# Proton elastic impact factors for 2,3 and 4 gluons

Leszek Motyka

*Hamburg University & Jagellonian University, Kraków*

## Overview

Motivation

Baryon light-cone wave functions

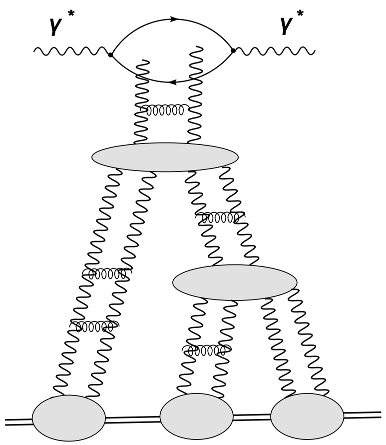
Baryon impact factors

Small $x$ evolution in the $t$-channel

Implications

Based on results obtained with Jochen Bartels

# Motivation




Small $x$ evolution of color dipole (photon) scattering is well known:
BFKL equation + unitarity corrections

Pattern of unitarisation for color dipole scattering was established:
BFKL Pomeron fan diagrams $(\gamma^* A)$ and BFKL Pomeron loops $(\gamma^* \gamma^*)$

Neccessary to describe DIS at low $Q^2$ and DDIS in HERA

In contrast, little is know about baryon scattering and small-$x$ evolution of baryon scattering amplitudes

Important problem: color dipole formalism justified for $N_c \to \infty$

Non-planar corrections get enhanced by number of quarks = $N_c$.

Dipole model approach does not seem to close [Praszałowicz, Rostworowski]

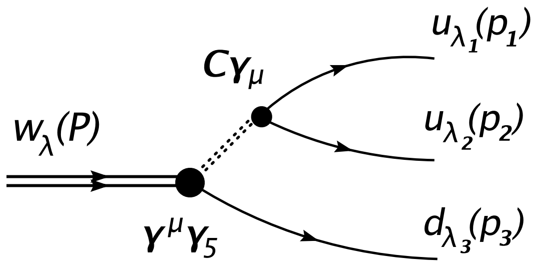
$\longrightarrow$ We try conventional $t$-channel evolution approach at $N_c = 3$

# Baryon wave function in infinite momentum frame

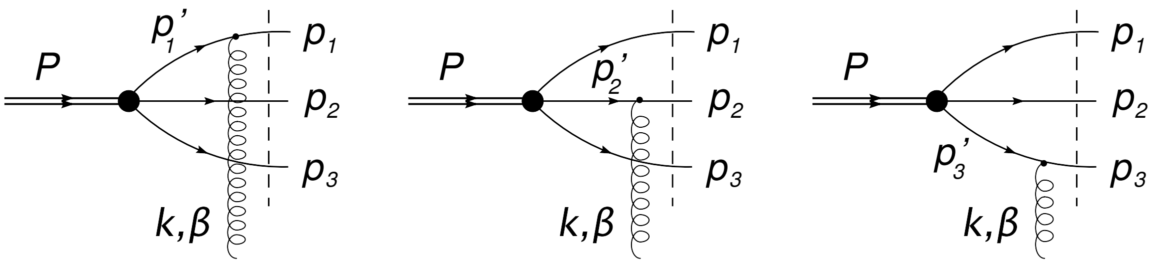
Ioffe current for the proton

$$\eta(x) = \varepsilon_{\kappa_1\kappa_2\kappa_3} \left[ (u^{\kappa_1}(x))^T C\gamma^\mu u^{\kappa_2}(x) \right] \gamma_\mu \gamma_5 \, d^{\kappa_3}(x)$$

Vertex: proton $\longrightarrow$ quarks




Gluon coupling




Evaluation in helicity basis in infinite momentum frame $\longrightarrow$ proton light cone wave function

# Baryon wave function from gluon coupling

In high energy limit all diagrams for proton $\longrightarrow$ quarks transition

$$p + \overbrace{g + g + \ldots + g}^{n \text{ gluons}} \quad \longrightarrow \quad u\, u\, d$$

with any number $n$ of gluons give universal amplitudes with quark momenta $\boldsymbol{p}_i$ evaluated at the vertex

These amplitudes need to be Borel-transformed in order to eliminate poles of quark propagators **[Balitsky, Lipatov]**

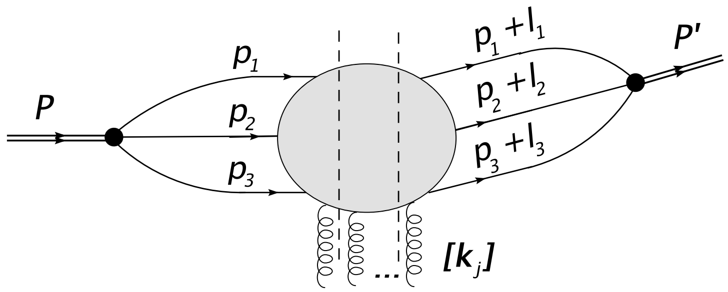
$$\mathcal{B}'\,\mathcal{B}\; f(P^2, P'^2) \quad \text{with} \quad f(P^2, P'^2) \;\sim\; \frac{1}{P^2 + \boldsymbol{P}^2 - M_X^2}\,\frac{1}{P'^2 + \boldsymbol{P}'^2 - {M'_X}^2}$$

$$\frac{1}{P^2 + \boldsymbol{P}^2 - \frac{\boldsymbol{p}_1^2}{\alpha_1} - \frac{\boldsymbol{p}_2^2}{\alpha_2} - \frac{\boldsymbol{p}_3^2}{\alpha_3}} \quad \longrightarrow \quad \exp\left[ - \left( \frac{\boldsymbol{p}_1^2}{\alpha_1} + \frac{\boldsymbol{p}_2^2}{\alpha_2} + \frac{\boldsymbol{p}_3^2}{\alpha_3} - \boldsymbol{P}^2 \right) \,/\, M^2 \right]$$

**[Brodsky, Lepage]**

$\longrightarrow$ Baryon wave function dependent on quark longitudinal and transverse momenta, and helicities

