The influence of the Al stabilizer layer thickness on the normal zone propagation velocity in high current superconductors

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Introduction

Normal Zone Propagation Velocity

A main issue in designing quench protection systems is the resistance growth rate in a superconducting magnet after a quench.

It can be determined by studying the dynamics of the normal zone, the region where the superconductor is in normal state.

The normal zone propagation in adiabatic conditions can be described by two parameters: the longitudinal velocity v_l, and the transverse velocity v_t.

v_l is easier to measure in an experiment and can be computed either numerically by some analytical formulae available in the literature.

As v_t can be linearly approximated using v_l, a satisfying description of the normal zone propagation can be obtained by knowing only v_l.

Introduction
A novel calculation

- A recent and new numerical calculation of v_l in NbTi/Cu Rutherford cables surrounded by a normal metal cladding

- The code solves a set of two coupled differential equations, the heat balance equation and the magnetic diffusion equation
- The model accounts for the current sharing process between super-conductor and stabilizer, and for the heat propagation over time and space.
- The material properties input is given as functions of both T and B
- The influence of the thickness of the cladding on v_l for varying magnetic field and operating current is studied as well
- To complete our analysis, we introduce an analytical formula to calculate v_l, following a previous idea by Mints et al. (R.G. Mints,T. Ogitsu and A. Devred,Cryogenics, **3,** 449, 1992)

Computational Model

Introduction

- Heat balance equation utilizing Ampere's law

$$\varrho C(T)\frac{\partial T}{\partial t} - \vec{\nabla}\cdot(k(T)\vec{\nabla}T) = \frac{\rho(T)}{\mu_0}\left|\vec{\nabla}\times\vec{B}(\vec{r},t)\right|^2 + q_{ext}(\vec{r},t)$$

- Magnetic diffusion

$$\vec{\nabla}\times\left(\rho(\vec{r},t)\vec{\nabla}\times\vec{B}(\vec{r},t)\right) + \mu_0\frac{\partial\vec{B}(\vec{r},t)}{\partial t} = 0$$

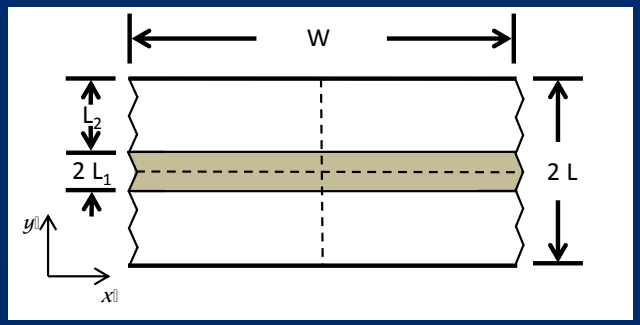
- The material properties are given as functions of both T and B. The different fits cover a magnetic field range of 0-5 T for a temperature range of 0-300 K
- We assume the Rutherford cable is made of a single, homogenous, material and average ρ, C and k over the cross-section of the cable
- The effective electrical resistivity is obtained by viewing the superconductor-copper system as two resistors connected in parallel

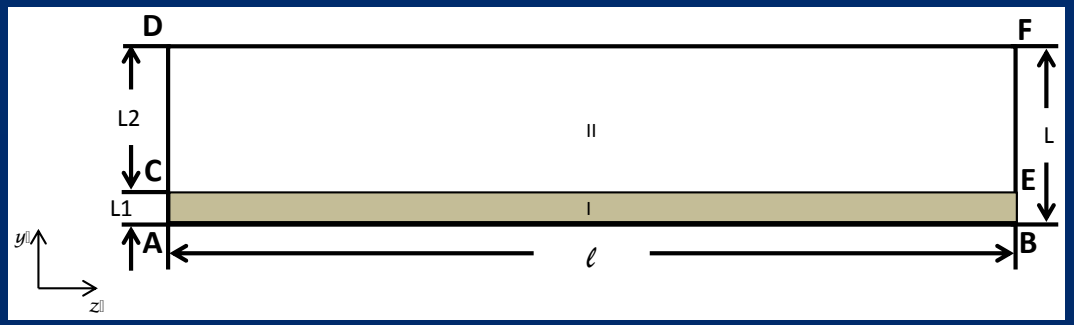
Computational Model

Model Geometry

$$\tau_H = \frac{L_{nzf}^2}{D_H} = \frac{k}{\varrho C v^2} \qquad \tau_M = \frac{\mu_0 L_2^2}{\rho_{st}}$$

- The current redistribution process can be considered as instantaneous only if the dimensionless parameter $\alpha = \tau_H / \tau_M$ is greater than one
- A sufficient understanding of the problem requires a 2D model
- We assume that the length scale in the x direction is infinitely large and the reduction to an equivalent 2D problem is thus done by considering a slice of the conductor on the yz plane.

