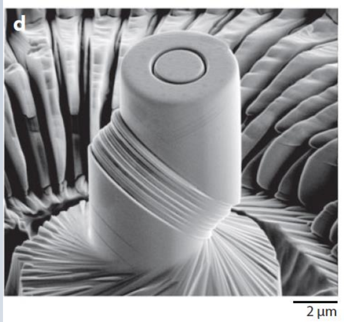
Early signals of breakdown through Stochastic modeling

Yinon Ashkenazy & Michael Assaf
Racah Institute of Physics,
Hebrew University, Jerusalem, Israel

Dislocation mediated – self organized criticality

Plasticity of Micrometer-Scale Single Crystals in Compression

Michael D. Uchic,[1] Paul A. Shade,[2] and Dennis M. Dimiduk[1]



Single crystal micro-pillar compression:

Dislocation mediated intermittent flow - size effects, hardening.

Dislocation density inside a plane as a controlling parameter.

Direct quantitative analysis of strain bursts (~20 micron).

Intermittency characterized by a universal Power law burst PDF

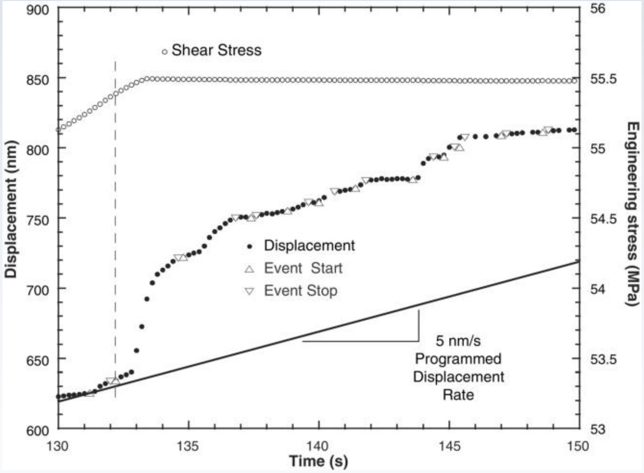
Acoustic emissions:

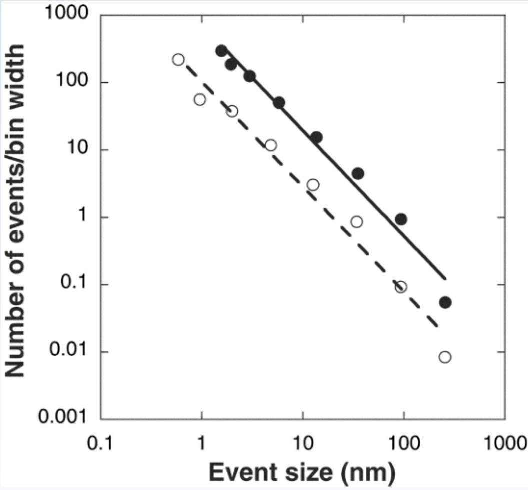
Similar + space and time coupling between events

(Weiss & Marsan, Scjence 2003)

Earthquakes show similar PDF and spatio-temporal correlation

(Kagan, Geopgysical J. (2007)

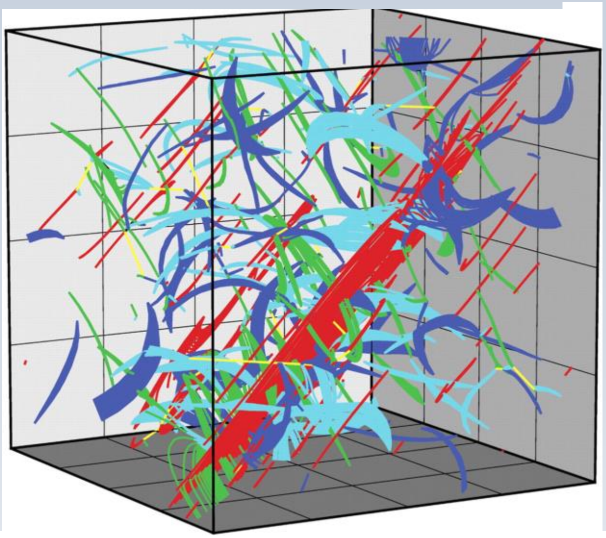


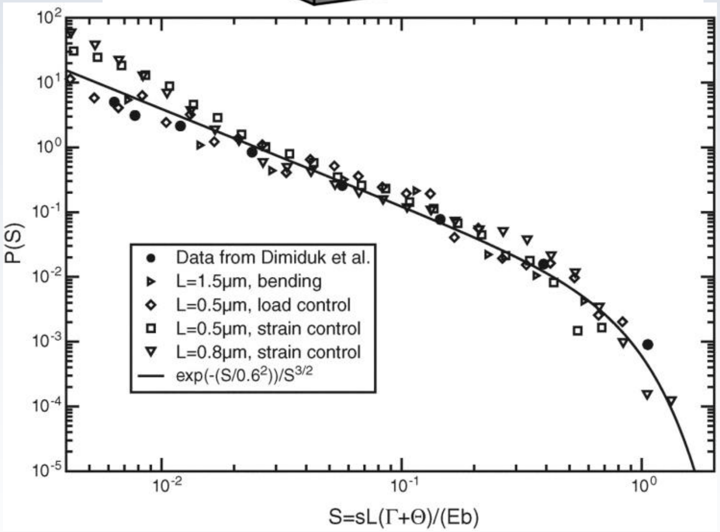


Uchic, Shade & Dimiduk, Annual Review of Materials Research (2009).
Dimiduk, Woodward, LeSar & Uchic: “Scale-Free Intermittent Flow in Crystal Plasticity.” Science (2006) 1188.

