Amplitude detuning measurement with AC dipole

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Outline

Amplitude detuning measurement

motivations, method and past measurements

Amplitude detuning with driven oscillations

theory, simulations

LHC experimental data

results of 2012 MD

Summary and outlook

possible improvements, experimental proof of theoretical approach

Motivations

- Amplitude detuning measurement provides a handle on how well the non-linear model is understood
- Lots of discussion last year regarding Landau damping and beam stability:
  - How does the machine detuning contributes to the overall detuning (long-range beam-beam , Landau octupoles)?
  - Is there any compensation effects?
- The tune kicker is effective at injection. Large amplitude can be reached and fresh beams can be injected when the emittance becomes too large
- This is not the case at top energy. The AC dipole provides a non-destructive alternative allowing for large amplitude excitation at any energy

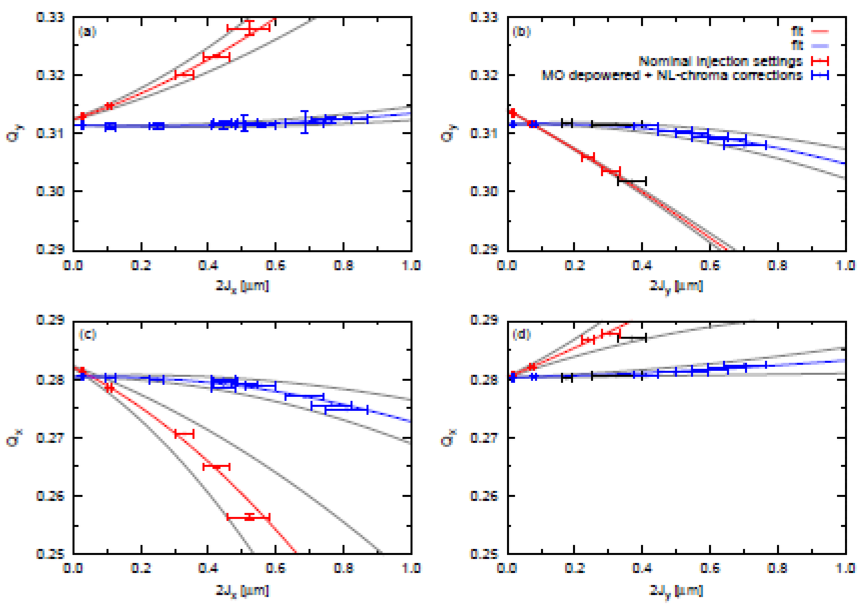
Principle

In the presence of non-linear fields the tune becomes amplitude dependent. Its behavior can be modeled by a polynomial of the form:

$$Q_x = Q_0 + \frac{\partial Q_x}{\partial \epsilon_x}\epsilon_x + \frac{\partial Q_x}{\partial \epsilon_y}\epsilon_y + \frac{1}{2}\frac{\partial^2 Q_x}{\partial \epsilon_x^2}\epsilon_x^2 + \frac{1}{2}\frac{\partial^2 Q_x}{\partial \epsilon_y^2}\epsilon_y^2 + \frac{\partial^2 Q_x}{\partial \epsilon_x \partial \epsilon_y}\epsilon_x \epsilon_y + \ldots$$

where $\epsilon = 2J$ and J is the action (normalized oscillation amplitude)

Measuring the tune as function of oscillation amplitude and applying a polynomial fit allows for a direct measurement of the detuning coefficients



→ Example of amplitude detuning measurement at the LHC

→ Done at injection energy using the tune kicker

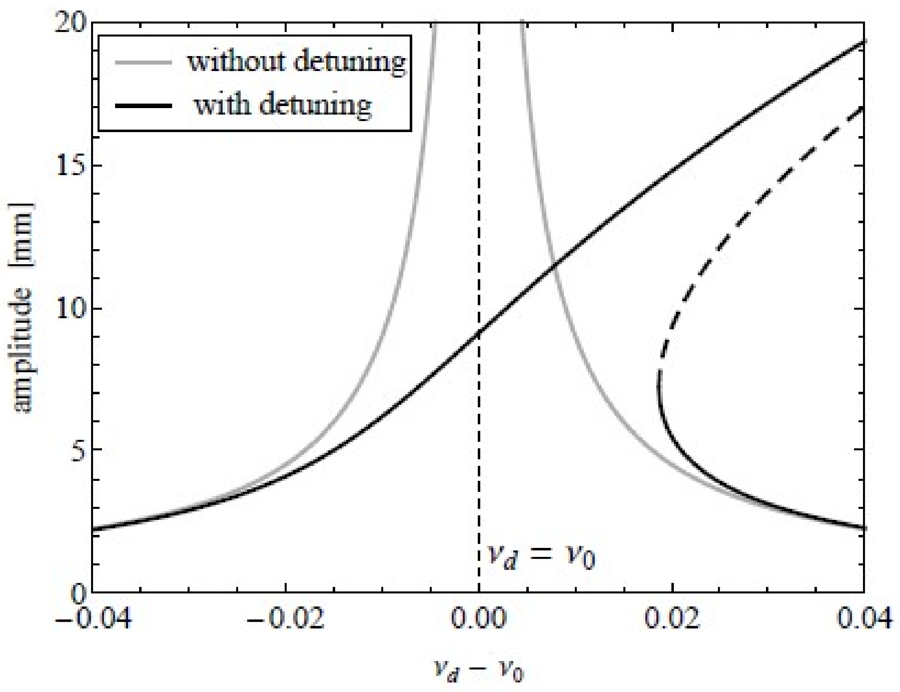
→ Reference: *E. Maclean et al. “Non-linear beam dynamics tests in the LHC: LHC dynamic aperture MD on Beam 2”*