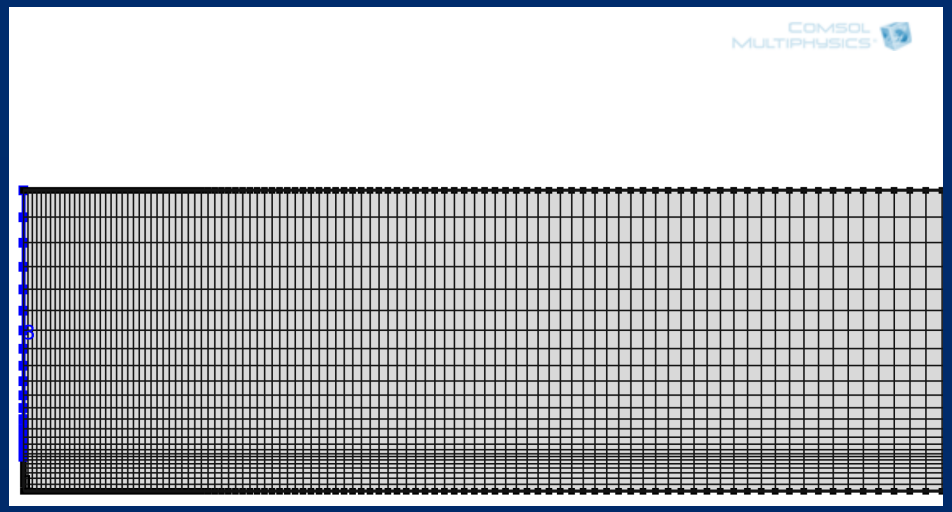




Computational Model

Mesh

- The mesh uses rectangular surface elements
- The elements are constructed so that they are finer in the vicinity of the boundary between the cable and the cladding, at L_1
- In the z direction the elements are getting coarser as z is increasing, so that near the origin, where the initial perturbation takes place and the temperature gradient is high, the mesh is considerably finer
- The model consists of 11000 domain elements and 2022 boundary elements for l = 2 m



Computational Model

Boundary Conditions and Initial Perturbation

- The conductor can be approximated by a set of infinite plates, where the cable is seen as an infinite current carrying sheet with initial total current I_{op}
- On the interface Γ between the composite material and the cladding we demand the continuity of the magnetic field and its flux
- On the external boundaries $B(y=0) = B_{\text{ext}}$, $B(y=L) = B_{\text{ext}} + \mu_0 J_{\text{eng}} L$
- The model assumes full adiabaticity. The initial value condition for the temperature reads $T(\vec{r}, t=0) = T_{op}$ in the bulk of the conductor
- The temperature and its flux are continuous along Γ
- The external energy input is given by $P_{\text{ext}} = P_0 \, \mathrm{e}^{-(z^2+y^2)\cdot\sigma} \mathrm{e}^{-t^2/\tau^2}$

Analytical Approach

Current Redistribution Included

- Due to the strong coupling between the heat and magnetic diffusion equations an exact solution is difficult to achieve.
- For low currents, where the current redistribution can be regarded as instantaneous:

$$v_{\mathrm{wil}} = \frac{I_{op}}{\varrho C A_{\mathrm{cd}}} \sqrt{\frac{\rho k}{T_{cs} - T_{op}}}$$

- To distinguish between the high and low current regimes according to:

$$\alpha = \frac{\tau_H}{\tau_M} = \frac{k \rho_{st}}{\varrho C v^2 \mu_0 L_2^2}$$

- A simple way to approximate this scenario is by considering the joule heating term as resulting from a uniform current flowing solely in a confined area within the conductor $A_{eff} = 2W\left[L_2\left(1 - e^{-\alpha}\right) + L_1\right]$

