Introduction to rf acceleration

Accelerating structures

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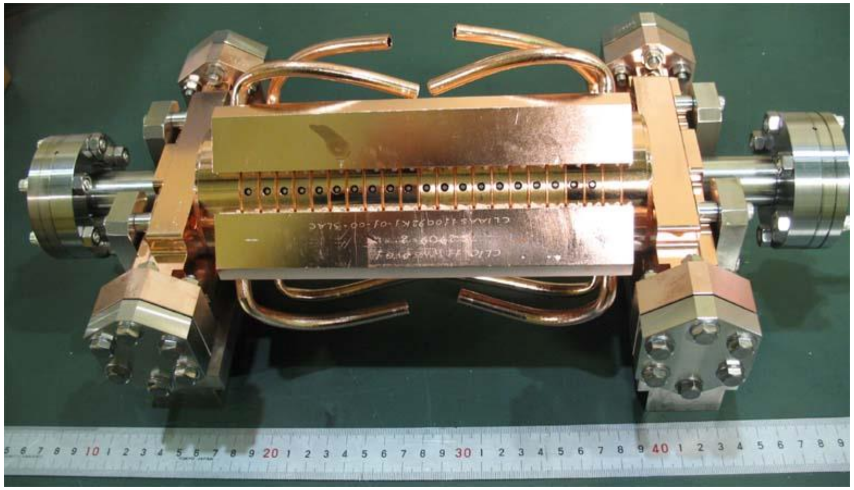
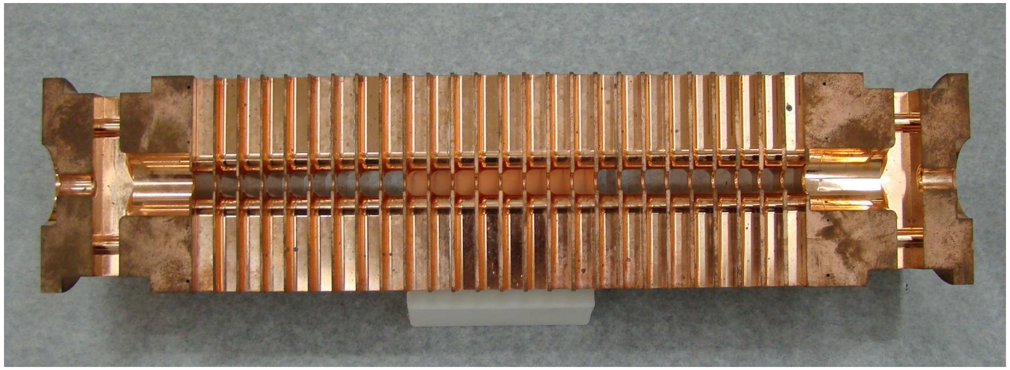
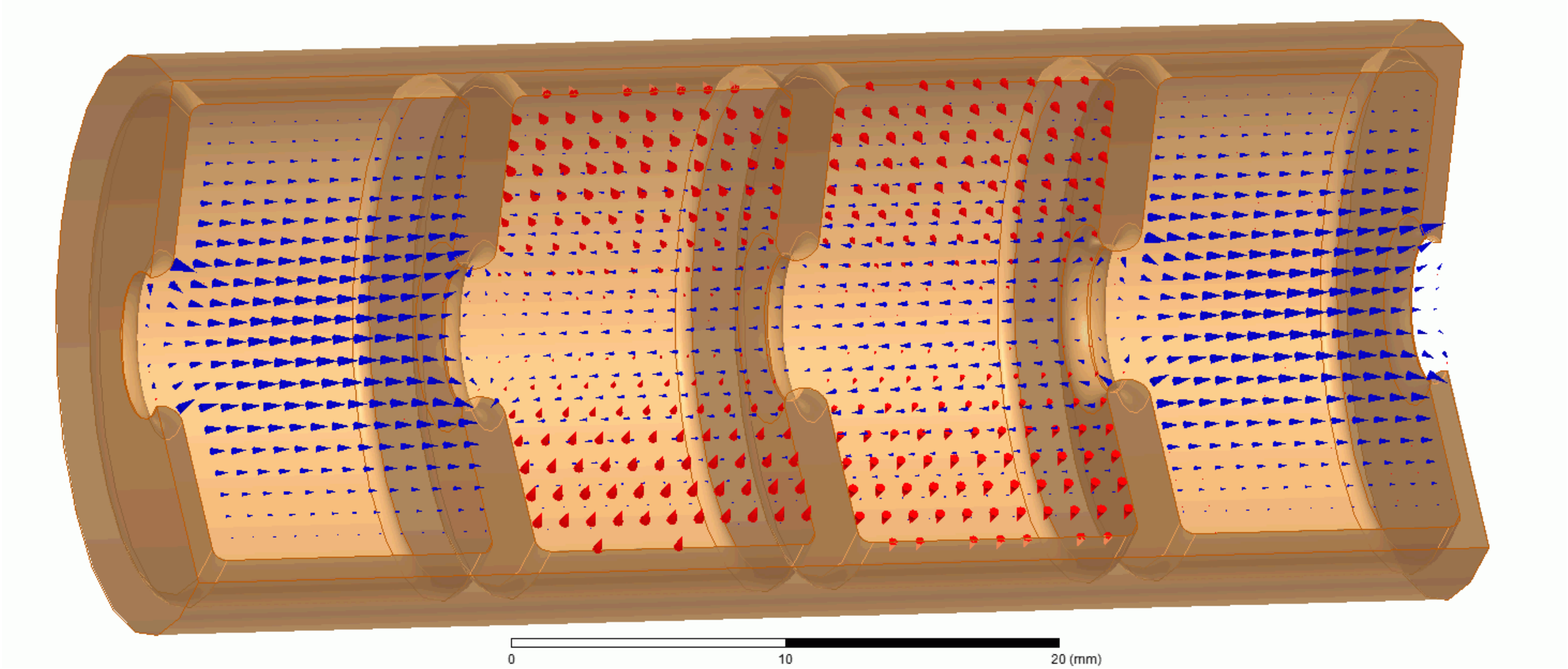




What the functional inside looks like.

A high-gradient accelerating structure.



The behaviour of the electromagnetic fields we would like to understand.

We have looked rf structures in order to understand how to get an interaction between an rf field and a relativistic beam – the issues were mainly getting synchronism and getting the electric field to point in the right direction.

Now we are going to push a little further into theory to describe how much acceleration the beam actually gets.

We are going to study how much energy you transfer to the beam from a certain stored energy in a standing wave cavity or power flow in a travelling wave cavity.

We approach this in steps.
• First look at a dc gap,
• then an rf gap

Acceleration is typically measured in units of MV/m, the CLIC target is 100 MV/m.

Let’s look together for a moment at a simple capacitor plate (big enough one so we don’t have to worry about edge effects) to make sure we are familiar with all the relevant quantities in a simple case.

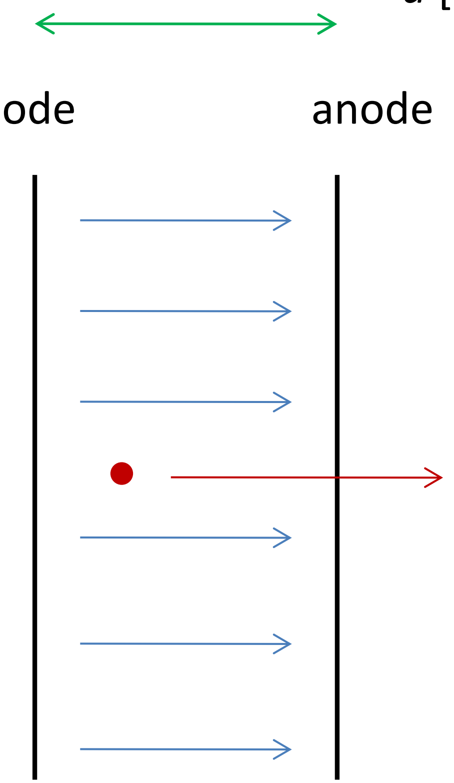
d [m] gap size, *A* [m^2]area

Electric field:

$$E = \frac{P}{d}\left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

cathode

anode



charge *q* [C]

Energy gained by bunch:

$$U = P[\mathrm{V}]q[\mathrm{C}] = Pq[\mathrm{J}]$$

Stored energy in electric field:

$$W = \int \frac{1}{2}\varepsilon_0 E^2 dV$$

\- *P* [V]

ground

Now an rf 'cavity' (without being specific about the details of what it is):

