$F_L(x, Q^2)$ Review

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We are finally about to have a real, and fairly accurate measurement of $F_L(x, Q^2)$ at HERA. What are the implications and what are the predictions?

Standard comment is that this gives an independent test of the gluon distribution at low x to go along with that determined from $dF_2(x, Q^2)/d\ln Q^2$. At present the fits to $F_2(x, Q^2)$ at low x are reasonably good (perhaps not perfect) but the gluon is free to vary to make them as good as possible. We need a cross-check.

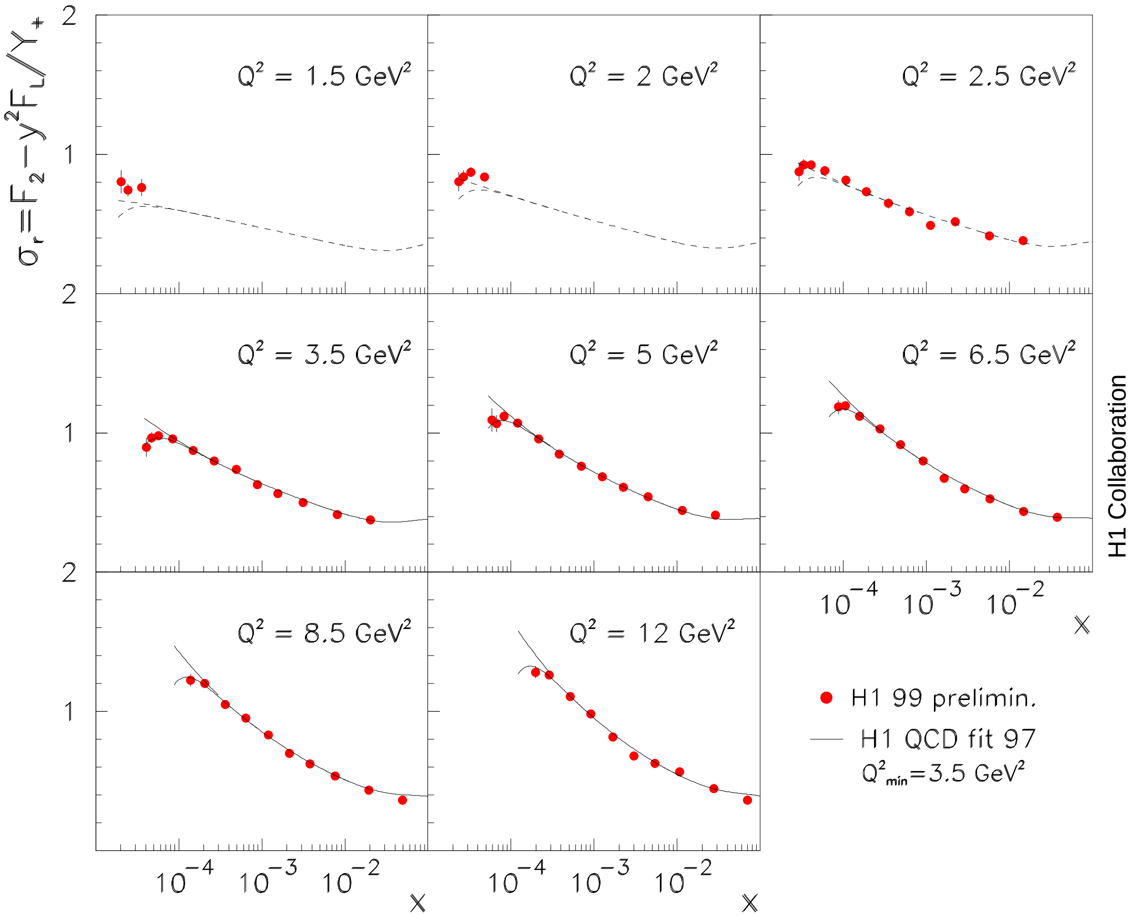
Similarly $F_L(x, Q^2)$ measurement is a direct test of success of different theories in QCD. Slightly different issue.

For which does $F_L(x, Q^2)$ have more discriminating power?

Currently have consistency checks on the relationship between $F_2(x, Q^2)$ and $F_L(x, Q^2)$ at high y where both contribute to the total cross-section.

Extrapolate in y using either NLO perturbative QCD or using $(d\sigma/d\ln y)_{Q^2}$ whilst making assumption about $(dF_2(x, Q^2)/d\ln y)_{Q^2}$.

For example, the NLO consistency check of $F_L(x,Q^2)$ for the H1 fit.

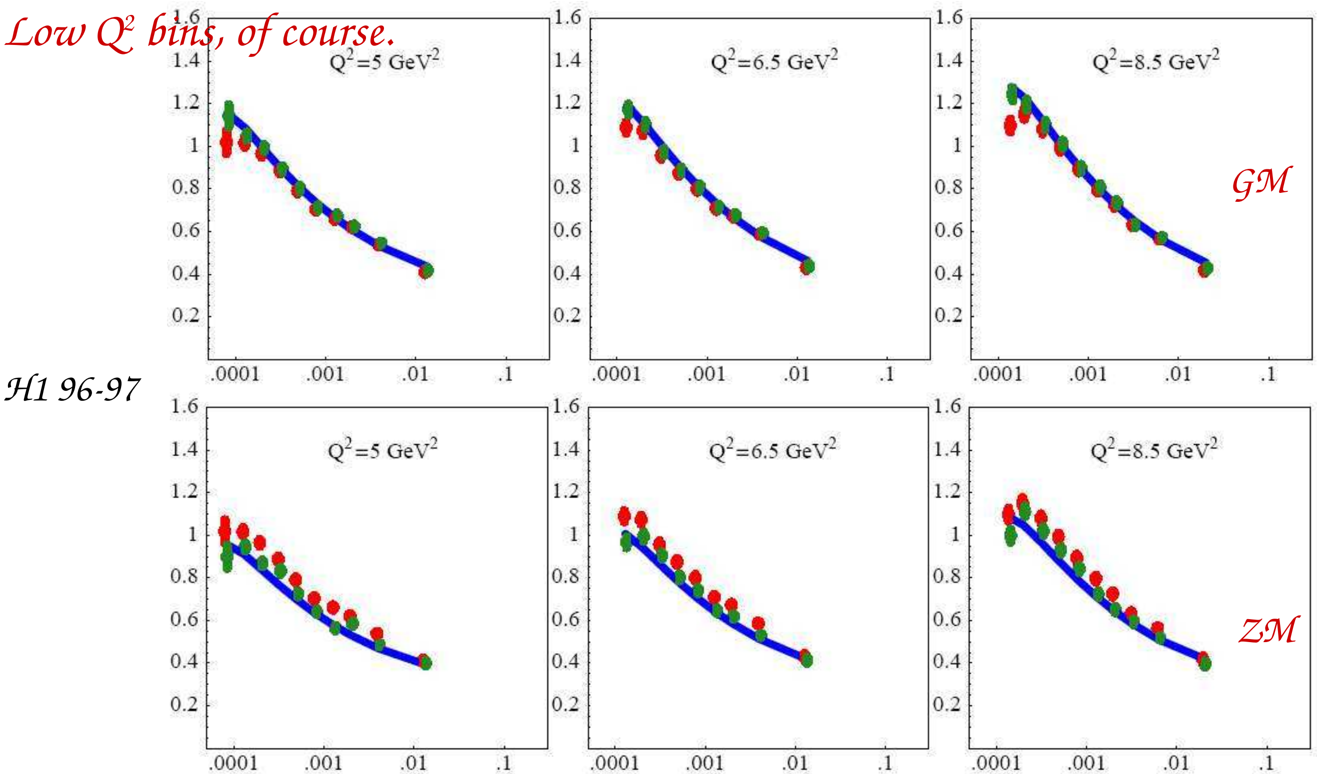


Turn over in $\tilde{\sigma}(x,Q^2) = F_2(x,Q^2) - y^2/(1+(1-y)^2)F_L(x,Q^2)$ clearly matched by $F_L(x,Q^2)$ contribution.

Consistency check works well for H1 NLO fit and some others though not as well for e.g. the MRST NLO fit.

