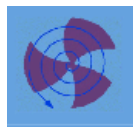


# Viscous hydrodynamic model for Relativistic Heavy Ion Collisions

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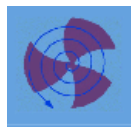


1. Introduction.
2. Why (viscous) hydrodynamics?
3. Hydrodynamical modeling of nuclear collisions.  
smooth hydrodynamics.

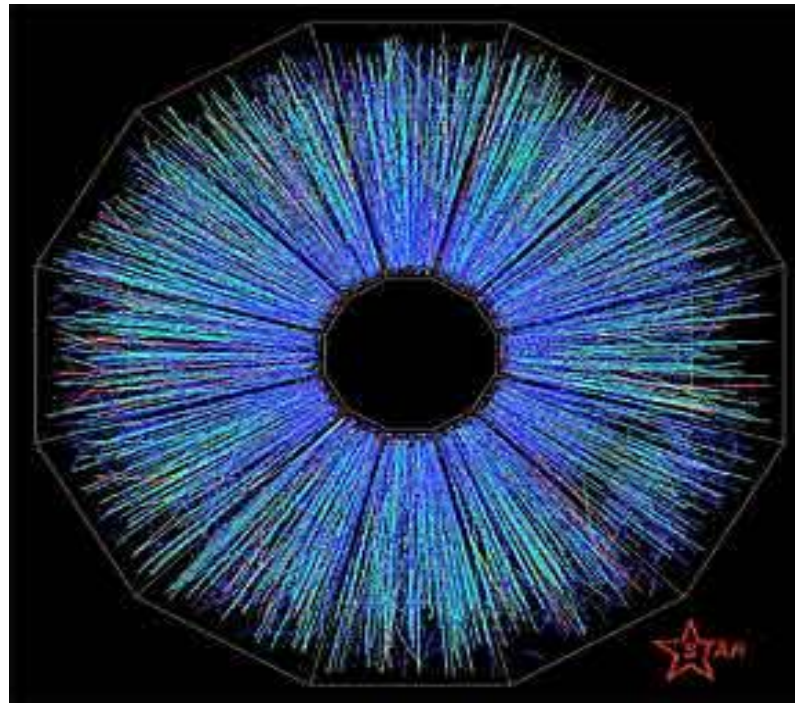
initial condition, freeze-out,  
representative simulation results.

granular (event-by-event) hydrodynamics.  
triangular flow, correlation.

4. Summary.

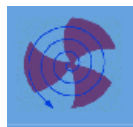


On June 12, 2000, scientists at Relativistic Heavy Ion Collider, Brookhaven National Laboratory recorded the first head-on collision between Gold nuclei.



STAR detector's view of particles streaming out from Au+Au collision.

Theoretical considerations suggest that the origin of these particles is a deconfined state of matter called Quark-Gluon Plasma (QGP). Characterising QGP is one of the major aim of current high energy nuclear physics. Hydrodynamics is a tool in the process of characterisation.



Hydrodynamic modeling in nuclear collisions have a long history.

1. Fermi's [1950] statistical model, assumed that hadrons produced in high energy collisions are in equilibrium. Energy density  $\sim \sqrt{s}$ .
2. Pommeranchuk [1951]: hadrons can decouple (freeze-out) only at low temperature  $T_F \sim m_\pi$ .
3. Landau [1953]: introduced hydrodynamical model describing the expansion of Fermi's equilibrium high density stage (the early stage) to Pomeranchuk's low density decoupling stage (the freeze-out).
4. Hagedorn [1965]: Bootstrap model (fireball in a fireball, in a fireball...), matter composed of hadrons has a maximum temperature,  $T_H \sim m_\pi$ .

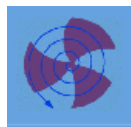
**Hagedorn model contradicts Fermi's idea that energy density can increase indefinitely with collision energy!**

Interest in hydrodynamical modeling was renewed after Bjorken's 1983 paper, when the idea of producing Quark-Gluon Plasma in heavy ion collisions gained some acceptance. Landau's model misses the inside-outside characteristic (high energy hadrons are produced later than the low energy ones) of particle production. It is corrected in Bjorken model.

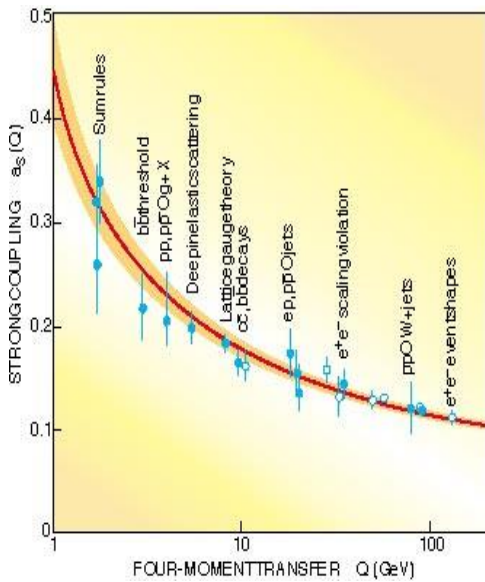


## Some important years in the history of QGP:

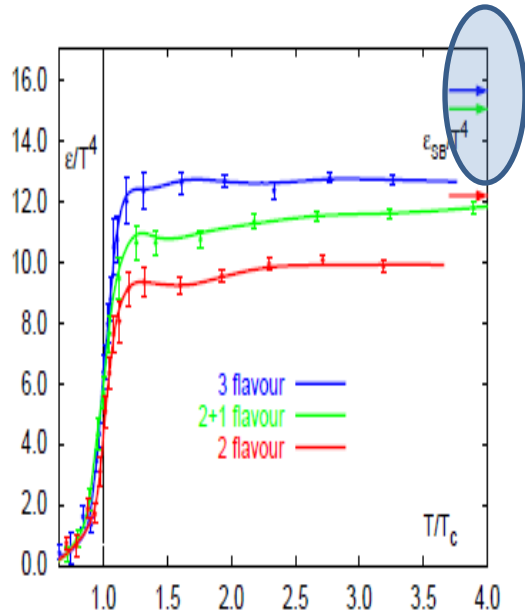
1. 1974-75: Lee & Wick [PRD9, Rec. Mod. Phys. 47] predicted abnormal nuclear states (nucleon mass zero inside a region, non-zero outside it). Argued that such states can be created in heavy ion collisions;  
Collins & Perry [PRL34]: super dense matter (found in neutron stars, exploding supernova, early universe) consists of quarks rather than of hadrons.
2. 1980: Shuryak [Phys. Reports 81]: 1st review paper, coined the term Quark-Gluon Plasma.
3. 1983: Bjorken [PRD27]: Hydrodynamic model for Relativistic Heavy Ion Collisions. Scaling solution. Chiu, Sudarshan & Wang [PRD12] obtained identical equations in 1975.
4. 1984: proposal to build Relativistic Heavy Ion Collider (RHIC) to search for a state of matter called the quark-gluon plasma.
5. 1988: VECC organize 1st ICPA-QGP, which paved the way for Indian Collaboration at SPS, RHIC, LHC and FAIR.
6. 2001: First physics results from RHIC.
7. 2005: BNL announce Perfect fluid at RHIC.
8. 2008: Large Hadron Collider (LHC) become operational. Heavy ion expt. by ALICE and ATLAS.



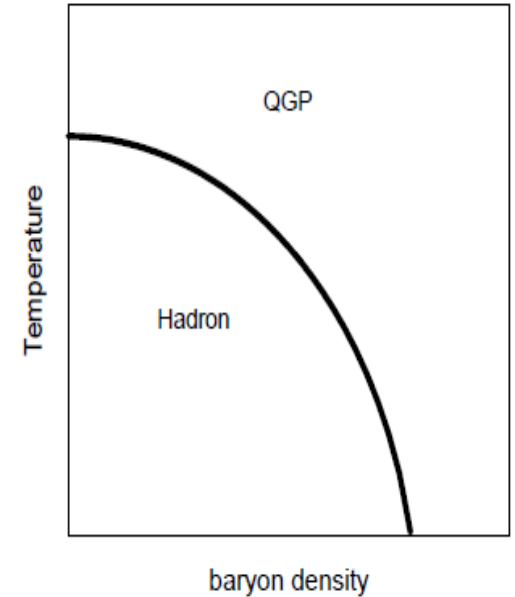
Paradigm before RHIC expt. **QGP, weakly interacting gas of quarks and gluons.**



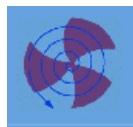
the change of phase is due to asymptotic property of QCD.



LQCD at high T is very close to Stefan-Boltzmann limit for free quark and gluons.



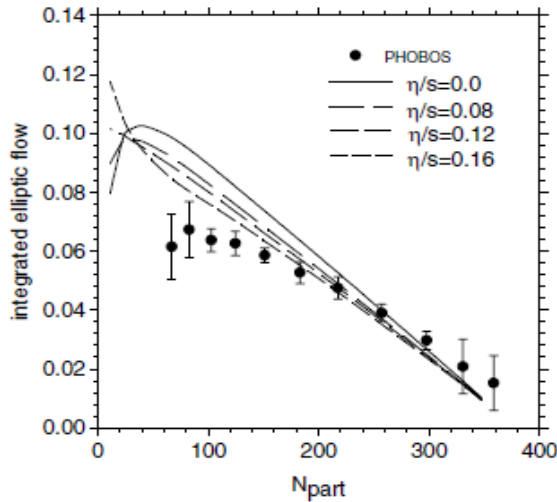
we also have a simple phase diagram.



## Two major discovering at RHIC:

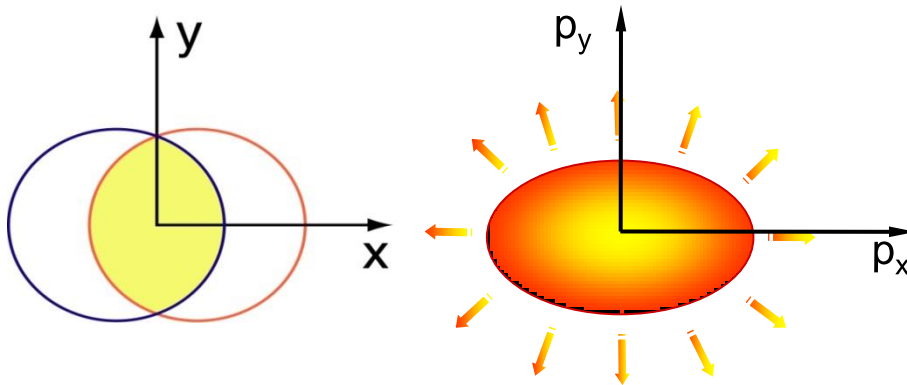
### (i) Elliptic flow

$$\frac{dN}{d\phi} = \frac{N}{2\pi} [1 + 2v_1 \cos(\phi - \psi_R) + 2v_2 \cos(2\phi - 2\psi_R) + \dots]$$

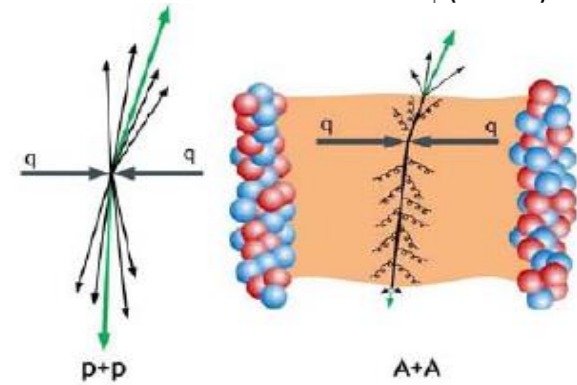
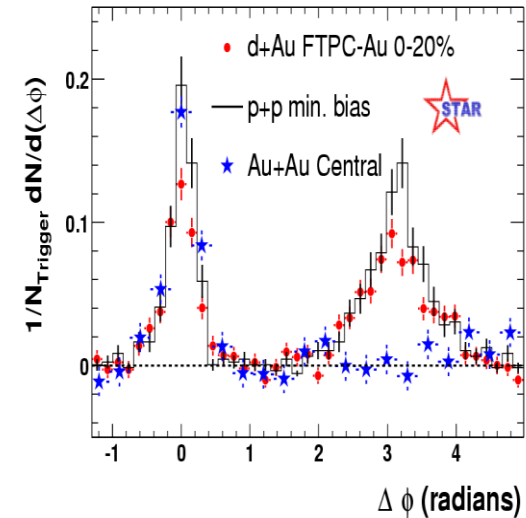


elliptic flow

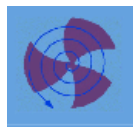
qualitatively/quantitatively understood if (thermal) medium is produced.



### (ii) Jet quenching: disappearance of away side jet / high $P_T$ suppression



**Paradigm shift: weakly coupled QGP  $\rightarrow$  strongly coupled QGP (sQGP).**





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## RHIC Scientists Serve Up "Perfect" Liquid

### New state of matter more remarkable than predicted – raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the [Relativistic Heavy Ion Collider \(RHIC\)](#) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In [peer-reviewed papers](#) summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

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Physics News  
Scientists Detect 'Fingerprint' of High-Temp

perfect fluid → small viscosity → large cross section  
large cross-section → strong couplings  
strong couplings → perturbation theory difficult.





