Minimum Quench Energy of Superconducting Wires/Cable with Lateral Cooling

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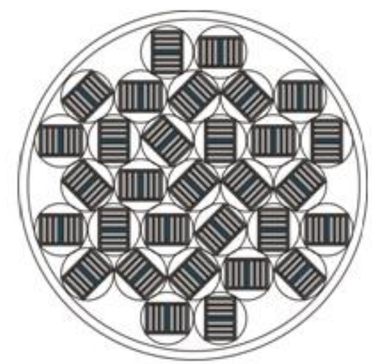
Motivation

1. Liquid cryogen cooling has been the norm for superconducting bus-bar/cables
2. Localised disturbances do not pose a quench risk due to high heat transfer coefficient
3. Gas cooled cables/bus-bars are now seriously considered to take advantage of the wide temperature range found in HTS and MgB_2
4. Heat transfer coefficient by gas cooling is much lower, local disturbance induced quench becomes a risk.

Novel *twisted-pair* cable concept optimized for **tape conductors** (MgB_2, Y-123 and Bi-2223). A. Ballarino “ Alternative design concepts for multi-circuit HTS link systems”. *IEEE Trans. on Applied Supercond.* **21** pp. 980-984, 2011



How to Determine Adiabatic MQE (1)

Embedded in the intrinsic instability of the quench equation

$$c_p(T(x,t))\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(T(x,T))\frac{\partial T(x,t)}{\partial x}\right) + J \cdot E(T(x,t),J)$$

with $T(x,0) = T_0$ and $T(x \to \pm\infty, t) = T_0$

MQE can be determined by

<u>Numerical solutions</u>: simple to implement but could be exhaustive

<u>Analytical approach</u>: more difficult but lead to deeper insight

The heat generation of *current sharing* $J \cdot E(T,J)$ is fundamental to the magnitudes and functional behaviour of MQE.

How to Determine Adiabatic MQE (2)

Methodology for Analytical MQE (following Dresner and Wilson)

1. Partial linearization: $k\big(T(x,T)\big) = k(T_0), c_p\big(T(x,t)\big) = c_p(T_0)$
2. Dimensionless transform

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \frac{E(u,j)}{(1-j)\ \rho_m J_C(T_0)} \text{ with } u(\xi,0) = 0, u(\xi \to \pm\infty, \tau) = 0$$

- ❑ Current: $j = \frac{J}{J_c(T_0)}$
- ❑ Length: $\xi = \frac{x}{l_{MPZ}}$ with $l_{MPZ} = \frac{\pi}{2}\sqrt{\frac{k(T_0)(T_C - T_0)}{\rho_m(T_C)\, j\, J_C^2(T_0)}}$
- ❑ Time: $\tau = \frac{k_0\, t}{c_p(T_0) l_{MPZ}^2}$
- ❑ Temperature: $u = \frac{\theta}{1-j}$ with $\theta = \frac{T - T_0}{T_c - T_0}$

3. Parameters reduced to just two dimensionless ones
- ❑ Current load factor *j*
- ❑ Current sharing voltage: $\epsilon(j) = \frac{E(u,j)}{(1-j)\rho_m(T_c) J_C(T_0)}$

How to Determine Adiabatic MQE (3)

Methodology for Analytical MQE (following Dresner and Wilson)

4. Dimensionless quench equation remains partial differential
   - more informative but not easier.
5. Dresner obtained MQE by finding the minimum enthalpy of a normal zone of an *assumed spatial profile* of the normal zone.
   - The method not easily applied to nonlinear current sharing of the power-law superconductors.
6. Wilson suggested that the MQE is given by the *stationary normal zone* equation: $\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \frac{E(u,j)}{(1-j)\,\rho_n J_C(T_0)} = 0, u(0) = u_0, u(L) = 0$
   - The problem becomes analytically manageable.
   - Results are consistent with Dresner.
   - No analytical proof yet for the underlying lemma.

How to Determine Adiabatic MQE (4)

<u>MQE from stationary normal zone</u>:

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \frac{E(u,j)}{(1-j)\,\rho_m J_C(T_0)} = 0, u(0) = u_0, u(L) = 0$$

- ❑ Solution of the *ordinary* differential equation
- ❑ Minimizing the thermal energy (enthalpy) of the normal zone (the area below $u(\xi)$):