Possible to get rid of $F_L(x, Q^2)$ turnover completely using correlated systematic uncertainties – DIS06 Tung

Where does the General Mass Formalism make a difference? Compare with CTEQ6.1M (ZM)



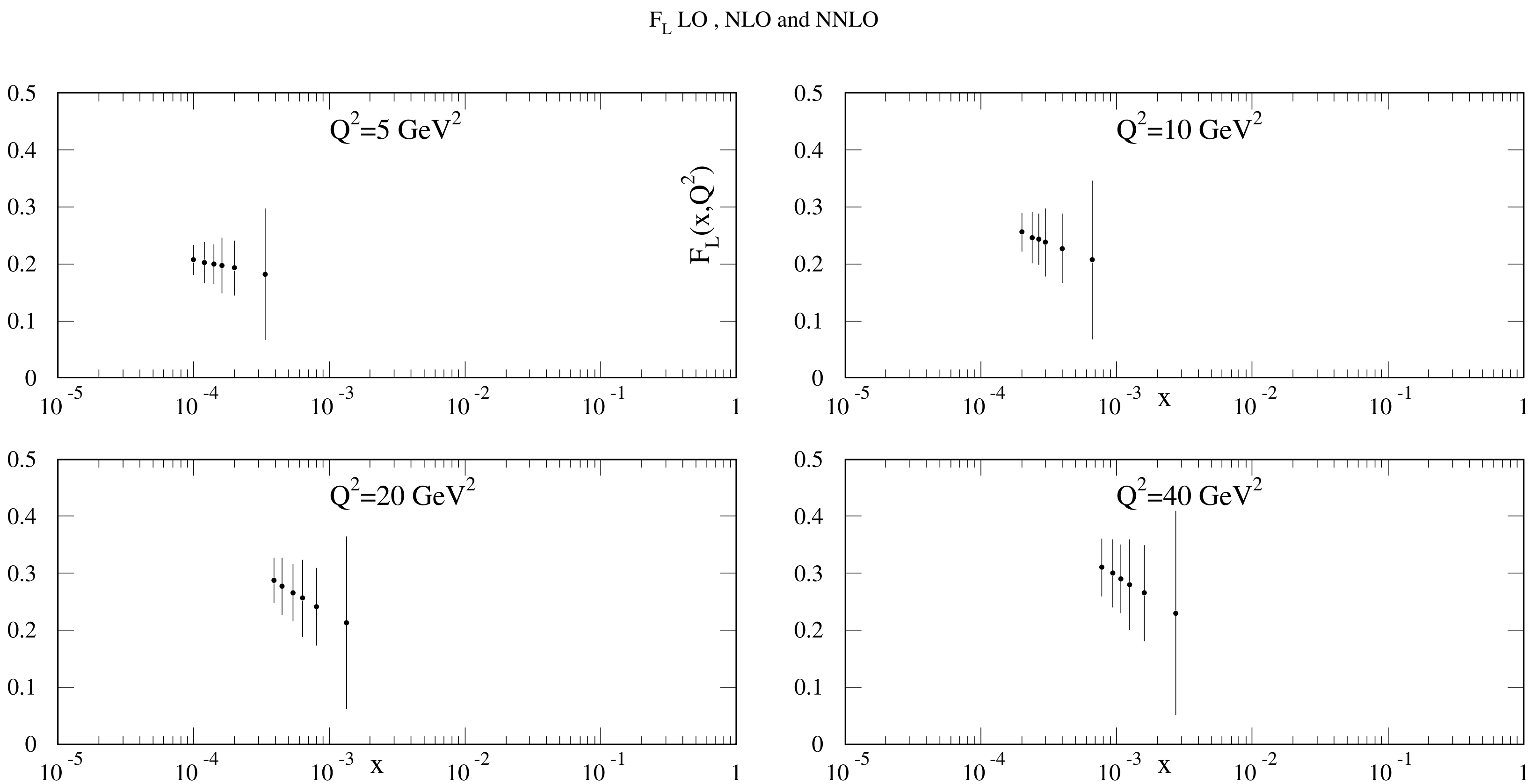
ZEUS 96-97 data show the same effects

Turn over in eliminated by variation in photo-production background uncertainty.

Not necessary for good fit, but symptomatic of difficulties in indirect method.

HERA have now run at two lower beam energies before finishing in order to make a direct measurement of $F_L(x, Q^2)$.

Measure data from $Q^2 \approx 5 - 800\mathrm{GeV}^2$ and $x = 0.0001 - 0.03$ with typical error of at best $12 - 15\%$. (H1 simulation, Klein).



Running went very well indeed. May obtain something like these results with full analysis.

Theoretical issues

Have to make sure we get a reliable theoretical prediction whatever framework we use.

Some issues with heavy flavours associated with $F_L(x, Q^2)$. Brief clarification.

What is the interplay between $F_L(x, Q^2)$ and the gluon distribution at fixed order perturbative QCD?

What about possible extensions? Various possibilities:

- Higher twist - renormalons.
- Resummations of large $\ln(1/x)$ corrections.
- Dipole model predictions.

Will concentrate on (more interesting) $lower$ Q^2 range.

Have to be careful about treatment of heavy flavours.

Dominated by $C_{Lg}(\alpha_S, x) \otimes g(x, Q^2)$ contributions. In massless quark approximation using charge weighting charm is nearly 40% of total.

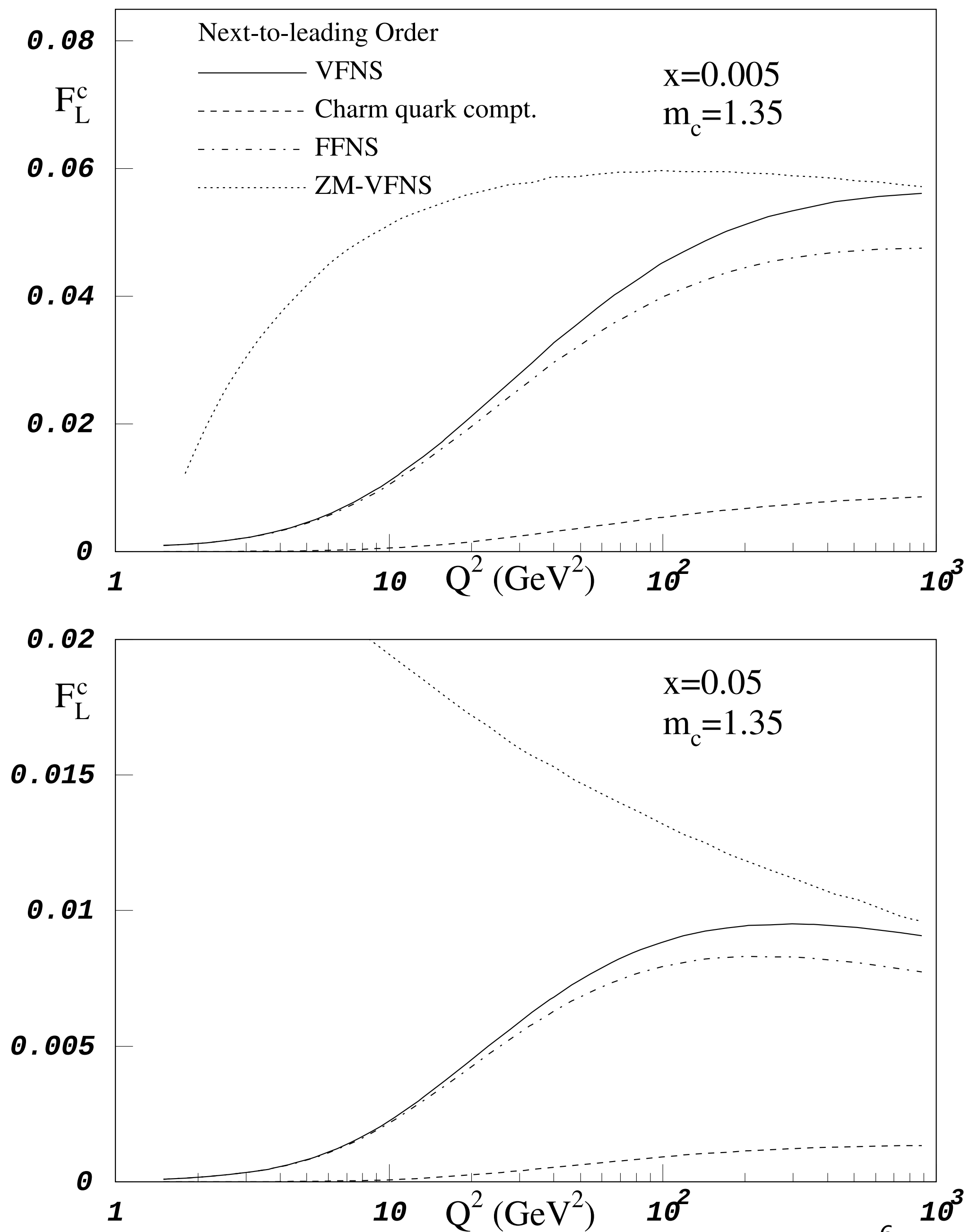
However, large massive quark suppression in heavy flavour coefficient functions.

$F_L^c(x, Q^2)$ is suppressed by a factor of v^3 where

$$v = 1 - \frac{4m_c^2 z}{Q^2(1-z)}$$

is the velocity of the heavy quark in the centre-of-mass frame. Also the limit of integration in the convolution is $\xi = x(1 + 4m_c^2/Q^2)$ rather than x.

$Q^2 >> m_c^2$ before massless limit starts to apply, not like $F_2^c(x, Q^2)$



Have to also consider ambiguities in definition of variable flavour scheme. If charm explicitly in proton zeroth order contribution to $C^c_L(x,Q^2) = \frac{4m_c^2}{Q^2}\delta(1 - z/(x(1+\frac{m_c^2}{Q^2}))$, which disappears at high Q^2.