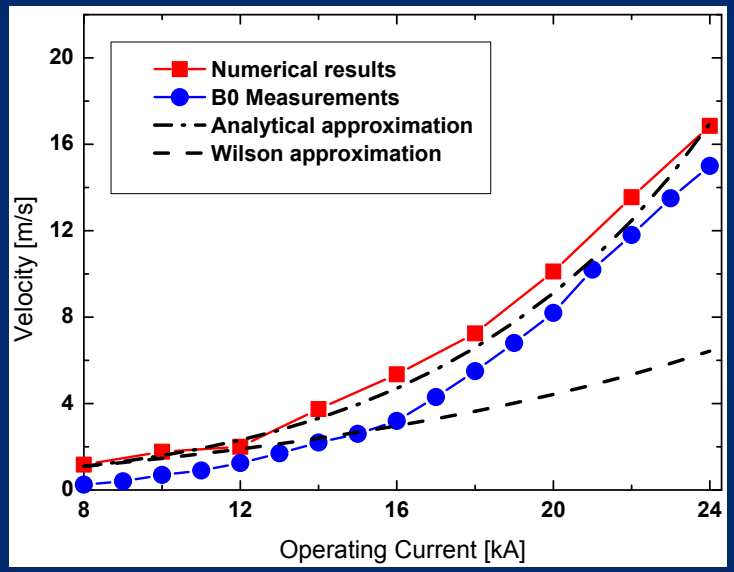
$$v_{MB} = \frac{I_{op}}{\varrho C \sqrt{A_{\mathrm{eff}} A_{\mathrm{cd}}}} \sqrt{\frac{k \rho_{\mathrm{eff}}}{T_{cs} - T_{op}}}$$

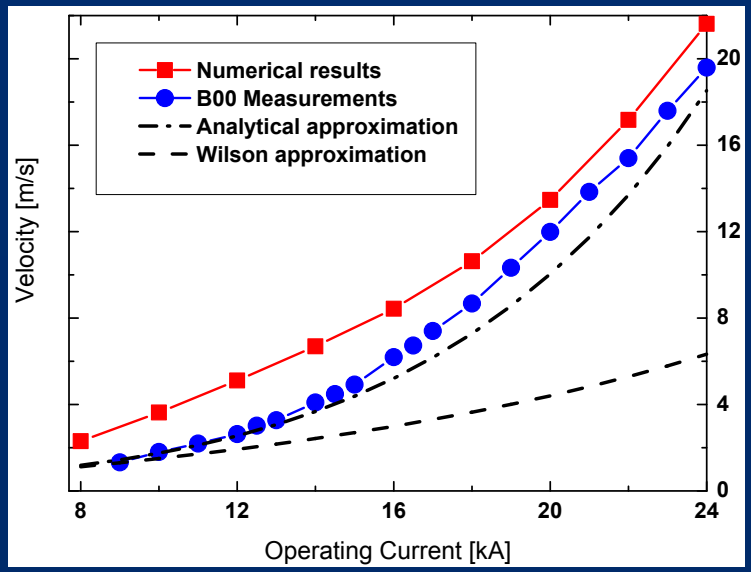
$$\rho_{eff} = A_{eff}\left(\frac{A_{sc}}{\rho_{NbTi}} + \frac{A_{Cu}}{\rho_{Cu}} + \frac{A_{eff} - A_R}{\rho_{st}}\right)^{-1}$$

Results

Velocity vs. Operating Current

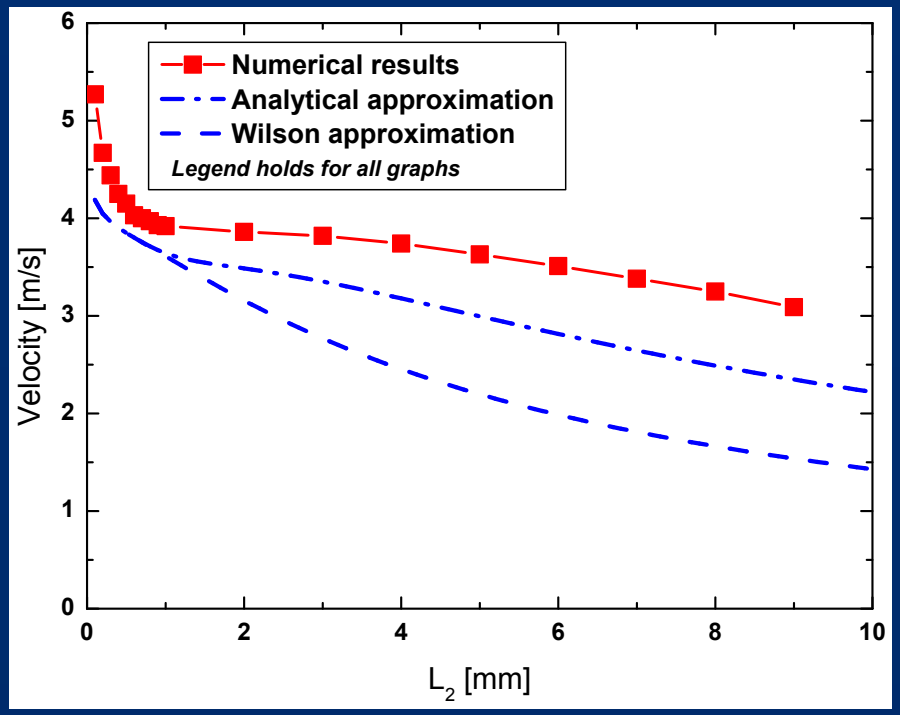
- Measurement of the propagation velocity for different currents gains insight in the normal zone behavior of the conductor.
- Where current redistribution is becoming significant, the MB analytical approximation provides a better match to the data
- Simulation results show generally higher values than measurements due to the adiabatic boundary conditions of the numerical model