Using dislocation dynamics to reproduce PDF

- 3D dislocation dynamics reproduce strain burst scaling $$P(s) = Cs^{-\tau} \exp\left[-\left(s/s_0\right)^2\right]$$
- where C is a normalization constant, τ is a scaling exponent, and s_0 is the characteristic strain of the largest avalanches.
- Intermittency – as a result of dislocation Interactions. Stochastic nature a result of varying initial conditions.
- Avalanche is a 2D event, with an upper cutoff due to structure and work-hardening. Strain is limited to about 10^-6 in a cm size sample.
- Recently (Chen, choi, papanikolaou & Sethna 2010 to 2013): scaling of structures using an advanced CDD code.





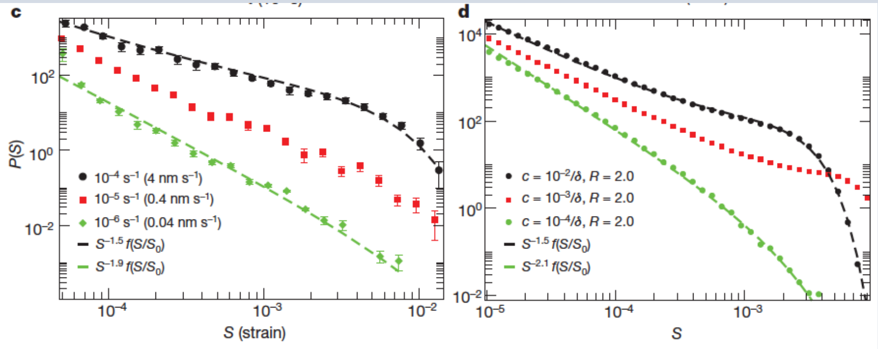
Csikor, Motz, Weygand, Zaiser & Zapperi, "Dislocation Avalanches, Strain Bursts, and the Problem of Plastic Forming at the Micrometer Scale" . Science (2007)

Mean field models for critical depining

Quasi-periodic events in crystal plasticity and the self-organized avalanche oscillator

Stefanos Papanikolaou[1], Dennis M. Dimiduk[2], Woosong Choi[3], James P. Sethna[3], Michael D. Uchic[2], Christopher F. Woodward[2] & Stefano Zapperi[4,5]

- Reproduce strain rate variation by modifying the mean field picture to include a competing relaxation mechanism. This lead to oscillation in avalanche size. (nature, 2012)



PRL 109, 105702 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

Distribution of Maximum Velocities in Avalanches Near the Depinning Transition

Michael LeBlanc,[1] Luiza Angheluta,[1,2] Karin Dahmen,[1] and Nigel Goldenfeld[1]

- Using a mean field model for interface depining and by solving Fokker-Planck eq. reproduced the power law decay of avalanche size and maximal velocity

$$\frac{dV}{dt} = -kV + F_c + \sqrt{V}x(t)$$

What are we trying to do…

- Use stochastic theory to allow for:
  - transferability of failure scenario analysis (across drive conditions)
  - Identify controlling mechanisms
  - Define critical experiments - model development / verification

  Such models serve as a link between the microscopic, short time scale problem which is accessible via simulation to the measured system to the real life scenario.

For now – demonstrate the basic method using a "spherical horse" model.

Not trying to do (at this stage):

Create a comprehensive consistent microscopic model

Describe the "real" mechanism at work

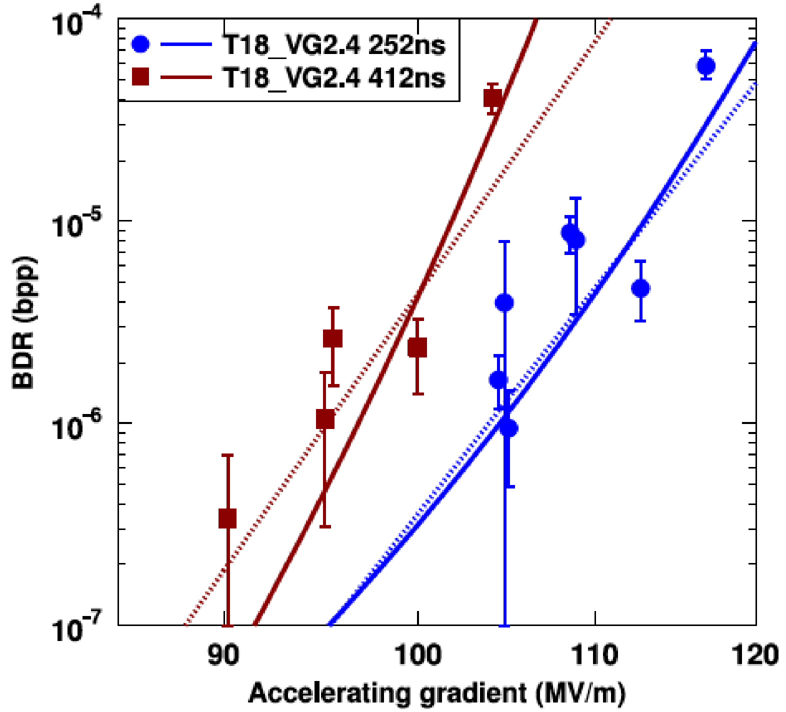
Solve the full field - structure – current response function