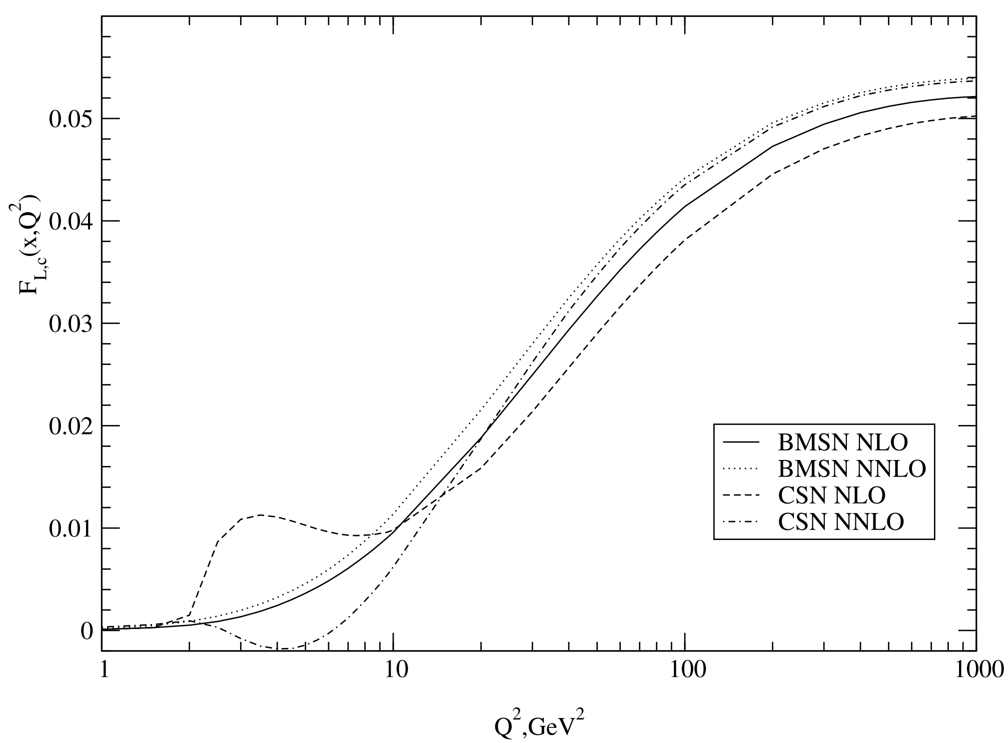
Cancels between orders in properly defined GM -VFNS. However, large near m_c^2 while other terms suppressed by v^3. If implemented can lead to peculiar behaviour for Q^2 slightly above m_c^2 – particularly at NNLO where charm distributions start off negative. Chosen to be absent in some GM-VFNS schemes (including T-Roberts).

Example of competing definitions of schemes for $F^c_L(x,Q^2)$ (Chuvakin, Smith, van Neerven).

One choice gives positive "bump" at NLO and negative "bump" at NNLO.

Other is smooth.

Fig 18



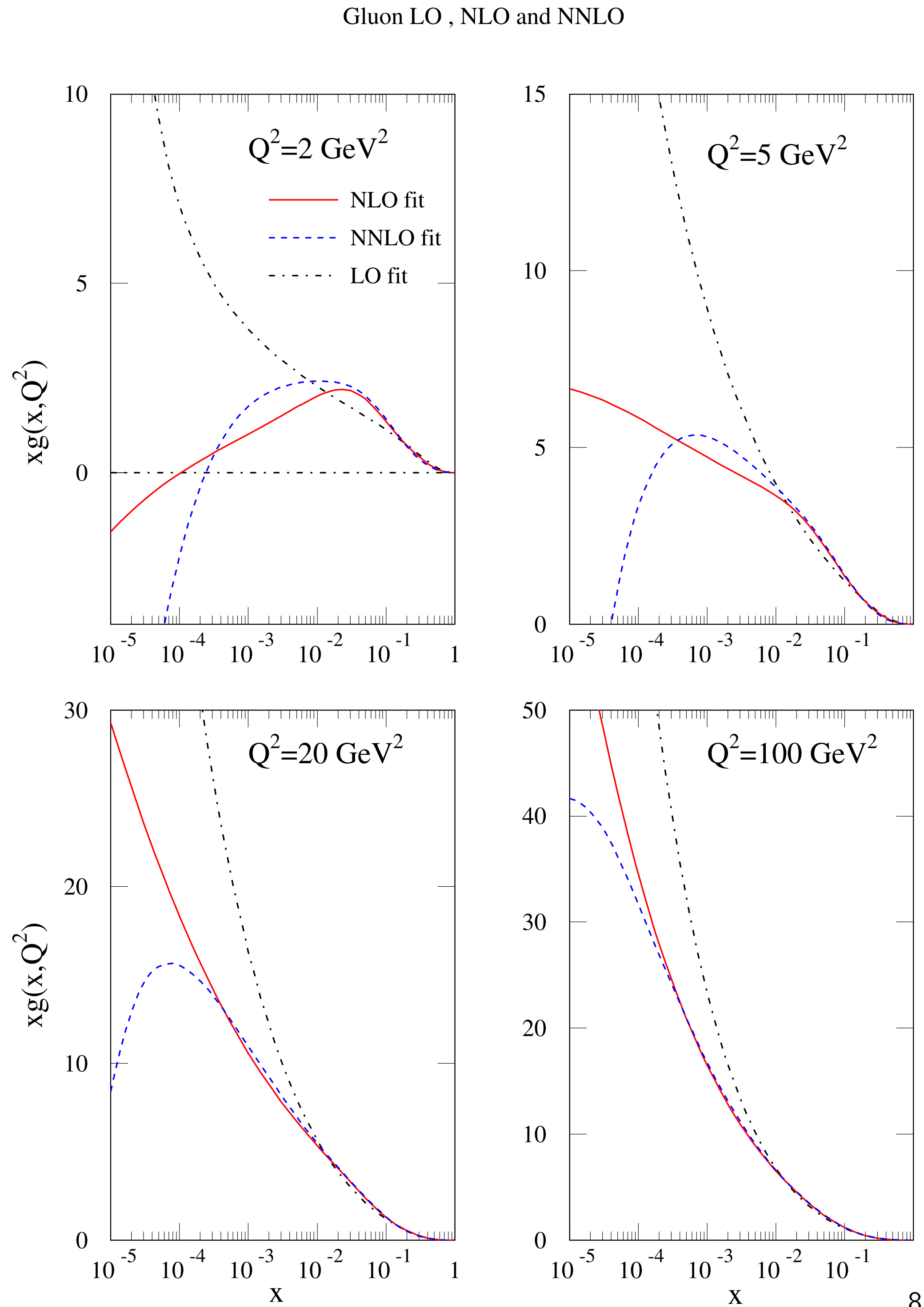
Physics probed by $F_L(x, Q^2)$.

Start by looking at fixed order QCD.

The gluon extracted from the global fit at LO, NLO and NNLO (prelim).

Additional and positive small-x contributions in P_{qg} at each order lead to smaller small-x gluon at each order.

Clearly poor stability.



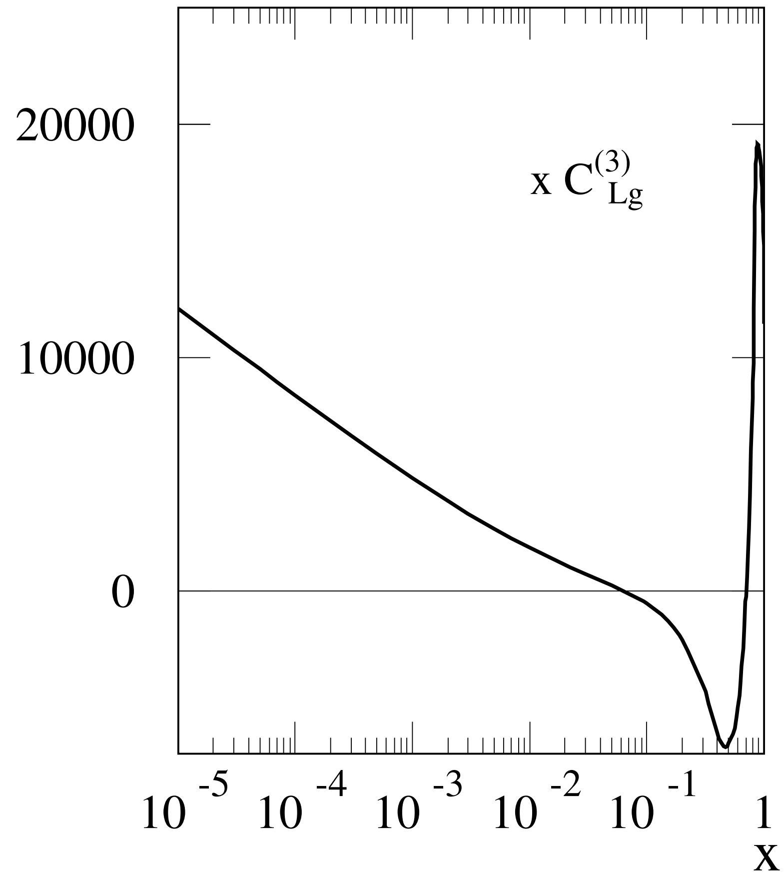
Splitting functions calculated at NNLO and coefficient functions for $F_L(x,Q^2)$ finished too – Moch, Vermaseren, Vogt.

The NNLO $\mathcal{O}(\alpha_s^3)$ longitudinal coefficient function $C^3_{Lg}(x)$ given by

$$C^3_{Lg}(x) = n_f\left(\frac{\alpha_S}{4\pi}\right)^3\left(\frac{409.5\ln(1/x)}{x} - \frac{2044.7}{x} - \cdots\right.$$

Clearly a significant positive contribution at small x.

Counters decrease in small-x gluon.

