News in jet algorithms:

SISCone and anti-k_t

Grégory Soyez

Brookhaven National Laboratory

G.P. Salam, G. Soyez, JHEP 05 (2007) 086 [arXiv:0704.0292]

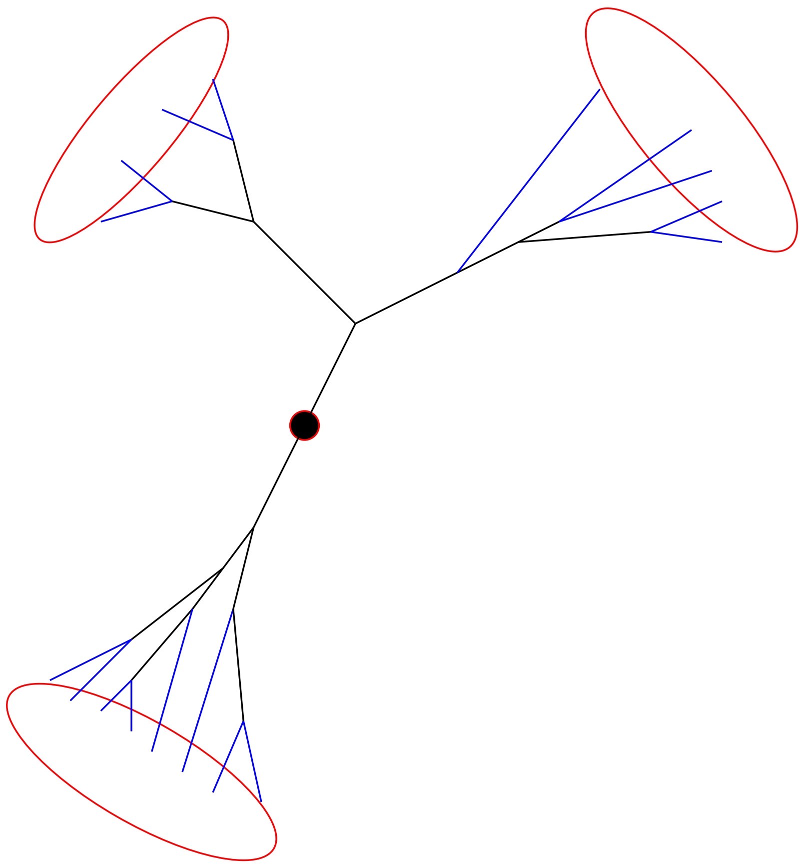
M. Cacciari, G.P. Salam, G. Soyez, to appear in JHEP [arXiv:0802.1189]

Jet definitions

Aim: Study hard processes

- QCD backgrounds, top quark physics
- Higgs, physics beyond the standard model

Define jets: parton $\leftrightarrow$ jet



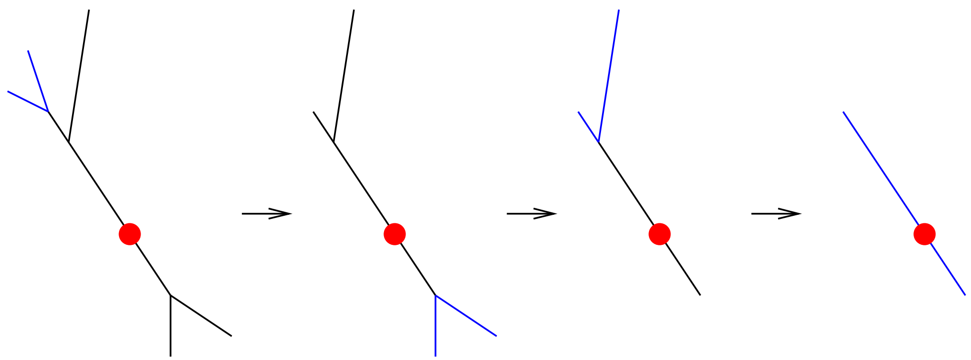
But: partons are ambiguous

Hence: Multiple definitions of a “jet”

Two classes of algorithms

Class 1: recombination

Successive recombinations of the “closest” pair of particle



- Distance:

$$\underline{k_t}: \quad d_{i,j} = \text{min}(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

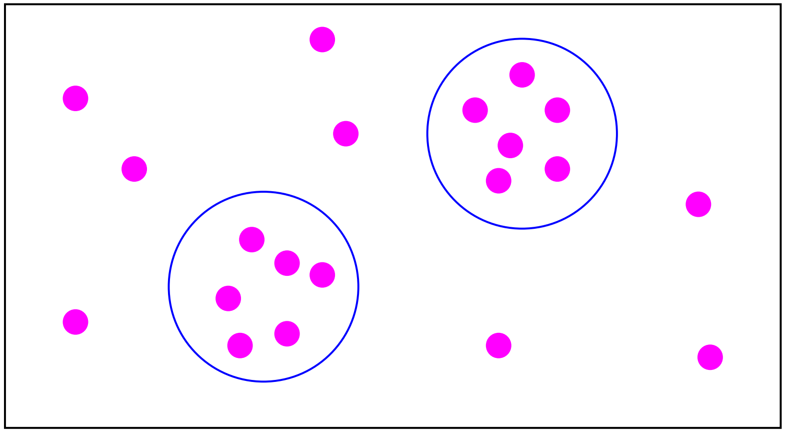
$$\underline{\text{Aachen/Cam.}}: \quad d_{i,j} = \Delta\phi_{i,j}^2 + \Delta y_{i,j}^2$$

- stop when $d_{\text{min}} > R$

Two classes of algorithms

Class 2: cone

Find directions of dominant energy flow $\equiv$ find ALL stable cones



for a cone of fixed radius R in the (y, ϕ) plane: **stable cones** such that:

centre of the cone $\equiv$ direction of the total momentum of its particle contents

Class 2: cone

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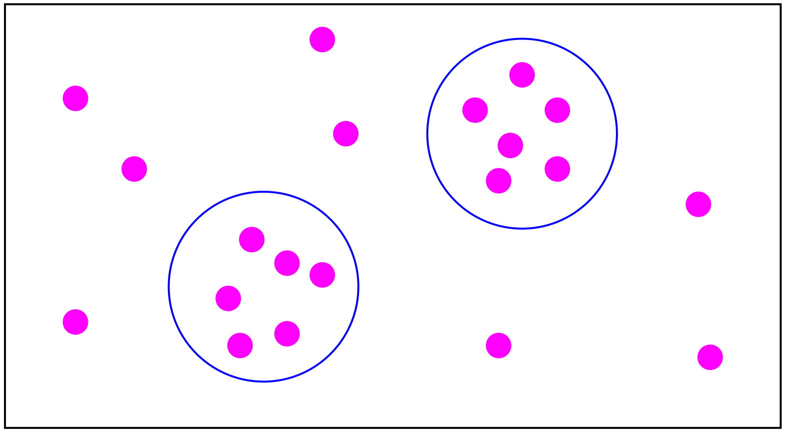


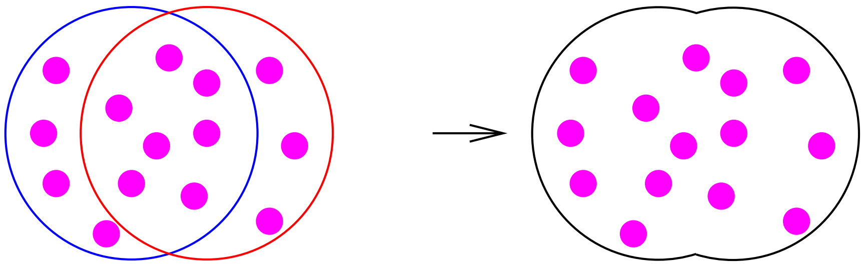
for a cone of fixed radius R in the (y, ϕ) plane: **stable cones** such that:
centre of the cone $\equiv$ direction of the total momentum of its particle contents

- Seeded/Iterative approaches:
  - seed = initial particle
  - seed = midpoint between stable cones found at first step
- One has to deal with overlapping stable cones: 2 subclasses

Two cone subclasses

Class 2(a): cone with split-merge

$\tilde{p}_{t,\text{shared}} > f\,\tilde{p}_{t,\text{min}}$



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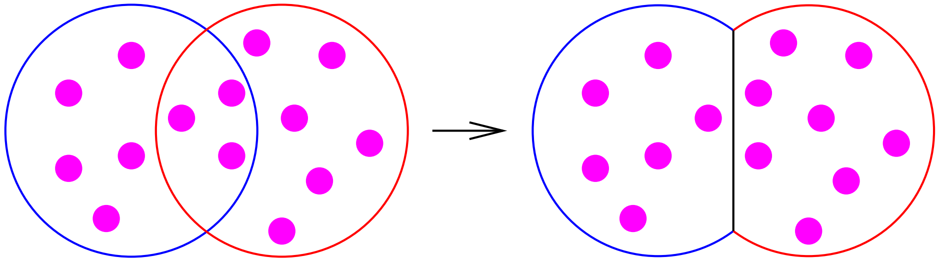

ex.: JetClu, MidPoint

Two cone subclasses

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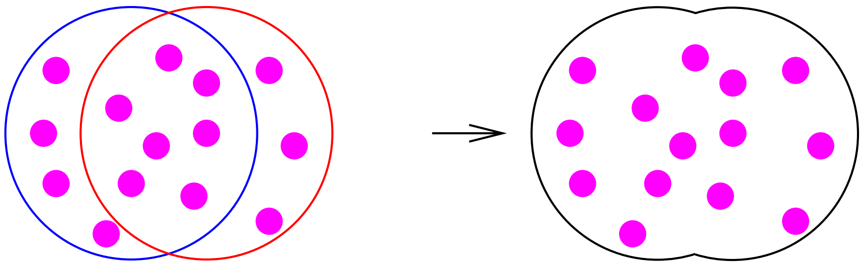

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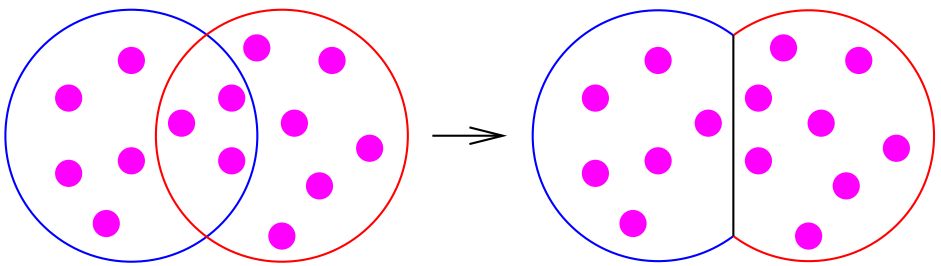


ex.: JetClu, MidPoint

Class 2(b): cone with progressive removal

- iterate from the hardest seed
- remove the stable cone as a jet and start again

ex.: Seeded Cone

Idea: “regular/circular” jets

SNOWMASS accords

SNOWMASS, Tevatron 1990 (i.e. old!):

Any jet algorithm must satisfy

1. Can be practically used in experimental analysis
2. Can be practically used in theoretical computations
3. Can be defined at any order of the perturbation theory
4. Yields finite cross-sections at any order
5. Has a small sensitivity to hadronisation corrections

i.e. usable by theoreticians (*e.g.* finite perturbative results)
and experimentalists (*e.g.* fast enough)

This talk:

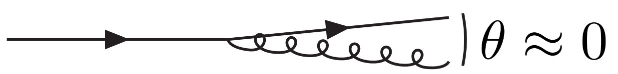
- Iterative cone algorithms miss stable cones $\Rightarrow$ theoretical problems
- That can be solved keeping experimental usefulness