Landau in Fluid Mechanics :In any fluid motion, some amount of irreversibility is present and leads to energy dissipation.

In Fluid dynamics, one encounters three type of dissipation,

(i) Shear viscosity: resistance to anisotropic expansion. Extensively studied.

(ii) bulk viscosity: resistance to isotropic expansion, very limited study.

(iii) conductivity: resistance to charge flow: minimum study.

AdS/CFT[Policastro,Son,Starinets,PRL87(2001)]:

$\eta/s$  is bounded from the lower side,

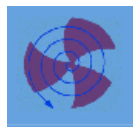
$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Nearly same result from uncertainty principle [Danielewicz&Gyulassy;PRD31(1985)]:

$$\eta = \frac{1}{3} np\lambda, \quad s = 4n$$

$$\frac{\eta}{s} \geq \frac{1}{4 \times 3}$$

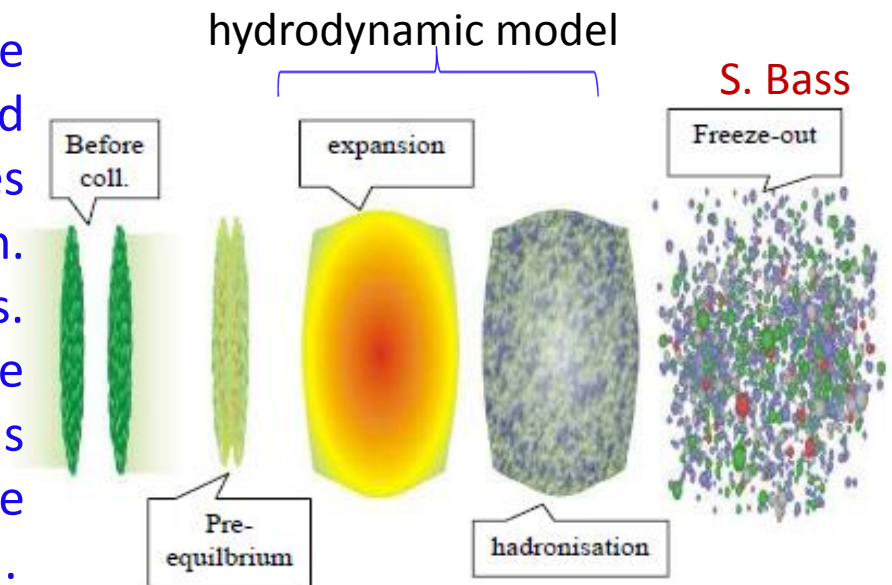
Here, n is the density of particles, transporting average momenta p over a momentum degrading length  $\lambda$ .  
From uncertainty principle,  $\lambda \geq 1/p$ .



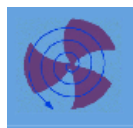
A nucleus-nucleus collision at relativistic energy passes through different stages

(i) **Pre-equilibrium stage:** Initial partonic collisions produce a fireball in a highly excited, non-equilibrium state. Frequent collisions between the constituents possibly establish a 'local equilibrium' state.

(ii) **expansion stage:** thermal pressure acts against the surrounding vacuum and the partonic medium undergoes collective (hydrodynamic) expansion. Energy density/temperature decreases. Below a critical temperature, there will be phase transition/cross-over and partons will be converted back to hadrons. The hadronic medium will continue to expand.



(iii) **Freeze-out:** a stage will come when inelastic collisions between the constituents become infrequent to keep up with the expansion. It is called chemical freeze-out. Finally, constituents will be far apart to interact even elastically and maintain local equilibrium. The stage is called kinetic freeze-out.



Hydrodynamic equations:  $\partial_\mu N^\mu = 0,$  solves in  $(\tau, x, y, \eta)$   $\tau = \sqrt{t^2 - z^2},$   
 $\partial_\mu T^{\mu\nu} = 0.$  coordinate system:  $\eta = \frac{1}{2} \ln \frac{t+z}{t-z}$

$$N^\mu = nu^\mu + \{V^\mu\},$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

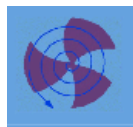
$$+ \{\pi^{\mu\nu} - \Pi(g^{\mu\nu} - u^\mu u^\nu) + W^\mu u^\nu + W^\nu u^\mu\}; \quad W^\mu = \frac{\varepsilon + p}{n} V^\mu + q^\mu$$

A out of equilibrium state tries to evolved into a equilibrium state. In the process dissipative flows are generated. In the first approximation,

dissipative flows are proportional to some thermodynamic forces.

The proportionality constants are called transport coefficients.

bulk viscosity:	$\Pi =$	$-\zeta\theta,$	} constitutive relations for 1st order theory.
conductivity:	$q^\mu =$	$-\lambda \frac{nT^2}{\varepsilon + p} \nabla^\mu (\mu/T),$	
shear viscosity:	$\pi^{\mu\nu} =$	$2\eta \nabla^{\langle\mu} u^{\nu\rangle}$	



In second order hydrodynamics dissipative flows are treated as extended thermodynamic variables with relaxation Equations. Many ways to derive rel.eqs. Some ambiguity!

[Jaisawal,Bhalerao,Pal,PRC87,PLB720, Betz, Henkel,Rischke: JPhysG36, Denicol,Koide,Rischke,1004.5013]

$$\begin{aligned}\Pi &= \Pi_{\text{NS}} - \tau_{\Pi} \dot{\Pi} \\ &+ \tau_{\Pi q} q \cdot \dot{u} - \ell_{\Pi q} \partial \cdot q - \zeta \hat{\delta}_0 \Pi \theta \\ &+ \lambda_{\Pi q} q \cdot \nabla \alpha + \lambda_{\Pi \pi} \pi^{\mu\nu} \sigma_{\mu\nu},\end{aligned}$$

$$\begin{aligned}q^{\mu} &= q_{\text{NS}}^{\mu} - \tau_q \Delta^{\mu\nu} \dot{q}_{\nu} \\ &- \tau_{q\Pi} \Pi \dot{u}^{\mu} - \tau_{q\pi} \pi^{\mu\nu} \dot{u}_{\nu} + \ell_{q\Pi} \nabla^{\mu} \Pi - \ell_{q\pi} \Delta^{\mu\nu} \partial^{\lambda} \pi_{\nu\lambda} + \tau_q \omega^{\mu\nu} q_{\nu} \frac{\kappa}{\beta} \hat{\delta}_1 q^{\mu} \theta \\ &- \lambda_{qq} \sigma^{\mu\nu} q_{\nu} + \lambda_{q\Pi} \Pi \nabla^{\mu} \alpha + \lambda_{q\pi} \pi^{\mu\nu} \nabla_{\nu} \alpha,\end{aligned}$$

$$\begin{aligned}\pi^{\mu\nu} &= \pi_{\text{NS}}^{\mu\nu} - \tau_{\pi} \dot{\pi}^{\langle\mu\nu\rangle} \\ &+ 2\tau_{\pi q} q^{\langle\mu} \dot{u}^{\nu\rangle} + 2\ell_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle} + 2\tau_{\pi} \pi_{\lambda}^{\langle\mu} \omega^{\nu\rangle\lambda} - 2\eta \hat{\delta}_2 \pi^{\mu\nu} \theta \\ &- 2\tau_{\pi} \pi_{\lambda}^{\langle\mu} \sigma^{\nu\rangle\lambda} - 2\lambda_{\pi q} q^{\langle\mu} \nabla^{\nu\rangle} \alpha + 2\lambda_{\pi\Pi} \Pi \sigma^{\mu\nu},\end{aligned}$$



15 variables, 14 equations. Hydrodynamic equations are closed only with an equation of state,  $p=p(\varepsilon,n)$ .

EOS connects the macroscopic hydrodynamic model to underlying microscopic physics. Hydrodynamic models are only relativistically covariant model that can account dynamically effect of phase transition.

EOS for QGP phase: (one generally use) lattice QCD results .

EOS for hadronic phase: non-interacting hadronic resonance gas.

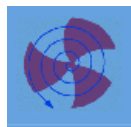
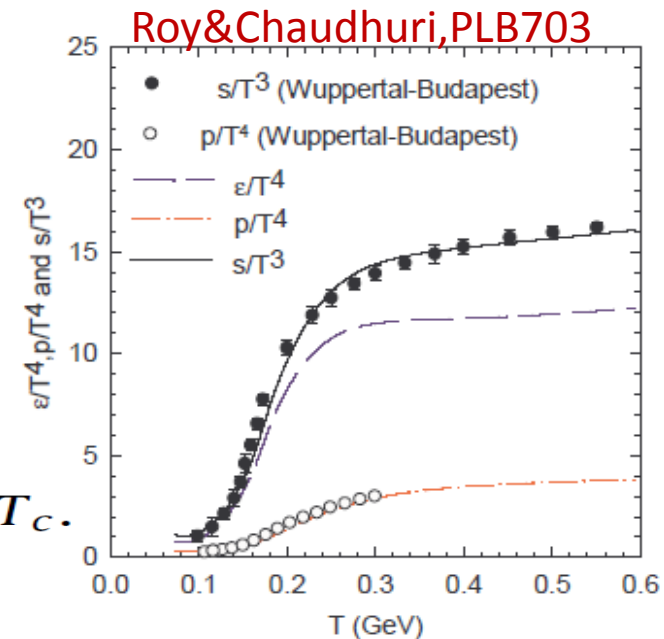
Current understanding : confinement-deconfinement transition is a cross-over(free energy is infinitely differentiable. )

A simple Lattice based cross-over EOS:

$$s = 0.5 \left[ (1 - \tanh(x)) s_{HRG} + 0.5 \left[ 1 + \tanh(x) \right] s_{lattice} \right]$$

$$p(T) = \int_0^T s(T') dT'$$

$$\varepsilon(T) = Ts - p. \quad x = \frac{T - T_c}{\Delta T_c}, \quad \Delta T_c = 0.1 T_c.$$



Hydrodynamics is an initial value problem.

initial time beyond which hydrodynamics is applicable.  $\tau_i$  small  $\sim 0.5-1.0$  fm!  
Not understood. Chromo-Weibel instability thermalise  $\sim 5$  fm [[arXiv:1301.7749](#)].

initial energy density distribution, fixes initial eccentricity (shape of the collision zone)

$$e(x, y, \eta_s) = \varepsilon(x, y) H(\eta_s) \quad H \sim \text{Gaussian form}$$

For transverse part, one uses Glauber/Color Glass Condensate (CGC) model.

Glauber: 
$$\epsilon(x, y) = \epsilon_0 [(1 - f) N_{part}(x, y) + f N_{coll}(x, y)]$$

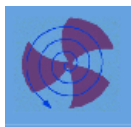
CGC: 
$$\frac{dN_g}{d^2r_T dY} = \frac{4\pi^2 N_c}{N_c^2 - 1} \int \frac{d^2p_T}{p_T^2} \int d^2k_T \alpha_s(k_T) \times \phi_A(x_1, p_T^2, r_T) \phi_B(x_2, (p_T - k_T)^2, r_T)$$

unintegrated gluon dist.  
KLN approach [[NPA730](#)]

$$e(x, y, b) = e_0 \left[ \frac{dN_g}{dx dy dY} \right]^{4/3}$$

for details of Glauber in hydro  
see [arXiv:nucl-th/0305084](#).

for details of CGC in hydro  
see [NPA743,305](#); [PRC86,014902](#).



initial baryon density distribution : similarly as the energy density.

initial fluid velocity distribution, generally assumed to be zero. Inside the collision zone, random velocities should cancel each other. Near the boundary non-zero values possible. initial velocity increase high  $p_T$  yield.

In dissipative dynamics, additionally, initialize

bulk viscous pressure: zero

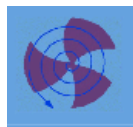
or boost-invariant value:  $\Pi(x,y)=-\zeta\theta$ .

shear stress tensor components: zero

or boost-inv. value:  $\pi^{xx}(x,y)=\pi^{yy}(x,y)=2\eta(x,y)/\tau_i$ ,  $\pi^{xy}=0$

also require: relaxation times, viscosity coefficients.

No theoretical model to constrain the initial parameters. They must be obtained from experimental data. **Leaves room for ambiguity.**



Freeze-out: Hydrodynamics give space-time evolution of the fluid. To connect with expt. a freeze-out prescription is needed.

Chemical freeze-out: from statistical model [JPhysG32,s21; PRC74,044905]. Statistical model predictions surprisingly agree with expt. hadron ratios.

Kinetic freeze-out: Most simulations assume constant temperature kinetic freeze-out. Other forms of freeze-out, e.g. isochronous (fixed time) is also possible. At kinetic freeze-out, invariant distribution of a hadron is obtained from Cooper-Frye prescription [PRD10],

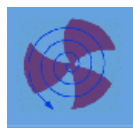
$$E \frac{dN}{d^3p} = \frac{dN}{dyd^2p_T} = \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f(x, p)$$

Cooper-Frye require non-interacting hadrons at freeze-out for energy conservation.

$$f^{neq}(x, p) = f^{eq}(x, p) [1 + r f^{eq}(x, p)] [1 + \phi(x, p)]$$

shear viscosity: 
$$\phi(x, p) = \frac{1}{2(\varepsilon + p)T^2} \pi_{\mu\nu} p^{\mu} p^{\nu}$$

there are ambiguities about the form of viscous correction!  
[Luzum,Ollitrault,arXiv:1004.2023;Dusling,Moore,Teany, PRC81]





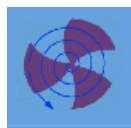
In general, one fixes most of the parameters from theoretical considerations and vary a limited number of parameters.