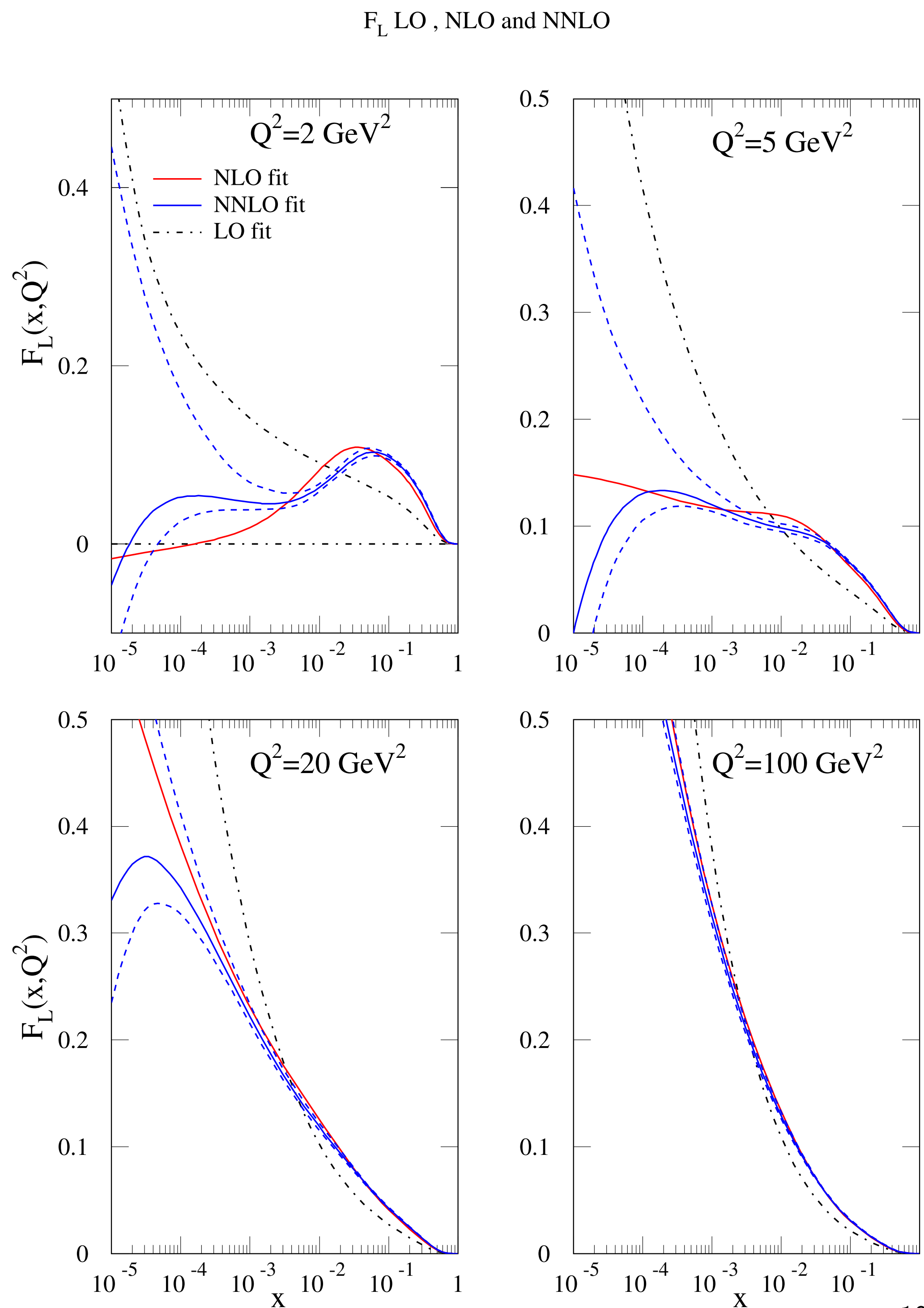


$F_L(x, Q^2)$ predicted from the MSTW global fit at LO, NLO and NNLO (prelim).

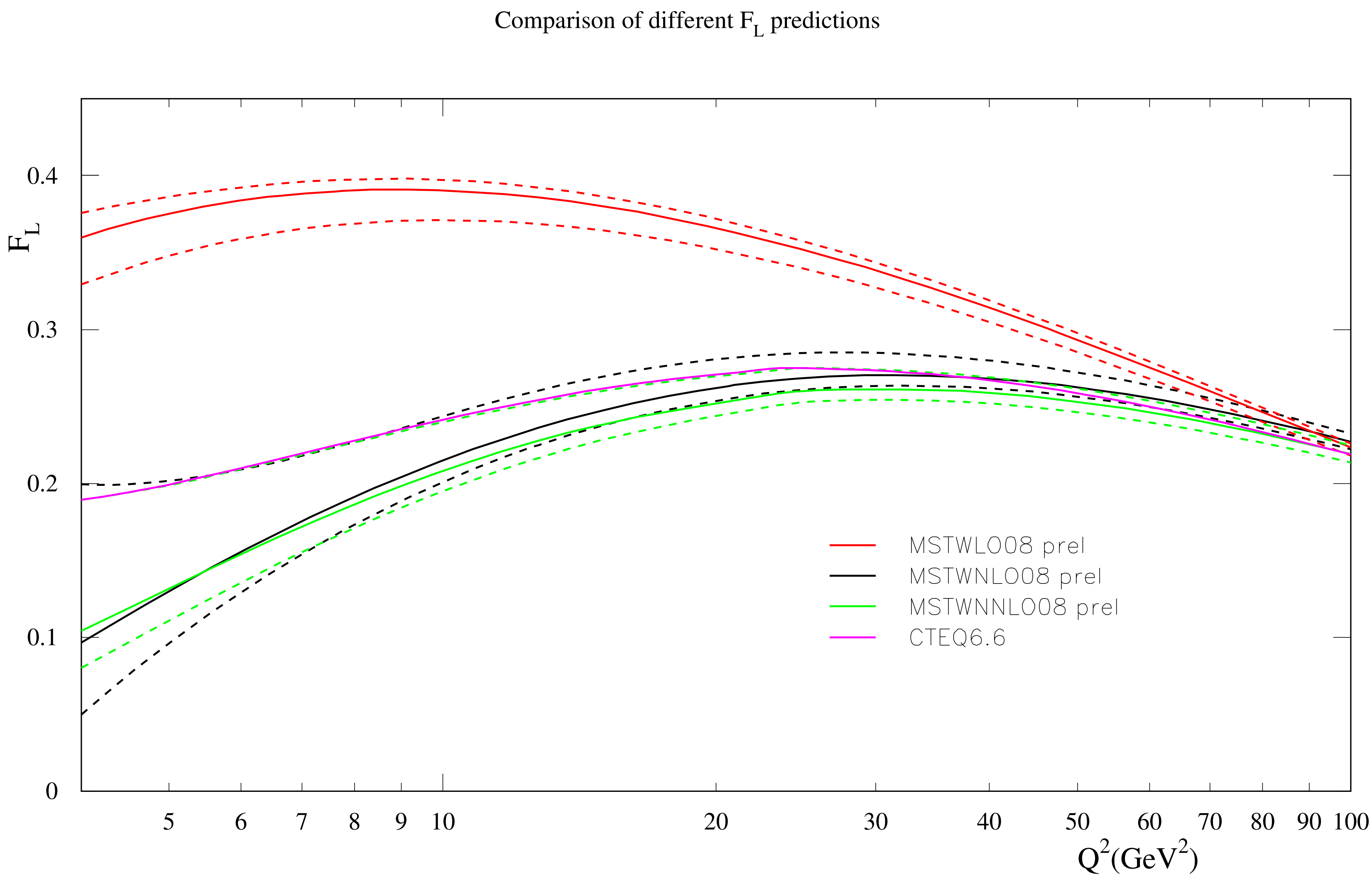
NNLO coefficient function compensates decrease in NNLO gluon.

$F_L(x, Q^2)$ prediction more stable than for gluon at small x.

However, current default negative at both NLO and NNLO at lowest x and Q^2, but enormous uncertainty here.



Look at variations in predictions for HERA range of measurement. Use $x = Q^2/35420$.



CTEQ6.6 curve (Nadolsky and Tung) at NLO, though uses different ordering definition $\rightarrow$ slight comparative increase. Within MSTW uncertainties.

Not too much variation between NLO and NNLO until lower Q^2 and x.

Can also look at explicit **higher twist** possibilities. For $F_2(x, Q^2)$ renormalon calculation of higher twist dies away at small x (from satisfying Adler sum rule).

Completely different picture for $F_L(x, Q^2)$. At small x contribution proportional to quark distributions, i.e. $F_L^{HT}(x, Q^2) \propto F_2(x, Q^2)$.

