Theoretical analysis on the influence of the cold end connecting tube in the U-type pulse tube cryocooler

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1、Introduction

For aerospace applications, U-type pulse tube cryocooler (UPTC) can improve the temperature uniformity and structural stability of the cold end which are very meaningful especially for the big cooling platform. Nevertheless, the influence of the cold end connecting tube on the performance of the UPTC is inevitable.

Previous researches put down the performance deterioration of the UPTC to the flow resistance of the cold end connecting tube. In this study, a simplified model of the cold end of the UPTC is developed based on Lagrangian viewpoint. With this model, the periodical thermodynamic processes of the gas parcels flowing through the cold end of the UPTC are obtained. In this way, the influence of the cold end connecting tube is analysed and quantified with the thermodynamic properties of the gas parcels.

2、The model of the cold end of the UPTC

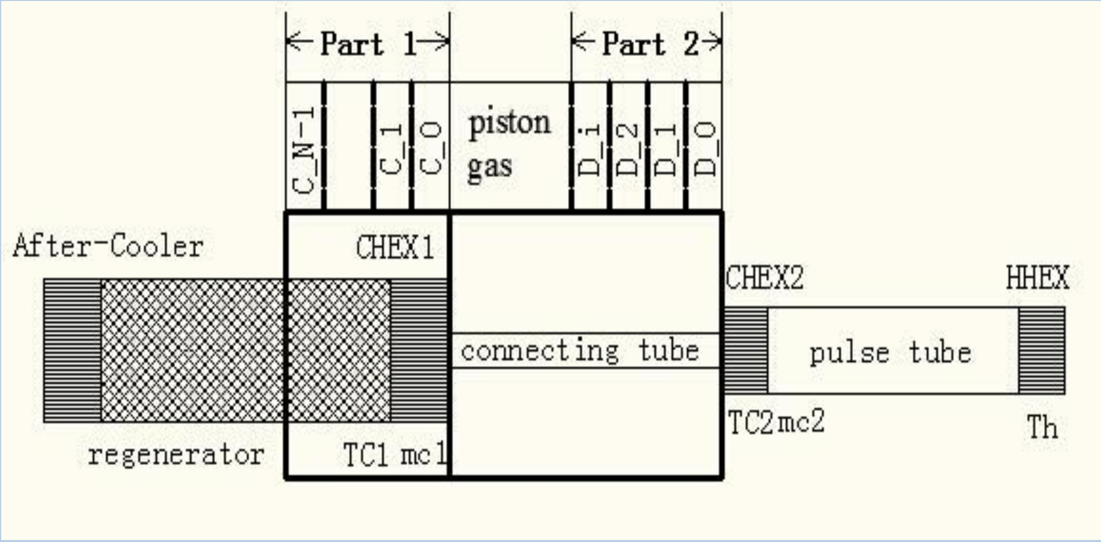

Figure 1. The simplified model of the UPTC

The simplified model of the the UPTC is displayed in figure 1. The time point t_0 is chosen as the beginning time of the calculation. According to the position at the beginning time, the gas involved in this model includes two parts as shown in figure 1. The following calculation is to achieve the thermodynamic processes of these gas parcels during a cycle.

The pressure is given by Eq.(1) and mass flow rate at the hot-end of the pulse tube is given by Eq. (2). According to the energy conservation equations in the pulse tube and the connecting tube, the mass flow rate at the CHEX2 and the CHEX1 are described by Eq. (3) and Eq. (4).

$$p = p_{av} + p_a * sin(\omega t) \qquad (1) \qquad \dot{m}_h = \dot{m}_{h\_a} * sin(\omega t + \vartheta_h) \qquad (2)$$

$$\dot{m}_{c2} = \frac{T_h}{T_{c2}}\dot{m}_h + \frac{V_{pt}}{n_{\rm pt}T_{c2}R_g}\frac{dp}{dt} \qquad (3) \qquad \dot{m}_{c1} = \frac{T_{c2}}{T_{c1}}\dot{m}_{c2} + \frac{V_{CECT}}{n_{\rm CECT}T_{c1}R_g}\frac{dp}{dt} \qquad (4)$$