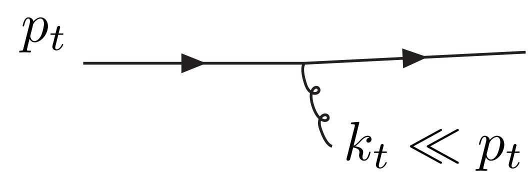
QCD divergences

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:





Collinear

Soft

QCD divergences

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

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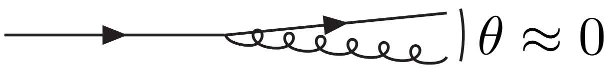

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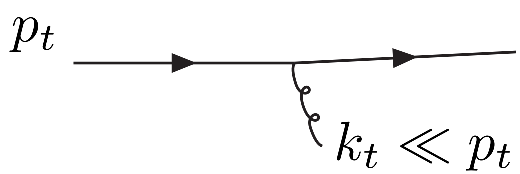


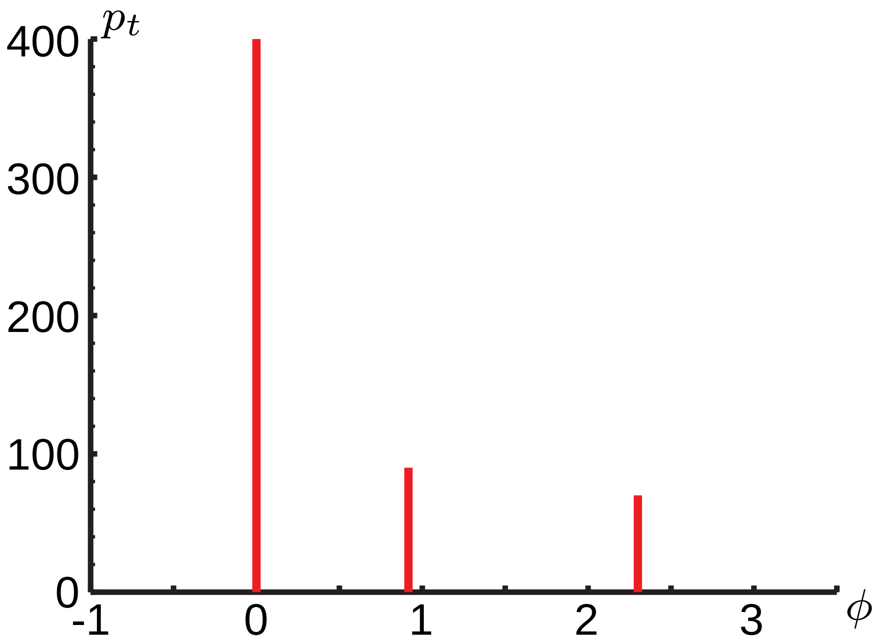
Soft

For QCD expansion to make sense

$\Rightarrow$ The (hard) jets (or stable cones) should not change when

- one has a collinear splitting
  i.e. replaces one parton by two at the same place
- one has a soft emission *i.e.* adds a very soft gluon

IR unsafety of the Midpoint alg



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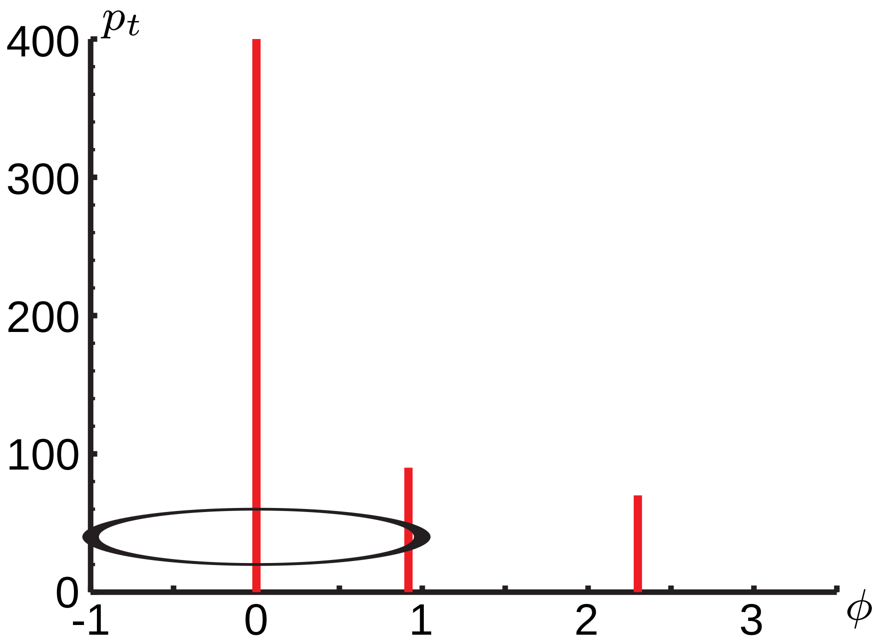

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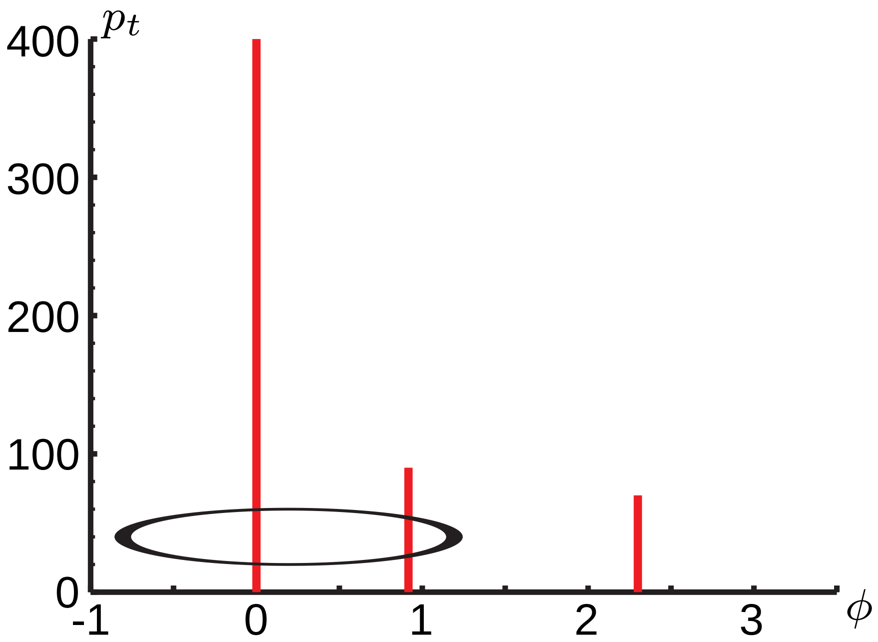

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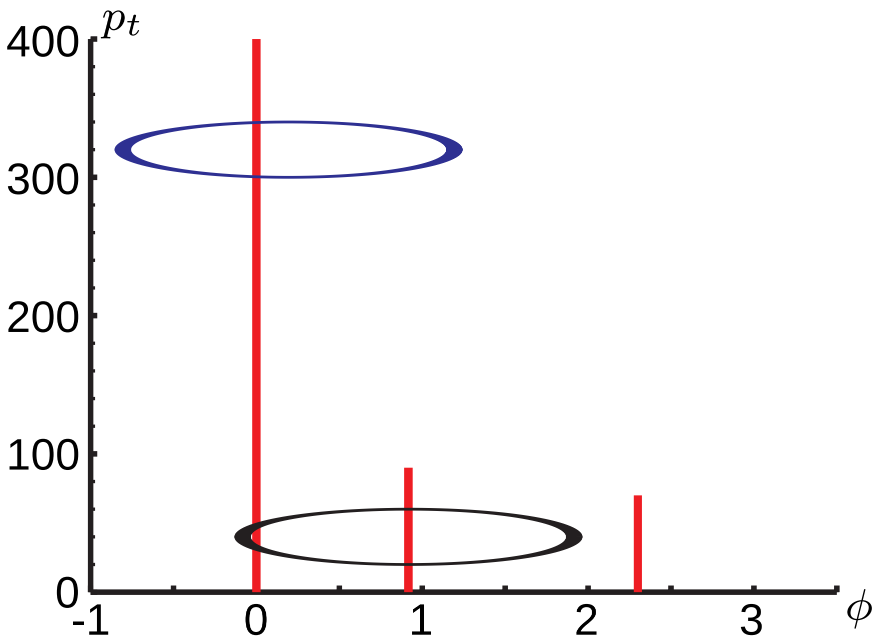

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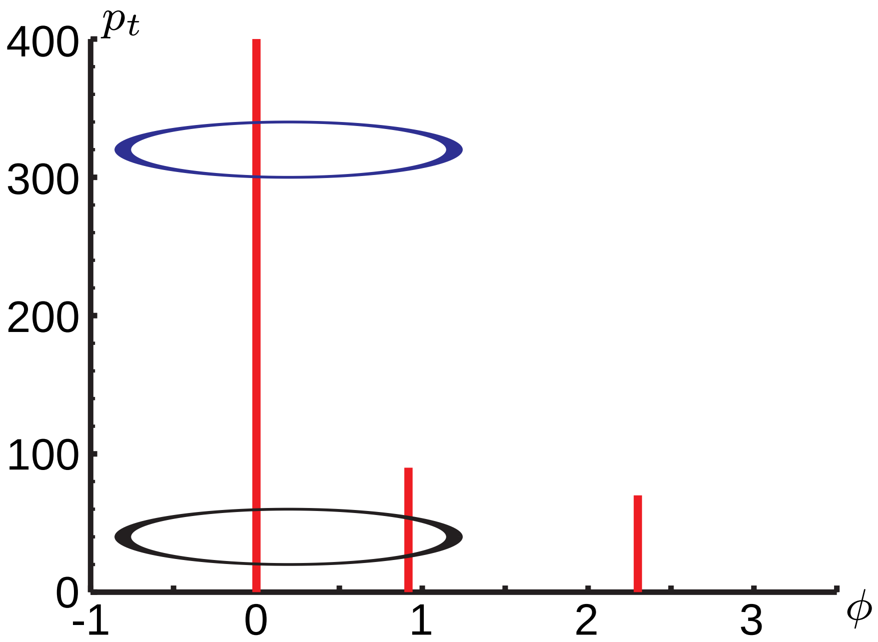

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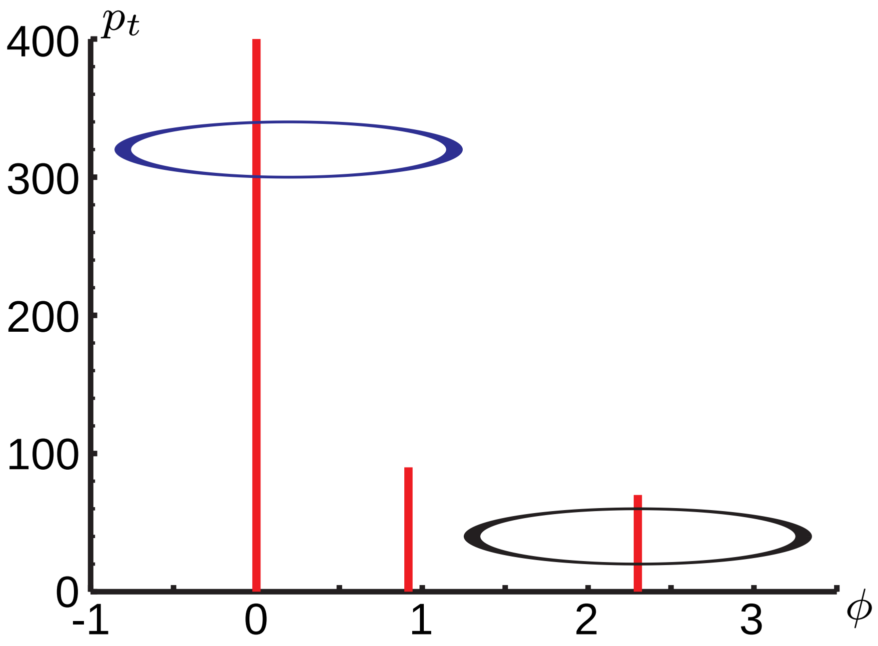