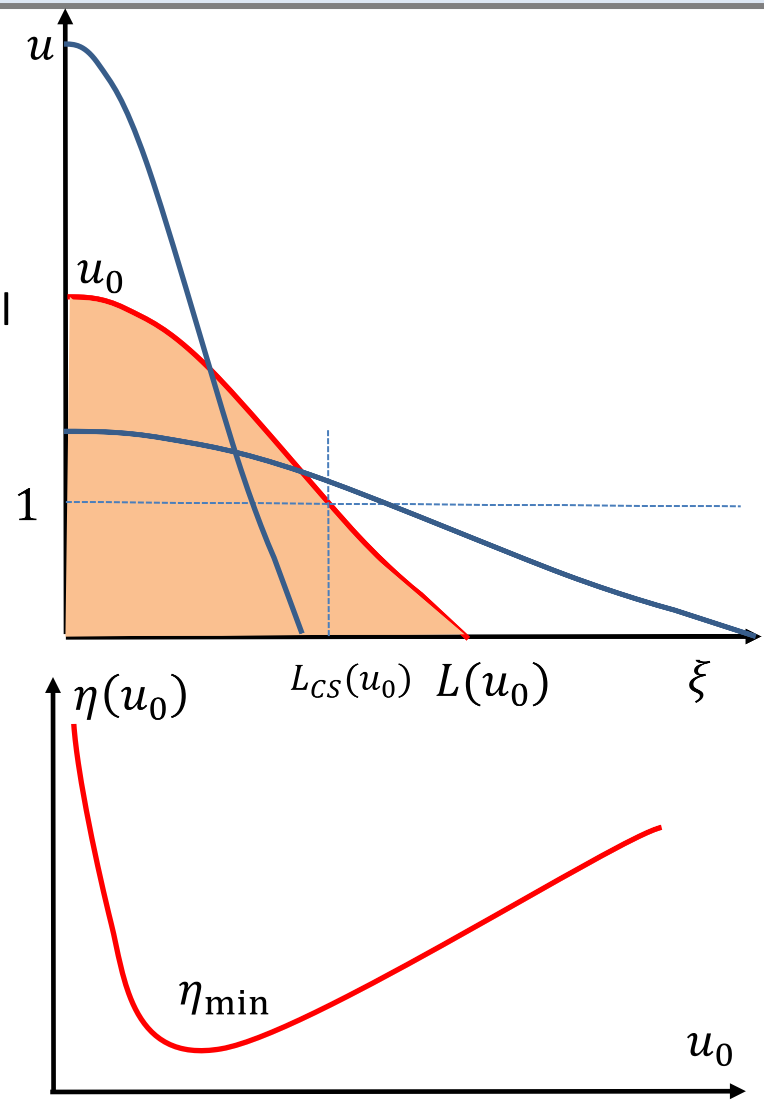
$$\eta(u_0, j, \rho_m J_C(T_0), \dots) = 2\int_0^{L(u_0)} u(\xi, j, \rho_m J_C(T_0), \dots)d\xi$$

<u>The existence of a minimum enthalpy</u> $\eta_{\min}$ <u>is self evident</u>:

- ❑ Normal zone length $L(u_0)$ reduces at higher u_0.
- ❑ $\eta(u_0) \to \infty$ at high and low temperatures, i.e., at $(u_0 \to \infty)$ and $L(u_0 \to 1) \to \infty$ respectively.
- ❑ The *dimensionless MQE* $\eta_{\mathrm{MQE}} = \eta_{\min}$ (Wilson)
- ❑ Dimensioned MQE from dimensionless η_{MQE}:

$$MQE(j) = \eta_{MQE}(j) \cdot \left(c_p(T_0) l_{MPZ}(1-j)(T_C - T_0)\right)$$

$$= \frac{\pi}{2} c_p(T_0)(T_C - T_0)\sqrt{\frac{k(T_0)(T_C - T_0)}{\rho_m(T_C)\, J_C^2(T_0)}}\,\eta_{MQE}(j)(1-j)j^{-0.5}$$



The Unique and Profound Case of Critical State with Linear $Jc(T)$

Linear $J_C(T)$ dependence + Critical State Current Sharing → A simple yet profound normal zone equation

$$J_C(T) = J_C(T_0)\cdot\frac{T_C - T}{T_C - T_0} = J_C(T_0)(1-\theta) = J_C(T_0)(1-(1-j)u)$$

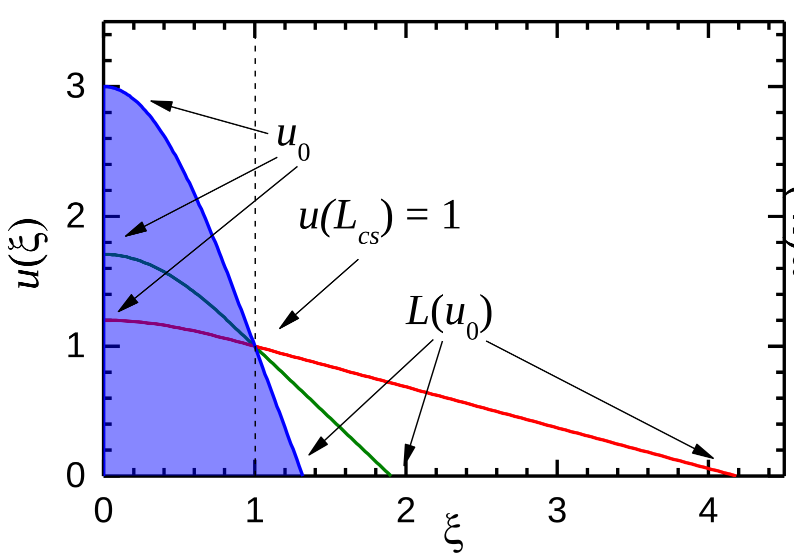
$$E(T,J) = \rho_m\left(J - J_C(T)\right) = \rho_m J_C(T_0)(1-j)(u-1)$$