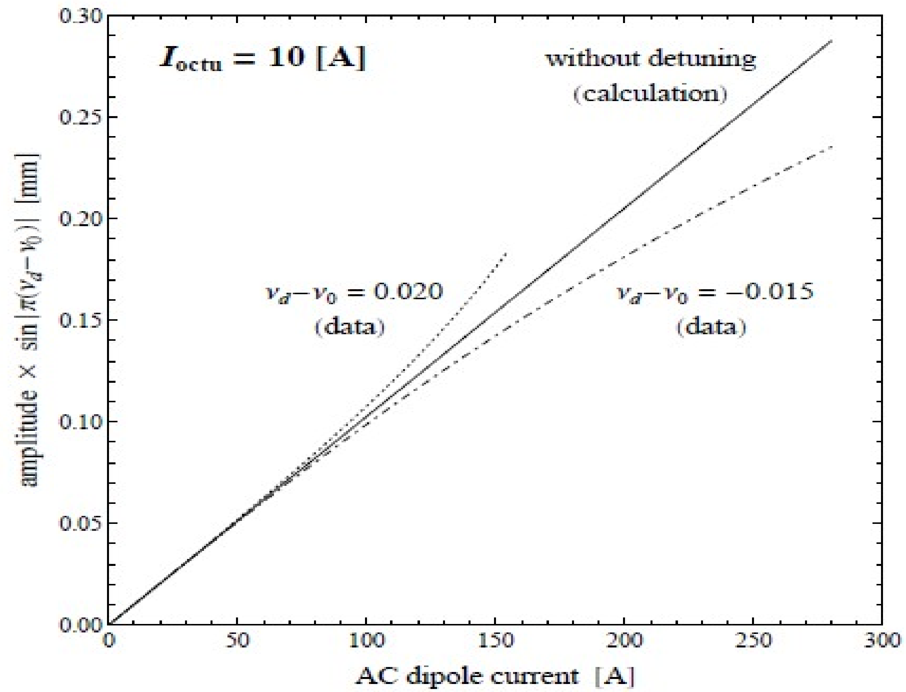
Tevatron experience

→ Reference: R. Miyamoto, PhD thesis

→ In the presence of detuning the beam response is non-linear with driven oscillation amplitude

→ This behavior provide information on detuning coefficient





→ Measurements taken with strong octupolar field

→ Plain curve is the linear case

→ Dashed curves represent a fit of the AC dipole ramp: could be used to derive the detuning – **indirect measurement**

Hamiltonian with AC dipole

The Hamiltonian describing the linear motion of a single particle in the presence of AC dipole can be expressed as:

$$H_0(x, p_x, s, t) = \frac{1}{2}p_x^2 + \frac{1}{2}K_x(s)x^2 + \delta(s,t)x$$

$K_x(s)$ is the focusing strength and $\delta(s,t)$ is the time dependent kick from the AC dipole

In the case of free oscillations the transverse coordinates can parametrized as:

$$x(s) = \sqrt{2J_x\beta_x(s)}\cos\phi_x$$
$$y(s) = \sqrt{2J_y\beta_y(s)}\cos\phi_y$$

For horizontal AC dipole excitation x(s) becomes:

$$x_D(s) = \sqrt{2J_x\beta_x(s)}\cos\phi_x + \sqrt{2A\beta'_x(s)}\cos\phi_D$$

→ The term J_x is driven by the non-adiabaticity of the ramping process of the AC dipole: in the presence of chromaticity of non-linearity it should be possible to observe the natural tune (*R. Tomas "Adiabaticity of the ramping process of an AC dipole" PRSTAB 8, 024401*).

Amplitude detuning

Using the perturbative approach, the Hamiltonian in the presence of non-linear field becomes H_0+H_1, where H_1 (perturbation Hamiltonian) is given by:

$$H_1 = \frac{q}{p}\mathrm{Re}\left[\sum_{m=3}^{\infty}\frac{1}{m}\left[B_m(s)+iA_m(s)\right](x+iy)^m\right]$$

$B_m(s)$ and $A_m(s)$ are the normal and skew coefficients of the expansion of the magnetic field of a multipole of order m. Considering the case of a normal octupole:

$$H_4 = \frac{q}{p}\frac{B_4(s)}{4}(x^4-6x^2y^2+y^4)$$

The detuning is given by: $$\Delta Q_{x,y} = \frac{1}{2\pi}\oint\frac{\partial}{\partial J_{x,y}} < H_4 > ds$$

Using the parametrized coordinates in the presence of AC dipole we get:

$$\Delta Q_x = \frac{q}{p}\frac{3B_4}{8\pi}(\beta_x^2 J_x + 2\beta_x\beta'_x A)$$ and $$\Delta Q_y = -\frac{q}{p}\frac{3B_4}{8\pi}(2\beta'_x\beta_y A + 2\beta_x\beta_y J_x)$$

→ The **direct term of the detuning measured with a single kick (*A=0*) is a factor 2 smaller than the one measured with AC dipole** (*A>>J*). Effect of the AC dipole on optics are assumed negligible ($\beta'=\beta$). **The cross term is not affected**.

Multipole of order $2n$

The perturbation Hamiltonian of a multipole of order $2n$ is expressed as:

$$H_{2n} = \frac{qB_{2n}}{2np}\mathrm{Re}\left[(x+iy)^{2n}\right]$$

This gives for the detuning coefficient in the case of free oscillations:

$$\Delta Q_x = \frac{qB_{2n}}{2np}\frac{2^{-n}}{2\pi}\binom{2n}{0}\binom{2n}{n}\beta_x^n\, nJ_x^{n-1}$$

$$\Delta Q_y = -\frac{qB_{2n}}{2np}\frac{2^{-n+1}}{2\pi}\binom{2n}{2}\binom{2n-2}{n-1}\beta_x^{n-1}\beta_y J_x^{n-1}$$