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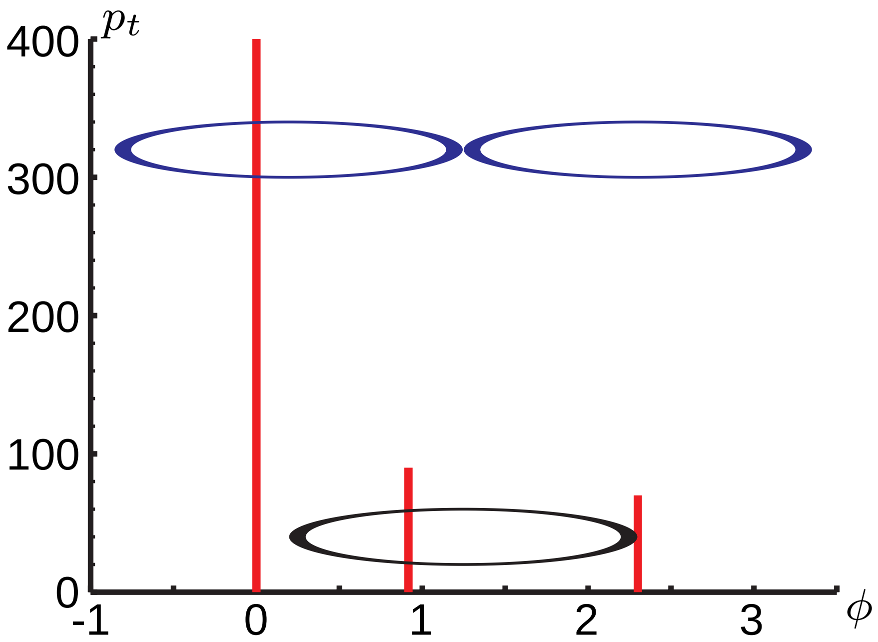

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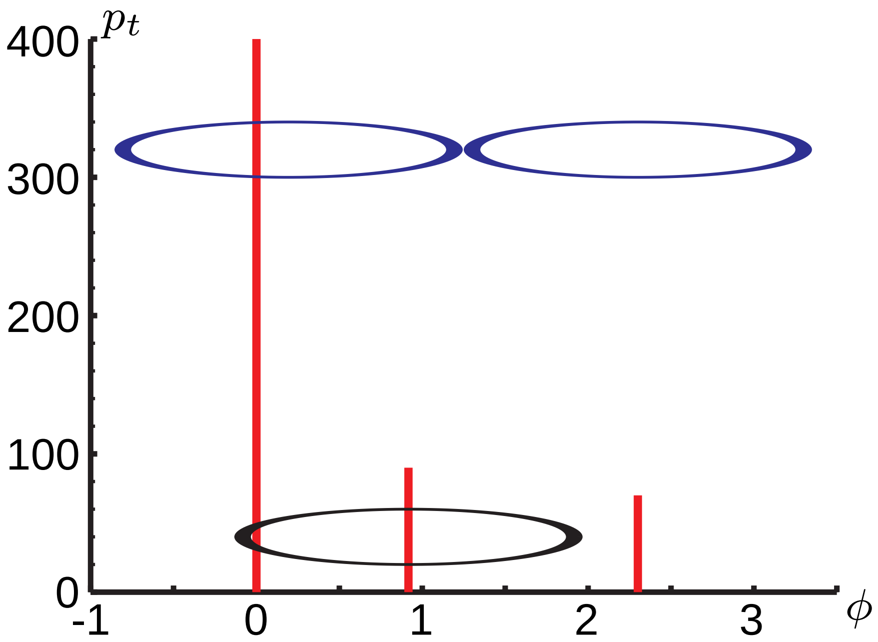

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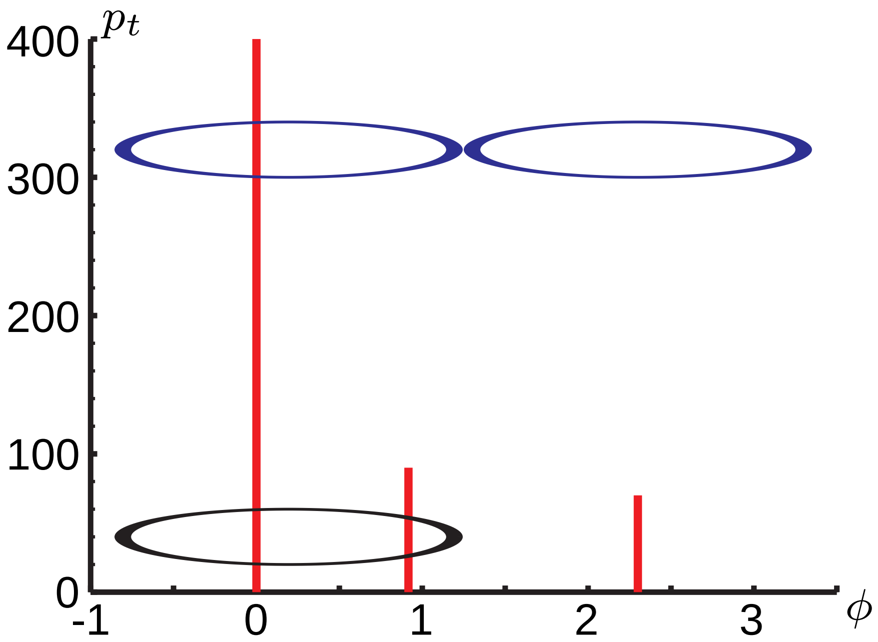

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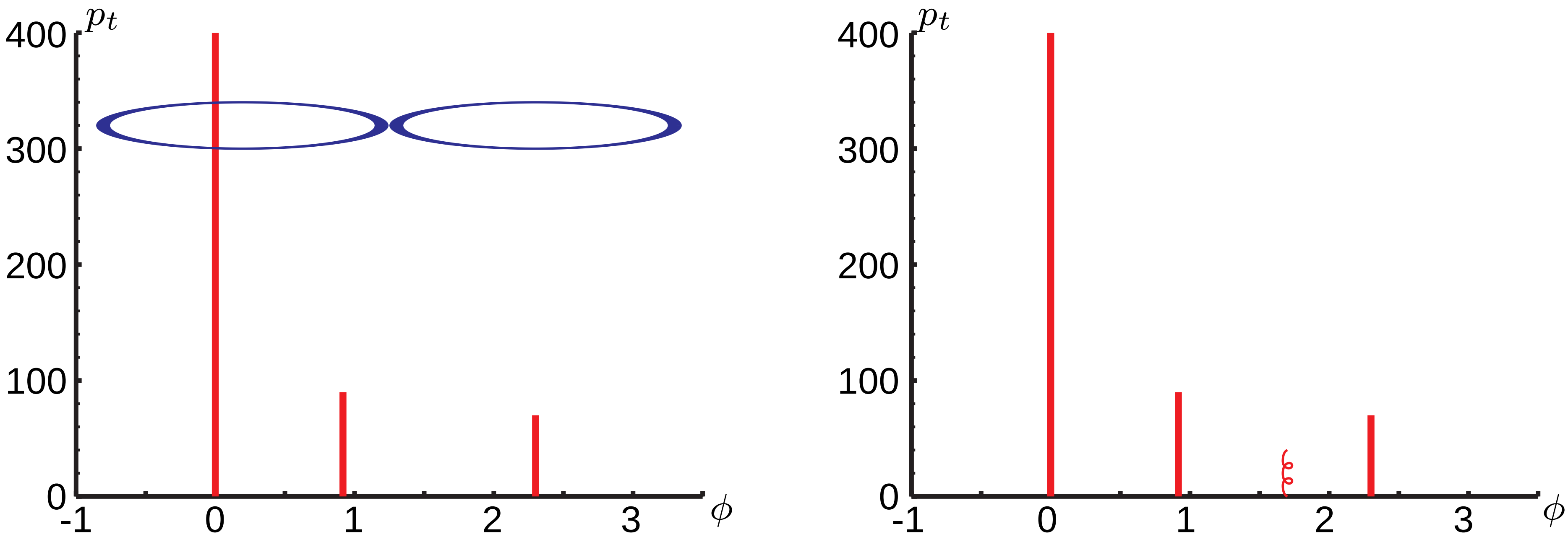

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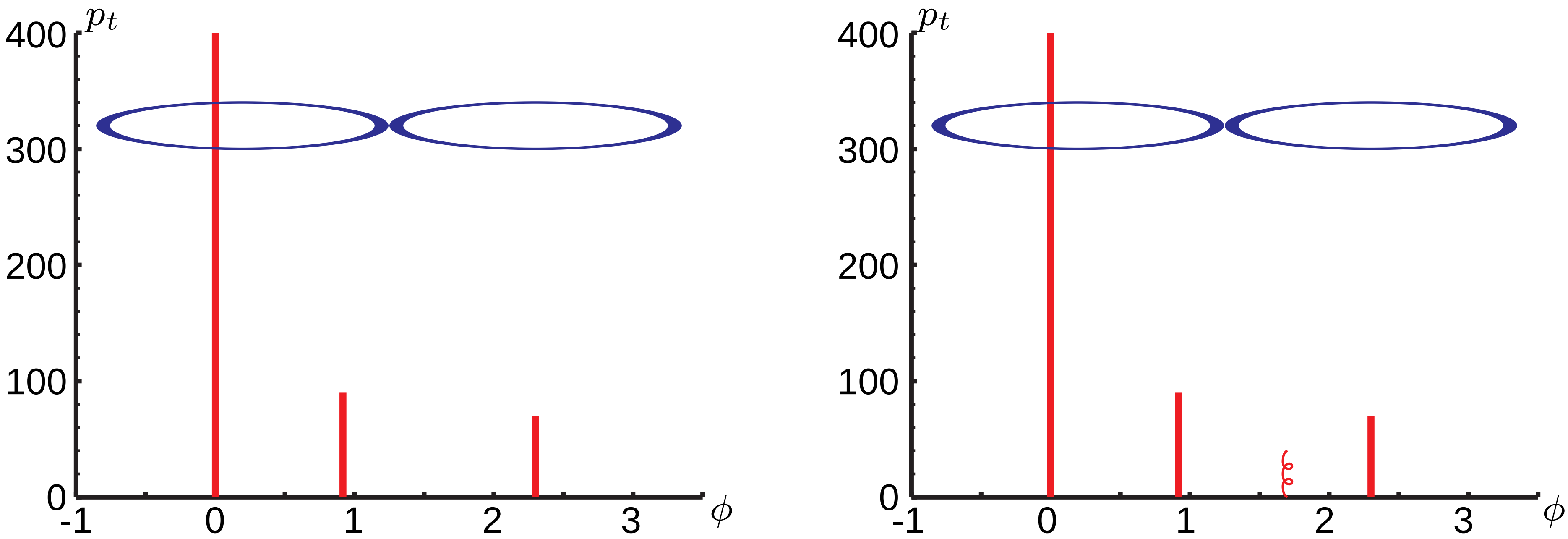

