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Basic operation of cryocoolers
and related thermal machines

Review article

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Wikipedia

program

1. thermodynamics

2. cryocoolers (AC)
   - Stirling coolers
   - GM coolers
   - pulse tube refrigerators

3. cryocoolers (DC)
   - JT coolers

4. modeling/thermoacoustics

5. dilution refrigerators

6. outlook

NOTATION

* (as in $\overset{*}{n}, \overset{*}{m}, \overset{*}{H}, \overset{*}{S}, \overset{*}{V}$) indicates a flow (e.g. $\overset{*}{H}$ is enthalpy flow in J/s)

over dot (as in $\dot{n}, \dot{m}, \dot{H}, \dot{S}, \dot{V}$) indicates a time derivative (e.g. $\dot{H} \overset{\mathrm{def}}{=} \mathrm{d}H/\mathrm{d}t$ is the rate of increase of the enthalpy of a system in J/s)
exceptions: heat flow ($\dot{Q}$) and entropy production ($\dot{S}_{\mathrm{i}}$)

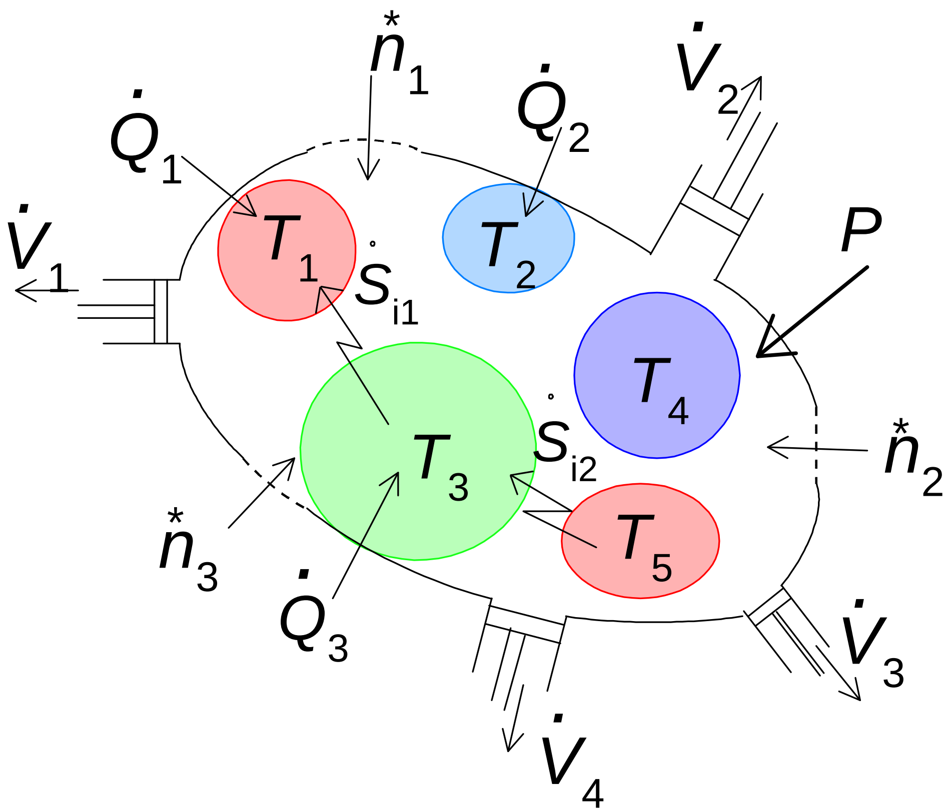
lower index m (as in $H_{\mathrm{m}}, S_{\mathrm{m}}, V_{\mathrm{m}}$) indicates molar quantities (e.g. H_{m} is the *molar* enthalpy in J/mol);
exeptions: $R = 8.31\,\mathrm{J}/(\mathrm{mol\,K})$ is *molar* ideal-gas constant, $C_{\mathrm{p}}, C_{\mathrm{V}}$

lower case characters (as h, s, c_{p}) are specific quantities (e.g. h is the *specific* enthalpy in J/kg)