Inter-relation between the parameters and their effect on final results are not well studied.

Several groups have solved hydrodynamics equations with dissipation

Ref.	dim.	IC	EOS	scheme	freezeout	Obs.
Romatschke[57]	2+1	G	lQCD	CD	single $T_F$	$v_2$
Dusling [132]	2+1	G	ideal gas	-	viscous correction	$v_2$
Luzum [47]	2+1	G,CGC	lQCD	CD	resonance decay	$v_2$
Schenke [112]	3+1	MC-G	lQCD	KT	viscous correction	$v_2, v_3$
Song [133]	2+1	MC-G, MC-CGC	lQCD	SHASTA	cascade(UrQMD)	$v_2$
Chaudhuri [58][42]	2+1	G, CGC, MC-G	lQCD	SHASTA	viscous correction	$v_2, v_3$
Bozek [134]	3+1	G	lQCD	-	THERMINATOR2	$v_1, v_2, \text{HBT}$
Denicol,Kodama,Koide[135]	3+1	G	lQCD	SPH	viscous correction	$v_2$

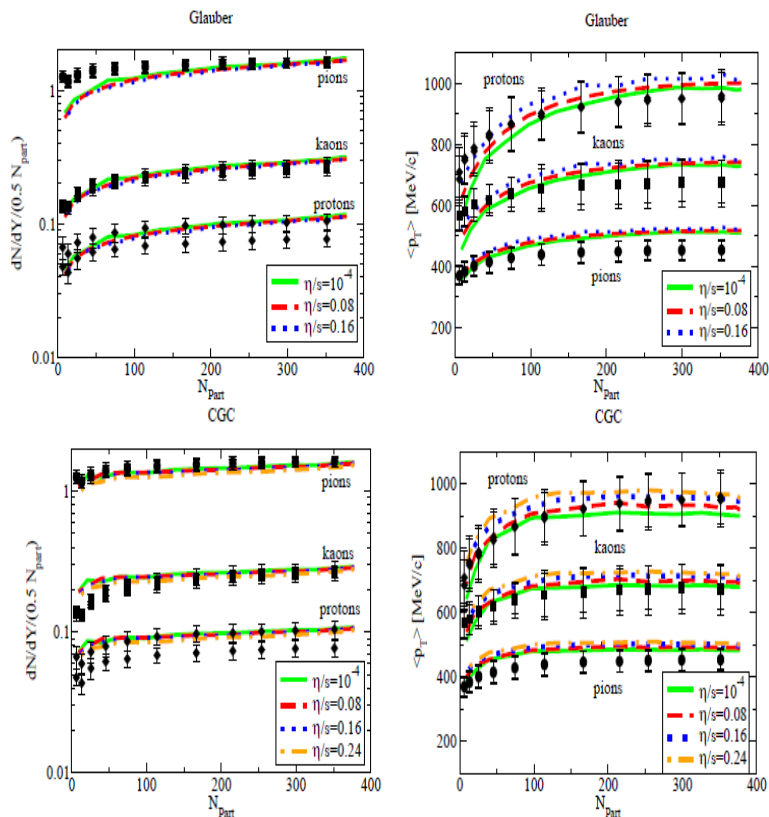
In the following, I will discuss some representative simulations.



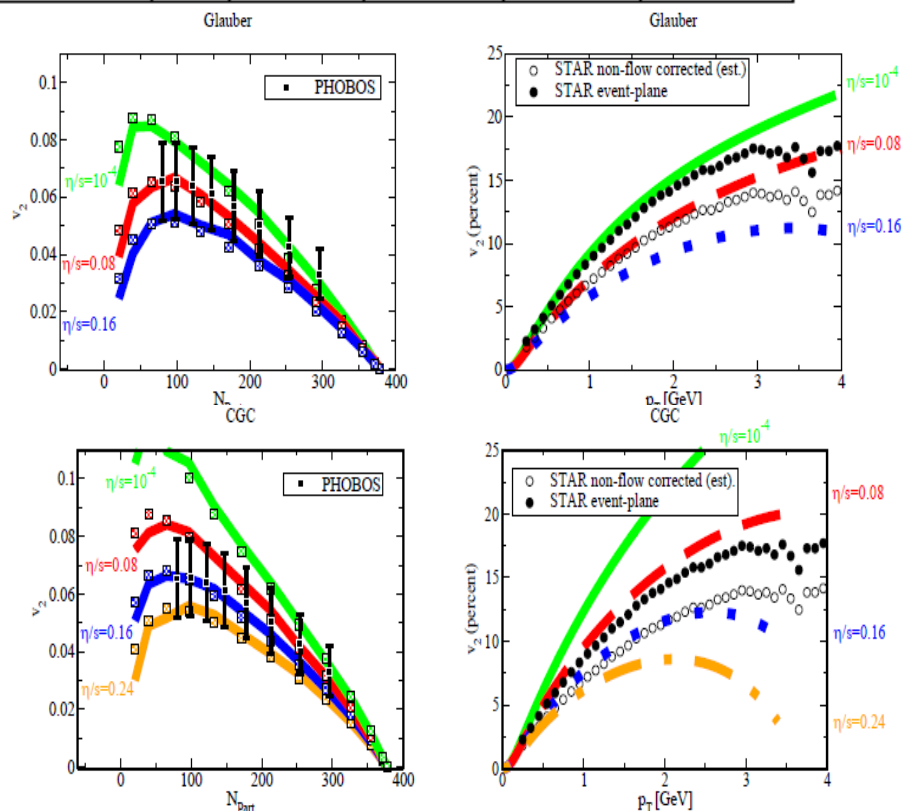
# Luzum& Romatschke [PRC78]

simulated Au+Au coll. with  
Glauber & CGC ini.cond.  
Analysis exclude  $\eta/s > 0.5$ .

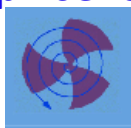
Initial condition	$\eta/s$	$T_i$ [GeV]	$T_f$ [GeV]	$\tau_0$ [fm/c]	$a$ [ $\text{GeV}^{-1}$ ]
Glauber	$10^{-4}$	0.340	0.14	1	2
Glauber	0.08	0.333	0.14	1	2
Glauber	0.16	0.327	0.14	1	2
CGC	$10^{-4}$	0.310	0.14	1	2
CGC	0.08	0.304	0.14	1	2
CGC	0.16	0.299	0.14	1	2
CGC	0.24	0.293	0.14	1	2



multiplicity insensitive to details of IC  
or viscosity. Av.  $p_T$  sensitive viscosity.

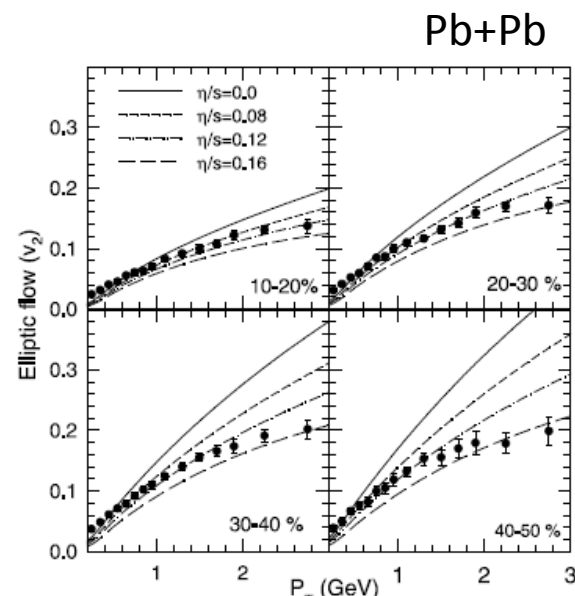
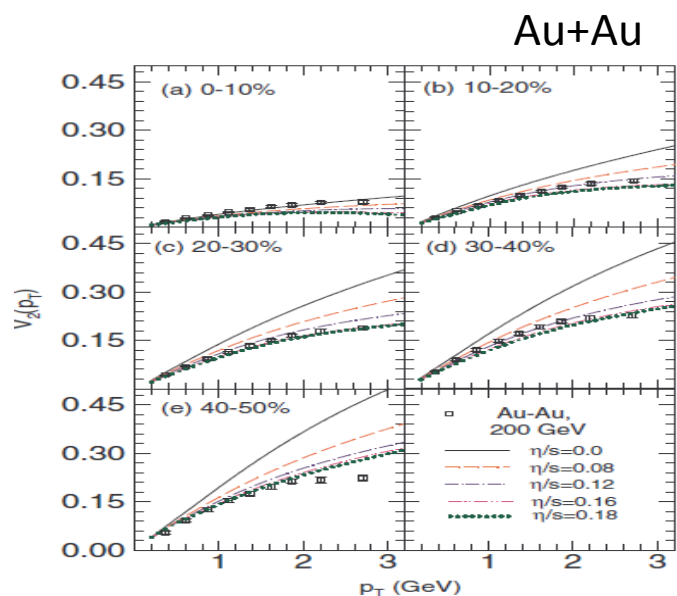


integrated and differential  $V_2$  shows  
sensitivity to details of IC and viscosity.

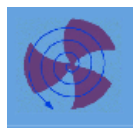


# AZHEDRO-KOLKATA simulations for $\sqrt{s}=200$ GeV Au+Au and $\sqrt{s}=2.76$ TeV Pb+Pb collisions.

$\eta/s$	0	0.08	0.12	0.16
$\varepsilon_i$ ( $\frac{\text{GeV}}{\text{fm}^3}$ )	$143.0 \pm 6.0$	$126.0 \pm 5.5$	$115.0 \pm 5.2$	$103.0 \pm 4.6$
$T_i$ (MeV)	$548 \pm 5$	$531 \pm 7$	$520 \pm 6$	$504 \pm 7$



$\sqrt{s}=2.76$  TeV Pb+Pb collisions [Roy&Chaudhuri,PLB703]  $\eta/s = 0.06 \pm 0.2$ .  
 $\sqrt{s}=200$  GeV Au+Au collisions [Chaudhuri,PLB681]  $\eta/s = 0.07 \pm 0.03$ .



systematic uncertainties in  $\eta/s$ : With Glauber Initial condition  
[Chaudhuri,PLB681] :

~90%:  $\tau_i=0.2-1.0$  fm

~100%:  $T_F=140-160$  MeV,

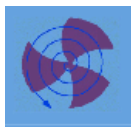
~55% hard scattering component 0-100%,

~100% initial velocity [ $v_r=\tanh(ar)$ , $a=0-0.06$ ]

~7% inaccuracy in AZHYDRO-KOLKATA code.

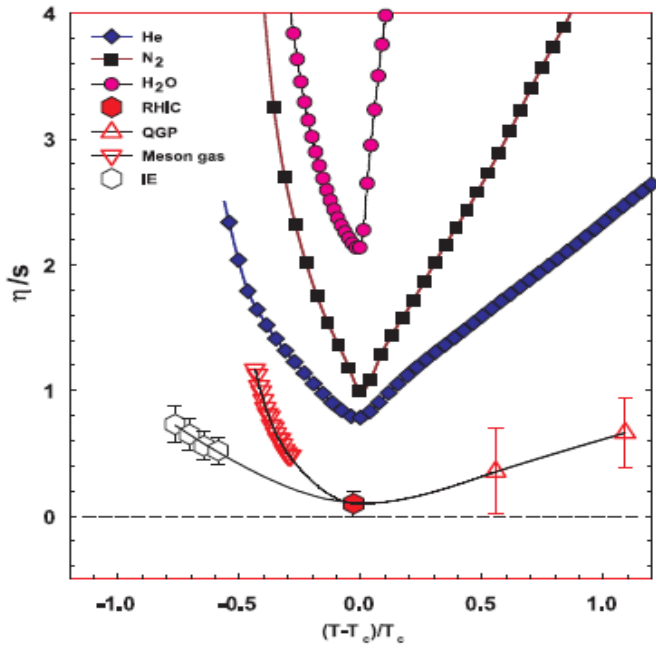
**Adding all in quadrature, systematic uncertainty in  $\eta/s$  ~175%.**

Note: all possible sources of uncertainties are still not accounted for, e.g. uncertainty due to IC, Glauber/CGC.



**Temperature dependence of  $\eta/s$ :**  
 present understanding: in QGP phase  $\eta/s$  increases with temp. In hadronic phase  $\eta/s$  decreases with temp. Minima (possibly cusp) at  $T=T_c$ .

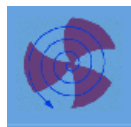
Niemi et al[PRL106]: elliptic flow at RHIC energy is dominated by the viscosity in the hadronic phase & largely insensitive to QGP viscosity. QGP viscosity dominates only at LHC energy.



**Bass&Heinz,Shen&Heinz[PRC83]**

Temperature dependence of QGP viscosity over entropy ratio cannot be constrained by fitting spectra and elliptic flow data alone.