DC vs RF

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **15**, 071002 (2012)

Defect model for the dependence of breakdown rate on external electric fields

K. Nordlund and F. Djurabekova



$$\frac{1}{t_{RBD}} \approx \exp\left[E^2 \Delta V/\left(k_B T\right)\right]$$

FIG. 4. Measured dependences of R_{BD} (in units of breakdown per pulse, bpp) versus electric field for the T18 accelerating structure [33,43] and fits of our model (solid lines) as well as power laws (dashed lines) to the data.

Formulation of a “well-mixed” 0d model

- Assumptions:
  - Breakdown currents are driven by formation of surface extrusion/intrusions.
  - Surface protrusions are formed due to multiple dislocation reaction leading to local geometric features
  - Sub-breakdown surface protrusion are not identified (true?).
    Breakdown rates do not go up with time (BDR even goes down...).
    Therefor we assume that gradual protrusion accumulation does not control breakdown:
    - surface relaxation, interaction between various slip systems, protrusion-dislocation interaction…
    - Field conditions are translated to an applied stress
      (AC thermal gradients ~ 100 Mpa, dc?)

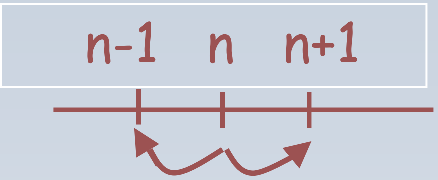
- Suggested controlling parameter -
  the number of mobile dislocations inside a band.
  - If large amount of dislocations reach the surface in unison – an instant extrusion/intrusion may lead to breakdown.
  - We avoid spatial interaction and assume gain-loss dynamics inside a specific band.

General gain-loss type Markovian processes

Rates for transition between states

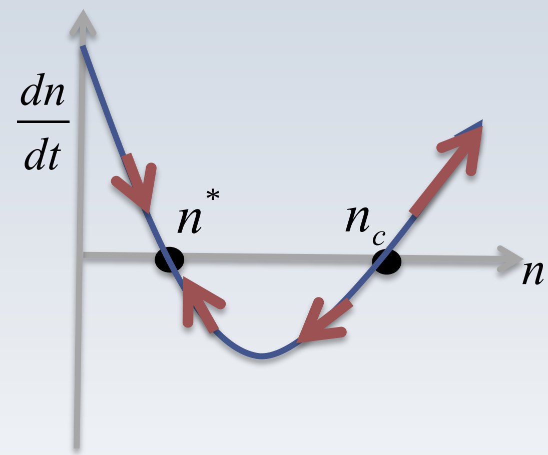
$$n \xrightarrow{\rho_n^+} n+1$$

$$n \xrightarrow{\rho_n^-} n-1$$



The master equation

$$\dot{P}_n = \rho^+_{n-1} P_{n-1} + \rho^-_{n+1} P_{n+1} - \left(\rho^+_n + \rho^-_n\right) P_n$$



can lead to bifurcation:
a metastable state and a critical one.

We look for the quasi-stationary probability distribution function
And the probability to cross the critical point (reach extinction)

Approximate solution based on WKB theory with 1/N being the small parameter.

$$\dot{P} = 0 \Rightarrow P(n) \equiv P(\rho N) \sim e^{-N[S(\rho)+O(1/N)]}$$

Assaf and Meerson, PRE 2010

“Minimal” model

- Define the “in-plane” density (in units of 1/nm).
- External stress (due to temp gradient on surface), range of 0.1 Gpa.
- Mobile dislocations can increase in number due to stress gradient (the driving force) as well as thermal activation of the multiplication reaction

