Parton Distributions, and QCD at LHCb

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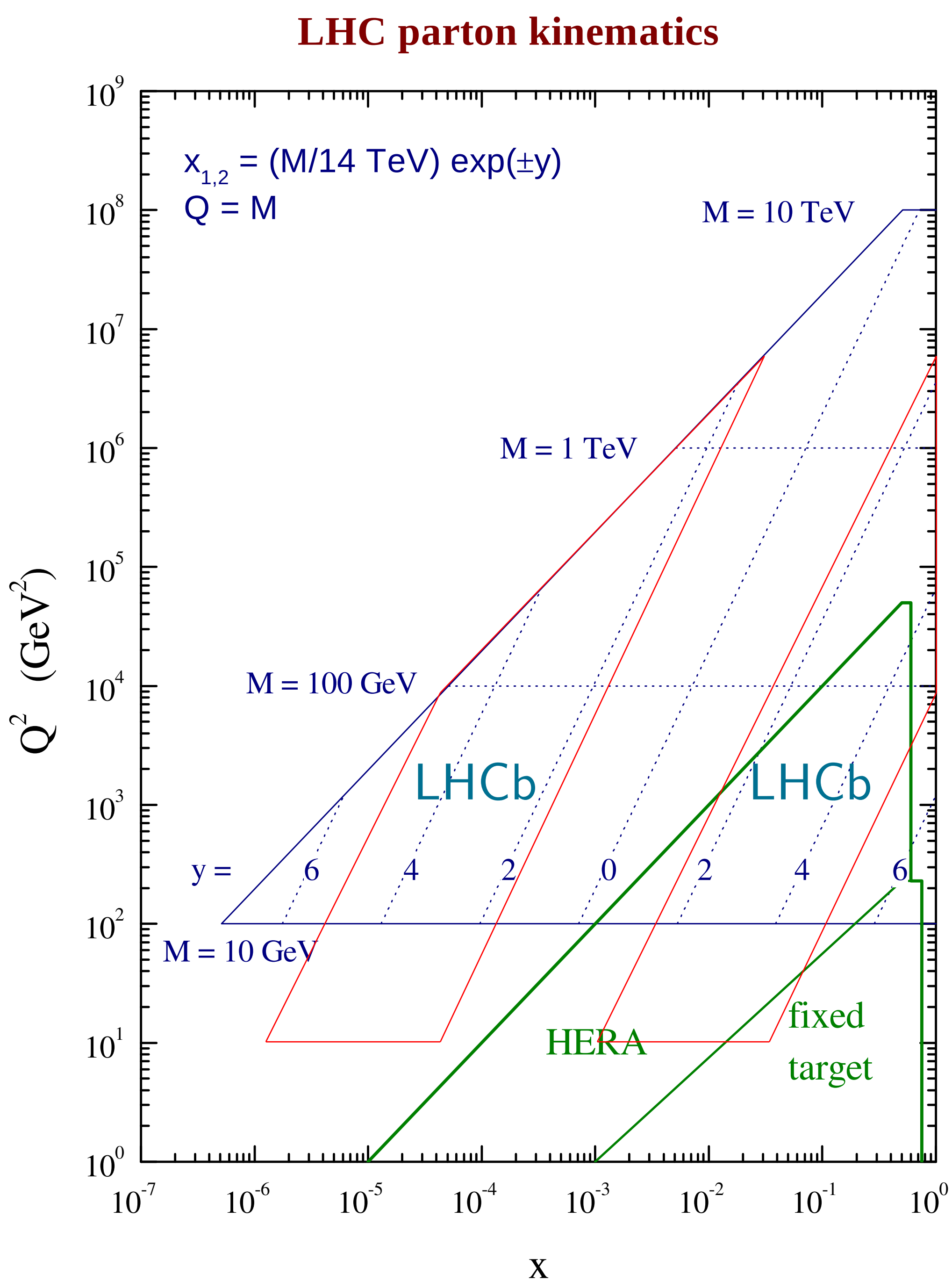
LHC **Physics**

The kinematic range for particle production at the LHC is shown.

$$x_{1,2} = x_0 \exp(\pm y), \quad x_0 = \frac{M}{\sqrt{s}}.$$

Smallish $x \sim 0.001 - 0.01$ parton distributions therefore vital for understanding the standard production processes at the LHC.

However, even smaller (and higher) x required when one moves away from zero rapidity, e.g. when calculating total cross-sections.



Particular Importance of LHCb

Cross-Section Ratios

Since $x_1 > x_2$,particularly at high y, and sea quarks die away at high x we assume

$$q_1(x_1)\bar{q}_2(x_2) + \bar{q}_2(x_1)q_1(x_1) \approx q_1(x_1)\bar{q}_2(x_2).$$

Also at small x_2 we assume that $\bar{u}(x_2) = \bar{d}(x_2)$. Very good for $x \leq 0.01$ for MSTW partons.

Will also assume cross-sections dominated by up and down contributions. Reasonable for obtaining general results, but not always all details.

For Z and $\gamma^\star$ the boson is essentially completely reconstructed.

For $W^\pm$ only one charged lepton is seen.

Defining angle of lepton in W rest frame

$$\cos^2\theta^* = 1 - 4E_T^2/M_W^2 \quad \rightarrow \quad y_{lep} = y_W \pm 1/2\log((1+\cos\theta^*)/(1-\cos\theta^*))$$

If $p_T = 30\text{GeV}$ – $y_{lep} = y_W \pm 0.8$, so $y_W = 1.0$ at minimum.

The appropriate ratios are

$$R_{Z/W} = \frac{\sigma(Z)}{\sigma(W^+) + \sigma(W^-)} \simeq \frac{Au(\tilde{x}_1)\bar{u}(\tilde{x}_2) + Bd(\tilde{x}_1)\bar{d}(\tilde{x}_2)}{u(x_1)\bar{d}(x_2) + d(x_1)\bar{u}(x_2)} \simeq \frac{Au(\tilde{x}_1) + Bd(\tilde{x}_1)}{u(x_1) + d(x_1),}$$

Where we have used our approximations.

This is very precisely predicted, but is equal to A plus small corrections.

$$A_{\pm} = \frac{(\sigma(W^+) - \sigma(W^-))}{(\sigma(W^+) + \sigma(W^-))} \simeq \frac{u(x_1)\bar{u}(x_2) - d(x_1)\bar{d}(x_2)}{u(x_1)\bar{d}(x_2) + d(x_1)\bar{u}(x_2)} \simeq \frac{u_V(x_1) - d_V(x_1)}{u(x_1) + d(x_1)}$$

so is a good test of valence quarks.

$$R_{\pm} = \frac{\sigma(W^-)}{\sigma(W^+)} \simeq \frac{d(x_1)\bar{d}(x_2)}{u(x_1)\bar{d}(x_2)} \simeq \frac{d(x_1)}{u(x_1)},$$

The simplest expression.

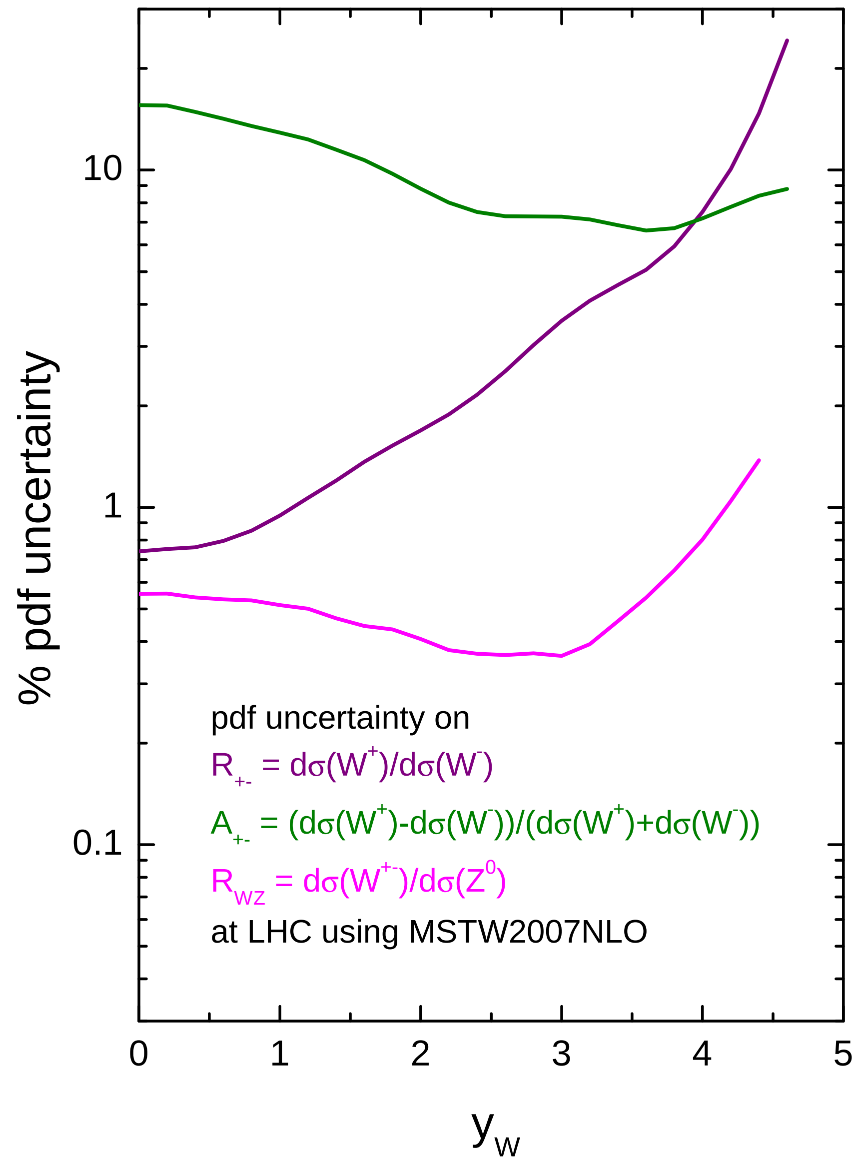
In the later cases minimum $x_1 = 0.015$. For Z minimum $x_1 = 0.039$.

Uncertainty on $R_{Z/W}$ is very small. Parton combinations highly correlated.

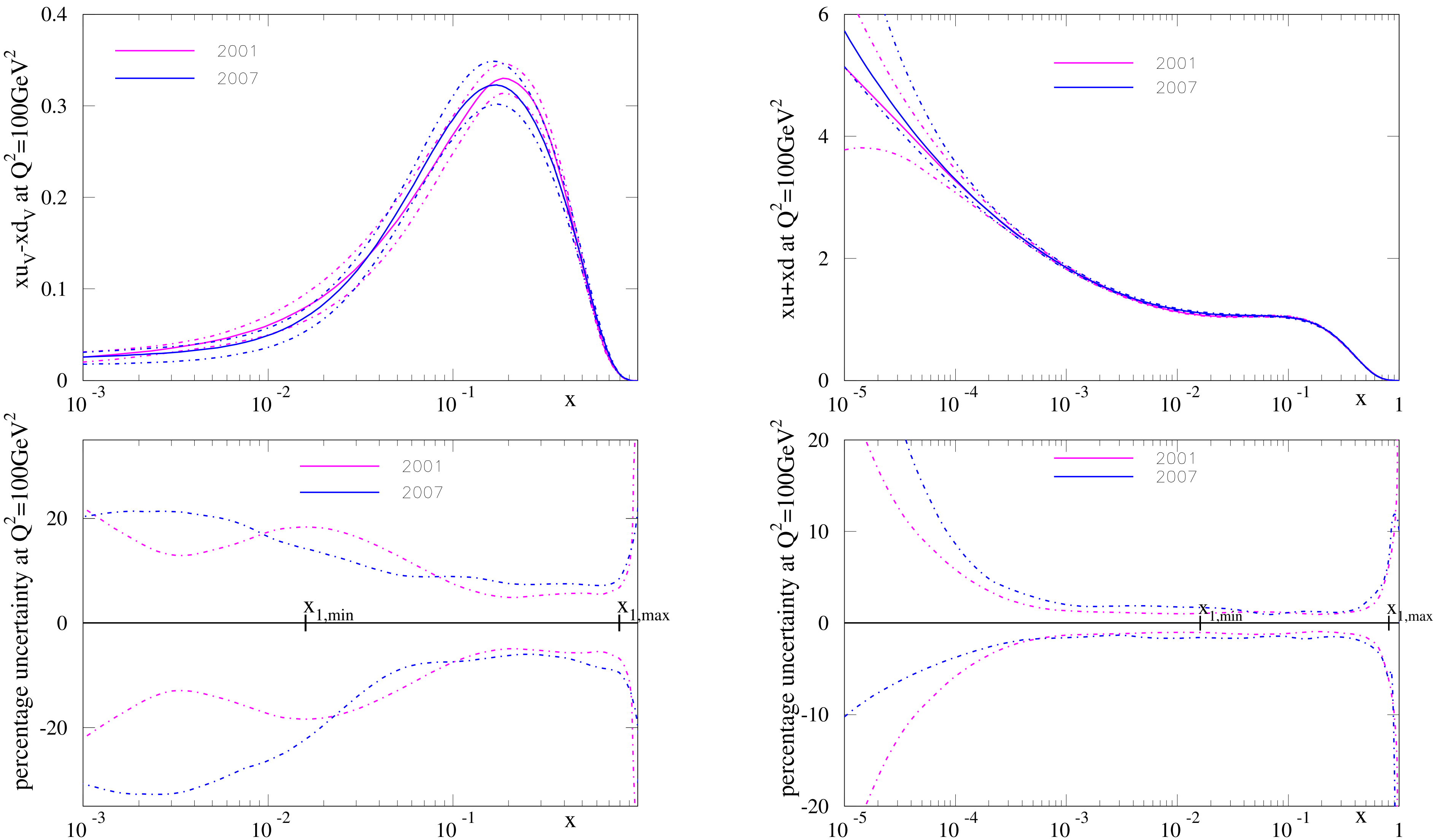
Assumes $\bar{u}(x_2) \approx \bar{d}(x_2)$. Easily checks if this is true.