The 'voltage' of an rf gap is of course more complicated because the fields are oscillating while the beam takes the time to cross the gap. Remember the definition of the transit time factor from section 1:

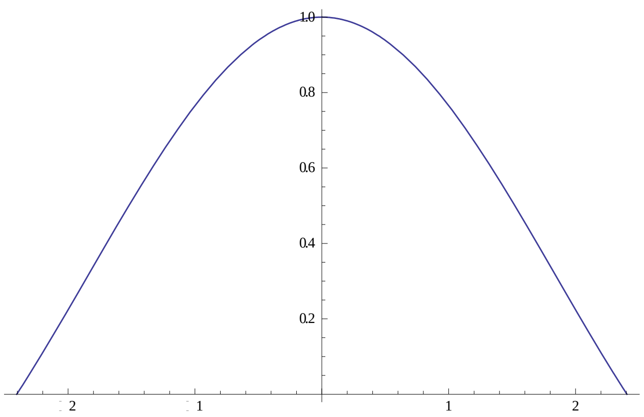
$$A = \frac{|V_{acc}|}{\int E_z dz} = \frac{\int E(z)dz}{\int E_z dz}$$

We will use the numerator again, which is the *effective* gap voltage:

$$|V_{acc}| = \int E(z)dz$$

The magnitude is the highest acceleration you get from the cavity.

remember this is a complex number



TM_{010} mode

For the stored energy in a cavity we need to include both the electric and magnetic field:

$$W = \int \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) dV$$

Putting the two terms we can define:

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{\omega W}$$

Which has units of Ω.

R/*Q* – relates the amount of acceleration (squared) you get for a given amount of stored energy. If the electric fields are concentrated along the central axis of a cavity this term is large. You can use computer programs to get actual values.

The numerator and denominator both scale with field squared, so it is independent of field level. It turns out that this term is independent of frequency as well for scaled geometries.

You can do lots of useful calculations knowing this term. But let's dig deeper.

Going a step further

Our goal now is to derive and understand the loss factor, *k*.

Accelerating a beam *extracts* energy from a cavity (and by the way that's what we need to do to get high rf to beam efficiency).

The beam gains energy when you accelerate so the rf fields must loose energy.

We'll attack this by considering the question "How much energy does a traversing beam leave behind in a particular mode of an empty cavity?" and then superimpose the solution on a filled cavity, which is how we normally think of acceleration.

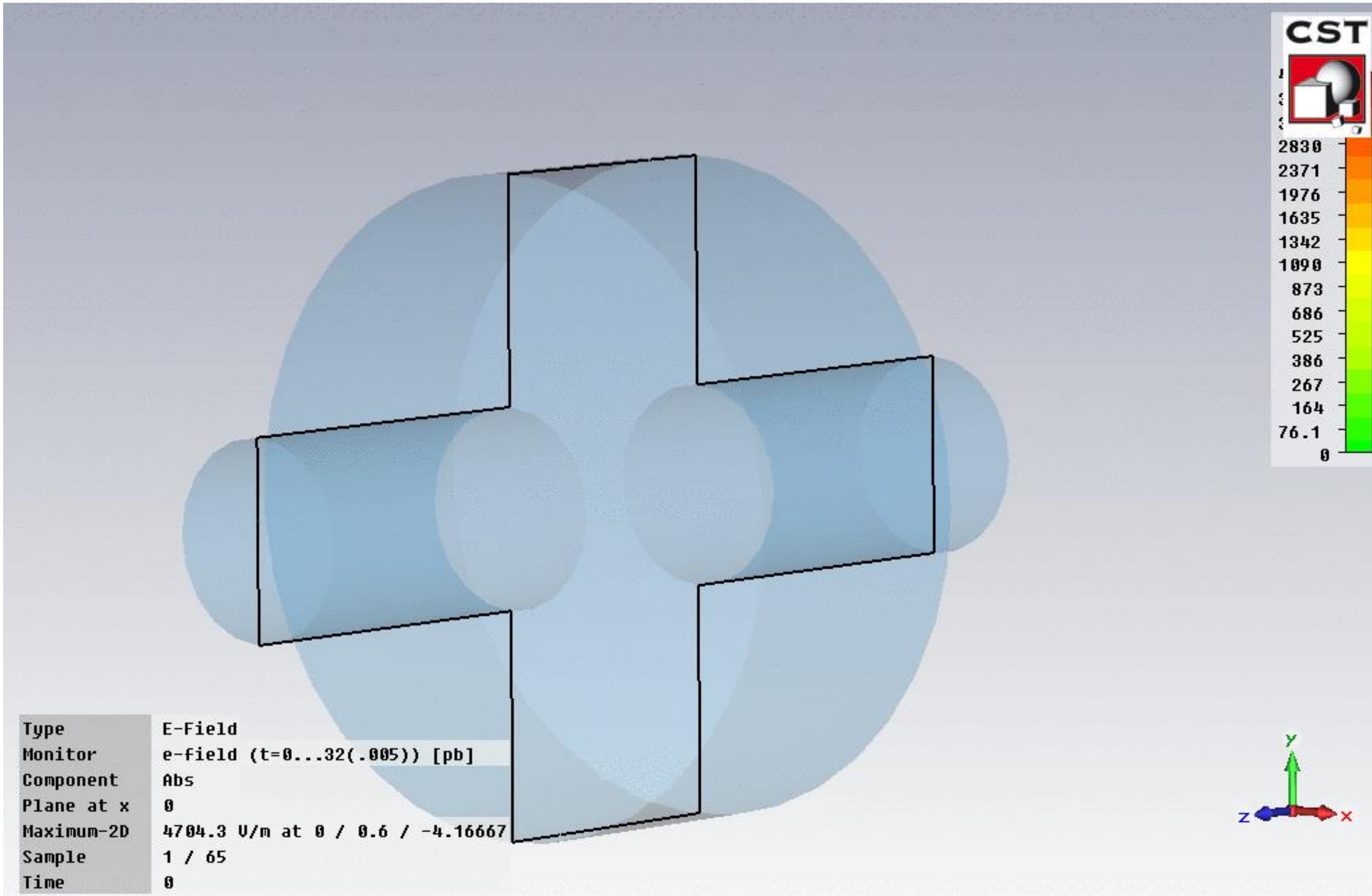
Concepts we will use

In this section we will often consider the driven rf fields (driven by a klystron or whatever), consider the fields the beam leaves behind and add the two together to get our final answer – **superposition** of *beam and rf fields*.

There is another subtlety we will use which is that you can break the problem up mode by mode, add them up and get the right answer. Another way of saying this is that all the eigenmodes of the cavity are orthogonal basis functions for all the possible fields in the cavity. You can expand reality as a **Fourier series** over all the *cavity modes*.

We will also consider **driving bunches**, these are the real bunches of the problem with finite amount of charge, and **witness charges**. Witness charges are basically just integrals over fields but it is useful to think of charges which follow the main one but have almost no charge so don't affect the fields themselves.

A charge passing through a cavity leaves behind it the cavity with voltage in it, and hence filled with energy. The beam loses the same amount of energy. The loses energy through interacting with an electric field, which in fact comes from itself.



Something to think about:

The charge interacting with the fields it makes itself is in direct analogy to the radiated electric field produced by a current that you see when discussing the retarded potential in free space. For a current in the y direction,

$$E_y(t) = -\frac{J\left(t - x/c\right)}{2\varepsilon_0}$$

You normally think of currents producing magnetic fields but of course to transfer energy to an electromagnetic field there has to be movement of an electrical charge in the direction of an electric field.

The fundamental theorem of beam loading

The fundamental theorem of beam loading says that the voltage seen by beam which has traversed a cavity is half the voltage it leaves behind, that is the one that a following witness bunch would see.

A non-rigorous way of seeing this, is that the cavity is empty when the beam enters and only full when it leaves – so on average it sees the cavity only half full (or half empty, like the proverbial glass!). A more rigorous understanding requires the formalism of longitudinal wakefields we will cover in section 4.

Is this easier to understand than the free-space case?

But in the mean time let's introduce a loss factor *k* which satisfies this factor of two. The voltage left is proportional to the charge so:

$$V_{seen} = kq$$

$$V_{left} = 2kq$$

The loss factor k

Let's now consider conservation of energy, what the bunch loses the cavity gains:

$$\Delta U_{beam} = V_{seen} q = kq^2$$

$$\Delta U_{left} = \Delta U_{beam}$$

$$\Delta U_{left} = kq^2$$

$$= \frac{V_{left}^2}{4k}$$

$$\Rightarrow \quad k = \frac{1}{4} \frac{V_{left}^2}{\Delta U_{left}} = \frac{\omega}{4} \frac{R}{Q}$$

$$V_{seen} = kq$$

$$V_{left} = 2kq$$

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{\omega W}$$

Equations we will use

So the higher the *R*/*Q* the more field left behind in a mode by a given charge.