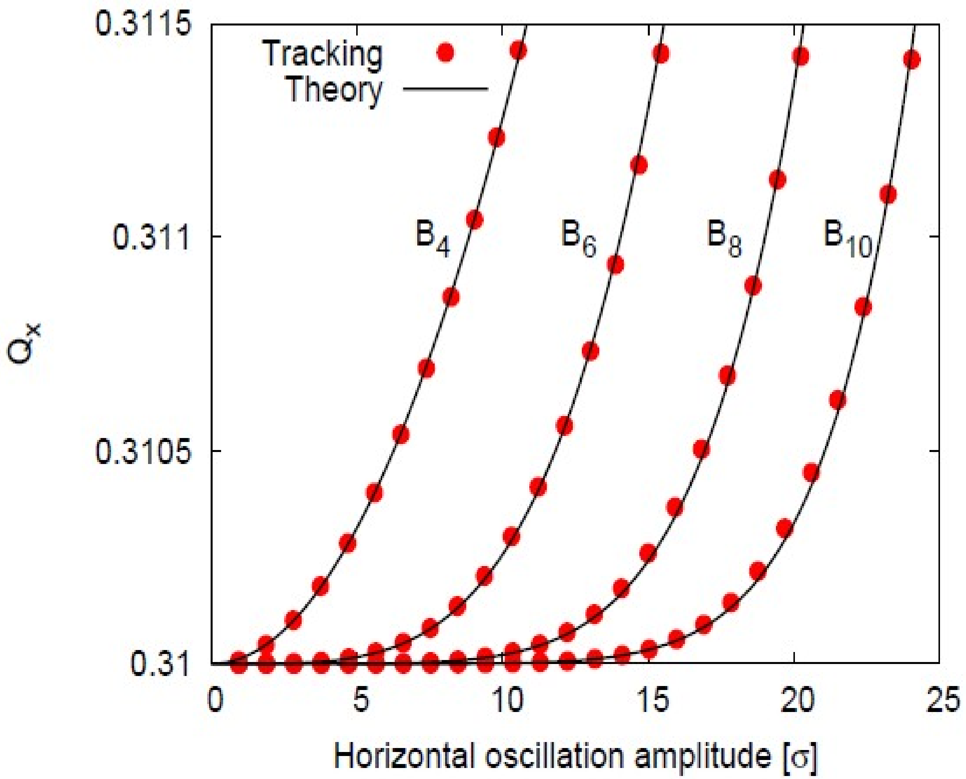
And in the case of driven oscillations

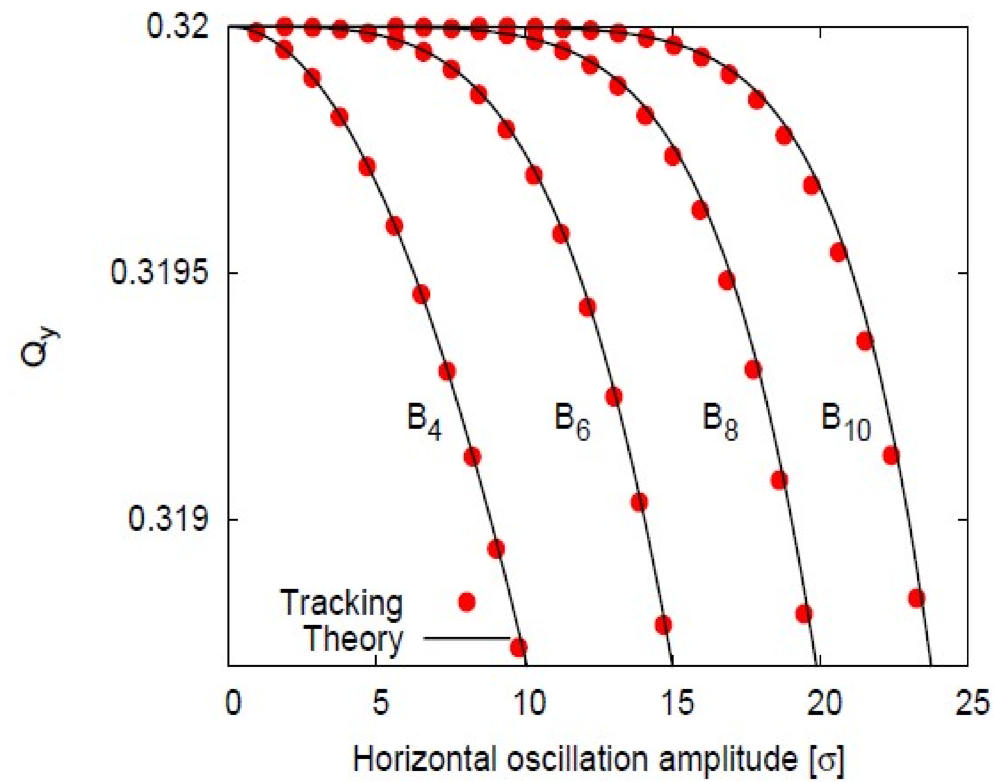
$$\Delta Q_x = \frac{qB_{2n}}{2np}\frac{2^{-n+1}}{2\pi}\binom{2n}{2n-2}\binom{2n-2}{n-1}\beta_x\beta_x'^{n-1}A^{n-1}$$

$$\Delta Q_y = -\frac{qB_{2n}}{2np}\frac{2^{-n+1}}{2\pi}\binom{2n}{2}\binom{2n-2}{n-1}\beta_y\beta_x'^{n-1}A^{n-1}$$

→ **Factor n difference between free and driven oscillation for the direct term**

