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The Physics of Particle Accelerators

an introduction

RF Systems for Particle Accelerators

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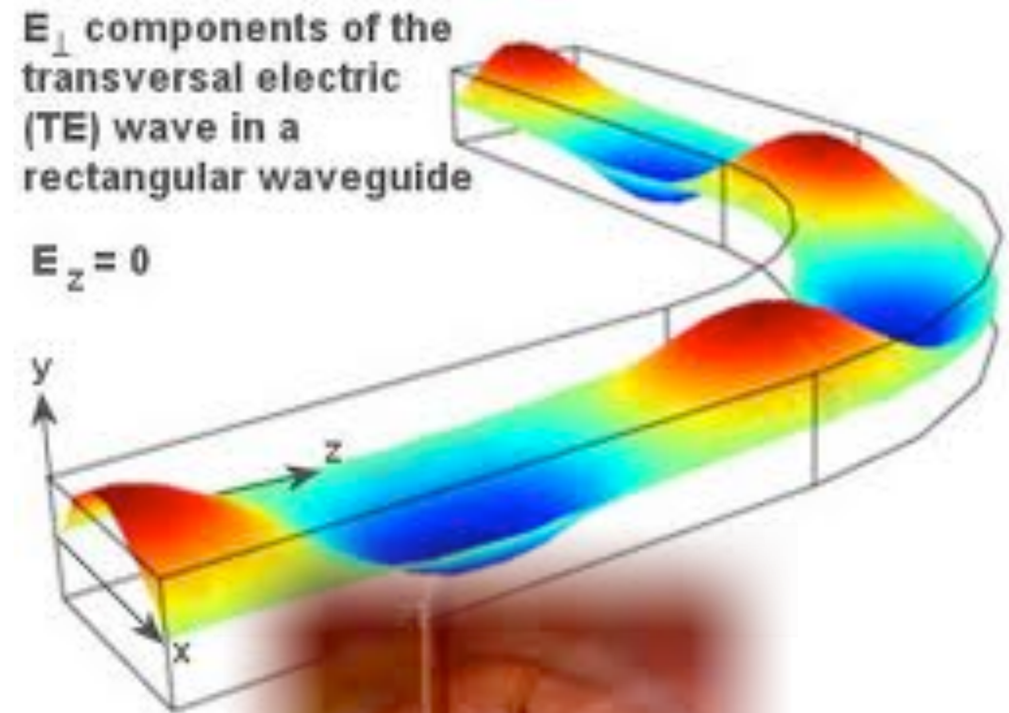
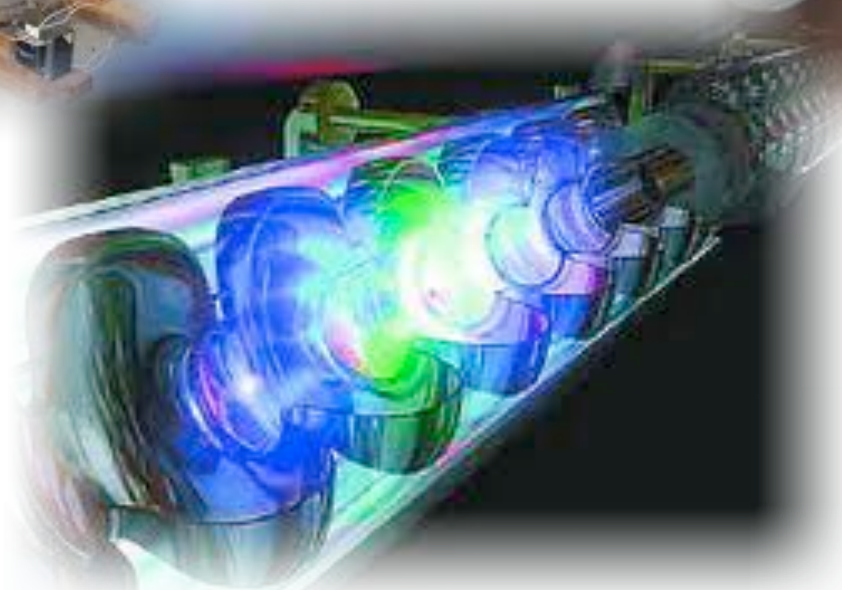
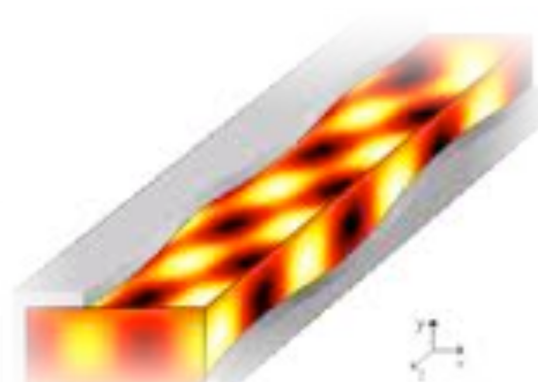
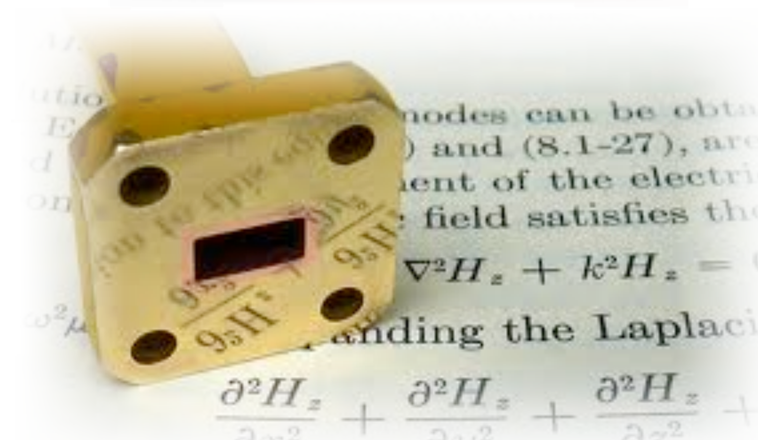
In this lecture...

