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The tiny black hole picture of perturbative saturation

1. Saturation in DIS
2. Critical gravitational collapse
3. Saturation/black hole holography

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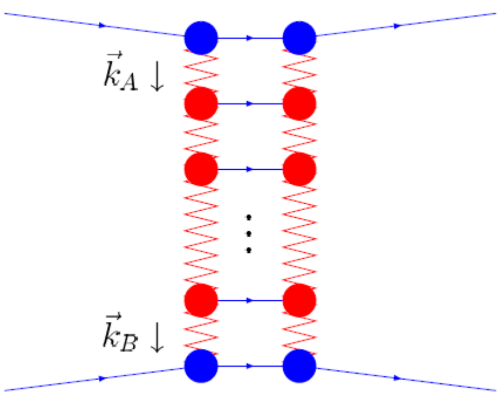
1. Saturation in DIS

Let us start with BFKL linear evolution



We can write the gluon´s Green function as

$$\bar{\varphi}\,(k_A, k_B, \mathrm{Y}) \;=\; \frac{1}{\pi k_A k_B} \int \frac{d\gamma}{2\pi i} \left(\frac{k_A^2}{k_B^2}\right)^{\gamma-\frac{1}{2}} e^{\chi(\gamma)\bar{\alpha}_s \mathrm{Y}}$$

At large energies the saddle point $\gamma = 1/2$ dominates

$$\chi(\gamma) \;\simeq\; 4\log 2 + 14\,\zeta_3 \left(\gamma - \frac{1}{2}\right)^2 + \cdots$$

We obtain

$$\bar{\varphi}\,(k_A, k_B, \mathrm{Y}) \;\simeq\; \frac{1}{2\pi k_A k_B} e^{\Delta \mathrm{Y}} \frac{1}{\sqrt{14\pi\zeta_3\bar{\alpha}_s \mathrm{Y}}} e^{\frac{-t^2}{56\zeta_3\bar{\alpha}_s \mathrm{Y}}}$$ with $t \equiv \log\left(k_A^2/k_B^2\right)$

This corresponds to IR/UV symmetric diffusion in transverse momenta for

$$\Phi\,(k_A, k_B, \mathrm{Y}) \;\equiv\; k_A\, k_B\, \bar{\varphi}\,(k_A, k_B, \mathrm{Y}) \qquad \frac{\partial \Phi}{\partial(\bar{\alpha}_s \mathrm{Y})} \;=\; 4\,\log 2\,\Phi + 14\,\zeta_3 \frac{\partial^2 \Phi}{\partial t^2}$$

$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma) \qquad \gamma \to 1-\gamma$$ invariant

Froissart bound violated by power-like growth

Non-linearities are needed to damp this growth

BK equation is a good candidate:

$$\frac{\partial \Phi\left(k_A, k_B, \mathrm{Y}\right)}{\partial(\bar{\alpha}_s \mathrm{Y})} = -\Phi\left(k_A, k_B, \mathrm{Y}\right)^2 + \int_0^1 \frac{dx}{1-x}\left[\Phi\left(\sqrt{x}k_A, k_B, \mathrm{Y}\right) + \frac{1}{x}\Phi\left(\frac{k_A}{\sqrt{x}}, k_B, \mathrm{Y}\right) - 2\Phi\left(k_A, k_B, \mathrm{Y}\right)\right]$$

Non-linearities can be introduced with weighted diffusion: [Gribov-Levin-Ryskin] [Mueller-Triantafyllopoulos]