# The impact factor




Baryon impact factor

$$\mathcal{B}^{\lambda\lambda'}(\{\boldsymbol{l}_i\};\boldsymbol{P},\boldsymbol{P}') = \underbrace{I_{qq}^{(l+m+n)}}_{\text{quark scattering}} \sum_{\text{diagrams}} \mathcal{F}^{\lambda\lambda'}(\{\boldsymbol{l}_i\};\boldsymbol{P},\boldsymbol{P}')\ \mathcal{C}(\text{diagram})$$

Color factor:

$$\mathcal{C}(\text{diagram}) = \frac{\varepsilon^{\kappa_1'\kappa_2'\kappa_3'}\varepsilon^{\kappa_1\kappa_2\kappa_3}}{3!}\left[t^{a_l}t^{a_{l-1}}\dots t^{a_1}\right]_{\kappa_1'\kappa_1}\left[t^{b_m}t^{b_{m-1}}\dots t^{b_1}\right]_{\kappa_2'\kappa_2}\left[t^{c_n}t^{c_{n-1}}\dots t^{c_1}\right]_{\kappa_3'\kappa_3}$$

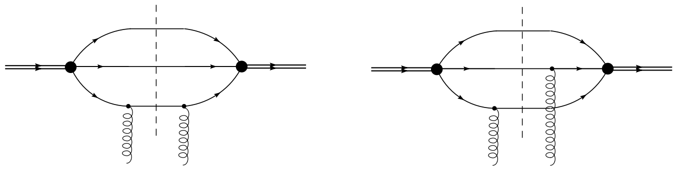
Kinematic part:

$$\mathcal{F}^{\lambda\lambda'}(\{\boldsymbol{l}_i\};\boldsymbol{P},\boldsymbol{P}') = \sum_{\lambda_1,\lambda_2,\lambda_3}\int[d^2\boldsymbol{p}_i]\,[d\alpha_i]\ \Psi_{\lambda}^{(\lambda_1,\lambda_2)\,\lambda_3}(\{\alpha_i\},\{\boldsymbol{p}_i\};\boldsymbol{P})\ \left[\Psi_{\lambda'}^{(\lambda_1,\lambda_2)\,\lambda_3}\Big(\{\alpha_i\},\{\boldsymbol{p}_i+\boldsymbol{l}_i\};\boldsymbol{P}'\Big)\right]^*$$

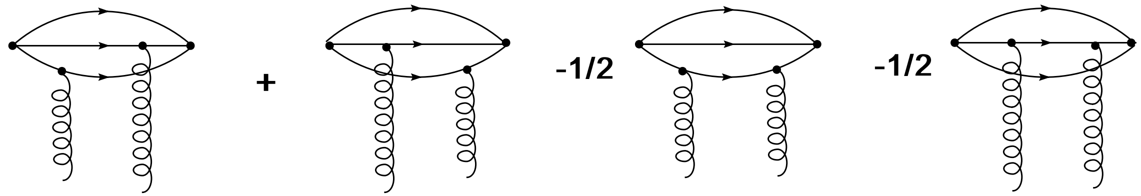
Baryon form-factor: $F(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3)$ depends on overall momentum transfers $\boldsymbol{l}_i$ to the quark lines

# 2 gluon baryon impact factor

Basic topologies:



Decomposition of amplitude into gauge invariant pieces — with one spectator quark




Structure of $D_{2;0}^{\{1,2\}}$ resembles color dipole, but with half of the coupling

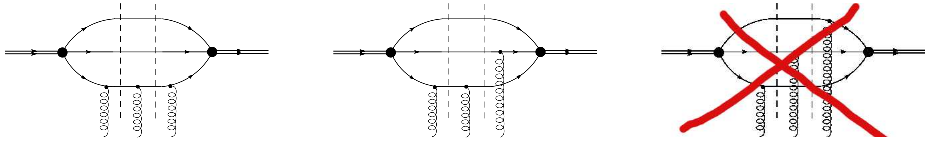
$$D_{2;0}^{\{1,2\}}(\boldsymbol{k}_1, \boldsymbol{k}_2) \;=\; \frac{-g^2}{12}\left[F(\boldsymbol{k},0,0) + F(0,\boldsymbol{k},0) - F(\boldsymbol{k}_1,\boldsymbol{k}_2,0) - F(\boldsymbol{k}_2,\boldsymbol{k}_1,0)\right]$$

Full impact factor: the sum over quasi-dipoles

$$B_{2;0}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \delta^{\,a_1 a_2}\left[D_{2;0}^{\{1,2\}}(\boldsymbol{k}_1, \boldsymbol{k}_2) + D_{2;0}^{\{1,3\}}(\boldsymbol{k}_1, \boldsymbol{k}_2) + D_{2;0}^{\{2,3\}}(\boldsymbol{k}_1, \boldsymbol{k}_2)\right]$$

# 3-gluon $C$-even baryon impact factor

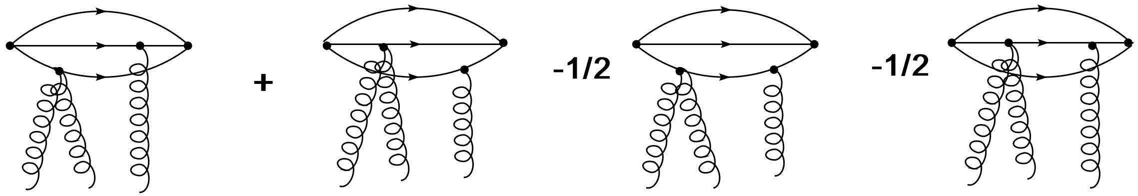
Topologies for even $C$:



3-gluon $C$-even impact factor may be decomposed

$$B_{3;0} = D_{3;0}^{\{1,2\}} + D_{3;0}^{\{1,3\}} + D_{3;0}^{\{2,3\}}$$

and $D_{3;0}^{\{i,j\}}$ have the color/momentum structure known from the photon/dipole