**Chaudhuri[JPhysG39]:** No unique linear temperature dependence from RHIC and LHC data.

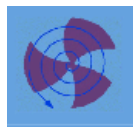


## Triangular flow in event-by-event hydrodynamics:

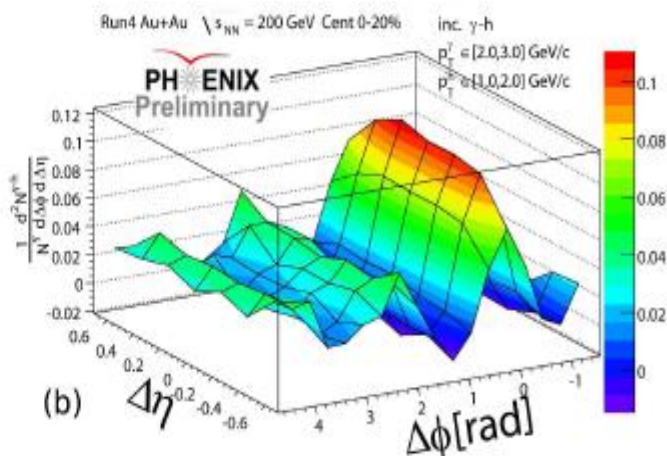
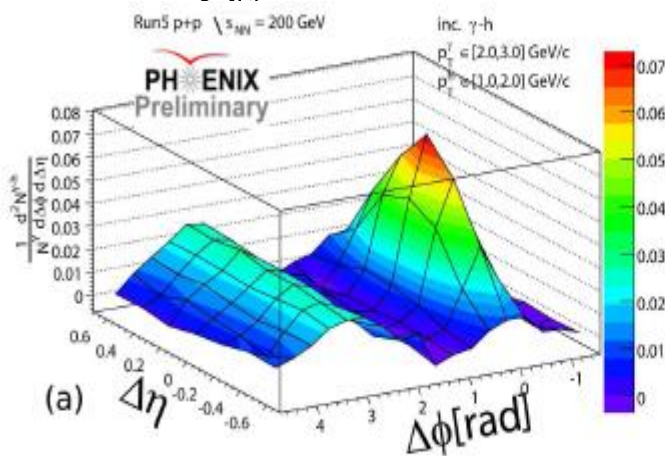
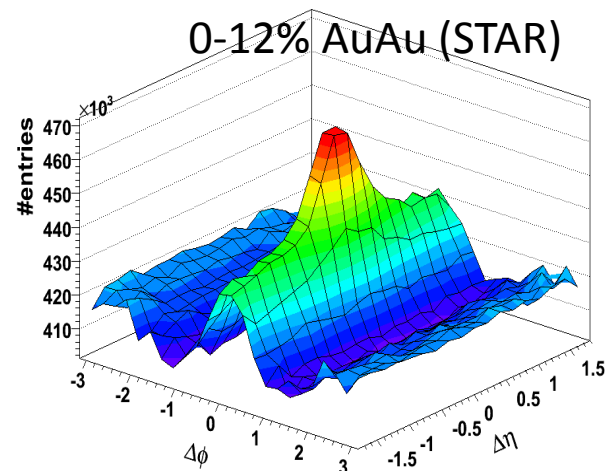
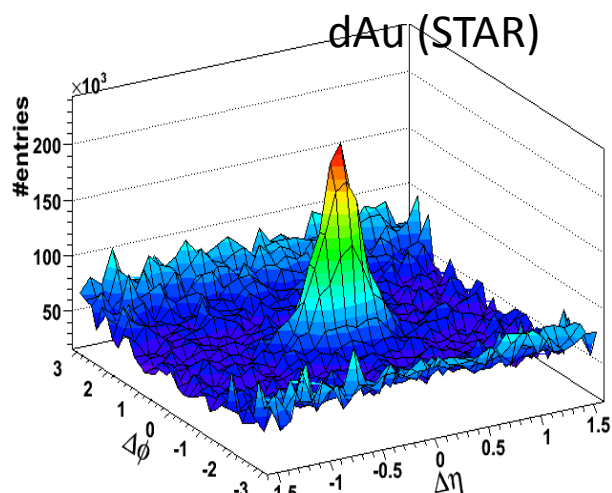
Importance of event-by-event analysis is best explained by the classic example by A. D. Jackson.

Put a sheet of paper outside the window on a rainy day. Look at it after-a-long time. It is uniformly soaked. **You conclude that the spatial distribution of rain is uniform.** However, if you continue to look into the paper, you come to a different conclusion, the spatial distribution of rain is far from uniform.

**Conclusion from (high statistics) single event data can be very different from the event averaged data. Interesting physics may reveal in event-by-event analysis.**



# Two particle correlation:

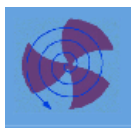


Long range correlation in  $\Delta\eta$ ;

(i) centered at  $\Delta\phi \approx 0^0$  called Ridge

(i) centered at  $\Delta\phi \approx 120^0$  called Shoulder

*A. K. Chaudhuri/Triggering discoveries in High Energy Physics*





Alver and Rolland [PRC81]: Ridge arise from collective response to fluctuating initial conditions. Specifically, if initial conditions is parameterised with quadrupole and triangular moments, response of the medium to these anisotropies is reflected in two body correlation as Ridge and shoulder.

Sorensen [J.PhysG37] also suggested higher order flow harmonics are responsible for the peculiar structures in two body correlation.

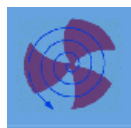
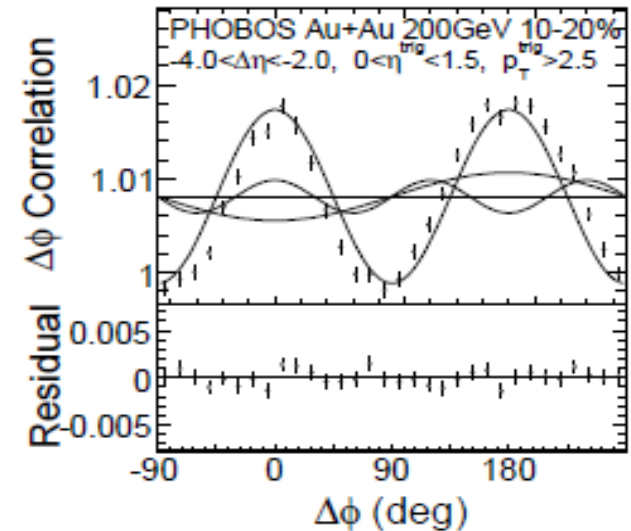
Azimuthal correlation can be Fourier decomposed,

$$\frac{dN}{d\phi} = \frac{1}{2\pi} \left[ 1 + \sum_n 2v_n \cos(n\phi) \right]$$

$$\frac{dN^{pair}}{d\Delta\phi} = \frac{1}{2\pi} \left[ 1 + \sum_n 2v_{n\Delta} \cos(n\Delta\phi) \right]$$

$V_{n\Delta}$  can be expressed in terms of flow coeff.  $V_n$ .

PHOBOS & STAR data on azimuthal correlation in Au+Au collisions are very well described by the first three Fourier components. **Third flow harmonic  $v_3$  is non-zero.**



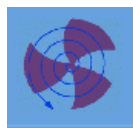
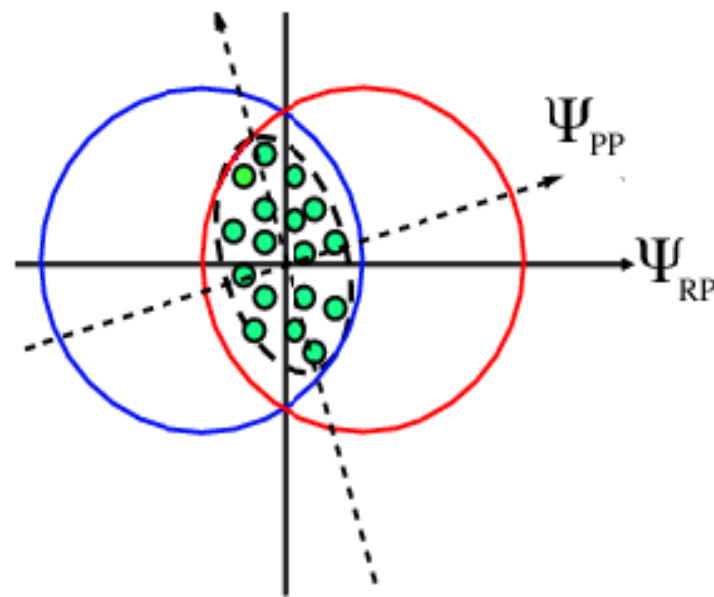


In traditional hydrodynamics, initial density distribution is smooth (obtained from overlap of Woods-Saxon density distributions). Symmetry plane of the collision zone coincides with the reaction plane. Odd harmonics  $v_3, v_5 \dots$  are zero by symmetry.

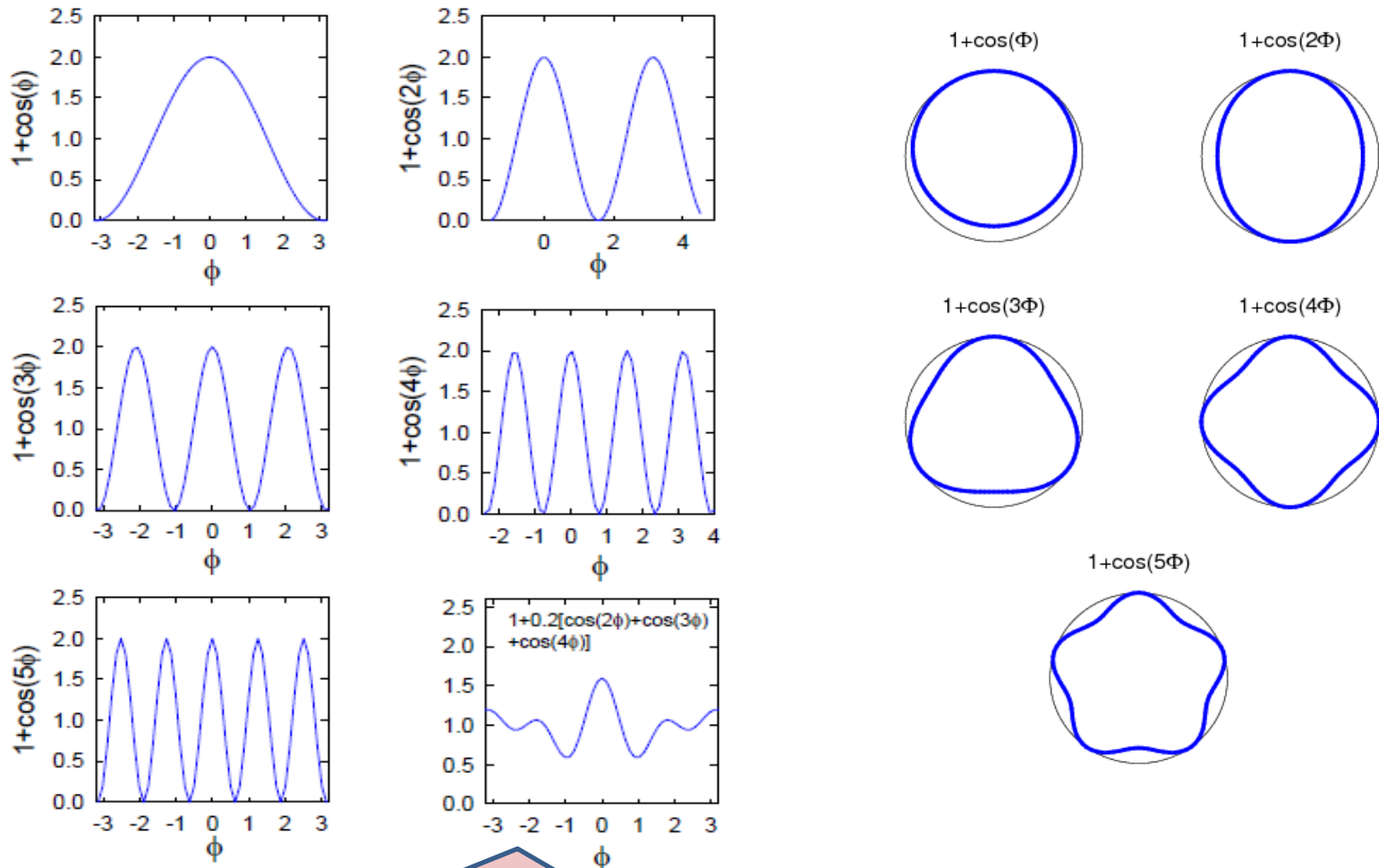
To understand non-zero higher harmonics, in terms of hydrodynamics, a revised understanding of collision geometry is required.

Revised understanding:

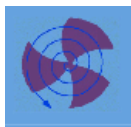
- (i) participating nucleons determine the symmetry plane for flow development.
- (ii) symmetry plane for participating nucleons may be tilted w.r.to the reaction plane.
- (iii) participating nucleon position fluctuates event-by-event.



