Review on Bell-Steinberger relation

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Workshop on T violation and CPT tests in neutral-meson systems
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- PDG minireview with M. Antonelli
- Based also on work with KLOE collaboration and Gino Isidori
- see also Physics with KLOE-2 arXiv:1003.3868, EPJ C6 8 (2010) 619

Outline

- Motivation
  - The CPT symmetry
- Bell-Steinberger: history
- Bell-Steinberger: update
- Conclusions

CPT symmetry

- Hermiticity of the Hamiltonian (probabilty conservation), QFT
- Locality
- Lorentz invariance

$\Longrightarrow$ CPT conservation

CPT violated at the Plank scale

- Quantum gravity may lead to CPT violation
- The low energy limit not known
- Interesting probe

$$|M_K - M_{\bar{K}}| < 10^{-18} M_K$$

CPT violation: Non-locality?

- Non-locality is enough? Barenboim,Lykken

Add to the Dirac lagrangian

$$\mathbf{S} = \frac{i\eta}{\pi} \int d^3x \int dt dt' \, \bar{\psi}(t, \mathbf{x}) \, \frac{1}{t - t'} \, \psi(t', \mathbf{x}).$$

Run into trouble with causality

We want still keep states that go from an initial state to a final state in a S-matrix approach

CPT violation: Break Lorentz invariance

Change the coefficient of the square of the magnetic field in the Lagrangian of quantum electrodynamics:

$$\vec{B}^2 \to (1+\epsilon)\vec{B}^2$$

This will cause the velocity of light c, given by $c^2 = 1+\epsilon$ to differ from the maximum velocity of particles, which remains equal to one.

$$\mathcal{L}^{eff}_{SU(3)\times SU(2)\times U(1)} = \frac{1}{2} i\overline{\psi}\Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \overline{\psi} M \psi$$

Spurions break Lorenz sym. Coleman-Glashow,Kostelecky et al.

$$\Gamma^\nu \equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + i f^\nu \gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu}$$

Other approaches, phenomenologically driven

- Departure from Wigner-Weisskopf approximation adding a decoherence parameter tested experimentally Lindblad, Gorini et al.

  Results from CPLEAR and KLOE

- Violation at the Planck scale of Poincare' symmetry codified by modifications of the type

$$m^2 = E^2 - p^2 + \eta \, p^2 \, \frac{E^n}{M_{Pl}^n} + ..$$

difficult to write an effective field theory Amelino-Camelia, Smolin et al.

QM mechanics violated in a perturbative way

- Quantum decoherence induced by Planck scale effects and concepts of local effective field theory, including CPT operator are ill defined Wald
- Pragmatic approach Ellis et al., Huet, Peskin
  paramaterisation in the neutral kaon system by means of an evolution of the reduced density matrix for kaons, which satisfies energy conservation on average, positivity of the density matrix and probability conservation

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \, \rho$$

$\delta \not{H}$ parametrized by α, β, γ determined experimentally

Decoherence, Ill-defined CPT Operator and new effects in Entangled States

Maybe Planck scale physics modify for instance the usual relation between particle and antiparticle resulting as a deviation of CPT theorem without requiring Lorentz violation Bernabeu et al., Kurkov Franke

Modification of EPR relations Bernabeu et al.

QM mechanics must be valid even if $C\not{P}T$ in the K's mass matrix

$$i\frac{d}{dt}\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right] = [M - i\Gamma/2]\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right]$$

For any superposition of K_S, K_L mass and width eigenstates

$$|\Psi\rangle = a|K_S\rangle + b|K_L\rangle$$

$$\sum_\Gamma |\langle\Gamma | T | \Psi\rangle|^2 = -\frac{d}{d\tau} | \Psi |^2$$

Bell Steinberger relations

Terms proportional to $\mid a \mid^2$ and $\mid b \mid^2$

$$\Gamma_L = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_L \rangle \mid^2 \qquad \Gamma_S = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_S \rangle \mid^2$$

Mixed terms, proportional to $ab^* \Longrightarrow$

$$-i(M_L^* - M_S)\langle K_L \mid K_S \rangle = \sum_\Gamma \int d\Gamma \overbrace{\left(\langle \Gamma \mid T \mid K_L \rangle \right)^* \langle \Gamma \mid T \mid K_S \rangle}^{\alpha_f} .$$