RF Systems for particle accelerators

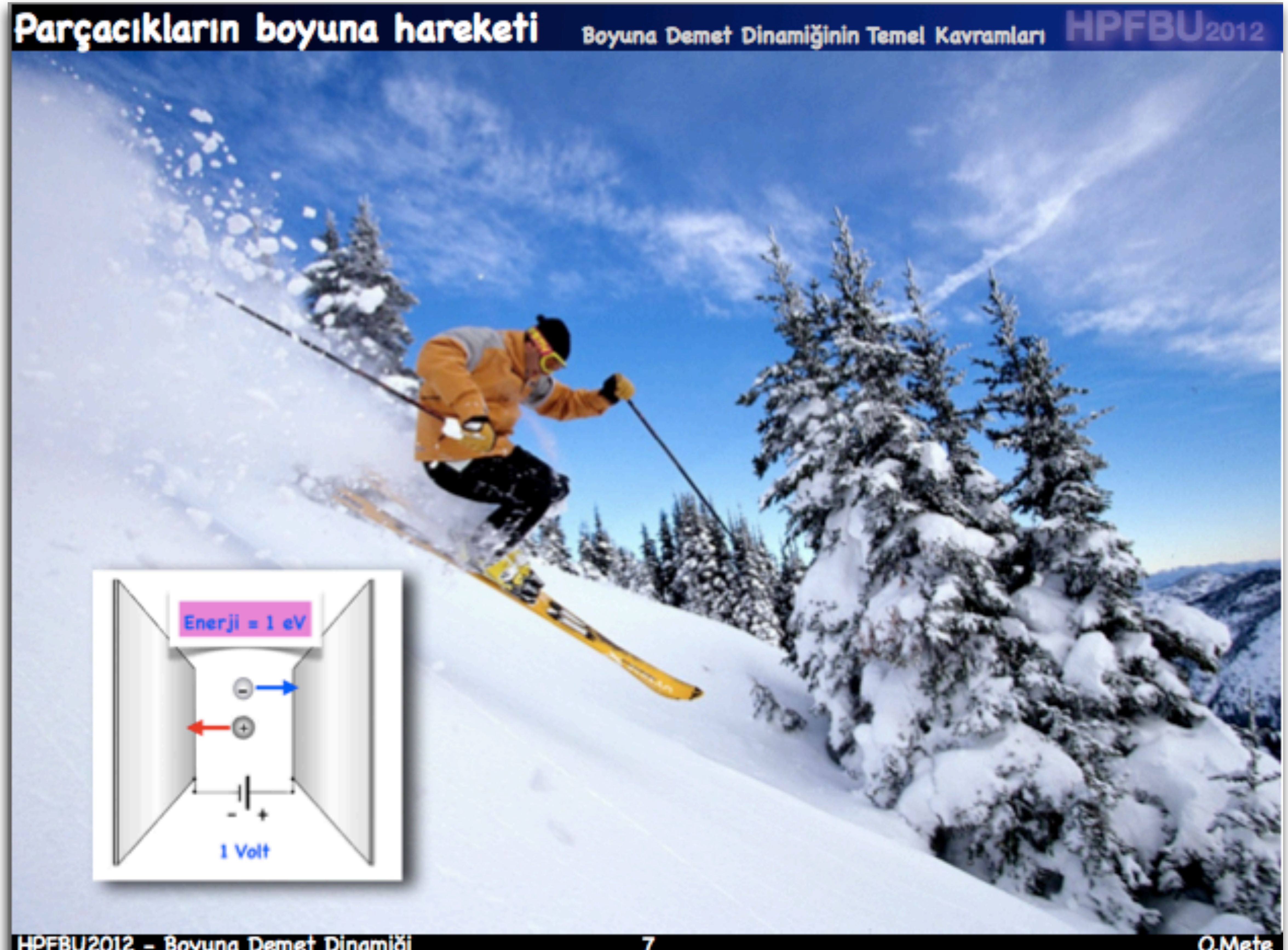








- Static acceleration: fundamental limitation to the final energy.
- High frequency to achieve high voltages.
- Modern accelerators use powerful radio-frequency systems (from MHz to GHz).


Parçacıkların boyuna hareketi
Boyuna Demet Dinamiğinin Temel Kavramları
HPFBU2012
Enerji = 1 eV
1 Volt
HPFBU2012 – Boyuna Demet Dinamiği
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O.Mete

Parçacıkların boyuna hareketi

- Parçacıkların hızlandırıcı alanın şiddeti, dolayısıyla kazanacakları enerji, **"kırılma (breakdown)"** denen bir olgu ile sınırlıdır.
- Kırılma sınırı düşünüldüğünde, daha yüksek gerilimlere durgun elektrik alanı çok kısa atmalar halinde uygulayarak ulaşılabilir.
  - Bu şekilde, kırılma sınırının altında, elde edilen en yüksek gerilim 10 MV mertebesindedir.
- Daha da yüksek gerilimler elde etmek istersek, o zaman yüksek frekanslı elektromagnetik alanlar kullanmalıyız[1,2,3].
  - İçlerinde bu şekilde alanlar uyarılabilecek özel RF kovukları tasarlamalıyız.
  - Parçacıklar RF kovuklarının içinden birkez ya da kazanmaları istenilen enerjiye göre birçok kez geçerler.
  - Yüksek frekanslı RF normal iletken kovuklar kullanılarak, kırılma noktasının altında elde edilebilen en yüksek elektrik alan **100 MV/m** 'dir. Bu değere ulaşabilen kovuklar **CERN'de CLIC projesi** dahilinde üretimekte, SLAC ve KEK işbirliğinde test eilmektedir.

[1] G. Ising
[2] R. Wideroe
[3] M.S. Livingstone

In the upcoming lectures:

- TEM, Transverse Electromagnetic wave mode: Electric and magnetic field components are perpendicular to each other, and both are transverse to the direction of propagation. No longitudinal electric field component.

- TM, Transverse Magnetic wave mode: Longitudinal electric field component.

- TE, Transverse Electric wave mode: Longitudinal magnetic field component.

- Both TE and T.M. modes have a characteristic cut-off frequencies. Waves of frequencies below the cut-off frequency can not propagate; power and signal transmission is only possible for frequencies above cut-off frequencies.

- Therefore, waveguides operating in TE of T.M. modes can be seen as high-pass filters.