Uncertainty on $A_{\pm}$ not strongly y-dependent.

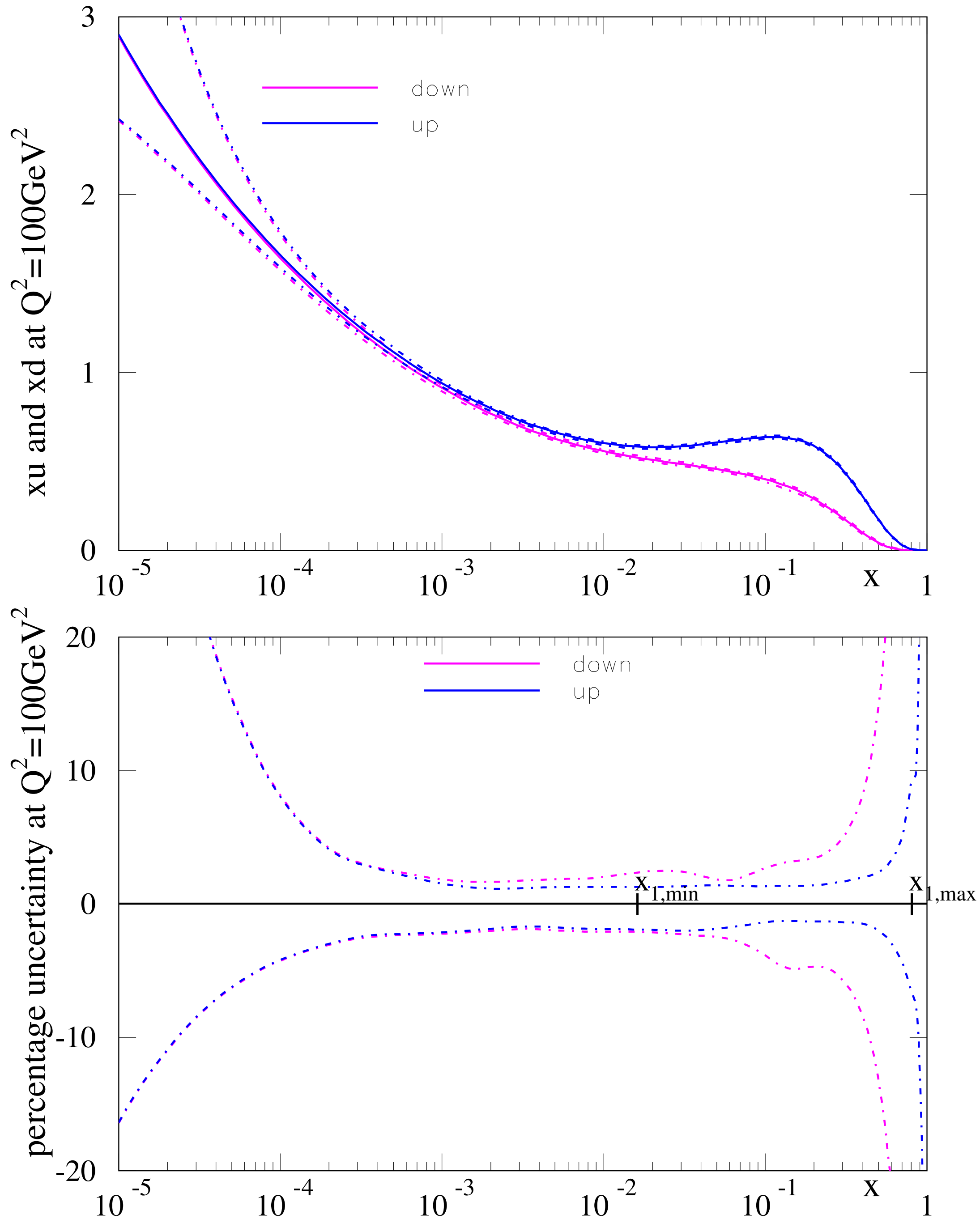
Uncertainty on $R_{\pm}$ increases strongly at high y.



Uncertainty on $A_{\pm}$ dominated by that on $u_V(x) - d_V(x)$. However, at very high y $A_{\pm} \rightarrow u_V/u_V$.



Uncertainty on $R_{\pm}$ dominated by that on $d(x)$, which at all $x \geq 0.05$ is much less than that on $u(x)$.



Total Cross-Sections

The different combinations are

$$\sigma_Z \simeq (v_u^2 + a_u^2) \times u(\tilde{x}_1)\bar{u}(\tilde{x}_2) + (v_d^2 + a_d^2) \times d(\tilde{x}_1)\bar{d}(\tilde{x}_2)$$

where $v_d > v_d$, so down contribution slightly enhanced

$$\sigma(W^+) \simeq u(x_1)\bar{d}(x_2)$$

so probes up valence and down sea

$$\sigma(W^-) \simeq d(x_1)\bar{u}(x_2),$$

sensitive to $d_V(x)$.

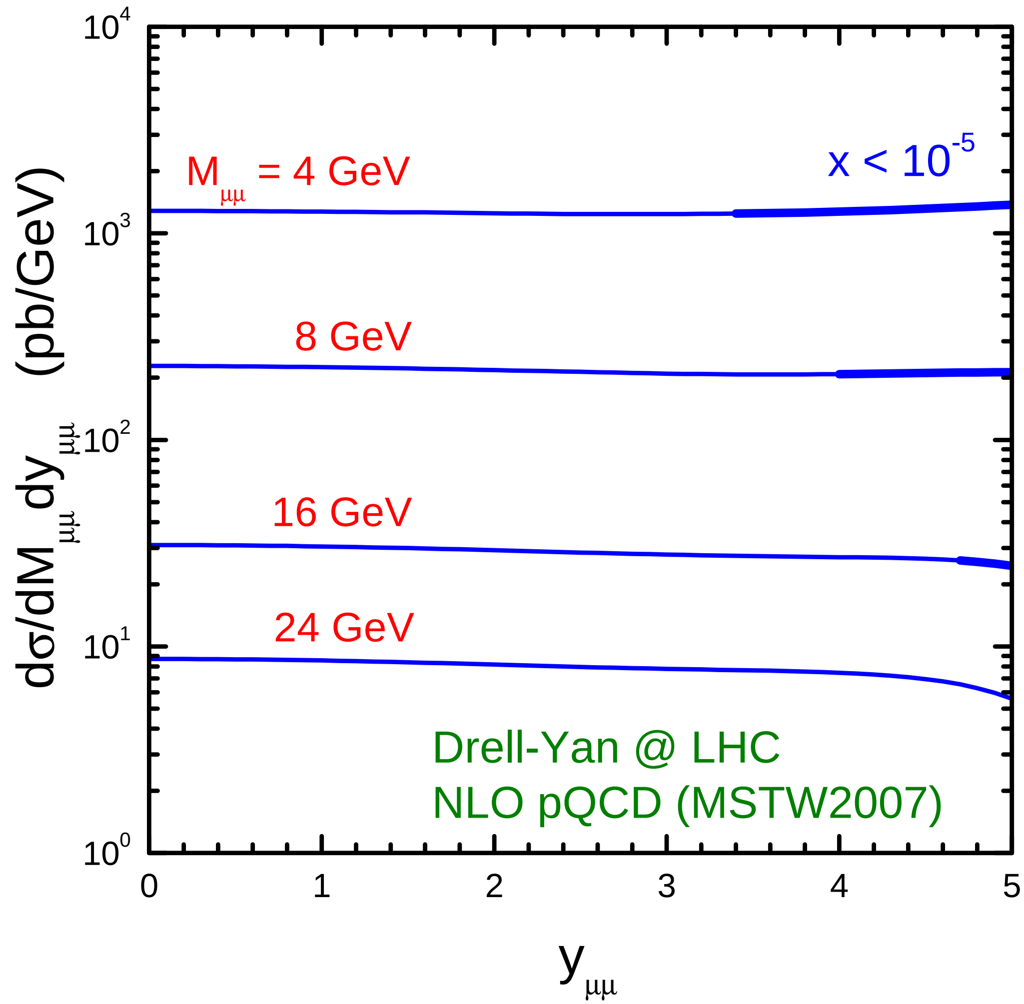
There is also Drell-Yan production of dimuon pairs from virtual photons

$$\sigma_{\gamma^\star} \simeq 4/9\, u(x_1)\bar{u}(x_2) + 1/9\, d(x_1)\bar{d}(x_2)$$

where $x_{1,2} = (M_\gamma/14,000)\exp(\pm y)$. If $M_\gamma \leq 20\mathrm{GeV}$ x_2 can be ≤ 0.00001 and x_1 stays away from the valence uncertainty at high x.

Possible to get to very low values of x at the LHC, particularly LHCb.

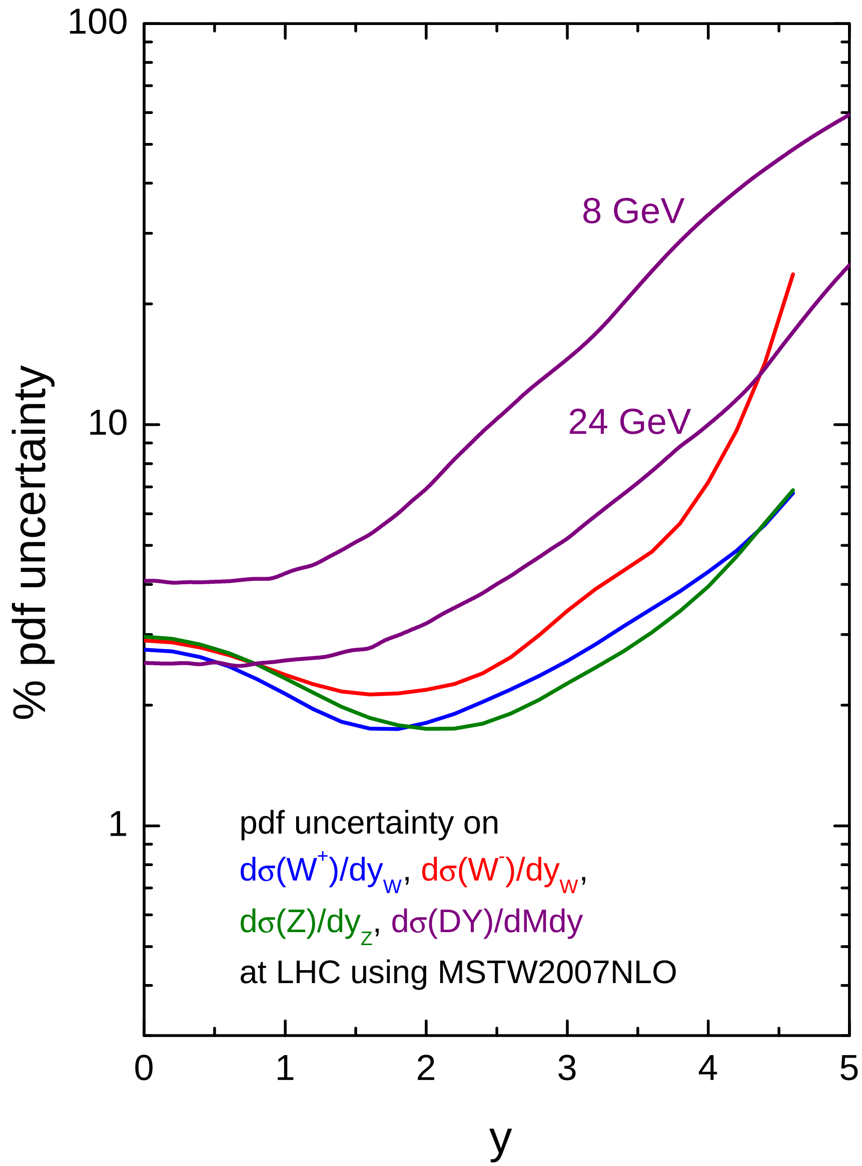
Can probe below $x = 10^{-5}$ - beyond range tested at HERA.