Now let's look at a cavity that already has fields in it

Everybody's first understanding is that the beam is just sees the accelerating fields that are there because we pump lots of microwaves into a cavity. But this is only true if the bunch charge is low, and we have negligible rf-to-beam efficiency.

In a linear collider we have rf-to-beam efficiency in the range of 30% to deal with the 10's of MW average power beams we need to accelerate.

So let's now look at a cavity with field in it that gives V_0 and currents which are leaving fields which are non-negligible.

The essential insight is that a passing bunch reduces the fields inside a filled cavity in exactly the same way as an empty cavity - superposition:

$$\Delta V = -2kq = -\frac{\omega}{2}\frac{R}{Q}q$$

Checking consistency through energy balance

Beam

$$\Delta U_{beam} = V_{seen} q$$
$$= (V_0 - kq)q$$
$$= V_0 q - kq^2$$

Cavity

Before bunch passage

$$U = \frac{V_0^2}{4k}$$

After bunch passage

$$U' = \frac{(V_0 - 2kq)^2}{4k}$$

$$U - U' = \frac{1\left(V_0^2 - V_0^2 + 4V_0 kq - 4k^2 q^2\right)}{4k}$$
$$= V_0 q - kq^2$$

Consistent!

In a real cavity, we of course have the losses we saw in section 1. To deal with this we introduce the *shunt impedance*. We start with *R*/*Q,* which is independent of any losses,

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{\omega W}$$

And take our definition of Q,

$$Q = \frac{\omega W}{P_{loss}}$$

To define the shunt impedance,

$$R = \frac{\left|V_{acc}\right|^2}{P_{loss}}$$

The units of *R* are again Ω, and a typical normal conducting cavity has an *R* in the range of M Ω. Note that both numerator and denominator scale with field squared. R is a measure of the acceleration to the losses and is often a quantity you optimize when designing an rf cavity.

Now travelling wave structures

We've just gone through an analysis where we have considered stored energy. This is straight forward to apply to standing wave structures

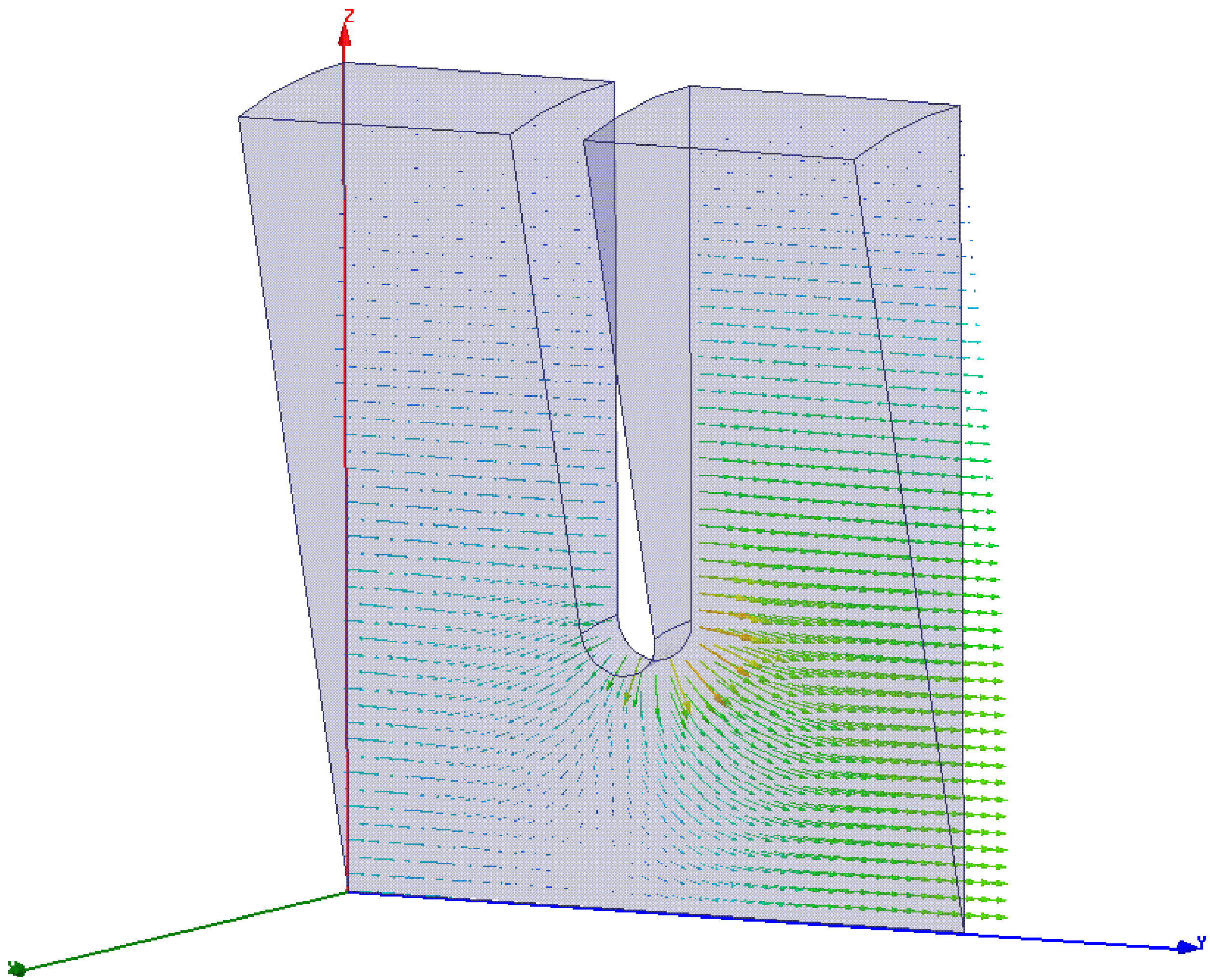
You will be doing some numerical examples in homework problems.

But the basic concepts remain the same for travelling wave structures. We have to extends things a bit and make sure we are accounting for all the energy going in and out of our problem.

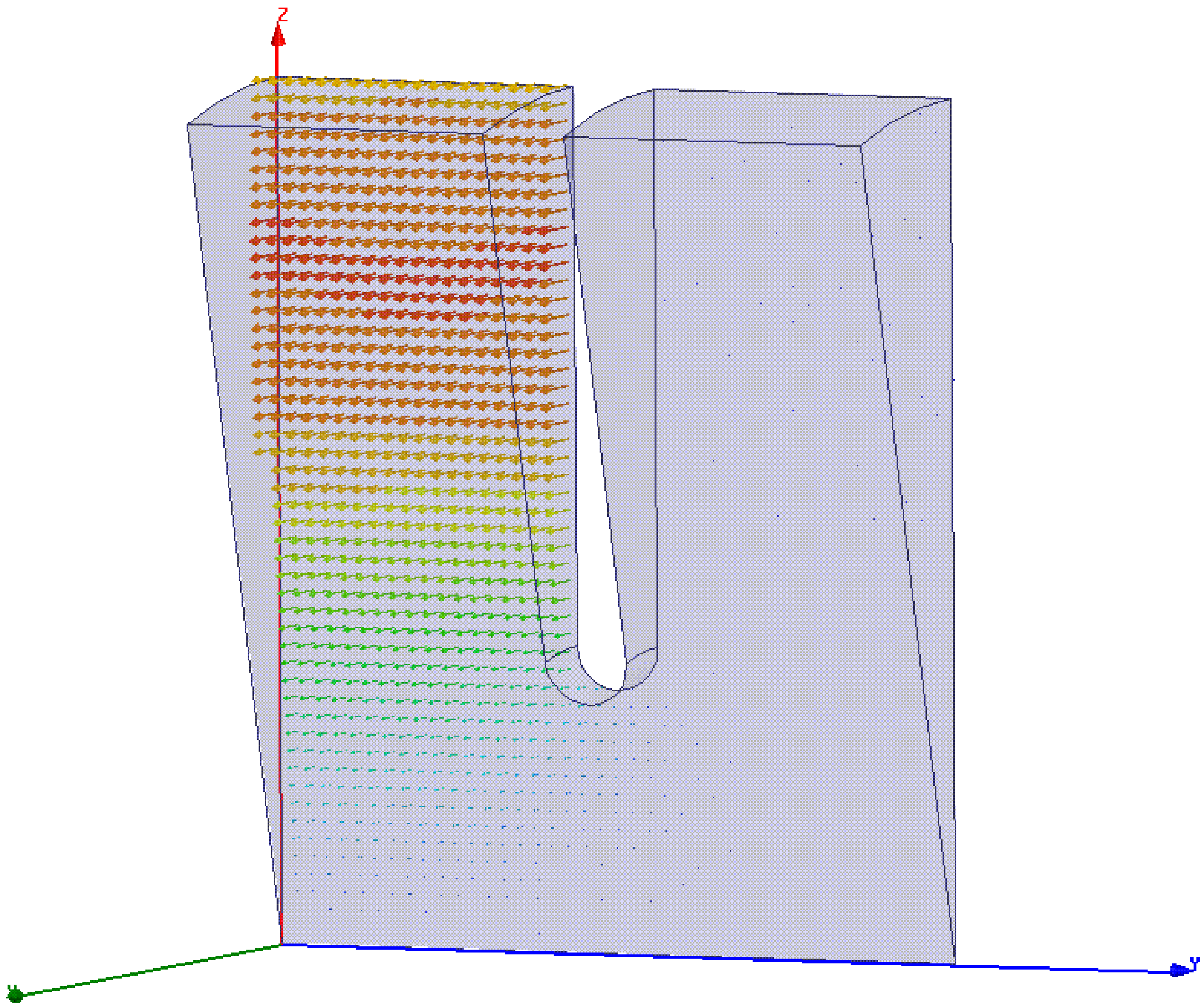
Firstly we are going to consider a single cell of an infinitely long periodic structure which has been tuned to v_{phase}=c, i.e. a synchronous wave. This is quite reasonable since tuned cells are usually what we deal with.