In the upcoming lectures:

- Parallel-plate waveguides for TE and T.M. modes,
  - All transverse field components can be expressed in terms of Ez for T.M. modes,
  - and in terms of Hz for TE mode.
- Attenuation constant due to imperfect conducting walls for T.M. and TE modes,
  - It depends on the mode and the frequency; for some modes attenuation can increase with frequency, for some other modes attenuation can reach a minimum as the frequency exceed the cut-off freq. by a some amount.
- Hollow metal pipes of arbitrary cross section,
  - Field, current, charge distribution, propagation and attenuation characteristics of rectangular and cylindrical waveguides (for T.M. and TE modes, no support for TEM).
- Propagation is possible by an open dielectric-slab waveguide. Fields are confined within the dielectric region and rapidly decays away from the slab surface. Therefore the the waves supported by the dielectric-slab waveguides are called the surface waves. (We will not study this option on detail.)
- Hollow conducting box with proper dimensions can be a resonance device. Box provides large areas for current flow. Such a box, which is a segment of a waveguide is called a cavity resonator.
  - Different mode patterns of the fields in the rectangular and cylindrical cavity resonators will be studied.

Assumptions :

> Waves propagate in the +z-direction,

> $\gamma = \alpha + i\beta$ propagation constant (to be determined),

> For an harmonic dependence with an angular frequency ω,

the dependence on z and t for all field components can be described by the exponential factor,

$$e^{-\gamma z} e^{j\omega t} = e^{(j\omega t - \gamma z)} = e^{-\alpha z} e^{j(\omega t - \beta z)}$$

> For cosine reference instantaneous $\vec{E}$ field in Cartesian coordinates,

$$\vec{E}(x,y,z;t) = \mathrm{Re}\left[\vec{E}^{\circ}(x,y)\, e^{(j\omega t - \gamma z)}\right]$$