The single Pomeron solution

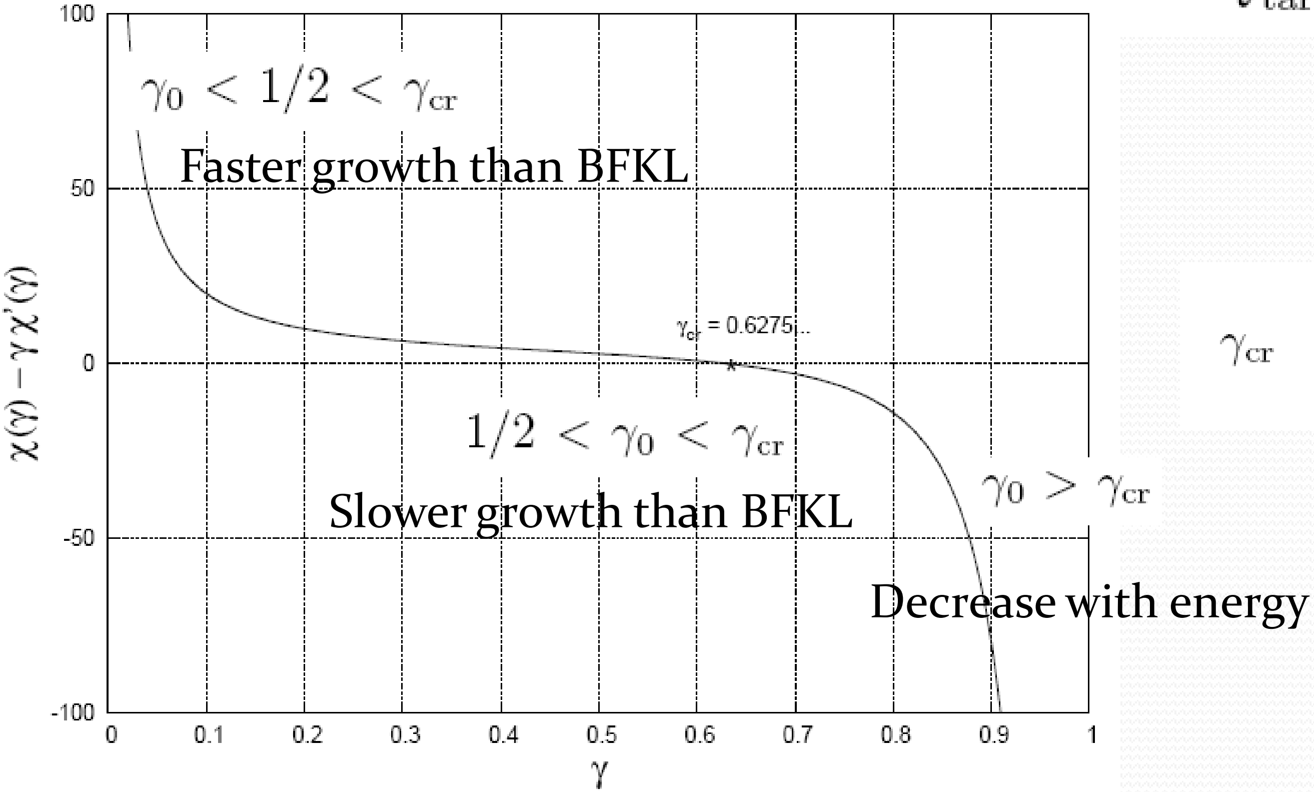
$$\bar{\varphi}\left(Q_{\rm targ}, Q_{\rm proj}, \mathrm{Y}\right) = \frac{1}{\pi Q_{\rm targ}^2}\int \frac{d\gamma}{2\pi i}\left(\frac{Q_{\rm targ}^2}{Q_{\rm proj}^2}\right)^{\gamma} e^{\chi(\gamma)\bar{\alpha}_s \mathrm{Y}}$$

is forced to have a different saddle point

$$\chi'(\gamma_0)\bar{\alpha}_s \mathrm{Y} + \log\left(\frac{Q_{\rm targ}^2}{Q_0^2}\right) = 0$$

$$\chi(\gamma) \simeq \chi(\gamma_0) + \chi'(\gamma_0)(\gamma-\gamma_0) + \frac{1}{2}\chi''(\gamma_0)(\gamma-\gamma_0)^2 + \cdots$$

$$\bar{\varphi}\left(Q_{\rm targ}, Q_{\rm proj}, {\rm Y}\right) \simeq e^{\gamma_0 t_0 + \bar{\alpha}_s {\rm Y}(\chi(\gamma_0) - \gamma_0 \chi'(\gamma_0))} \frac{e^{\frac{-t_0^2}{2\chi''(\gamma_0)\bar{\alpha}_s {\rm Y}}}}{\pi Q_{\rm targ}^2 \sqrt{\chi''(\gamma_0) 2\pi \bar{\alpha}_s {\rm Y}}}$$



$$\gamma_{\rm cr} = \frac{\chi(\gamma_{\rm cr})}{\chi'(\gamma_{\rm cr})} \simeq 0.6275...$$