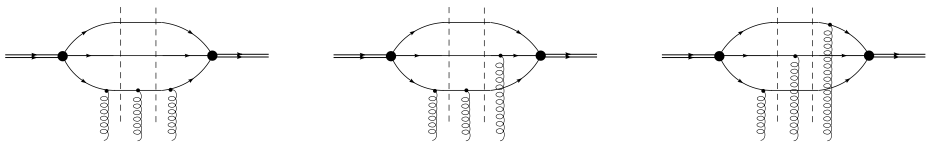



$$D_{3;0}^{\{i,j\}}(1,2,3) = \frac{1}{2} g f^{a_1 a_2 a_3} \left[ D_{2;0}^{\{i,j\}}(12,3) - D_{2;0}^{\{i,j\}}(13,2) + D_{2;0}^{\{i,j\}}(23,1) \right]$$

Suggestive of independent Reggeization of dipole-like components $D_{3;0}^{\{i,j\}}$

# 3-gluon $C$-odd baryon impact factor

All topologies contribute:



$C$-odd baryon impact factor

$$\tilde{B}_{3;0}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \;=\; d^{\,a_1 a_2 a_3}\, E_{3;0}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

Impact factor

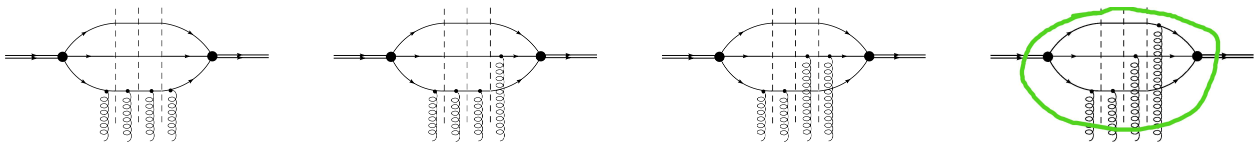
$$E_{3;0}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \;=\; \frac{g^3}{24} \sum_{\sigma} \left[ 2F^{\sigma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) - \sum_{i=1}^{3} F^{\sigma}(\boldsymbol{k}_i, \boldsymbol{k} - \boldsymbol{k}_i, 0) + F^{\sigma}(\boldsymbol{k}, 0, 0) \right]$$

is Bose symmetric: $\quad E_{3;0}\left(\boldsymbol{k}_{\sigma(1)}, \boldsymbol{k}_{\sigma(2)}, \boldsymbol{k}_{\sigma(3)}\right) \;=\; E_{3;0}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ for any $\sigma$

and gauge invariant: $\quad E_{3;0}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \;=\; 0$ for any $\boldsymbol{k}_j \longrightarrow 0.$

# 4-gluon $C$-even baryon impact factor

Topologies:

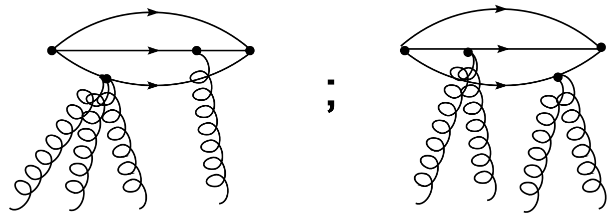


Dipolar terms $D_{4;0}$ found again in $C$-even impact factor

$$B_{4;0} = D_{4;0}^{\{1,2\}} + D_{4;0}^{\{1,3\}} + D_{4;0}^{\{2,3\}} + Q_{4;0}$$

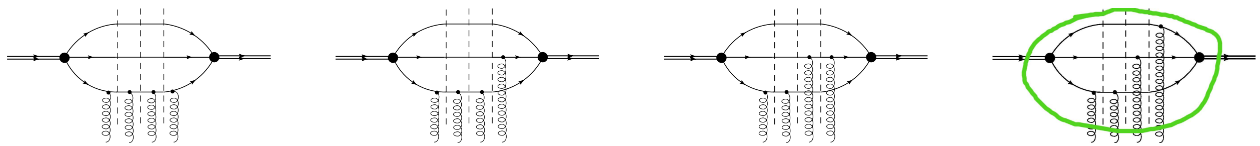
'Dipole-like' components again follow the pattern found for the photon scattering

Reggeizing terms:

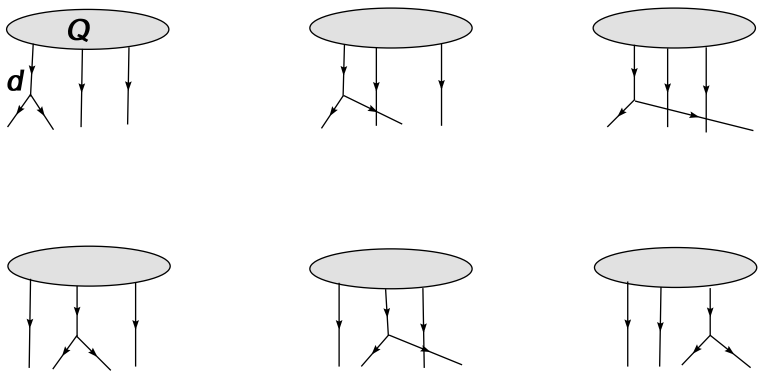


# New structure found in baryon impact factor

All topologies contribute:



$$
\begin{aligned}
Q_{4,0}(1,2,3,4) \quad = \quad & \frac{1}{2}\,g\,\left[d^{\,a_1a_2b}\,d^{\,ba_3a_4} - \frac{1}{3}\,\delta^{\,a_1a_2}\,\delta^{\,a_3a_4}\right]\left[\,E_{3,0}(12,3,4) + E_{3,0}(34,1,2)\,\right] + \\
& \frac{1}{2}\,g\,\left[d^{\,a_1a_3b}\,d^{\,ba_2a_4} - \frac{1}{3}\,\delta^{\,a_1a_3}\,\delta^{\,a_2a_4}\right]\left[\,E_{3,0}(13,2,4) + E_{3,0}(24,1,3)\,\right] + \\
& \frac{1}{2}\,g\,\left[d^{\,a_1a_4b}\,d^{\,ba_2a_3} - \frac{1}{3}\delta^{\,a_1a_4}\,\delta^{\,a_2a_3}\right]\left[\,E_{3,0}(14,2,3) + E_{3,0}(23,1,4)\,\right]
\end{aligned}
$$

