Analytical Study on Multi-stream Heat Exchanger Include Longitudinal Heat Conduction and Parasitic heat Loads

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Background

High performance heat exchanger is a critical component in many cryogenic systems and its performance is typcially very sensitive to longitudinal heat conduction, parasitic heat loads and property variations. This paper gives an analytical study on 1-D model for multi-stream parallel-plate fin heat exchanger by using the method of decoupling transformations. The results obtained in the present paper are valuable for the reference on optimization for heat exchanger design.

Objectives

- ❖ Cryognic multi-stream parallel-plate fin heat exchanger

Equtions and Methods

Equations and boundary conditions

According to Figs.1 and 2 and energy balance, the stream's governing equation is

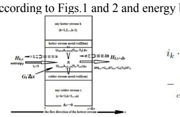


Fig.1 The schematic of stream k's heat exchange

$$i_k \cdot C_k(T_k) \cdot \frac{dT_k}{dx} = \sum_{h=1}^{k-1} U_{hm,k}(T_k) \cdot A_{s,hm,k} \cdot (T_{hm} - T_k) - \sum_{c=k+1}^{n} U_{k,cm}(T_k) \cdot A_{s,k,cm} \cdot (T_k - T_{cm}) + G'_k(T_k)$$

the metal wall's governing equation is

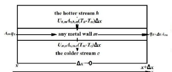


Fig.2 The schematic of metal wall m's heat exchange

$$A_{c,m} \cdot \frac{d}{dx}[\lambda_m(T_m)\frac{dT_m}{dx}] + U_{h,m}(T_h) \cdot A_{s,h,m} \cdot (T_h - T_m) - U_{m,c}(T_c) \cdot A_{s,m,c} \cdot (T_m - T_c) = 0,$$

Equations and boundary conditions

When all coefficients are constants, the governing equations can be rearanged and written in matrix form as

$$\frac{d[T]_{n^2\times 1}}{dx} = [A]_{n^2\times n^2}[T]_{n^2\times 1} + [f]_{n^2\times 1}$$

the boundary conditions are

$$T_k|_{x=\left(\frac{1-i_k}{2}\right)L} = T_{k,in}$$

$$\frac{dT_m}{dx}|_{x=0} = \frac{dT_m}{dx}|_{x=L} = 0$$

Conclusion

- ❖ This paper presents a mathematical model for a multi-stream heat exchanger. In this model, both longtudinal heat conduction and parasitic heat loads are concerned. By using the method of decoupling transformations, the analytical solutions of the model is obtained.
- ❖ In this paper, heat exchanger is in steady-state. When some stream′s mass flow or inlet temperature is changed, heat exchanger is in dynamic state. This will be investigated in the near future.

Samples

Analytical solutions

The governing equations can be solved by the method of decoupling transformations Assuming $[\chi]^{-1}[A][\chi] = [D]$, where [D] is a diagonal matrix containing the Eigenvalues of [A] and $[\chi]$ contains the Eigenvectors of [A]. Then the governing equations becomes

$$\frac{d[Z]}{dx} = [D][Z] + [g]$$

The solution is

$$[Z] = \begin{bmatrix} C_1 \cdot e^{-D_1 \cdot x} - \frac{g_1}{D_1} \\ C_2 \cdot e^{-D_2 \cdot x} - \frac{g_2}{D_2} \\ \cdots\cdots \\ C_n \cdot e^{-D_n \cdot x} - \frac{g_n}{D_n} \end{bmatrix}$$

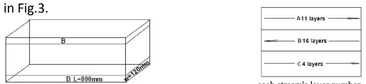
Analytical solutions

According to $[Z] = [\chi]^{-1}[T]$, there is $[T] = [\chi][Z]$. Inserting boundary conditions into $[T] = [\chi][Z]$, we can get the values of $C_j(j = 1; 2; ...; n^2)$ and then the temperature distribution in the heat exchanger can be obtained.

Results

Sample Heat Exchanger

The examples are as follow: a heat exchanger's framework and each stream's layer number is as shown in Fig.3.



a. heat exchanger's dimension

each stream's layer number

The layers' arrangement [BA]/5 [BC]/2 BAB [CB]/2 [AB]/6

Fig.3.heat exchanger's framework

Its plate-fins are as shown in the following fig, the dimension are as shown in Fig.4 and Table.1.

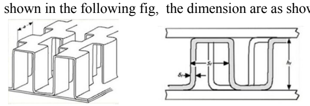


Fig.4.heat exchanger's plate-fins' shape

Table 1: The plate-fins' dimension of example heat exchanger 1

s_f(mm)	h_f(mm)	a(mm)	δ_f(mm)	Δ(mm)
1.4	4.7	3	0.2	1.1

Table.1.heat exchanger's plate-fins' dimension

Inlet parameters

Each stream's mass flow, inlet temperature and inlet pressure are as shown in the following Table.2.

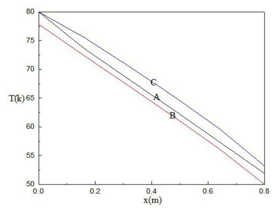
Table 2: Each stream's inlet parameters and mass flow

stream	T_{in}(K)	P_{in}(bar)	q_m(g/s)
A	80	13	18
B	50	12	28
C	80	6	10

Temperature distribution

Using the method and the analytical solutions in the present paper, we can get that the temperature distribution in this example heat exchanger is as shown in the Fig.5.

Fig.5.Each stream's temperature distribution of the example heat exchanger



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