Explicit renormalon calculation (Stein *et al*) gives

$$F_L^{HT}(x, Q^2) = \frac{A}{Q^2}(\delta(1-x) - 2x^3) \otimes \sum_f Q_f^2 q_f(x, Q^2).$$

where estimate for A is

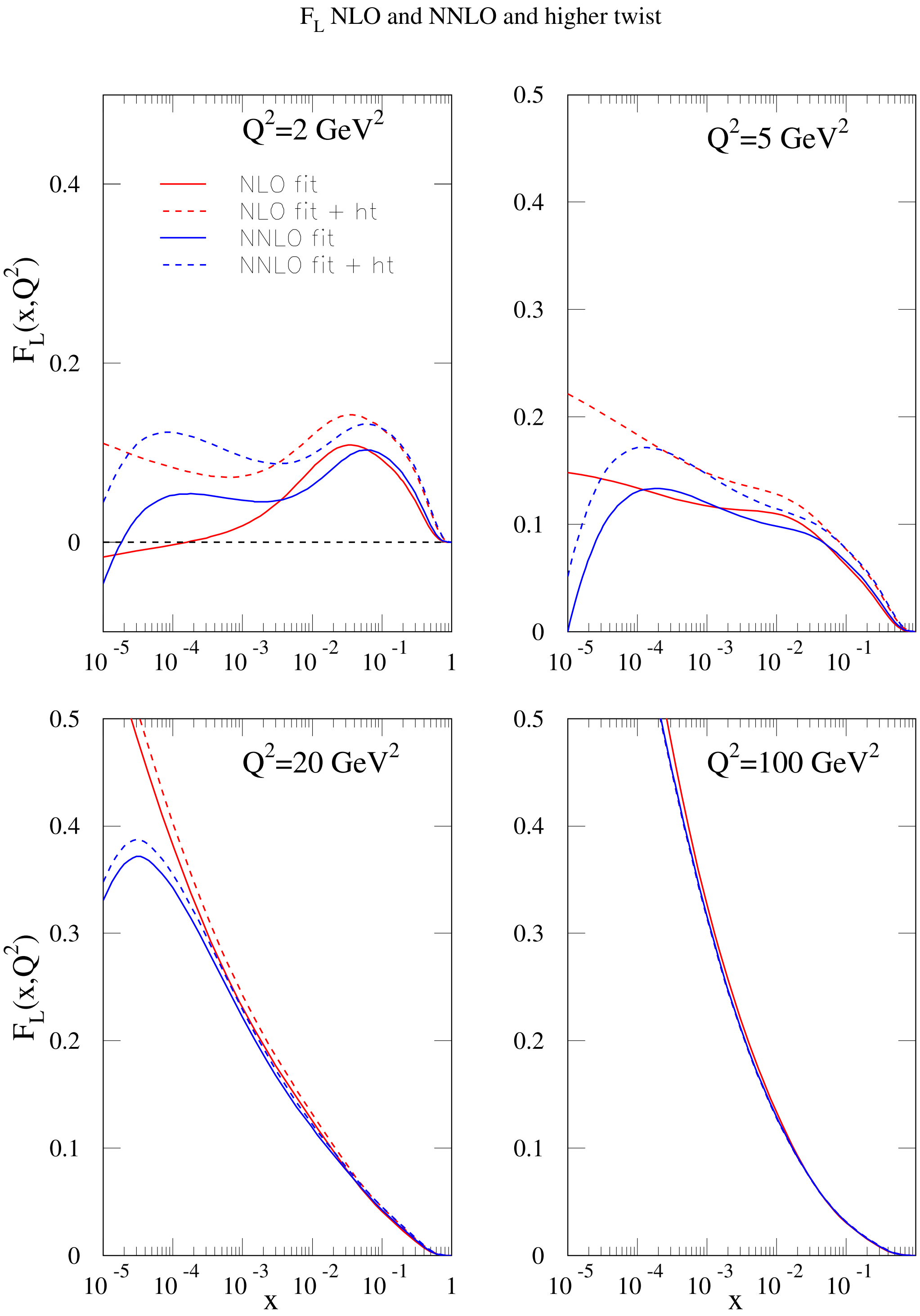
$$A = \frac{4C_f \exp(5/3)}{\beta_0}\Lambda_{QCD}^2 \approx 0.4 - 0.8\,\mathrm{GeV}^2.$$

This effect is nothing to do with the gluon distribution, and is not part of the higher twist contribution in the dipole approach.

At small x becomes effectively

$$F_L^{HT}(x, Q^2) = \frac{A}{3Q^2}\delta(1-x) \otimes \sum_f Q_f^2 q_f(x, Q^2) \approx \frac{A}{3Q^2} F_2(x, Q^2).$$

Corrections to leading twist NLO and NNLO when $A = 0.6\mathrm{GeV}^2$ is used.



Small-x resummations

If the NNLO correction is itself rather large, might not other corrections on top of this - e.g. higher orders still – be important?

Leading $\ln(1/x)$ terms of the form

$$P_{gg}(x) \sim \frac{\alpha_S^n \ln^{n-1}(1/x)}{x} \qquad P_{qg}(x) \sim \frac{\alpha_S^n \ln^{n-2}(1/x)}{x} \qquad C_{Lg}(x) \sim \frac{\alpha_S^n \ln^{n-2}(1/x)}{x}$$

A fit which performs a double resummation of leading $\ln(1/x)$ and β_0 terms leads to a better fit to small-x data than a conventional perturbative fit (C White, RT).

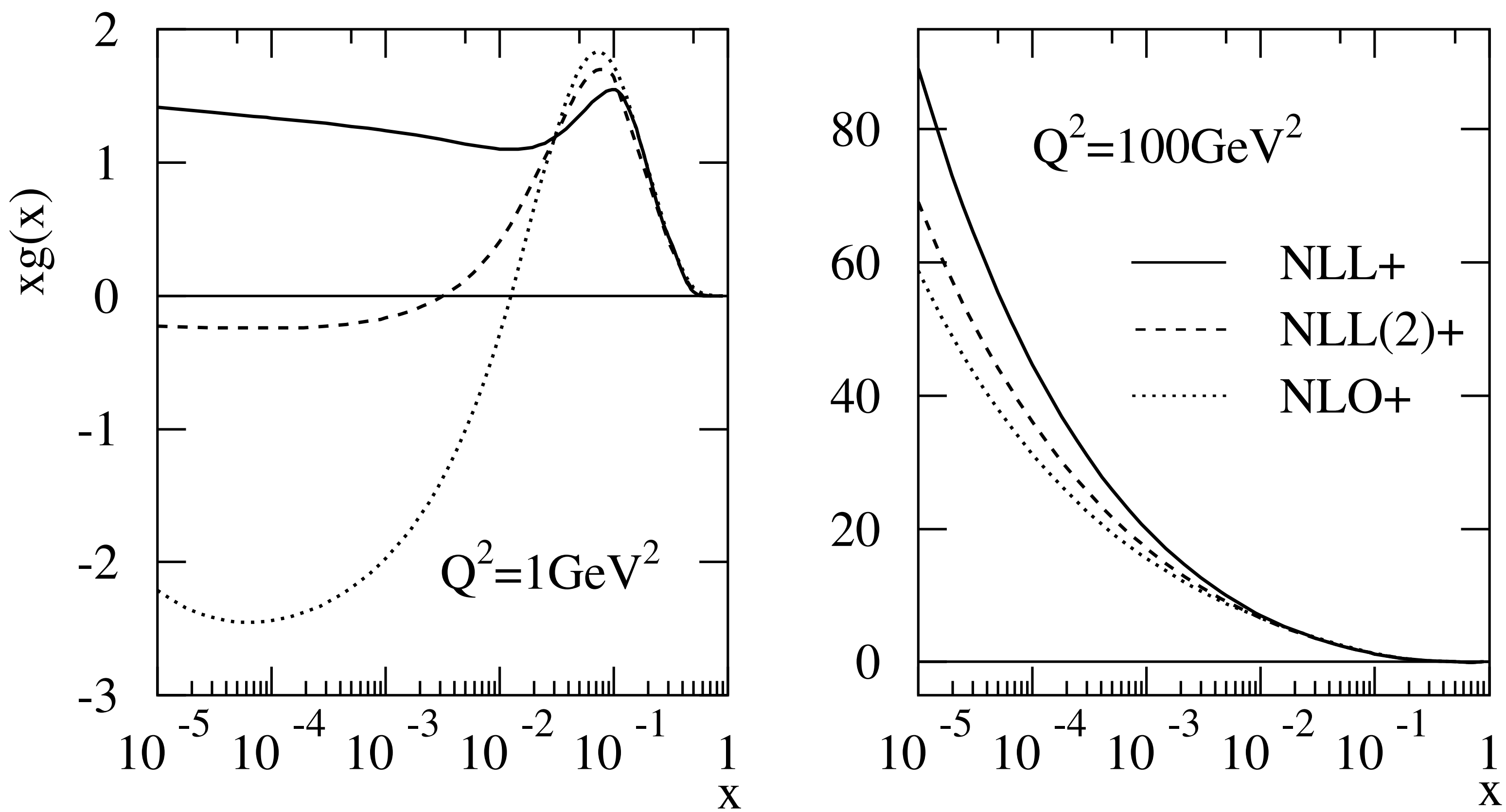
NLO in all leading logs, e.g.

$$P_{gg}(x) \sim \frac{\alpha_S^n \ln^{n-2}(1/x)}{x} \qquad C_{Lg}(x) \sim \frac{\alpha_S^n \ln^{n-3}(1/x)}{x}$$

where approximation based on kinematic corrections required for NLL coefficient functions (sub-dominant effect).