$$\frac{dr^{+}}{dt} = n_0 G A \left(s(r)\right)^{k} e^{-(f_0 - sW)/k_B T}$$

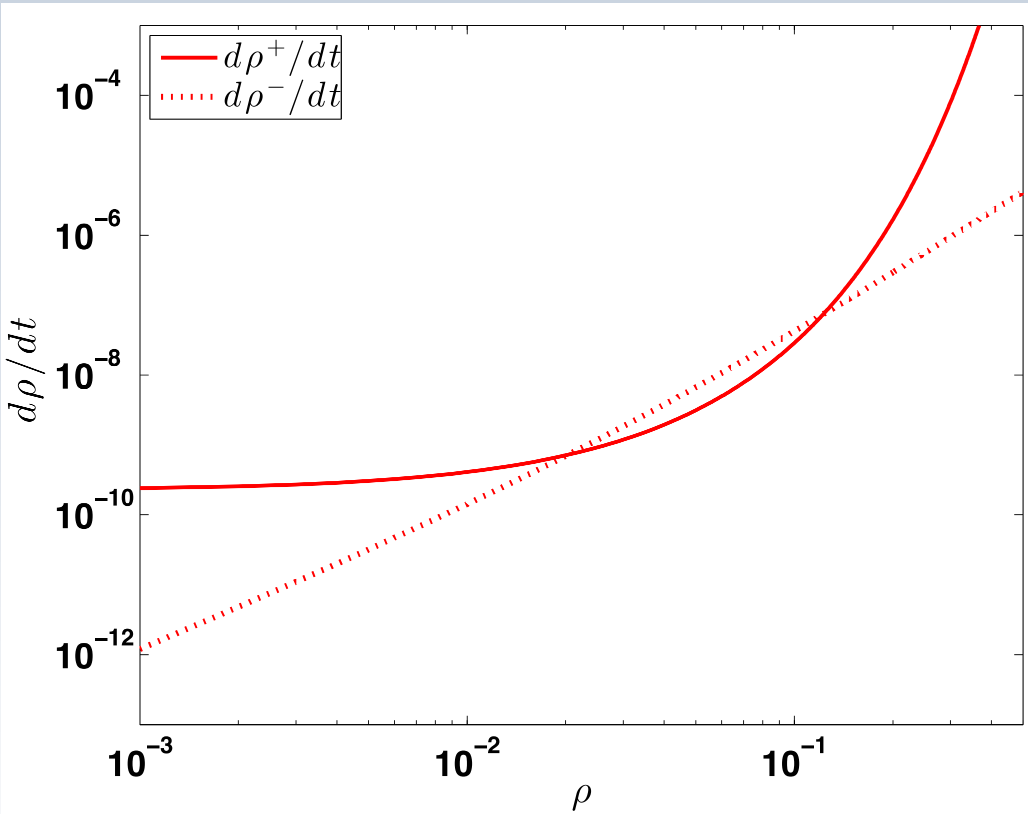
- Moving dislocations can become sessile at:
  - Pre-existing barriers (concentration - C)
  - “collisions” with other moving dislocations

$$\frac{dr^{-}}{dt} = r V(s) c + r 2V(s) r$$

- Properties dependence:
  - Velocity increase with stress, independent of the number of moving dislocations
  - Stress increase with dislocation content

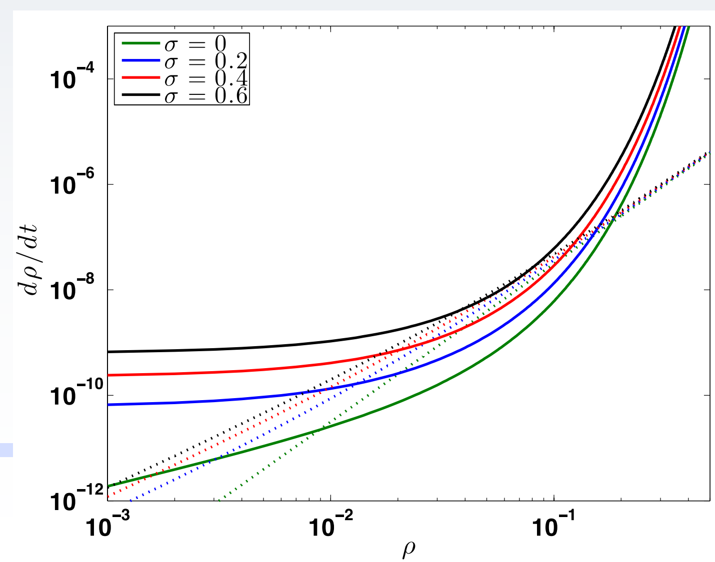
$$s = s_E + G r b \qquad V = B n_0 b^2 s$$