$C\not{P}T$ in the K's mass matrix

$$\text{Diagonalize} \qquad \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$$

$$K_{S,L} = \frac{1}{\sqrt{2\,(1+|\epsilon_{S,L}|^2)}} \left[(1+\epsilon_{S,L})\,K^0 + (1-\epsilon_{S,L})\,\bar{K}^0 \right]$$

$$\begin{aligned} \epsilon_{S,L} &= \frac{-i\Im\,(M_{12}) - \frac{1}{2}\Im\,(\Gamma_{12}) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\,(\Gamma_{11} - \Gamma_{22}) \right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &= \epsilon_M \mp \Delta \end{aligned}$$

$$\epsilon_M \equiv |\epsilon_M| e^{i\varphi_{SW}} \qquad \tan\varphi_{SW} = \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$$

$C\not{P}T$ in semileptonic decays

$$
\begin{aligned}
A(K^0 \to l^+ \nu \pi^-) &= a + b = a(1-y) \\
A(K^0 \to l^- \nu \pi^+) &= c + d = a^*(x_+ - x_-)^* \\
A(\bar{K}^0 \to l^- \nu \pi^+) &= a^* - b^* = a^*(1+y)^* \\
A(\bar{K}^0 \to l^+ \nu \pi^-) &= c^* - d^* = a(x_+ + x_-)
\end{aligned}
$$

$b, d \quad (y, x_-) \qquad C\not{P}\not{T} \qquad\qquad c, d \quad (x_+, x_-) \qquad \Delta S = -\Delta Q$

$$
A_{S,L} = \frac{\Gamma^{l^+}_{S,L} - \Gamma^{l^-}_{S,L}}{\Gamma^{l^+}_{S,L} + \Gamma^{l^-}_{S,L}} = 2\Re(\epsilon_{S,L}) + 2\Re\left(\frac{b}{a}\right) \mp 2\Re\left(\frac{d^*}{a}\right)
$$

- $A_S - A_L = 4(\Re(\Delta) + \Re(x_-)) \qquad A_S + A_L = 4(\Re(\epsilon_M) - \Re(y))$

$C\not{P}T$ **in** $K \to \pi\pi$

$$A(K^0 \to \pi\pi(I)) \equiv (A_I + B_I)e^{i\delta_I}$$
$$A(\bar{K}^0 \to \pi\pi(I)) \equiv (A_I^* - B_I^*)e^{i\delta_I}$$

- B_I is $C\not{P}T$ as $\left(\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}} \quad \eta_{00}=|\eta_{00}|e^{i\phi_{00}}\right)$

$$\phi_{+-} - \phi_{00} = 0.2 \pm 0.4^o \qquad \text{KTEV,NA48}$$

$$\phi_{+-} = 43.51 \pm 0.05 \text{Assuming CPT} \quad 43.4 \pm 0.5 \text{ Not Assuming CPT}$$

Bell Steinberger relations: CPLEAR,KTeV,NA48,KLOE

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2}-i\Im(\Delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f \overbrace{A_L(f)A_S^*(f)}^{(\alpha_f)}$$

Also a new analysis by Gino+KLOE and me

Actual SM expectations for Bell Steinberger relations, KLOE+Gino

Channel	$B(K_S)$	$B(K_L)$	$10^5\ \alpha_f^{\rm SM}$
$\pi^+\pi^-(\gamma)$	0.69	2.1×10^{-3}	$110.8+105.1i$
$\pi^0\pi^0$	0.31	9.3×10^{-3}	$49.2+46.6i$
$\pi^\pm e^\mp\nu$	6.7×10^{-4}	0.39	$0.22+0.00i$
$\pi^\pm\mu^\mp\nu$	4.7×10^{-4}	0.27	$0.17+0.00i$
$\pi^0\pi^0\pi^0$	1.9×10^{-9}	0.21	$0.06+0.06i$
$\pi^+\pi^-\pi^0$	2.7×10^{-7}	0.12	$0.04+0.04i$
$\pi^+\pi^-\gamma_{\rm DE}$	10^{-5}	10^{-5}	<0.01

α_f determinations for Bell Steinberger relations

$$\alpha_{\pi^+\pi^-} = ((1.112 \pm 0.013) + i(1.061 \pm 0.014)) \times 10^{-3}$$
$$\alpha_{\pi^0\pi^0} = ((0.493 \pm 0.007) + i(0.471 \pm 0.007)) \times 10^{-3}$$
$$\alpha_{\pi^+\pi^-\pi^0} = ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6}$$