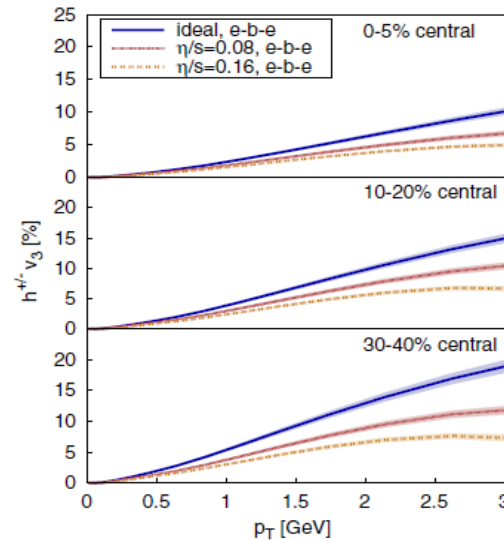
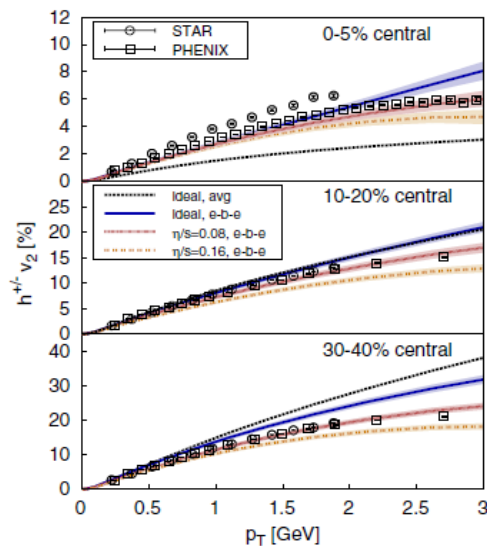
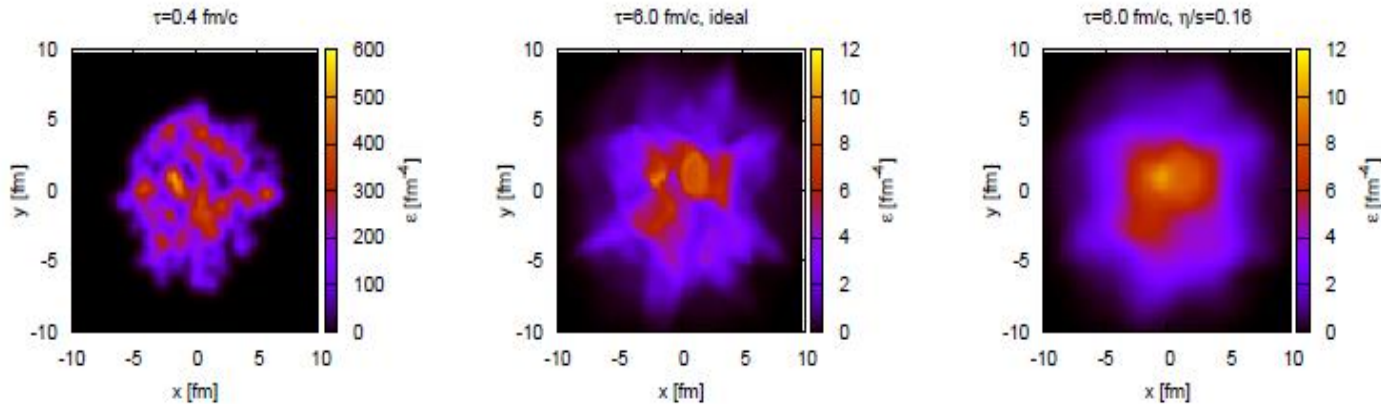
# Azimuthal distribution and its relation to shape of the collision zone:



regular structures will be modified when flow coefficients mix.



# Schenke,Jeon,Gale: [PRL106] 3+1D hydrodynamics, MC-Glauber model.



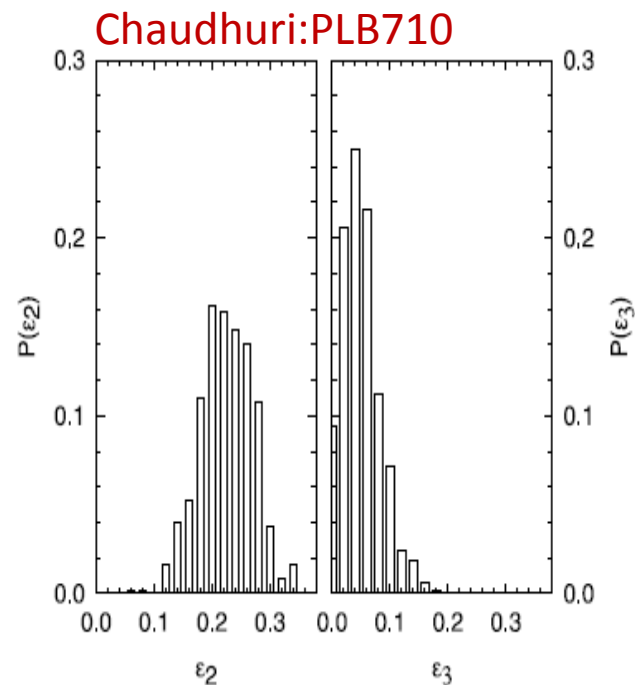
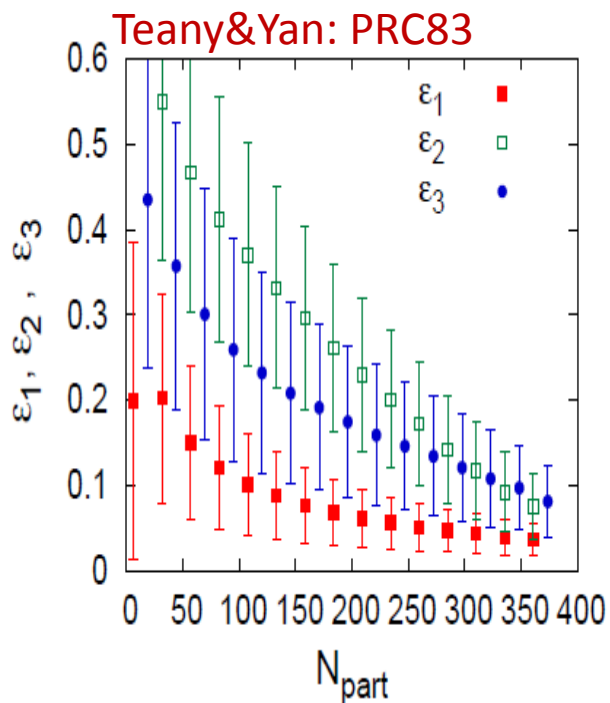
**simultaneous measurements of  $v_2$  and  $v_3$  can determine  $\eta/s$  more precisely.**

Only average value is shown. Such conclusions can not be reached from the average values only. Fluctuations can be large.



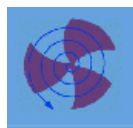
initial asymmetry

$$\epsilon_n e^{in\psi_n} = -\frac{\int \int \epsilon(x, y) r^n e^{i2\phi} dx dy}{\int \int \epsilon(x, y) r^n dx dy}, n = 2, 3, 4, 5$$



Initial asymmetry parameters have large fluctuations. Expect large fluctuations in flow coefficients as they depend on the asymmetry measures.

**In PLB710, it was shown that sensitivity of flow coefficients towards viscosity is markedly reduced in e-by-e hydrodynamics.**

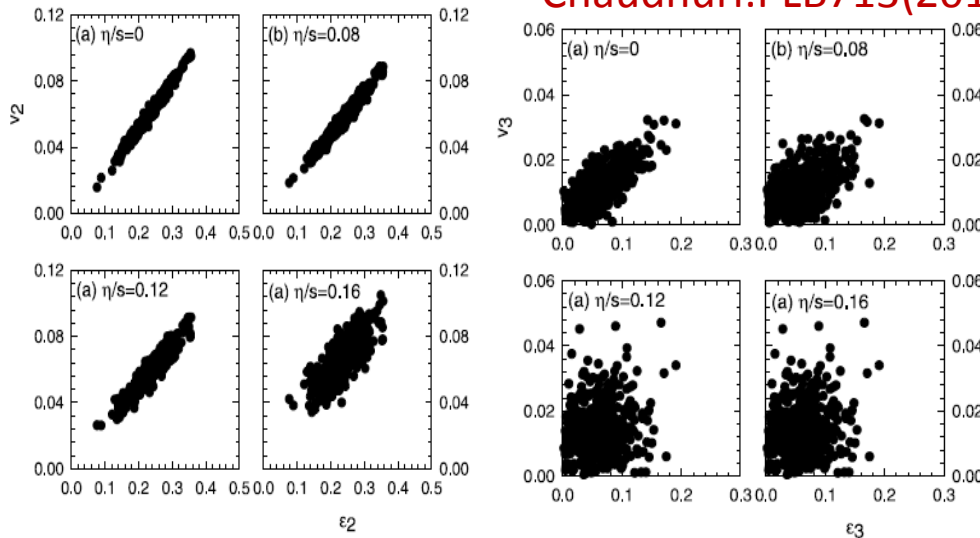


## Correlation between flow and initial asymmetry parameter:

In smooth, hydrodynamics,  $v_2 = \text{const.} \cdot \epsilon_2$ . Does viscosity affect the relation? Does a similar correlation exist between  $v_3 = k \epsilon_3$ ?

Theoretically, since viscosity introduces additional length scales, one expects reduced correlations.

Chaudhuri:PLB713(2012)



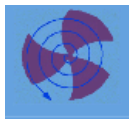
In id. fluid, elliptic flow is strongly correlated. Viscosity weakens the correlation. In id. fluid, triangular flow is weakly correlated. Viscosity weakens it further.

perfect corr.:  $C_{\text{measure}} = 0$   
 random corr.:  $C_{\text{measure}} = 1$

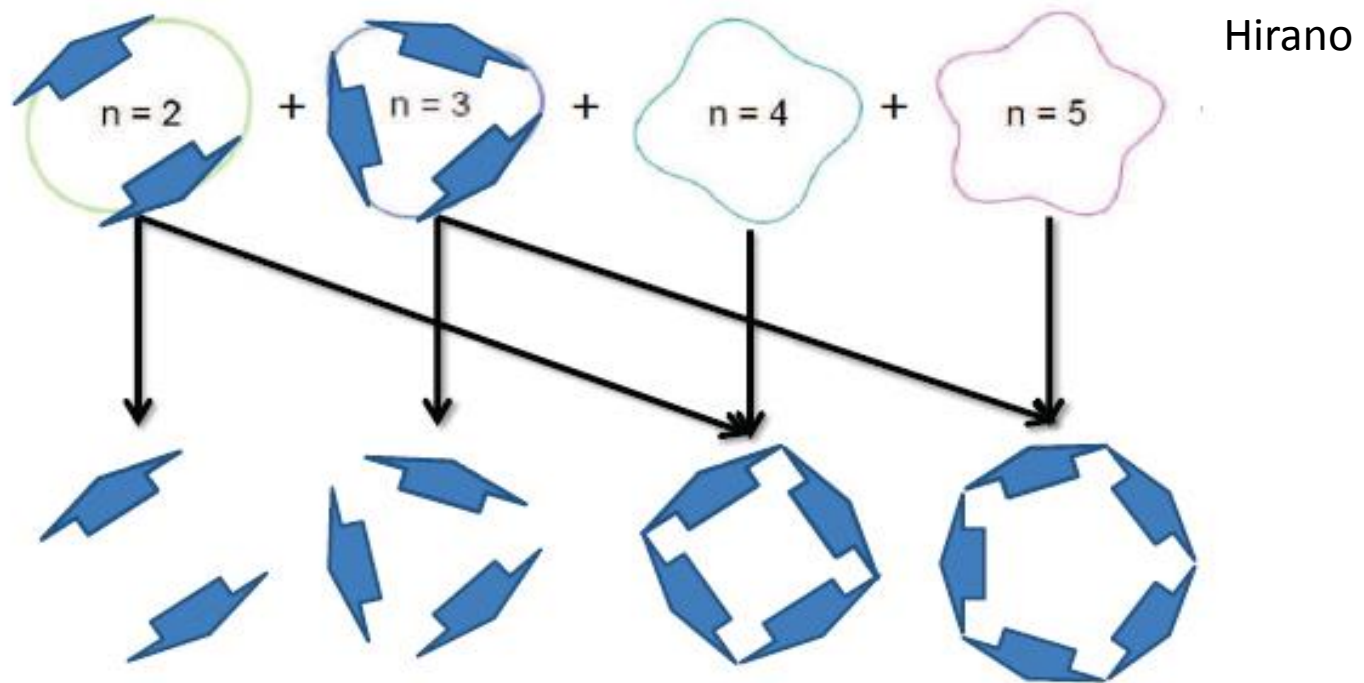
We also introduce correlation measure,

$$C_{\text{measure}}(v_n) = \frac{\sum_i [v_{n,\text{sim}}^i(\epsilon_n) - v_{n,\text{st.line}}(\epsilon_n)]^2}{\sum_i [V_{\text{random}}^i(\epsilon) - v_{\text{st.line}}(\epsilon)]^2}$$

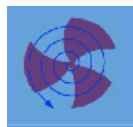
	$\frac{\eta}{s} = 0$	$\frac{\eta}{s} = 0.08$	$\frac{\eta}{s} = 0.12$	$\frac{\eta}{s} = 0.16$
$C_{\text{measure}}(v_2)$	0.052	0.060	0.105	0.202
$C_{\text{measure}}(v_3)$	0.280	0.336	0.446	0.513



## Understanding correlation in e-by-e hydrodynamics:



Higher flows can have contributions from lower order eccentricities.  
Response to deformation is non-linear.