Single particle tracking

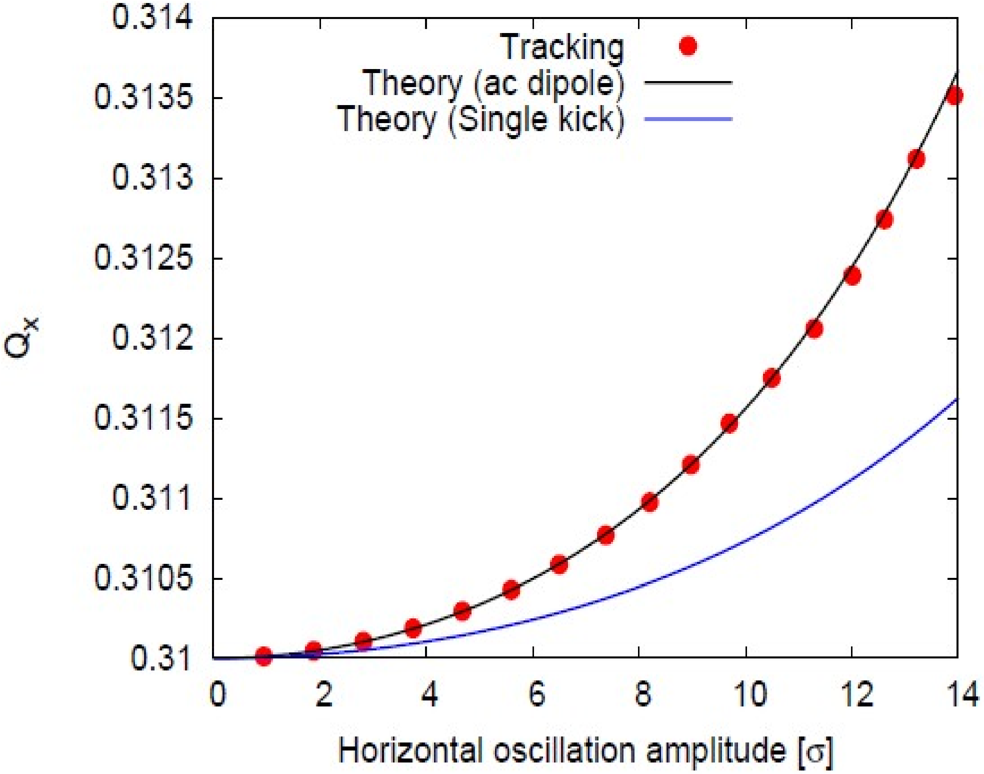


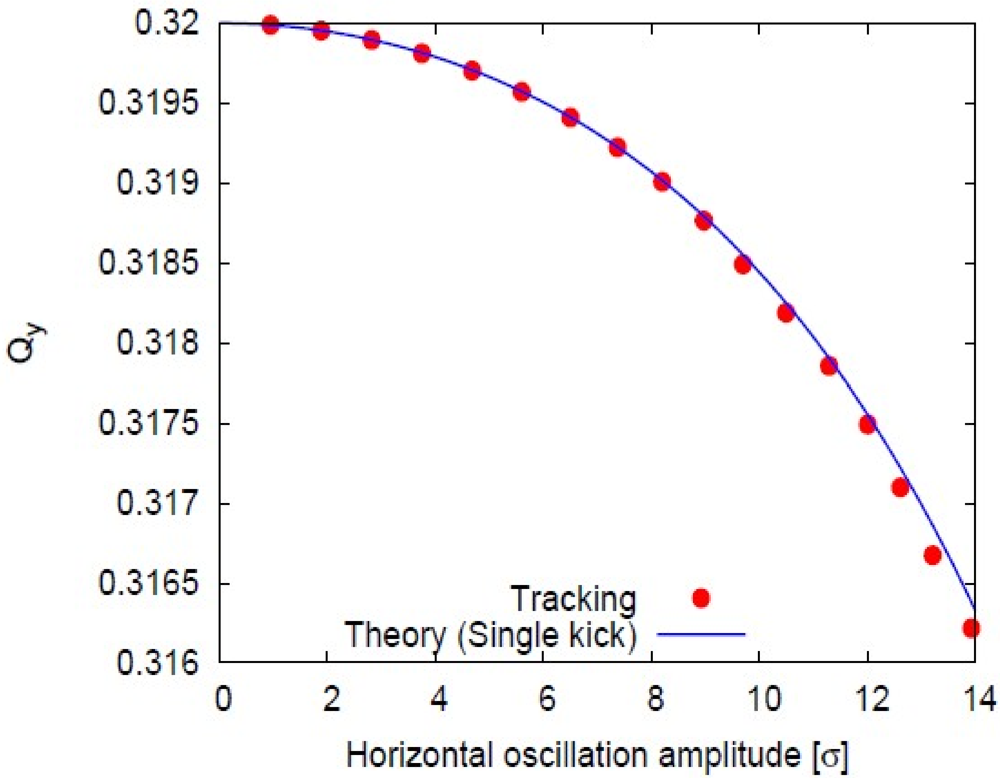


→ Built a simple tracking code allowing to study driven oscillation in the presence of arbitrary non-linear field components (behavior also checked with MADX, much slower)

→ Start with a single multipole: **excellent agreement between theory and tracking** up to a B_{10} magnetic field component