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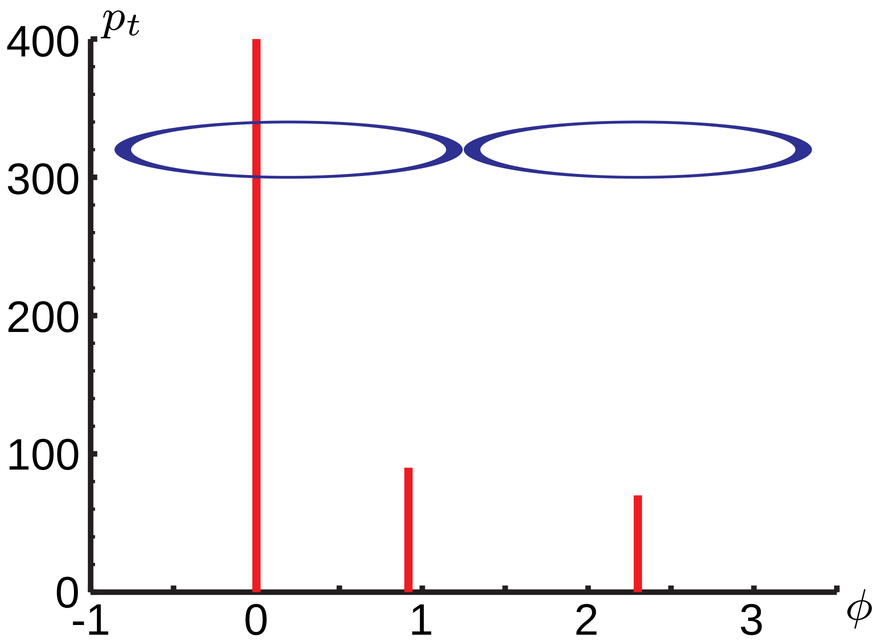

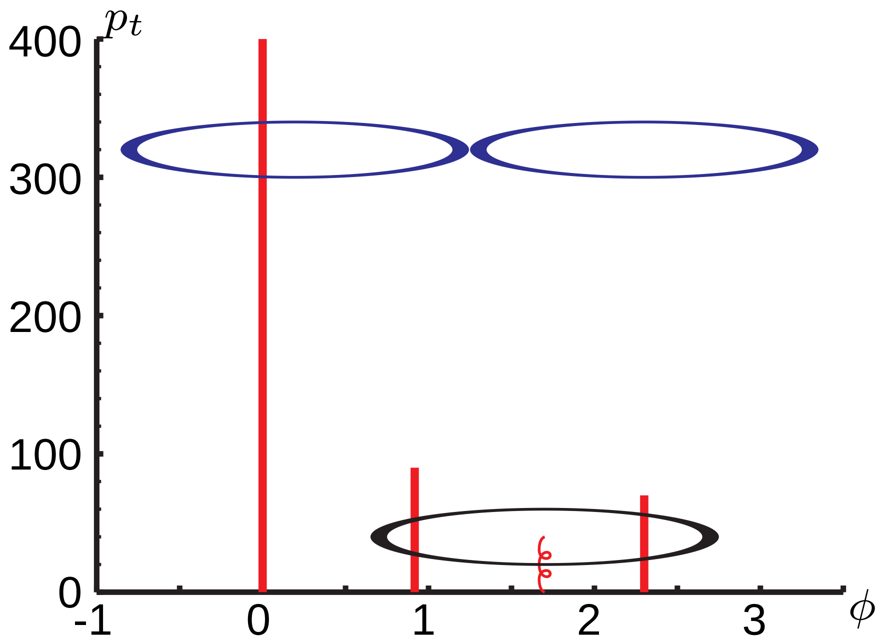

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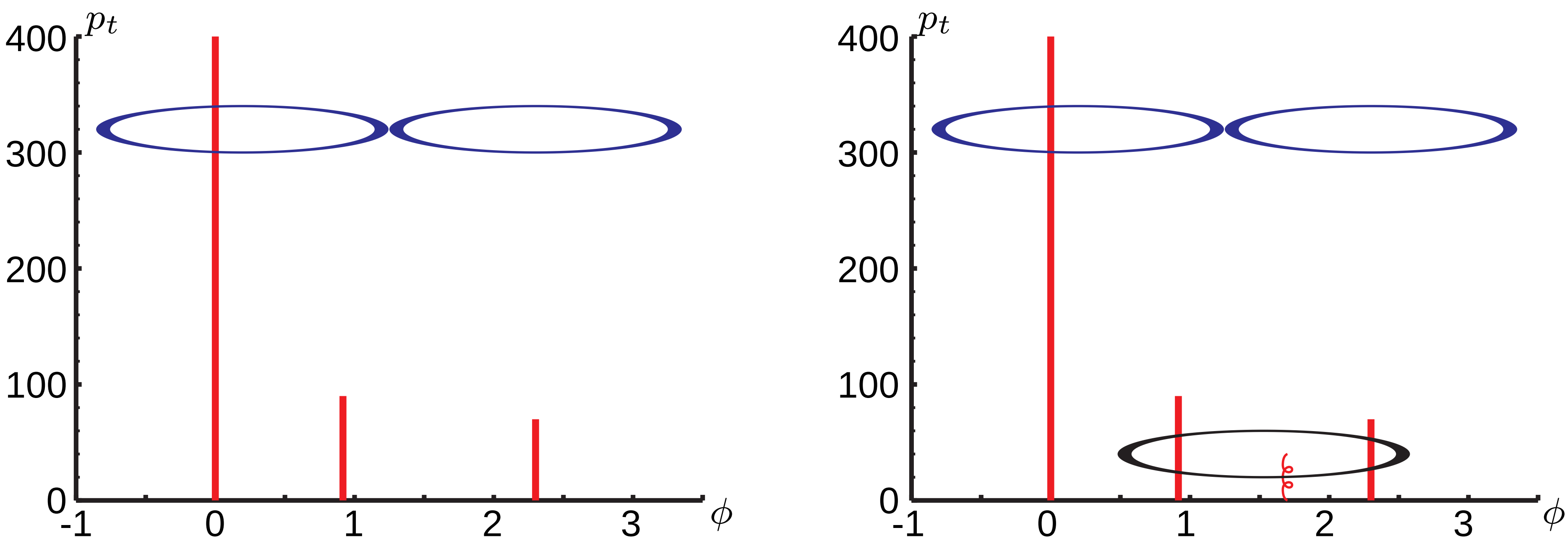

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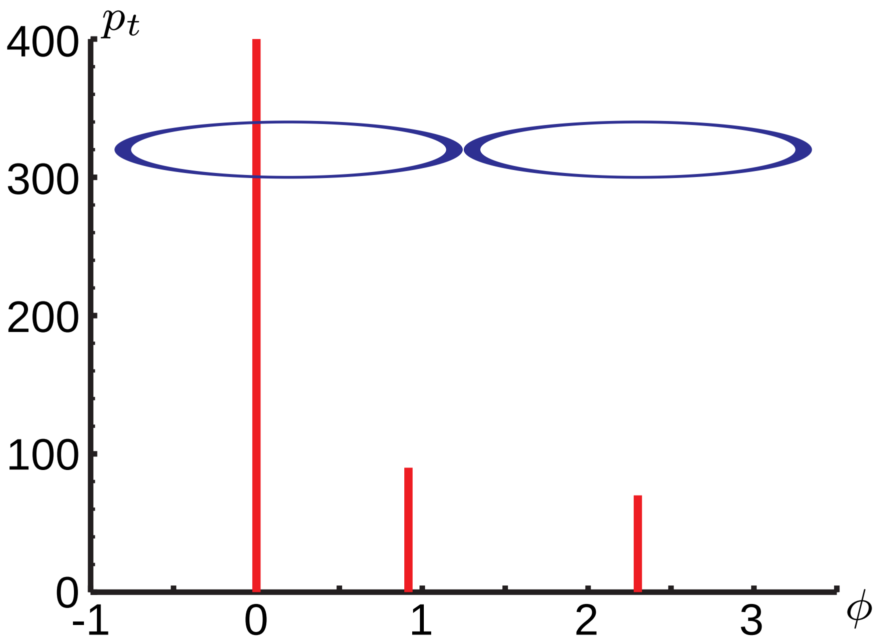
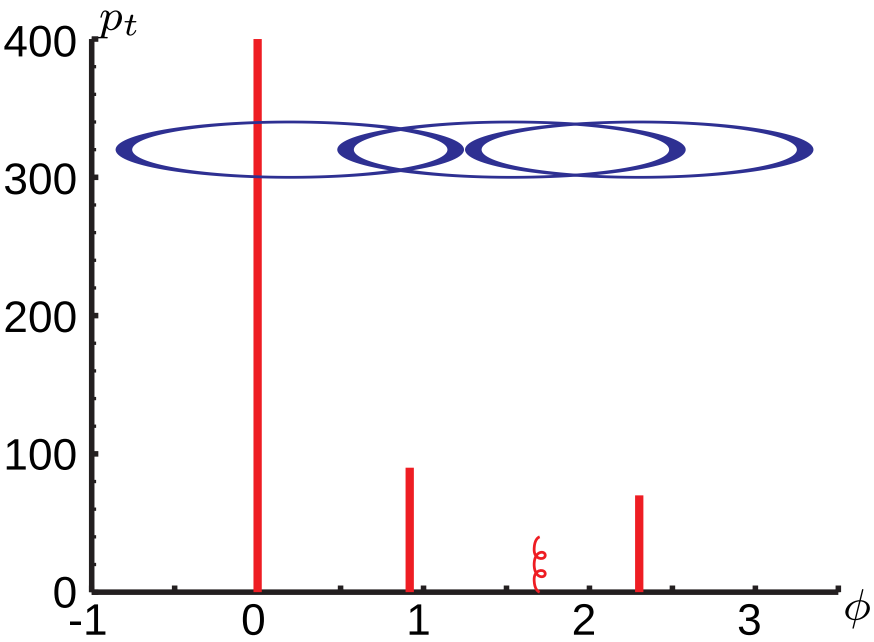
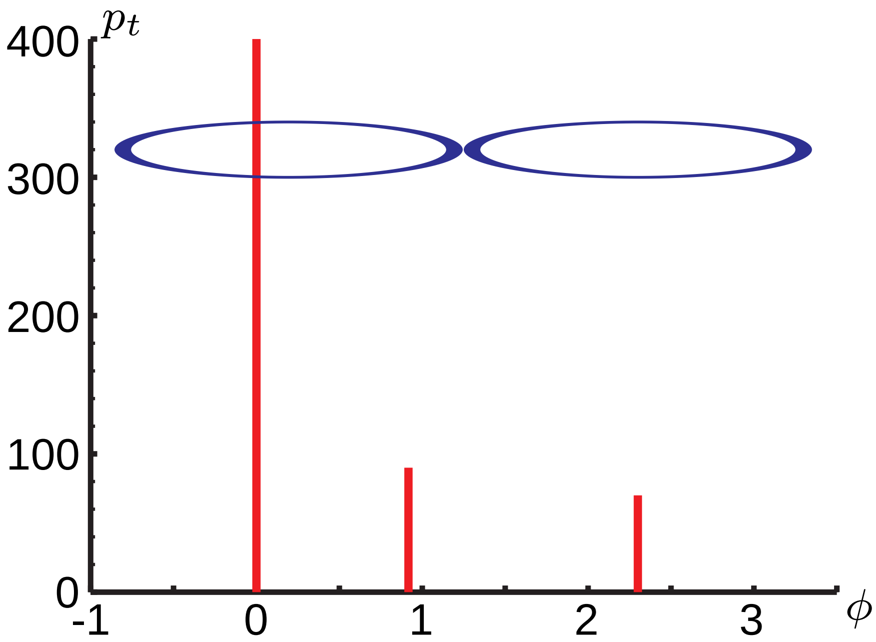