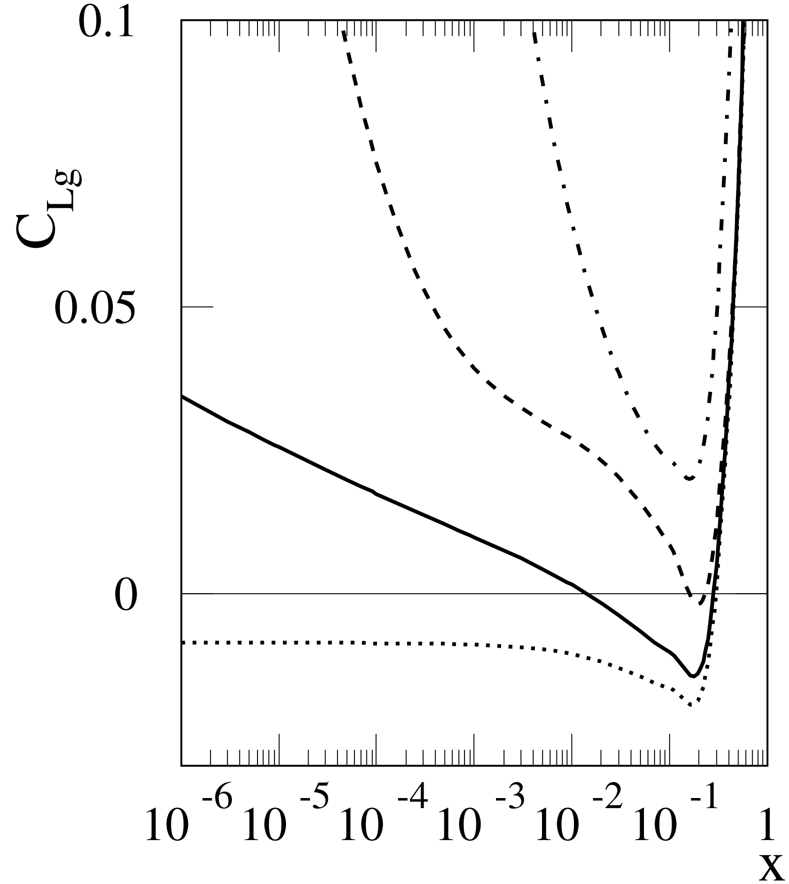
Approaches similar, with some differences, by Ciafaloni et al and Altarelli, et al. Rather similar results between groups for calculated splitting functions. No other fits yet.

Gluon distribution from resummed fit larger at small x and Q^2.



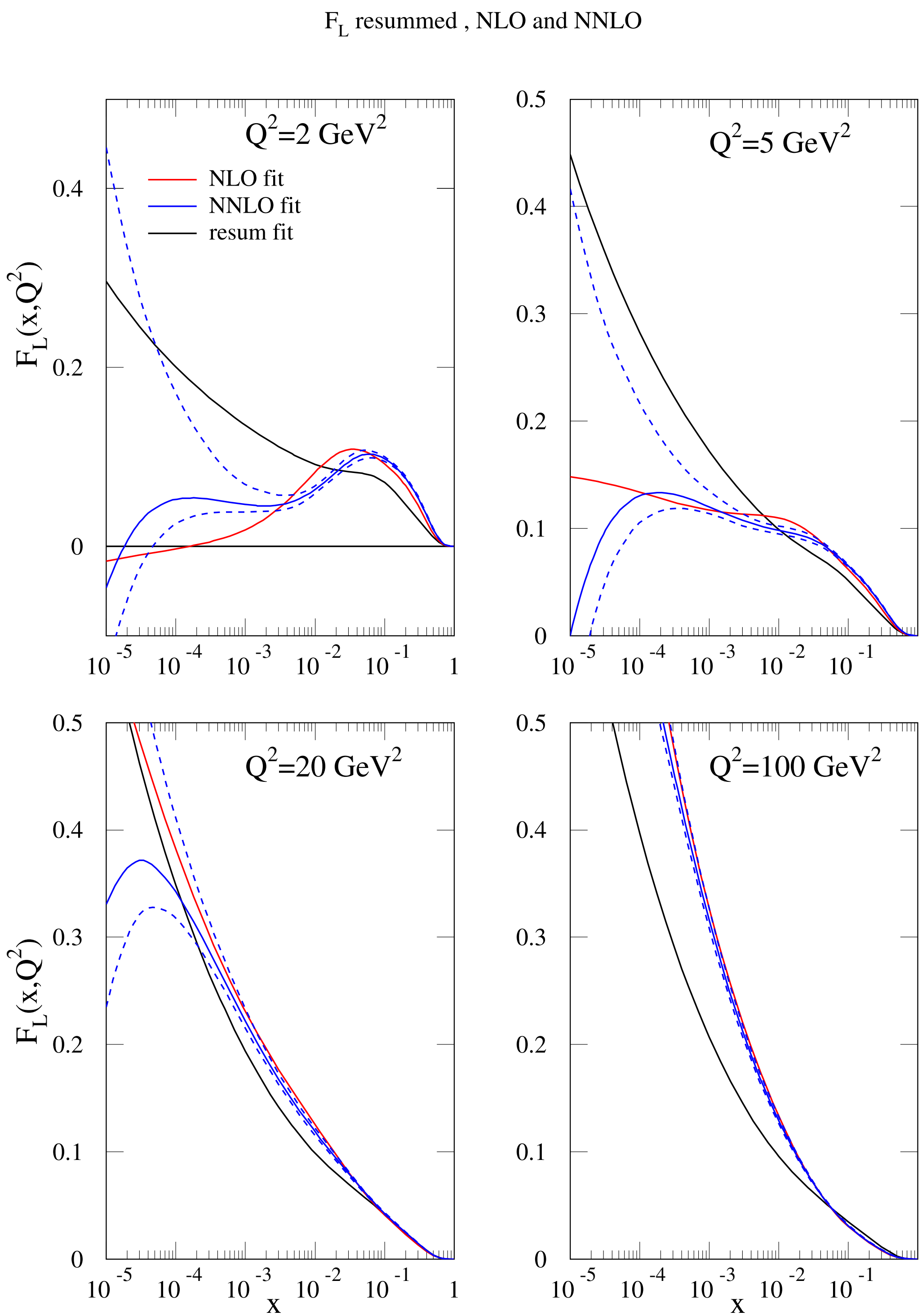
Full NLL resummed $C_{Lg}(x)$ (solid) together with LL (dashed) and LL without running coupling corrections (dash-dotted) and fixed-order NLO (dotted).

Factor of $(\alpha_S^3/4\pi)^3 \sim 10^{-5}$ compared to previous plot, so rise of resummed coefficient function at small x less than fixed order NNLO (cuts axis at ~ 0.11).



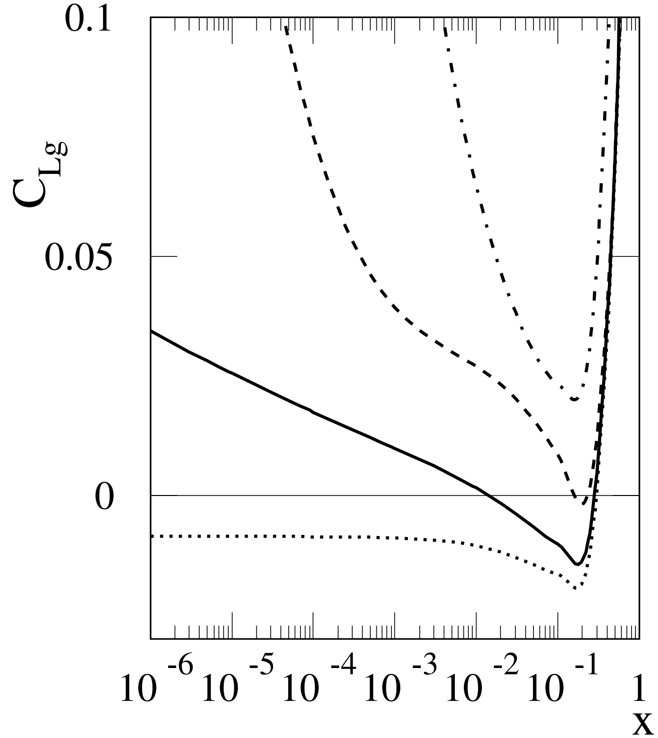
Overall predicted resummed NLL $F_L(x,Q^2)$ steeper at small x than fixed order.