$\alpha_{\pi\pi\pi}$ from CPLEAR, NA48, KLOE

Time dependent studies: CPLEAR

CPLEAR Study of tagged $K^0(\bar{K}^0)$

$$\frac{\Gamma(K^0(t) \to f) - \Gamma(\bar{K}^0(t) \to f)}{\Gamma(K^0(t) \to f) + \Gamma(\bar{K}^0(t) \to f)}$$

$$\left[\Re\left(A_L^f A_S^{f*}\right)\cos(\Delta m t) + \Im\left(A_L^f A_S^{f*}\right)\sin(\Delta m t)\right]$$

$$\Longrightarrow \Delta m, \qquad A(K_{L,S} \to \pi^+\pi^-\pi^0)\,, \qquad A(K_{L,S} \to \pi^0\pi^0\pi^0)\,,$$

$$A(K_{L,S} \to \pi l \nu)$$

KLOE determination of semileptonic $\alpha_{\pi l\nu}$

KLOE adds the measurement of $A_S - A_L = 4[\Re(\delta) + \Re(x_-)] = (-2 \pm 10) \times 10^{-3}$. The results, referred to as the $K_{\ell 3}$ average, are given in

	value	Correlation coefficients				
$\Re(\Delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1				
$\Im(\Delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	-0.27	1			
$\Re(x_-)$	$(-0.3 \pm 0.21) \times 10^{-2}$	-0.23	-0.58	1		
$\Im(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	-0.35	-0.12	0.57	1	
$A_S + A_L$	$(-0.4 \pm 0.83) \times 10^{-2}$	-0.12	-0.62	0.99	0.54	1

Bell-Steinberger determination

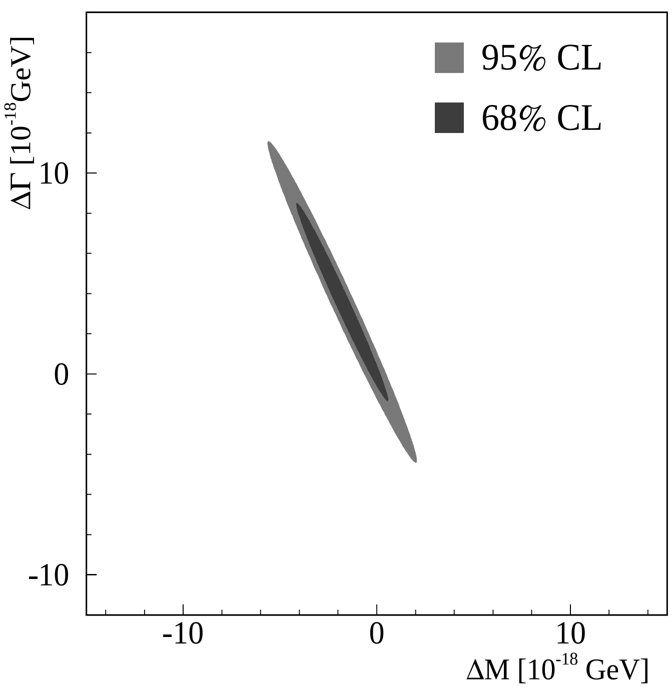
Unitarity allows a determination of $\Re(\epsilon)$ not requiring CPT