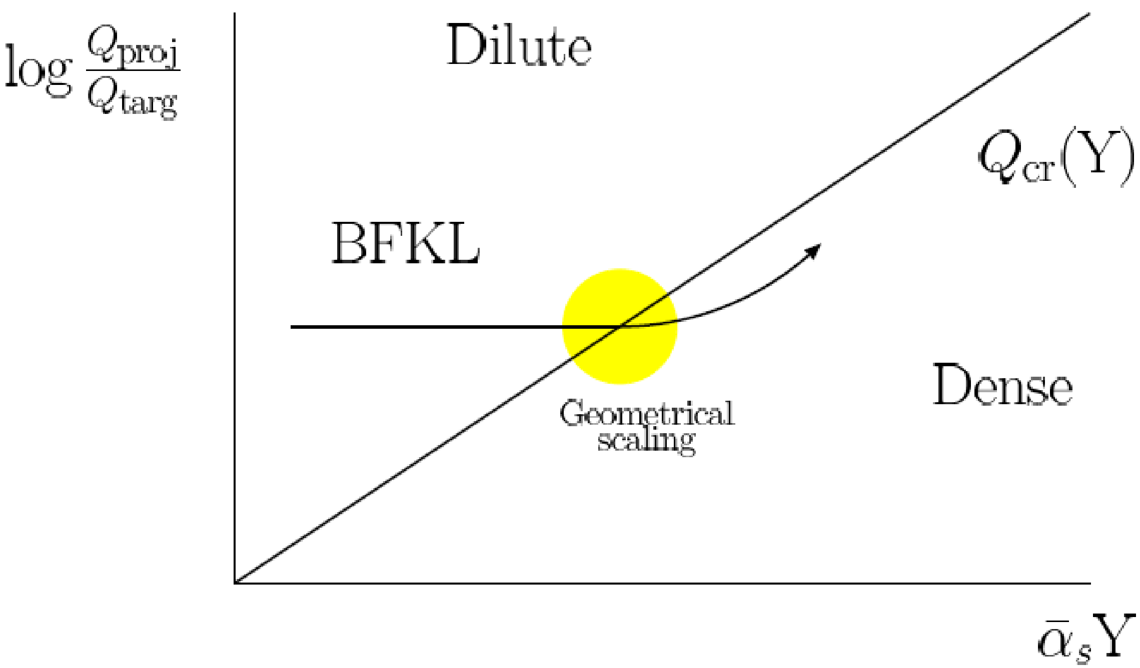
Exactly for $\gamma_0 = \gamma_{\rm cr}$ there is no growth with energy

$$\bar{\varphi}\left(Q_{\rm targ}, Q_{\rm proj}, {\rm Y}\right) \simeq \left(\frac{Q_{\rm cr}({\rm Y})}{Q_{\rm proj}}\right)^{2\gamma_{\rm cr}} \frac{e^{\frac{-t_{\rm cr}^2}{2\chi''(\gamma_{\rm cr})\bar{\alpha}_s {\rm Y}}}}{\pi Q_{\rm targ}^2 \sqrt{\chi''(\gamma_{\rm cr}) 2\pi \bar{\alpha}_s {\rm Y}}}$$

$$Q_{\rm cr}({\rm Y}) \;=\; Q_{\rm targ} \exp\left[\frac{\chi'(\gamma_{\rm cr})}{2}\bar{\alpha}_s {\rm Y}\right]$$ is a critical line