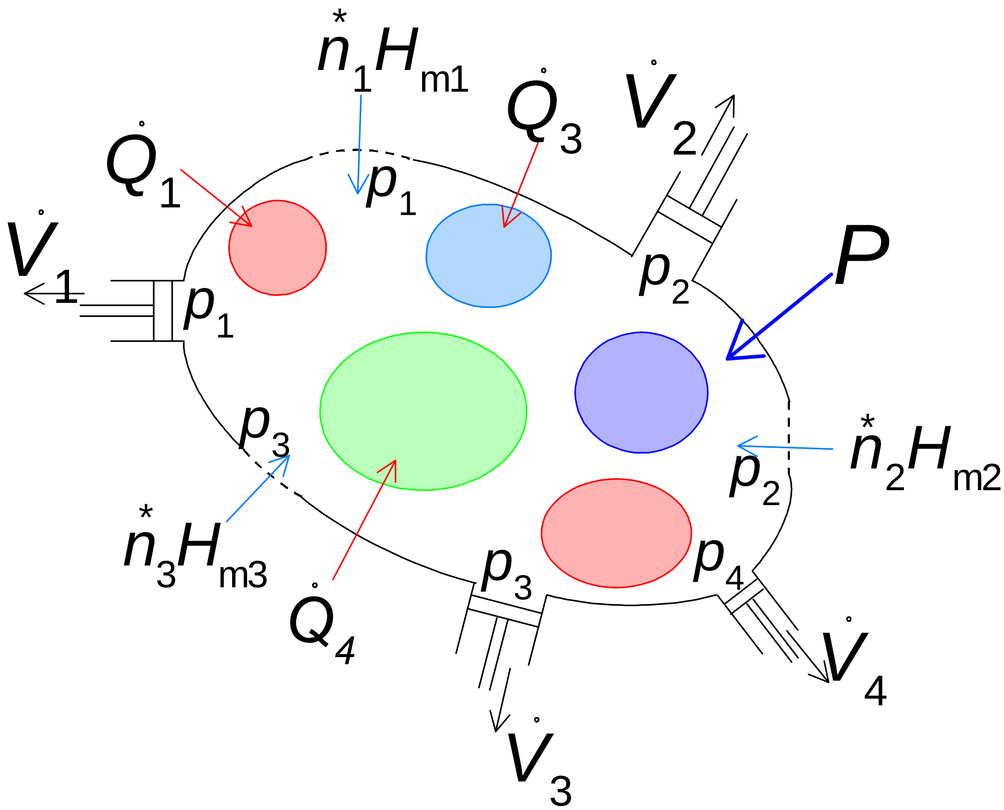
over bar (as in $\overline{\overset{*}{H}}$) indicates time average (some authors use $\left\langle \dot{H} \right\rangle$)