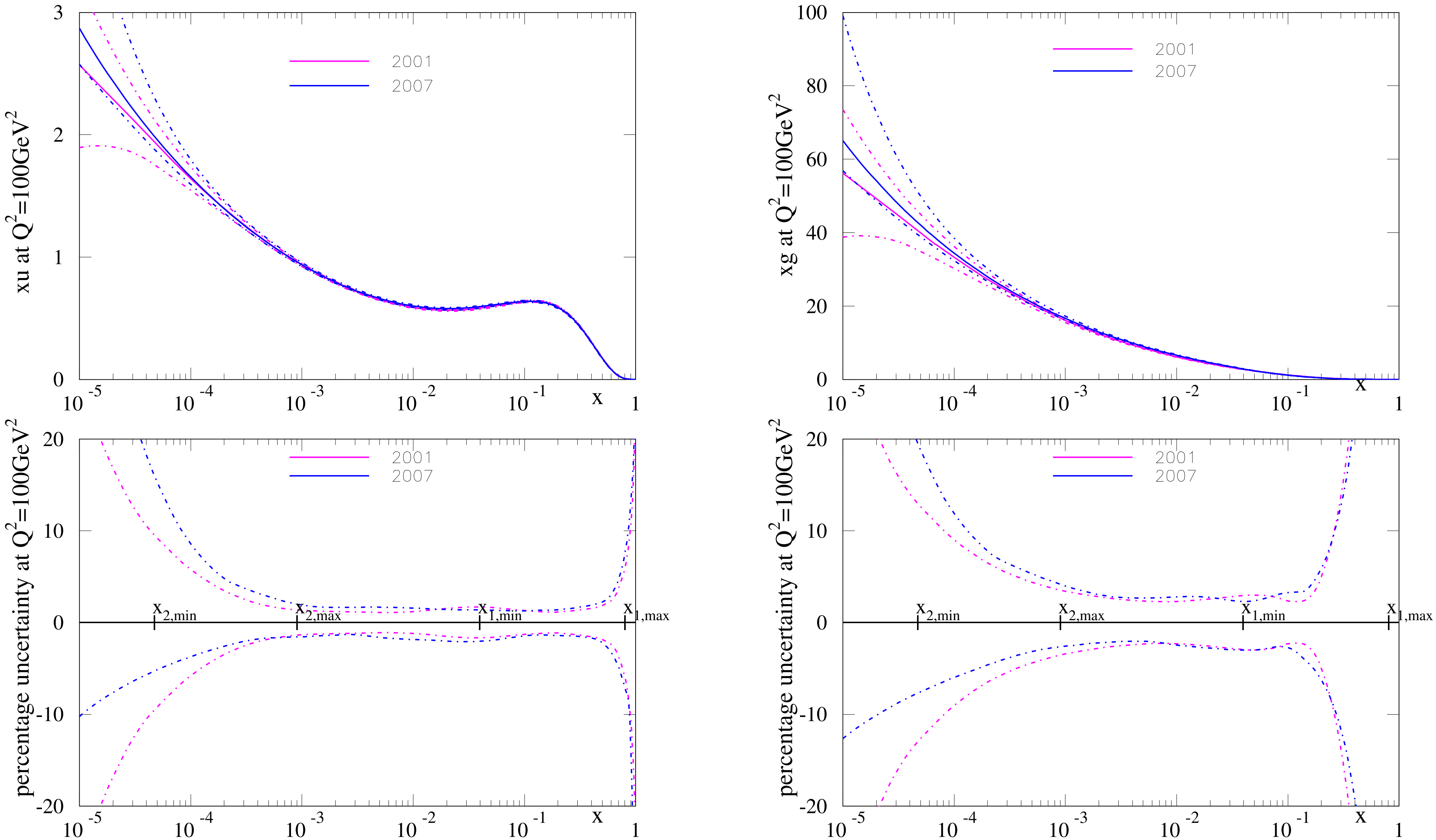
Uncertainty on $\sigma(Z)$ and $\sigma(W^+)$ grows at high rapidity. Converges – both dominated by $u(x_1)\bar{u}(x_2)$ at very high y.

Uncertainty on $\sigma(W^-)$ grows more quickly at very high y.

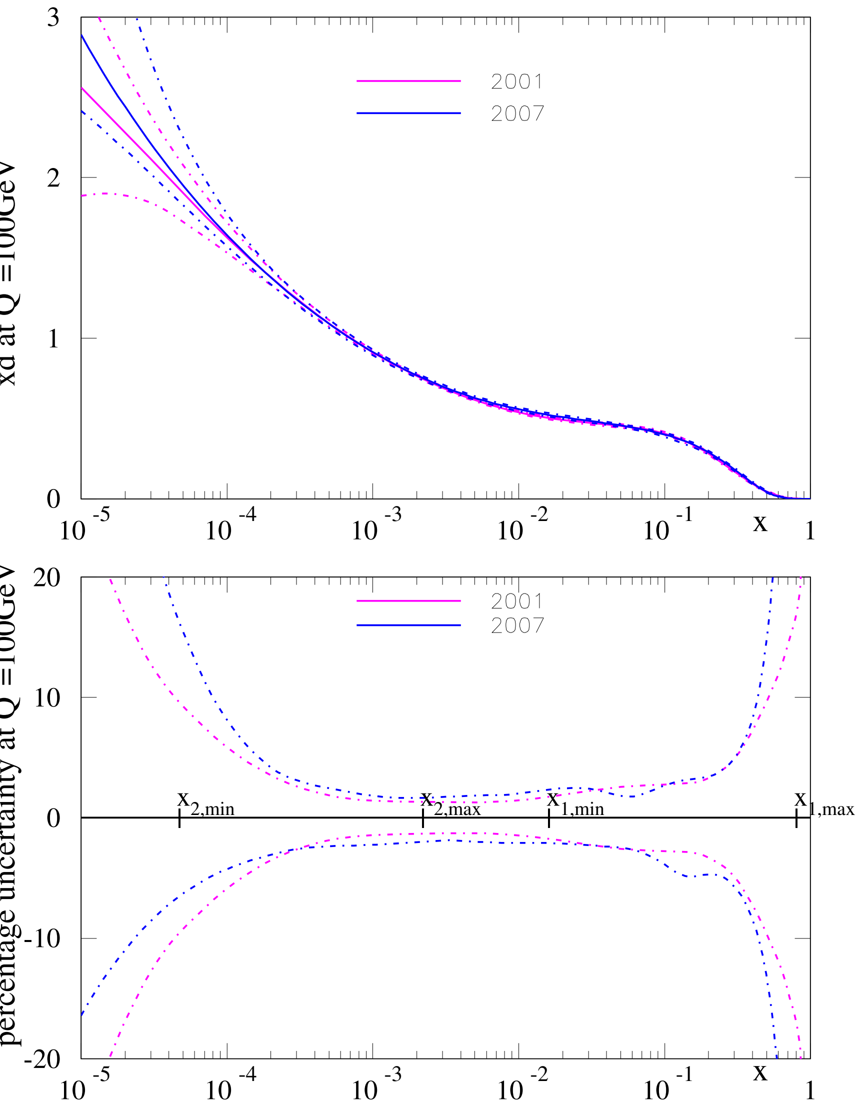
Uncertainty on $\sigma(\gamma^\star)$ is greatest as y increases.

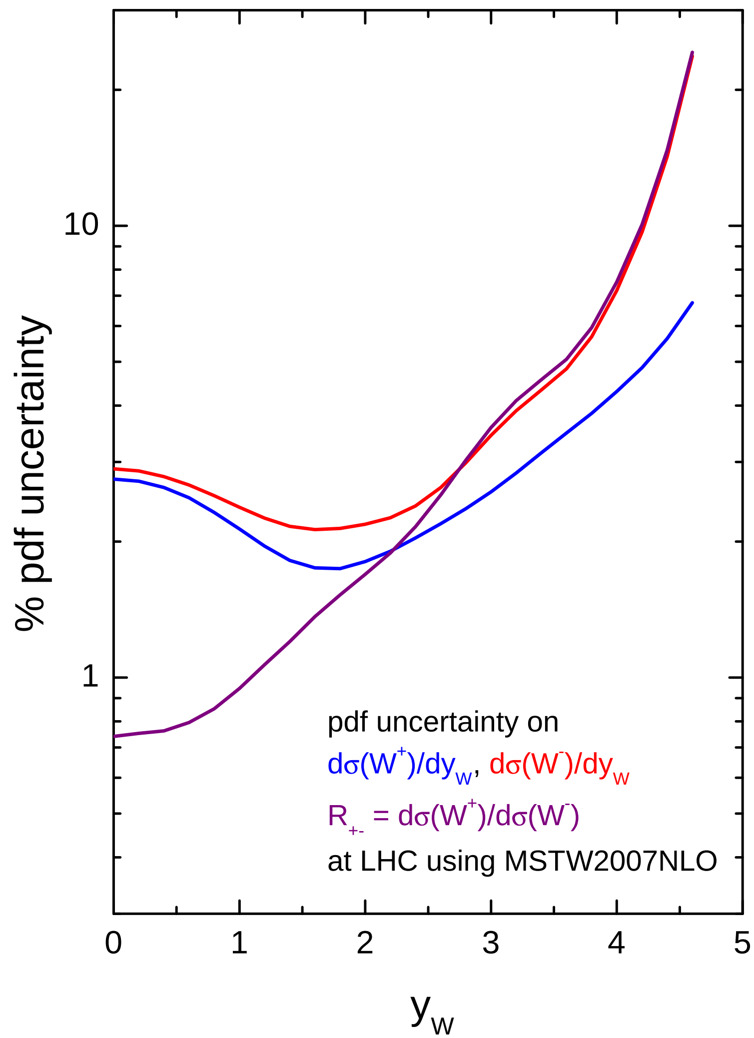


Uncertainty on $\sigma(Z)$ and $\sigma(W^+)$ dominated at high y by sea quark small-x uncertainty rather than high-x $u_V(x)$. Related to evolution and gluon distribution.

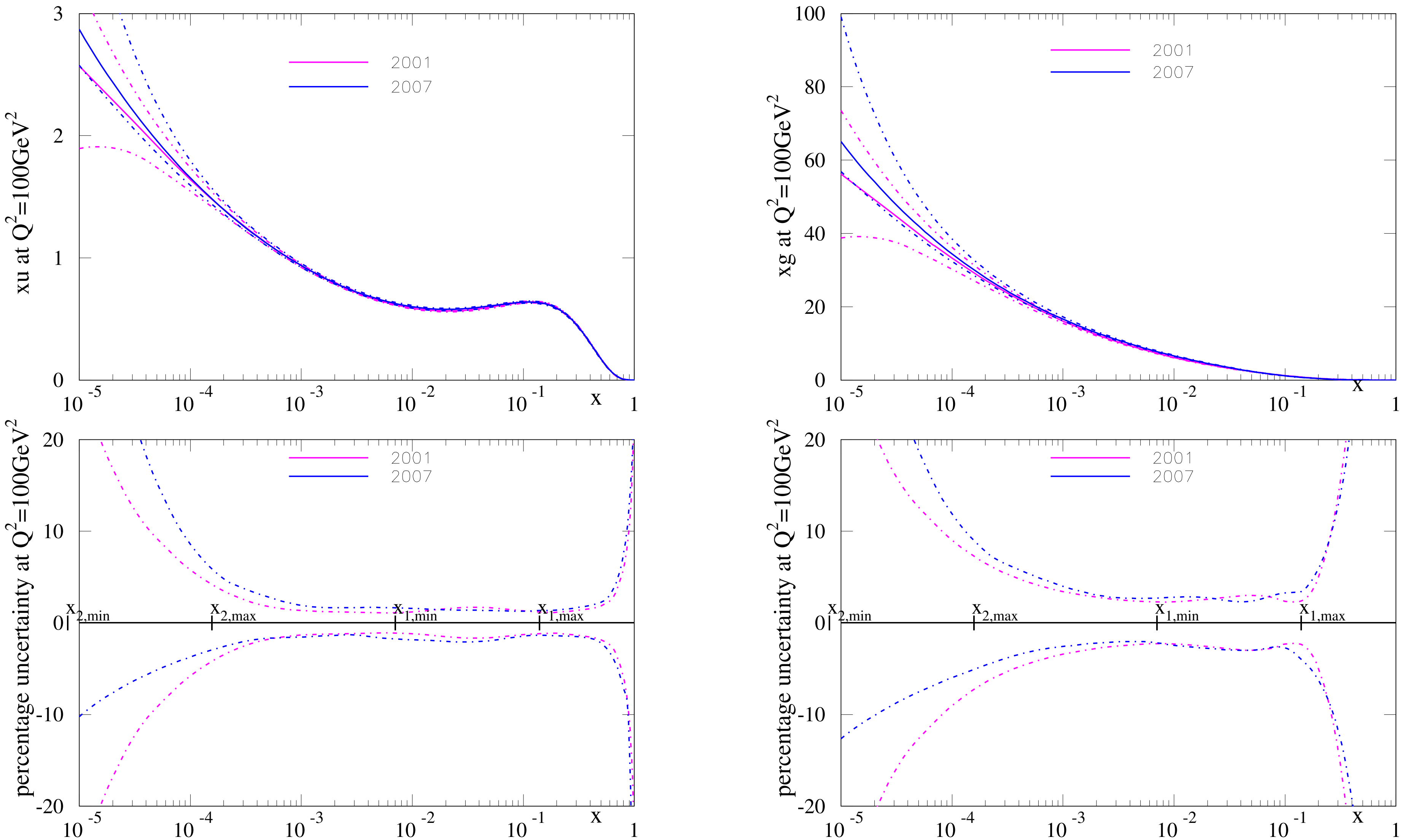


Uncertainty on $\sigma(W^-)$ dominated at very high y by high-x $d_V(x)$. Cleaner probe in ratio.





Uncertainty on $\sigma(\gamma^\star)$ driven by very small-x parton distributions not very well determined by HERA. Dominated by evolution, gluon distribution and small-x physics. Consider $M_{\gamma^\star} = 14\text{GeV}$ as example.