Solution invariant under geometrical scaling:

$$\bar{\alpha}_s {\rm Y} \;\rightarrow\; \bar{\alpha}_s {\rm Y} + \log\lambda,$$
$$\frac{Q_{\rm proj}}{Q_{\rm targ}} \;\rightarrow\; \frac{Q_{\rm proj}}{Q_{\rm targ}} \lambda^{\frac{\chi'(\gamma_{\rm cr})}{2}}.$$



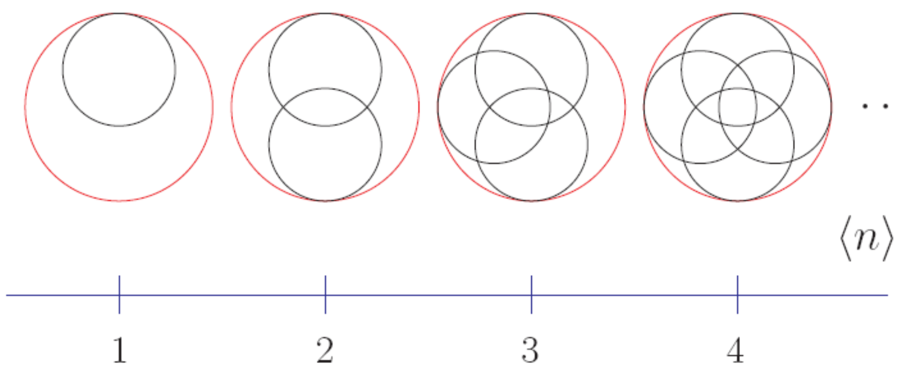
with critical exponent

$$\chi'(\gamma_{\rm cr})/2 \simeq 2.4417\ldots$$

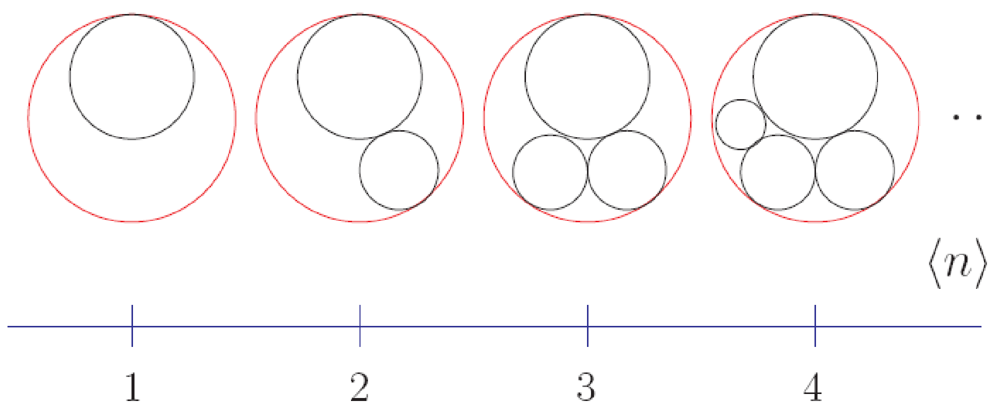
for the crossover dilute-dense transition

Main features of saturation:

1. Dilute/dense transition
2. Scaling symmetry
3. Critical exponent 2.44
4. IR/UV competition



$$\mathcal{T}_{\rm cr} \;=\; \mathcal{T}_{\rm targ}\exp\left[-\frac{\chi'(\gamma_{\rm cr})}{2}\bar{\alpha}_s \mathrm{Y}\right]$$



2. Critical gravitational collapse

We have studied the gravitational collapse of a perfect fluid in any dimension

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p\, g_{\mu\nu}$$

With barotropic equation of state: $p = k\,\rho, \qquad 0 \leq k \leq 1.$

At initial time a density of matter is distributed in the radial coordinate r

There is spherical symmetry to avoid gravitational waves:

$$ds^2 = -\alpha(t,r)^2 dt^2 + a(t,r)^2 dr^2 + R(t,r)^2 d\Omega^2_{d-2}$$

This type of collapse was studied exactly by Choptuik in a classical work in numerical relativity.

