BFKL Pomeron, reggeized gluons and Bern-Dixon-Smirnov amplitudes

Lev N. Lipatov
Petersburg Nuclear Physics Institute

J. Bartels, L. N. Lipatov, A. Sabio Vera
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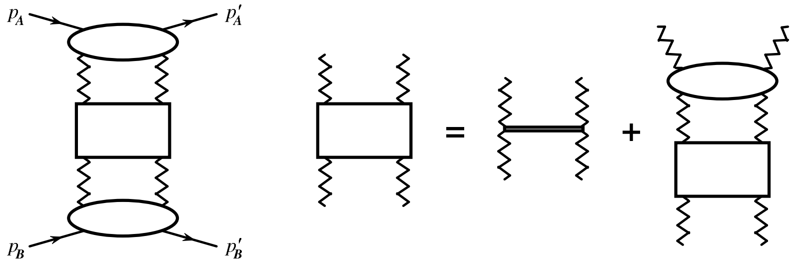
1 BFKL equation (1975)

Diagrams for the total cross-section



Balitsky-Fadin-Kuraev-Lipatov equation

$$E\,\Psi(\vec{\rho}_1,\vec{\rho}_2) = H_{12}\,\Psi(\vec{\rho}_1,\vec{\rho}_2)\;,\;\; \sigma_t \sim s^{\Delta}\,,\;\; \Delta = -\frac{\alpha_s N_c}{2\pi}\,E$$

BFKL Hamiltonian

$$H_{12} = \ln\,|p_1p_2|^2 + \frac{1}{p_1p_2^*}(\ln|\rho_{12}|^2)p_1p_2^*$$

$$+\frac{1}{p_1^*p_2}(\ln|\rho_{12}|^2)p_1^*p_2 - 4\psi(1)\,,\;\; \rho_{12} = \rho_1 - \rho_2$$

Möbius invariance and conformal weights (L. (1986))

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d}\,,$$

$$m = \gamma + n/2\,,\;\; \widetilde{m} = \gamma - n/2\,,\;\; \gamma = 1/2 + i\nu$$

2 Integrability at $N_c \to \infty$

Bartels-Kwiecinski-Praszalowicz equation (1980)

$$E\,\Psi(\vec{\rho}_1,...,\vec{\rho}_n) = H\,\Psi(\vec{\rho}_1,...,\vec{\rho}_n)\;,\;\; H = \sum_{k<l} \frac{\vec{T}_k \vec{T}_l}{-N_c}\,H_{kl}$$

Holomorphic factorization at large N_c (L. (1988))

$$\Psi(\vec{\rho}_1,\vec{\rho}_2,...,\vec{\rho}_n) = \sum_{r,s} a_{r,s}\,\Psi_r(\rho_1,...,\rho_n)\,\Psi_s(\rho_1^*,...,\rho_n^*)$$

Transfer and monodromy matrices (L. (1993))

$$T(u) = tr\,t(u)\;,\;\; t(u) = L_1 L_2 ... L_n = \sum_{r=0}^{n} u^{n-r}\,q_r\;,$$

$$L_k = \left(\begin{array}{cc} u + \rho_k\,p_k & p_k \\ -\rho_k^2\,p_k & u - \rho_k\,p_k \end{array} \right)\;,\;\; \hat{l} = u\,\hat{1} + i\,\hat{P}$$

Yang-Baxter equation (L. (1993))

$$t_{r_1'}^{s_1}(u)\,t_{r_2'}^{s_2}(v)\,l_{r_1 r_2}^{r_1' r_2'}(v-u) = l_{s_1' s_2'}^{s_1 s_2}(v-u)\,t_{r_2}^{s_2'}(v)\,t_{r_1}^{s_1'}(u)$$

Heisenberg spin model (L. (1994); F.,K. (1995))