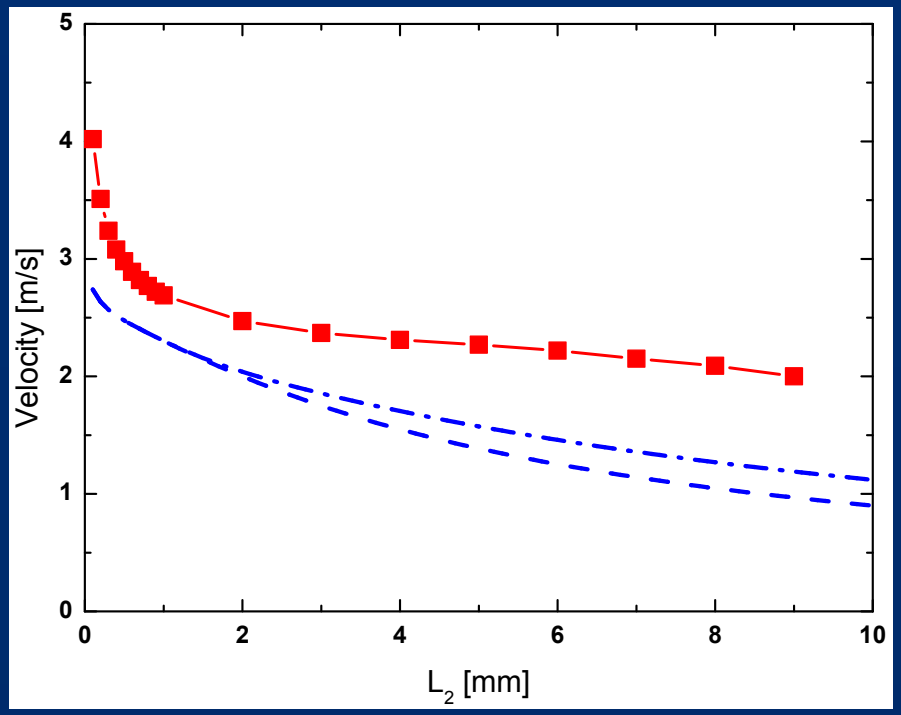


Results

Velocity vs. Cladding Thickness



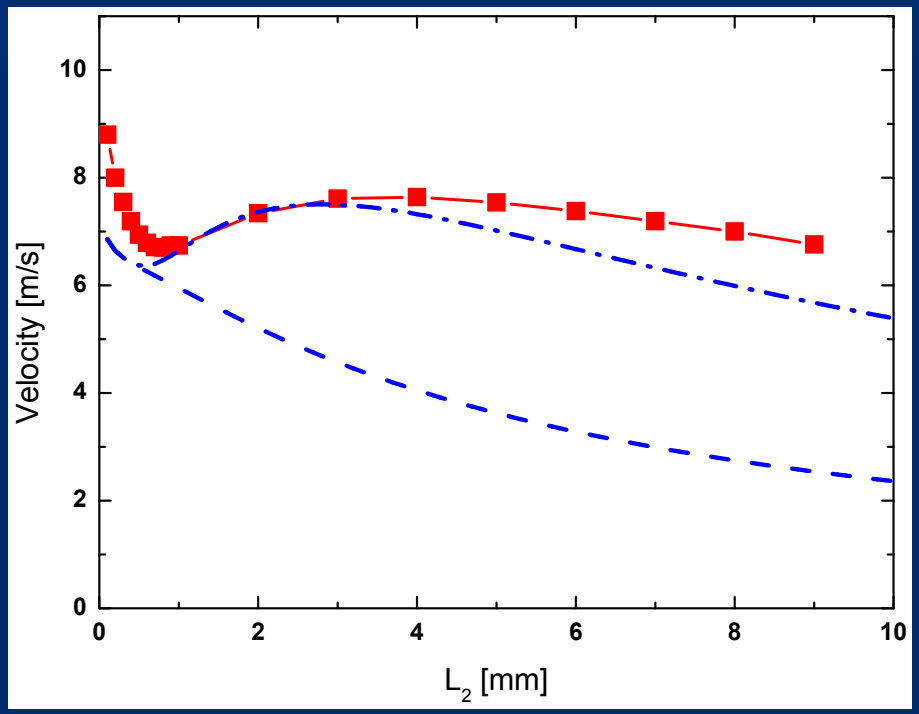
I_{op} = 10 kA, B_{ext} = 4.5 T



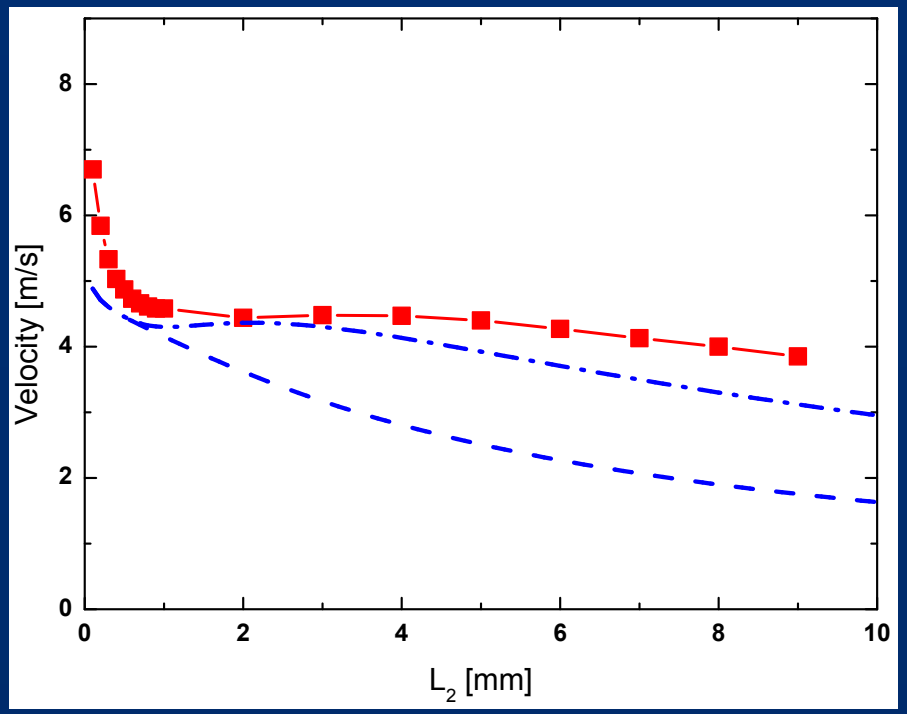
I_{op} = 10 kA, B_{ext} = 1.6 T

Results

Velocity vs. Cladding Thickness



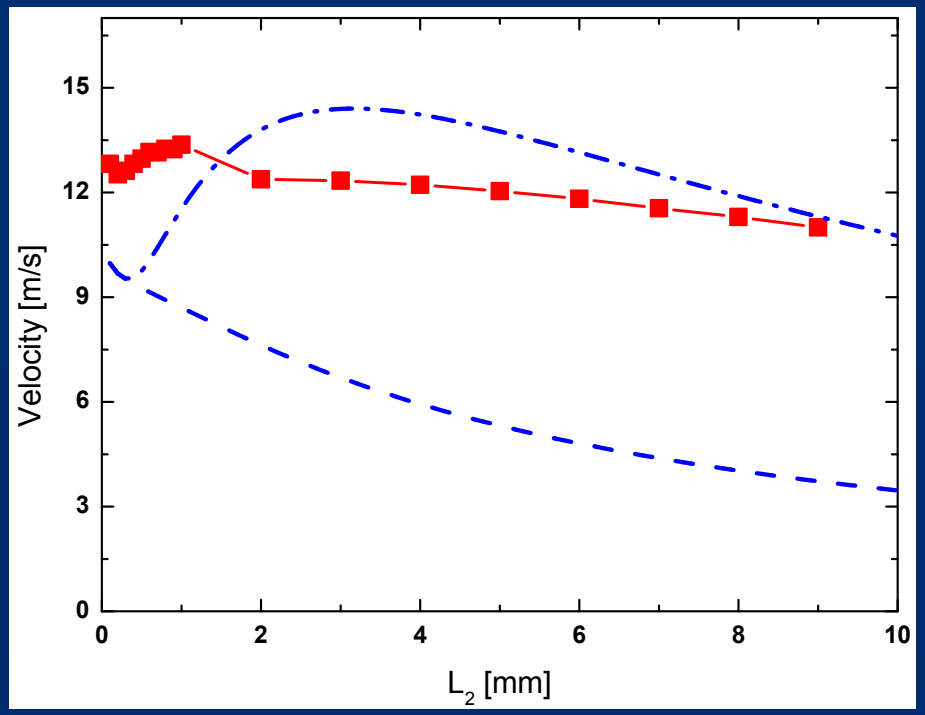
I_{op} = 15 kA, B_{ext} = 4.5 T



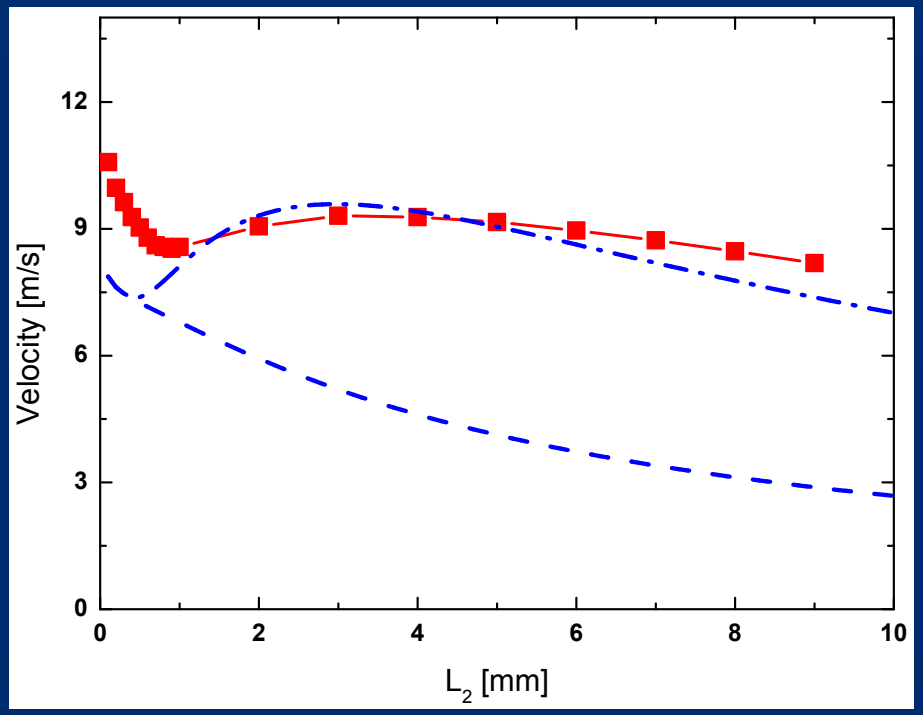
I_{op} = 15 kA, B_{ext} = 2.7 T

Results

Velocity vs. Cladding Thickness



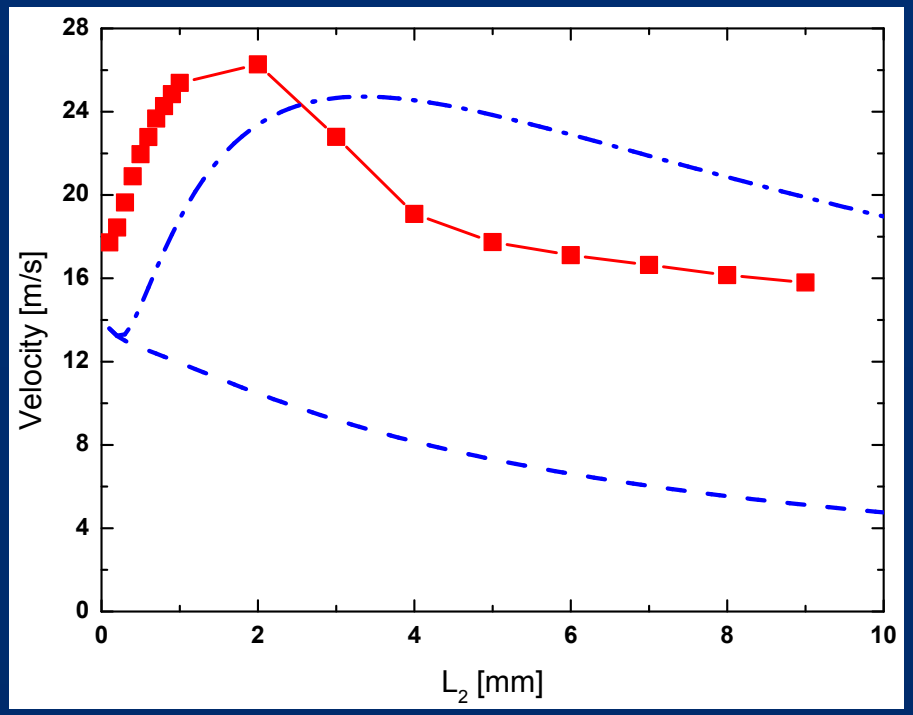