Model characteristics

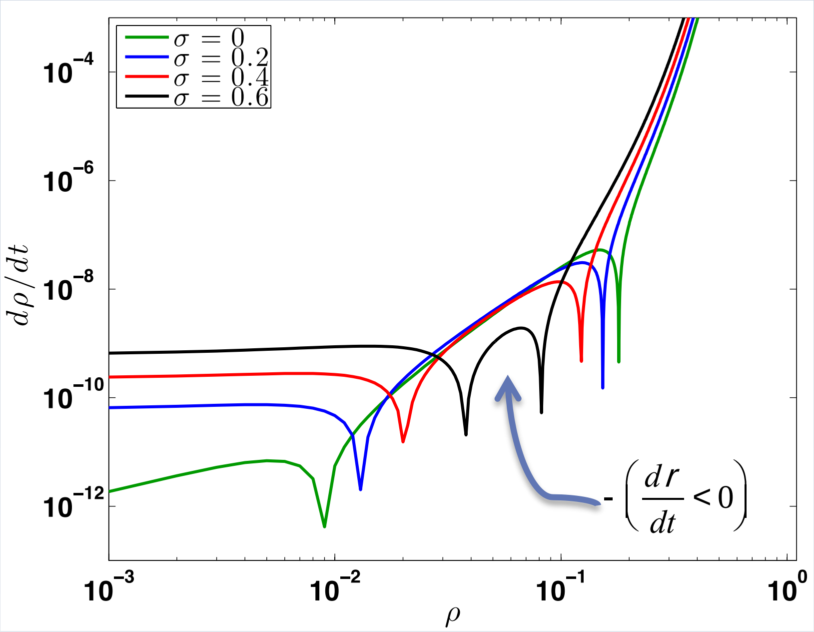


Stable point:
Dislocation generation and annihilation identical

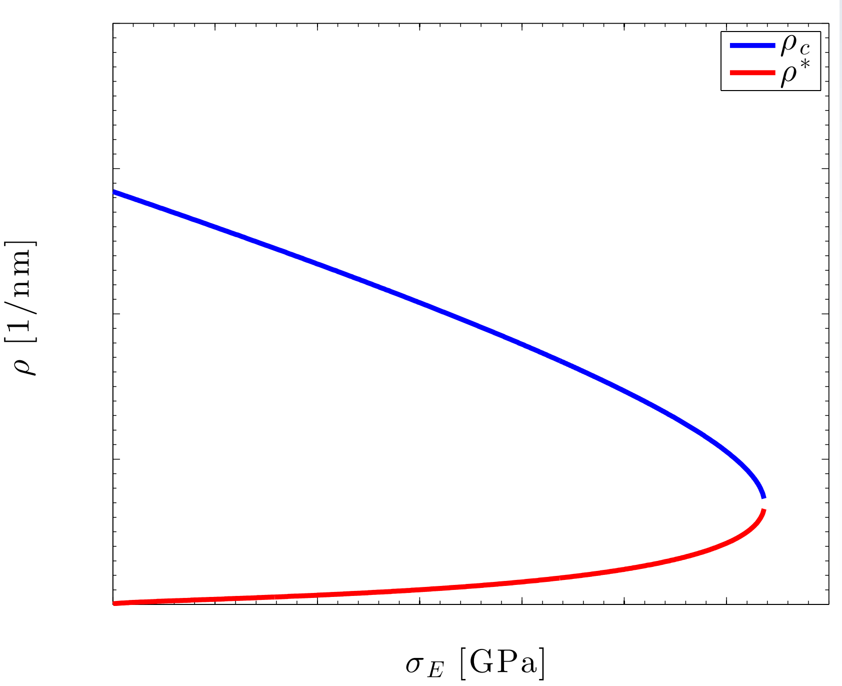
Critical point – threshold for dislocations avalanche



Low stresses:
Mobile dislocation density remains in Metastable solution.
Dynamic barrier decreases with increasing stress.



Up to a critical stress – bifurcation to two solutions.
Above it - no stable solution.



Define : $$s(\rho) = -\int_{\rho} \ln \frac{\rho^{+}(x)}{\rho^{-}(x)} dx$$

For (k=2): $$s(r) = -r \ln\left[\frac{r^{+}(r)}{r^{-}(r)}\right] - \frac{s_E}{bm} \ln\left[s(r)\right] + \frac{c}{2}\ln(c+2r) - r\left(1 - \frac{r\,b\,mW}{2k_B T}\right)$$

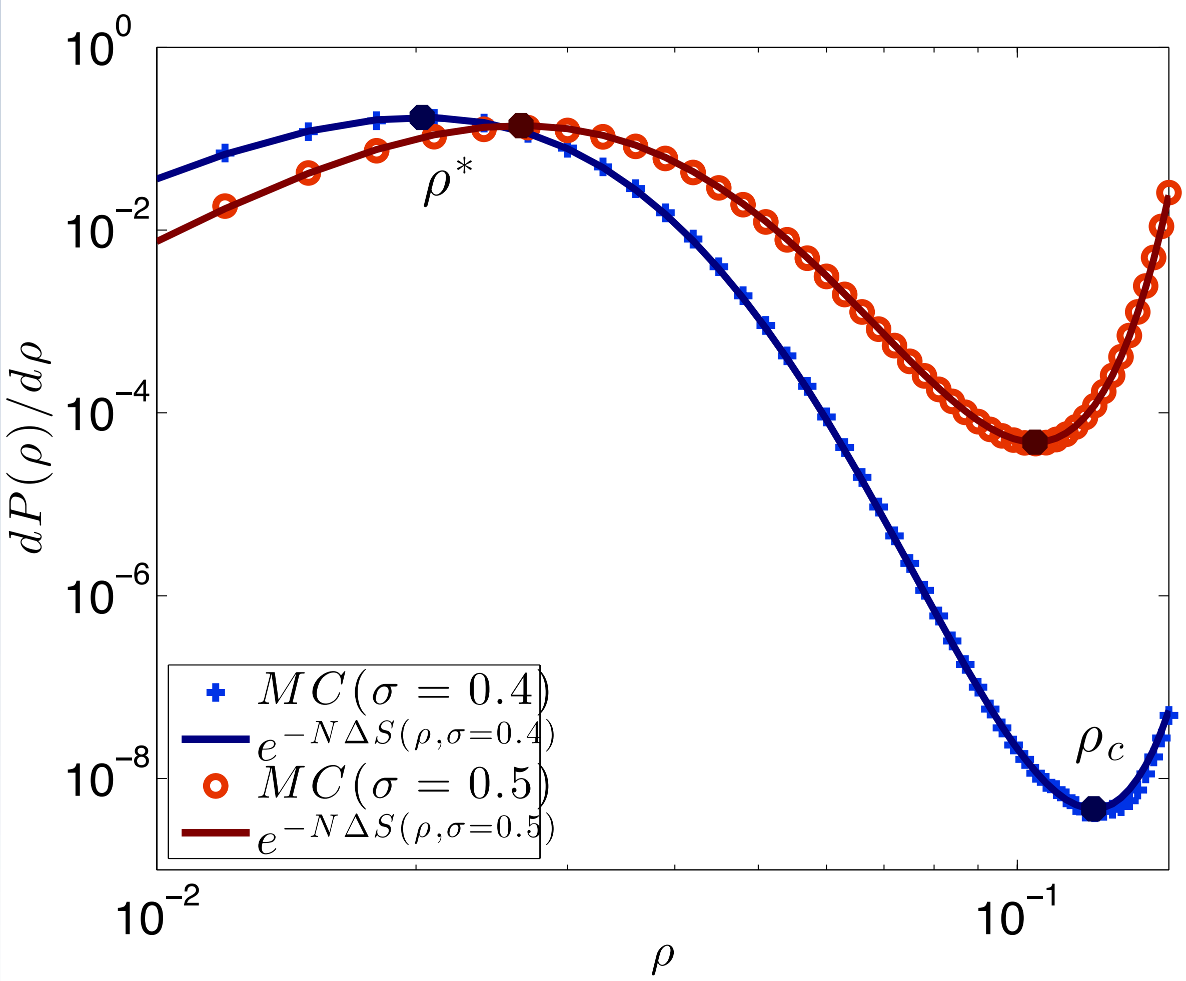
Using*: $$P(n = N r) = P(r) \sim e^{-Ns(r)}$$