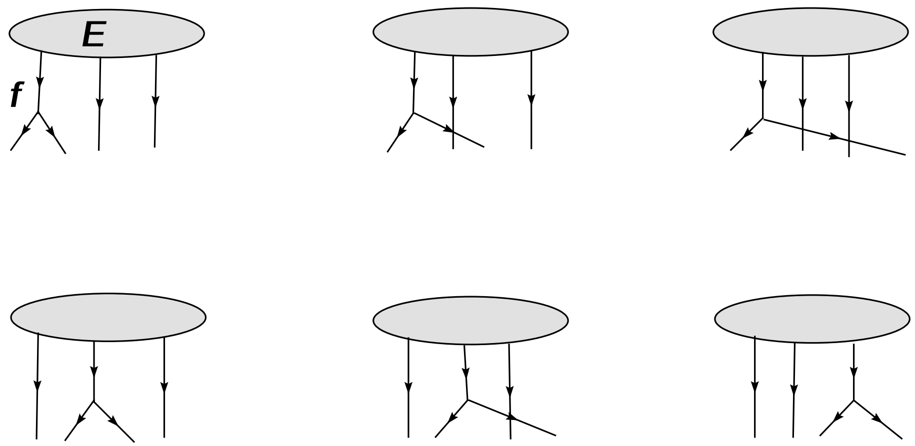



Bose symmetry and gauge invariance: $Q_{4;0}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \; = \; 0$ for any $\boldsymbol{k}_j \longrightarrow 0$

# 4-gluon $C$-odd impact factor

Impact factor suggestive of Reggeizing form

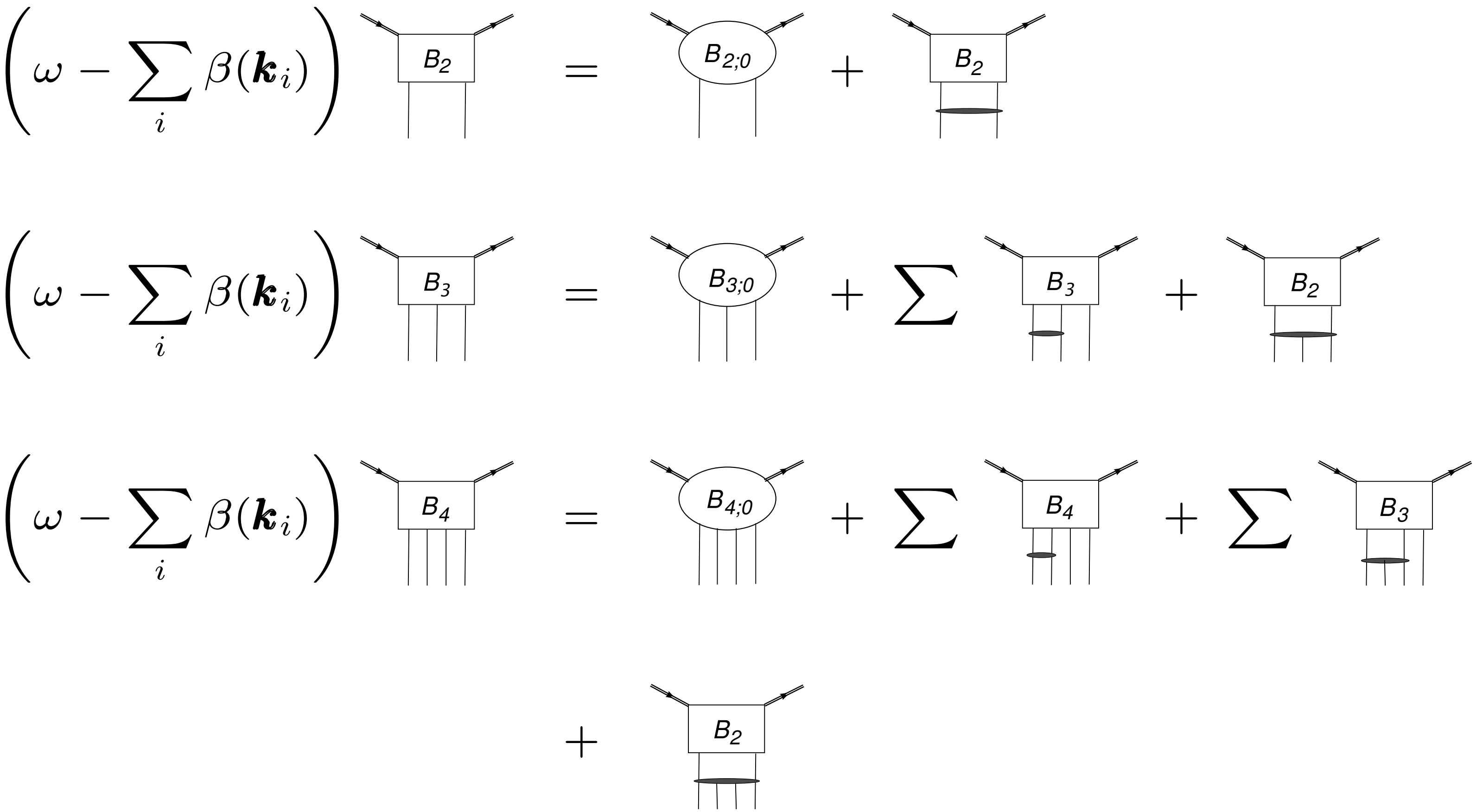
$$\begin{aligned}\tilde{B}_{4;0}(1,2,3,4) \quad &= \quad f^{a_1a_2b}\,d^{ba_3a_4}\,E_{3;0}(12,3,4) + f^{a_1a_3b}\,d^{ba_2a_4}\,E_{3;0}(13,2,4) + \\ &+ \quad f^{a_1a_4b}\,d^{ba_2a_3}\,E_{3;0}(14,2,3) + f^{a_2a_3b}\,d^{ba_1a_4}\,E_{3;0}(23,1,4) + \\ &+ \quad f^{a_2a_4b}\,d^{ba_1a_3}\,E_{3;0}(24,1,3) + f^{a_3a_4b}\,d^{ba_1a_2}\,E_{3;0}(34,1,2)\end{aligned}$$