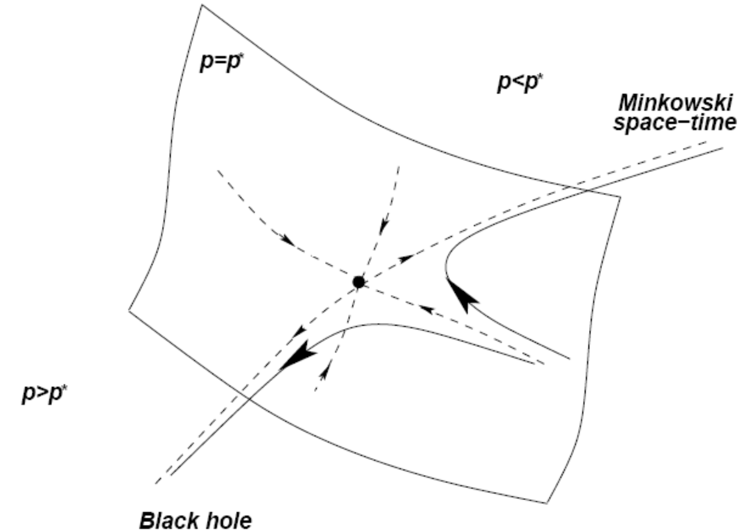
For a generic initial density, parametrized by p, there is no collapse

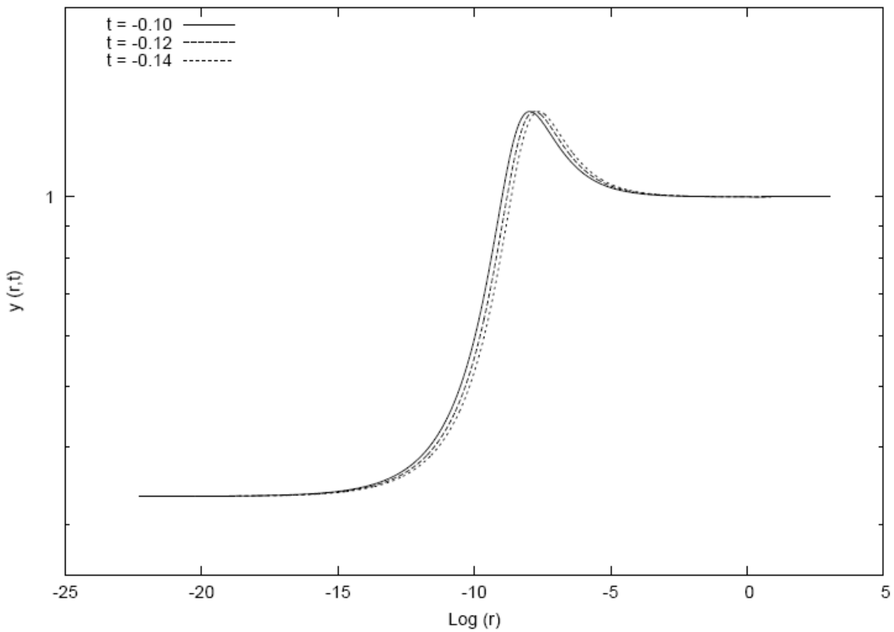
For critical initial density, p*, a small fraction of matter goes through a region dominated by a continuous self-similar scaling law and forms a tiny black hole