3 Pomeron in $N = 4$ SUSY

BFKL kernel in two loops (F., L. (1998))

$$\omega = 4\,\hat{a}\;\chi(n,\gamma) + 4\;\hat{a}^2\,\Delta(n,\gamma)\,,\;\hat{a} = g^2 N_c/(16\pi^2)\,,$$

$$\chi(n,\gamma) = 2\Psi(1) - \Psi(\gamma + |n|/2) - \Psi(1 - \gamma + |n|/2)$$

Non-analytic terms (K.,L. (2000))

$$\Delta_{QCD}(n,\gamma) = c_0\delta_{|n|,0} + c_2\delta_{|n|,2} + \ldots\,,\;c_0^{N=4} = c_2^{N=4} = 0$$

Hermitian separability in $N = 4$ SUSY (K.,L. (2000))

$$\Delta(n,\gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2\hat{a}/\omega}\,,\;M = \gamma + \frac{|n|}{2}\,,$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2)\,,\;\beta'(z) = \frac{1}{4}\left[\Psi'\left(\frac{z+1}{2}\right) - \Psi'\left(\frac{z}{2}\right)\right]$$

Maximal transcendentality (K.,L. (2002))

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 2\Phi(M) + 2\beta'(M)\Big(\Psi(1) - \Psi(M)\Big),$$

$$\Phi(M) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+M}\left(\Psi'(k+1)\;-\;\frac{\Psi(k+1) - \Psi(1)}{k+M}\right)$$