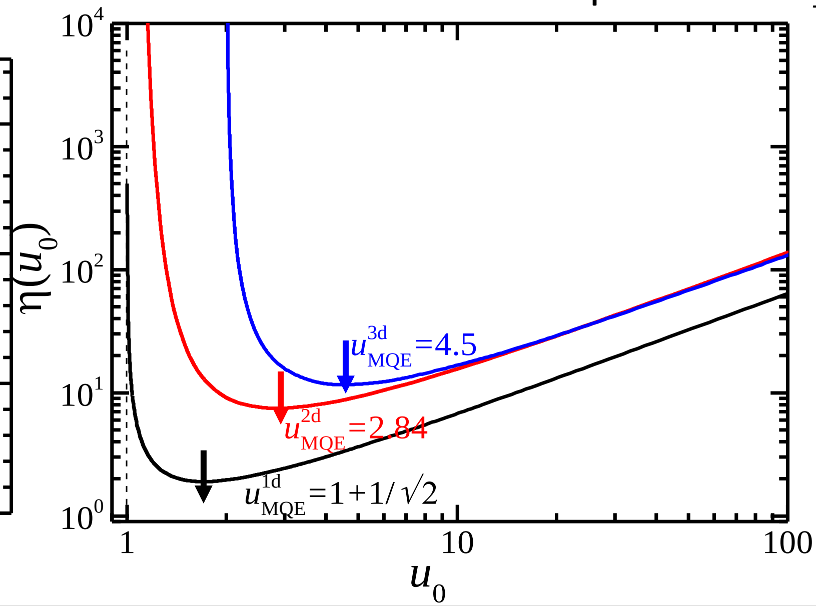
$$\epsilon(u,j) = (u-1)$$

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 (u-1) = 0$$

<u>All parameters eliminated</u>: The *non-dimensional normal zone* and *minimum enthalpy* independent of j, J_c, ρ_m

MPZ is well defined with the current sharing length $L_{CS} = 1$ and $u(L_{CS}) = 1$.





The Unique and Profound Case of Critical State with Linear *Jc*(*T*)

Linear $J_c(T)$ dependence + Critical State Current Sharing → A simple yet profound normal zone equation

$$J_C(T) = J_C(T_0)\cdot\frac{T_C - T}{T_C - T_0}$$
$$= J_C(T_0)(1-\theta)$$
$$= J_C(T_0)(1-(1-j)u)$$

$$E(T,J) = \rho_m\left(J - J_C(T)\right)$$
$$= \rho_m J_C(T_0)(1-j)(u-1)$$

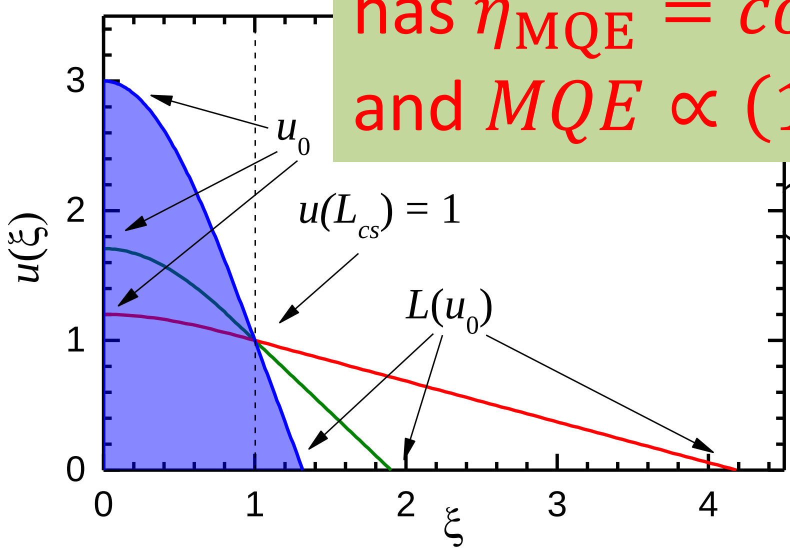
$$\epsilon(u,j) = (u-1)$$

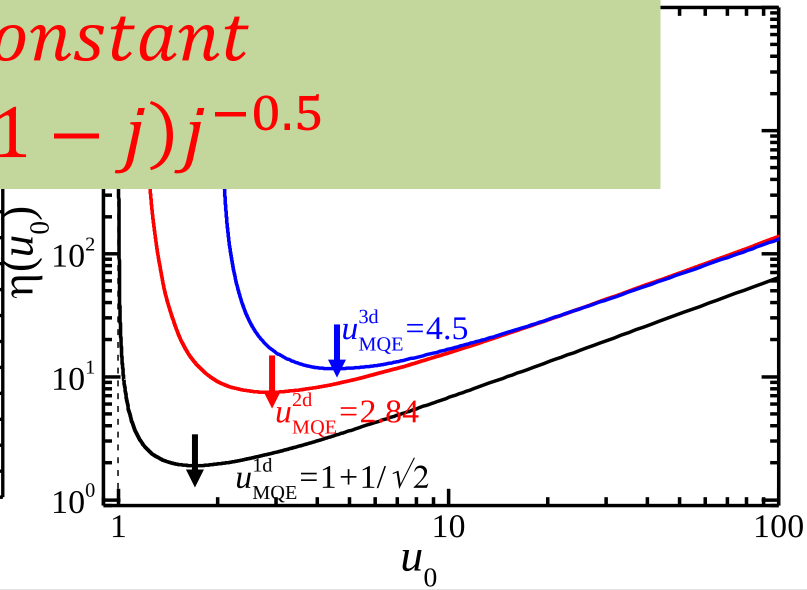
$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 (u-1) = 0$$

<u>All parameters eliminated</u>:
The *non-dimensional normal* ... *minimum enthalpy* ... dent of j, J_c, ρ_m

MPZ is well define... sharing length L_C...