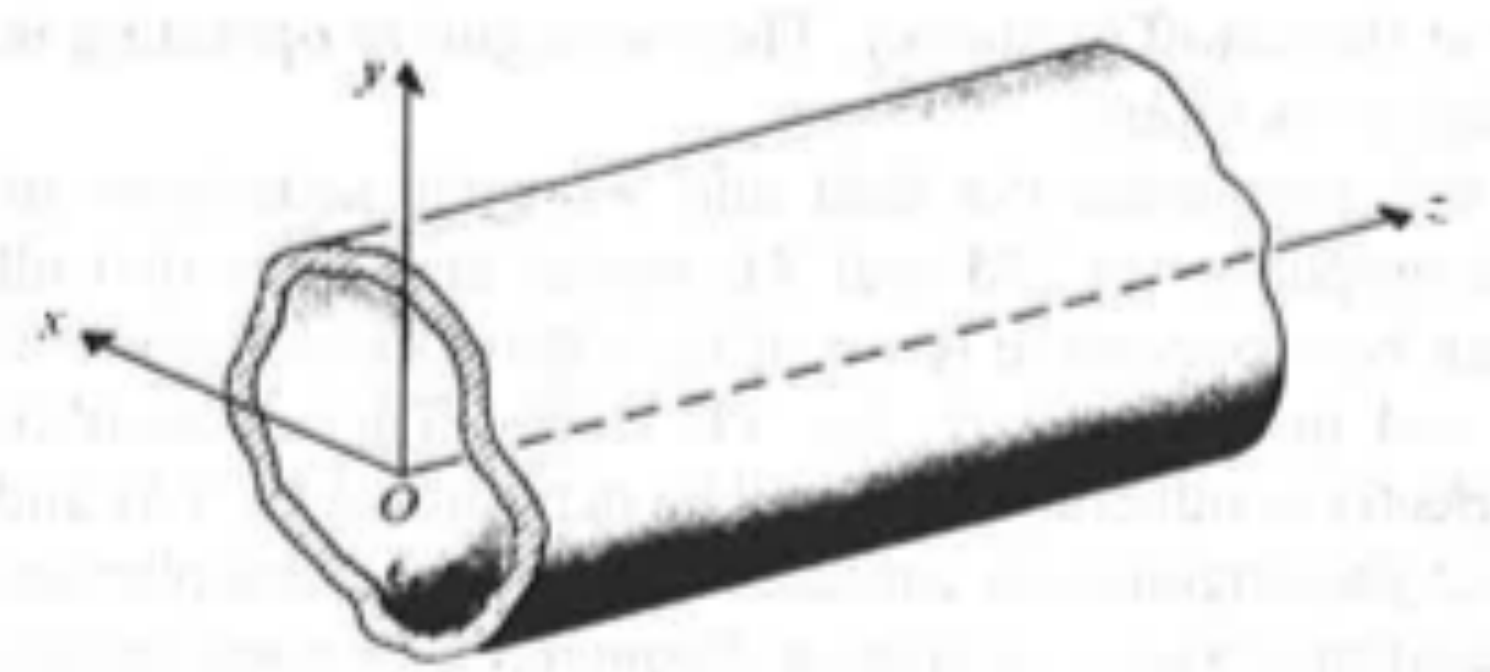
> Similarly for $\vec{H}$,

$$\vec{H}(x,y,z;t) = \mathrm{Re}\left[\vec{H}^{\circ}(x,y)\, e^{(j\omega t - \gamma z)}\right]$$

> Here $\vec{E}^{\circ}$ and $\vec{H}^{\circ}$ are phasors only depending on the ~~trans~~ cross sectional coordinates.

> Using a phasor representation in equations relating field quantities, replace partial derivatives with respect to t and z with $(j\omega)$ and $(-\gamma)$.

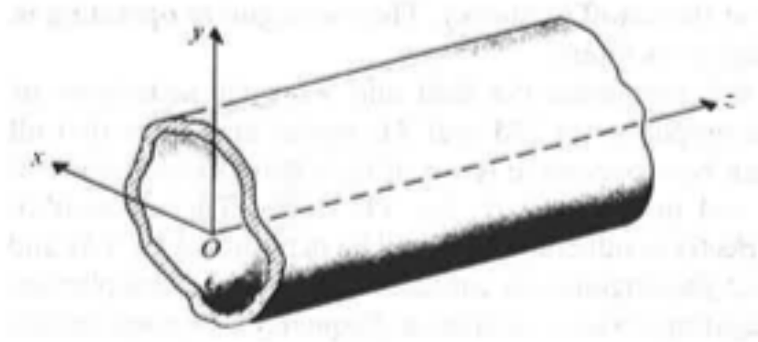
> Consider a straight [guide] in the form of a dielectric-filled metal tube with an arbitrary cross section, lying along z-axis.



> Electric and magnetic field intensities in the charge-free dielectric region satisfy the following homogeneous vector Helmholtz's equations:

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 H + k^2 H = 0$$



> E, H : three dimentional vector phasors, k: wavenumber.

$$k = \omega\sqrt{\mu\epsilon}$$

> Three dimentional Laplacian operator, ∇^2, may be broken into two parts:

$$\nabla^2 \longrightarrow \nabla^2_{u_1 u_2} \, , \, \nabla^2_z$$

> For waveguides with rectangular cross section we use Cartesian coordinates.

$$\nabla^2 E = \left(\nabla_{xy}^2 + \nabla_z^2\right) E = \left(\nabla_{xy}^2 + \frac{\partial^2}{\partial z^2}\right) E$$

$$= \nabla_{xy}^2 E + \gamma^2 E$$

$$= -k^2 E \qquad \text{(from Helmholtz's equation)}$$

$$\boxed{\begin{aligned} &\nabla_{xy}^2 E + (\gamma^2 + k^2) E = 0 \\ &\nabla_{xy}^2 H + (\gamma^2 + k^2) H = 0 \end{aligned}}$$

Governing equations for waveguides with rectangular crosssection.

> These equations are three second-order partial differential equations (one for each component of E and H),

> The exact solutions of ~~thesecomp~~ these component equations depend on

- The cross sectional geometry,
- Boundary conditions that a particular field component must satisfy at conductor-dielectric interfaces.

> Note: By writing $\nabla^2_{r\phi}$ for the transversal operator ∇^2_{xy}, the equations become the governing equations for waveguides with a circular cross section.

> Various components of $\vec{E}$ and $\vec{H}$ are not all independent.

> So it is not necessary to solve all six second-order partial differential equations for six components of $\vec{E}$ and $\vec{H}$.