Potential Results from LHCb

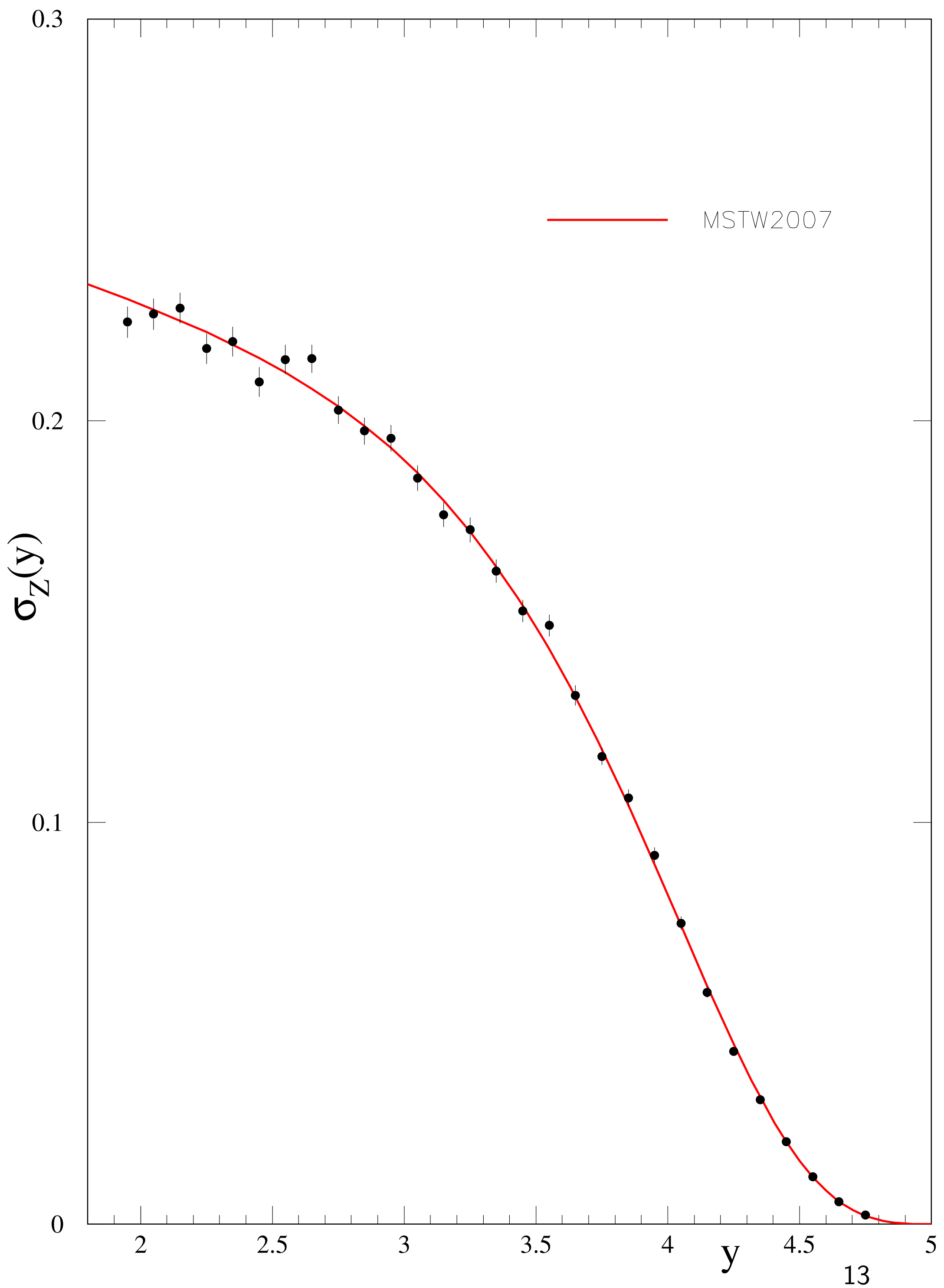
With $1fb^{-1}$ of data LHCb expects to obtain 212100 events for $Z \to \mu^+\mu^-$ for $1.9 < y < 4.9$.

Can correspond to 30 equal rapidity bins with $\sim 1\%$ statistical error at lower rapidity becoming higher as data falls of at high y.

Systematic uncertainties also $\sim 1\%$ with fairly high correlation.

Luminosity uncertainty similarly projected at $\sim 1\%$. Since this is a common factor less important in parton determination/QCD test since no impact on shape.

Possible data if completely consistent with current prediction shown opposite.

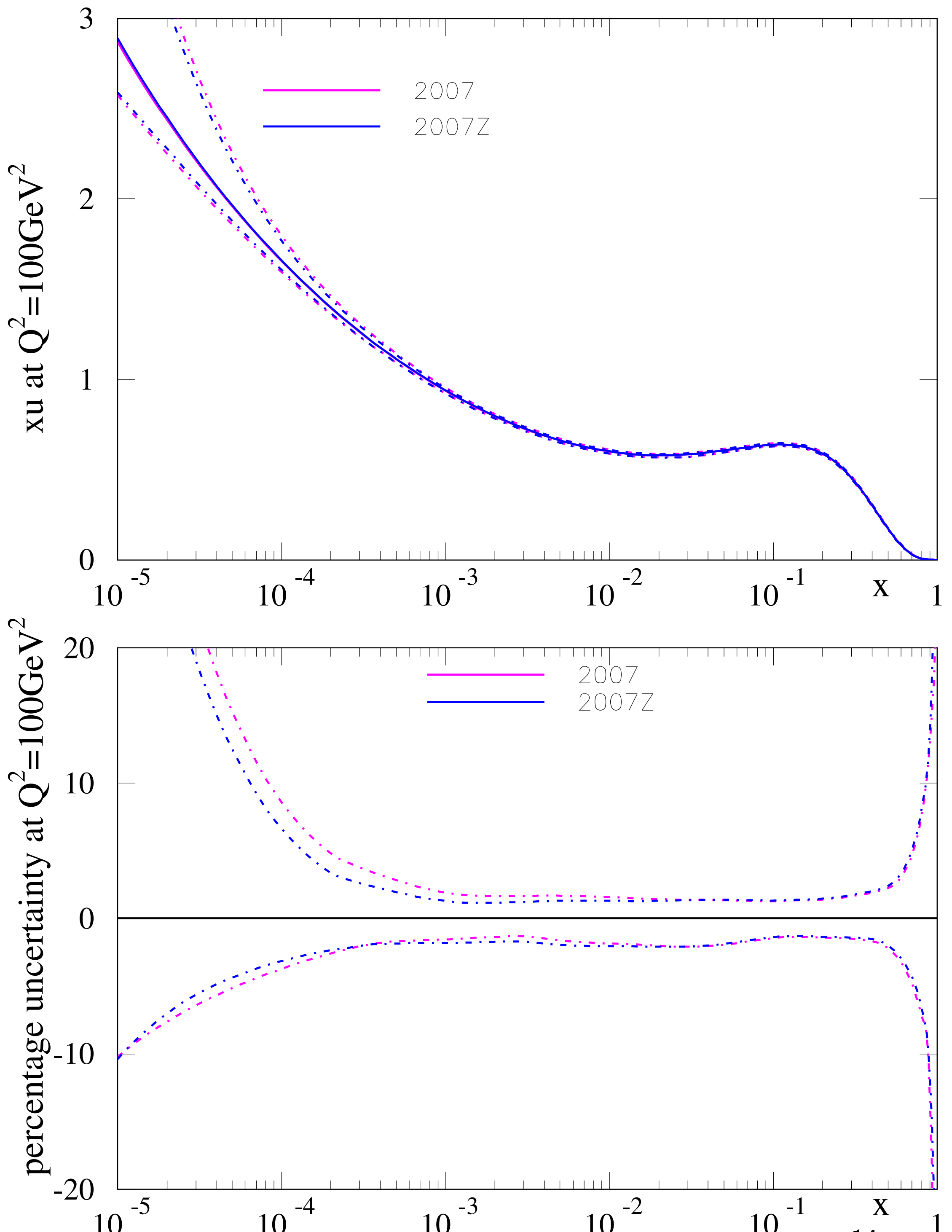


Constrains parton distributions almost entirely at small x.

With previous data improvement on uncertainty on $u(x,Q^2)$ shown. Essentially identical for $d(x,Q^2)$.

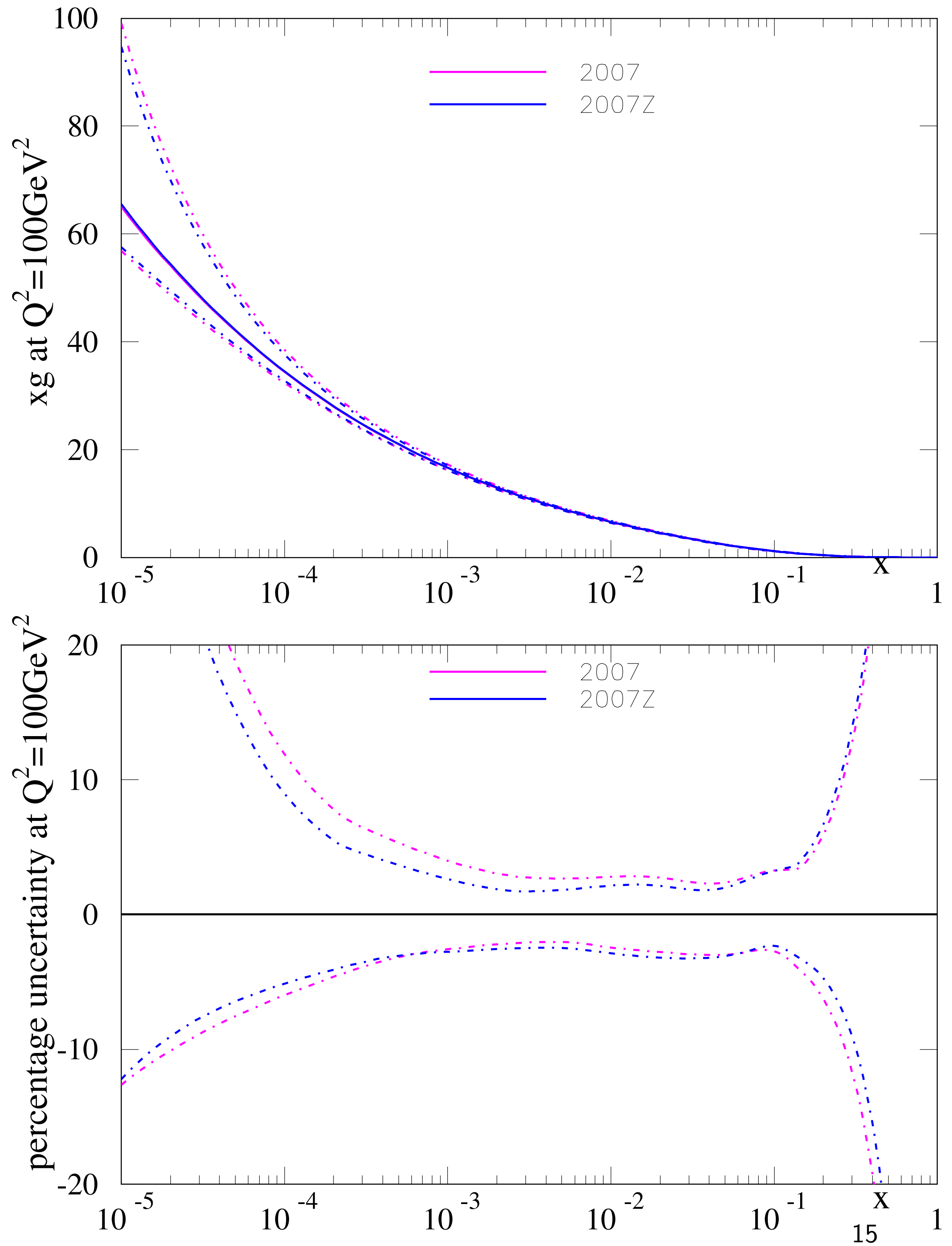
Definite narrowing of uncertainty band for $x < 0.01$.

Maximum reduction in uncertainty of $\sim 20\%$. However, can improve to perhaps $\sim 50\%$ with $10 fb^{-1}$ data.



Actually even more effect on $g(x, Q^2)$, and less confined in x.

Illustrates that at small x everything depends on evolution. Driven mainly by gluon.



However, main discriminating power in this type of data if result is not exactly what is expected.

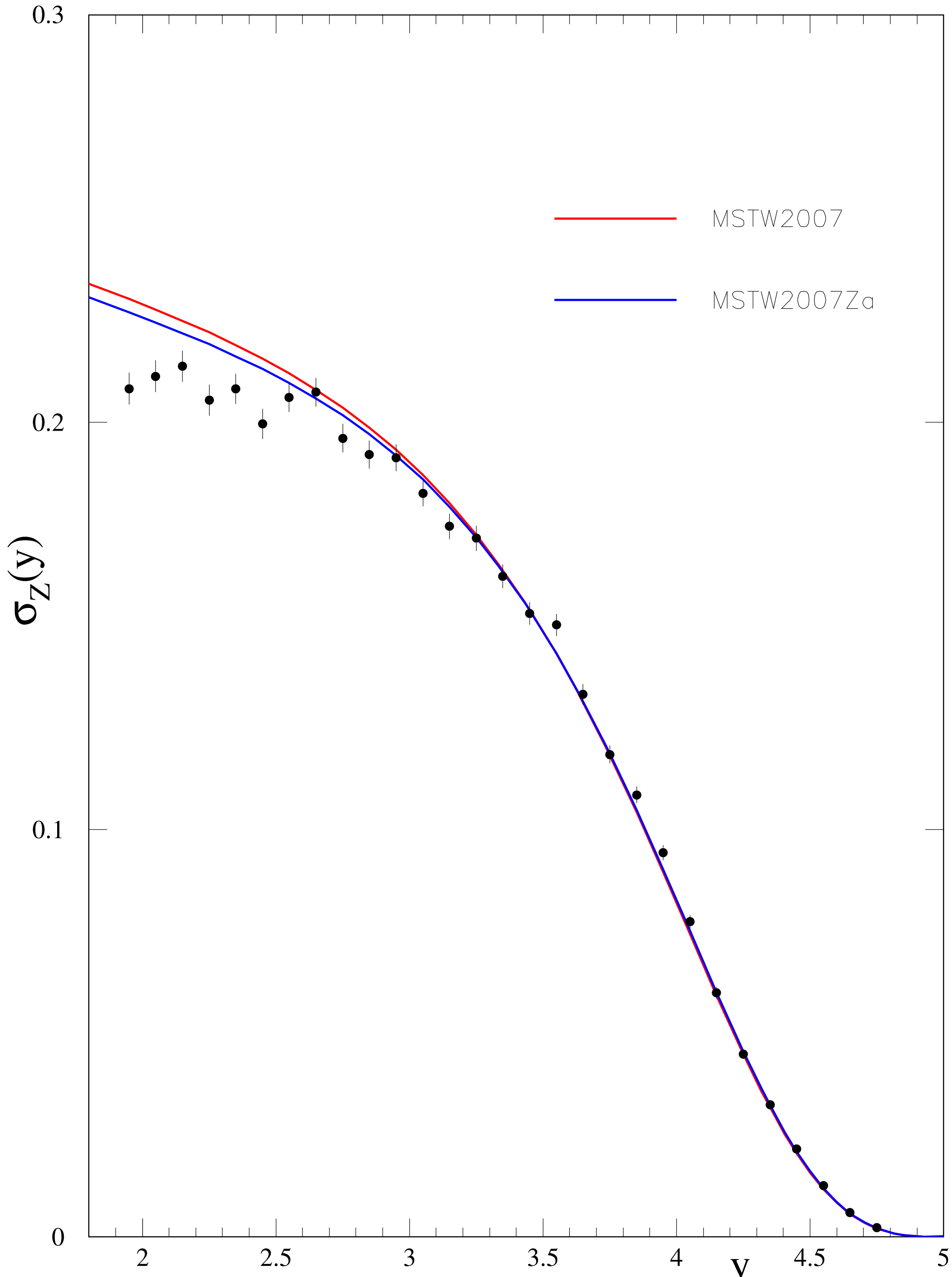
Illustrated opposite is data shifted compared to current prediction where data shifted by factor $0.05(y-3.4)$. Relatively small shift.