4 Three loops for the BFKL kernel

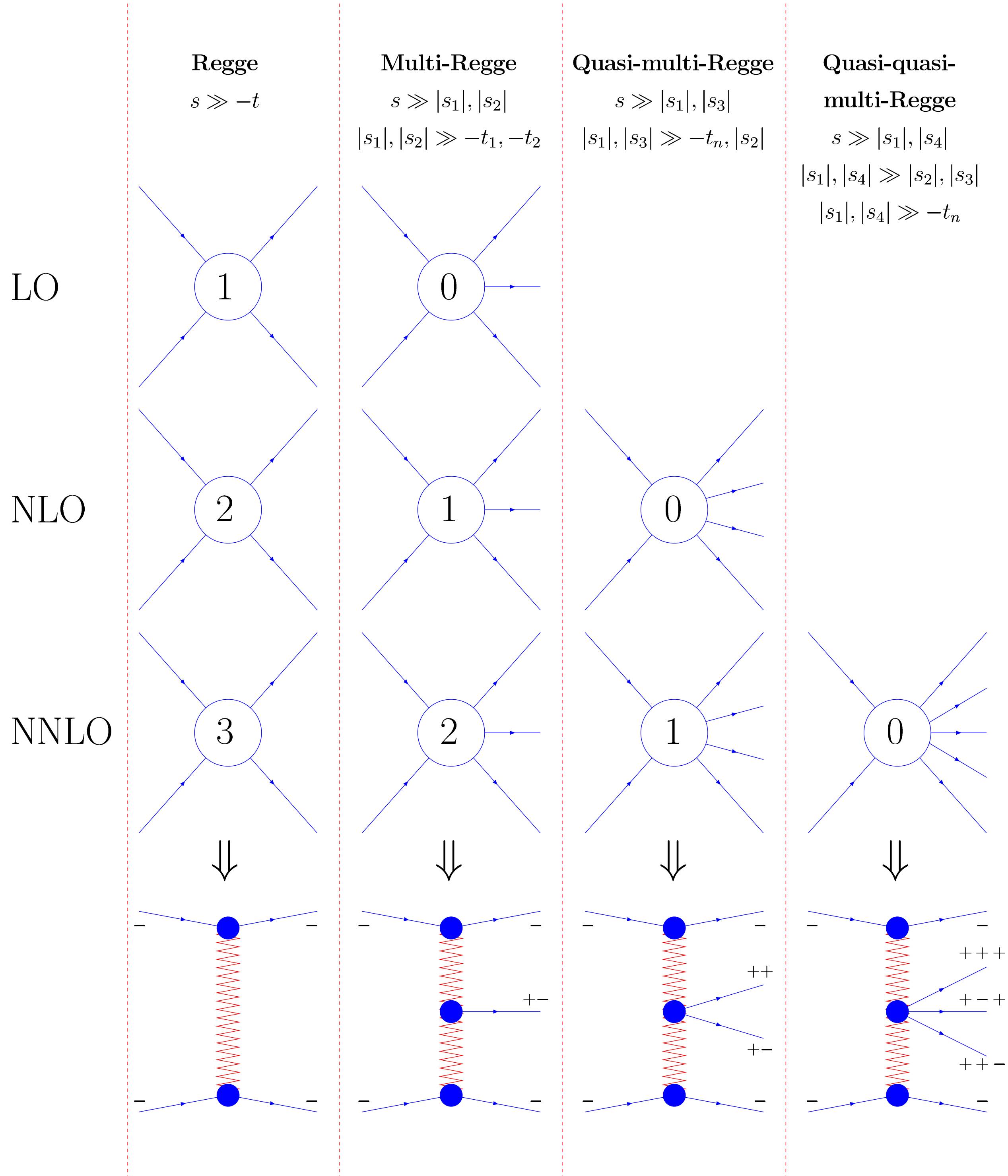


Figure 1: Diagrams contributing to the BFKL kernel

5 Effective action approach

Locality in the rapidity

$$y=\frac{1}{2}\ln\frac{\epsilon_k+|k|}{\epsilon_k-|k|}\,,\;\;|y-y_0|<\eta\,,\;\;\eta<<\ln\,s$$

Gluon and Reggeized gluon fields

$$v_\mu(x)=-iT^a v_\mu^a(x)\,,\;\;A_\pm(x)=-iT^a A_\pm^a(x)$$

Local gauge transformations

$$\delta v_\mu(x)=\frac{1}{g}[D_\mu,\chi(x)],\;\delta\psi(x)=-\chi(x)\,\psi(x),\;\delta A_\pm(x)=0$$

Effective action for reggeized gluons (L., 1995)

$$S=\int d^4x\,(L_0+L_{ind})\;,\;\;L_0=i\bar{\psi}\hat{D}\psi+\frac{1}{2}\,Tr\,G_{\mu\nu}^2$$

$$L_{ind}=Tr\,(L_{ind}^k+L_{ind}^{GR})\,,\;\;L_{ind}^k=-\partial_\mu A_+^a\partial_\mu A_-^a\;,$$

$$L_{ind}^{GR}=-\frac{1}{g}\partial_+\,P\exp\left(-g\frac{1}{2}\int_{-\infty}^{x^+}v_+(x')d(x')^+\right)\,\partial_\sigma^2 A_-$$

$$-\frac{1}{g}\partial_-\,P\exp\left(-g\frac{1}{2}\int_{-\infty}^{x^-}v_-(x')d(x')^+\right)\,\partial_\sigma^2 A_+$$

6 Maximal helicity violation

BDS amplitudes in $N=4$ SUSY at $N_c \gg 1$ (2005)

$$A^{a_1,\ldots,a_n} = \sum_{\{i_1,\ldots,i_n\}} Tr\, T^{a_{i_1}} T^{a_{i_2}} \ldots T^{a_{i_n}} \, f(p_{i_1}, p_{i_2}, \ldots, p_{i_n}) \,,$$

$$f(p_{i_1}, p_{i_2}, \ldots, p_{i_n}) = f_B(p_{i_1}, p_{i_2}, \ldots, p_{i_n}) M_n(p_{i_1}, p_{i_2}, \ldots, p_{i_n})$$

Invariant amplitudes

$$\ln M_n = \sum_{l=1}^{\infty} a^l \left(f^{(l)}(\epsilon) \left(\hat{I}_n^{(l)}(l\epsilon) + F_n^{(1)}(0) \right) + C^{(l)} \right),$$

$$C^{(1)} = 0 \,, \;\; C^{(2)} = -\zeta_2^2/2 \,, \;\; f^{(l)}(\epsilon) = f_0^{(l)} + \epsilon f_1^{(1)} + \epsilon^2 f_2^{(l)}$$

Cusp anomalous dimension

$$f_0^{(l)} = \frac{1}{4} \gamma_K^{(l)} \,, \;\; f_1 = -a\zeta_3/2 + a^2(2\zeta_5 + 5\zeta_2\zeta_3/3) + \ldots$$