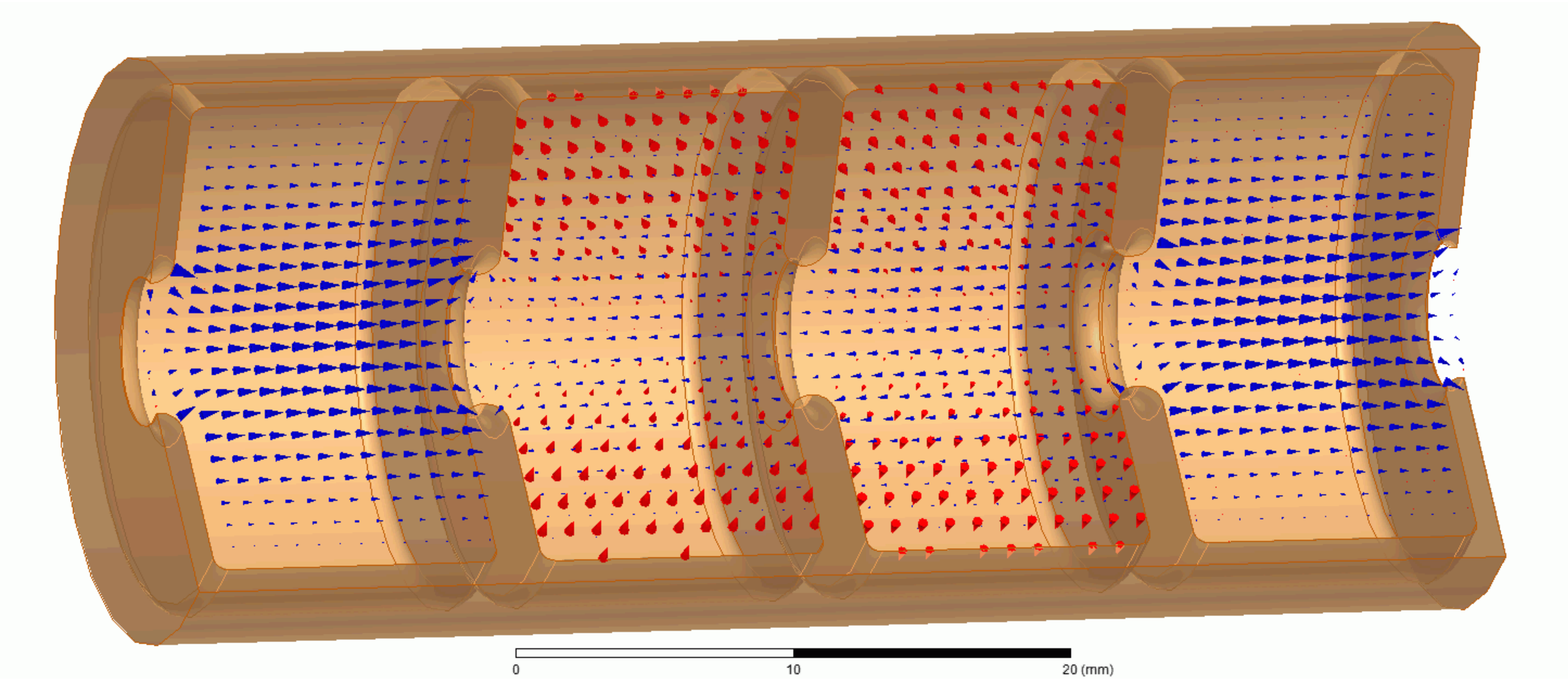
The fact that the phase and beam velocities are the same gives us the periodicity to easily do all of our calculations on a single cell.

Single cell electric field pattern $2\pi/3$ phase advance



Single cell magnetic field pattern $2\pi/3$ phase advance




0
10
20 (mm)

Standing to travelling wave

We will take over our definition of *R/Q* and shunt impedance and define it per cell, but then divide by the length of the cell, *l*, to get *R'/Q* and *R'* which are per unit length.

The other thing we will do is to put these quantities in terms of power flow rather than stored energy since this is the natural quantity for travelling wave structures.

The relationship between power flow and stored energy is,

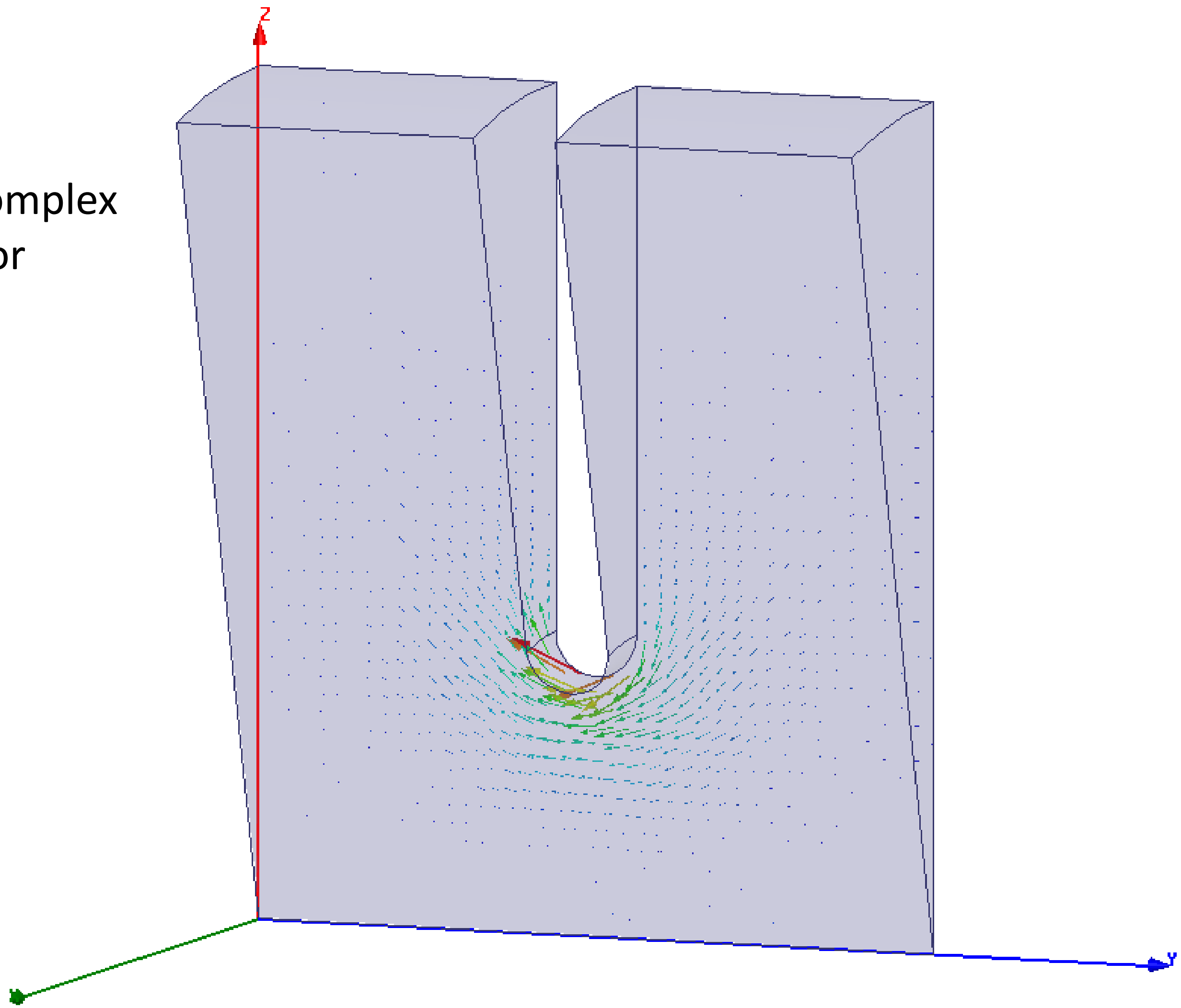
$$P = v_g W$$

And we can get the relationship between accelerating gradient *G*, voltage per unit length (which is valid over one cell length), and power flow,