Comparing to prediction $\chi^2 = 153/30$.

Blue line shows result of new fit. Not possible to obtain good agreement $\chi^2 = 103/30$.

HERA data and Tevatron high-E_T jet data do not allow enough movement for good fit.

Discrepancy with theory discovered.



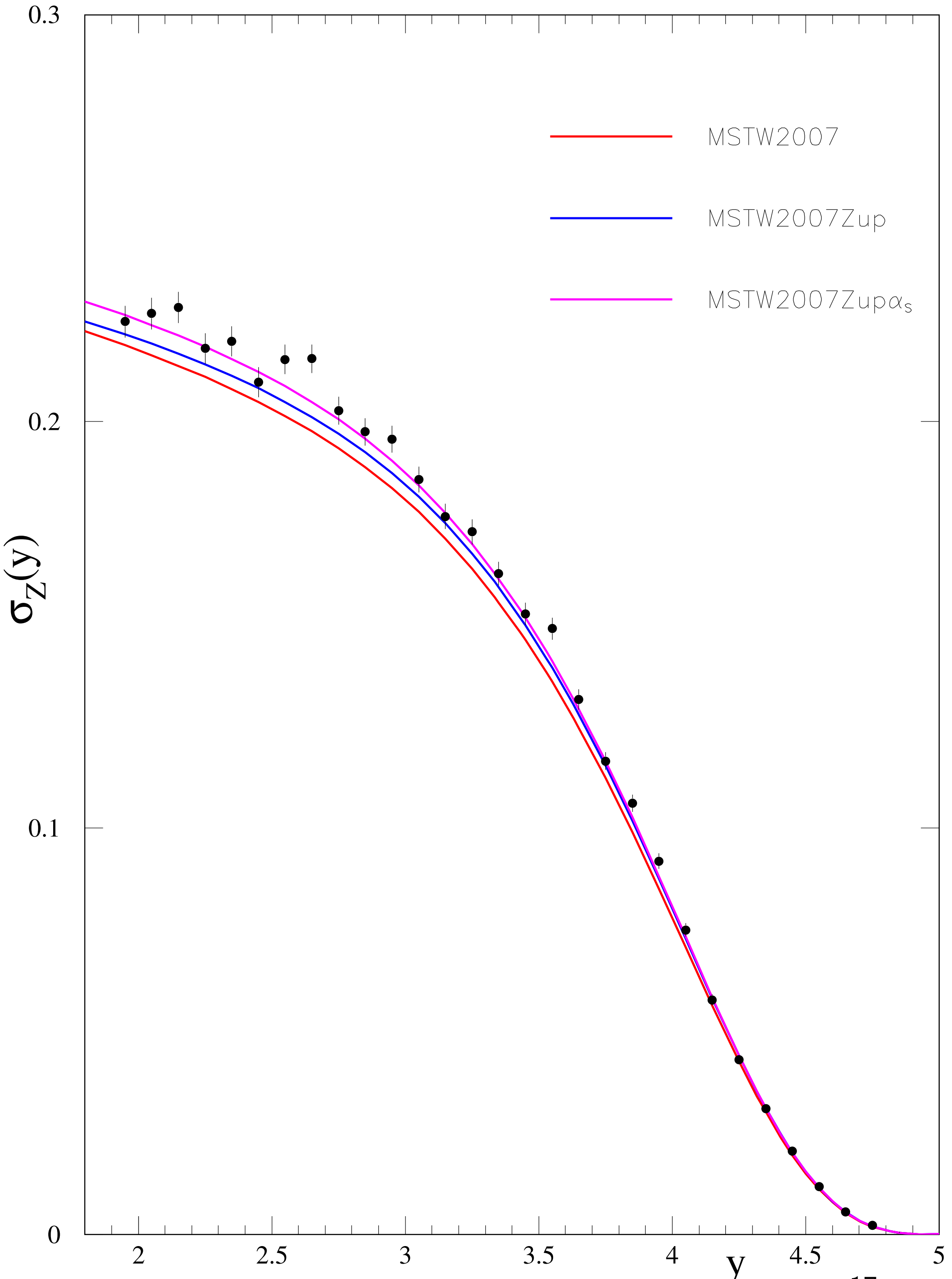
Alternatively if luminosity is very well determined what if data 1.05 greater than expected?

Comparing to prediction $\chi^2 = 204/30$.

Blue line shows result of new fit. Not possible to obtain very good agreement $\chi^2 = 92/30$.

H1 data do not allow enough movement for good fit. Shape wrong, not normalization. To fit this data small-x evolution, not low-Q^2 quarks, must be bigger.

Allowing both α_S and m_c to vary a best fit where we obtain $\chi^2 = 55/30$ is possible, but for very high coupling (0.122) and high mass.



Also look at influence of data from LHCb on $\sigma(W^-)/\sigma(W^+)$.

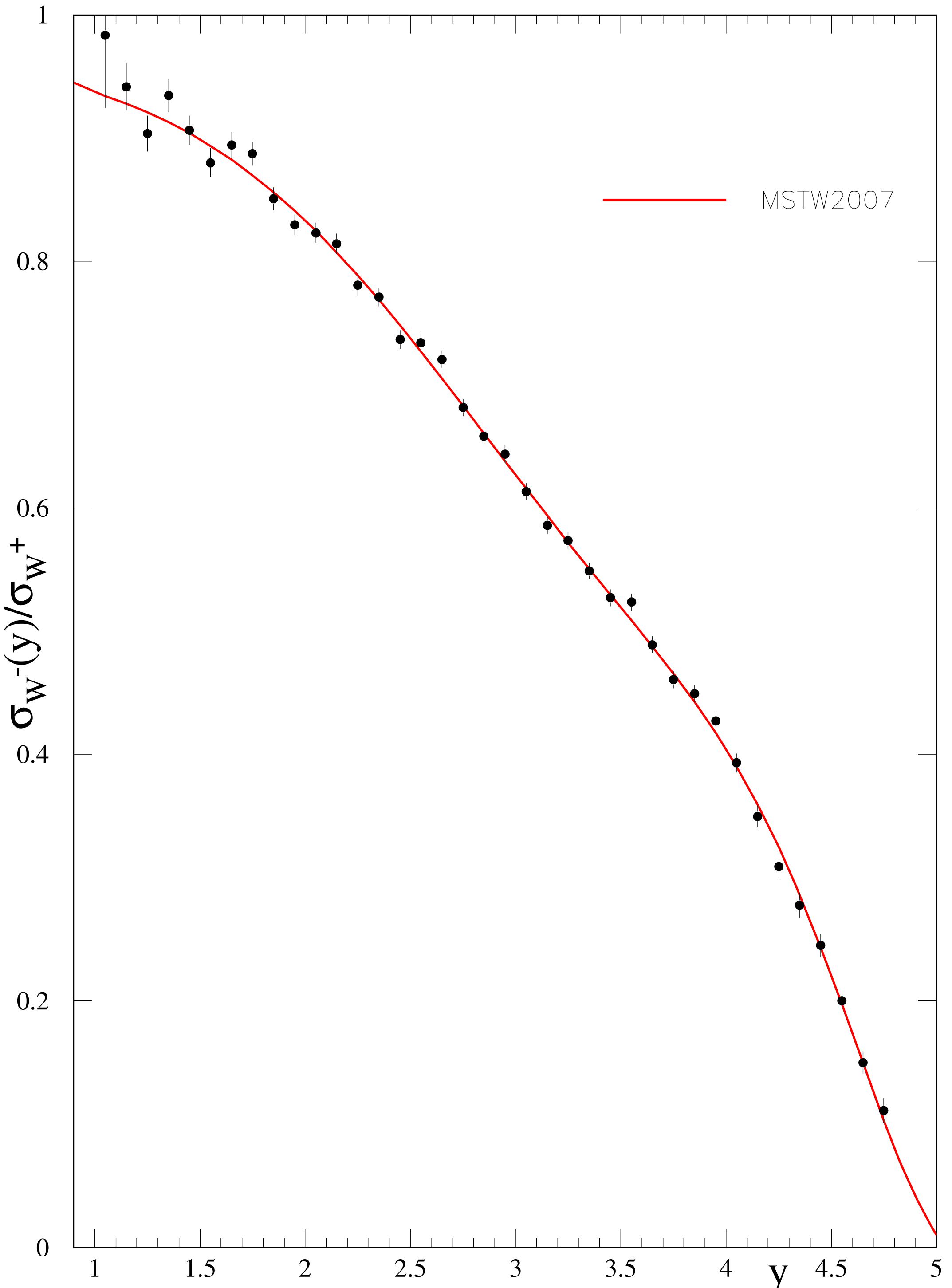
Cross-section for $W \rightarrow \mu\nu_\mu$ ten times $Z \rightarrow \mu^+\mu^-$, but more difficult and more systematics.

In particular measure lepton rapidity not W. Ignore this here. (CDF now work back to W).

Systematics also cancel in ratio.

Assume that for $1fb^{-1}$ error $\sim 1\%$ at low y with similar decrease to before at high y. Perhaps a bit better than this.

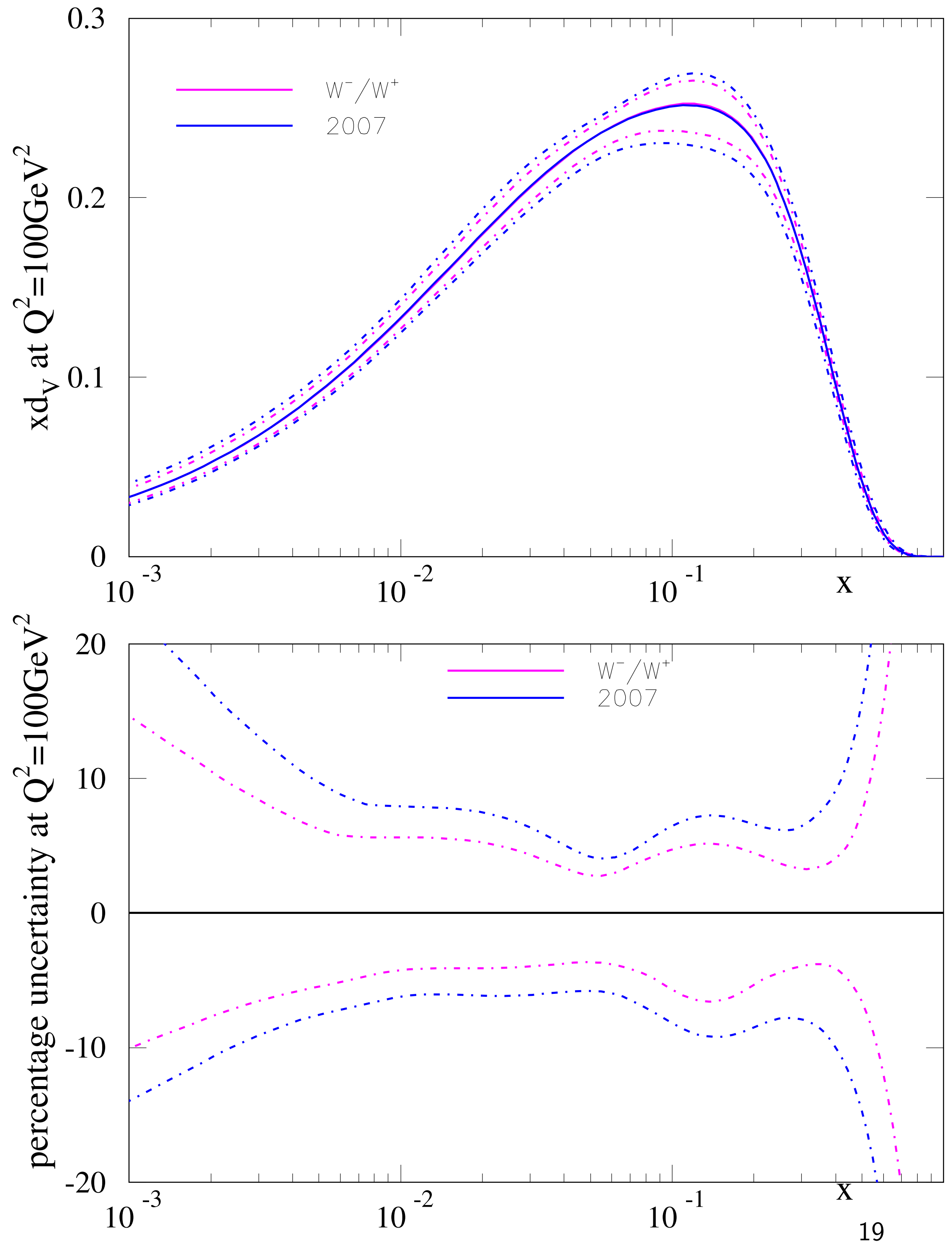
Data compared with prediction shown.