The critical state with linear Jc(T) has $\eta_{\mathrm{MQE}} = constant$ and $MQE \propto (1-j)j^{-0.5}$

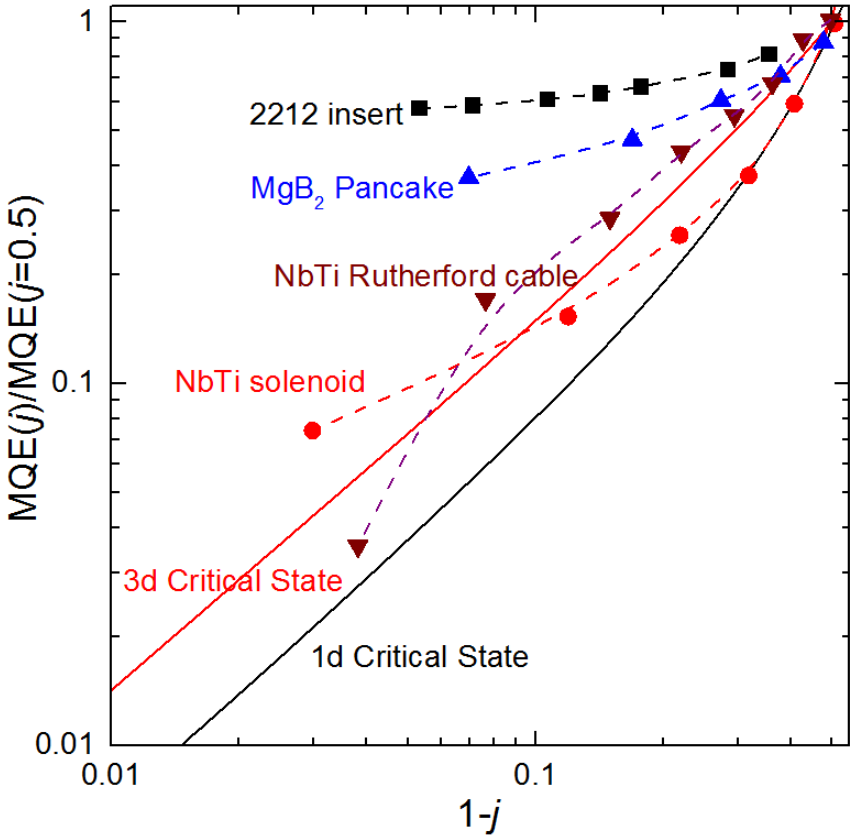




Experimental MQE do not vanish with (1-*j*)

Critical state MQE vanishes with $1 - j$

- High current $j > 0.9$ is more sensitive for ascertaining the current scaling of MQE.
- Although MQE measurements at high current are difficult, data do exist and show clearly the experimental MQE deviates from the critical state:
  - Most notably a slower reduction at high current load *j*>0.9;
  - In both LTS and HTS;
  - **1d**: Rutherford cable (L Shirshov)
  - **2d**: MgB_2 pancake (J Pelegrin)
  - **3d**: NbTi (Dresner and Scott) and 2212 (Y Yang) solenoids



MQE of Power-law Superconductors
An example of the methodology

- Nonlinear current sharing $\epsilon(u,j)$:

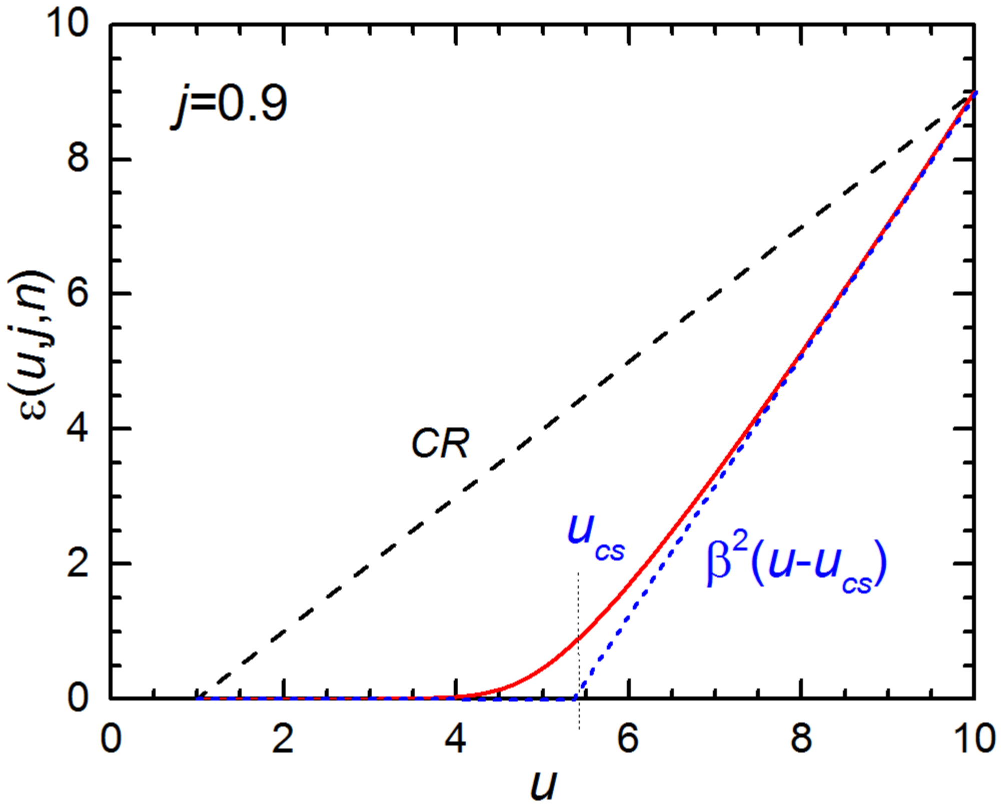
$$(1-(1-j)u)\left(\frac{E(u,j))}{E_0}\right)^{\frac{1}{n}}+\frac{E(u,j)}{J_C(T_0)\rho_m}=j$$