$$G = \sqrt{\omega \frac{1}{v_g} \frac{R'}{Q} P}$$

Power flow in disk loaded waveguide 2π/3 phase advance

Real part of complex
Poynting vector



Now to meaningfully deal with travelling wave accelerating structures we will derive a differential equation which accounts for:

- power flowing along the structure
- power being transferred to the beam (acceleration)
- power being lost to the cavity walls

Seeing the derivation of the differential equation will give you insight into how to approach specific problems and give you practice using the terms we have introduced.

In our initial analysis, we are only going to consider steady state conditions. We will generalize later.

The general differential equation in terms of gradient

$$\frac{dG}{dz} = -\frac{G}{2}\left[\frac{1}{v_g}\frac{dv_g}{dz} + \frac{1}{Q}\frac{dQ}{dz} - \frac{1}{R'}\frac{dR'}{dz} + \frac{\omega}{v_g Q}\right] - \frac{IR'}{2}\frac{\omega}{v_g Q}$$

$$G(0) = \sqrt{\frac{P_{in} R' \omega}{v_g Q}}$$

Solutions in closed form

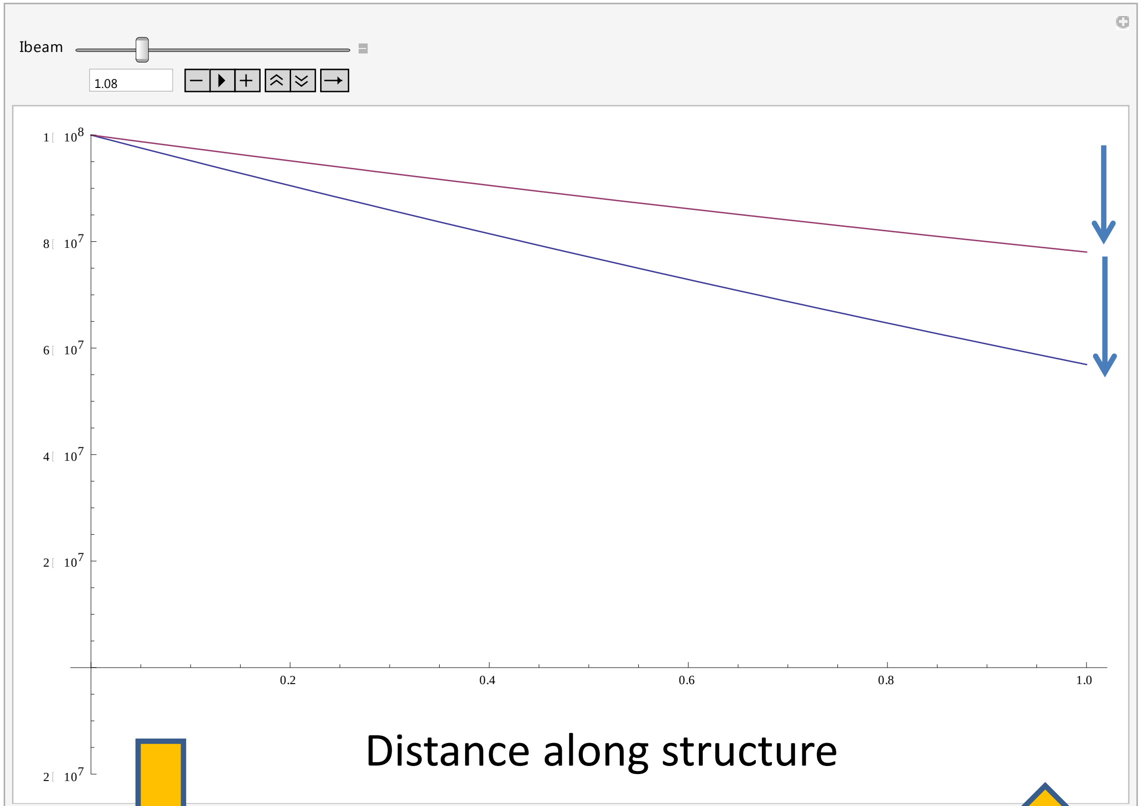
$$G(z) = G(0)\sqrt{\frac{v_g(0)}{v_g(z)}}\sqrt{\frac{Q\ (0)}{Q(z)}}\sqrt{\frac{R'(z)}{R'(0)}}\,e^{-\frac{1}{2}\int_0^z \frac{\omega}{Q(z)v_g(z)}dz}$$

$$G_{loaded}(z) = G(z)\left[1 - \int_0^z \frac{I}{G(z)}\frac{\omega R'(z)}{Q(z)v_g(z)}dz\right]$$

A. Lunin, V. Yakovlev, A. Grudiev PRSTAB, 14, 052001 (2011)

An example of the solutions to this equation for constant gradient (all cells are the same):

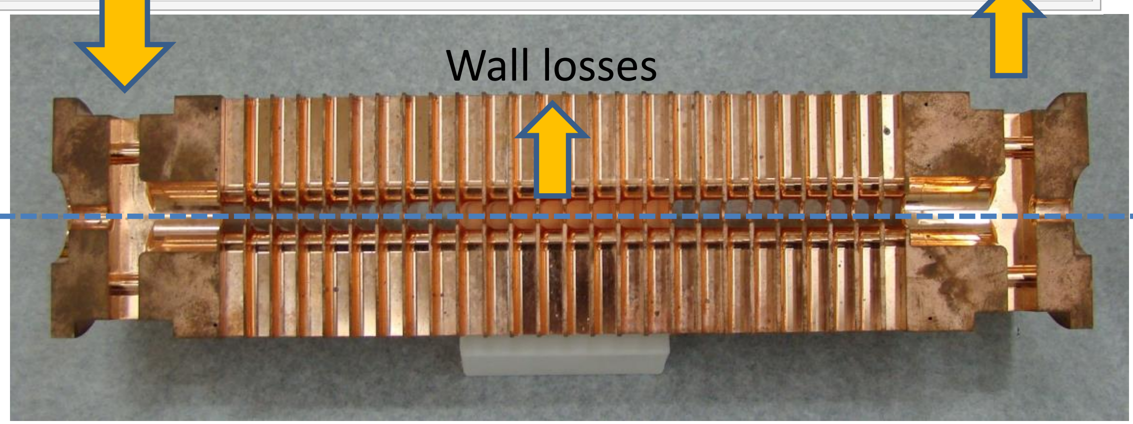
100 MV/m



Field goes down because of wall losses.

Field goes down because power goes into beam.