Leads to $$P(r) \propto \left[\frac{r^{+}(r)}{r^{-}(r)}\right]^{Nr} \frac{\left[s(r)\right]^{Ns_E/bm}}{(c+2r)^{Nc/2}} e^{Nr\left(1-\frac{r\,b\,mW}{2k_B T}\right)}$$

And the normalized PDF $$P(r) = \underbrace{\sqrt{\frac{S''(r^{*})}{2pN}}}_{P(r*)} e^{-N[s(r)-s(r^{*})]}$$

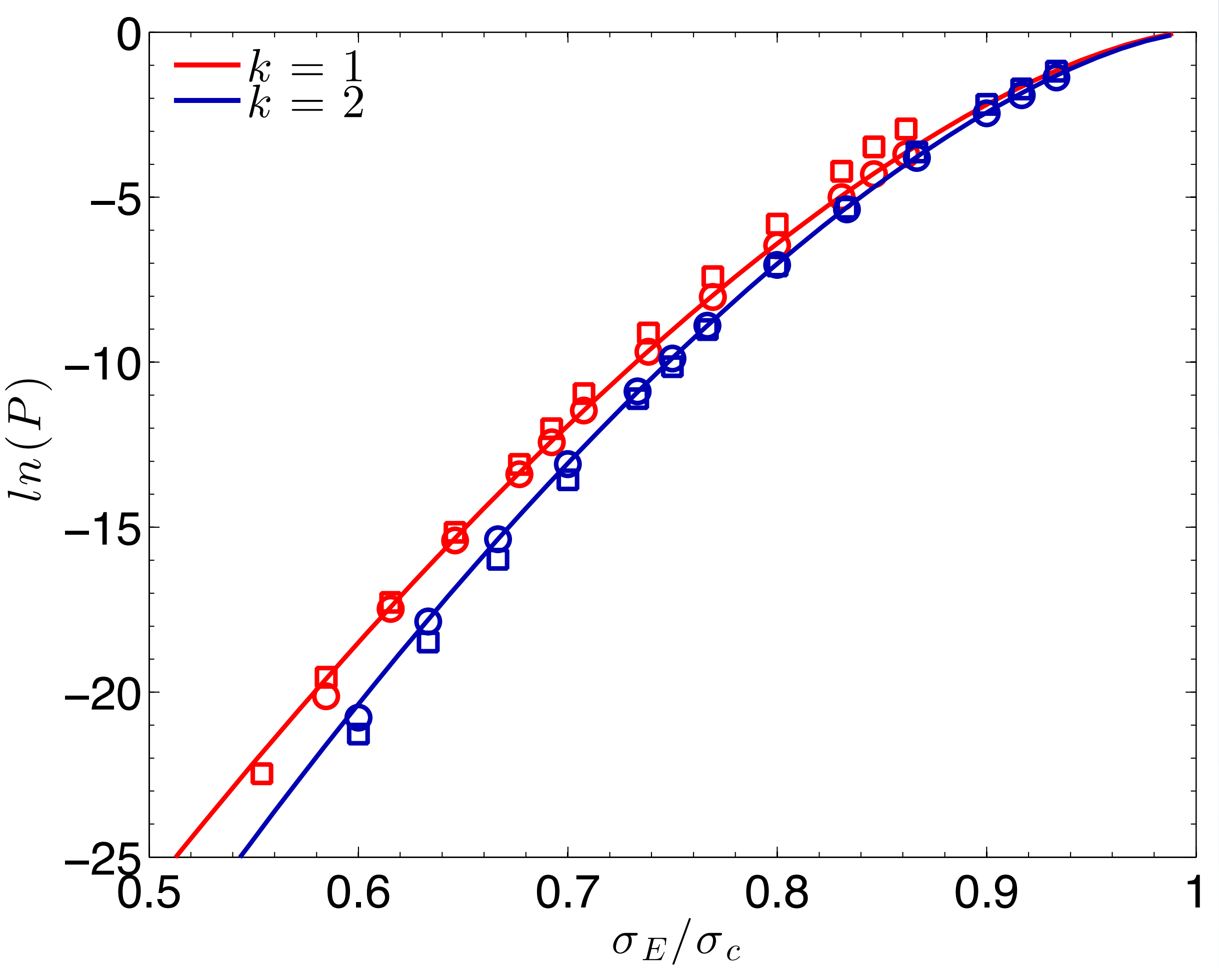
*Assaf and Meerson, Phys Rev. E 21116 (2010)