Combination of several orders

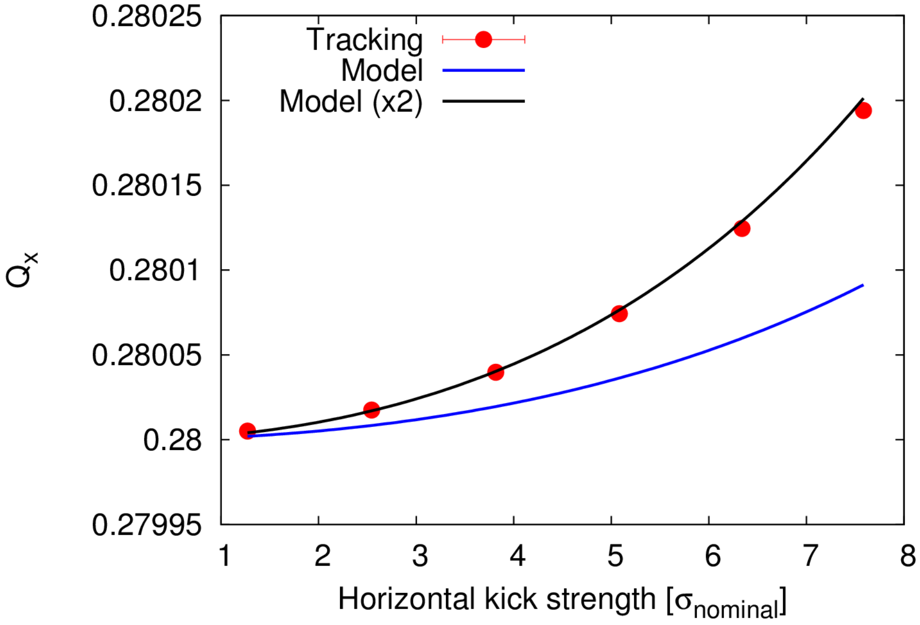


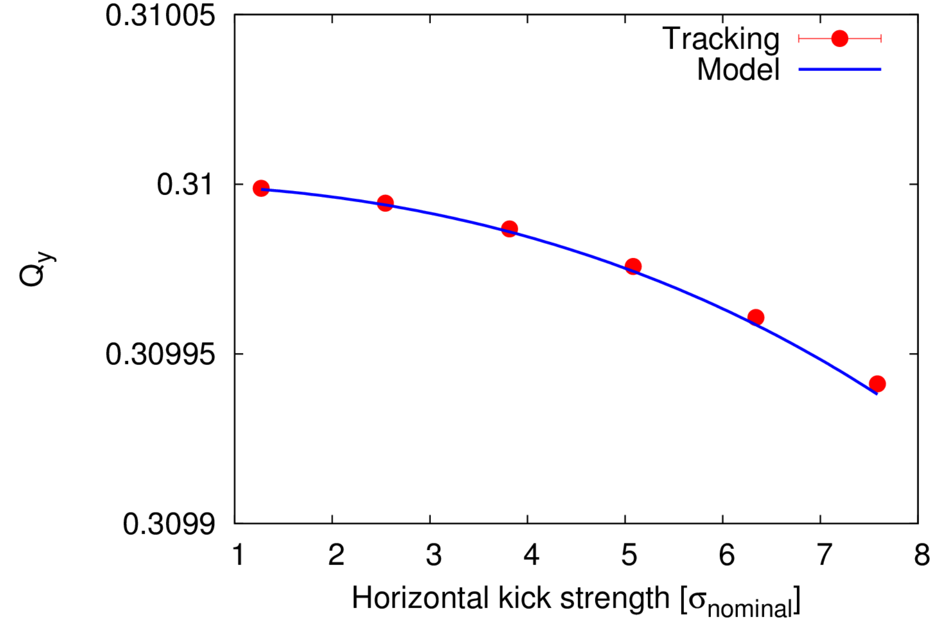


→ Repeat the same study with the combination of several multipoles from B_4 to B_{10}

→ Again an excellent agreement is observed between theory and tracking, **the under-estimation using free oscillation model is clearly observed**

Full LHC non-linear model



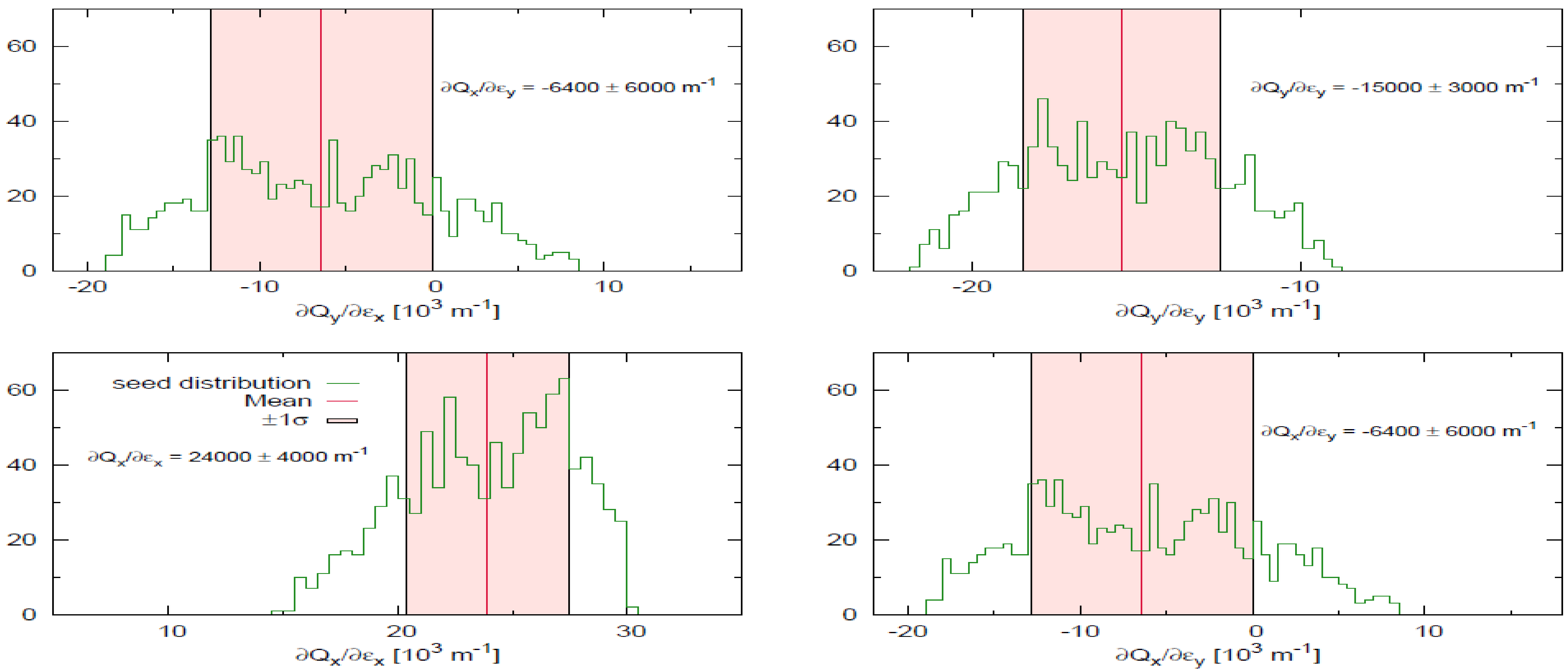


→ MADX model constructed using a thin lattice with magnetic field errors from WISE of order from (B_3, A_3) up to (B_{15}, A_{15}). Includes second order effects. Example of flat top optics in this case