# Small-$x$ evolution of multiple discontinuities

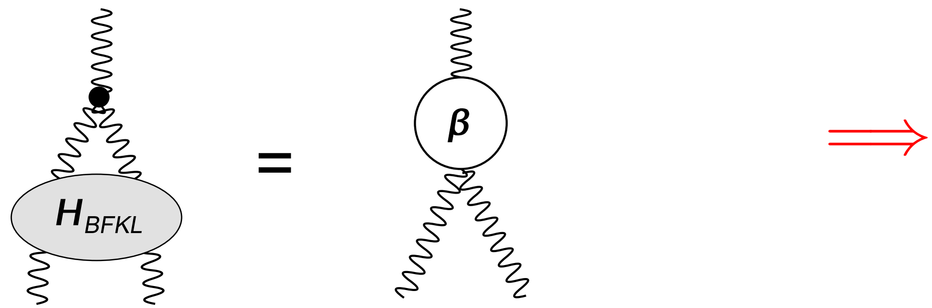
Ingredients of EGGLA formalism [I. Balitsky, V. Fadin, E. Kuraev, L. Lipatov; J. Bartels; J. Kwieciński, M. Praszałowicz, M. Wüsthoff, C. Ewerz]:

Reggeized gluon trajectory $\beta(\boldsymbol{k})$ and $2 \to 2$ BFKL interaction kernel, $2 \to n$ transition kernels

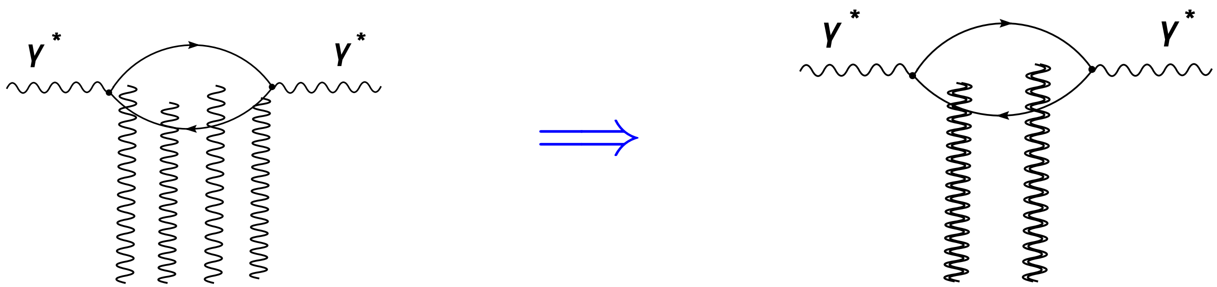


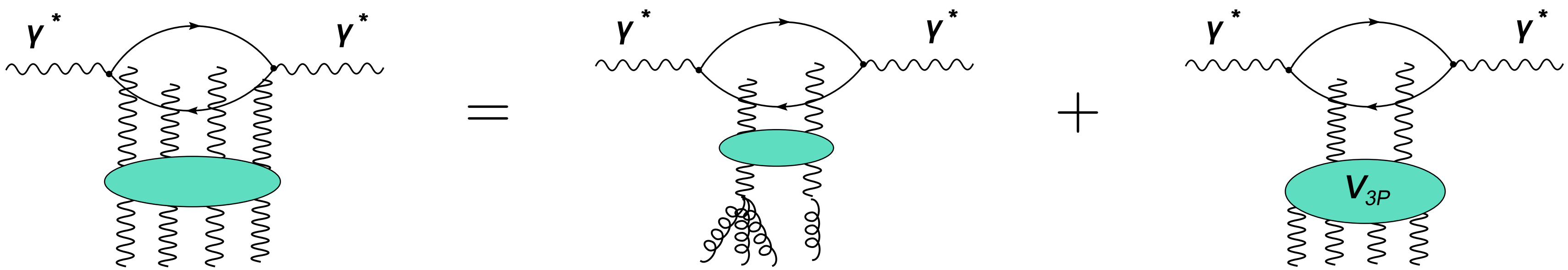
# Bootstrap in gluon trajectory and 3-Pomeron vertex

Bootstrap:




Reduction:



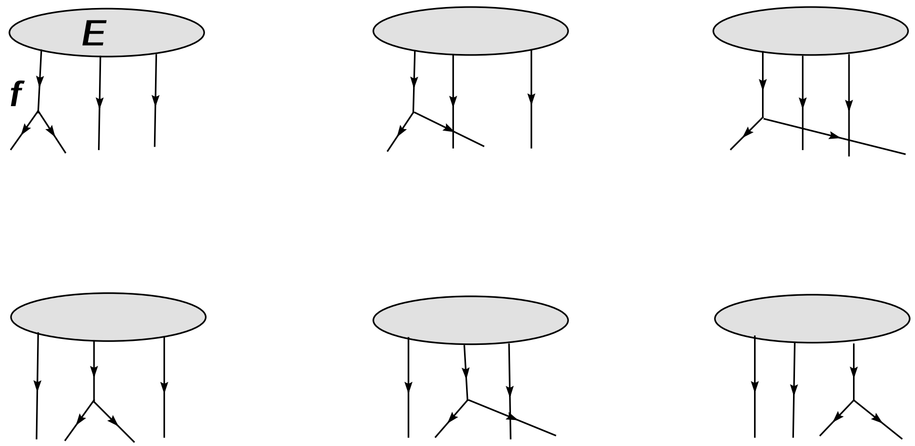

Decomposition: Amplitude = Reggeizing part + Irreducible part

# Solutions: Odderon

$\tilde{B}_3 = E_3$ — solution of BKP equation with initial condition $E_{3;0}$ (Janik–Wosiek solution)

$$\left(\omega - \sum_i \beta(\boldsymbol{k}_i)\right) E_3 \; = \; E_{3;0} + \sum_{(r,s)} K_{2\to 2} \otimes E_3$$