Infraredly divergent contribution

$$\hat{I}_n^{(1)}(\epsilon) = -\frac{1}{2\epsilon^2} \sum_{i=1}^{n} \left(\frac{\mu^2}{-s_{i,i+1}} \right)^{\epsilon} , \;\; a = \frac{\alpha N_c}{2\pi} \left(4\pi e^{-\gamma}\right)^{\epsilon}$$

Finite remainder

$$F_n^{(1)}(0) = \lim_{\epsilon \to 0} F_n^{(1)}(\epsilon)$$

7 BDS amplitude at $s/t \to \infty$

Elastic amplitude

$$M_{2\to 2} = \Gamma(t) \left(\frac{-s}{\mu^2}\right)^{\omega(t)} \Gamma(t)$$

Reggeized gluon trajectory

$$\omega(t) = -\frac{\gamma(a)}{4} \ln\frac{-t}{\mu^2} + \int_0^a \frac{da'}{a'} \left(\frac{\gamma_K(a')}{4\epsilon} + \beta(a')\right)$$

$$= \left(-\ln\frac{-t}{\mu^2} + \frac{1}{\epsilon}\right) a + \left(\zeta_2 \left(\ln\frac{-t}{\mu^2} - \frac{1}{2\epsilon}\right) - \frac{\zeta_3}{2}\right) a^2$$

$$+ \left(-\frac{11}{2}\zeta_4 \left(\ln\frac{-t}{\mu^2} - \frac{1}{3\epsilon}\right) + \frac{6\zeta_5 + 5\zeta_2\zeta_3}{3}\right) a^3 + \ldots$$

Reggeon residues

$$\ln\Gamma(t) = \ln\frac{-t}{\mu^2} \int_0^a \frac{da'}{a'} \left(\frac{\gamma(a')}{8\epsilon} + \frac{\beta(a')}{2}\right) + \frac{C(a)}{2} + \frac{\gamma(a)}{2} \zeta_2$$

$$- \int_0^a \frac{da'}{a'} \ln\frac{a}{a'} \left(\frac{\gamma(a')}{4\epsilon^2} + \frac{\beta(a')}{\epsilon} + \delta(a')\right)$$