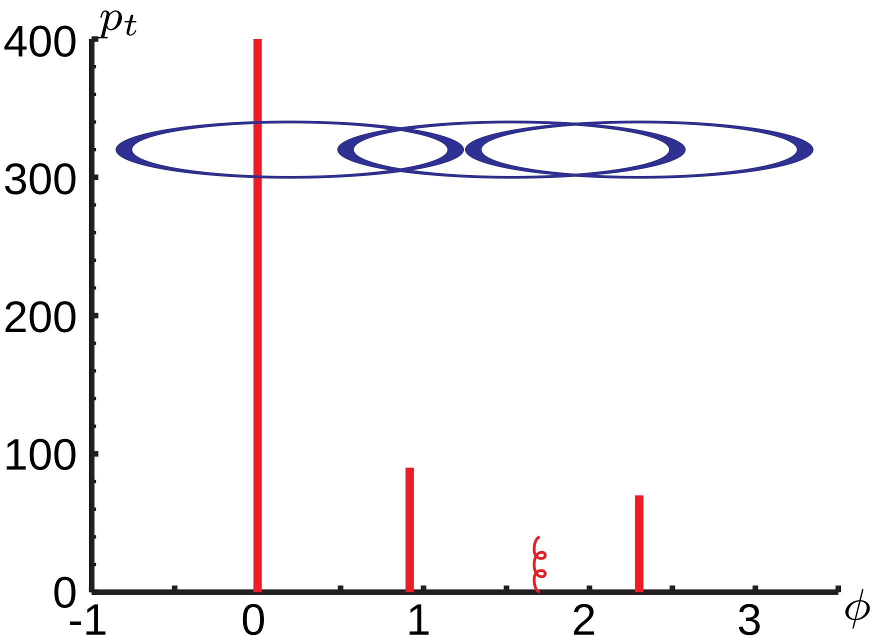


Stable cones:

Midpoint: {1,2} & {3} {1,2} & {3} & {2,3}

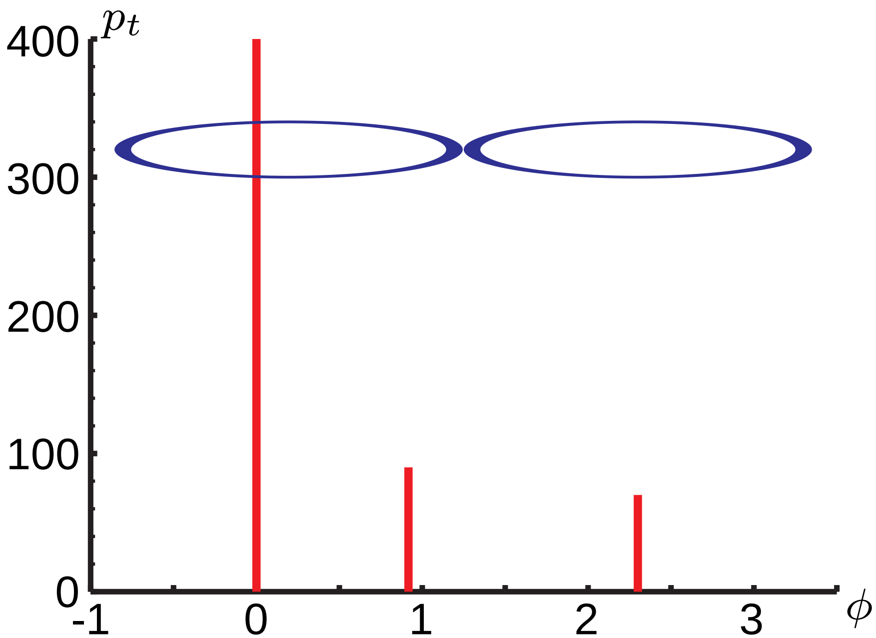
IR unsafety of the Midpoint alg

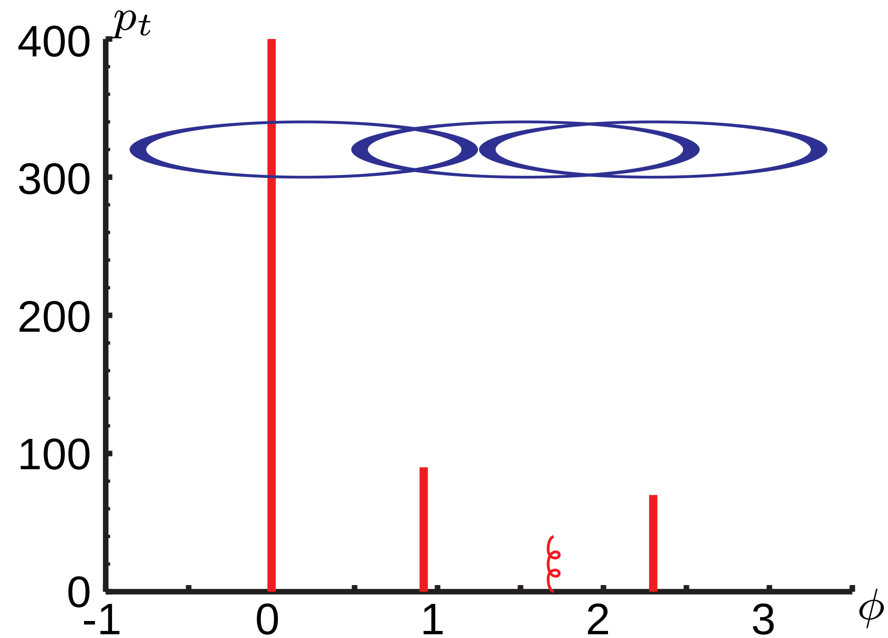




Stable cones:		
Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Jets: ($f = 0.5$)		
Midpoint:	{1,2} & {3}	{1,2,3}

IR unsafety of the Midpoint alg





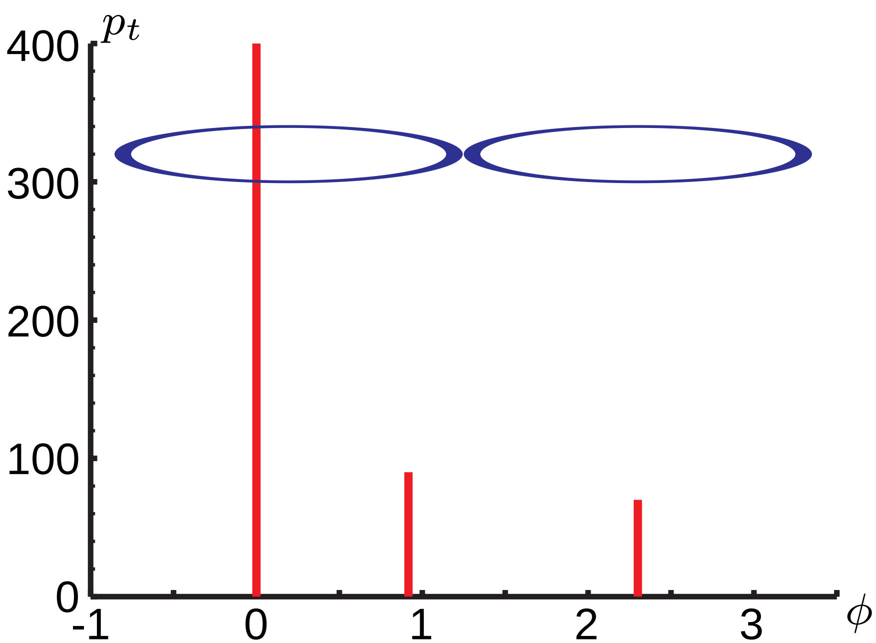
Stable cones:

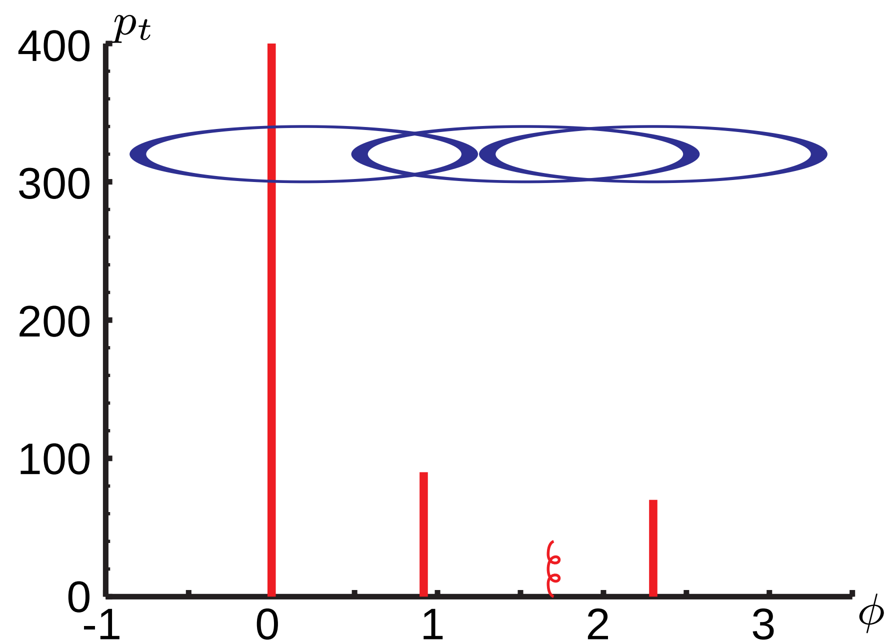
Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

IR unsafety of the Midpoint alg





Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

Stable cone missed $\longrightarrow$ IR unsafety of the midpoint algorithm

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones

- Naive approach: check stability of each subset of particle

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Naive approach: check stability of each subset of particle Complexity is $\mathcal{O}\left(N2^N\right)$
  $\Rightarrow$ definitely unrealistic: 10^{17} years for $N = 100$
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

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Idea: use geometric arguments

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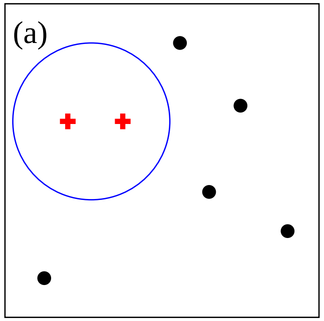

- Enumerate enclosures and check if they are stable

Solution: SISCone

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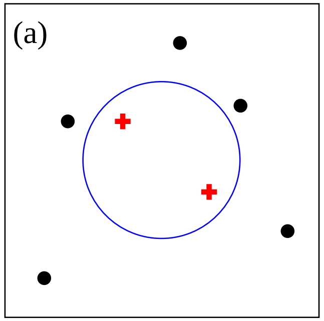

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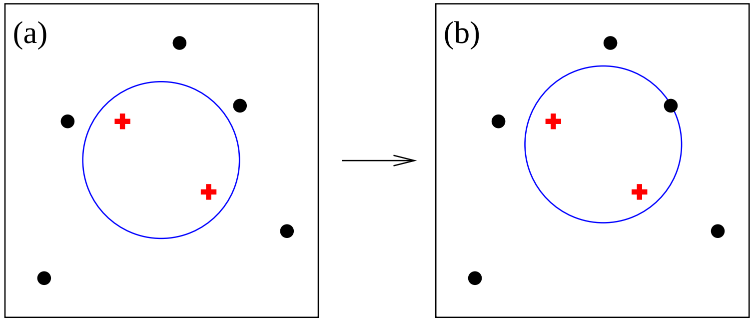

- Enumerate enclosures and check if they are stable
- Each enclosure can be moved (in any direction) until it touches a point