Laws of Thermodynamics

thermodynamic system



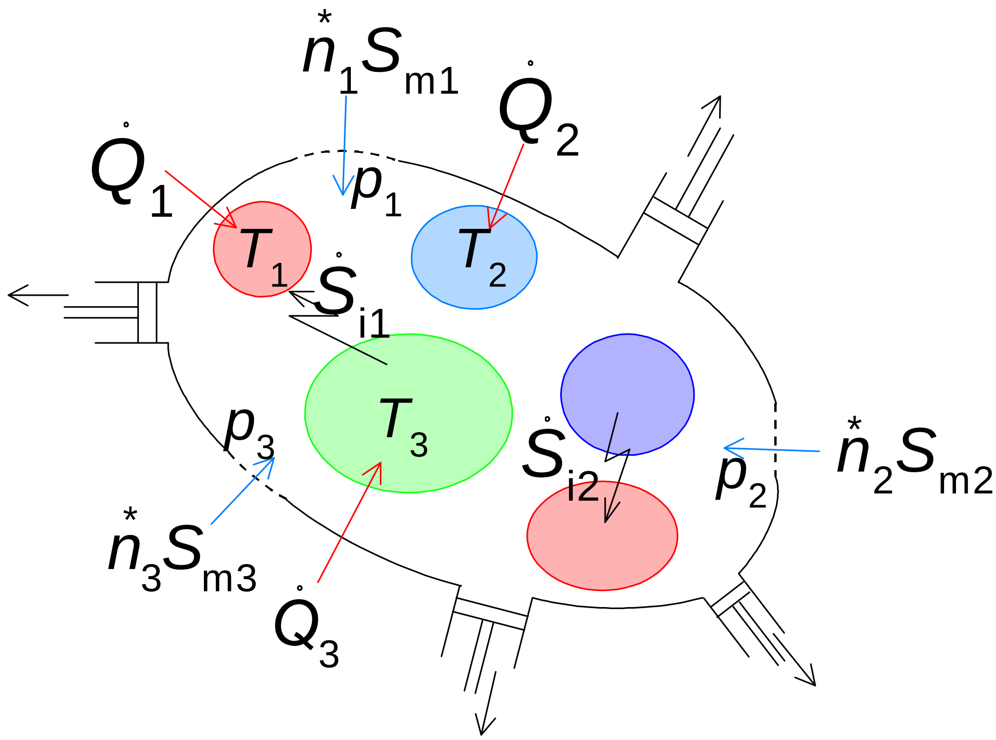
first law



$$\frac{\mathrm{d}U}{\mathrm{d}t} = \sum_k \dot{Q}_k + \sum_k \overset{*}{H}_k - \sum_k p_k \frac{\mathrm{d}V_k}{\mathrm{d}t} + P$$

enthalpy flow $\overset{*}{H}_k = \overset{*}{n}_k H_{\mathrm{m}k} = \overset{*}{m}_k h_k$

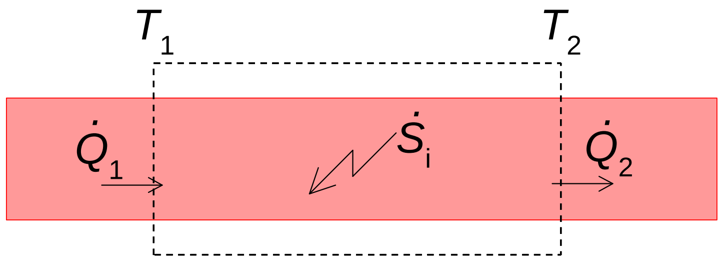
second law



$$\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_k \frac{\dot{Q}_k}{T_k} + \sum_k \overset{*}{S}_k + \sum_k \dot{S}_{\mathrm{i}k} \quad \text{with} \quad \dot{S}_{\mathrm{i}k} \geq 0$$

entropy flow $\overset{*}{S}_k = \overset{*}{n}_k S_{\mathrm{m}k} = \overset{*}{m}_k s_k$

entropy production by heat conduction



first law

$$\dot{Q}_1 = \dot{Q}_2 = \dot{Q}$$

second law

$$0 = \frac{\dot{Q}}{T_1} - \frac{\dot{Q}}{T_2} + \dot{S}_i \text{ with } \dot{S}_i \geq 0$$

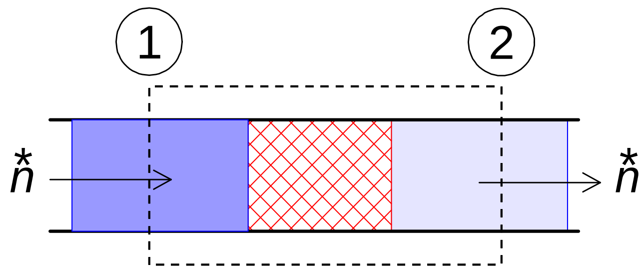
or

$$\dot{S}_i = \left(\frac{1}{T_2} - \frac{1}{T_1}\right)\dot{Q} = \frac{T_1 - T_2}{T_1 T_2}\dot{Q} \geq 0$$