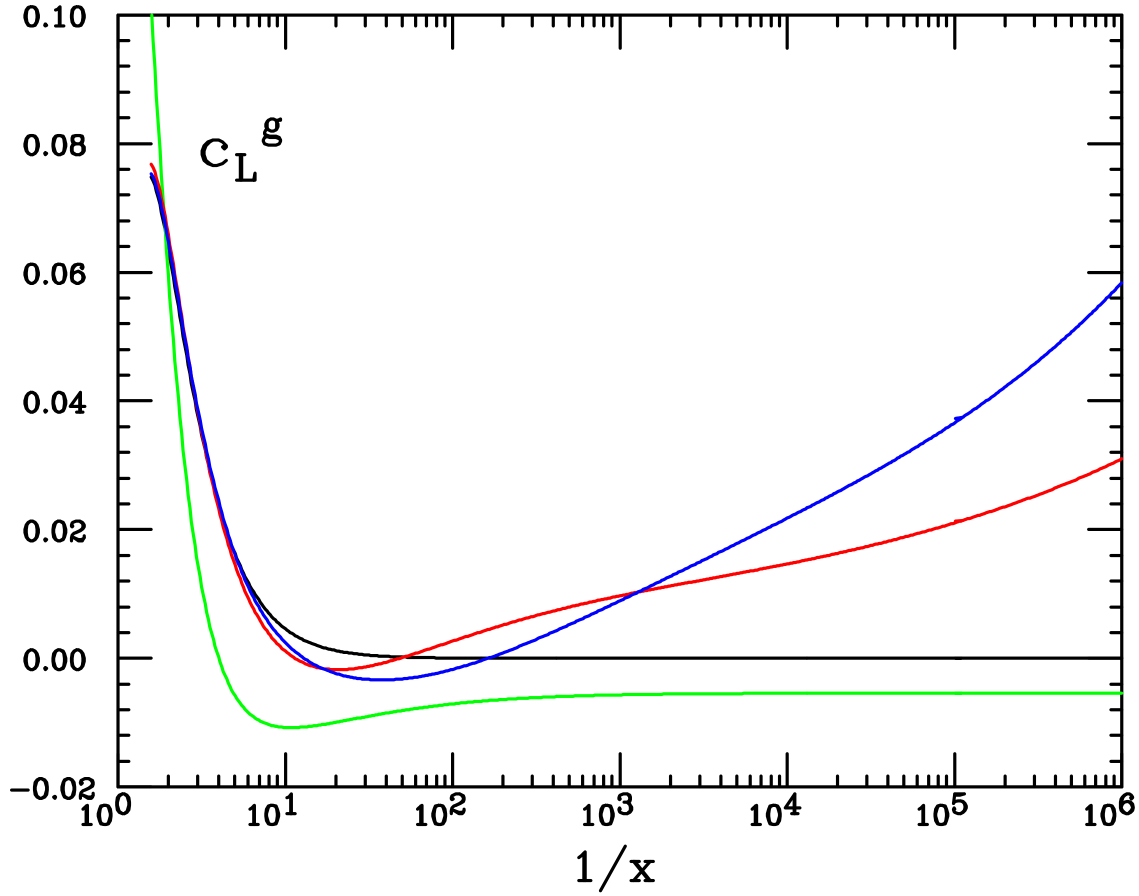
Reasonably far outside uncertainties.



All groups produce similar splitting functions.

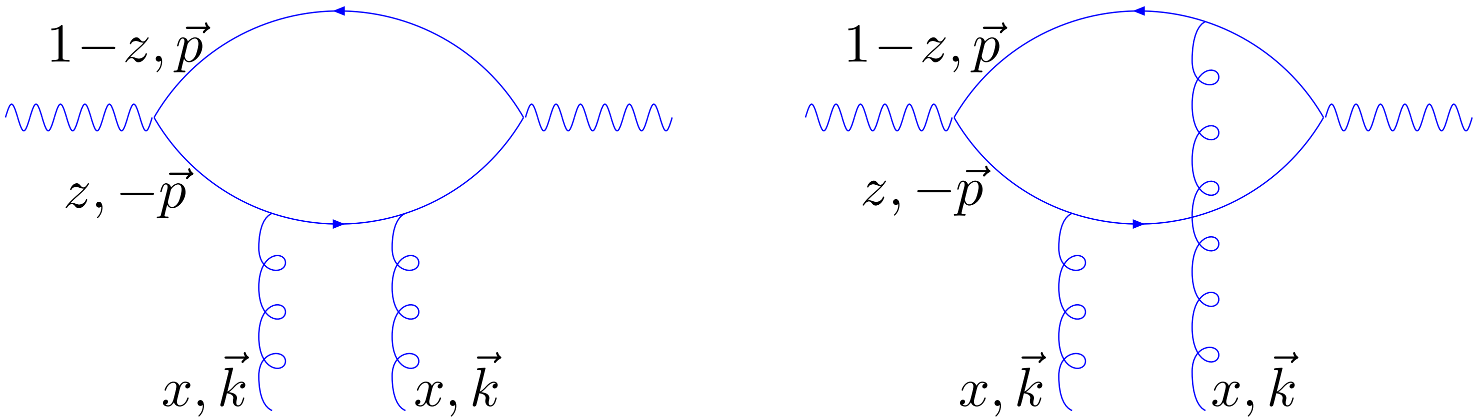
Coefficient functions from Altarelli et al (blue, red different schemes for resummation) very similar in form.





Presumably predictions will be fairly similar.

Dipole Cross-section.



Within LO k_T-factorization theory can write $\gamma^\star p$ cross-section (Bialas, Navelet, Peschanski) as

$$\sigma_L \propto \int_0^1 dz[z(1-z)]^2 \int \frac{d^2k}{k^4} \int d^2p \left(\frac{1}{\hat{Q}^2 + p^2} - \frac{1}{\hat{Q}^2 + (p+k)^2} \right)^2 f(x, k^2)$$

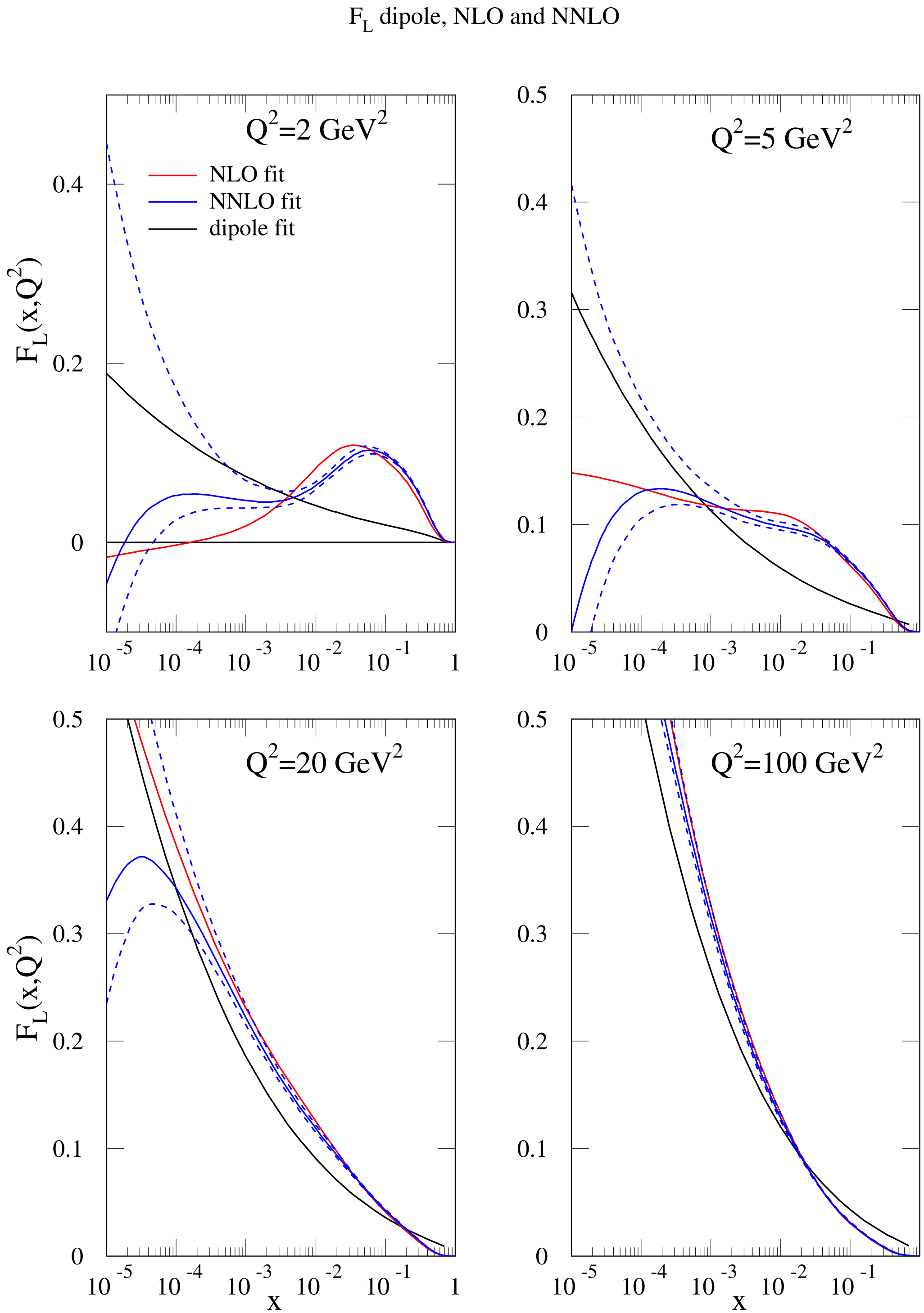
where $f(x, k^2)$ is the unintegrated gluon distribution, $\hat{Q}^2 = z(1-z)Q^2$.

Includes some of the resummation effects, and also higher twist – different contributions to renormalon calculation.