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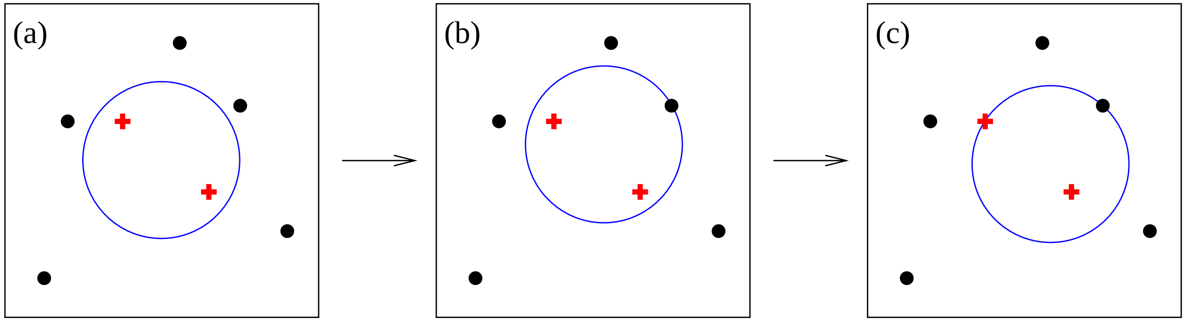

- Enumerate enclosures and check if they are stable
- Each enclosure can be moved (in any direction) until it touches a point
- ... then rotated until it touches a second one

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments

$\Rightarrow$ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
$\longrightarrow$ find all enclosures

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
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- Complexity: $\mathcal{O}\left(N^3\right)$, with improvements: $\mathcal{O}\left(N^2 \log(N)\right)$

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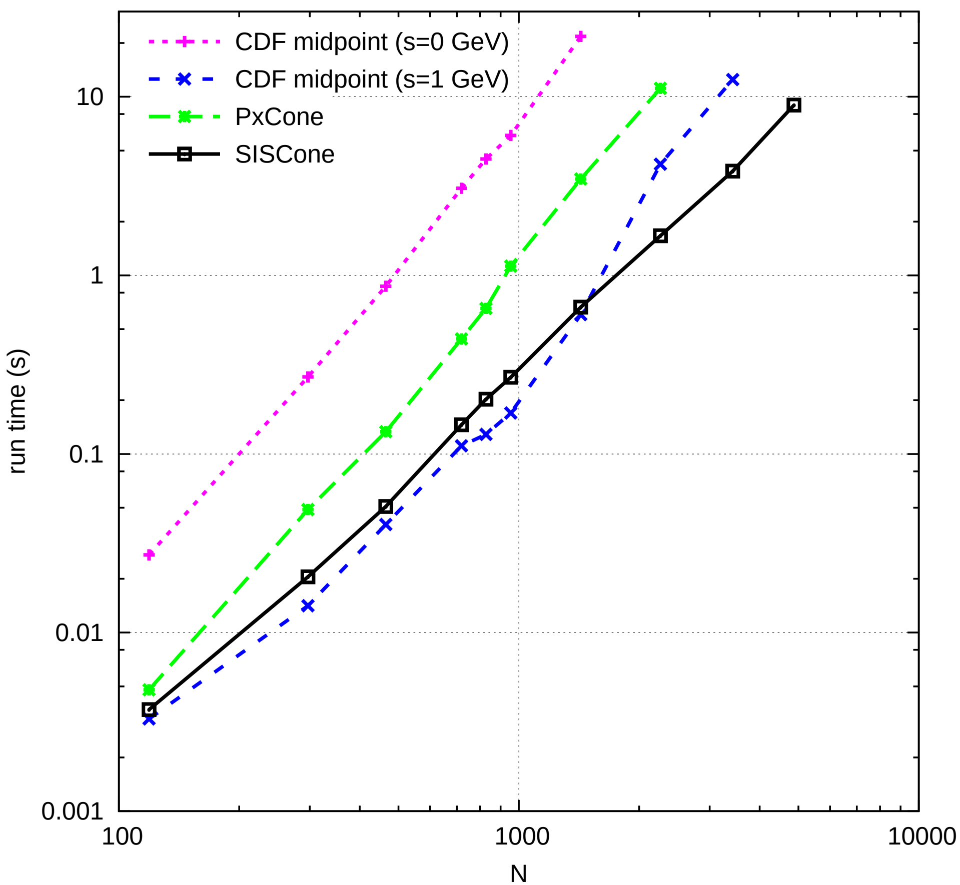
- Complexity: $\mathcal{O}\left(N^3\right)$, with improvements: $\mathcal{O}\left(N^2 \log(N)\right)$

$\longrightarrow$ C++ implementation: Seedless Infrared-Safe Cone algorithm (SISCone)
G.Salam, G.S., JHEP 04 (2007) 086; http://projects.hepforge.org/siscone

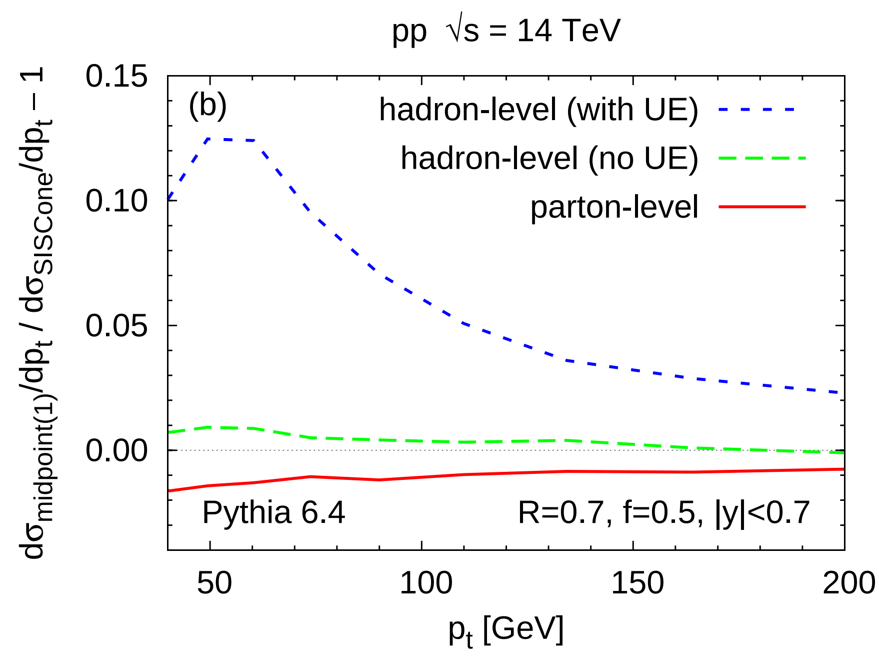
NB.: also available from FastJet
[M.Cacciari, G.Salam, G.S.]; http://www.lpthe.jussieu.fr/~salam/fastjet

Execution timings

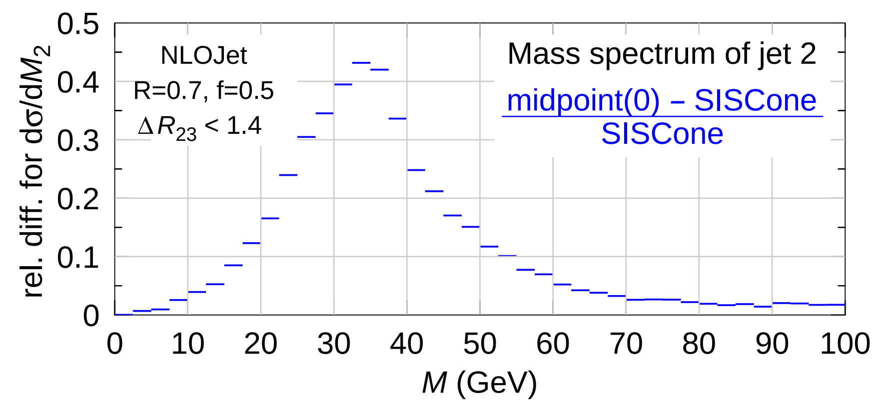


- at least as fast as midpoint cones
- effect from a few percents (incl.) to $\sim 45\%$ (excl.)

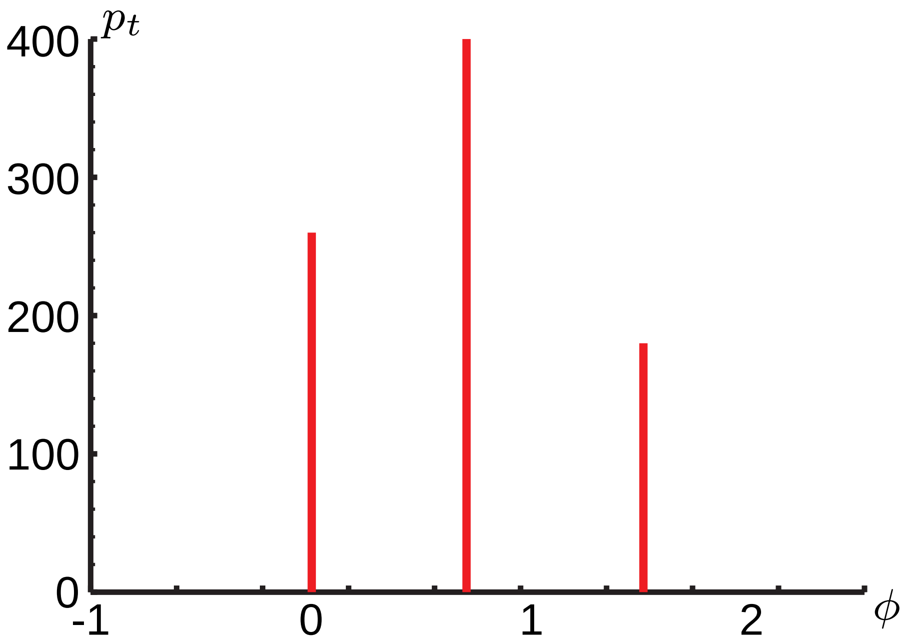
Inclusive (midpoint/SISCone-1)