Dual representation

$$M_{2\to 2} = \Gamma(s) \left(\frac{-t}{\mu^2}\right)^{\omega(s)} \Gamma(s)$$

8 Production amplitude

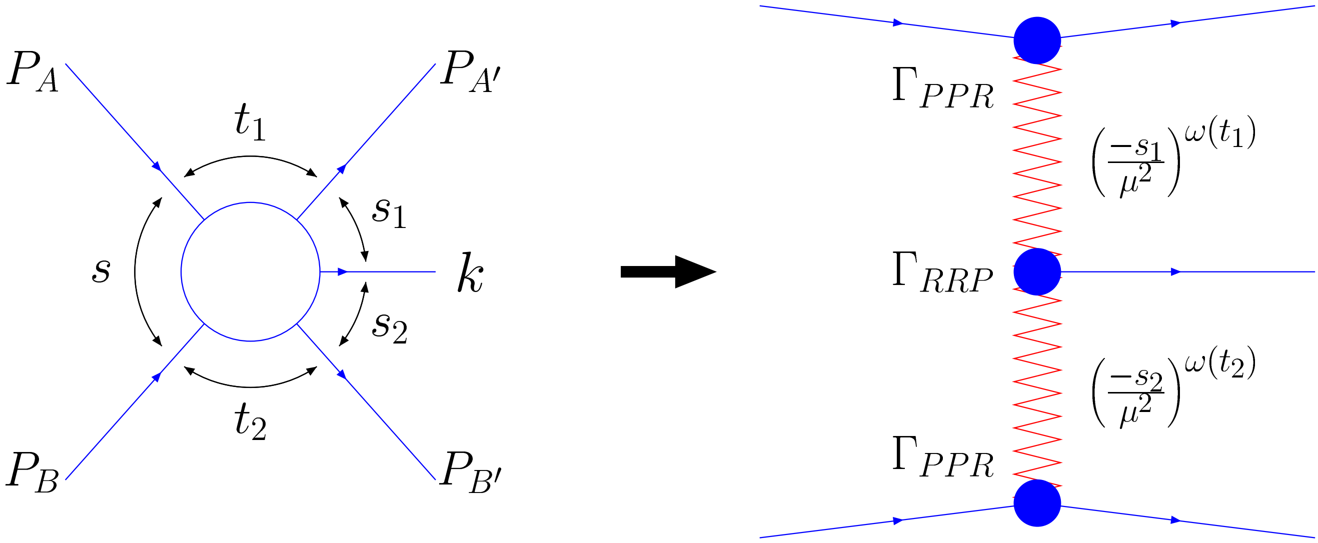


Figure 2: Production in the multi-Regge regime

Reggeon-Reggeon-gluon vertex

$$\ln\Gamma(t_2,t_1,\ln -\kappa) = -\frac{\gamma_K(a)}{16}\left(\ln^2\frac{-\kappa}{\mu^2} - \ln^2\frac{-t_1}{-t_2} - \zeta_2\right)$$

$$-\frac{1}{2}\int_0^a \frac{da'}{a'}\ln\frac{a}{a'}\left(\frac{\gamma_K(a')}{4\epsilon^2} + \frac{\beta(a')}{\epsilon} + \delta(a')\right) -$$

$$\frac{1}{2}\left(\omega(t_1)+\omega(t_2) - \int_0^a \frac{da'}{a'}\left(\frac{\gamma_K(a')}{4\epsilon} + \beta(a')\right)\right)\ln\frac{-\kappa}{\mu^2},$$