→ The model detuning coefficients are computed using PTC, the tracking is performed using the AC dipole module in MADX

→ The detuning is dominated by the octupolar field components: **a factor 2 difference is observed between PTC and AC dipole tracking – consistent with theory**

Uncertainty due to coupling

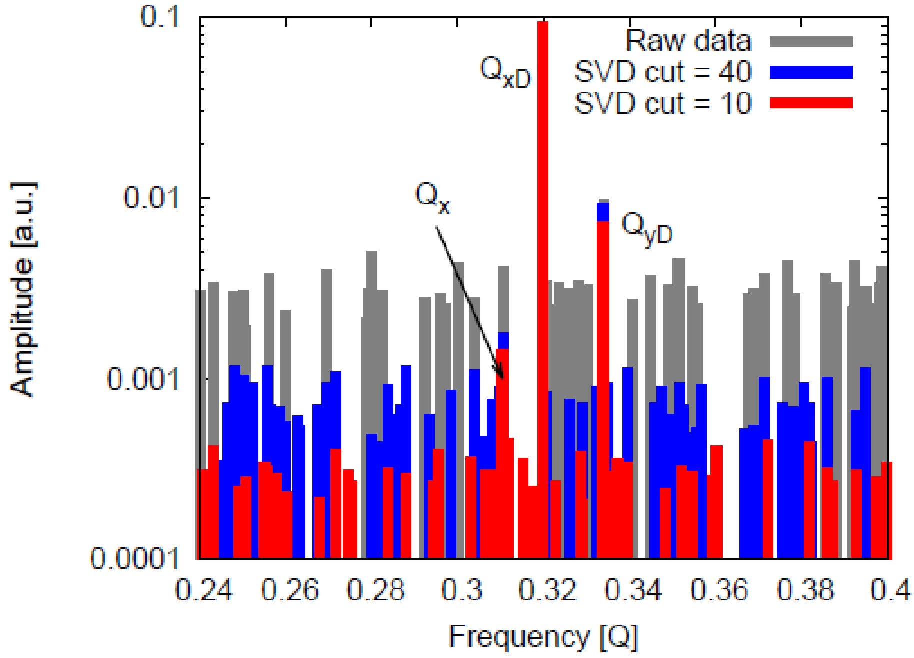


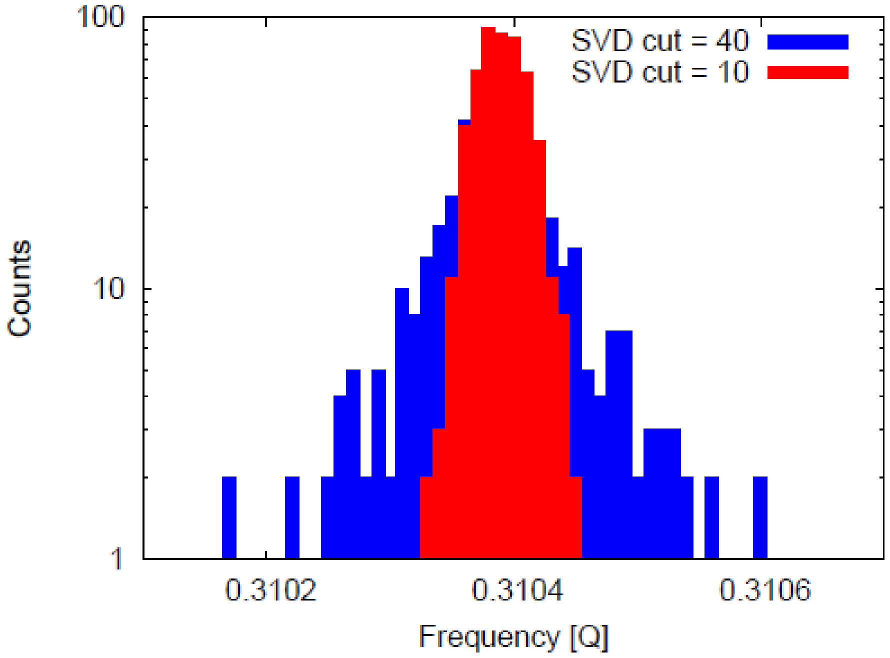
→ Uncertainties on the model apply in the presence of coupling or misalignments. **Due to technical limitation in MADX and PTC only the effects of coupling could be studied**

→ Distribution generated from random distribution of coupling amplitude and phase

→ **Uncertainties of the order of 20% for the direct term and 100% for cross term were derived from these simulations**