The first half cycle is divided into N equal time periods. The gas in part 1 flowing through CHEX1 from t_i to $t_{i+1}$is defined as C_i. The gas in part 2 flowing through CHEX2 from t_i to $t_{i+1}$is defined as D_i. Therefore, the mass of the C_i and D_i can be defined by Eq. (5) and the their mass can be obtained by Eq. (6) and Eq. (7). The phase shift between the mass flow and pressure at the two cold end heat exchangers is described by Eq.(8).

$$M_i = \int_{t_i}^{t_{i+1}} \dot{m} dt \qquad (5)$$

$$M_{D_i} = 2sin\frac{\omega\Delta t}{2}\sqrt{(A * sin\vartheta_h + B)^2 + (A * cos\vartheta_h)^2}sin(\omega t_i + \frac{\omega\Delta t}{2} + \vartheta_{c2}) \qquad (6)$$

$$M_{C_i} = 2sin\frac{\omega\Delta t}{2}\sqrt{(C * sin\vartheta_h + D)^2 + (C * cos\vartheta_h)^2}sin(\omega t_i + \frac{\omega\Delta t}{2} + \vartheta_{c1}) \qquad (7)$$

$$\theta_{c2} = arctan(tan\vartheta_h + \frac{B}{A*COS\vartheta_h}), \quad \vartheta_{c1} = arctan(tan\vartheta_h + \frac{D}{C*COS\vartheta_h}) \qquad (8)$$

Where $A = \frac{T_h}{T_{c2}} * \dot{m}_{h\_a} * \frac{1}{2\pi f}$, $B = \frac{V_{pt}}{\gamma_2 T_{c2} R_g} * p_a$, $C = \frac{T_h}{T_{c1}} * \dot{m}_{h\_a} * \frac{1}{2\pi f}$, $D = (\frac{V_{CECT}}{r_1 T_{c1} R_g} + \frac{V_{pt}}{r_2 T_{c1} R_g}) * p_a$

According to Eq. (6) and Eq. (7), the mass of the gas parcels of different time periods are also sinusoidal. Supposing the gas parcels are at their leftmost position at the time t_0. If the proper beginning time t_0 is chosen, all the gas parcels move towards right from t_0 to $t_0 + \tau/2$. And then in the following half cycle, these gas parcels move back to the left. Since A and B are positive values, the phase angle at the CHEX2 is bigger than that at the HHEX while it is smaller than that in the CHEX1.This result verifies the capacitive of the CECT and the pulse tube.

3、Results and analysis

As shown in figure 1, whether there is piston gas in the CECT depends on the length of the CECT. Nevertheless, according to our analysis and calculating results, for the most of the applications of the UPTC, the length of the CECT is smaller than the critical length. So, the following researches will focus on CECT within the critical length.

The basic parameters of the UPTC are listed in table 1. Every research involves only one variable which means the value of the remaining parameters are kept constant at the value listed in table 1.

Table 1. Basic parameters of the UPTC

Length of the CECT	L_{CECT}	0.02 m
Length of the pulse tube	L_{pt}	0.07 m
Length of the regenerator	L_{rg}	0.07 m
Inner Diameter of the regenerator	D_{rg}	0.018 m
Inner Diameter of the CECT	D_{CECT}	0.002 m
Inner Diameter of the pulse tube	D_{pt}	0.009 m
Amplitude of mass flow rate at the hot-end of pulse	$\dot{m}_{h\_a}$	5.38e-4 kg/s
Phase shift at the hot-end of the pulse tube	ϑ_h	-52.2^0
Charge pressure	p_{av}	3.7E6 Pa
Dynamic pressure amplitude	p_a	2E5 Pa
Frequency	f	50 Hz
Temperature of CHEX1	T_{c1}	80 K
Temperature of CHEX2	T_{c2}	80 K
Temperature of HHEX	T_h	300 K