$$-\kappa = \frac{(-s_1)(-s_2)}{(-s)}, \ |\kappa| = \vec{k}_\perp^2$$

9 Two particle production

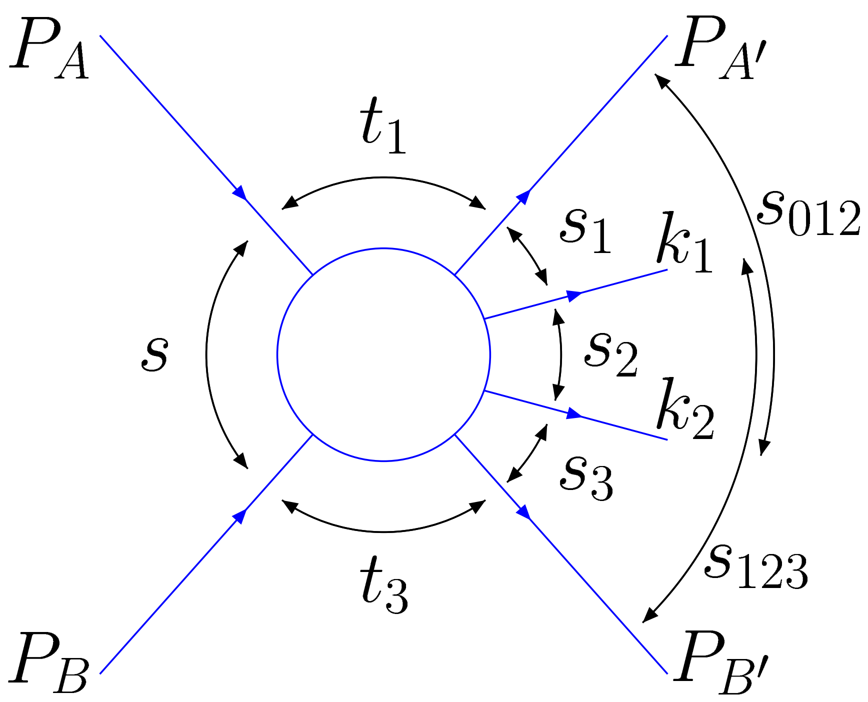


Figure 3: Multi-Regge kinematics

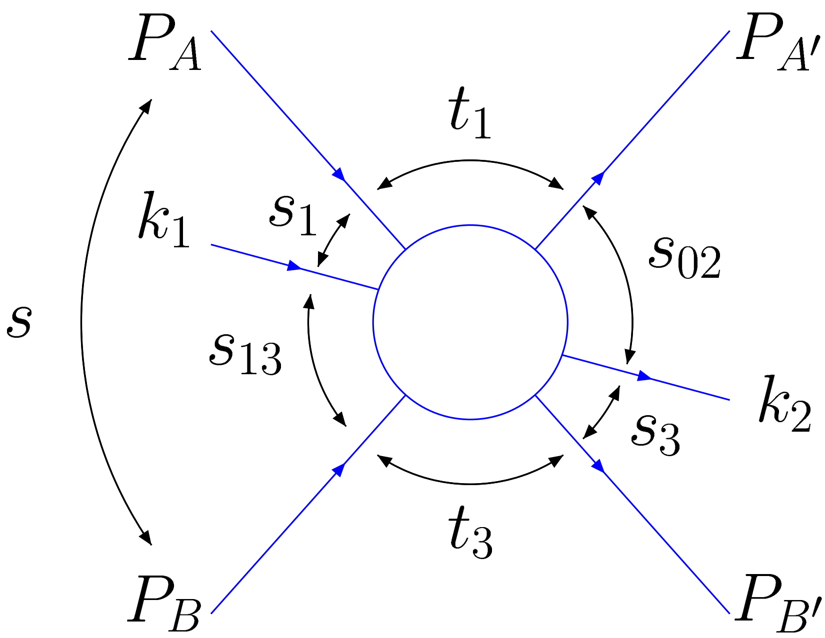
Regge factorization violation at

$s, s_2 > 0;\ \ s_1, s_3, s_{012}, s_{123} < 0$

$$\frac{M_{2\to 4}}{\Gamma(t_1)\Gamma(t_3)} == \exp\left[\frac{\gamma_K(a)}{4}\, i\pi \left(\ln \frac{t_1 t_2}{(\vec{k}_1+\vec{k}_2)^2\mu^2} - \frac{1}{\epsilon}\right)\right]$$

$$\times \left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)} \Gamma(t_2,t_1) \left(\frac{-s_2}{\mu^2}\right)^{\omega(t_2)} \Gamma(t_3,t_2) \left(\frac{-s_3}{\mu^2}\right)^{\omega(t_3)}$$

10 Tree particle scattering



Regge factorization violation at

$s, t_2' > 0;\ s_1, s_3, s_{13}, s_{02} < 0$

$$\frac{M_{3\to 3}}{\Gamma(t_1)\Gamma(t_3)} == \exp\left[\frac{\gamma_K(a)}{4}\, i\pi\, \ln \frac{(\vec{q}_1+\vec{q}_3-\vec{q}_2)^2\, \vec{q}_2^{\,2}}{(\vec{q}_1-\vec{q}_2)^2\,(\vec{q}_2-\vec{q}_3)^2}\right]$$

$$\times \left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)} \Gamma(t_2, t_1, \ln\kappa_{12} + i\pi) \left(\frac{t_2' e^{-i\pi}}{\mu^2}\right)^{\omega(t_2)}$$

$$\times\Gamma(t_2, t_1, \ln\kappa_{23}+i\pi) \left(\frac{-s_3}{\mu^2}\right)^{\omega(t_3)}\ ,\ \ \kappa_{12} = \vec{k}_1^{\,2}\,,\ \ \kappa_{23} = \vec{k}_2^{\,2}$$