## Summary:

I have briefly discussed ingredients of hydrodynamical modeling of nuclear collisions. Hydrodynamics has been highly successful in explaining a variety of experimental data.

Central temperature in Au+Au:  $T \sim 380$  MeV at  $\tau_i \sim 0.6$  fm

Central temperature in Pb+Pb:  $T \sim 550$  MeV at  $\tau_i \sim 0.6$  fm

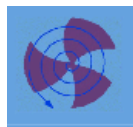
Viscosity over entropy ratio:  $\eta/s \sim 1/4\pi$ , but systematic uncertainty large.

However some issues remain unanswered.

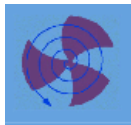
Is there thermal equilibration in HI collisions? How the various parameters of hydro models are inter-correlated? How this inter-correlations affect estimate of viscosity? What is the correct form of dissipative corrections?

Freeze-out, specially with granular IC. Some interior part may freeze-out early but hadrons can get absorbed and re-heat the system.

Origin of decorrelation of higher harmonics?



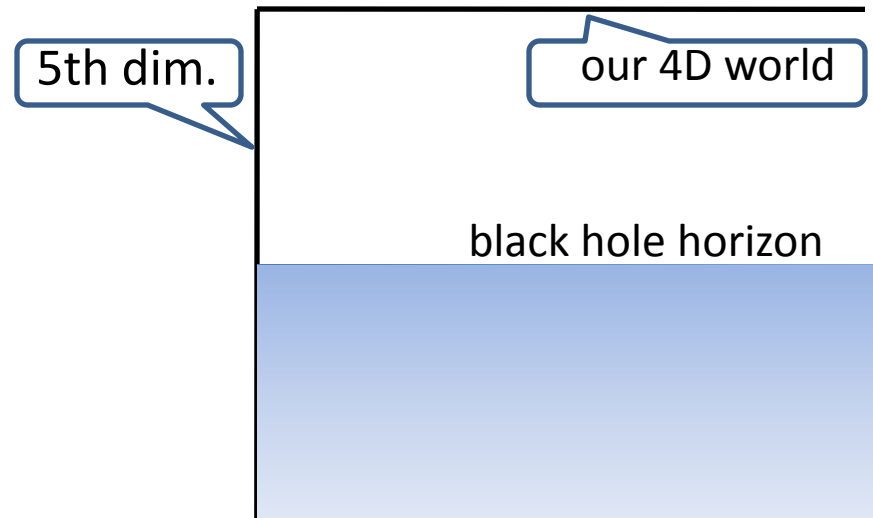
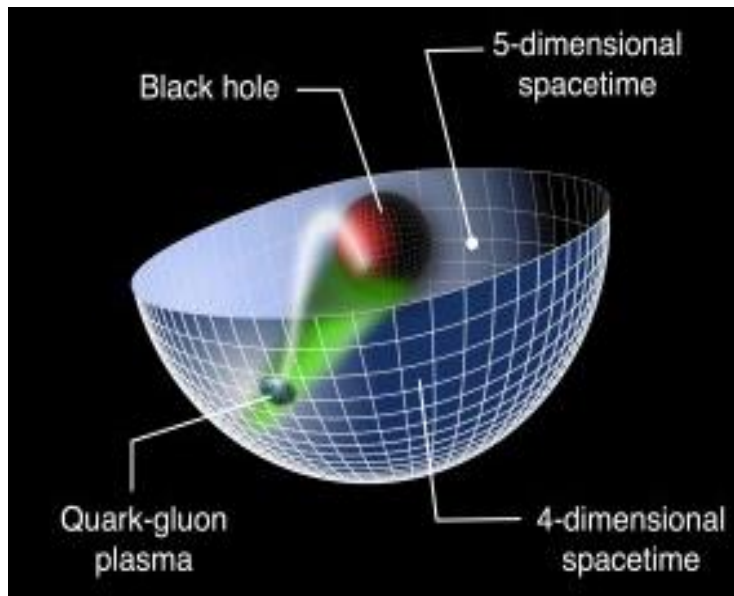
# BACKUP





AdS/CFT correspondence is based on Malcedena conjecture,  
 $AdS_5 \times S^5$  is dual to N=4 supersymmetric Yang-Mills Fields in 4-dimension.  
 $AdS_5$ : Anti-deSitter space: maximally symmetric space with negative curvature.  
 $S^5$ : 5D sphere.

The conjecture is based on Holographic principle (t'Hooft&Susskind):  
Information thrown in BH is encoded on its surface.



## Viscosity over entropy ratio in AdS/CFT:

$$\text{Kubo relation: } \eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle$$

Gauge-Gravity duality [Phys.Reports.323,183]: Stress energy tensor  $T_{xy}$  couples to graviton at the boundary. Absorption cross section of a graviton of frequency  $\omega$ , polarised in  $xy$  direction is the imaginary part of the retarded Green function of  $T_{xy}$ .

$$\sigma_{\text{abs}}(\omega) = -\frac{2\kappa^2}{\omega} \text{Im} G^{\text{R}}(\omega) = \frac{\kappa^2}{\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle \quad \kappa = \sqrt{8\pi G}$$
$$\eta = \frac{\sigma_{\text{abs}}(0)}{2\kappa^2} = \frac{\sigma_{\text{abs}}(0)}{16\pi G}$$

One can show that absorption cross section of graviton is equal to that of a minimally coupled scalar. In the low energy limit [Das,Gibbons,Mathur,PRL78,417] absorption cross-section of a scalar:

$$\sigma_{\text{abs}}(0) = A \text{ (area of the horizon)}$$

$$\text{black brane (hole) entropy: } S = \frac{A}{4G} \quad \text{Viscosity over entropy ratio: } \frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

One also argues that it is the lower bound of the ratio [PRL94,111601, PRL81,081601].

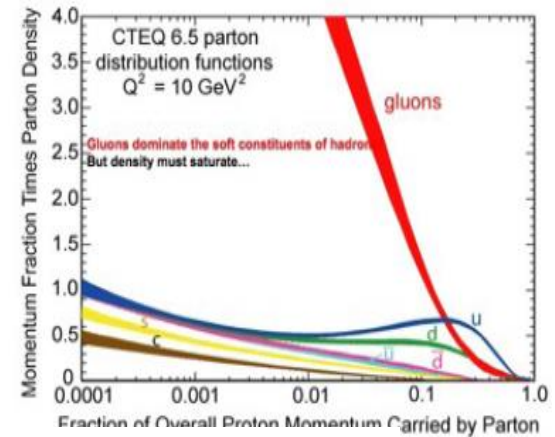
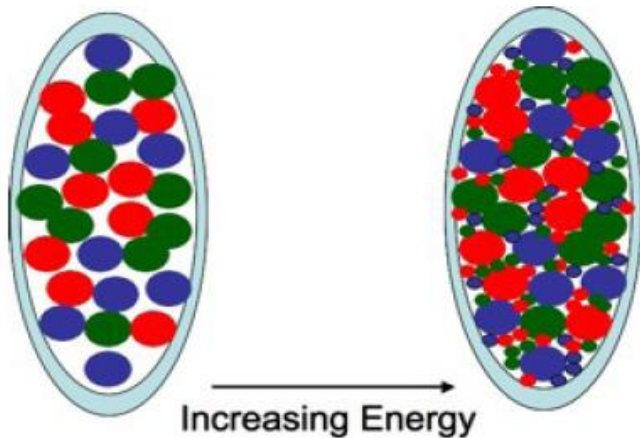
## Color Glass Condensate (CGC):

McLerran and co-workers [PRD52(1995)3809,6231] proposed CGC, a new form of matter.

Gluon density rapidly grows at small  $x=E_g/E_h$ . Gluons control high energy limit of hadron wave function.

Density can not grow indefinitely (unitary limit). Above a certain density repulsive force takes over and on the balance gluon density saturates.

imagine a proton is being packed with fixed size gluons



Phase space density:  $\rho = \frac{dN}{dyd^2p_T d^2x_T}$

Effective energy functional:  $E = -\kappa\rho + \kappa'\alpha_S\rho^2$

Extremum:  $\rho \sim 1/\alpha_S$

$$\frac{dN}{dy} = \int d^2x_T \int d^2q_T \rho \sim \frac{1}{\alpha_S} \pi R^2 Q_{sat}^2$$

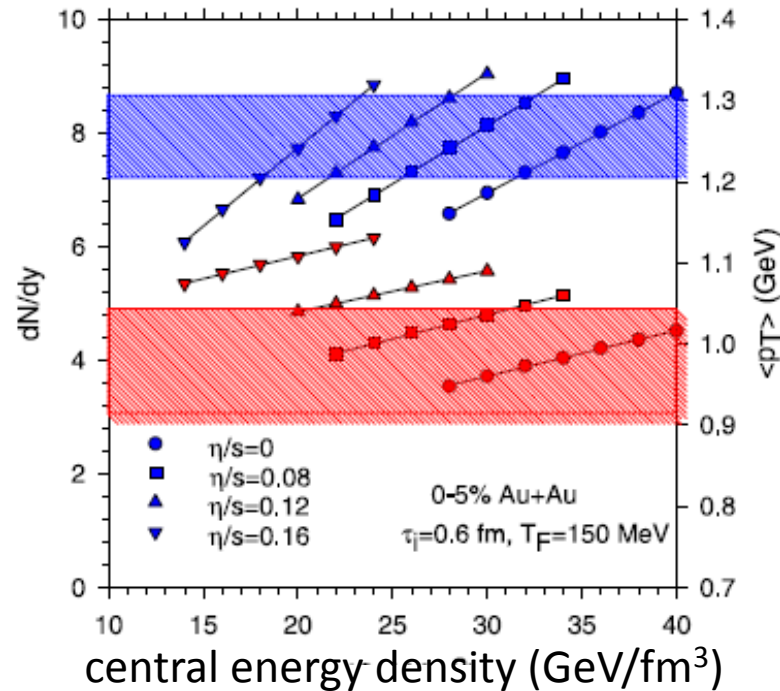
Saturation momentum

**Color:** gluons are colored.

**Glass:** small x gluons have long time scale compared to natural time scale. Small x gluons are generated from fast gluons. Fast gluons time scale is dilated. They pass it to small x gluons.

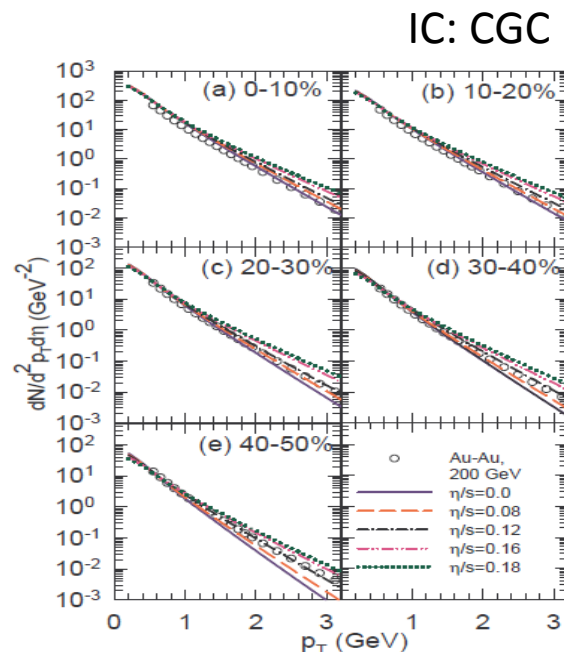
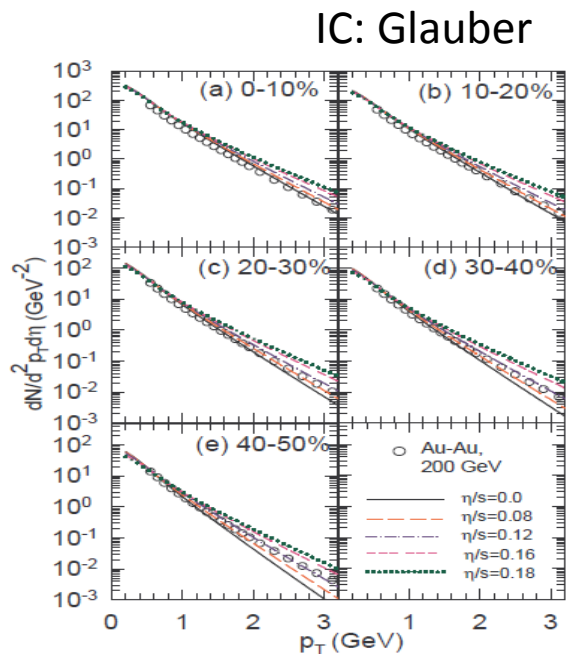
**Condensate:**  $\alpha_S \sim 1/\rho$ , typical of condensate phenomena.

# sensitivity of multiplicity data to initial central energy density and viscosity



irrespective of viscosity, multiplicity data can be fitted and central energy density can be obtained with reasonable accuracy of  $\sim 15\%$

av.  $p_T$  data are more sensitive!



$p_T$  spectra: sensitive to viscosity but not to the details of IC.