> We can find interrelationships among six components in Cartesian coordinates by expanding the two source-free curl equations,

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

Note: Maxwell equations governing electromagnetic phenomena in source-free media:

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

$$\nabla \times H - \frac{\partial D}{\partial t} = 0$$

For external sources in vacuum,

$$D = \epsilon_0 E$$

$$B = \mu_0 H$$

In dielectric media: $\epsilon_0 \rightarrow \epsilon_r \epsilon_0 = \epsilon$

$\mu_0 \rightarrow \mu_r \mu_0 = \mu$

In phasor representation $\frac{\partial}{\partial t} \longrightarrow (j\omega)$

From $\nabla \times E = -j\omega\mu H$

$$\frac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0 \quad (1a)$$

$$-\gamma E_x^0 - \frac{\partial E_z^0}{\partial x} = -j\omega\mu H_y^0 \quad (1b)$$

$$\frac{\partial E_y^0}{\partial x} - \frac{\partial E_x^0}{\partial y} = -j\omega\mu H_z^0 \quad (1c)$$

$$\nabla \times \mathbf{A} = \mathbf{e}_1\left(\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}\right) + \mathbf{e}_2\left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right) + \mathbf{e}_3\left(\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}\right)$$

from $\nabla \times H = j\omega\epsilon E$

$$\frac{\partial H_z^o}{\partial y} + \gamma H_y^o = j\omega\epsilon E_x^o \qquad (2a)$$

$$-\gamma H_x^o - \frac{\partial H_z^o}{\partial x} = j\omega\epsilon E_y^o \qquad (2b)$$

$$\frac{\partial H_y^o}{\partial x} - \frac{\partial H_x^o}{\partial y} = j\omega\epsilon E_z^o \qquad (2c)$$

$$\nabla \times \mathbf{A} = \mathbf{e}_1\left(\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}\right) + \mathbf{e}_2\left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right) + \mathbf{e}_3\left(\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}\right)$$

Note:

> partial derivatives with respect to $z \rightarrow (-\gamma)$,

> all the component field quantities in the equations are phasors that depend on x and y,

> the common $e^{-\gamma z}$ factor for z dependence having been omitted.

> In the equations $h^2 = \gamma^2 + k^2$.

Example : Take 1a and 2b, eliminate E_y° and obtain H_x°

$$\frac{\partial E_z^\circ}{\partial y} + \gamma E_y^\circ = -j\omega\mu H_x^\circ$$

$$-\gamma H_x^\circ - \frac{\partial H_z^\circ}{\partial x} = j\omega\epsilon E_y^\circ$$

$$-\gamma H_x = j\omega\epsilon E_y^\circ + \frac{\partial H_z^\circ}{\partial x}$$

$$E_y = \left(-j\omega\mu H_x - \frac{\partial E_z^\circ}{\partial y}\right)\frac{1}{\gamma}$$

$$-\gamma H_x = \frac{j\omega\epsilon}{\gamma}\left(-j\omega\mu H_x - \frac{\partial E_z}{\partial y}\right) + \frac{\partial H_z}{\partial x}$$

$$-\gamma H_x = \frac{\omega^2\epsilon\mu}{\gamma} H_x - \frac{j\omega\epsilon}{\gamma}\frac{\partial E_z}{\partial y} + \frac{\partial H_z}{\partial x}$$

$$H_x\left(-\gamma - \frac{\omega^2\epsilon\mu}{\gamma}\right) = -\frac{j\omega\epsilon}{\gamma}\frac{\partial E_z}{\partial y} + \frac{\partial H_z}{\partial x}$$

$$H_x\underbrace{\left(\frac{-\gamma^2 - \omega^2\epsilon\mu}{\cancel{\gamma}}\right)}_{-h^2} = -j\omega\epsilon\frac{\partial E_z}{\partial y} + \gamma\frac{\partial H_z}{\partial x}$$

$$-\gamma^2 - \omega^2\epsilon\mu$$

$$k = \omega\sqrt{\mu\epsilon}$$

$$k^2 = \omega^2\mu\epsilon$$

$$H_x^0 = -\frac{1}{k^2}\left(\gamma \frac{\partial H_z^0}{\partial x} - j\omega\epsilon \frac{\partial E_z^0}{\partial y}\right),$$