$$\epsilon(u,j,n)=\frac{E(j,u)}{(1-j)J_C(T_0)\rho_m}$$
$$=\frac{1}{(1-j)e_\rho}\frac{E(u,j)}{E_0}$$

- Approximate linearization of $\epsilon(u,j)$:

$$\frac{E(u,j)}{E_0}\sim e_\rho(1-j)\beta^2\left(u-\frac{1-j\beta^{-2}}{1-j}\right)$$

With $\beta^2=\frac{(je_\rho)^{\frac{1}{n}}}{1+\frac{1}{n\cdot j}(je_\rho)^{\frac{1}{n}}}, u_{CS}(j,\beta)=\frac{1-j\beta^{-2}}{1-j}$



- Transform to effective critical state

$$\epsilon(u,j)=\left(\frac{u}{u_{cs}(j,\beta)}-1\right)\beta^2u_{cs}(j,\beta)\xrightarrow{v=\frac{u}{u_{CS}},\zeta=\beta\xi}\frac{\partial^2v}{\partial\zeta^2}+\left(\frac{\pi}{2}\right)^2(v-1)=0$$

MQE of Power-law Superconductors
An example of the methodology

- <u>Nonlinear current sharing $\epsilon(u,j)$:</u>

$(1-(1-j)u)\left(\frac{1}{-}\right.$ …

$\epsilon(u,j,n) =$ …

$= \frac{1}{(1-j)e\ldots}$

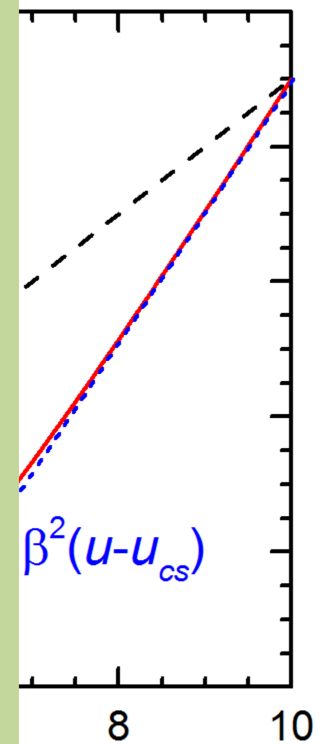
- <u>Approximate</u> …

$\frac{E(u,j)}{E_0} \sim e_\rho(1 - \ldots$

With $\beta^2 = \frac{(j\ldots}{1+\frac{1}{n\cdot}\ldots}$

- <u>Transform to</u> …

$$\epsilon(u,j) = \left(\frac{u}{u_{cs}(j,\beta)} - 1\right)\beta^2 u_{cs}(j,\beta) \xrightarrow{v=\frac{u}{u_{CS}},\zeta=\beta\xi} \frac{\partial^2 v}{\partial \zeta^2} + \left(\frac{\pi}{2}\right)^2 (v-1) = 0$$



Power-law approximated to the critical state with an increased current sharing temperature u_{cs}:

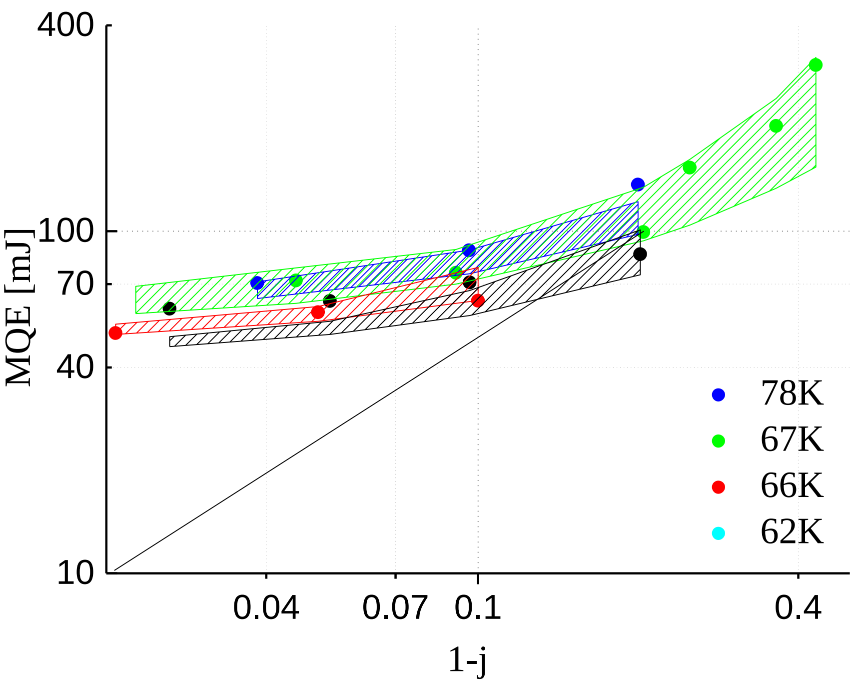
$$MQE \propto (1-j)j^{-0.5}\beta^{-1}u_{CS}(j,\beta)$$