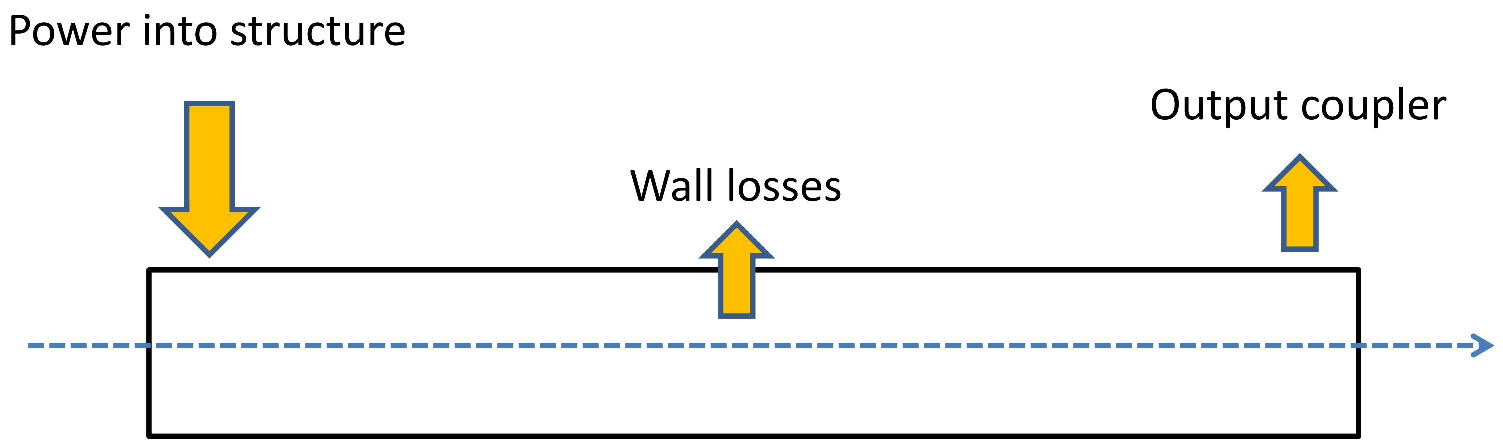
rf power in



Less power out

Beam accelerated

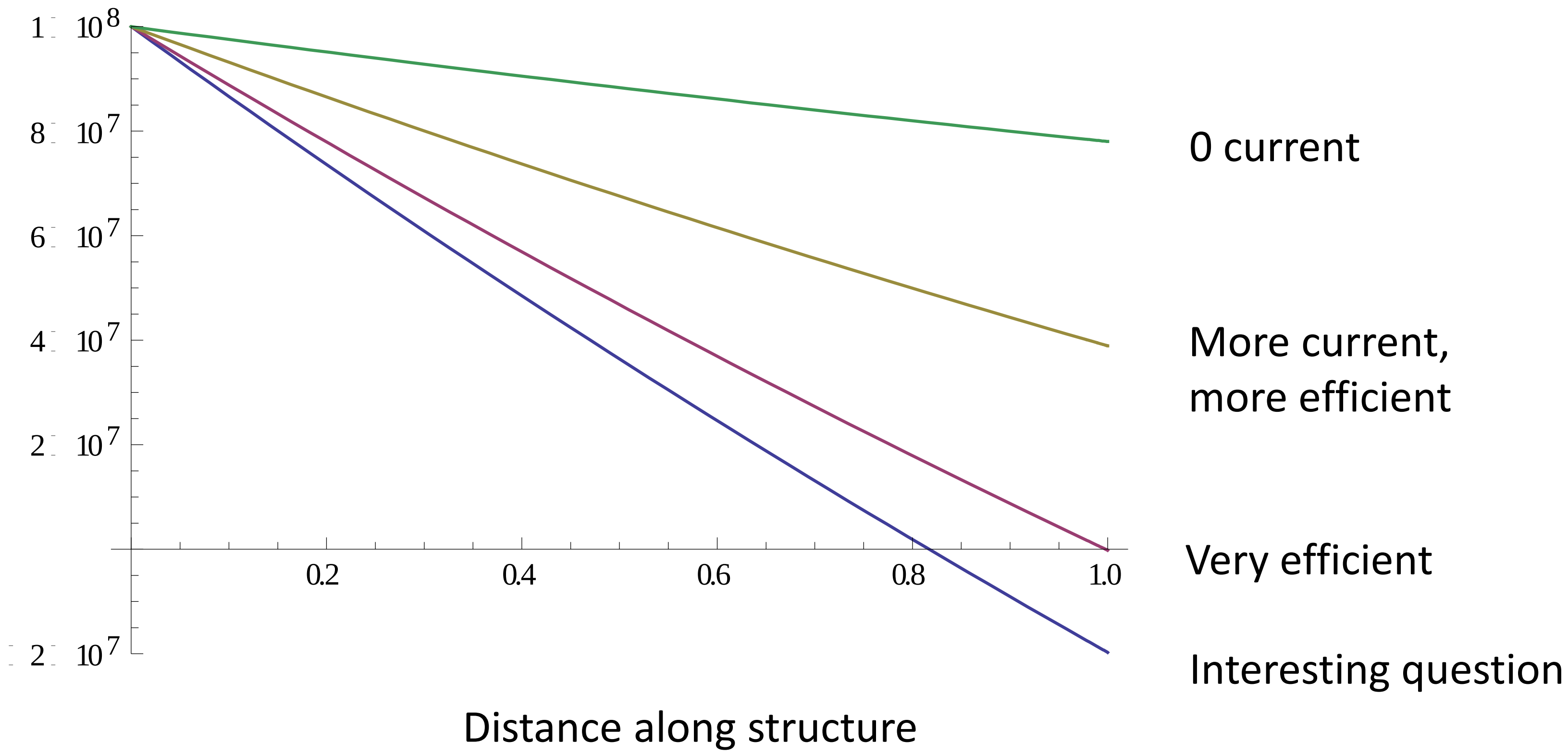
Now we ask ourselves – How efficiently have we converted rf power into beam power?
To ask it using accelerator jargon – What is the rf-to-beam efficiency?
This is one of the most important performance issues for a normal conducting linear collider since it directly affects the overall performance.

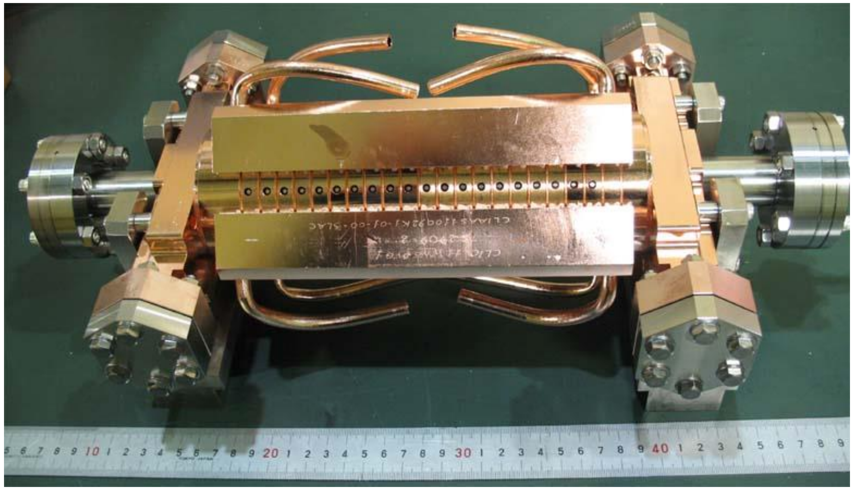


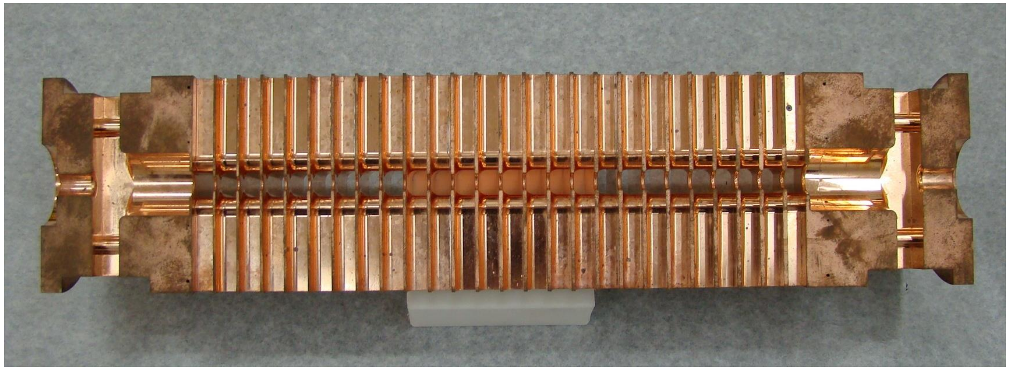
Power into beam $\Delta P_{beam} = I\langle G\rangle L$