if $\dot{Q} = \kappa \frac{A}{L}(T_1 - T_2)$ then

$$\dot{S}_i = \kappa \frac{A}{L}\frac{(T_1 - T_2)^2}{T_1 T_2}$$

entropy production by throttling (Joule-Thomson expansion)



first law

$$0 = \overset{*}{n} H_{\mathsf{m1}} - \overset{*}{n} H_{\mathsf{m2}}$$

so

$$H_{\mathsf{m1}} = H_{\mathsf{m2}} \text{ or } h_1 = h_2$$

ideal gas $H_{\mathsf{m}} = C_{\mathsf{p}} T$ so $T_1 = T_2$

second law

$$0 = \overset{*}{n} S_{\mathsf{m1}} - \overset{*}{n} S_{\mathsf{m2}} + \dot{S}_{\mathsf{i}}$$

so

$$\dot{S}_{\mathsf{i}} = \overset{*}{n} \left(S_{\mathsf{m2}} - S_{\mathsf{m1}} \right) \geq 0$$

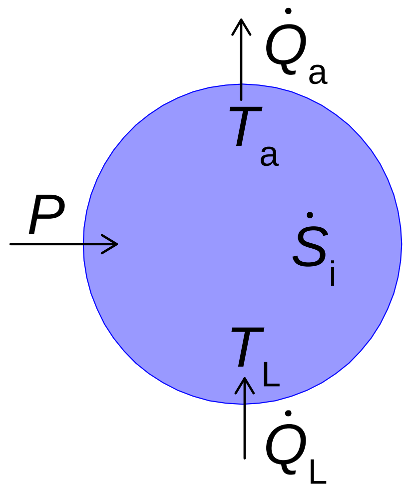
with

$$\mathsf{d}H_\mathsf{m} = T\mathsf{d}S_\mathsf{m} + V_\mathsf{m}\mathsf{d}p$$

small pressure drop with $\mathsf{d}H_\mathsf{m} = 0$ gives

$$\dot{S}_\mathsf{i} = \frac{\overset{*}{\dot{V}}\delta p}{T}$$

refrigerators



first law

$$\frac{dU}{dt} = \sum_k \dot{Q}_k + \sum_k \overset{*}{H}_k - \sum_k p_k \frac{dV_k}{dt} + P$$

reduces to

$$0 = \dot{Q}_L - \dot{Q}_a + P$$

second law

$$\frac{dS}{dt} = \sum_k \frac{\dot{Q}_k}{T_k} + \sum_k \overset{*}{S}_k + \sum_k \dot{S}_{ik} \text{ with } \dot{S}_{ik} \geq 0$$

reduces to

$$0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_a}{T_a} + \dot{S}_i \text{ with } \dot{S}_i \geq 0$$

or

$$\dot{S}_{\mathsf{i}} = \frac{\dot{Q}_{\mathsf{a}}}{T_{\mathsf{a}}} - \frac{\dot{Q}_{\mathsf{L}}}{T_{\mathsf{L}}}$$

with $\dot{Q}_{\mathsf{a}} = P + \dot{Q}_{\mathsf{L}}$

$$\dot{S}_{\mathsf{i}} = \frac{P + \dot{Q}_{\mathsf{L}}}{T_{\mathsf{a}}} - \frac{\dot{Q}_{\mathsf{L}}}{T_{\mathsf{L}}}$$

or

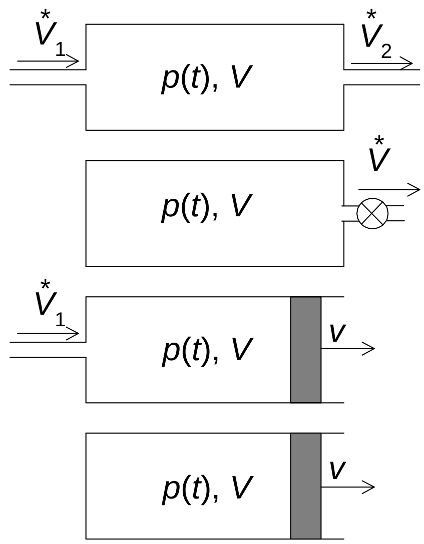
$$P = \frac{T_{\mathsf{a}} - T_{\mathsf{L}}}{T_{\mathsf{L}}}\dot{Q}_{\mathsf{L}} + T_{\mathsf{a}}\dot{S}_{\mathsf{i}}$$