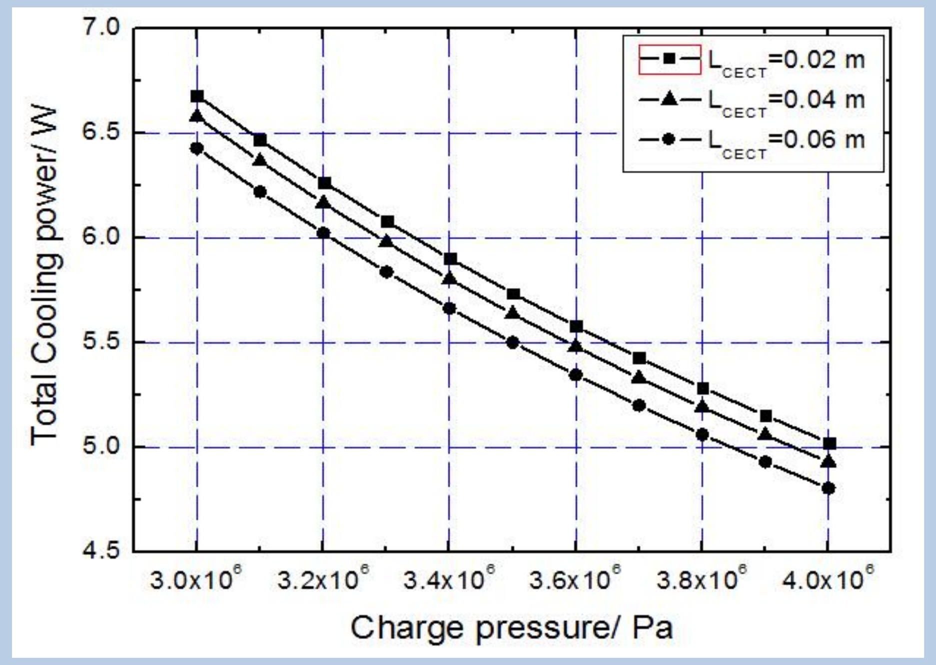

Figure 2(a). Cooling power vs. charge pressure

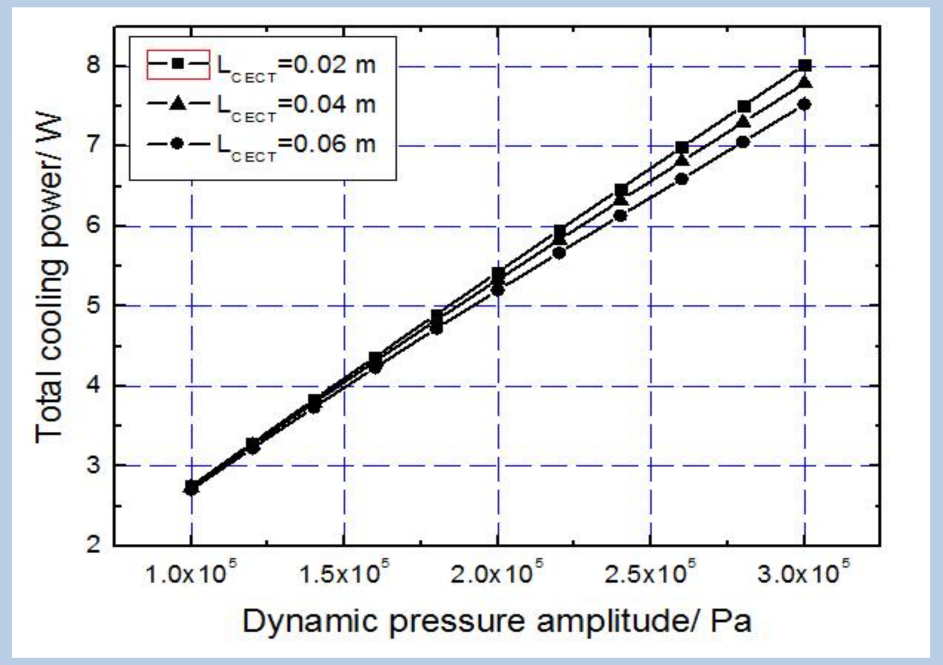

Figure 2(b). Cooling power vs. dynamic pressure amplitude

In figure 2, The total cooling power decreases with the increase of the charge pressure while the increase of the dynamic pressure amplitude leads to the increase of the total cooling power. The results demonstrate that higher pressure ratio is good for the increase the cooling power because of the increase of the temperature fluctuation.

