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A BLUE implementation in BAT

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Overview

- The problem: combining several measurements with Gaussian uncertainties
- Solutions: BLUE and the Maximum Likelihood method / Bayesian inference
- The implementation in BAT
- Conclusions

The problem

The problem:

- Aim: combine several measurements
- Context: TOPLHC WG
- Difficulties:
  - One or several observables, e.g. top-quark mass, W-helicity fractions, etc.
  - Correlations between measurements, e.g. via sources of systematic uncertainty, partially overlapping data sets, etc.
  - Correlations between observables, e.g. W-helicity fractions (sum=1)
  - Technical problems

Solutions

- BLUE: Best Linear Unbiased Estimator
  - Construct weighted average which is
    - linear,
    - unbiased, and
    - has best (smallest) variance

L. Lyons, D. Gibaut, and P. Clifford,
How to combine correlated estimates of a single physical quantity,
Nucl. Instrum. Meth. A 270 (1988) 110.

A. Valassi,
Combining correlated measurements of several different physical quantities,
Nucl. Instrum. Meth. A 500 (2003) 391.

- ML Method/Bayesian inference:
  - Construct statistical model with Gaussian PDFs including correlations
  - Calculate maximum and estimate uncertainty
  - Traditionally done using Minuit

Example: W-helicity combination

Technical problems encountered:

- Limited to 20 input measurements
  → cannot combine all single measurements
- No information about correlation due to individual sources of uncertainty
  → Can not easily combine combinations (arithmetic mean of corr. coefficients)
- Not possible to include constraints (or priors)
  → sum of fractions equals unity
- No proper propagation of uncertainty
  → rely on Gaussian estimate for right-handed fraction (also neg. values)
- No measure for consistency of measurements
  → Quantify agreement of measurements "by eye"
- No uncertainty on correlation coefficients
  → Simple stability checks

All these issues are solved in the BAT implementation

BAT
BAYESIAN ANALYSIS TOOLKIT
http://www.mppmu.mpg.de/bat/

Repeat combination with BAT implementation:

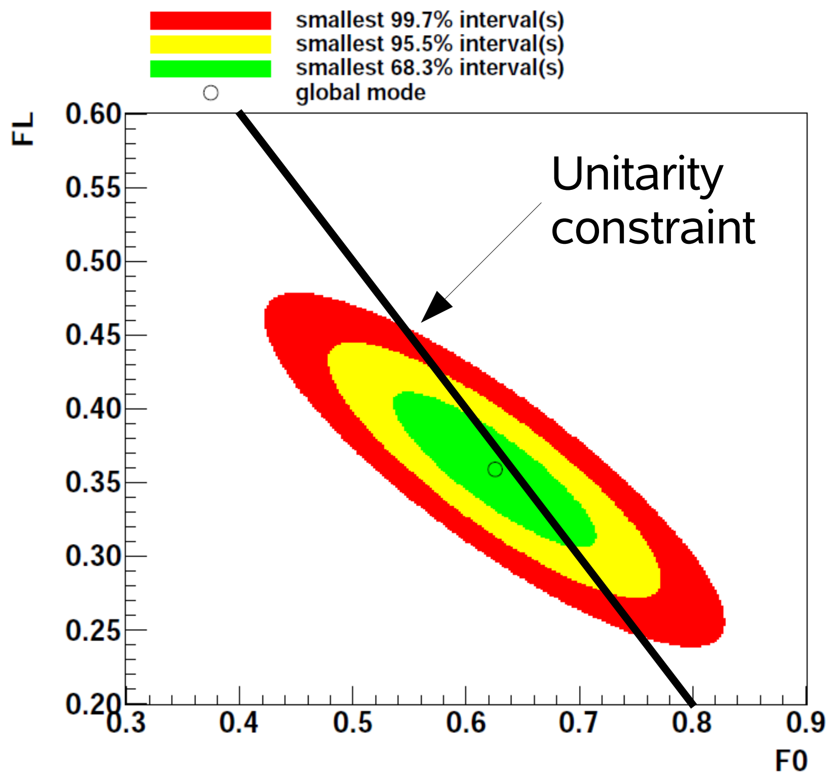
- BLUE results:
  - F0: 0.626 ± 0.034 (stat.) ± 0.048 (syst.)
  - FL: 0.359 ± 0.021 (stat.) ± 0.028 (syst.)
- Weights [%]:

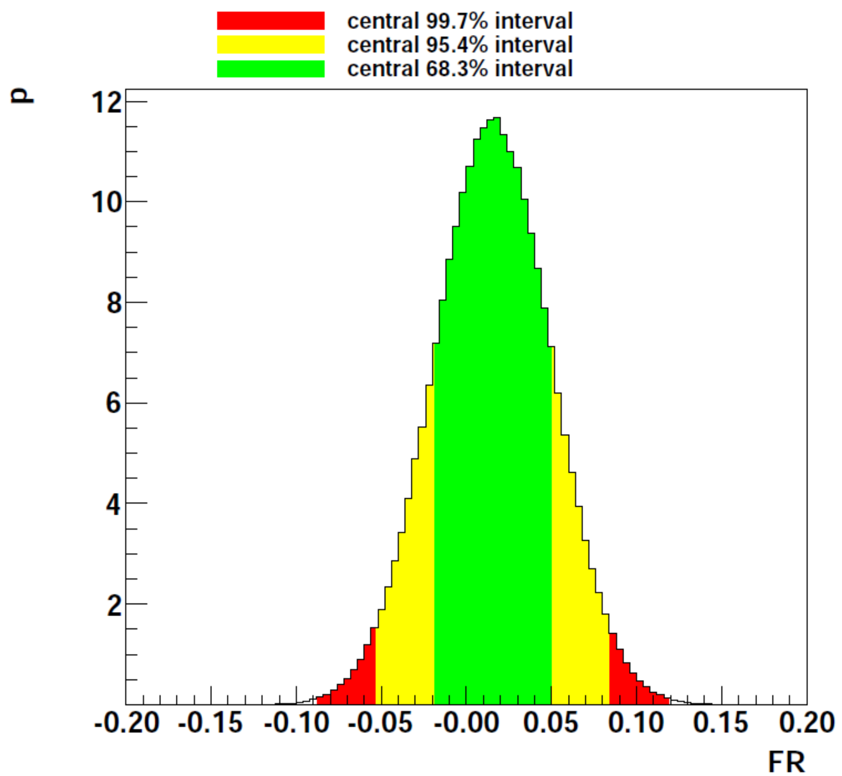
A10lj	F0:	+12.2	+7.4
A10lj	FL:	+19.0	+11.6
A11lj	F0:	+39.4	-8.4
A11lj	FL:	-16.0	+35.4
A11dil	F0:	+13.0	+2.8
A11dil	FL:	+4.9	+15.2
C11lj	F0:	+35.4	-1.8
C11lj	FL:	-7.9	+37.8

Same as is note (rounding)

Repeat combination with BAT implementation:

- ML/Bayesian inference without constraints: same as BLUE



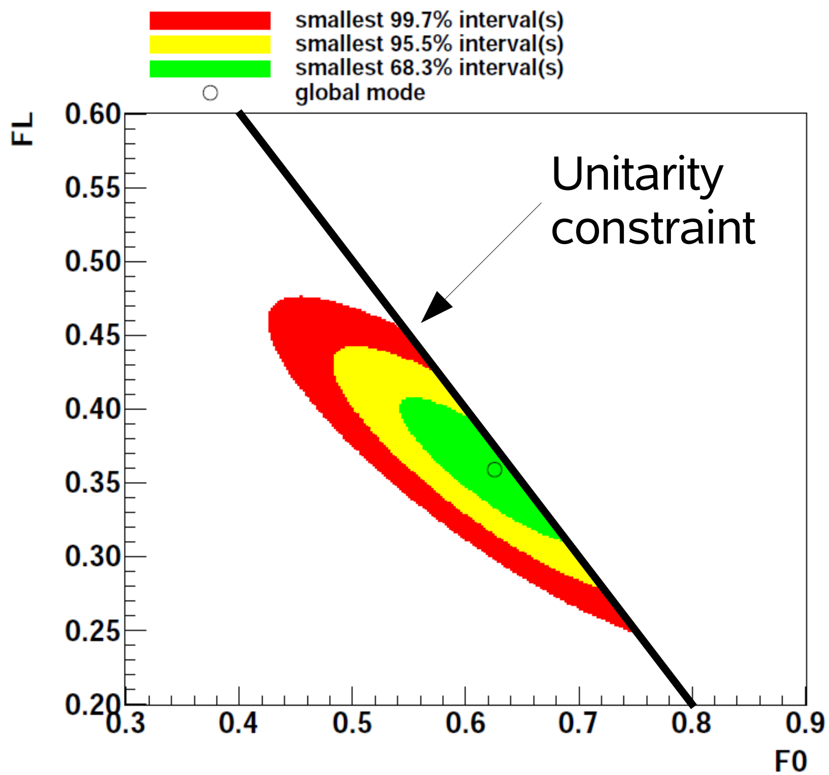


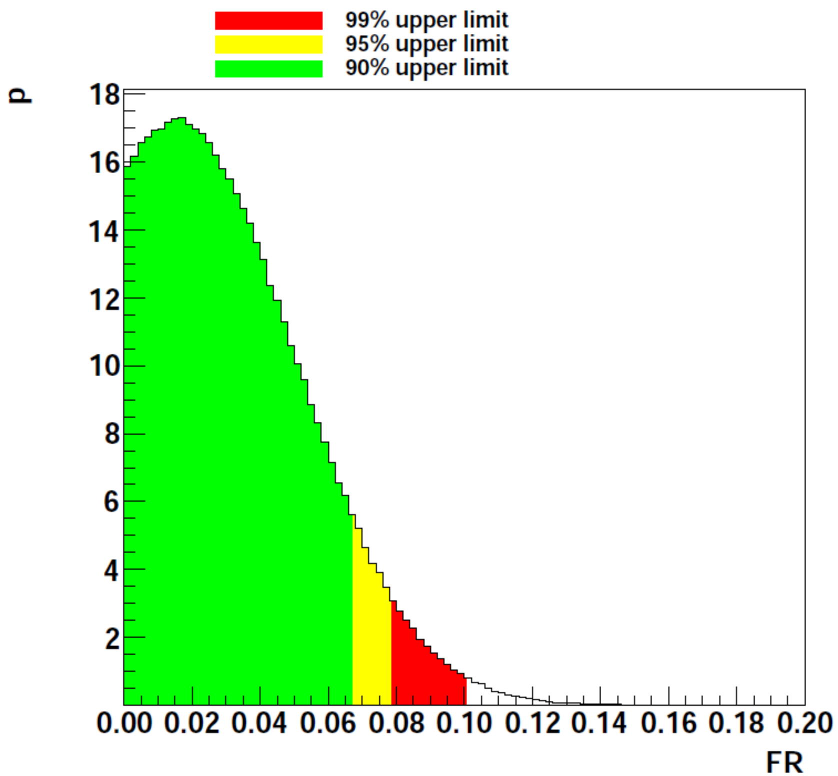
Global mode = BLUE solution
Propagation of uncertainty: FR can be negative

Repeat combination with BAT implementation:

- ML/Bayesian inference with constraints

NEW



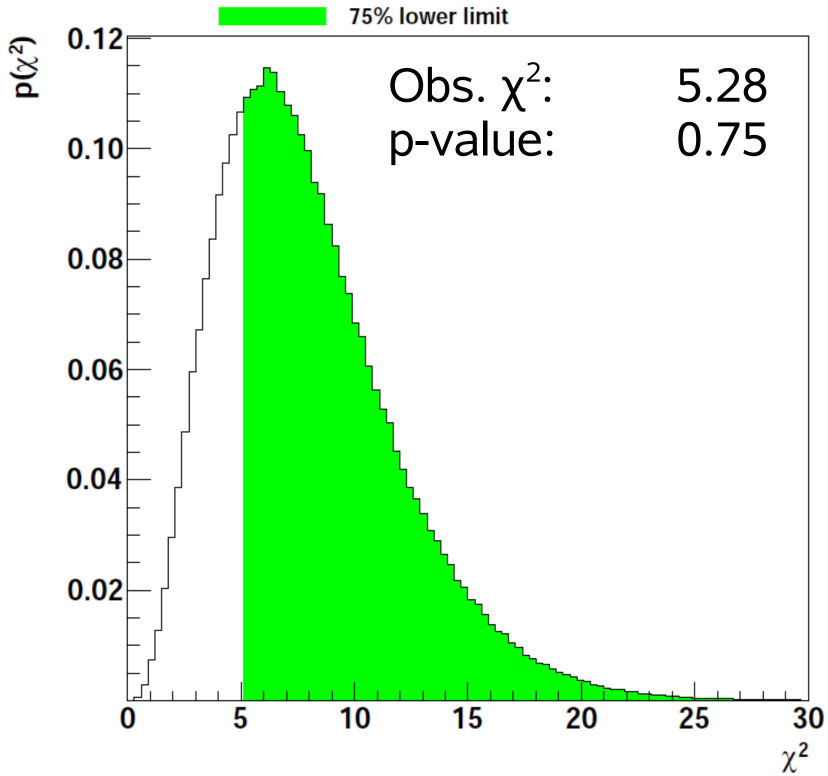


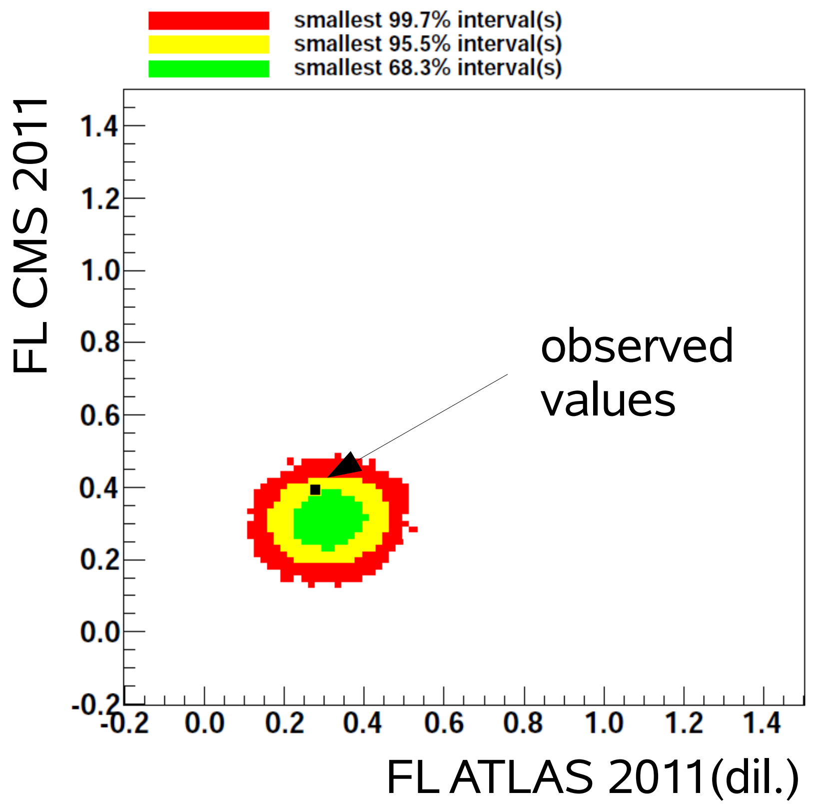
Non-trivial shape close to the boundary
Set limits on FR>0, do not quote a measurement

Repeat combination with BAT implementation:

NEW

- Goodness-of-fit: assume SM values and throw toys



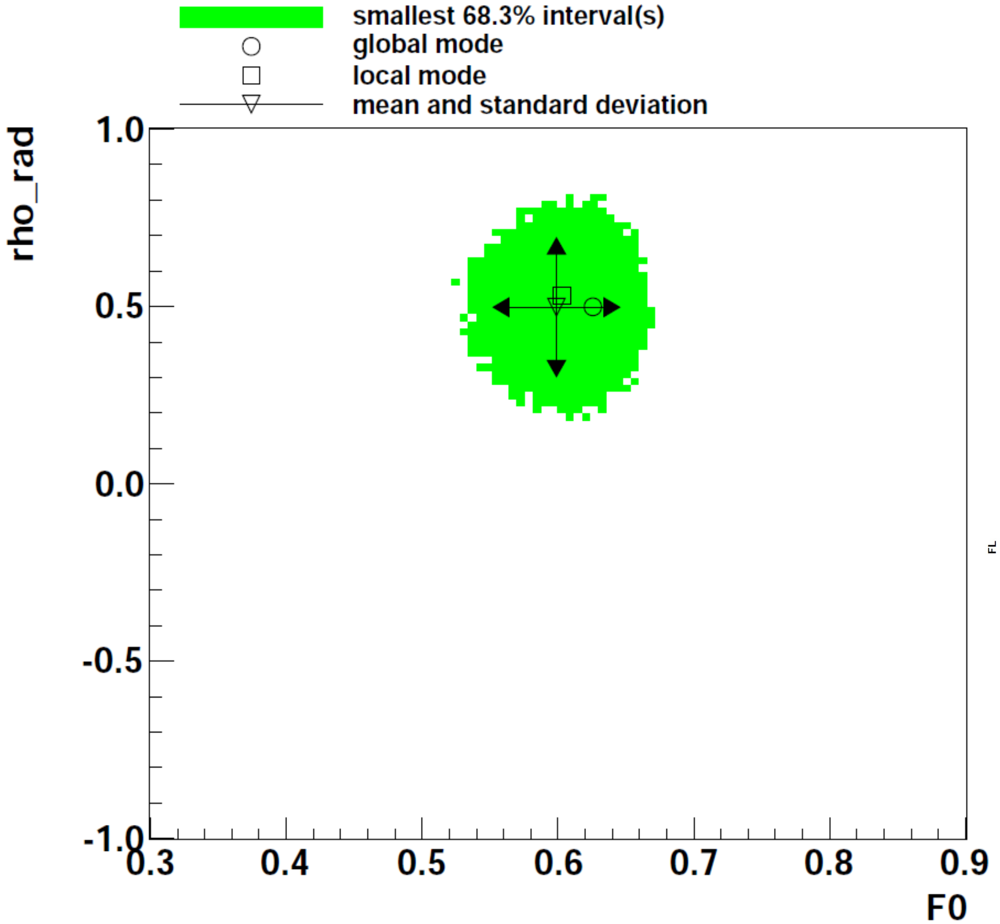


Good overall agreement
Agreement among measurements

NEW

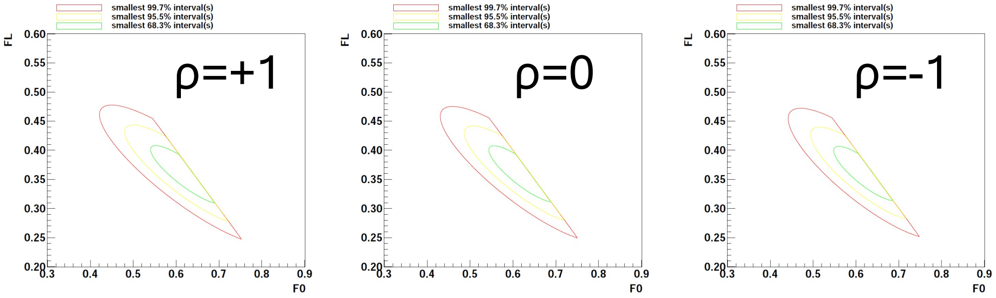
Repeat combination with BAT implementation:

- Parametrize correlation coefficient “Radiation” with nuisance parameter (prior: Gaussian with mean 0.5 and std. 0.2)



No strong dependence on correlation coefficient

Can also produce “slices” with fixed correlation coefficient



Conclusions:

- Features of BAT-implementation:
  - Not limited in the number of input measurements
  - Possible to include constraints and priors
  - Information about correlation due to individual sources of uncertainty
  - (non-Gaussian) Propagation of uncertainty
  - Goodness-of-fit tests:
    - Overall agreement
    - Agreement of individual measurements
  - Correlation coefficients can be parametrized with nuisance parameters
  - Interface is a simple text file (like in previous implementation)
- Code will be part of next BAT release
- If interested now: write me an email

Example: W-helicity combination

Physics case:

- Polarization of W bosons in top-quark decays:
  - Two observables, F0 and FL

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{8}(1+\cos\theta^*)^2 F_R + \frac{3}{8}(1-\cos\theta^*)^2 F_L + \frac{3}{4}\sin^2\theta^* F_0$$