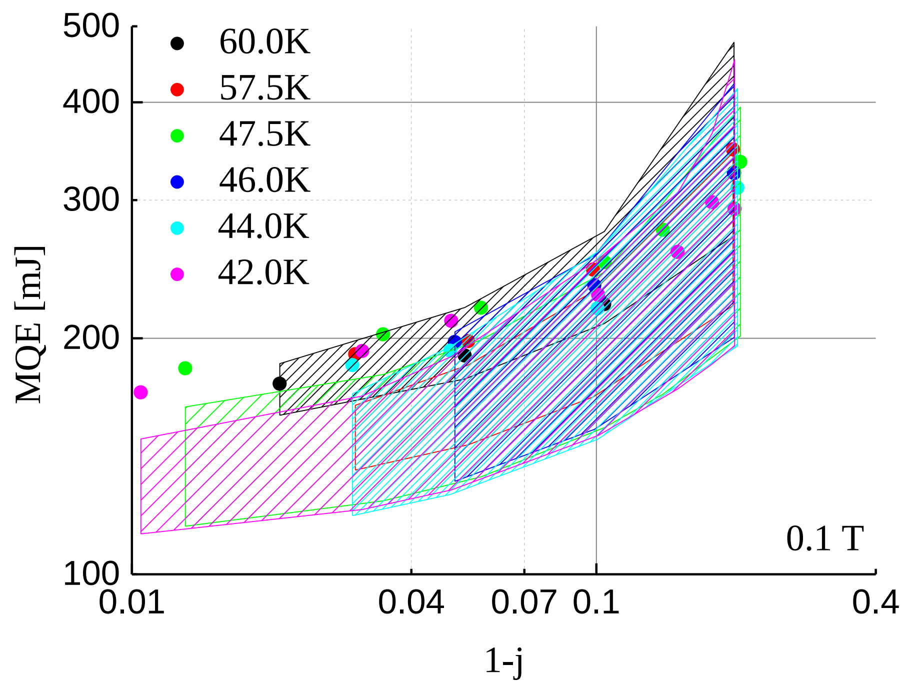
$$= (1-j\beta^{-2})\beta^{-1}j^{-0.5}$$

Slower reduction than 1-*j* when approaching full current load!

MQE of Power-law Superconductors
An example of the methodology

Experimental MQE of Adiabatic 2G YBCO Tapes does not vanish with (1-*j*)





Account for lateral cooling (1)

Add the lateral heat transfer term

$$c_p(T(x,t))\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(T(x,T))\frac{\partial T(x,t)}{\partial x}\right) + J\cdot E(T(x,t),J) - \frac{hP}{A}(T(x,t)-T_0)$$

with $T(x,0) = T_0$ and $T(x\to\pm\infty,t) = T_0$

Maintain the same non-dimensional transformation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \epsilon(u,j) - \frac{hPl_{MPZ}^2}{k(T_0)A}u$$

Hence

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 (\epsilon(u,j) - \mathrm{Cg}\, j^{-1}\, u) \text{ with } \frac{hPl_{MPZ}^2}{k(T_0)A} = \left(\frac{\pi}{2}\right)^2 \frac{\frac{hP}{A}(T_c - T_0)}{J_{C(T_0)}^2\rho_m} = \left(\frac{\pi}{2}\right)^2 \mathrm{Cg}$$

Account for lateral cooling (2)

Introducing a new dimensionless number:

$$\mathrm{Cg} = \frac{\frac{hP}{A}(T_c - T_0)}{J^2_{C(T_0)}\rho_m}$$

which is the ratio between lateral cooling and current sharing heat generation.

Consider single 2G tape (4mm width):

$$P = 8\mathrm{mm}, A = 0.4\mathrm{mm}^2, \frac{P}{A} = 2\times 10^4\mathrm{m}^{-1}$$