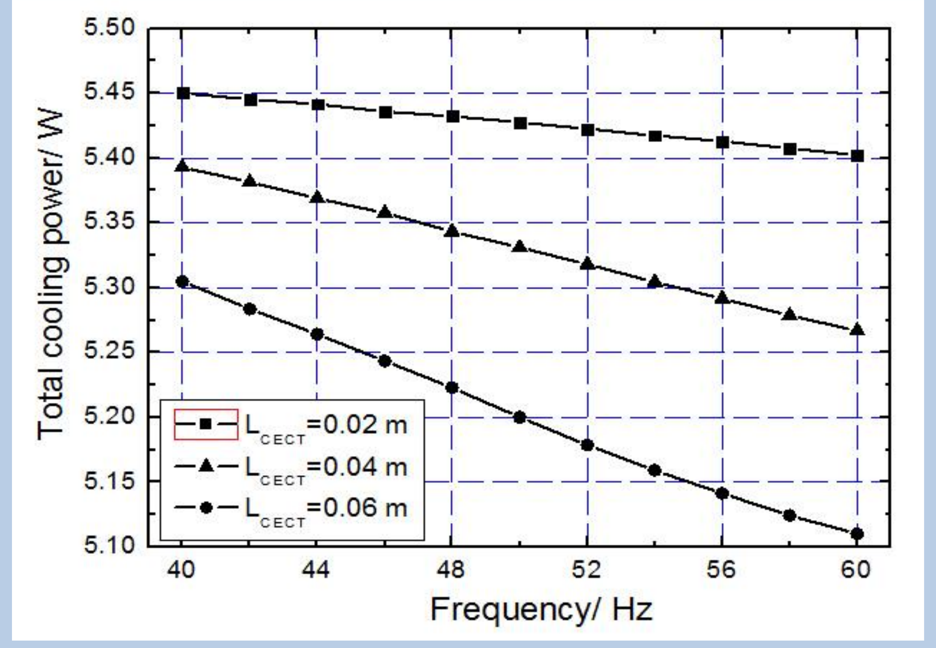

Figure 3(a). Cooling power vs. frequency

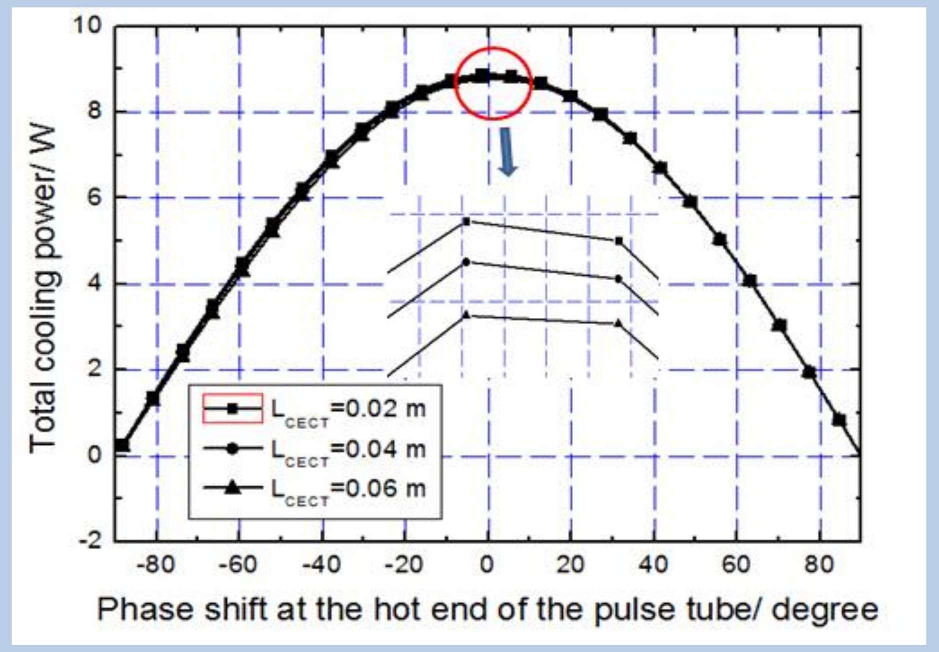

Figure 3(b) Cooling power vs. phase shift at the hot end of the pulse tube

Figure 3(a) shows lower frequency benefits for the total cooling power. Figure 3(b) displays the total cooling power with the change of phase shift. The results illustrate that for the different length of the CECT, the influences of these parameters have the same tendency. Among these parameters shown in figure 2 and 3, the total cooling power is most sensitive with the phase shift. Compared with the results of different length of the CECT, the CECT really has disadvantageous influence on the performance of the UPTC. With the increase of the length of the CECT, the performance deterioration becomes more and more serious.