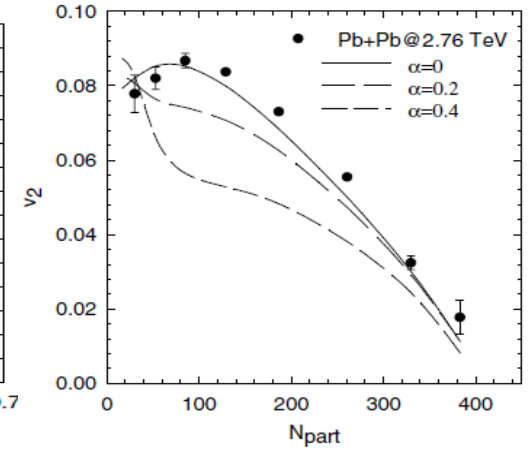
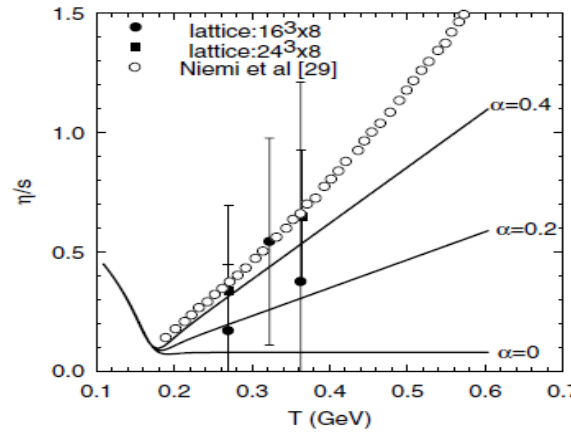
# Temperature dependence of $\eta/s$ :

parametrise

$$\left(\frac{\eta}{s}\right)_{QGP} = \frac{1}{4\pi} + \alpha \frac{T - T_c}{T_c}$$

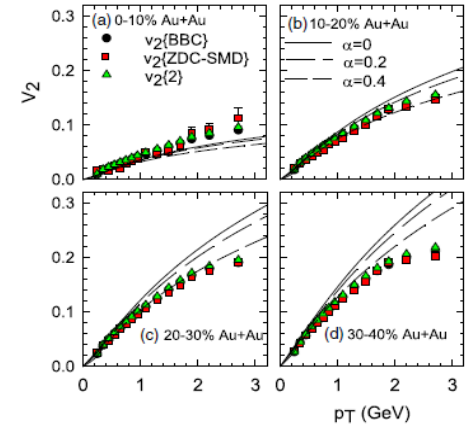
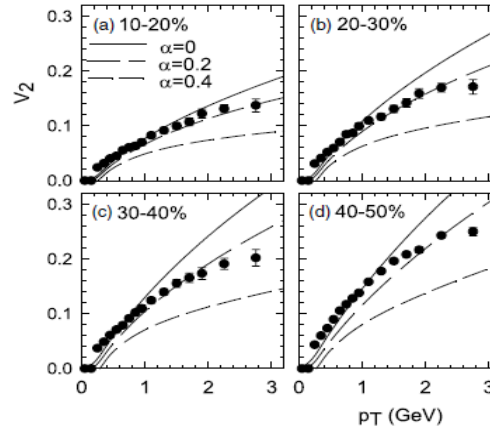
$$\left(\frac{\eta}{s}\right)_{HAD} = \text{from Hadron Resonance Gas}$$

Jphys.G39(2012)125102



ALICE data on Pb+Pb:  
prefer  $\alpha \sim 0-0.2$ ,  
disfavor  $\alpha \sim 0.4$ .

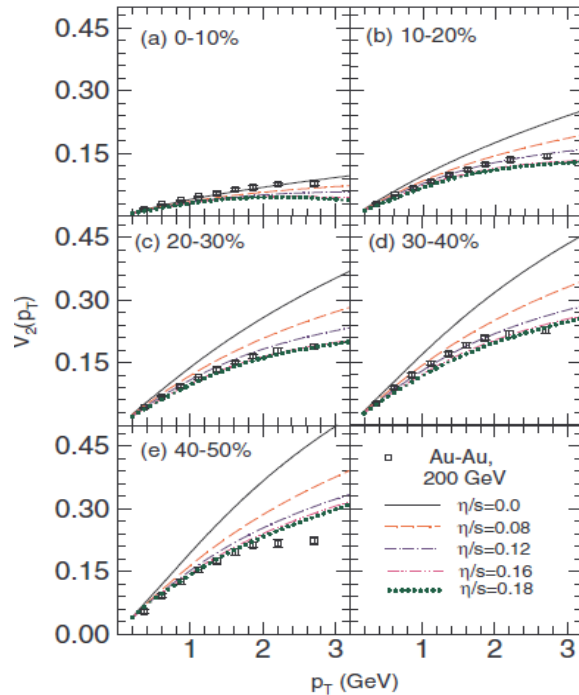
RHIC data on Au+Au:  
prefer  $\alpha \sim 0.4$ ,  
disfavor  $\alpha \sim 0-0.2$



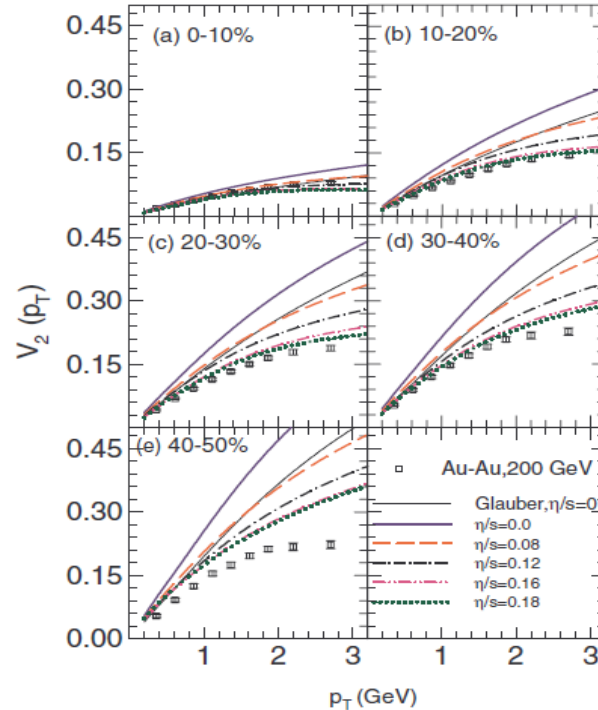
no unique 'linear' temperature dependence.

# AZHYDRO-KOLKATA simulations for $V_2$ and $V_4$ [Roy,Chaudhuri,Mohanty,PRC86]

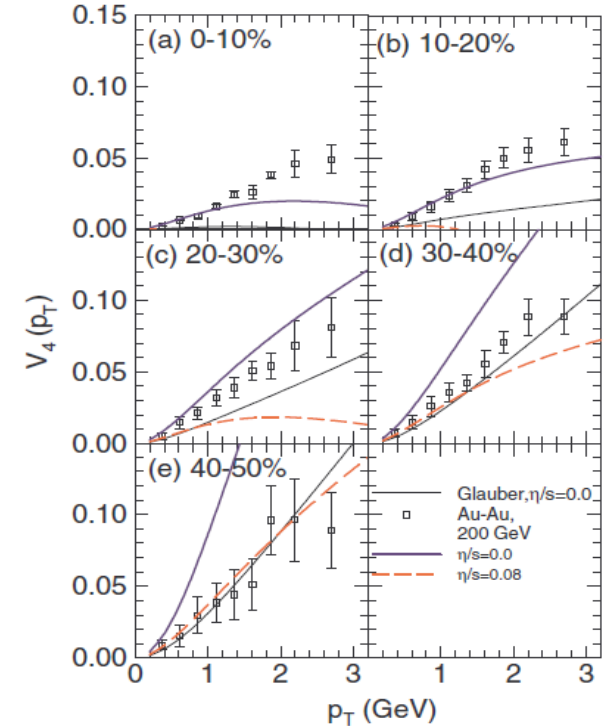
## Glauber model



## CGC model



## Glauber/CGC model



1. peripheral collisions prefer higher viscosity.
2. CGC IC demand higher viscosity to explain data than Glauber IC.
3. With the exception of 40-50% collision, Glauber model do not explain  $V_4$ .
4. with CGC,  $V_4$  require  $\eta/s \sim 0-0.08$ .  $V_2$  however require higher value (inconsistency?).

## Viscous effects on EM probes:

For em probes, production rates are convoluted over the fluid space-time evolution.

photon rate: 
$$E \frac{d^3 R}{d^3 p} = \sum_i \frac{\mathcal{N}}{(2\pi)^7} \frac{1}{16E} \int ds dt |\overline{\mathcal{M}}_i|^2 \int dE_1 dE_2 f_1(E_1) f_2(E_2)$$

dilepton rate: 
$$\times [1 \pm f_3(E_1 + E_2 - E)] \frac{\theta(E_1 + E_2 - E)}{\sqrt{(aE_1^2 + bE_1 + c)}}$$

$$\frac{dR}{d^4 q} = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} f_{\text{neq},1}(E_1, T) f_{\text{neq},2}(E_2, T) v_{12} \sigma(M^2) \delta^4(q - p_1 - p_2)$$

rate eq. are complicated  
in viscous hydro

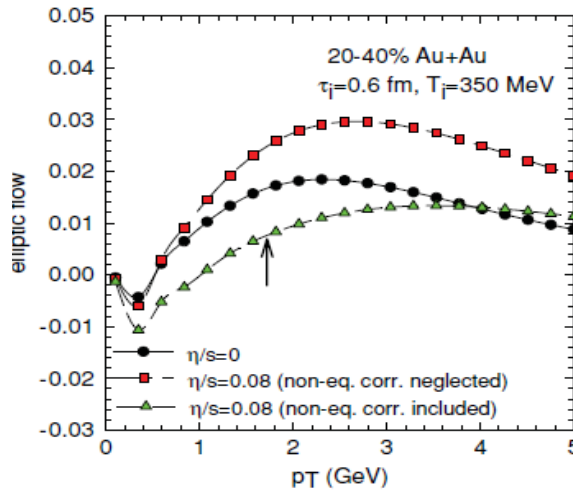
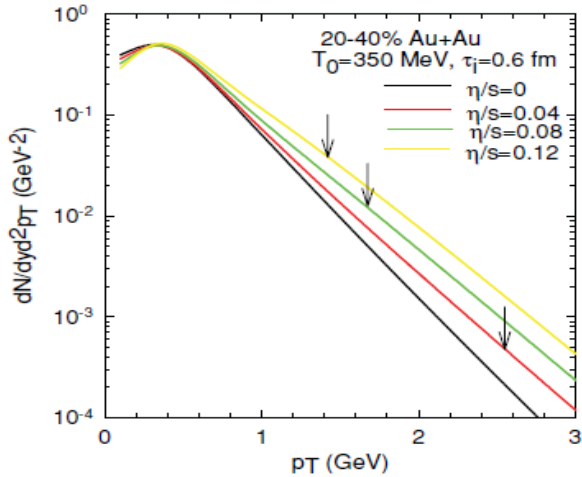
$$f_{\text{neq}} \rightarrow f_{\text{eq}}(1 + \delta f_{\text{neq}}), \delta f_{\text{neq}} \ll 1.$$

in the QGP phase, rate equations can be simplified. With simplified rate, viscous effect on photon and dilepton were studied in NPA809, NPA839, PRC83, JPhysG40.

Recently, Gale&co-worker calculated photon rate equations for the hadronic sector [PRC84,1308.2440,1308.2111].



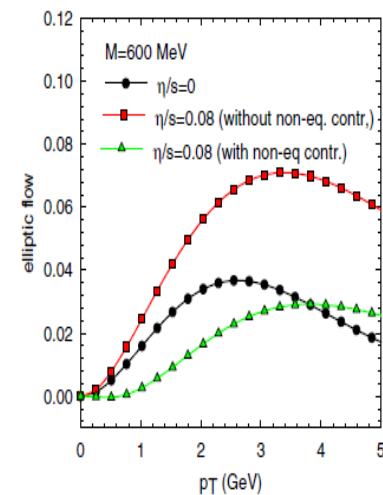
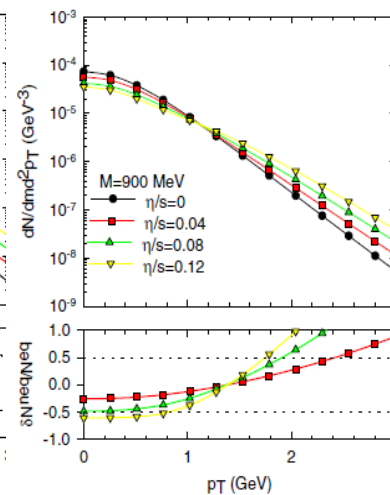
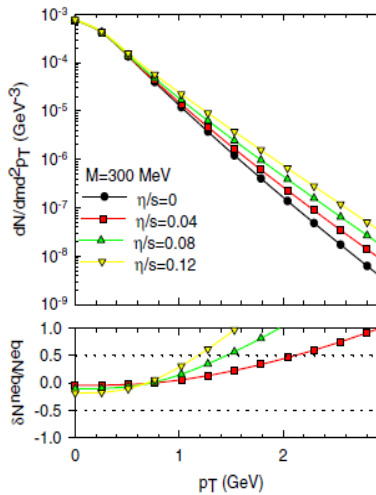
## Direct photon [Chaudhuri, Sinha, PRC83]:



strong viscous effect. Reliable simulation is possible only in a limited  $p_T$  range. Non-eq. correction in rate Eq. more important than correction in fluid evolution.