11 Regge cuts in j_2-plane

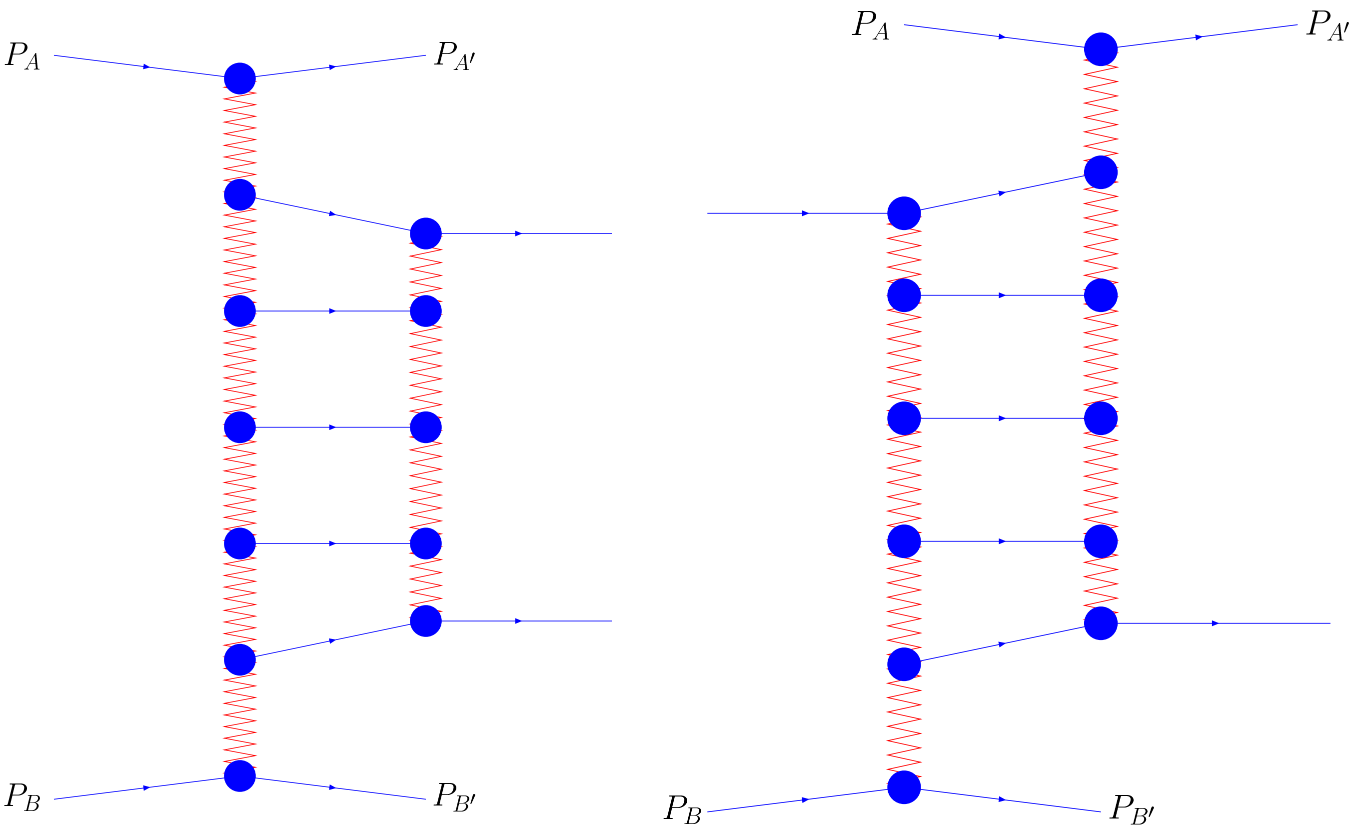


Figure 4: BFKL ladders in $M_{2\to 4}$ and $M_{3\to 3}$

BFKL equation in octet channels

$$\Delta_2 = -a\left(E - \frac{1}{\epsilon}\right)\,,\;\; E\Psi = -a\left(\frac{1}{2}\ln\frac{|p_1p_2|^2}{\mu^4} + \frac{H}{2}\right)\Psi\,,$$

$$H = \ln|p_1p_2|^2 + \frac{1}{p_1p_2^*}\ln|\rho_{12}|^2 p_1p_2^* + \frac{1}{p_1^*p_2}\ln|\rho_{12}|^2 p_1^*p_2 + 4\gamma$$