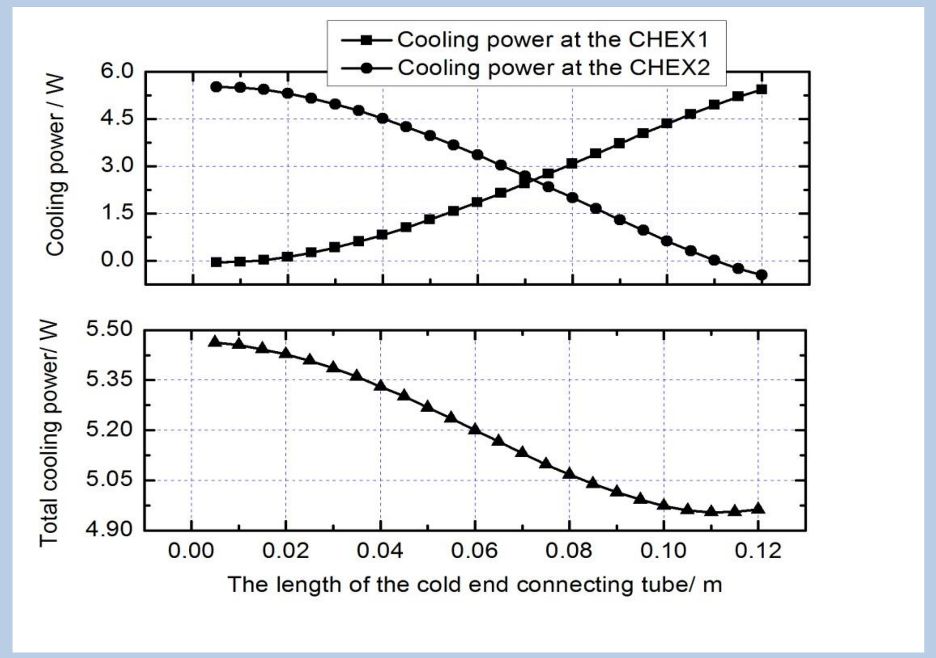

Figure 4. Cooling power vs. the length of the CECT

figure 4 shows the cooling power at the two cold end heat exchangers as well as the total cooling power with the different length of the CECT. Part of PV work offsets the cooling power because of the existence of the CECT. So, the total cooling power decreases with increase of the length of the CECT.

Since how many gas parcels in part 1can flow through the CHEX2 mainly depends on the length of the CECT, the length of the CECT influences the amount of the heat absorbed (released) by the gas parcels at the two cold end heat exchangers. So, the CECT plays an important role on the distribution of the cooling power.