4 gluons: solution saturated by Reggeizing contribution



# Solutions: Pomeron

Two-gluon impact factors $D_2^{\{i,j\}}$ are solutions of BFKL equation with initial conditions $D_{2;0}^{\{i,j\}}$

$$\left(\omega - \sum_{i=1}^{2} \beta(\boldsymbol{k}_i)\right) \; D_2^{\{i,j\}} \;=\; D_{2;0}^{\{i,j\}} \;+\; K_{2\to 2} \otimes D_2^{\{i,j\}}$$

For three gluons — impact factor is also superposition of evolving dipolar pieces:

$$B_3 \;=\; D_3^{\{1,2\}} \;+\; D_3^{\{1,3\}} \;+\; D_3^{\{2,3\}}$$

and solutions for $D_3^{\{i,j\}}$ have Reggeizing form

$$D_3^{\{i,j\}}(1,2,3) \;=\; \frac{1}{2}\, g\, f^{a_1 a_2 a_3} \; \left[ D_2^{\{i,j\}}(12,3) - D_2^{\{i,j\}}(13,2) + D_2^{\{i,j\}}(23,1) \right]$$

For 3 gluons — evolved baryon is a superposition of three possible BFKL solutions

## Solutions for Pomeron: four gluons

Decomposition of 4-gluon evolving baryon impact factor $B_4$

$$B_4 \;=\; \underbrace{D_4^{\{1,2\}} + D_4^{\{1,3\}} + D_4^{\{2,3\}}}_{\text{dipole}-\text{like}} + Q_4$$

Dipole-like pieces $D_4^{\{i,j\}}$ similar to photon case: sum of Reggeizing and irreducible contributions

$$D_4^{\{i,j\}} \;=\; D_4^{\{i,j\}\,;R} + D_4^{\{i,j\}\,;I}$$

Reggeizing contribution preserves the color–momentum structure of the bare impact factor while...

Irreducible contribution

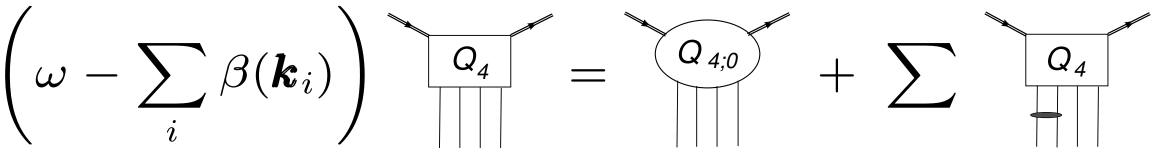
$$D_4^{\{i,j\}\,;I} \;=\; V_{2\to 4} \otimes D_2$$

defines $2 \longrightarrow 4$ transition vertex $V_{2\to 4}$ (triple Pomeron vertex)

# Solutions for Pomeron: four gluons — new piece

Contributions from the dipole-like pieces of are saturated by $D_4^{\{i,j\}}$

Remaining part of linear integral equations $\longrightarrow$ BKP evolution equation for $Q_4$




$Q_4$ is decomposed into Reggezing part $Q_4^R$ and irreducible part $Q_4^I$:

$$Q_4 \;=\; Q_4^R + Q_4^I$$

The Reggezing piece $Q_4^R$ preserves the structure of $Q_{4;0}$, but $E_{3;0} \longrightarrow E_3$

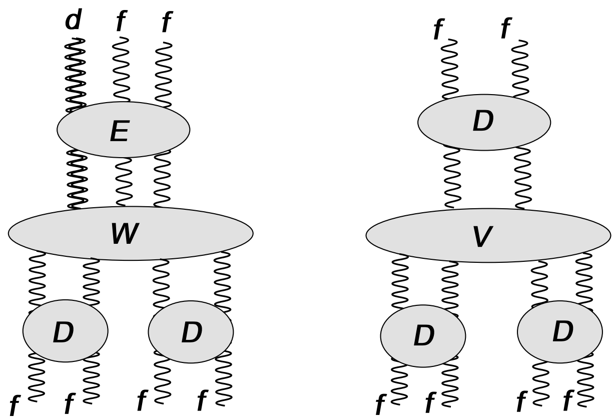
$$\begin{aligned} Q_4^R(1,2,3,4) \;=\; & \frac{1}{2}g\left[d^{\,a_1a_2b}\,d^{\,ba_3a_4} - \frac{1}{3}\delta^{\,a_1a_2}\,\delta^{\,a_3a_4}\right][\,E_3(12,3,4) + E_3(34,1,2)\,] + \\ & \frac{1}{2}g\left[d^{\,a_1a_3b}\,d^{\,ba_2a_4} - \frac{1}{3}\delta^{\,a_1a_3}\,\delta^{\,a_2a_4}\right][\,E_3(13,2,4) + E_3(24,1,3)\,] + \\ & \frac{1}{2}g\left[d^{\,a_1a_4b}\,d^{\,ba_2a_3} - \frac{1}{3}\delta^{\,a_1a_4}\,\delta^{\,a_2a_3}\right][\,E_3(14,2,3) + E_3(23,1,4)\,] \end{aligned}$$

Presence of $C$-even $d$-Reggeon $\longrightarrow$ Similar to Bartels-Lipatov-Vacca Odderon solution

# Transition vertices

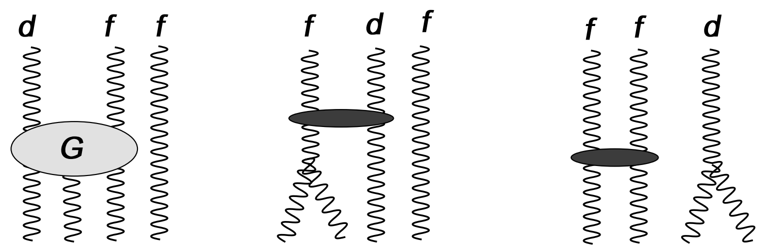
Irreducible part of $Q_4$ defines new $3 \longrightarrow 4$ vertex: $Q_4^I(1,2,3,4) \;=\; (W \;\otimes\; E_3)(1,2,3,4)$

2 triple Pomeron vertices in baryon evolution:




$W$: transition from $(d f\, f)$ BKP Pomeron state $Q$ into $(f\, f\, f\, f)$ BKP Pomeron state
$V$: transition from $(f\, f)$ BFKL Pomeron state $D$ into $(f\, f\, f\, f)$ BKP Pomeron state

$W \;\Rightarrow$




Similar vertex $W$ was found in analysis of jet production [Bartels, Salvadore, Vacca]

## Interpretation

Baryon small $x$ evolution driven by a Hamiltonian $\mathcal{H}$ in the $t$-channel

$$\frac{\partial|\mathcal{B}\rangle}{\partial y} = \mathcal{H}|\mathcal{B}\rangle$$

⟶ Basic quanta: Reggeized gluons with odd $|f\rangle$ and even signatures $|d\rangle$.