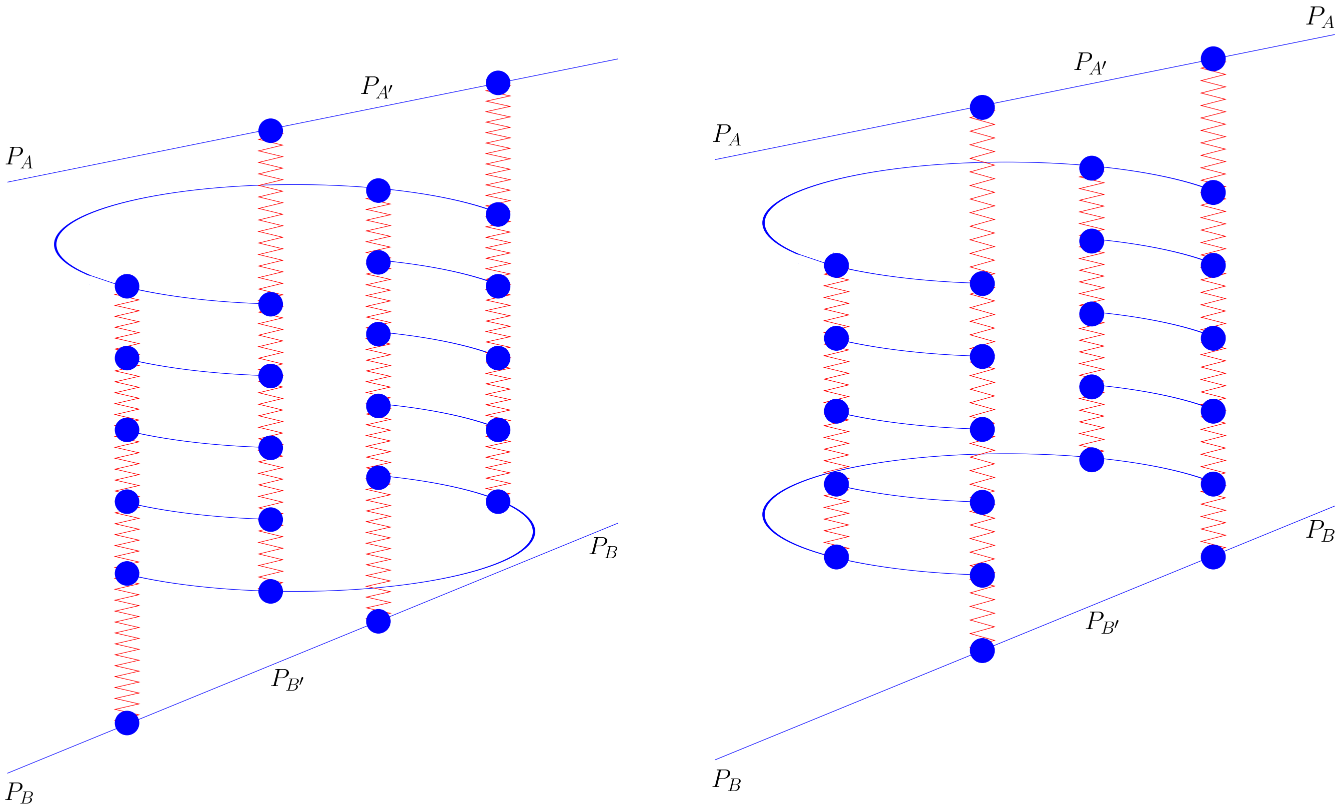


Figure 5: Cylinder-type topologies in the unitarity sums for the total cross section: the intermediate states (discontinuity cuts) are obtained by slicing the cylinders in all possible ways across the intermediate momenta $p_{A'}$ and $p_{B'}$

12 Discussion

1. Pomeron as a composite state of reggeized gluons.
2. Integrability of the BFKL dynamics.
3. Properties of the BFKL kernel in $N = 4$ SUSY.
4. Next-to-next-to-leading corrections.
5. Effective action for Reggeized gluon interactions.
6. BDS amplitudes in the multi-Regge kinematics.
7. Breakdown of the Regge factorization.
8. BFKL cuts in the planar amplitudes M_n for $n > 5$.