Analytical vs MC



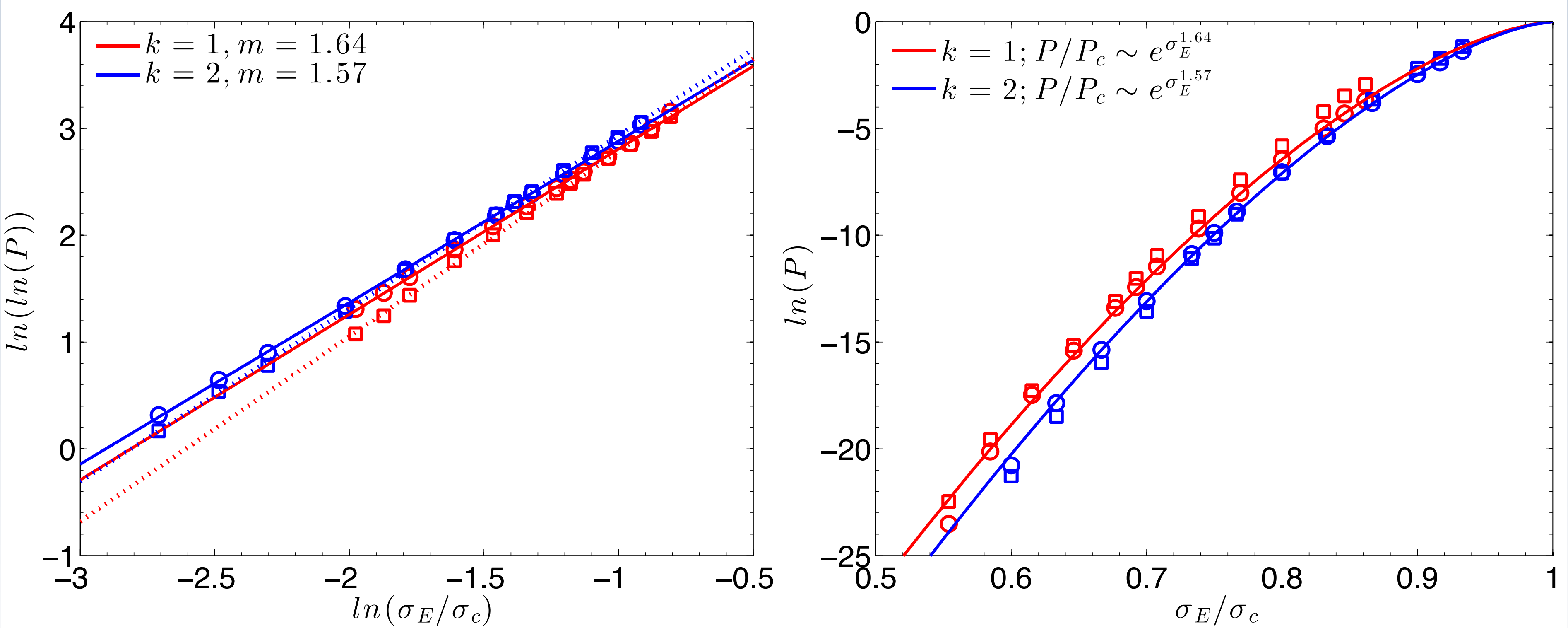
- Analytical analysis reproduces full PDF.

Getting BDR curves



- Analytical solution for relative probability to reach critical point.
- Normalized probability and rate for reaching the critical state.