⟶ Physical states: multi-Reggeon states in color singlet, gauge invariant, Bose symmetric

⟶ Number of Reggeons is not conserved

Initial condition decomposition

$$|\mathcal{B}\rangle = \overbrace{|\mathcal{D}_{2;0}^{\{1,2\}}\rangle + |\mathcal{D}_{2;0}^{\{1,3\}}\rangle + |\mathcal{D}_{2;0}^{\{2,3\}}\rangle + |\mathcal{Q}_{4;0}\rangle}^{C\ \text{even}} + \overbrace{|\mathcal{E}_{3;0}\rangle}^{C\ \text{odd}}$$

indicates that only single Pomeron (BFKL or BKP) couples to proton (valence d.o.f.)

$$\mathcal{H} = \overbrace{\mathcal{H}_{2\to2}}^{BFKL} + \overbrace{\mathcal{H}_{3\to3}}^{BKP} + \overbrace{\mathcal{H}_{4\to4}}^{BKP} + \overbrace{\mathcal{H}_{2\to4}^{(+)}}^{\text{vertex } V} + \overbrace{\mathcal{H}_{3\to4}^{(+)}}^{\text{vertex } W} + \ldots$$

## Summary

- A model for a proton light cone wave function was constructed: dependent on quark momenta (longitudinal and transverse) and helicities

- Proton (valence) couples to BFKL Pomerons, 3-Reggeon BKP Odderon, and 3-Reggeon BKP Pomeron. No indications of direct two Pomeron coupling

  Proton resembles superposition of color 3 dipoles. . . but there is an extra component

- New triple Pomeron vertex: BKP Pomeron $\longrightarrow$ 2 BFKL Pomerons

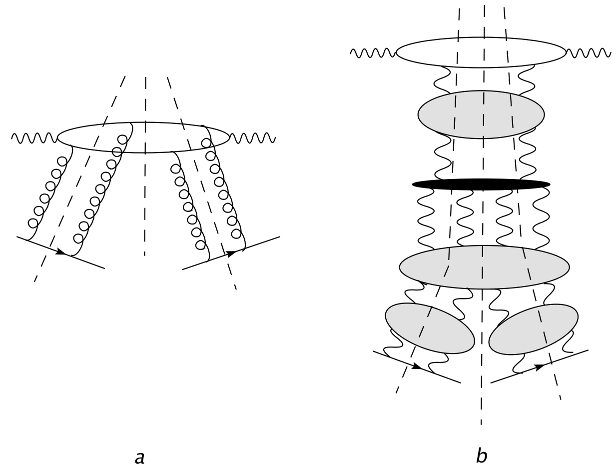
- Unitarisation of scattering amplitudes on valence proton:

  Pomeron loops — not fans!

- Phenomenological applications?

# Backup

# Strategy

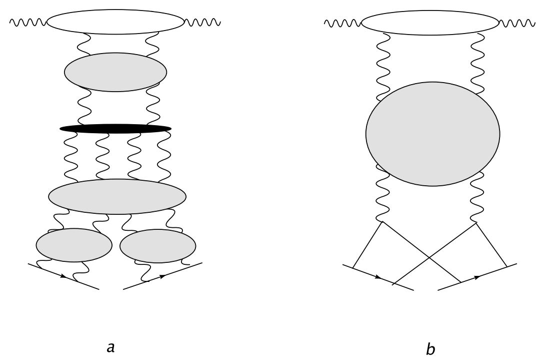



Basic objects: multiple discontinuities

⟶ May be constructed using $M \rightarrow N$ scattering amplitudes

⟶ Obey small $x$ integral evolution equations

⟶ Contain information about full amplitudes



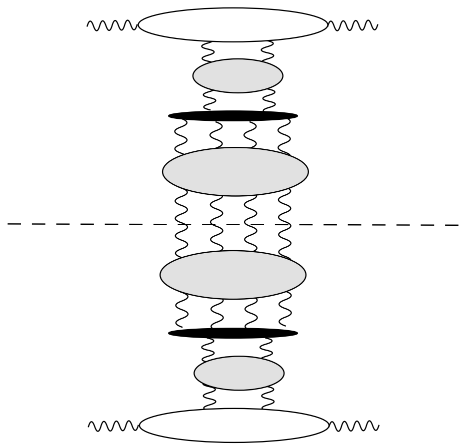

Bootstrap for gluon trajectory

⟶ Reduction of discontinuities to $t$-channel physical states

Bose symmetric, gauge invariant BKP states (e.g. BFKL)

⟶ Isolation of irreducible pieces

⟶ Gauge invariant transition vertices e.g. 3–Pomeron vertex



Regge factorisation

⟶ Physical amplitudes may be built from the gauge invariant states and transition vertices between them