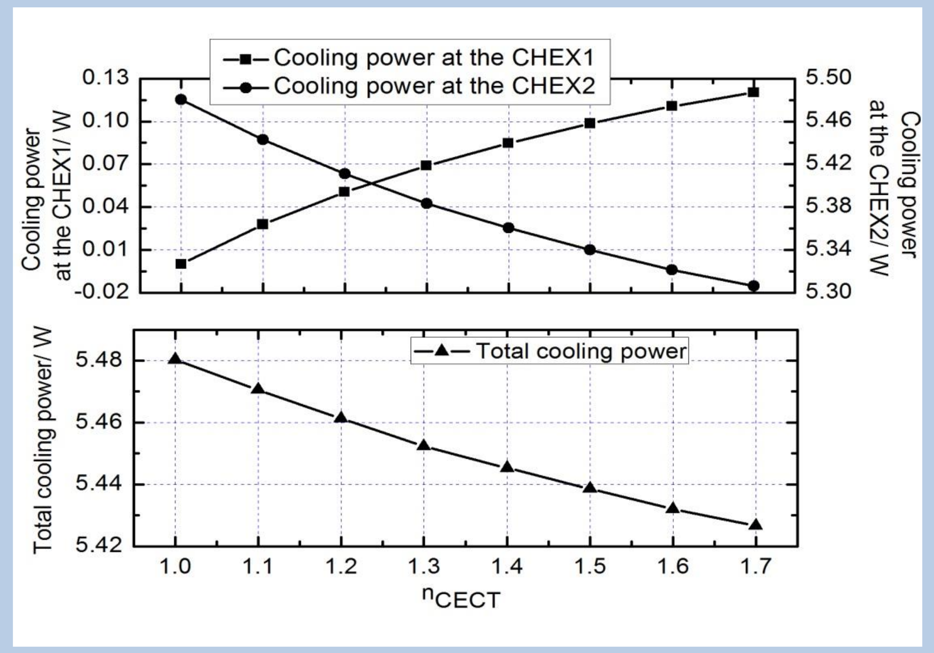

Figure 5. Cooling power vs. the polytroopic compression index

In figure 5, the increase of the polytropic compression index means the thermodynamic processes change from isothermal processes to adiabatic processes. The changes of the cooling power at the CHEX1 and the CHEX2 are opposite and the total cooling power decreases. The results imply that the heat convection between the gas and the CECT is beneficial for the improvement of the cooling power in the UPTC.

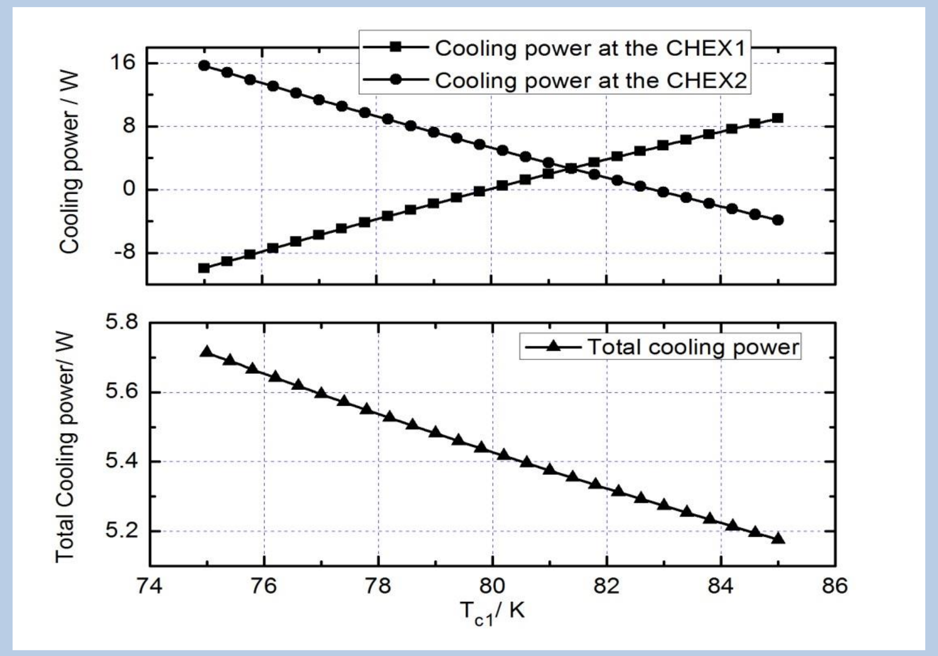

Figure 6. Cooling power vs. T_{c1}

Figure 6 shows the influence of the temperature difference between the two cold end heat exchangers. The cooling power at the CHEX1 keeps increasing while the cooling power at the CHEX2 keeps decreasing with the increase of the T_{c1}. The total cooling power keeps decreasing. On the contrary, with the change of T_{c2} , the results are just the opposite. In a conclusion, the total cooling power tend to increase with the increase of the value T_{c2} minus T_{c1} .

4、Conclusion

a. The CECT has disadvantageous influences on the performance of the UPTC. The influence becomes more and more serious with the increase of the length of the CECT.
b. The CECT have significant influence on the distribution of the cooling power between the two cold end heat exchangers. With the increase of length of the CECT, more and more cooling power transfers from the CHEX 2 to the CHEX1.
c. The heat convection between the gas and the CECT is beneficial for the improvement of the performance of the UPTC. The total cooling power tend to increase with the increase of the value that T_c2 minus T_c1.

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