Natural tune measurement

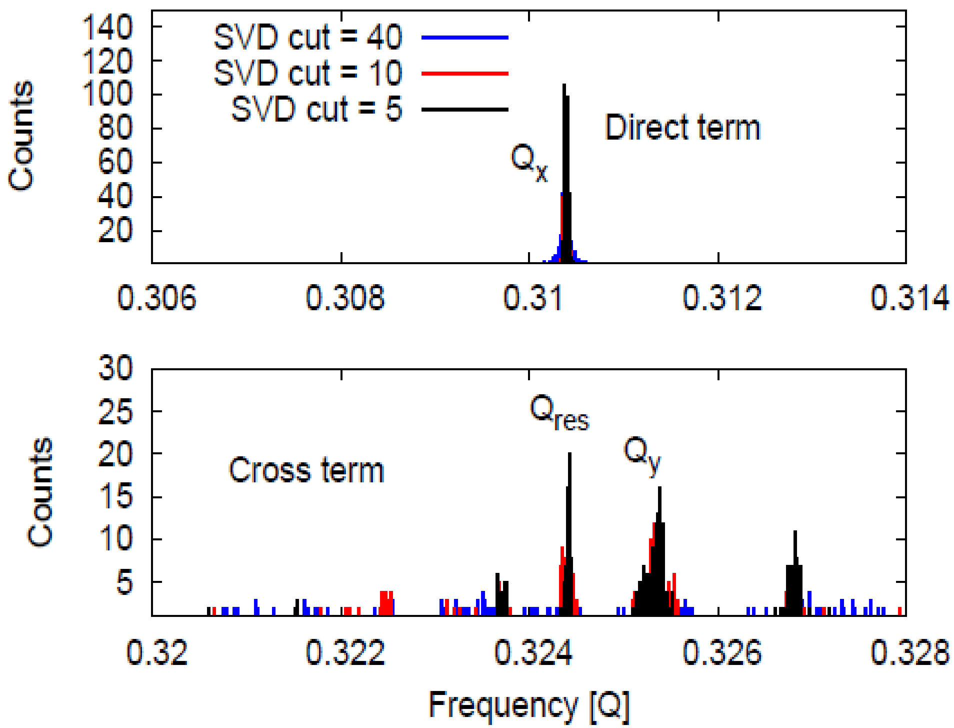


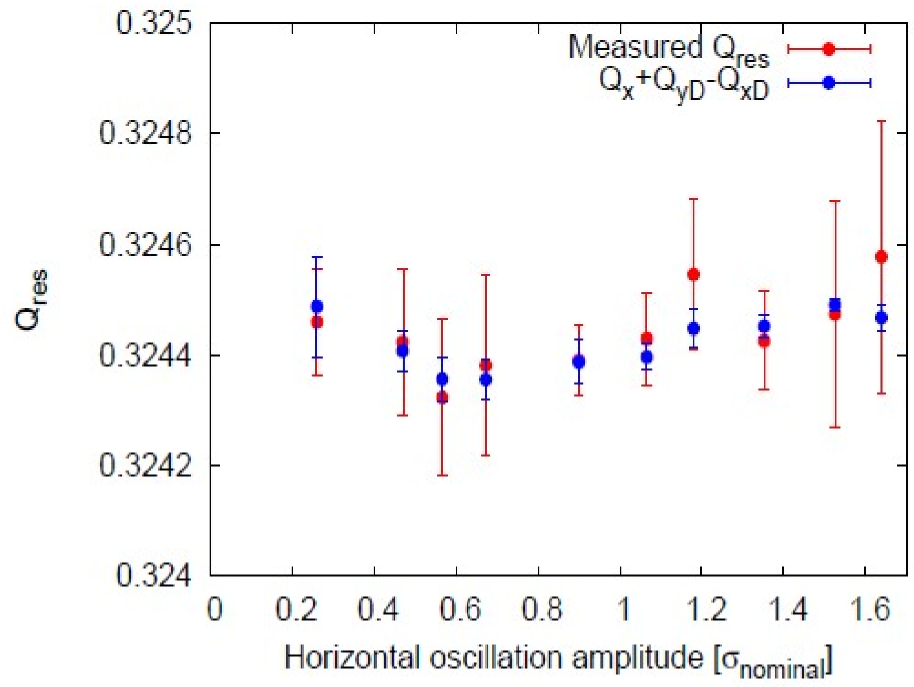


→ Although the natural tune spectral should be excited during the ramping process due to the non-linearities its amplitude remains much lower than the drive spectral line and most of the time below the noise floor

→ Noise removal is possible using SVD post-processing of the data keeping only the modes with physical meaning for data analysis: **SVD "cleaning" of the raw data allows to clearly observe the natural tune** spectral line (left) and reduces the spread from all BPMs (right)

Non-linear resonances





→ **Non-linearities not only allow to observe the natural tune but also excite non-linear resonances: should be avoided if possible**

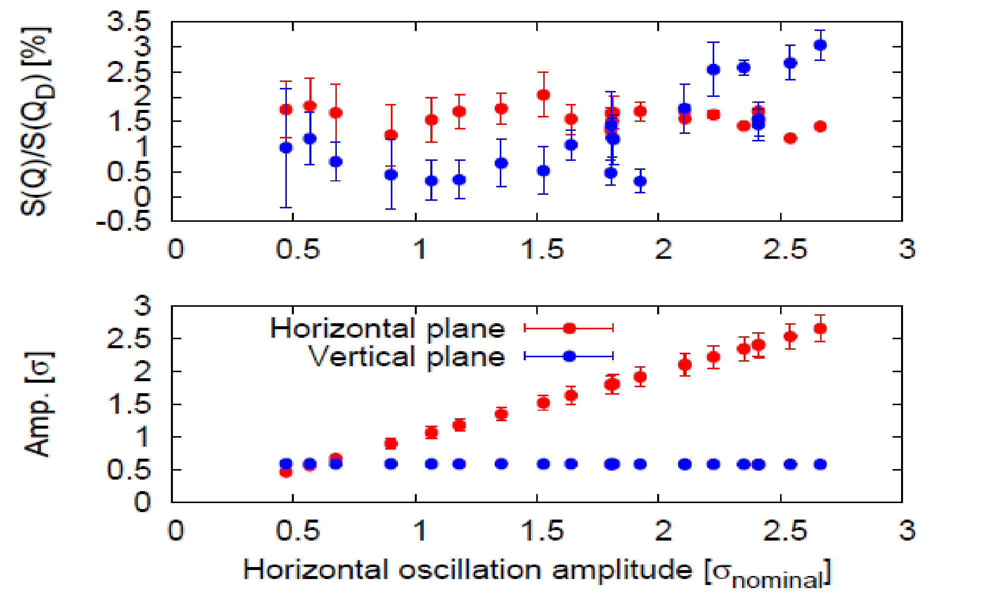
→ This was strongly observed in the cross plane data and degraded the data quality and our ability to cleanly measure the natural tune

→ Here an example of an octupolar resonance which frequency was measured during the amplitude scan. Expected and measured frequencies are in good agreement

