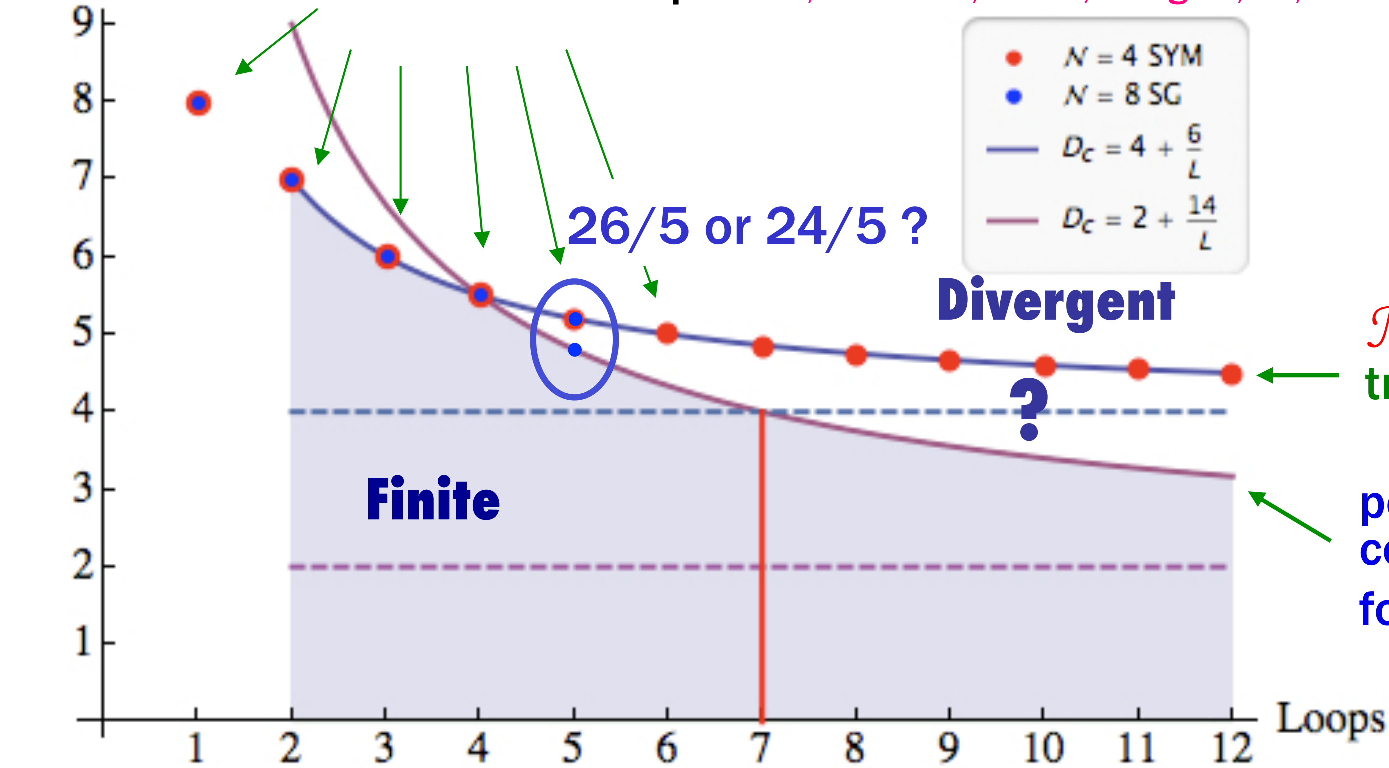
1) Question of supergravity finiteness…

Parameter space for UV divergences in $\mathcal{N}=8$ SUGRA and $\mathcal{N}=4$ SYM

1-2 loops: Green, Schwarz, Brink; Marcus and Sagnotti
3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban
6 loops: Bern, Carrasco, Dixon, Douglas, HJ, von Hippel

Dimension

calculations:



$\mathcal{N}=4$ SYM and current trend for $\mathcal{N}=8$ SUGRA

pessimistic power counting prediction for $\mathcal{N}=8$ SUGRA

5-loop UV calc. will give strong indication of $\mathcal{N}=8$ finiteness/divergence

2) Color-Kinematics Duality

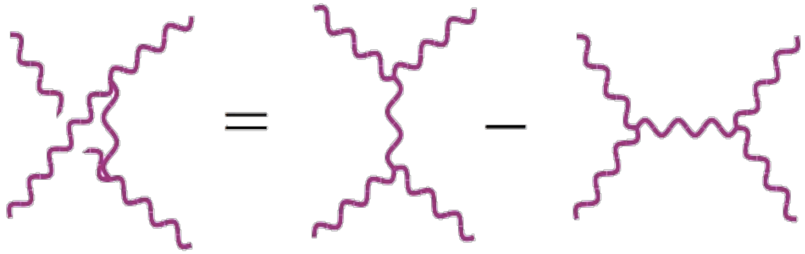
Yang-Mills theories are controlled by a kinematic Lie algebra

- Amplitude represented by cubic graphs:

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

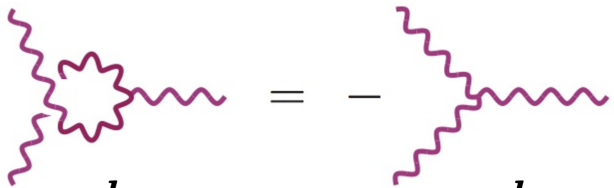
numerators (n_i), color factors (c_i), propagators ($p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2$)

Color & kinematic numerators satisfy same relations:



$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$

Jacobi identity



$$f^{bac} = - f^{abc}$$

antisymmetry

Duality: color ↔ kinematics

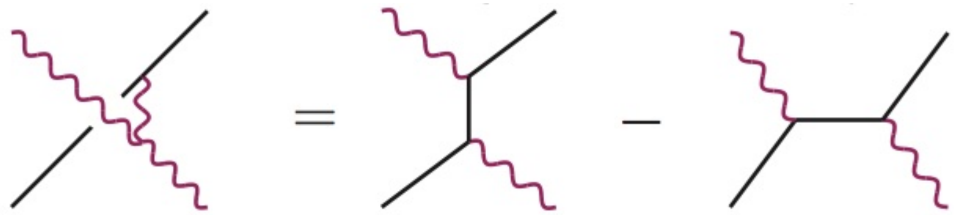
Bern, Carrasco, HJ

Some details of color-kinematics duality

Bern, Carrasco, HJ

can be checked for 4pt on-shell ampl. using Feynman rules

Example with two quarks:



$$\varepsilon_2 \cdot (\overline{u}_1 \not{V} u_3) \cdot \varepsilon_4 \;=\; \overline{u}_1 \not{\varepsilon}_4 \not{p}_t \not{\varepsilon}_2 u_3 \;-\; \overline{u}_1 \not{\varepsilon}_2 \not{p}_s \not{\varepsilon}_4 u_3$$

$$f^{cba} T^c_{ik} = T^b_{ij} T^a_{jk} - T^a_{ij} T^b_{jk}$$

1. $(A^\mu)^4$ contact interactions absorbed into cubic graphs
   - by hand 1=*s*/*s*
   - or by auxiliary field $B \sim (A^\mu)^2$
2. Beyond 4-pts duality not automatic ➔ Lagrangian reorganization
3. Known to work at tree level: all-*n* example Kiermaier; Bjerrum-Bohr et al.
4. Enforces (BCJ) relations on partial amplitudes ➔ (*n*-3)! basis
5. Same/similar relations control string theory S-matrix

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger

Gravity is a double copy

- Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

BCJ

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can belong to different theories:

n_i $\tilde{n}_i$

($\mathcal{N}$=4) × ($\mathcal{N}$=4) → $\mathcal{N}$=8 sugra

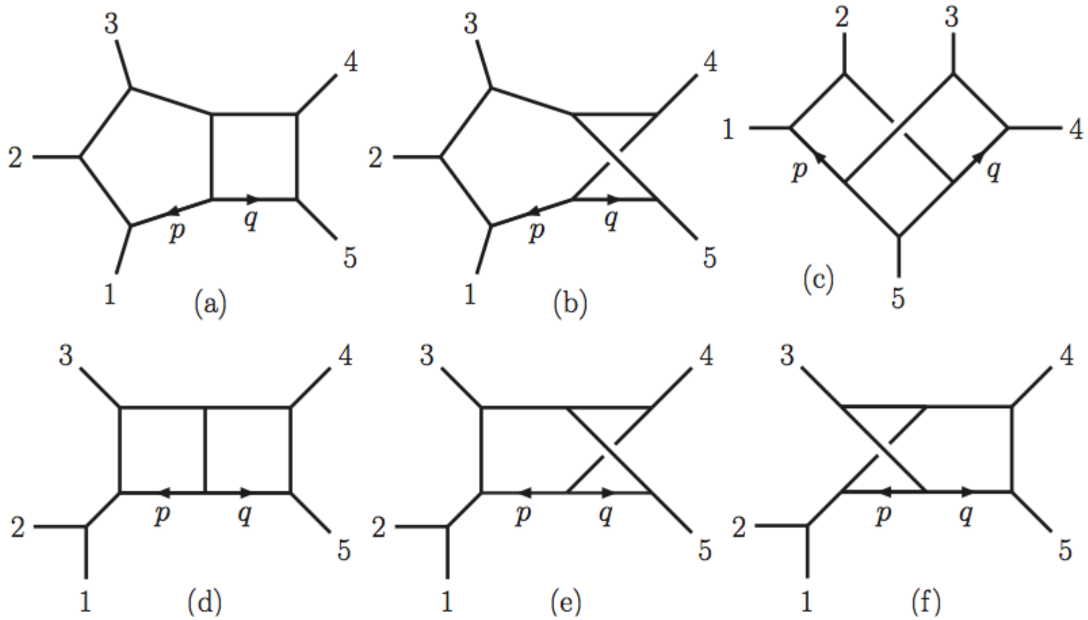
($\mathcal{N}$=4) × ($\mathcal{N}$=2) → $\mathcal{N}$=6 sugra

($\mathcal{N}$=4) × ($\mathcal{N}$=0) → $\mathcal{N}$=4 sugra

($\mathcal{N}$=0) × ($\mathcal{N}$=0) → Einstein gravity + axion+ dilaton

similar to Kawai-Lewellen-Tye but works at loop level

Example: 2-loop 5-pts $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG



Carrasco, HJ
1106.4711 [hep-th]

The 2-loop 5-point amplitude with duality exposed

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p})$ $+2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q})$ $+\gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$

$\mathcal{N}$= 8 SG obtained from numerator double copies

$$\tau_{ip} = 2k_i \cdot p$$

3) Two-loop QCD automation

Maximal Unitarity at two loops **H.J. Kosower, Larsen**

- **Complete two-loop integral basis (massless particles)**

$$\text{Ampl} = \sum_{j \in \text{Basis}} c_j \, \text{Int}_j + \text{Rational}$$

- **Refined unitarity method**
  - **Maximal cut method**
  - **Integral contour ←→ IBP relations**
  - **projectors:** $\text{Cut}_j[\text{Int}_i] = \delta_{ij}$
  - **coefficients:** $\text{Cut}_j[\text{Ampl}] = c_j$