1. In liquid nitrogen pool $T_0 = 77\mathrm{K}$:
   $h = 1 - 3\ \mathrm{Wcm}^{-2}\mathrm{K}{\sim}2\times 10^4\mathrm{Wm}^{-2}\mathrm{K}$,
   $T_C - T_0{\sim}10\mathrm{K}, I_C(T_0) = 100\mathrm{A}$,
   $J_C(T_0) = 2.5\times 10^8\mathrm{Am}^{-2}, \rho_m = 3.2\times 10^{-9}\Omega\mathrm{m}$

   $$\mathrm{Cg} = 2$$

2. Helium gas cooled $T_0 = 20\mathrm{K}$:

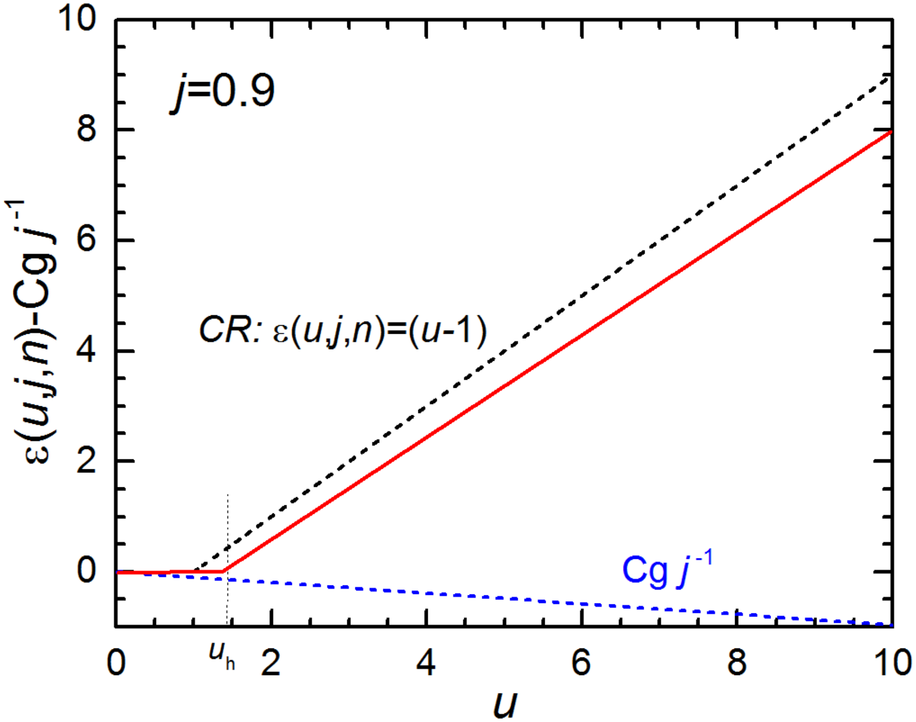
   $$h = \frac{\mathrm{Nu}k_{He}}{D}\ {\sim}40\mathrm{Nu}\ \mathrm{Wm}^{-2}\mathrm{K}$$
   $$T_C - T_0{\sim}70\mathrm{K}, I_C(T_0) = 800\mathrm{A},$$
   $$J_C(T_0) = 2\times 10^9\mathrm{Am}^{-2}, \rho_m = 3.2\times 10^{-10}\Omega\mathrm{m}$$
   $$\mathrm{Cg} = 0.1$$

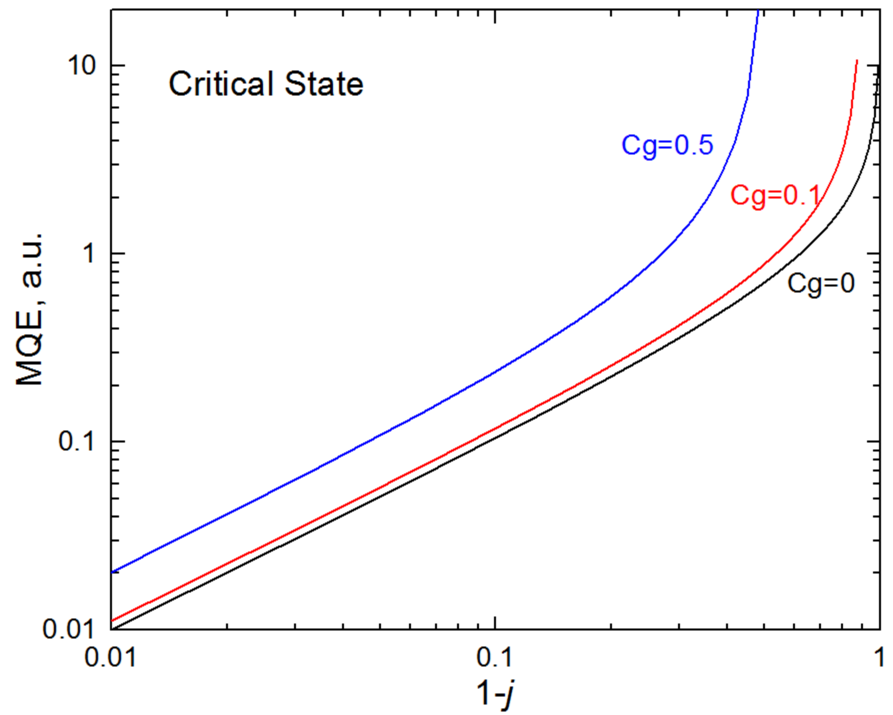
Critical State with lateral cooling

- Approximate transformation to effective critical state

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \left(\left(1 - \frac{\text{Cg}}{j}\right) u - 1\right) = 0 \xrightarrow{v=\frac{u}{u_h}, u_h = \frac{1}{1-\text{Cg}\, j^{-1}}} \frac{\partial^2 v}{\partial \zeta^2} + \left(\frac{\pi}{2}\right)^2 (v-1) = 0$$

$$MQE \propto (1-j)j^{-0.5} u_h(j) = \frac{1-j}{1-\text{Cg}\, j^{-1}} j^{-0.5} = \frac{MQE_{CR}}{1-\text{Cg} j^{-1}}$$