$$H_y^0 = -\frac{1}{k^2}\left(\gamma \frac{\partial H_z^0}{\partial y} + j\omega\epsilon \frac{\partial E_z^0}{\partial x}\right),$$

$$E_x^0 = -\frac{1}{k^2}\left(\gamma \frac{\partial E_z^0}{\partial x} + j\omega\mu \frac{\partial H_z^0}{\partial y}\right),$$

$$E_y^0 = -\frac{1}{k^2}\left(\gamma \frac{\partial E_z^0}{\partial y} - j\omega\mu \frac{\partial H_z^0}{\partial x}\right).$$

Conclusions:

> The wave behaviour in a waveguide can be analysed by ~~using the equations~~ solving the equations for the longitudinal components, E_z^0 and H_z^0,

$$\nabla_{xy}^2 E + (\gamma^2 + k^2) E = 0$$

$$\nabla_{xy}^2 H + (\gamma^2 + k^2) H = 0$$

> Then equations 1(a,b,c) and 2(a,b,c) can be solved to determine the other components.

Conclusions:

> It is convenient to classify the propagating waves in a uniform waveguide into three types according to whether E_z or H_z exists.

- Transverse electromagnetic (TEM) waves:
  Neither E_z nor H_z.
- Transverse magnetic (TM) waves:
  Nonzero E_z but $H_z = 0$.
- Transverse electric (TE) waves:
  Nonzero H_z but $E_z = 0$.

Next lecture :

> TEM, TM, TE waves, propagation characteristics.

> Some exercises.

Vector Formulas

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

$$(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = (\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d}) - (\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})$$

$$\nabla\times\nabla\psi = 0$$

$$\nabla\cdot(\nabla\times\mathbf{a}) = 0$$

$$\nabla\times(\nabla\times\mathbf{a}) = \nabla(\nabla\cdot\mathbf{a}) - \nabla^2\mathbf{a}$$

$$\nabla\cdot(\psi\mathbf{a}) = \mathbf{a}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{a}$$

$$\nabla\times(\psi\mathbf{a}) = \nabla\psi\times\mathbf{a} + \psi\nabla\times\mathbf{a}$$

$$\nabla(\mathbf{a}\cdot\mathbf{b}) = (\mathbf{a}\cdot\nabla)\mathbf{b} + (\mathbf{b}\cdot\nabla)\mathbf{a} + \mathbf{a}\times(\nabla\times\mathbf{b}) + \mathbf{b}\times(\nabla\times\mathbf{a})$$

$$\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\nabla\times\mathbf{a}) - \mathbf{a}\cdot(\nabla\times\mathbf{b})$$

$$\nabla\times(\mathbf{a}\times\mathbf{b}) = \mathbf{a}(\nabla\cdot\mathbf{b}) - \mathbf{b}(\nabla\cdot\mathbf{a}) + (\mathbf{b}\cdot\nabla)\mathbf{a} - (\mathbf{a}\cdot\nabla)\mathbf{b}$$

If $\mathbf{x}$ is the coordinate of a point with respect to some origin, with magnitude $r = |\mathbf{x}|$, $\mathbf{n} = \mathbf{x}/r$ is a unit radial vector, and $f(r)$ is a well-behaved function of r, then

$$\nabla\cdot\mathbf{x} = 3 \qquad \nabla\times\mathbf{x} = 0$$

$$\nabla\cdot[\mathbf{n}f(r)] = \frac{2}{r}f + \frac{\partial f}{\partial r} \qquad \nabla\times[\mathbf{n}f(r)] = 0$$

$$(\mathbf{a}\cdot\nabla)\mathbf{n}f(r) = \frac{f(r)}{r}[\mathbf{a} - \mathbf{n}(\mathbf{a}\cdot\mathbf{n})] + \mathbf{n}(\mathbf{a}\cdot\mathbf{n})\frac{\partial f}{\partial r}$$

$$\nabla(\mathbf{x}\cdot\mathbf{a}) = \mathbf{a} + \mathbf{x}(\nabla\cdot\mathbf{a}) + i(\mathbf{L}\times\mathbf{a})$$

where $\mathbf{L} = \frac{1}{i}(\mathbf{x}\times\nabla)$ is the angular-momentum operator.