$$\eta = \frac{\Delta P_{beam}}{P_{in}} = \frac{I}{P_{in}}\int_0^L G(z)dz$$

Different amounts of beam loading and efficiency

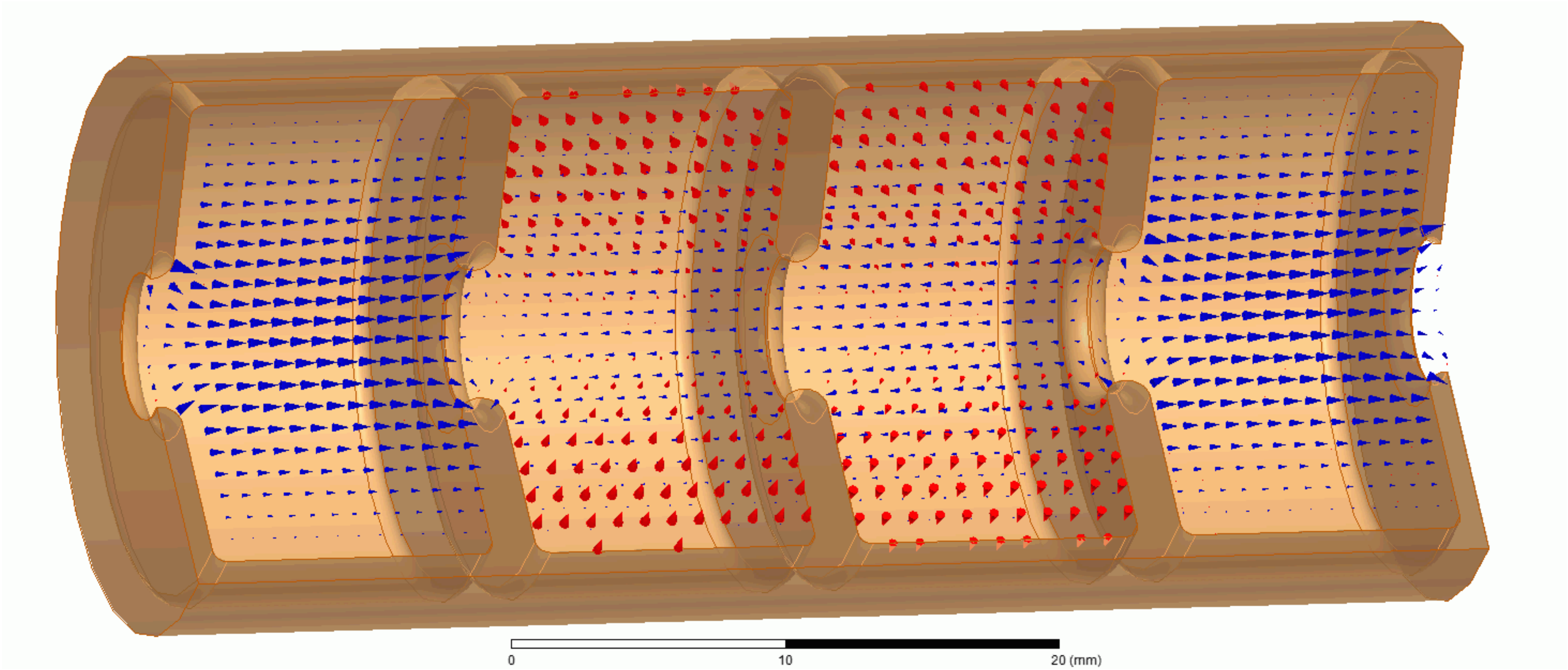






What the functional inside looks like.

A high-gradient accelerating structure.

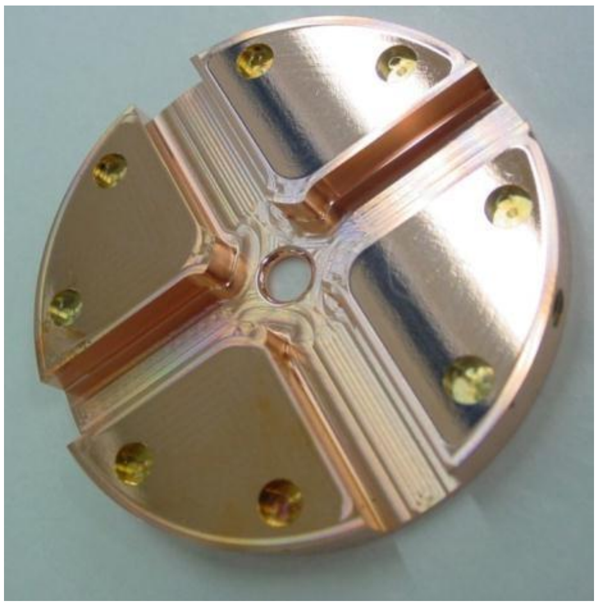


The behaviour of the electromagnetic fields we would like to understand.

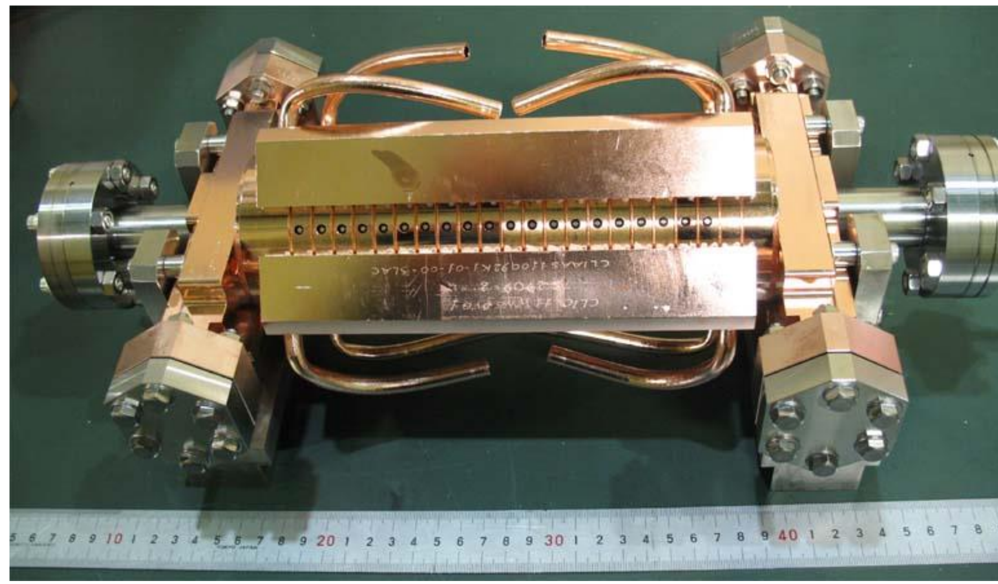
The CLIC accelerating structure

Now that you have a feeling for the basic mechanisms which underlie high-efficiency acceleration, we will look into the main features of the CLIC rf system.

Let’s start by looking at the CLIC accelerating structure:

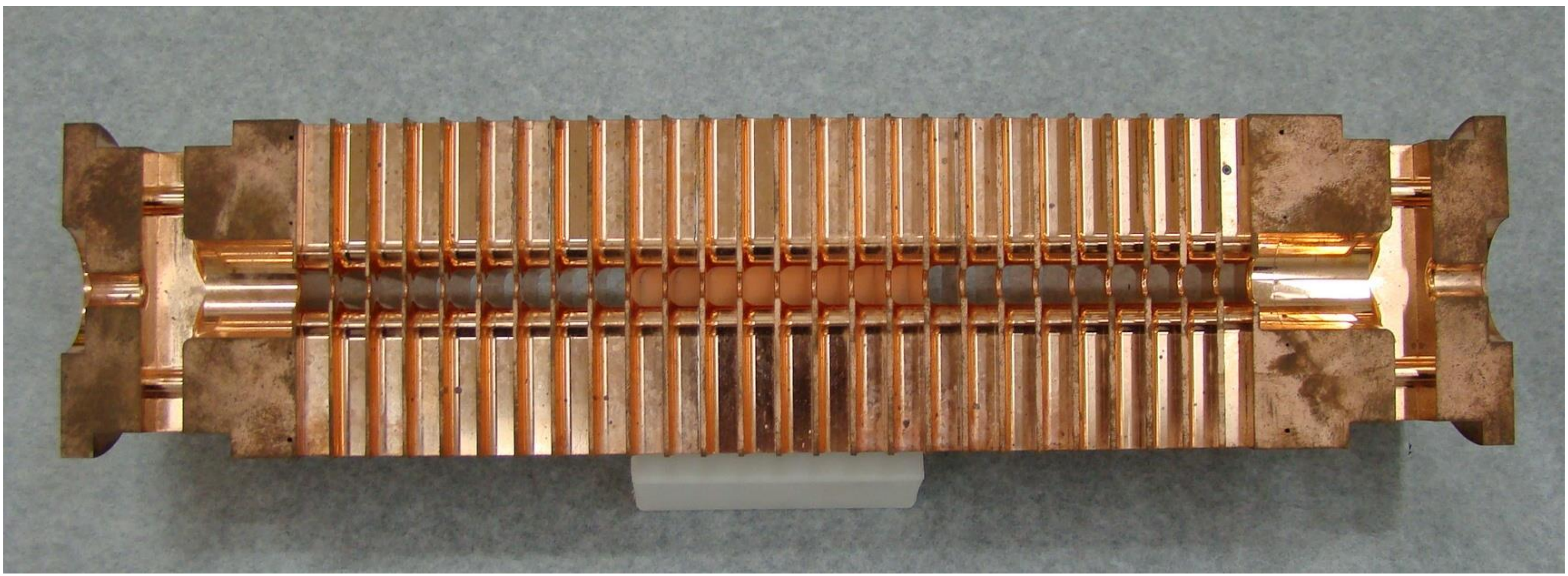


The basic component: diamond turned and milled disk. We form a periodic structure by stacking them. The radial lines are damping waveguides