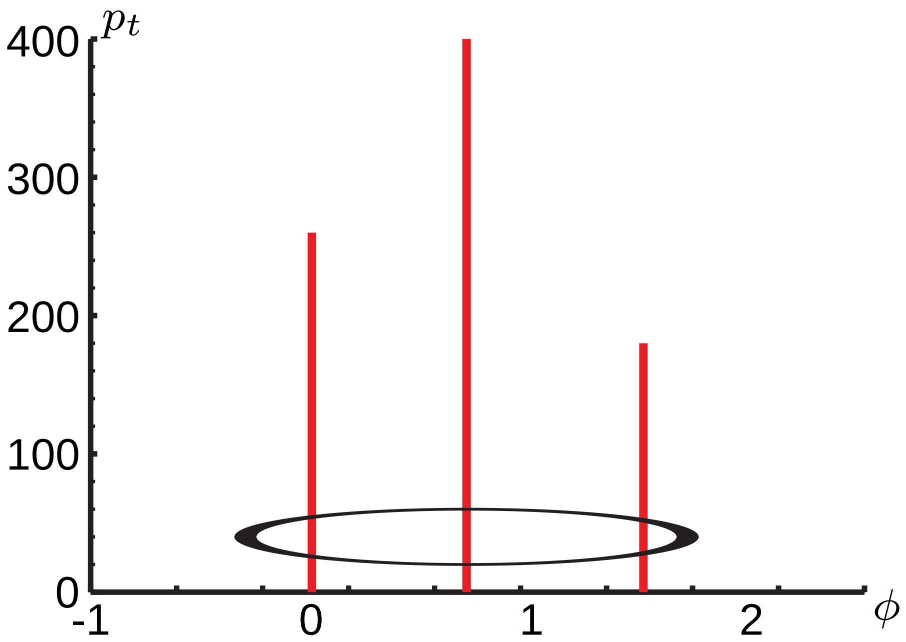
Masses in 3-jet events



Coll. unsafety of the iterative cone



Coll. unsafety of the iterative cone



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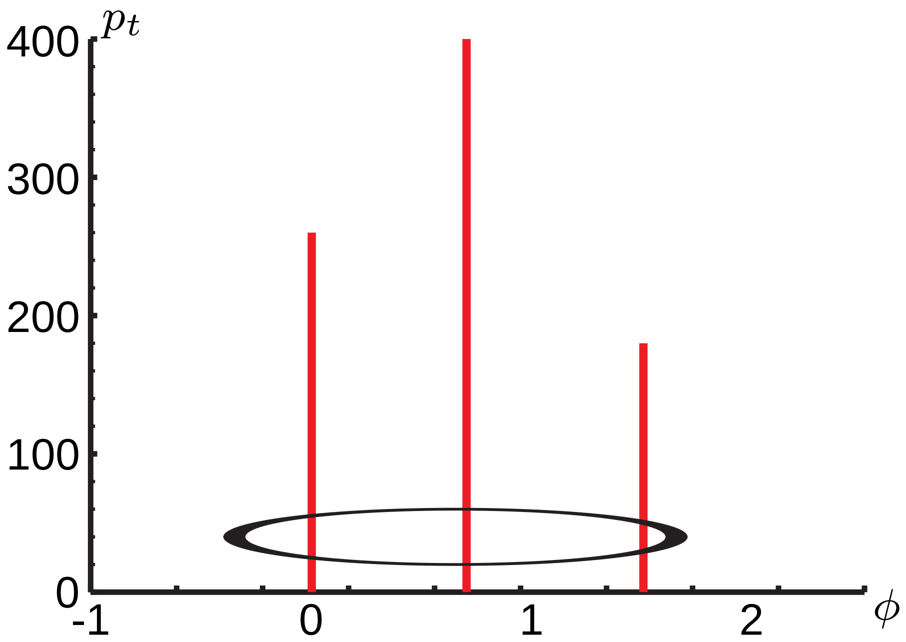

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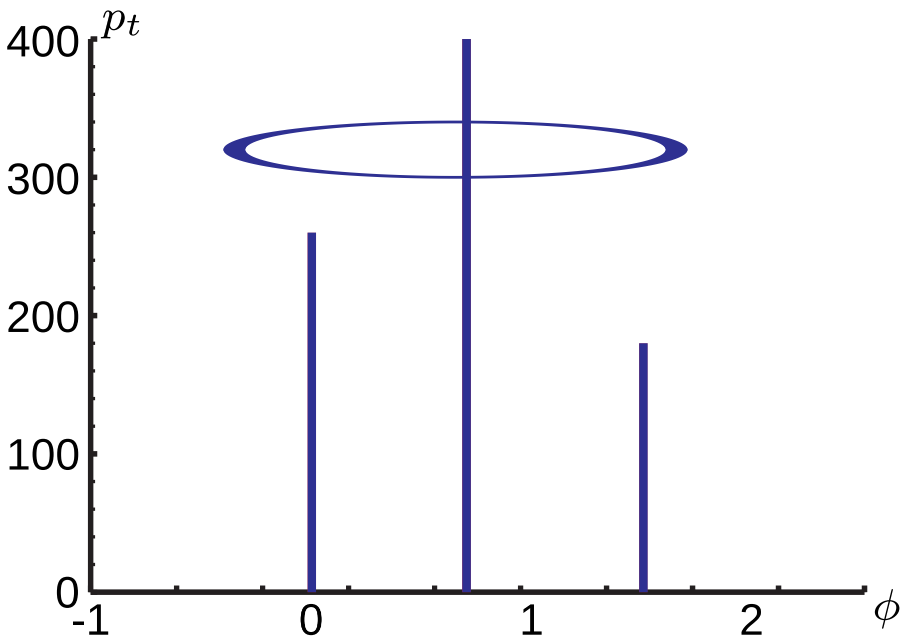

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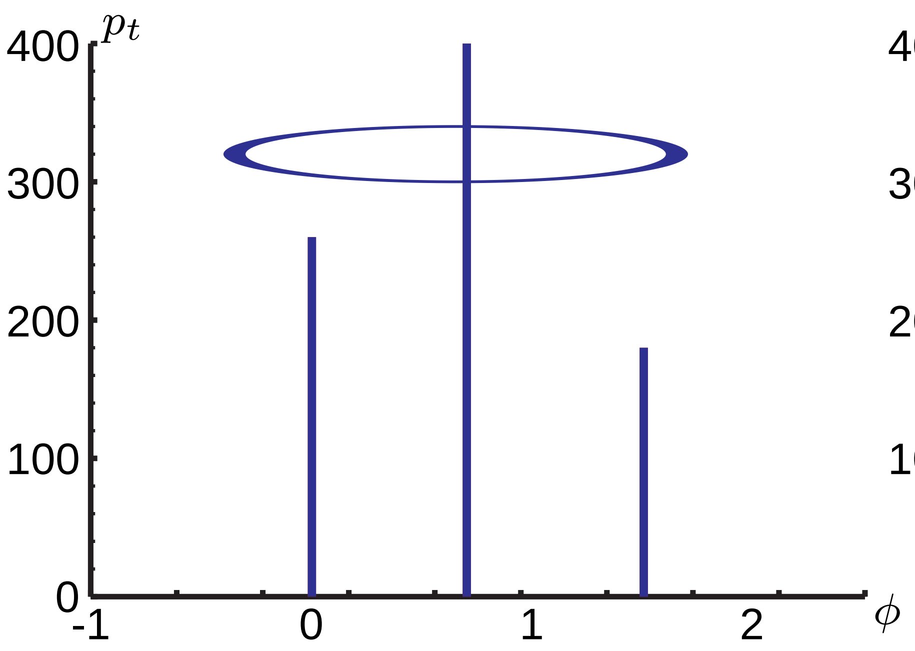

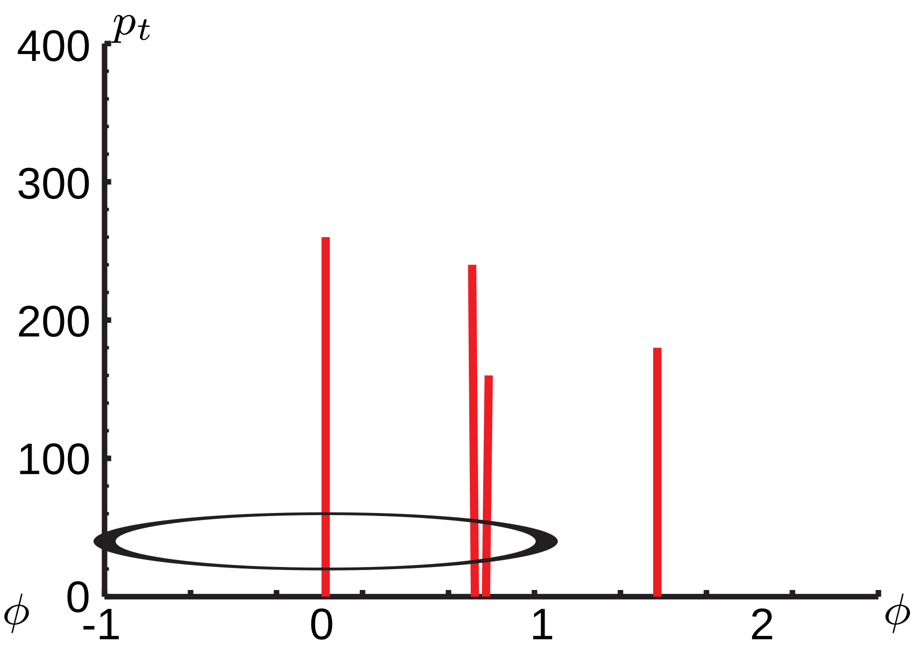

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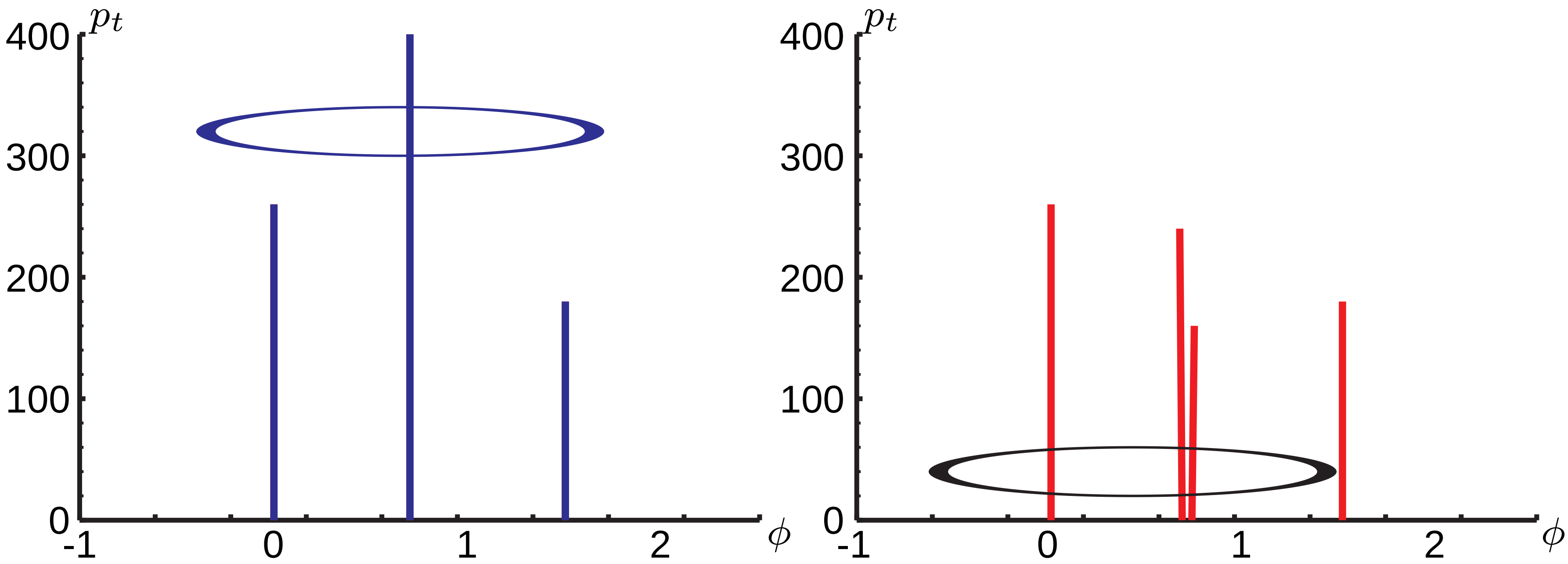