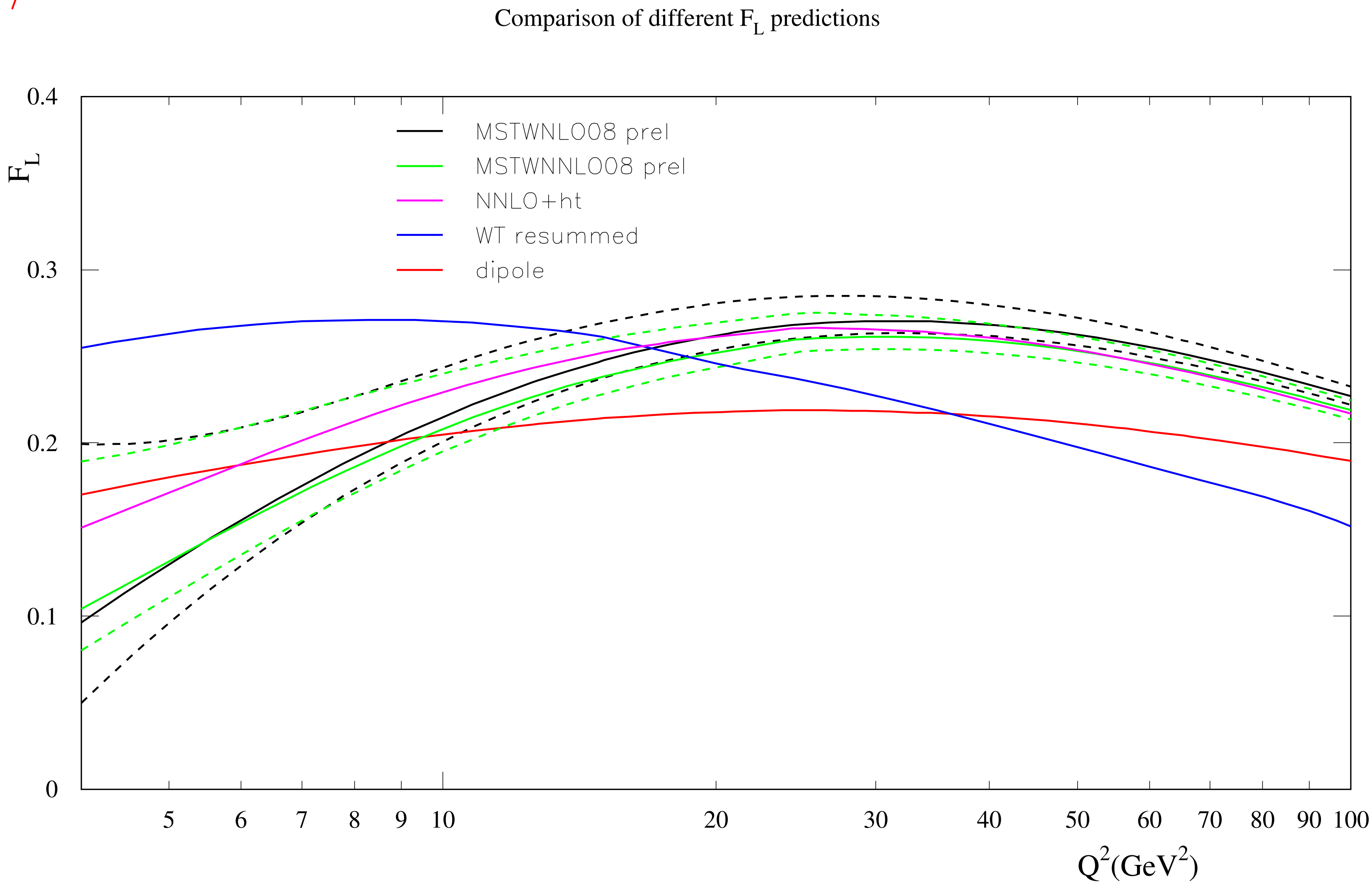
Misses quark and higher-x contributions.

Overall predicted dipole $F_L(x, Q^2)$ steeper at small x than fixed order, and automatically stable at lowest Q^2.

Smaller than fixed order predictions at high Q^2 but within uncertainties at lower Q^2 in probed x range at HERA.



Look at variations in additional predictions for HERA range of measurement. Use $x = Q^2/35420$.



Within range higher twist corrections smaller than uncertainties at NLO and NNLO. Resummations and dipole model different shape. Possible to distinguish former at lower Q^2 perhaps. Is measurement accurate enough at higher Q^2?

Conclusions

A measurement of $F_L(x, Q^2)$ is a direct way to determine the gluon distribution at low x particularly at low Q^2, and to determine whether fixed order calculations are sufficient or whether resummations, or other theoretical extensions, may be needed.

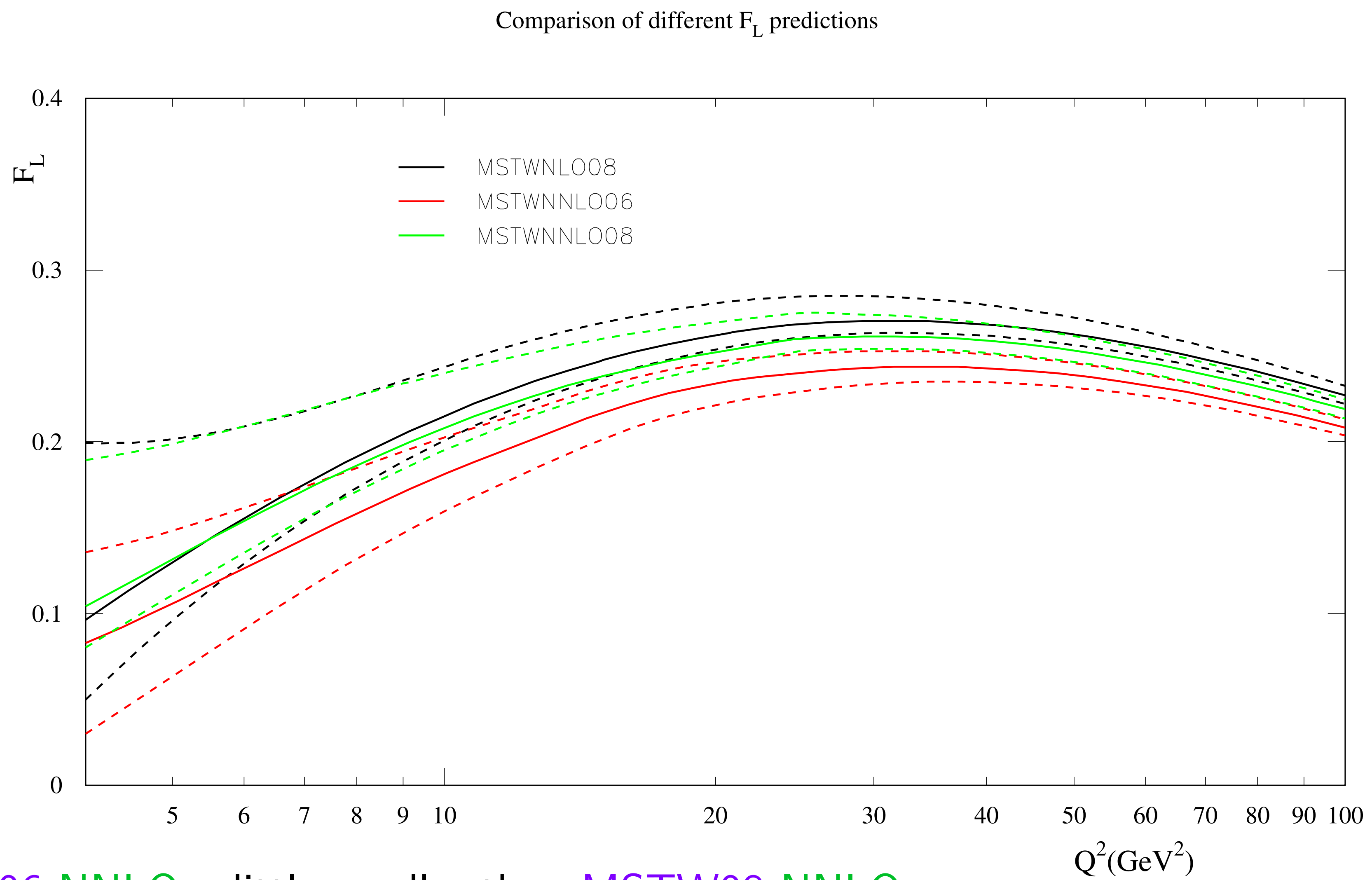
Clear differences between NLO and NNLO predictions appear at lower Q^2 than measurements.

At $Q^2 \geq 10GeV^2$ (for the x probed) the uncertainty on fixed order predictions is a few percent. An $F_L(x, Q^2)$ measurement will not add to the direct constraint on the gluon. However, there may be deviations from NLO/NNLO predictions of $20 - 30\%$ due to e.g. resummations or dipole models. Data may see some sign of deviations.

For $Q^2 \leq 10GeV^2$ the uncertainty in NLO/NNLO predictions for $F_L(x, Q^2)$ due to gluon uncertainty increases to $> 20\%$. A good measurement of $F_L(x, Q^2)$ will automatically improve the gluon determination.

Resummations/dipole models suggest a higher low-Q^2 $F_L(x, Q^2)$ by an absolute value of up to 0.15. This is well outside even the large fixed-order uncertainties. A good measurement of $F_L(x, Q^2)$ will start to discriminate between theories.

Look at variations in predictions for HERA range of measurement. Use $x = Q^2/35420$.



MRST06 NNLO a little smaller than MSTW08 NNLO.

Additionally, Alekhin performed fits to DIS data, using reduced cross-section for HERA data, and allows higher-twist corrections to be determined phenomenologically.

Finds unambiguous positive correction for $F_L(x,Q^2)$, i.e. consistency check fails for perturbative fit (though errors smaller than I believe).

My view, although higher twist may be important for $F_L(x,Q^2)$ at low x, there are other important effects. Have to consider higher orders in perturbation theory as well as possible higher twist.