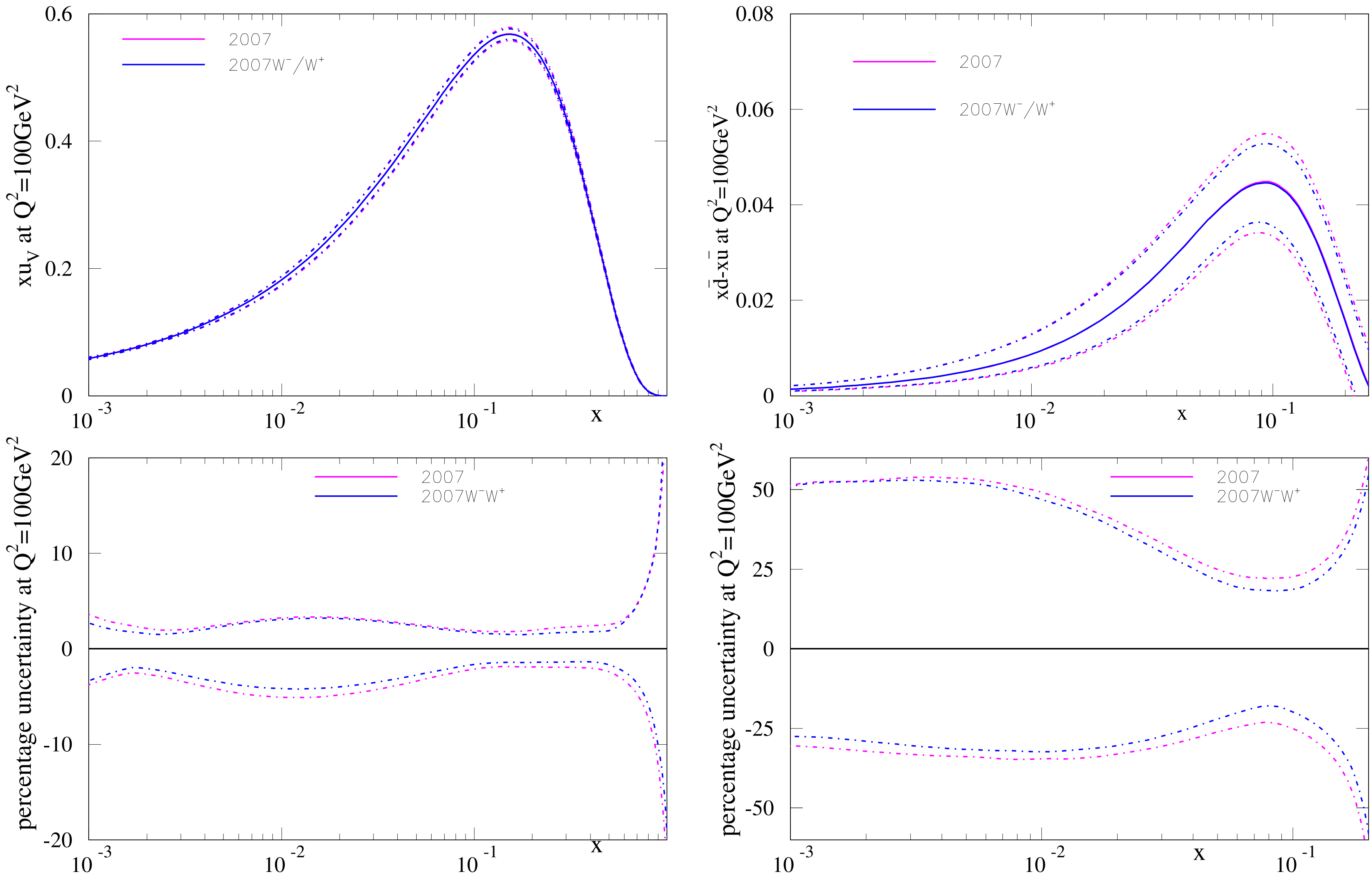
This time data most sensitive to high-x down distribution, i.e. $d_V(x,Q^2)$.

Significant reduction in this at all x (helped by sum rule).

Immediately just about main constraint on this parton.



Additional constraint on $d_V(x,Q^2)$ also helps constrain, directly and indirectly, $u_V(x,Q^2)$. Also reduces uncertainty in $\bar{d}(x,Q^2) - \bar{u}(x,Q^2)$ from small $d(x_2)\bar{u}(x_1)$ and $d(x_2)\bar{u}(x_1)$ contributions



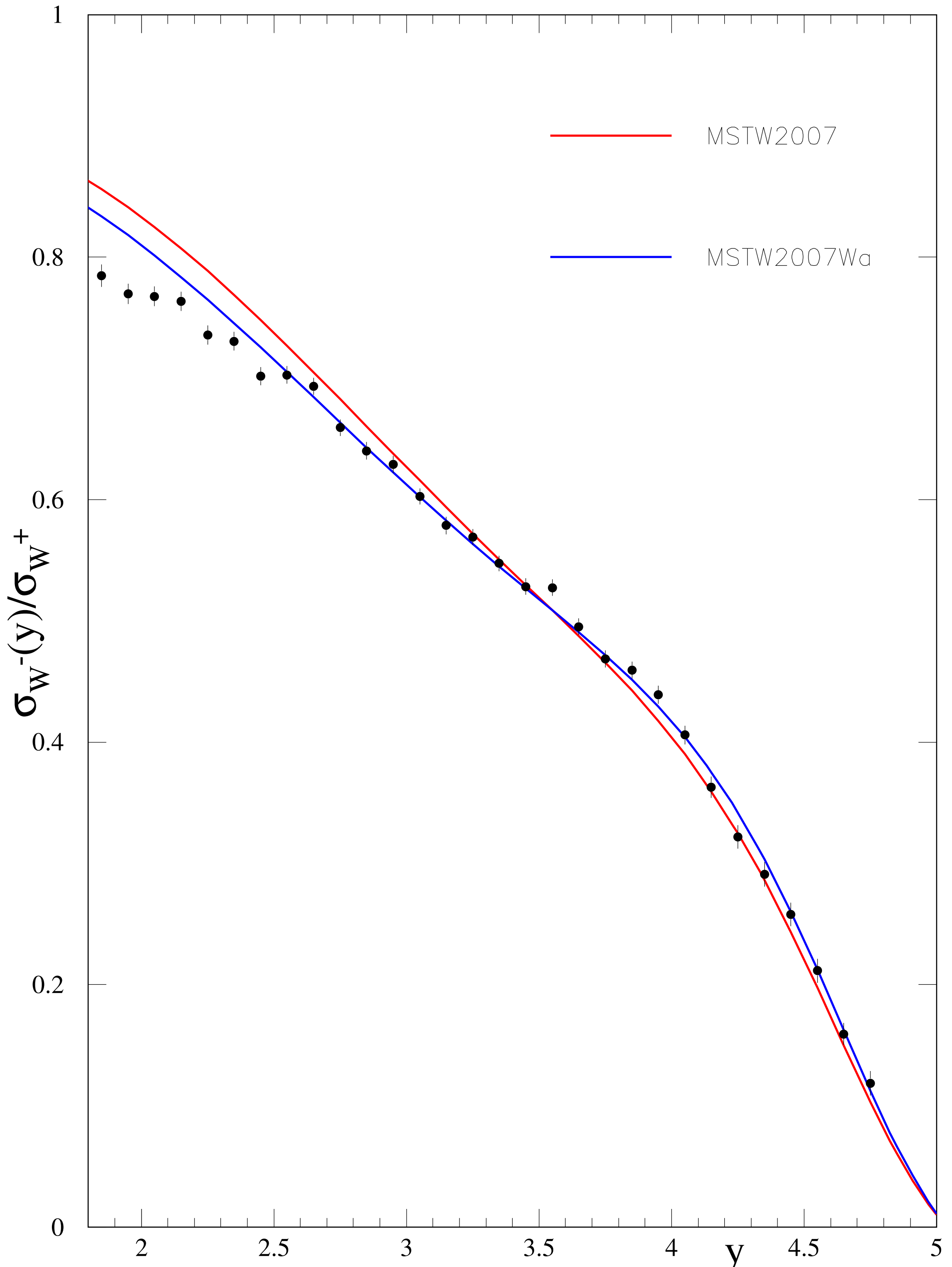
Again we obtain even more information if the measurement is not as predicted.

As before shift predicted data by by factor $0.05(y - 3.4)$.

Comparing to prediction $\chi^2 = 707/30$.

Blue line shows result of new fit. Not possible to obtain good agreement $\chi^2 = 184/30$.

Overall – discrepancy with theory discovered. Not so easily interpreted as QCD effect in this case.

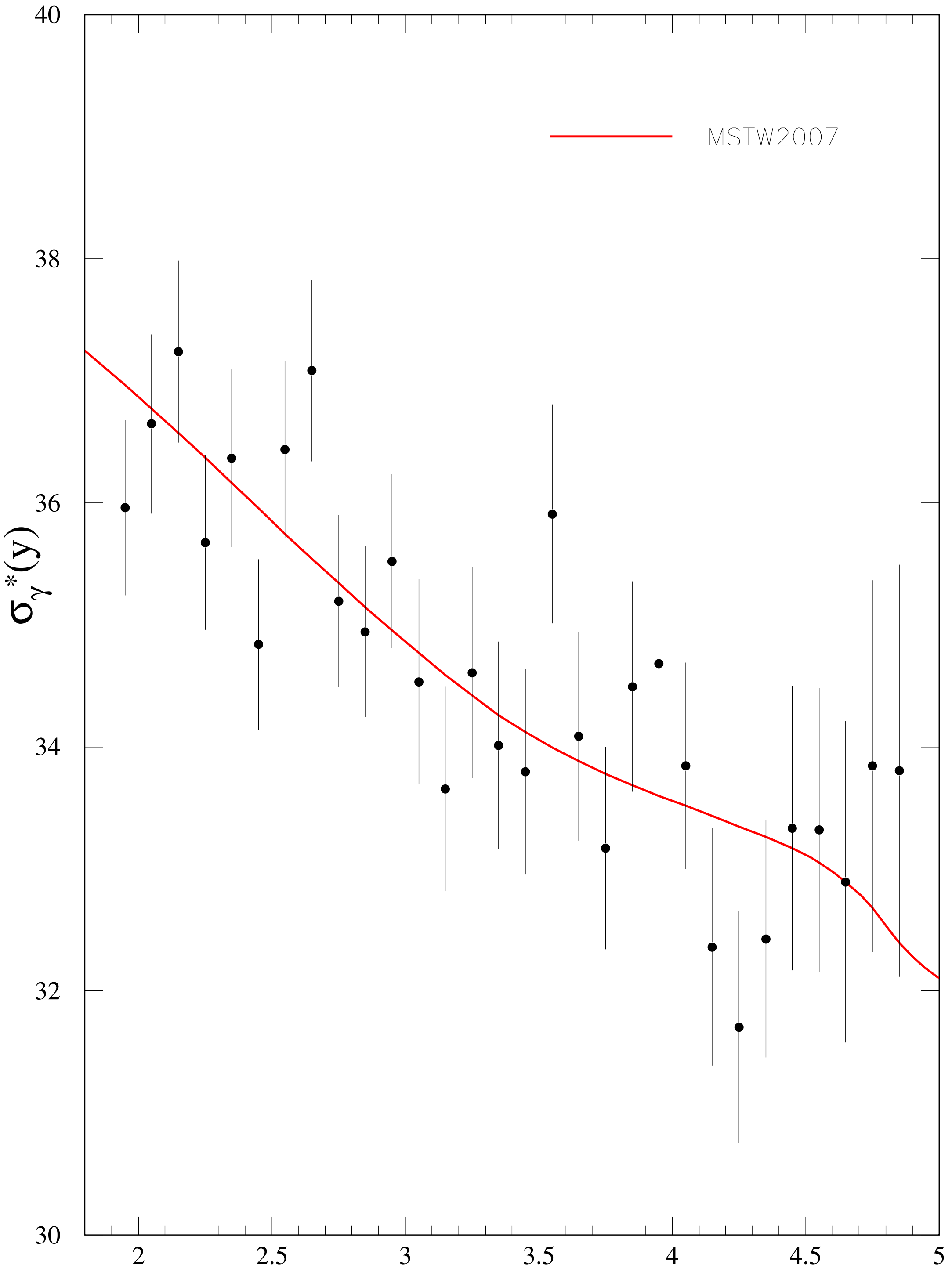


Finally look at influence of data from LHCb on $\sigma(\gamma^\star)$ for $M_{\gamma^\star} = 14\mathrm{GeV}$.

$d\sigma/dMdy$ for $\gamma^\star$ for this virtuality similar to that for $Z \to \mu^+\mu^-$, at the Z peak.

Assume that for $1fb^{-1}$ error about twice that for $Z \to \mu^+\mu^-$ at lowest y but much less decrease at high y since cross-section is not falling off – not reaching valence quark fall off.