# Baryon wave function: gluon coupling

In high energy limit all diagrams

$$p + \overbrace{g + g + \ldots + g}^{n \text{ gluons}} \longrightarrow u\,u\,d$$

with any $n$ give universal amplitudes with quark momenta $\boldsymbol{p}_i$ evaluated at the vertex

$$\Theta_{\lambda}^{(\lambda_1,\lambda_2)\,\lambda}(\{\alpha_i\},\{\boldsymbol{p}_i\};\,\boldsymbol{P}) = \lambda\,\mathcal{N}\,\frac{2\sqrt{\alpha_1\alpha_2\alpha_3}}{M^2+\boldsymbol{P}^2-\frac{\boldsymbol{p}_1^2}{\alpha_1}-\frac{\boldsymbol{p}_2^2}{\alpha_2}-\frac{\boldsymbol{p}_3^2}{\alpha_3}}\;\delta^{(2)}\Big(\sum \boldsymbol{p}_i-\boldsymbol{P}\Big)\;\times$$

$$\delta_{-\lambda_1,\,\lambda_2}\;\left\{\delta_{\lambda_1,\,\lambda}\left(\frac{p_2}{\alpha_2}-P\right)\left(\frac{p_1}{\alpha_1}-\frac{p_3}{\alpha_3}\right)^{*}+\;\delta_{\lambda_2,\,\lambda}\left(\frac{p_1}{\alpha_1}-P\right)\left(\frac{p_2}{\alpha_2}-\frac{p_3}{\alpha_3}\right)^{*}\right\}^{\mathcal{C}(\lambda)}$$

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$$\Theta_{\lambda}^{(\lambda_1,\lambda_2)\,-\lambda}(\{\alpha_i\},\{\boldsymbol{p}_i\};\,\boldsymbol{P}) = \mathcal{N}\,\frac{2M\sqrt{\alpha_1\alpha_2\alpha_3}}{M^2+\boldsymbol{P}^2-\frac{\boldsymbol{p}_1^2}{\alpha_1}-\frac{\boldsymbol{p}_2^2}{\alpha_2}-\frac{\boldsymbol{p}_3^2}{\alpha_3}}\;\delta^{(2)}\Big(\sum \boldsymbol{p}_i-\boldsymbol{P}\Big)\;\times$$

$$\delta_{-\lambda_1,\,\lambda_2}\;\left\{\delta_{\lambda_1,\,\lambda}\left(\frac{p_3}{\alpha_3}-\frac{p_2}{\alpha_2}\right)+\;\delta_{\lambda_2,\,\lambda}\left(\frac{p_3}{\alpha_3}-\frac{p_1}{\alpha_1}\right)\right\}^{\mathcal{C}(\lambda)}$$

## Baryon wave function in infinite momentum frame

Point-like baryon–quarks vertex is unrealistic — baryon is a bound state

⟶ Severe ultraviolet divergences, on-shell poles in energy denominators

⟶ Need to perform Borel transform (QCD sum rules)

$$\mathcal{B}f(s) \;=\; \lim_{n\to\infty} \frac{s^{n+1}}{n!}\left(-\frac{d}{ds}\right)^{n} f(s), \qquad s\to\infty, \quad s/n \to M^2$$

We apply two independent Borel transforms w.r.t. virtualities of incoming and outgoing baryon

$$\mathcal{B}'\,\mathcal{B}\;f(P^2,P'^2) \quad \text{with} \quad f(P^2,P'^2) \;\sim\; \frac{1}{P^2+\boldsymbol{P}^2-M_X^2}\,\frac{1}{P'^2+\boldsymbol{P}'^2-{M'_X}^2}$$

**[Balitsky, Lipatov]**

$$\frac{1}{P^2+\boldsymbol{P}^2-\frac{\boldsymbol{p}_1^2}{\alpha_1}-\frac{\boldsymbol{p}_2^2}{\alpha_2}-\frac{\boldsymbol{p}_3^2}{\alpha_3}} \quad\longrightarrow\quad \exp\left[-\left(\frac{\boldsymbol{p}_1^2}{\alpha_1}+\frac{\boldsymbol{p}_2^2}{\alpha_2}+\frac{\boldsymbol{p}_3^2}{\alpha_3}-\boldsymbol{P}^2\right)\,/\,M^2\right]$$

# Baryon scattering in position space

$$\mathcal{B} \;=\; \sum_{\text{diagrams}} \int [d^2\boldsymbol{r}_i] \;\; \left[\tilde{\Psi}^{\;\lambda_i}\left(\{\boldsymbol{r}_i\}\right)\right] \;\; \left[\mathcal{C}(\text{diagram}) \;\prod_j \exp\left(-\,i\,\boldsymbol{l}_j\cdot\boldsymbol{r}_j\,\right)\right] \;\; \left[\tilde{\Psi}^{\lambda_i}_{\lambda}\left(\{\boldsymbol{r}_i\}\right)\right]^*$$

$$\mathcal{C}(\text{diagram}) \;\sim\; \varepsilon^{\kappa_1\kappa_2\kappa_3}\left[t^{a_l}t^{a_{l-1}}\ldots t^{a_1}\right]_{\kappa'_1\kappa_1}\;\left[t^{b_m}t^{b_{m-1}}\ldots t^{b_1}\right]_{\kappa'_2\kappa_2}\;\left[t^{c_n}t^{c_{n-1}}\ldots t^{c_1}\right]_{\kappa'_3\kappa_3}\varepsilon^{\kappa'_1\kappa'_2\kappa'_3}$$

Wilson lines

$$W(\boldsymbol{r}) \;=\; \mathcal{P}\;\exp\left(ig\int dr^{\mu}A_{\mu}\right)$$

Color dipole scattering:

$$S(\boldsymbol{r}_1,\boldsymbol{r}_2) \;\sim\; \left\langle\;[W(\boldsymbol{r}_1)]_{\kappa\kappa'}\;[W(\boldsymbol{r}_2)]^*_{\kappa'\kappa}\;\right\rangle$$

Baryon scattering

$$S(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3) \;\sim\; \left\langle\varepsilon^{\kappa_1\kappa_2\kappa_3}\;\;[W(\boldsymbol{r}_1)]^*_{\kappa_1\kappa'_1}\;[W(\boldsymbol{r}_2)]^*_{\kappa_2\kappa'_2}\;[W(\boldsymbol{r}_3)^*]_{\kappa_3\kappa'_3}\;\;\varepsilon^{\kappa'_1\kappa'_2\kappa'_3}\right\rangle$$

# Dipole vs baryon scattering