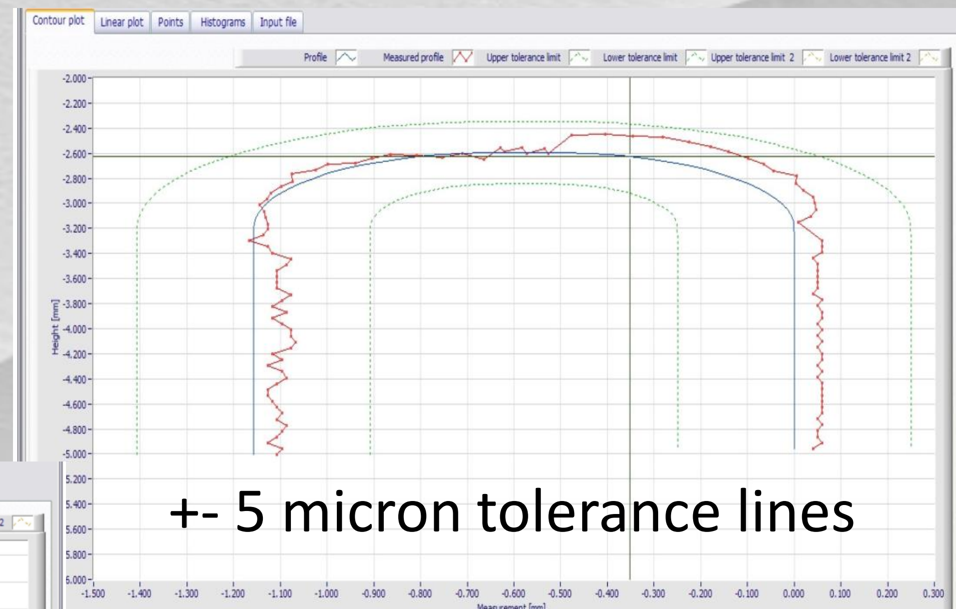
An assembled high-power test structure. Made in a collaboration between CERN, KEK and SLAC.

How it looks

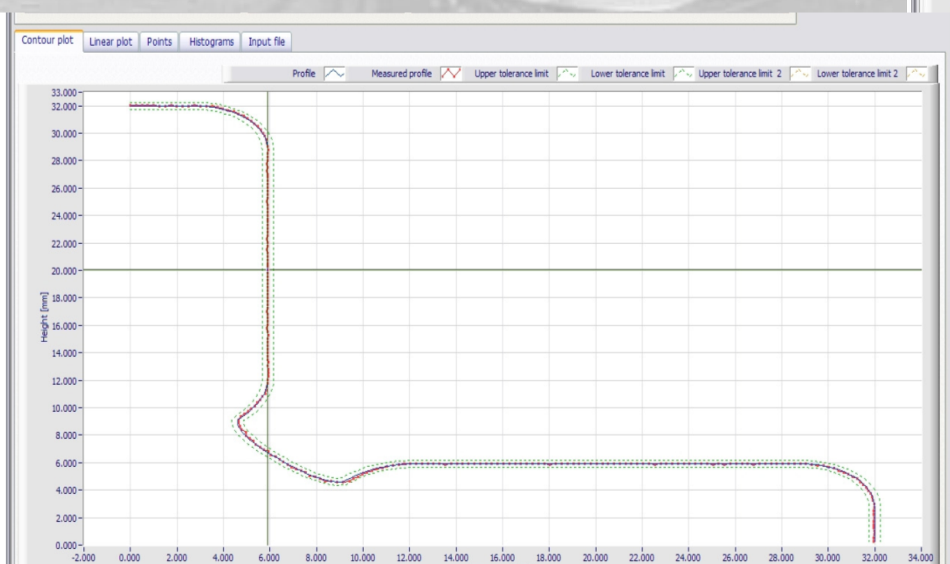


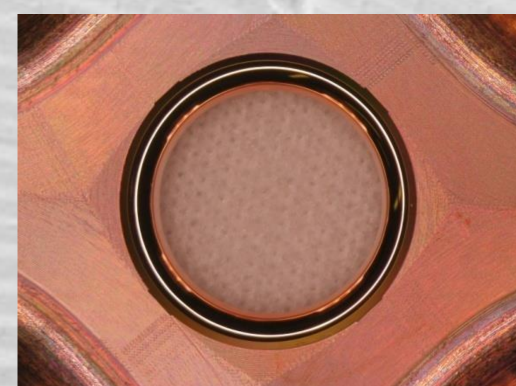
How to make 'em

Machining: OFHC copper diamond milled and turned disks with micron precision.



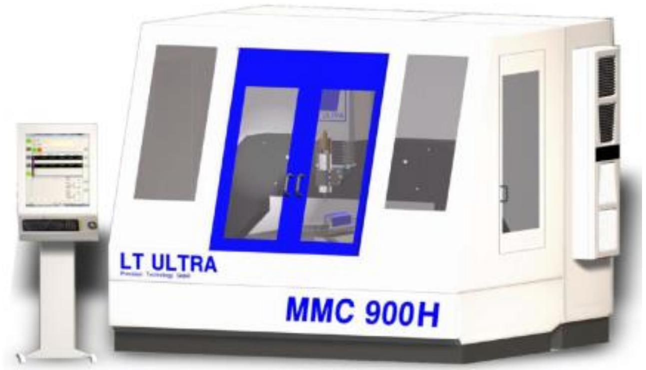
+- 5 micron tolerance lines



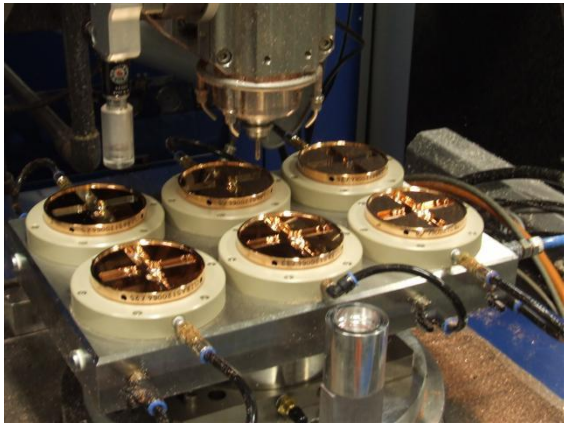


Micron precision turning and milling

- Accelerating structure tolerances drive transverse wakefields and off-axis rf induced kicks which in turn leads to emittance growth – micron tolerances required.
- Multi-bunch trains require higher-order-mode wakefield suppression – cells require milled features.
- High-speed diamond machining also seems to be beneficial for high-gradient performance through minimizing induced surface stresses.



Development done "in industry"



Evolution of machining capability

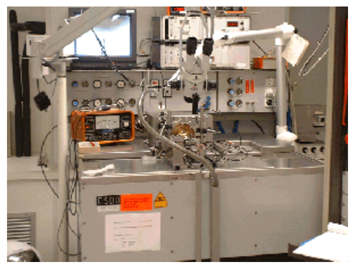
Single point diamond turning

Up to the 1980's

First machines at research institutes and universities

1980's - 1990's

Start of industrialization
- *Optical recording*
- *contact lenses*



2000's - 2010's

- *Larger machines*
- *Multiple axis (X/Y/Z and C)*

Future ?

- *Intelligent machines ?*
- *Robotisation ?*

Ultra precision diamond milling *(lagging more than a decade behind on turning)*

Up to the 1990's

Limited to fly cutting
- *mirror optics*
- *Laser scanner mirrors*

1990's - 2000's

Milling as add-on on lathes
- *Lens arrays*
- *Intra ocular lenses*

2010's

First proto type machines
- *Micro fluidics*
- *Accelerator parts*

Future ?

- *Pallet machining?*
- *Robotisation ?*