I_{op} = 20 kA, B_{ext} = 4.5 T



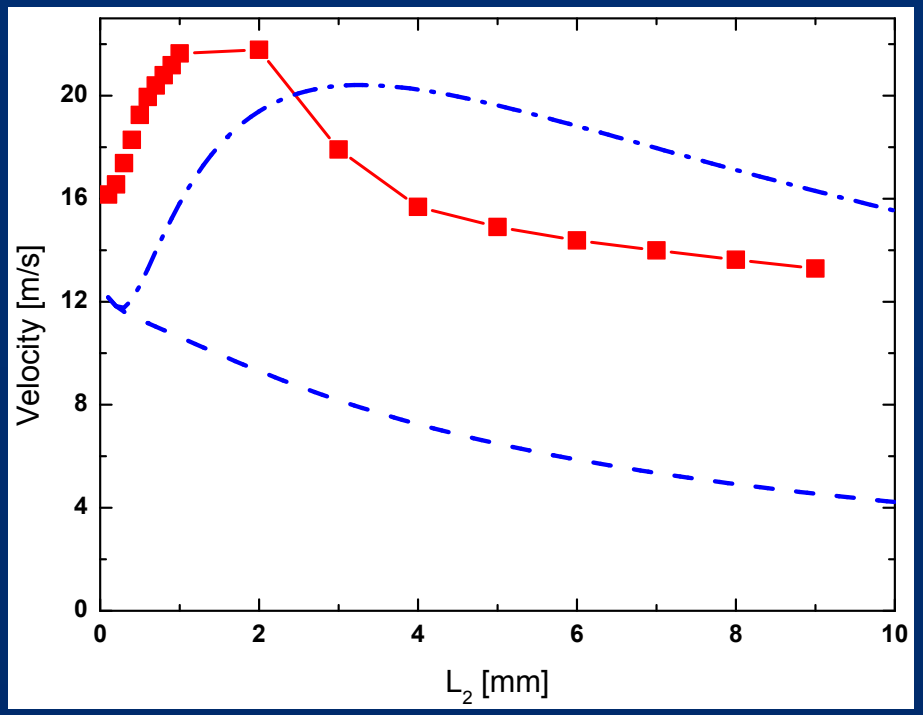
I_{op} = 20 kA, B_{ext} = 3.2 T

Results

Velocity vs. Cladding Thickness



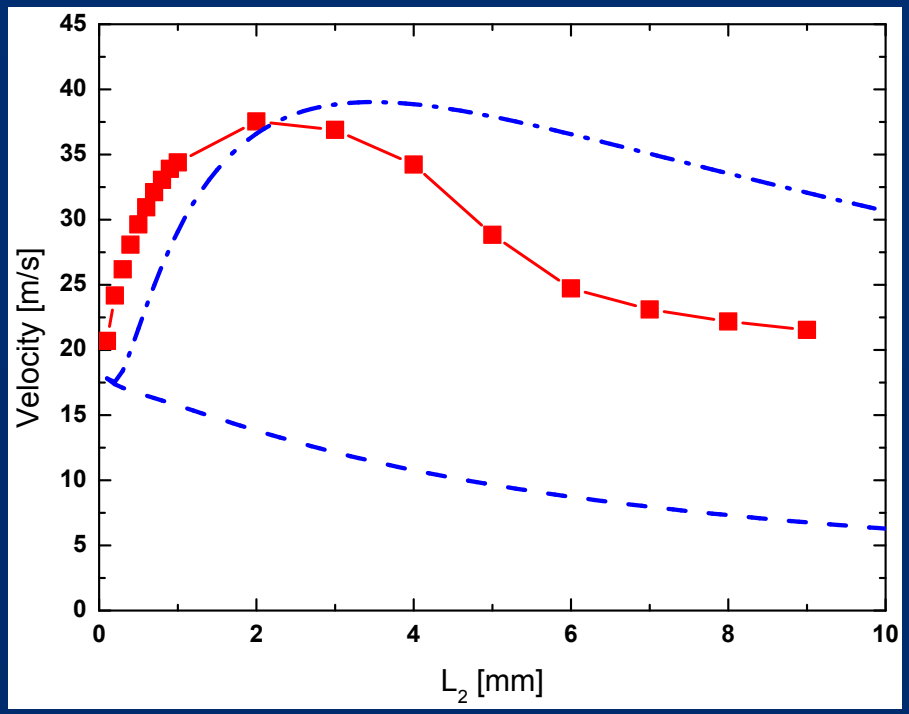
I_{op} = 25 kA, B_{ext} = 4.5 T



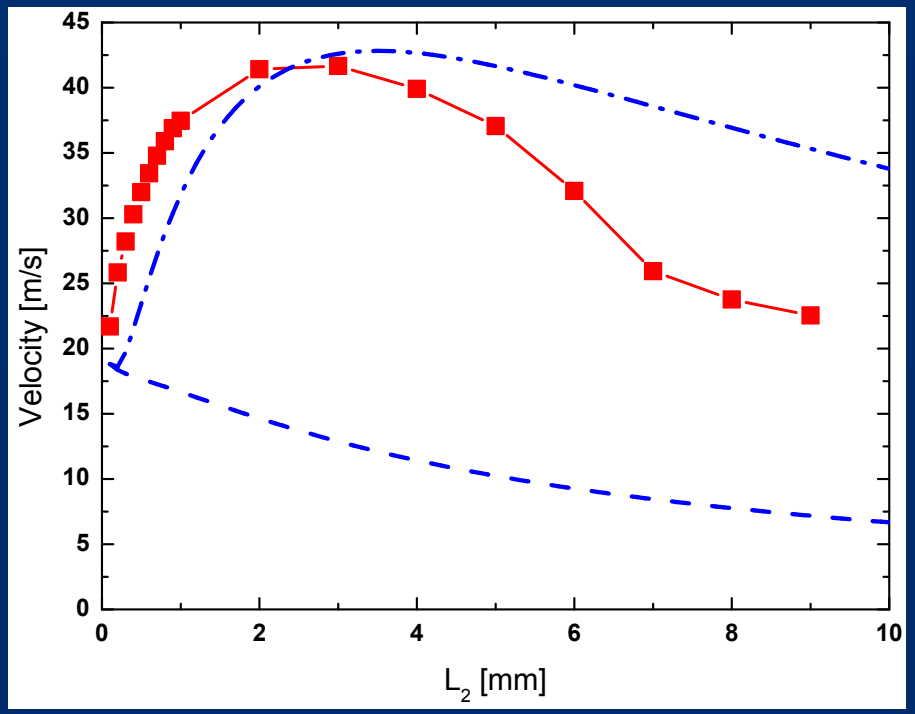
I_{op} = 25 kA, B_{ext} = 4.0 T

Results

Velocity vs. Cladding Thickness



I_{op} = 30 kA, B_{ext} = 4.5 T



I_{op} = 30 kA, B_{ext} = 4.7 T

Results

Velocity vs. Cladding Thickness

- For low currents, there is a very good agreement between our analytical and numerical models. For higher currents, this agreement breaks and some deviations between the two models appear
- A general behavior of the plots can be explained, however, by examining the analytical formula. This has three characteristics

1. For $L_2 \ll 1$ the dominant term is the current density. As L_2 increases from 0, J decreases, thus leading to a small decrease in propagation velocity
2. When $\alpha < 1$, and for small enough L_2, the ongoing decrease in J is compensated by a change in material properties. The cladding metal acts as a heat sink, thereby increasing the velocity
3. For large L_2, the cladding matrix becomes the dominant element in the conductor. Then, the current density becomes again the dominant factor and thus reduces again the velocity

Summary

- Our analysis provides a good estimation of the normal zone propagation for a variety of superconducting magnets
- We have shown that for $\alpha < 1$ the current remains confined to a small area around the composite material, limiting the cladding's contribution to the normal zone propagation.
- Since the cladding is thick enough to act as a heat sink, the heat generated in the composite material quickly propagates into the cladding.
- This, in turn, accelerates the normal zone propagation because the heat generation that contributes to the normal zone propagation is formed almost exclusively within the Rutherford cable, which carries almost all of the current