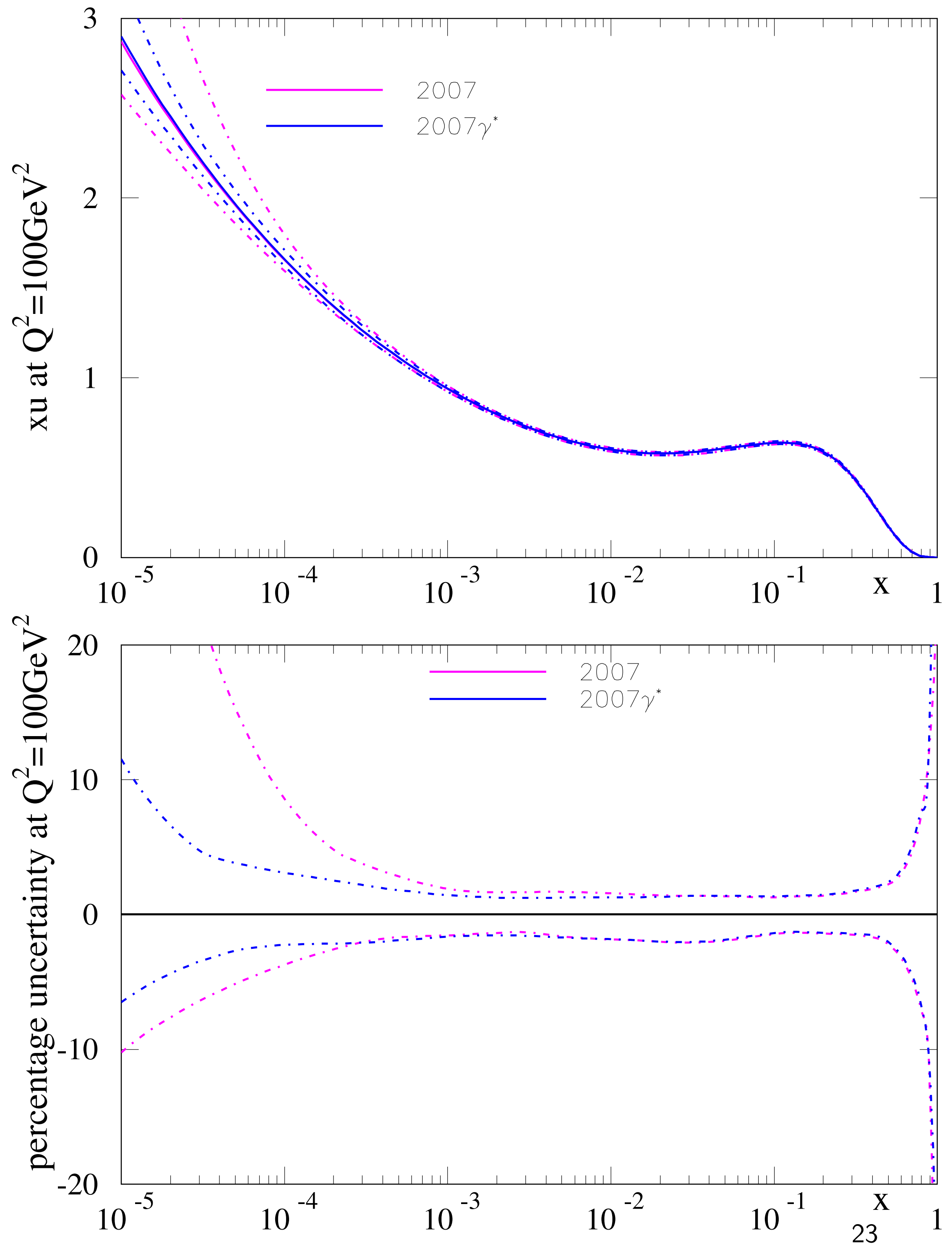
Data compared with prediction shown.



This time data most sensitive to very low-x quark distributions.

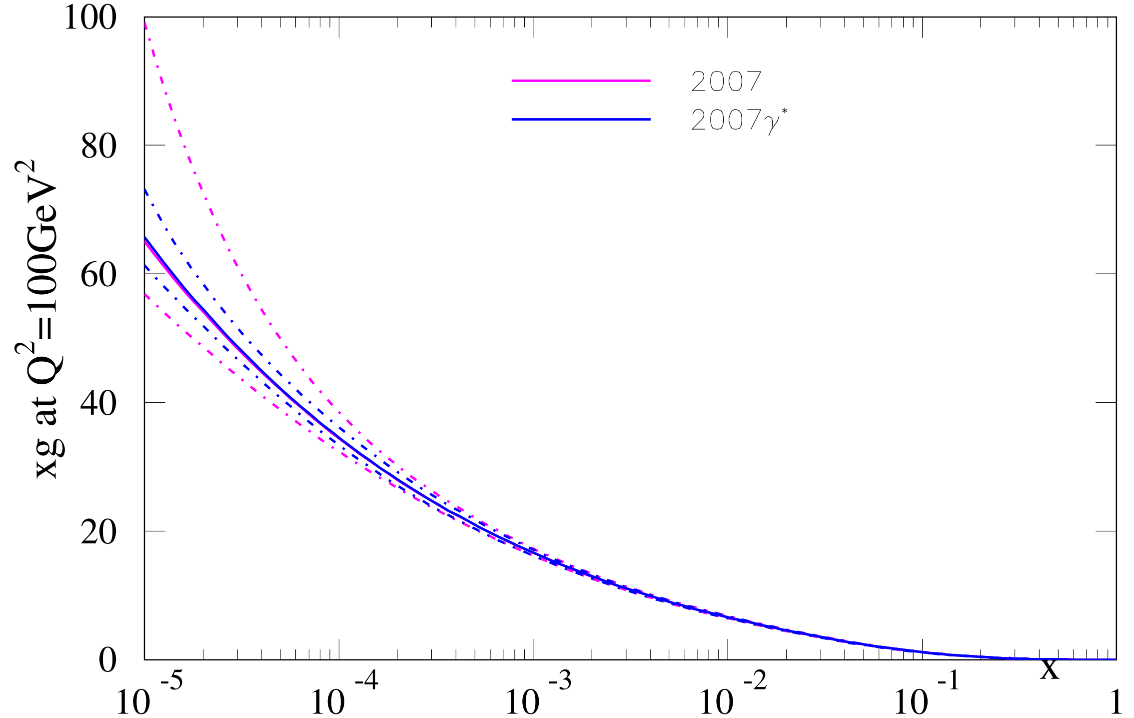
Very significant reduction in uncertainty.

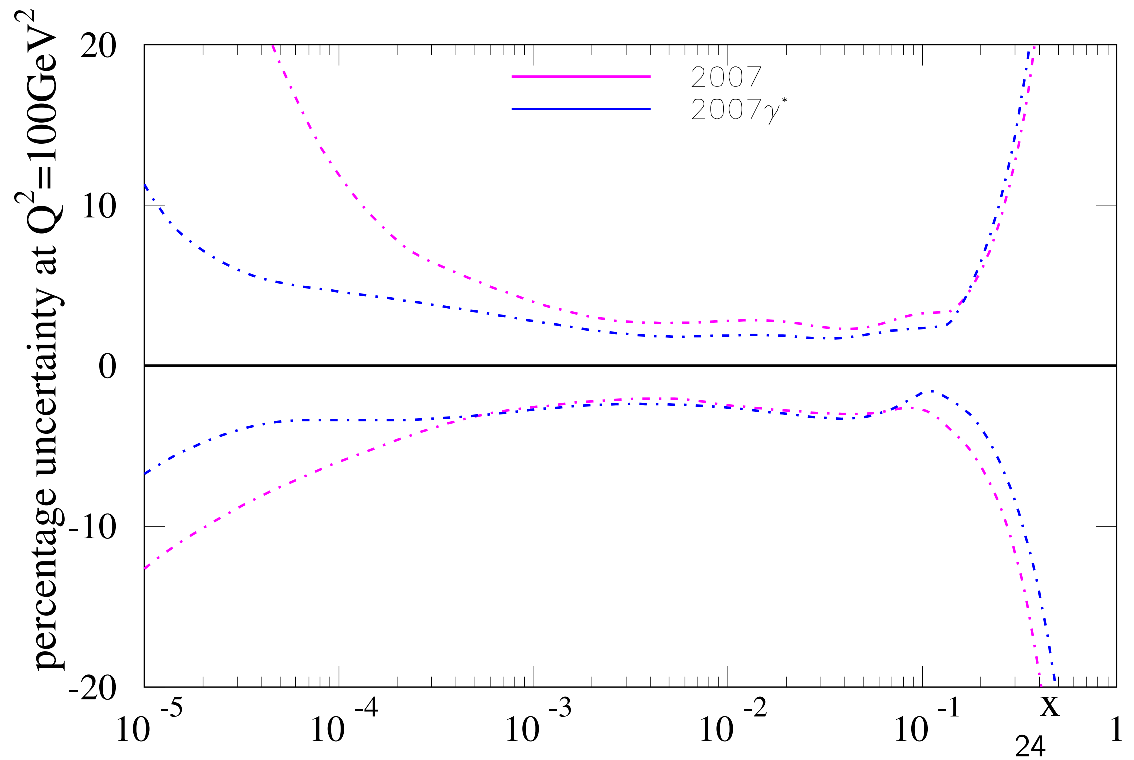
Immediately just about main constraint quarks for $x \leq 0.0003$.



However, again quarks largely determined by evolution.

Similar improvement in uncertainty on small-x gluon distribution.





As before shift predicted data by by factor $0.05(y-3.4)$.

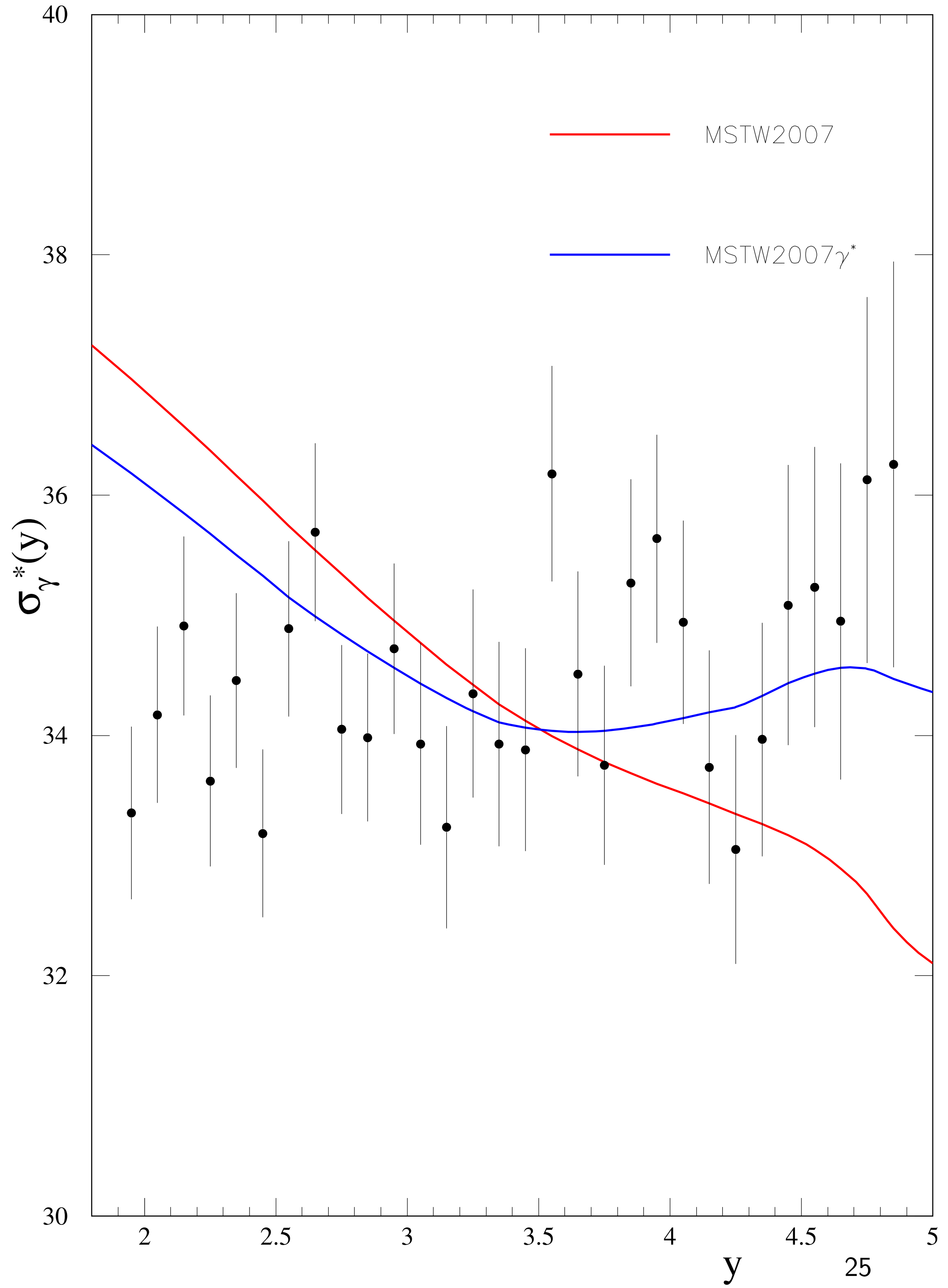
Initially $\chi^2 = 128/30$.

Blue line shows result of new fit. Possible to obtain moderate agreement $\chi^2 = 66/30$.

Deterioration in fit to other data too bad to accommodate this. Main effect in HERA small-x data.

Larger intrinsic initial uncertainty means data initially more constraining than showing discrepancies.