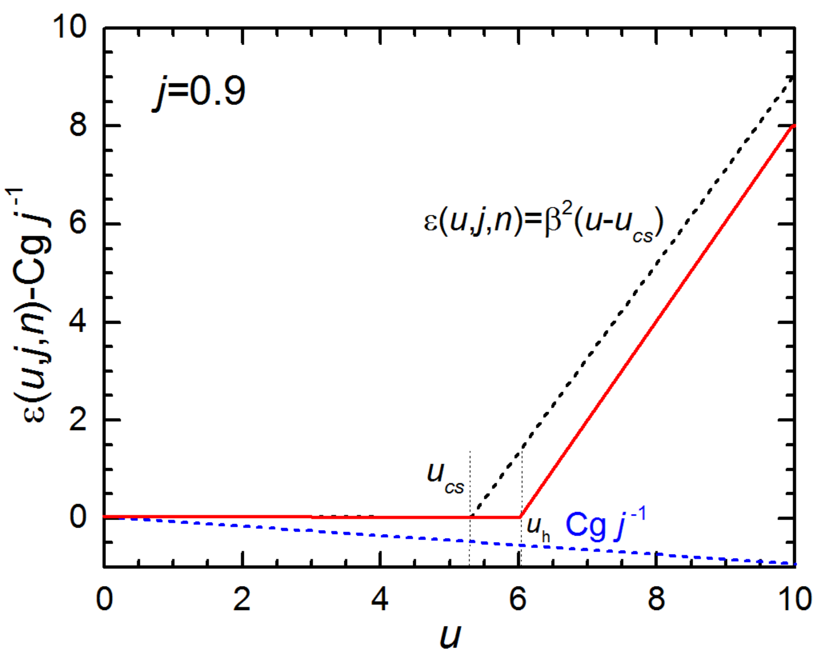


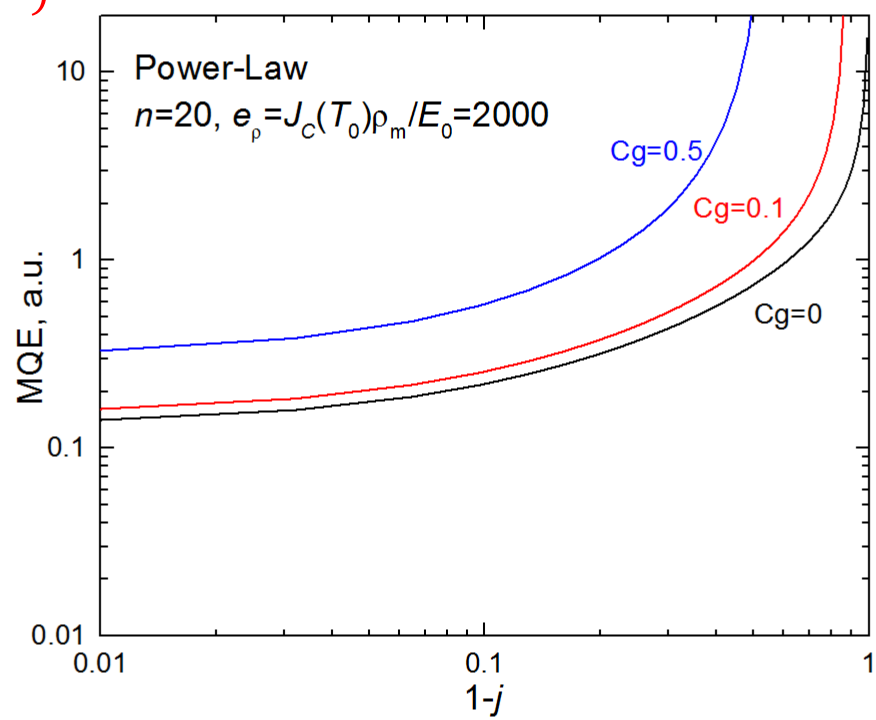
Power-law superconductors with lateral cooling

- Approximate transformation to effective critical state

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \left(\beta^2 (u - u_{cs}(j,\beta)) - \frac{\text{Cg}}{j} u\right) = 0 \xrightarrow{v=\frac{u}{u_h}, u_h=\frac{u_{cs}}{1-\text{Cg}\,\beta^{-2} j^{-1}}, \zeta=\sqrt{\beta^2-\text{Cg}\, j^{-1}}\xi} \frac{\partial^2 v}{\partial \zeta^2} + \left(\frac{\pi}{2}\right)^2 (v-1) = 0$$

$$MQE \propto \frac{1-j}{\sqrt{\beta^2 - \text{Cg} j^{-1}}} u_h j^{-0.5} = \frac{1 - j\beta^{-2}}{\beta(1-\text{Cg}\beta^{-2} j^{-1})^{1.5}} j^{-0.5}$$





Conclusions

1. Explicit current scaling of MQE obtained for power-law superconductors shows MQE does not vanish with (1-*j*)
2. Scaling consistent with experimental results.
3. Explicit current scaling of MQE obtained for superconducting cables/conductors at different heat transfer coefficient.
4. A single dimensionless number is sufficient to quantify the cooling as a proportion of current sharing heat generation.