Large invariant mass dileptons, due to their lower velocity, are less affected by viscosity than the low invariant mass dileptons.

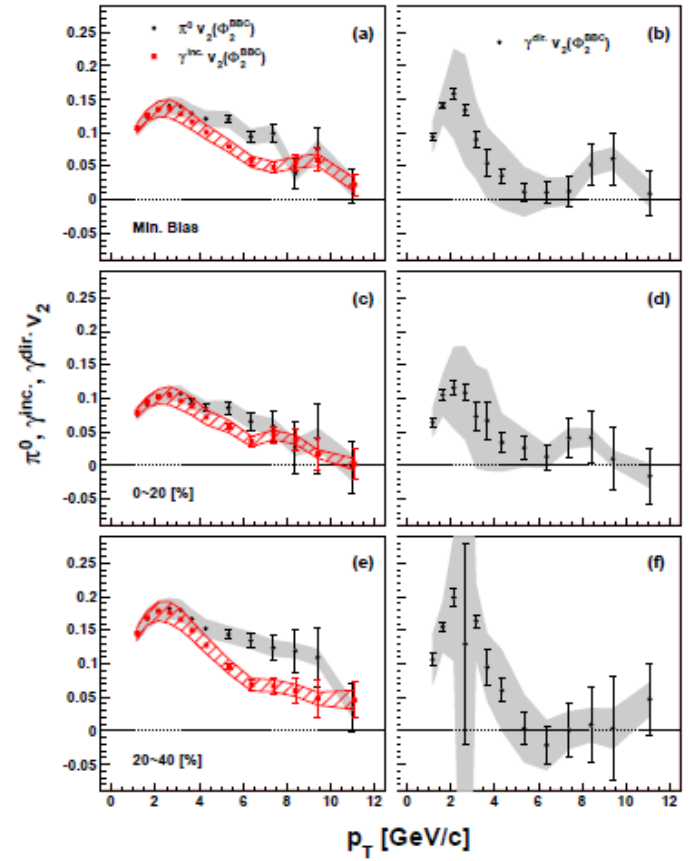
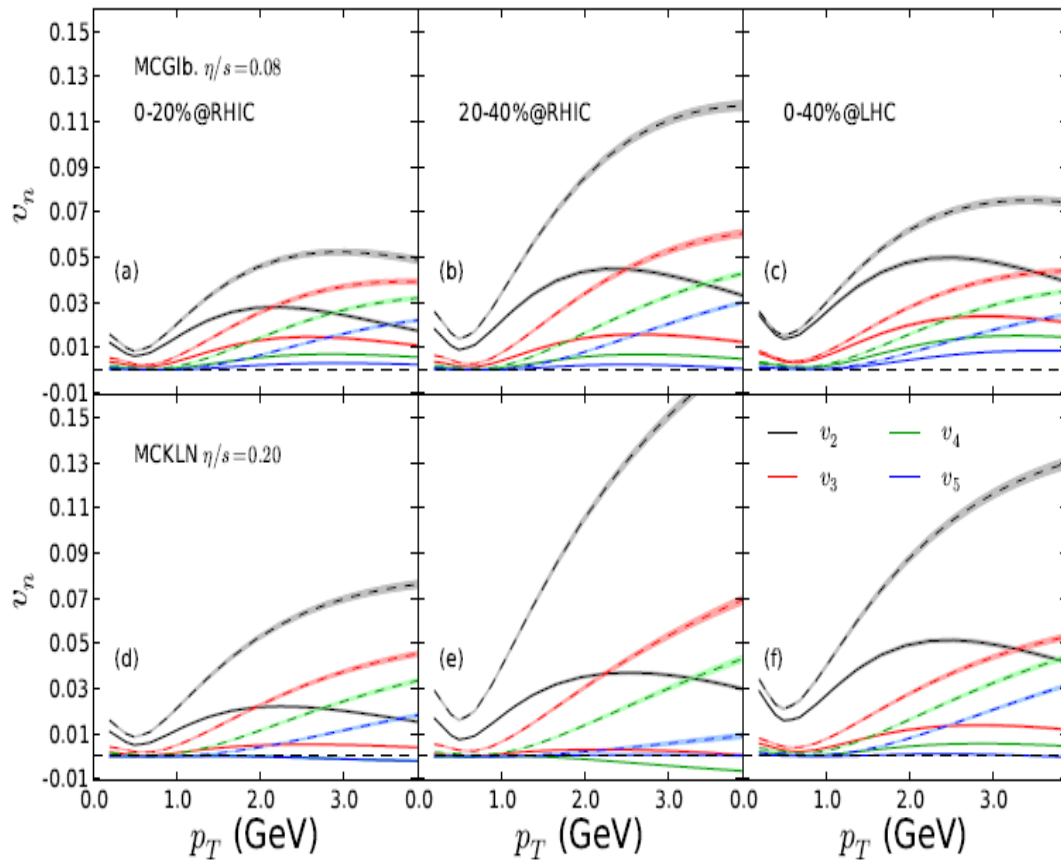
## Dilepton [Chaudhuri, Sinha, JPhysG40]



# e-by-e simulations for Photons

Shen, Heinz, Paquet, Kozlov, Gale: arXiv/1308.2111

PHENIX: PRL109(2012)



Viscous corrections to the rates have a larger effect on the  $v_n$  coefficients than the viscous suppression of hydrodynamic flow anisotropies. Simulated  $v_2$  is less than in experiment.

# Viscous effects in e-by-e hydro [Chaudhuri:PLB710]:

Hot spot model (captures the spirit of Glauber Monte-Carlo model)

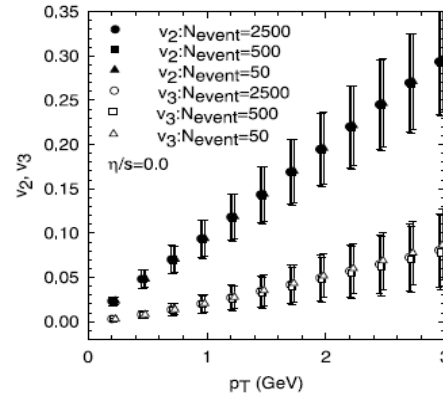
flow coefficients [Qiu&Heinz,PRC84(2011)]

$$v_n(y, p_T) e^{in\psi_n(y, p_T)} = \frac{\int d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\frac{dN}{dy p_T dp_T}}$$

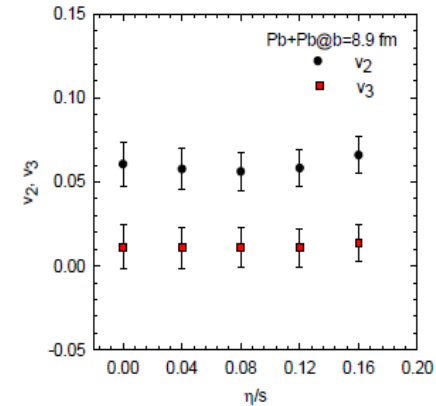
$$v_n(y) e^{in\psi_n(y)} = \frac{\int p_T dp_T d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\frac{dN}{dy}}$$

simulate b=8.9 fm Pb+Pb coll. Each event is solved by AZHYDRO-KOLKATA.

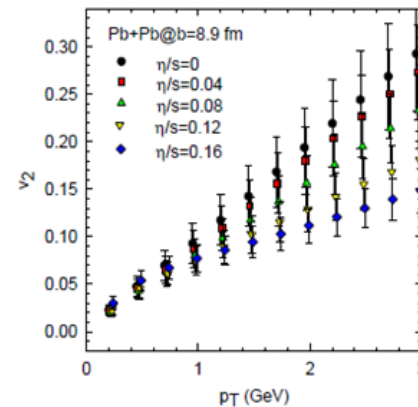
**With fluctuating initial conditions, sensitivity of flow coefficients towards viscosity is markedly reduced.**



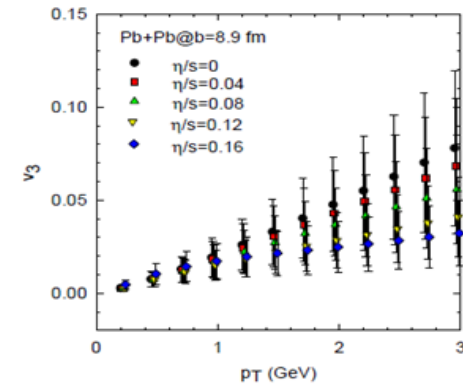
strong non-statistical fluctuation.



$\eta/s=0-0.16$  not distinguished



$\eta/s=0-0.08$  are not distinguished

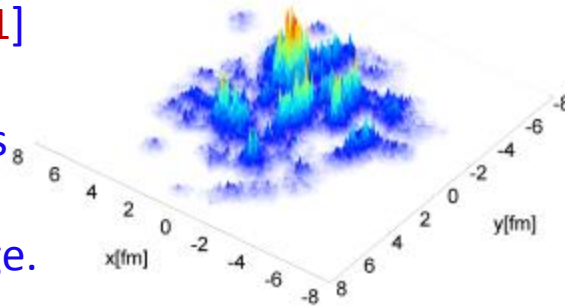


$\eta/s=0-0.16$  are not distinguished

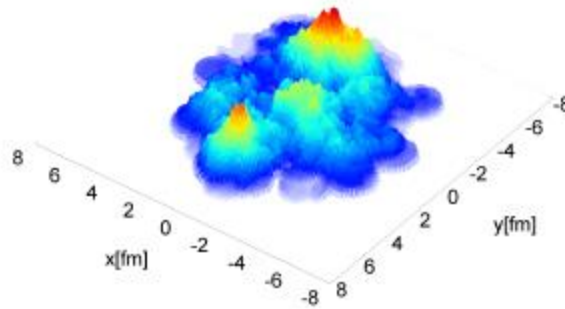
# Initial condition in event-by-event hydrodynamics:

## IP Glasma [PRL108,252301]

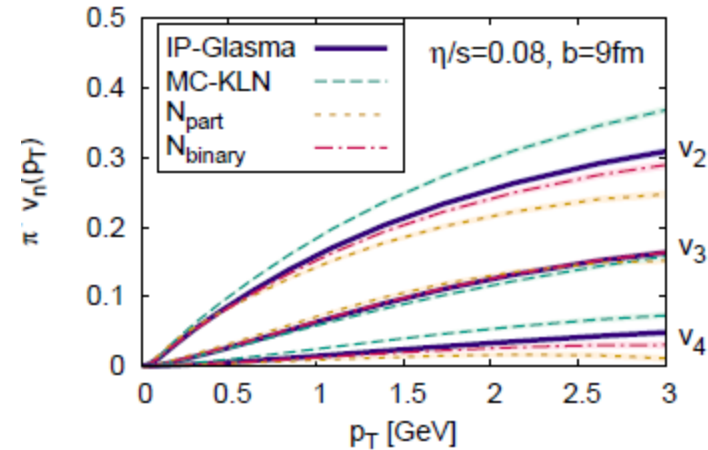
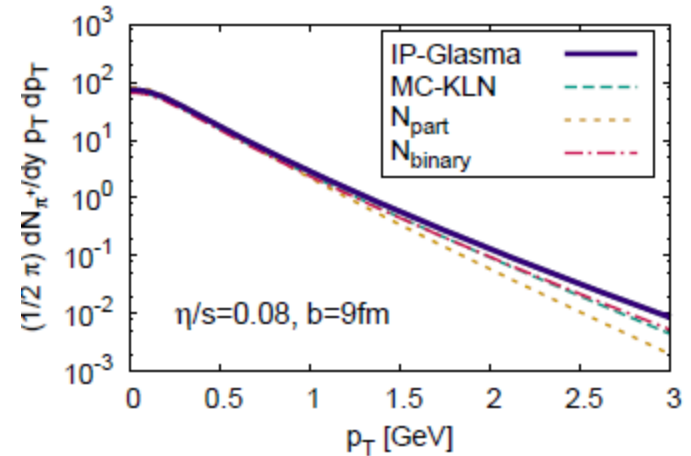
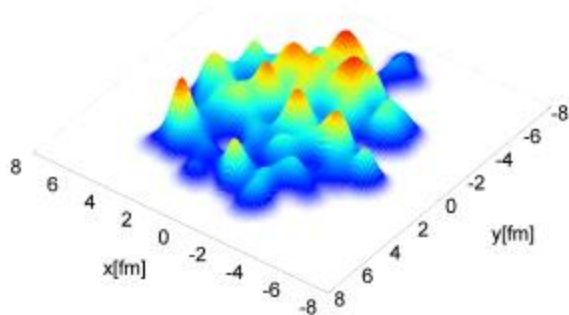
In addition to fluctuations in nucleon positions, takes into account quantum fluctuations of color charge.



MC-CGC: fluctuations in nucleon positions. fail to explain  $v_2$  &  $v_3$  simultaneously.



MC-Glauber: fluctuations in nucleon positions. Can be tuned to explain  $v_2$  &  $v_3$  simultaneously.



MC-CGC improved with color fluctuations expected to better explain the data.