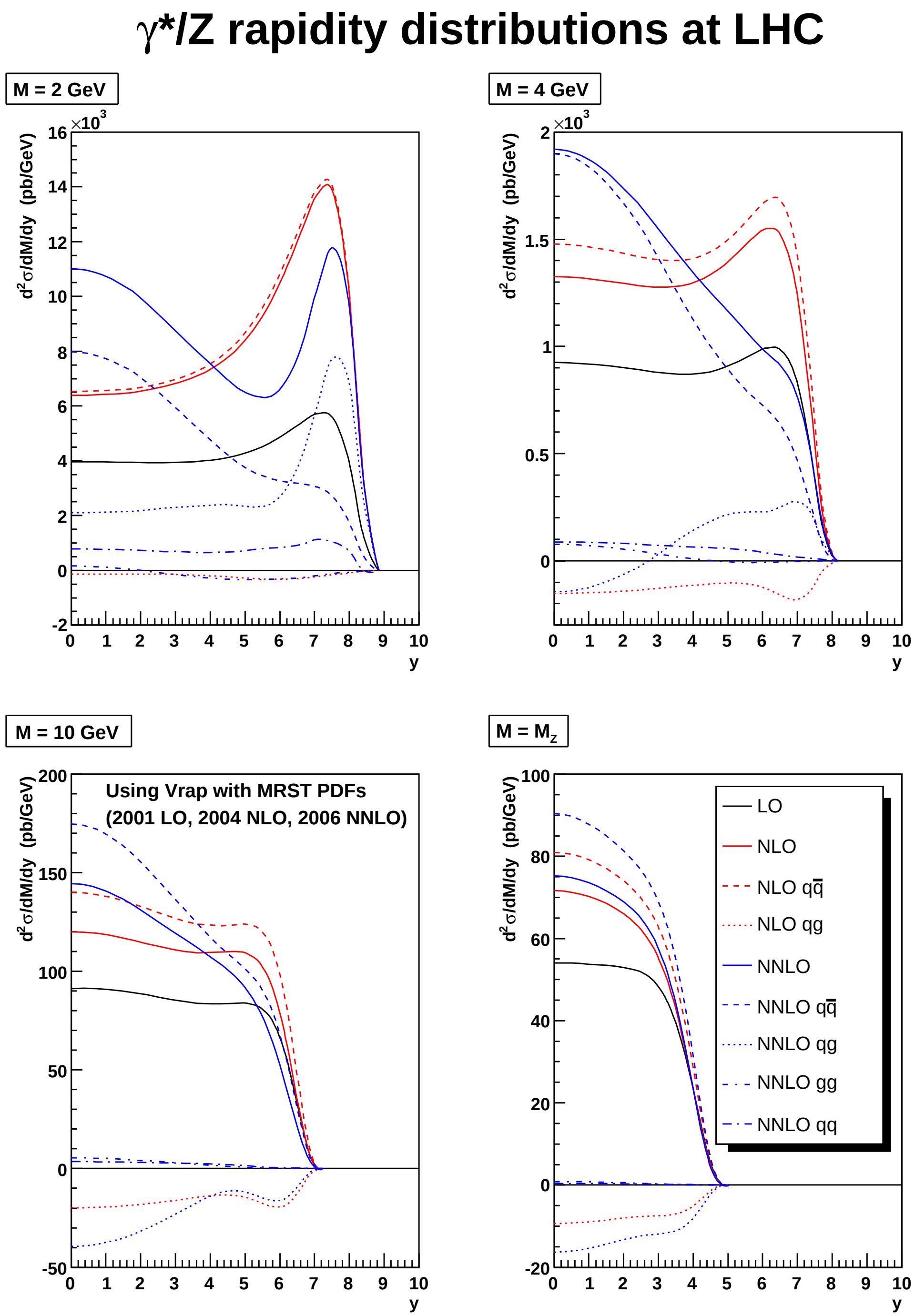
However, measurements for various $M_{\gamma^\star}$ test evolution, which is more tightly constrained at small x than absolute value of partons at one scale.



However, this assumes perturbative prediction of Drell-Yan production is reliable.

As seen very large change in prediction from order to order, particularly for low M and high y.

Problem with perturbative stability. Is this due to partons or cross-sections?



At LO $d\sigma_{\gamma^\star} = q(x_1)\bar{q}(x_2) + q(x_2)\bar{q}(x_1)$, but beyond have contributions roughly like

$$\alpha_S^n(M^2)q(x_1)\int_{x_2}^1 dz\, C_{qi}(z)\,\tilde{f}_i(x_2/z), \qquad \tilde{f}(y) = yf(y),$$

where $f_i(x)$ spans all partons.

In principle double convolution, and $x_1 \leftrightarrow x_2$, but dominant effects roughly of this form. Also at NNLO gluon-gluon contribution, but tiny.

$C_{qq}(z)$ contributions positive and concentrated near $z = 1$ – probes quark at $z = x_2$.

At NLO – $C_{qg} \sim a\ln(1-z) + b\ln(1/z)$. Dominant contribution negative near $z = 1$.

At NNLO – $C_{qg} \sim a\ln^3(1-z) + b/z$. Dominant contribution negative near $z = 1$ but also positive at small z. Latter probes gluon for $z \gg x_2$ - important for large y.

At small z $C_{q\bar{q}} \sim 4/9\, C_{qg}$.

Can use this to help explain the features – particularly large positive qg contribution at small M and large y.

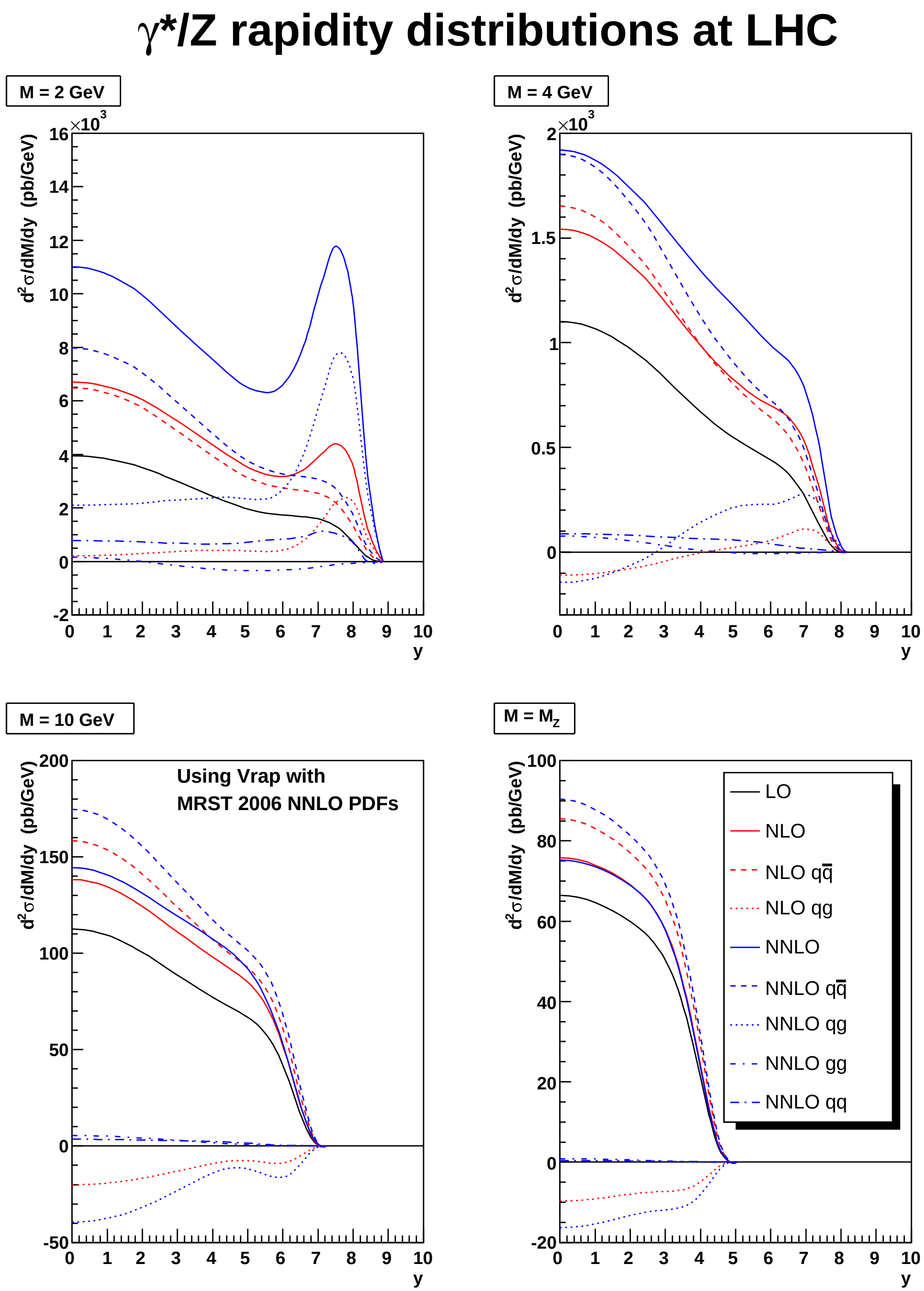
Know that at higher orders pick up additional $\ln^2(1-z)$ at each order at high z (also in $C_{q\bar{q}}$ - remaining positive) and an additional $\ln(1/z)$ at each order at low z.

Keeping partons fixed while changing cross-sections (using MRST2006 NNLO partons) also shows part of instability due to partons. Unusual behaviour in very small x partons at NNLO. Due to similar high and low z terms in splitting functions.

Overall most obvious effect – large change in quark-gluon (and quark-quark) contributions at NNLO due to $1/z$ divergences in cross-sections appearing at this order.

Reminiscent of behaviour of $F_L(x, Q^2)$ which has similar large logs.

Cross-section may be sensitive to resummations (high and low z) at lowest M and highest y. In region where measurements can be made?



Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Uncertainties rather small – $\sim 1 - 5\%$ for many quantities, but larger if sensitive to small x or something other than up quark at high x. There are also theory uncertainties, particularly at small x.

LHCb has a unique handle into these regions of uncertainty in QCD at LHC.

Ratios involving W production significantly improve quark uncertainty at high x, or spot new physics.

Total cross-sections more sensitive to small-x physics. Low-mass Drell-Yan production allows investigation into value of x not reached (in perturbative realm) before. (p_T distributions also interesting at small-x (Stirling – Dublin LHCb meeting)). Improve uncertainties, test whether we understand QCD evolution in complicated regime and also feed back into searches for new physics.

Total b cross-section would probe similar physics, but is theoretically more complicated.

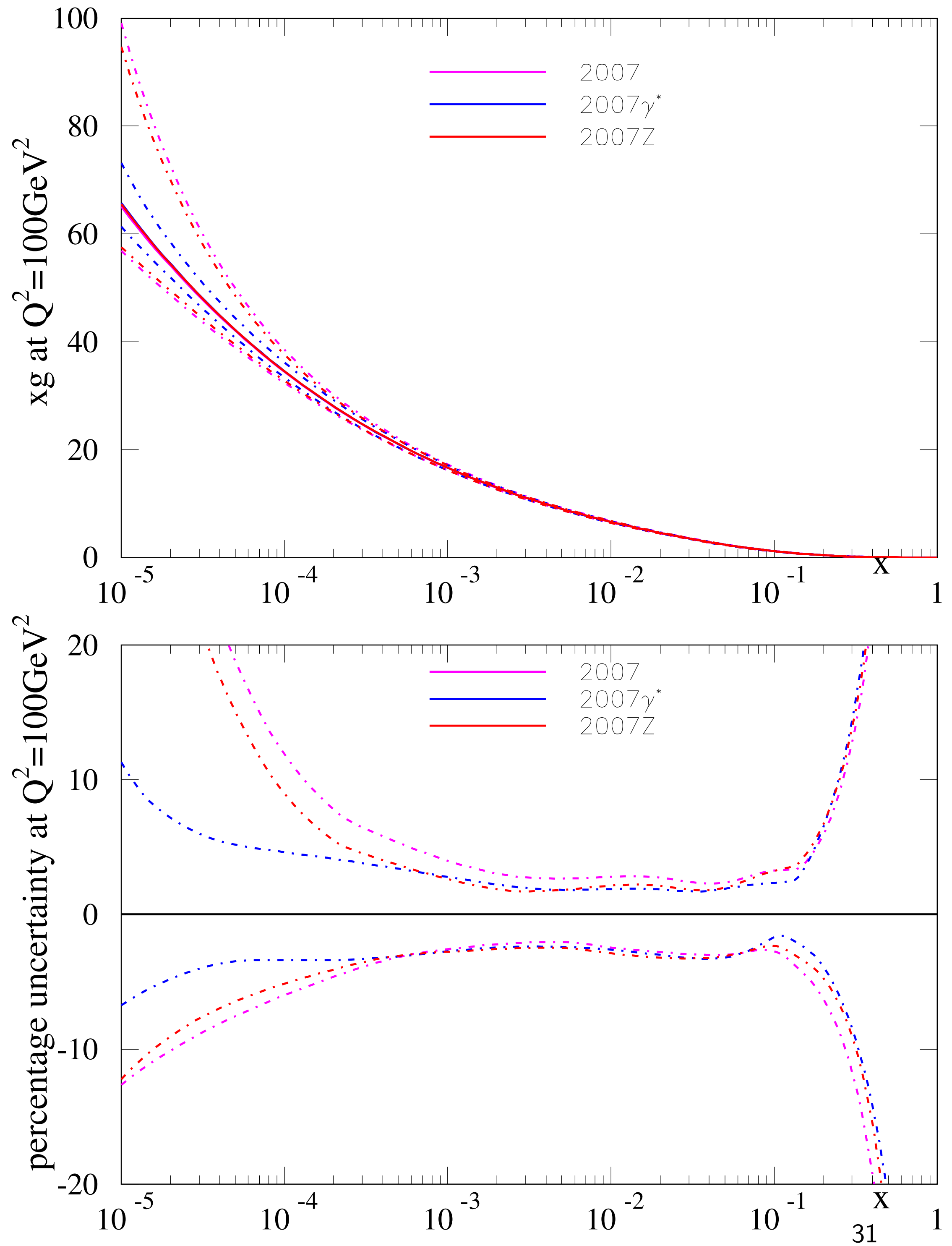
LHCb has enormous potential for this type of physics. At very worst help significantly extend our understanding of hadrons. Could be rather more than this.

Wide variety of data are badly fit to accommodate modified $\sigma(w^-)/\sigma(W^+)$ data.

Surprisingly main change in $u_V(x, Q^2)$ and $\bar{u}(x, Q^2)$ (while $u(x, Q^2)$ largely constant), and particularly in $s(x, Q^2)$ and $\bar{s}(x, Q^2)$. $s(x, Q^2) - \bar{s}(x, Q^2)$ changes sign.

Although not dominant contributors have terms like $u(x_1)\bar{s}(x_2)$ for W^+ and $\bar{u}(x)s(x)$ for W^-, and these parton contributions less constrained.

Effect on uncertainty on $g(x, Q^2)$ at small x from $Z \rightarrow \mu^+\mu^-$ data and $\gamma^\star \rightarrow \mu^+\mu^-$ data, with $M_{\gamma^\star} = 14\mathrm{GeV}$, compared.



Effect on uncertainty on $u(x,Q^2)$ at small x from $Z \rightarrow \mu^+\mu^-$ data and $\gamma^\star \rightarrow \mu^+\mu^-$ data, with $M_{\gamma^\star} = 14\mathrm{GeV}$, compared.