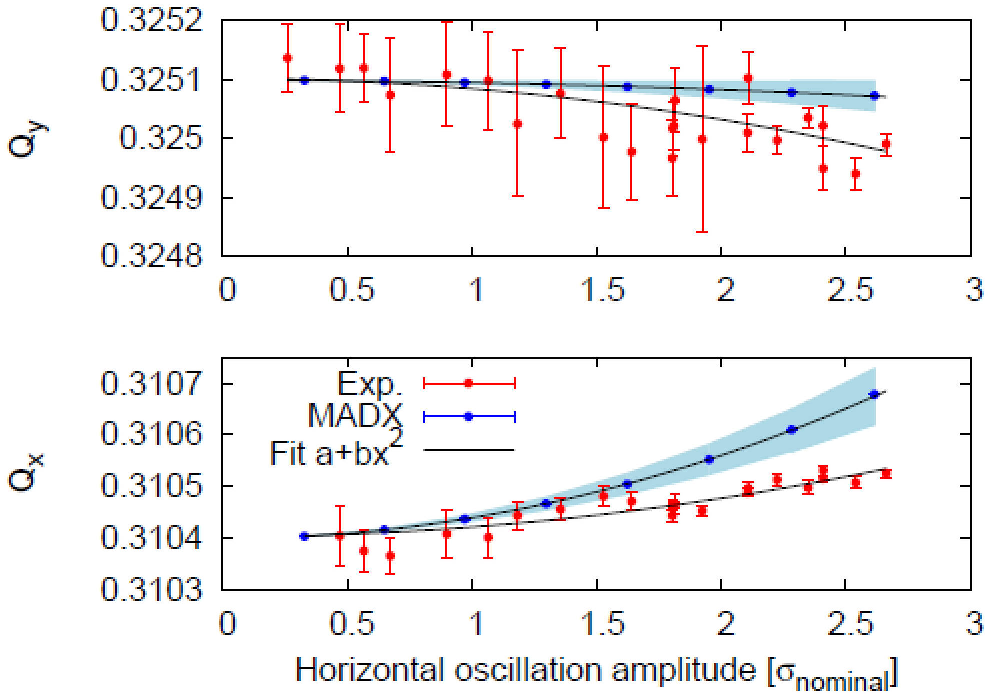
Squeezed optics measurements

→ The importance of non-adiabaticity can be estimated by looking a the ratio of drive and natural tune spectral line

→ Horizontal plane is constant. Increase in the vertical plane is attributed to coupling



	$\partial Q_x/\partial \epsilon_x$ [m^{-1}]	$\partial Q_y/\partial \epsilon_x$ [m^{-1}]
Data	18528 ± 10%	-17387 ± 23%
AC dipole model	45757 ± 1%	-4492 ± 2%
Free oscillation model	22666	-5493



→ **Disagreement of a factor 2-3 between the data and the model**

→ **Measurement taken after non-linear correction: only very small errors left: How well are they understood**?

Summary

- **Amplitude detuning equations in the presence of AC dipole were derived using the perturbation Hamiltonian:**
  - Valid for multipoles of arbitrary order
  - Driven oscillations cannot be approximated by the free oscillation model :correction factors are required
  - Direct amplitude detuning measurement with AC dipole is possible

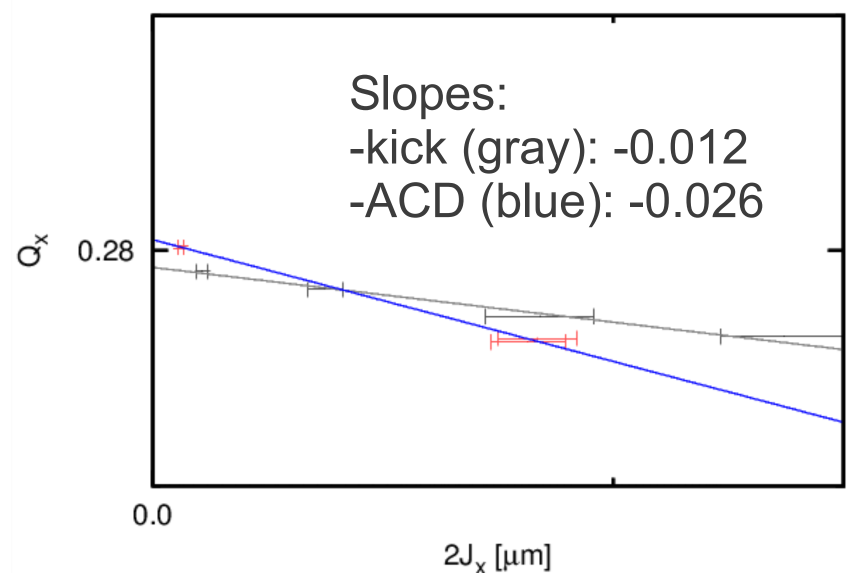
- **Experimental protocol well defined:**
  - SVD cleaning essential for natural tune measurement
  - Some optimization required (working point, excitation of resonances,...) - see next slide

- **2012 MD results:**
  - First direct amplitude detuning measurement using AC dipole performed at the LHC
  - Disagreement of a factor 2-3 between measurement and model
  - Done after non-linear corrections: remaining errors are very small and possibly not well understood: difficulties to build an accurate model
  - Uncertainties associated to misalignments could not be assessed

- Very nice achievement for a first try, lots of lessons learned

Outlook

- **Possible improvements:**
  - Coupling: keep the tunes well separated avoid driving in between Q_x and Q_y (as was done in 2012)
  - Non-linear resonances: single plane excitation – carefully choose the working point
  - Drive to larger amplitudes: have a single bunch in the machine would reduce the losses
  - Model: perform the measurement without non-linear corrections as a first check-effect of misalignments?
- **Experimental proof of theoretical derivations:**
  - Repeat at injection with strong octupoles – scan octupole current
  - Compare single kick data with AC dipole data: model independent



→ Maybe a first hint of experimental proof at the LHC?
→ Data taken during the non-linear MD at injection

Thank you for your attention!