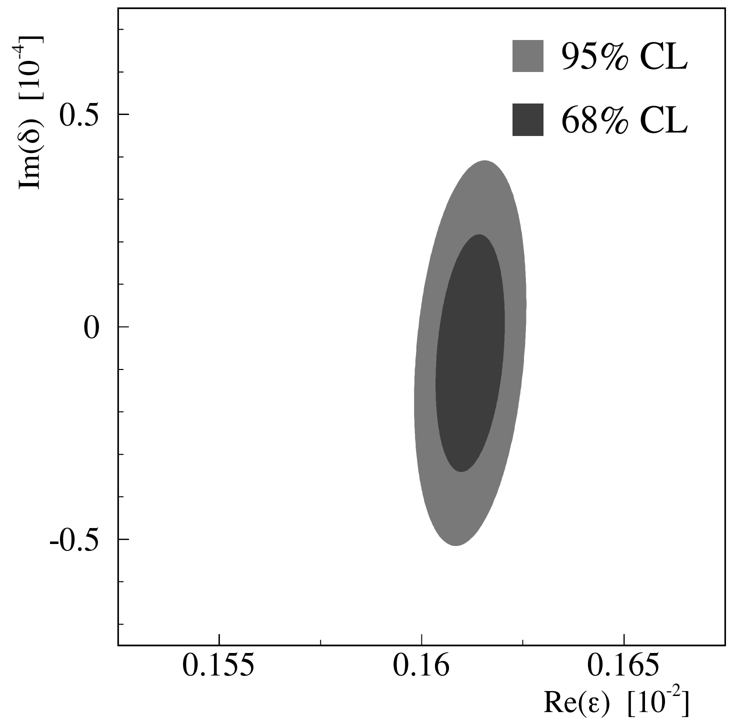
$$\Re(\epsilon) = (161.1 \pm 0.5) \times 10^{-5}, \qquad \Im(\Delta) = (-0.7 \pm 1.4) \times 10^{-5}$$

$$\Delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] \, .$$

$$-4.0 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\overline{K}^0} < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at 95 \% CL} \, .$$

improving CPLEAR $|m_{K^0} - m_{\overline{K}^0}| < 12.7 \times 10^{-19}$ GeV at 90% CL


95% CL
68% CL
ΔΓ [10⁻¹⁸GeV]
ΔM [10⁻¹⁸ GeV]
10
0
-10
-10
0
10


95% CL
68% CL
Im(δ) [10⁻⁴]
Re(ε) [10⁻²]
0.5
0
-0.5
0.155
0.16
0.165

Conclusions

- We are seeing the determination of the unitarity of the row of the CKM matrix at 0.2% fighting for small possible NP contributions; HERE we do not even know how large is NP contributions: could be very large..

- Remember CP lesson (not theoretically predicted)

- Scaling argument correct?

CPLEAR determination of semileptonic $\alpha_{\pi l\nu}$

$$\sum_{\pi\ell\nu}\langle\mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle = 2\Gamma(K_L\to\pi\ell\nu)\left(\Re(\epsilon)-\Re(y)-i(\Im(x_+)+\Im(\Delta))\right)$$
$$= 2\Gamma(K_L\to\pi\ell\nu)\left((A_S+A_L)/4-i(\Im(x_+)+\Im(\Delta))\right)$$

Time asymm. allow CPLEAR to obtain $\alpha_{\pi l\nu}$ from

	value	Correlation coefficients			
$\Re(\Delta)$	$(3.0\pm 3.4)\times 10^{-4}$	1			
$\Im(\Delta)$	$(-1.5\pm 2.3)\times 10^{-2}$	0.44	1		
$\Re(x_-)$	$(0.2\pm 1.3)\times 10^{-2}$	-0.56	-0.97	1	
$\Im(x_+)$	$(1.2\pm 2.2)\times 10^{-2}$	-0.60	-0.91	0.96	1

backup

$$\left(\frac{\Gamma_S+\Gamma_L}{2}-i\Delta m\right)\frac{2\mathrm{Re}(\tilde{\epsilon})}{1+\mid\tilde{\epsilon}\mid^2}=\sum_\Gamma\int d\Gamma\left(\langle\Gamma\mid T\mid K_L\rangle\right)^*\langle\Gamma\mid T\mid K_S\rangle$$

Using the Schwartz inequality

$$\left|\frac{\Gamma_S+\Gamma_L}{2}-i\Delta m\right|\frac{2\mathrm{Re}(\tilde{\epsilon})}{1+\mid\tilde{\epsilon}\mid^2}\leq\sqrt{\Gamma_L\Gamma_S}\implies\frac{2\mathrm{Re}\tilde{\epsilon}}{1+\mid\tilde{\epsilon}\mid^2}\leq 2.9\times 10^{-2}$$

$\implies$ BS relation among $\Gamma_L, \Gamma_S, \Delta m, p, q$ and α_f

BS assume $\langle K_L \mid K_S \rangle = \dfrac{\mid p \mid^2 - \mid q \mid^2}{\mid p \mid^2 - \mid q \mid^2} \overset{BS}{=} \dfrac{2\mathrm{Re}(\tilde{\epsilon})}{1 + \mid \tilde{\epsilon} \mid^2}$