Theorems from Vector Calculus

In the following ϕ, ψ, and $\mathbf{A}$ are well-behaved scalar or vector functions, V is a three-dimensional volume with volume element d^3x, S is a closed two-dimensional surface bounding V, with area element da and unit outward normal $\mathbf{n}$ at da.

$$\int_V \nabla \cdot \mathbf{A}\, d^3x = \int_S \mathbf{A} \cdot \mathbf{n}\, da \qquad \text{(Divergence theorem)}$$

$$\int_V \nabla\psi\, d^3x = \int_S \psi\mathbf{n}\, da$$

$$\int_V \nabla \times \mathbf{A}\, d^3x = \int_S \mathbf{n} \times \mathbf{A}\, da$$

$$\int_V (\phi\nabla^2\psi + \nabla\phi \cdot \nabla\psi)\, d^3x = \int_S \phi\mathbf{n} \cdot \nabla\psi\, da \qquad \text{(Green's first identity)}$$

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, d^3x = \int_S (\phi\nabla\psi - \psi\nabla\phi) \cdot \mathbf{n}\, da \qquad \text{(Green's theorem)}$$

In the following S is an open surface and C is the contour bounding it, with line element $d\mathbf{l}$. The normal $\mathbf{n}$ to S is defined by the right-hand-screw rule in relation to the sense of the line integral around C.

$$\int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n}\, da = \oint_C \mathbf{A} \cdot d\mathbf{l} \qquad \text{(Stokes's theorem)}$$

$$\int_S \mathbf{n} \times \nabla\psi\, da = \oint_C \psi\, d\mathbf{l}$$

Explicit Forms of Vector Operations

Let $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ be orthogonal unit vectors associated with the coordinate directions specified in the headings on the left, and A_1, A_2, A_3 be the corresponding components of $\mathbf{A}$. Then

Cartesian $(x_1, x_2, x_3 = x, y, z)$

$$\nabla\psi = \mathbf{e}_1 \frac{\partial\psi}{\partial x_1} + \mathbf{e}_2 \frac{\partial\psi}{\partial x_2} + \mathbf{e}_3 \frac{\partial\psi}{\partial x_3}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3}$$

$$\nabla\times\mathbf{A} = \mathbf{e}_1\left(\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}\right) + \mathbf{e}_2\left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right) + \mathbf{e}_3\left(\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}\right)$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} + \frac{\partial^2\psi}{\partial x_3^2}$$

Cylindrical (ρ, ϕ, z)

$$\nabla\psi = \mathbf{e}_1 \frac{\partial\psi}{\partial\rho} + \mathbf{e}_2 \frac{1}{\rho}\frac{\partial\psi}{\partial\phi} + \mathbf{e}_3 \frac{\partial\psi}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_1) + \frac{1}{\rho}\frac{\partial A_2}{\partial\phi} + \frac{\partial A_3}{\partial z}$$

$$\nabla\times\mathbf{A} = \mathbf{e}_1\left(\frac{1}{\rho}\frac{\partial A_3}{\partial\phi} - \frac{\partial A_2}{\partial z}\right) + \mathbf{e}_2\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial\rho}\right) + \mathbf{e}_3\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_2) - \frac{\partial A_1}{\partial\phi}\right)$$

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$$

Spherical (r, θ, ϕ)

$$\nabla\psi = \mathbf{e}_1 \frac{\partial\psi}{\partial r} + \mathbf{e}_2 \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \mathbf{e}_3 \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_1) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_2) + \frac{1}{r\sin\theta}\frac{\partial A_3}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \mathbf{e}_1\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_3) - \frac{\partial A_2}{\partial\phi}\right] + \mathbf{e}_2\left[\frac{1}{r\sin\theta}\frac{\partial A_1}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_3)\right] + \mathbf{e}_3\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_2) - \frac{\partial A_1}{\partial\theta}\right]$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\left[\text{Note that } \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi).\right]$$

Further Reading

- The physics of particle accelerators, Klaus Wille, Chapter 5,
- Field and Wave Electromagnetics, David K. Cheng, Chapter 10,
- Classical Electrodynamics, J. D. Jackson, Chapter 8.