$T_{\mathsf{a}}\dot{S}_{\mathsf{i}}$ is the dissipated power

coefficient of performance (*COP*)

$$\xi = \frac{\dot{Q}_{\mathsf{L}}}{P} \leq \frac{T_{\mathsf{L}}}{T_{\mathsf{a}} - T_{\mathsf{L}}} = \xi_{\mathsf{C}}$$

the volume-flow equation



$$\overset{*}{V}_{\mathbf{1}} = \overset{*}{V}_{\mathbf{2}} + \frac{V}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} \text{ pulse tube}$$

$$0 = \overset{*}{V} + \frac{V}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} \text{ buffer volume}$$

$$\overset{*}{V}_{\mathbf{1}} = Av + \frac{V}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} \text{ compressor}$$

$$0 = \frac{\mathrm{d}V}{\mathrm{d}t} + \frac{V}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} \Longrightarrow pV^{\gamma} = \text{ constant, Poisson?}$$

derivation of the volume-flow equation

internal energy of a gas in a volume V

$$U = \int_V \frac{U_\mathsf{m}}{V_\mathsf{m}} \mathsf{d}V$$

ideal gas: $U_\mathsf{m} = C_\mathsf{V} T$ and $V_\mathsf{m} = RT/p$ so

$$U = \frac{C_\mathsf{V}}{R} \int_V p \mathsf{d}V$$

if p is homogeneous

$$U = \frac{C_\mathsf{V}}{R} pV$$

this relation holds even if T is not homogeneous

with $\overset{*}{H}_k = \overset{*}{n}_k C_\mathsf{p} T_k$ and $\overset{*}{n}_k = p\overset{*}{V}_k / RT_k$ we get

$$\overset{*}{H}_k = \frac{C_\mathsf{p}}{R} p \overset{*}{V}_k$$

the system is adiabatic and $P = 0$ so the first law gives

$$\frac{\mathsf{d}U}{\mathsf{d}t} = \sum \overset{*}{H}_k - p \sum \frac{\mathsf{d}V_k}{\mathsf{d}t}$$

so

$$\frac{C_\mathsf{V}}{R} V \frac{\mathsf{d}p}{\mathsf{d}t} + \frac{C_\mathsf{V}}{R} p \frac{\mathsf{d}V}{\mathsf{d}t} = \frac{C_\mathsf{p}}{R} p \sum \overset{*}{V}_k - p \sum \frac{\mathsf{d}V_k}{\mathsf{d}t}$$

with

$$\frac{\mathsf{d}V}{\mathsf{d}t} = \sum \frac{\mathsf{d}V_k}{\mathsf{d}t}$$

and $C_\mathsf{V} + R = C_\mathsf{p}$

$$\frac{C_\mathsf{V}}{R} V \frac{\mathsf{d}p}{\mathsf{d}t} = \frac{C_\mathsf{p}}{R} p \sum \overset{*}{V}_k - \frac{C_\mathsf{p}}{R} p \sum \frac{\mathsf{d}V_k}{\mathsf{d}t}$$

with $\gamma = C_\mathsf{p}/C_\mathsf{V}$ we get our final result

$$\frac{V}{\gamma p} \frac{\mathsf{d}p}{\mathsf{d}t} = \sum \overset{*}{V}_k - \sum \frac{\mathsf{d}V_k}{\mathsf{d}t}$$

equation holds for an adiabatic, ideal gas with homogeneous pressure, but the temperature can be inhomogeneous!