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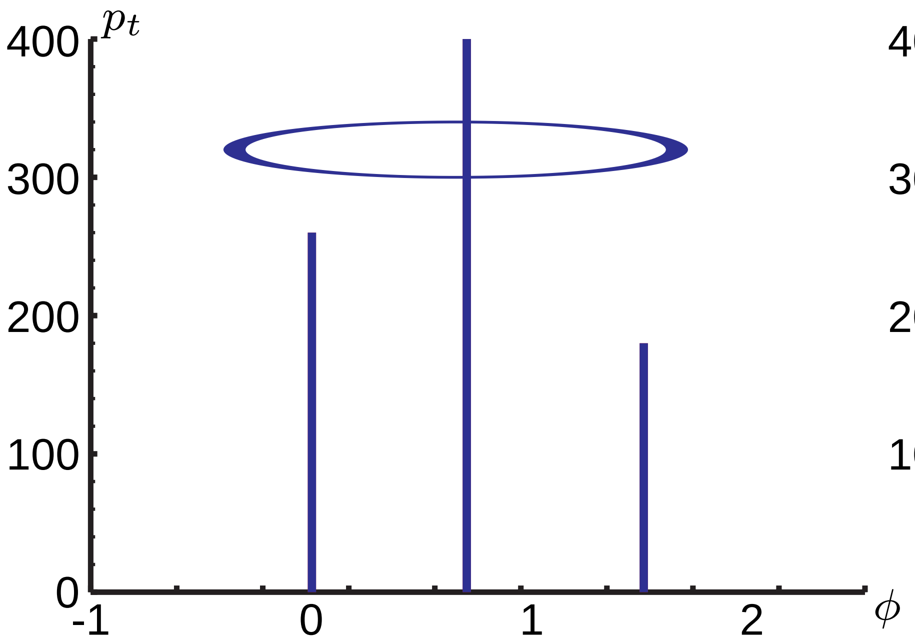

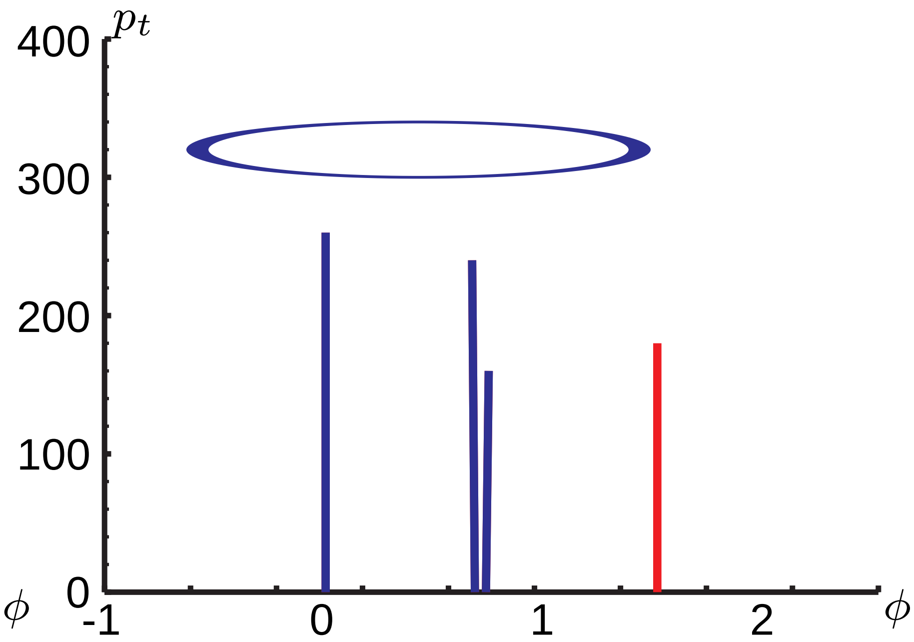

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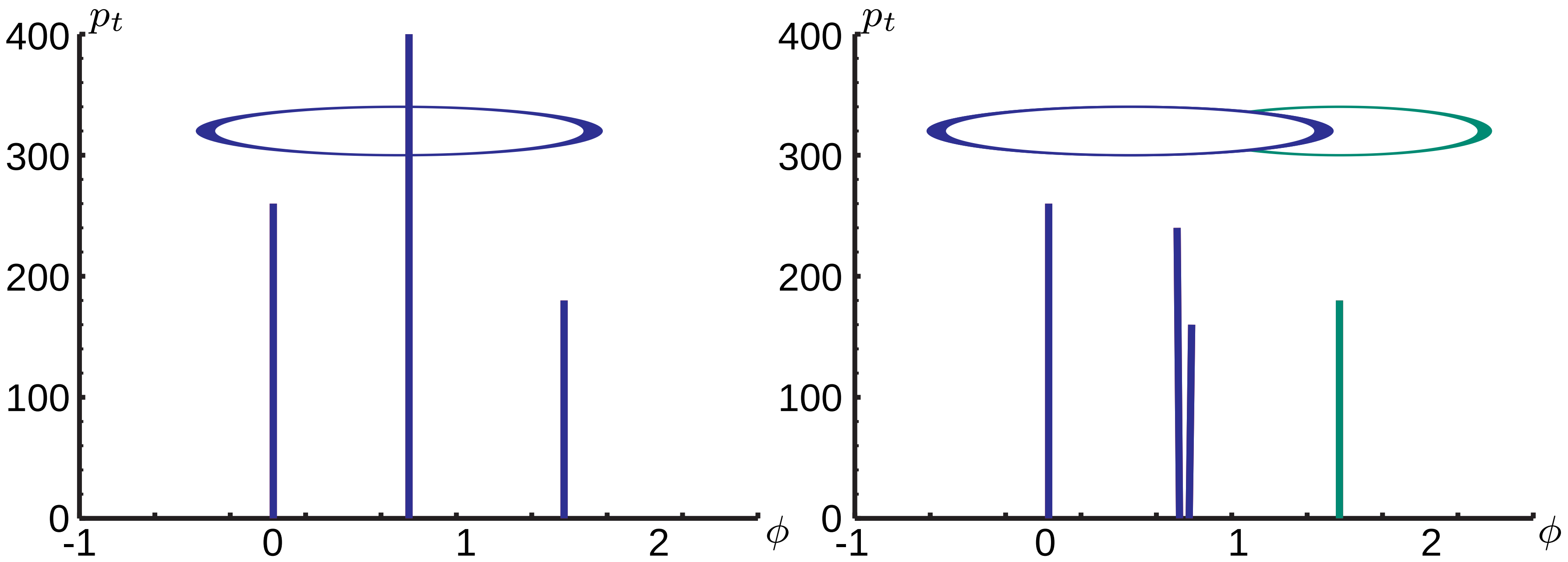

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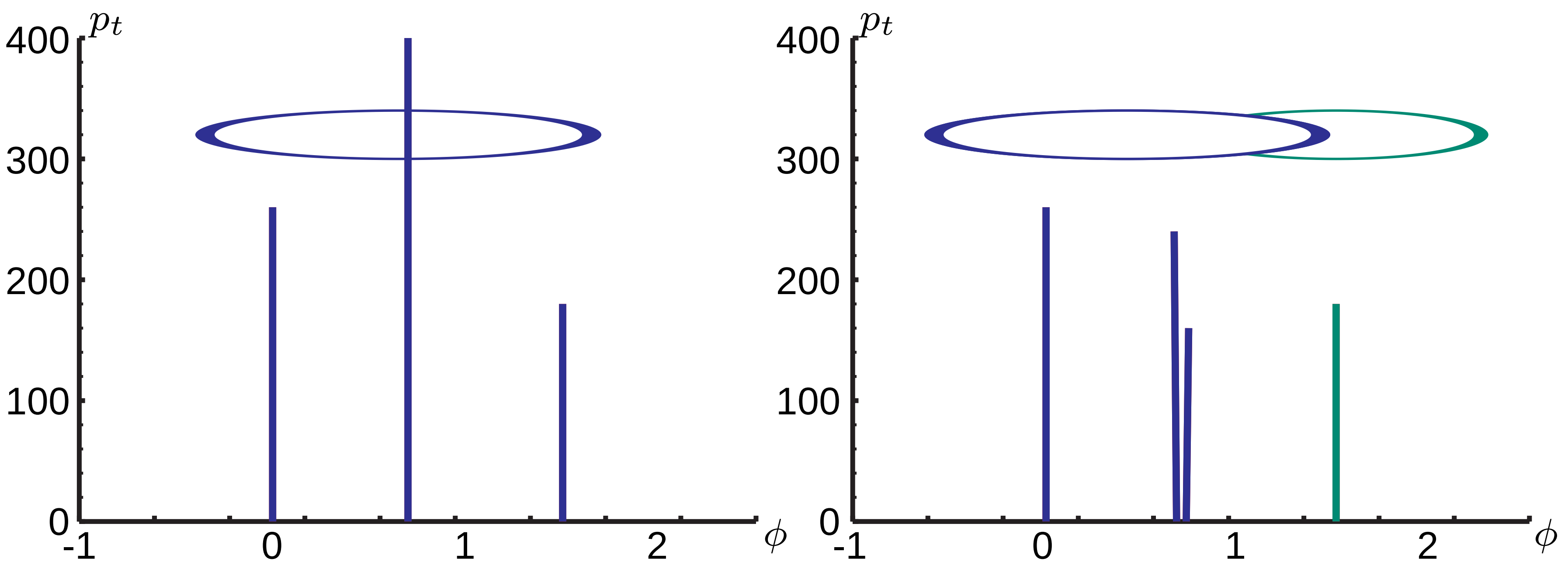

- Before collinear spliting: 1 jet
- After collinear spliting: 2 jets

$\longrightarrow$ **collinear unsafety of the iterative cone algorithm**

Anti-k_t

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ \left(\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2\right)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm

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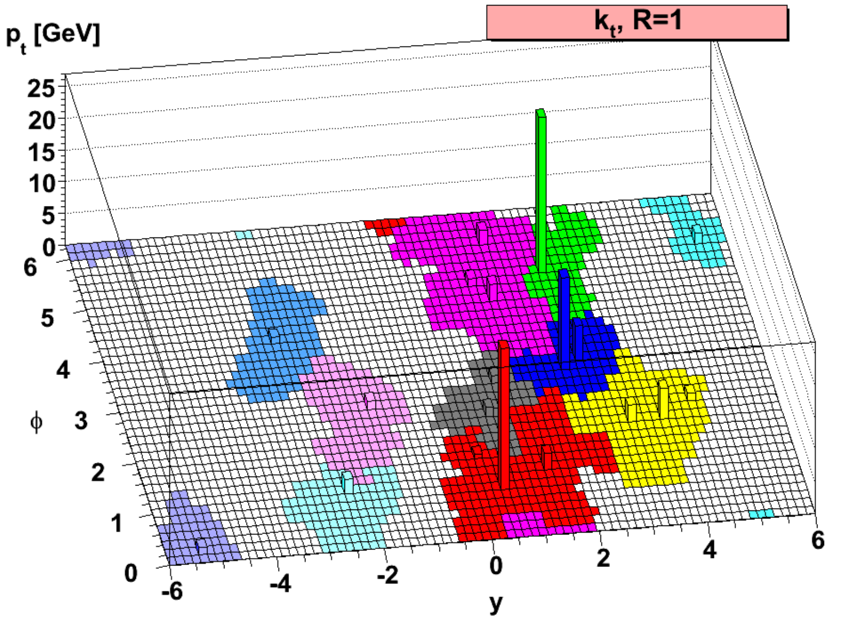
Why should that be related to the iterative cone ?!?

- "large $k_t \Rightarrow$ small distance"
  i.e. hard partons "eat" everything up to a distance R
  i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

Hard event + homogeneous soft background



anti-k_t is soft-resilient

more in Matteo Cacciari's talk...

Conclusions

- Midpoint and the iterative cone IR or Collinear unsafe (at $\mathcal{O}(\alpha_s^4)$)

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)

- The IR-unsafety issue will matter at LHC

  + **We do not want the theoretical efforts to be wasted**

- SISCone is a natural replacement for Midpoint (as fast, IRC safe)
- anti-k_t could replace the iterative cone (regular, IRC safe)
- Available from `FastJet` (http://www.lpthe.jussieu.fr/∼salam/fastjet)
  SISCone: http://projects.hepforge.org/siscone
- Algorithms at play: see Juan Rojo's talk