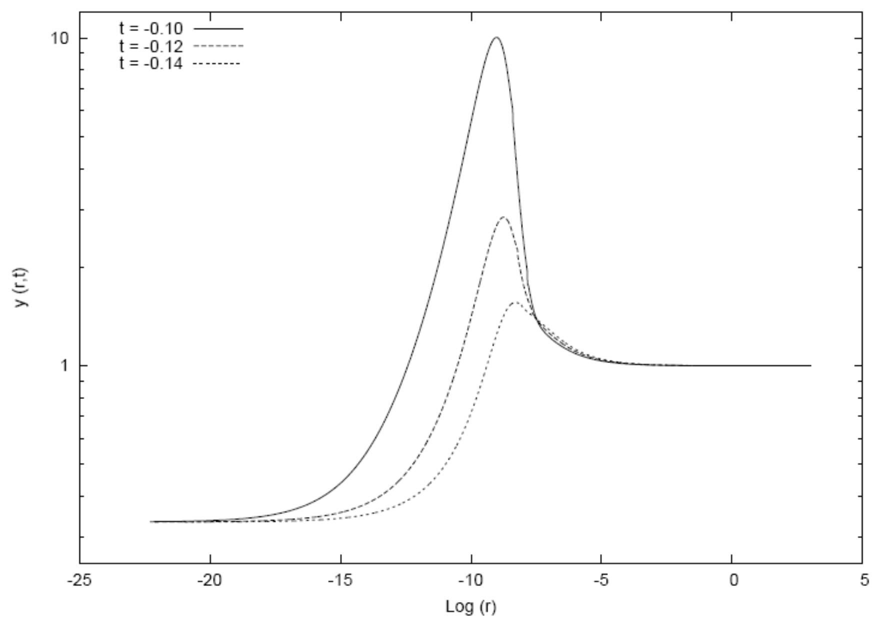
The size of this black hole scales with the formula $\boxed{r_{\mathrm{BH}} \sim \ell_0 \left|p - p^*\right|^{1/\lambda_{\mathrm{BH}}}}$

Our approach for any dimension is more modest

We impose CSS in Einstein´s equations, this is our critical solution which only depends on the variable z=r/t: Z(r,t)=Z(z)







Then we look for an inestable mode in a Liapunov expansion:

$$Z(\tau, z) = Z(z)\left[1 + \epsilon\, e^{\lambda\tau}\, Z_1(z) + \ldots\right]$$

This mode breaks CSS

The Liapunov´s mode coincides with Choptuik´s critical exponent

$$r_{\mathrm{BH}} \sim \ell_0 |p - p^*|^{1/\lambda_{\mathrm{BH}}}$$

The one of interest to us is the case of conformal fluid and dimension five.

$$k = 1/4, \ \lambda = 2.58$$

Main features of critical gravitational collapse:

1. Flat/black hole transition
2. Scaling symmetry
3. Critical exponent 2.58
4. Gravity/kinetic competition

k	$\lambda_{d=4}$	$\lambda_{d=5}$	$\lambda_{d=6}$	$\lambda_{d=7}$
0.01	8.747	4.435	3.453	3.026
0.02	8.140	4.288	3.376	2.974
0.03	7.617	4.152	3.302	2.924
0.04	7.163	4.027	3.233	2.876
0.05	6.764	3.911	3.169	2.831
0.06	6.412	3.804	3.107	2.788
0.07	6.099	3.703	3.049	2.746
0.08	5.818	3.609	2.993	2.706
0.09	5.565	3.521	2.940	2.668
0.10	5.334	3.438	2.890	2.631
0.11	5.124	3.360	2.841	2.595
0.12	4.932	3.286	2.795	2.561
0.13	4.756	3.216	2.751	2.527
0.14	4.593	3.149	2.708	2.494
0.15	4.442	3.086	2.667	2.464
0.16	4.301	3.026	2.627	2.433
0.17	4.170	2.968	2.589	2.414
0.18	4.048	2.913	2.552	2.377
0.19	3.933	2.860	2.517	2.348
0.20	3.825	2.809	2.482	2.321
0.21	3.723	2.760	2.449	2.297
0.22	3.627	2.713	2.417	2.272
0.23	3.536	2.668	2.386	2.246
0.24	3.449	2.625	2.355	2.224
0.25	3.367	2.583	2.325	2.202

3. Saturation/Black hole holography

4d Perturbative QCD

1. Dilute/dense transition
2. Geometric scaling
3. Critical exponent 2.44
4. IR/UV competition

5d Tiny Black hole

1. Flat/black hole transition
2. CSS
3. Critical exponent 2.58
4. Gravity/kinetic competition