“Universal” BDR exponent

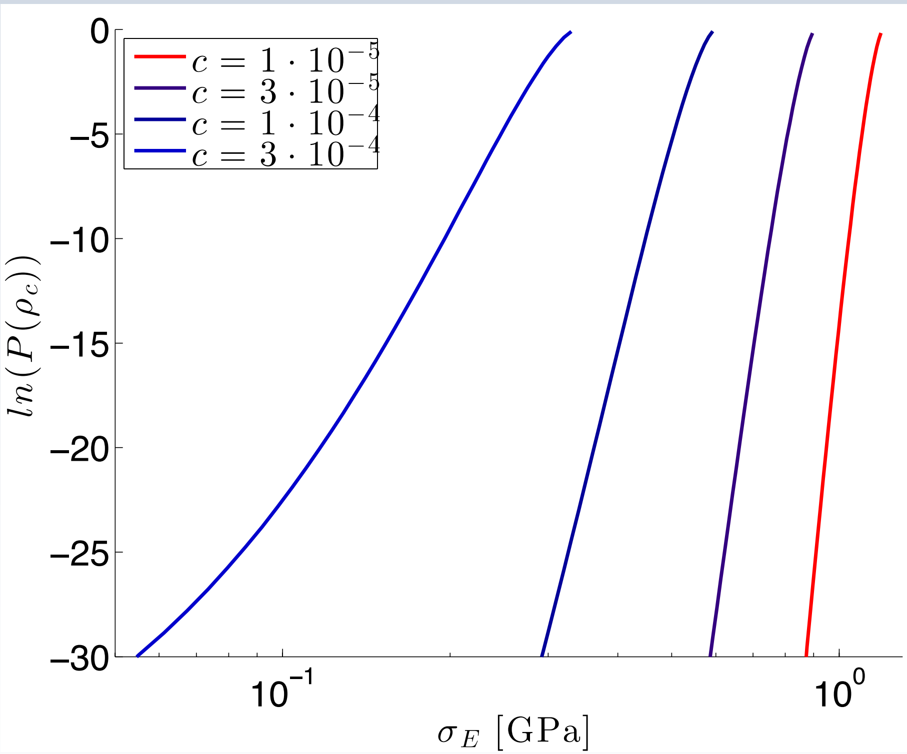


- fit to experimental results

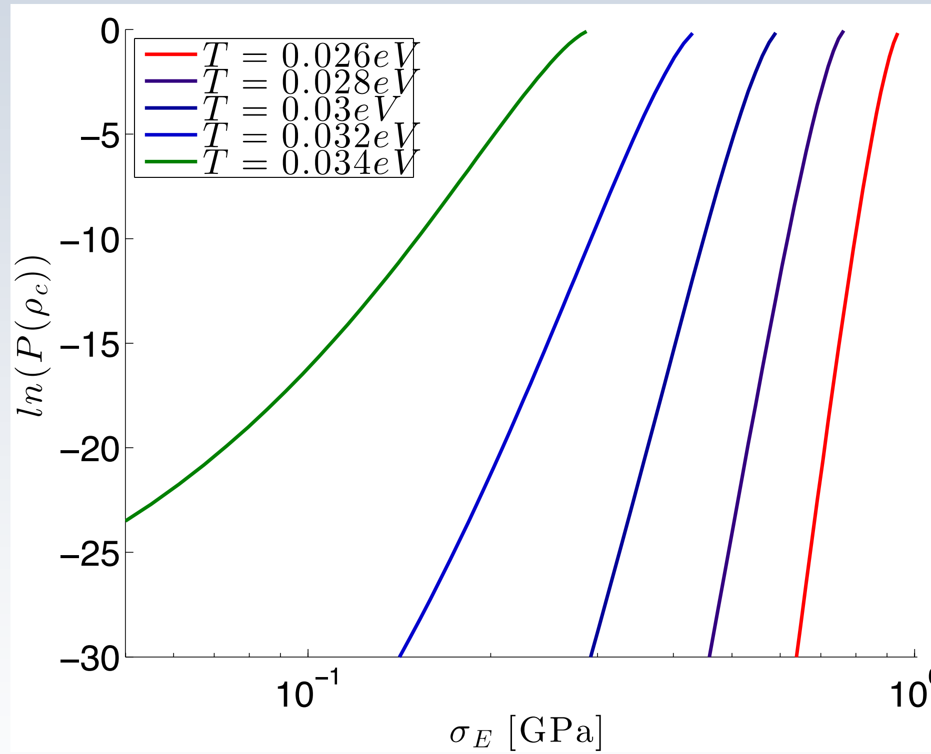
$$P \sim \frac{1}{\tau} \sim \exp\left(\sigma_E^{1.6}\right)$$

Other dependencies

Employ analytical solution to various scenarios



Dependency on mobile dislocation generation pre-factor



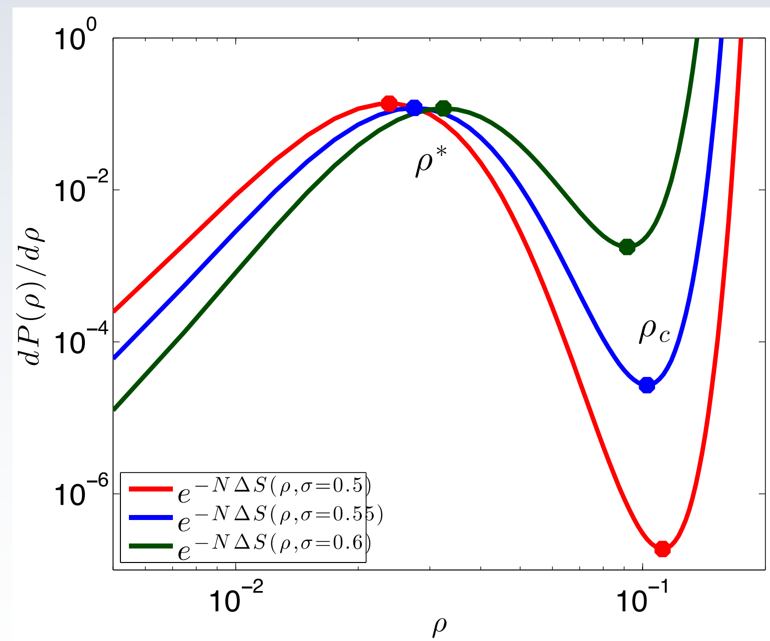
Temperature dependence.

Signs of criticality

- Adiabatically moving between quasi-stationary PDF:
  Change in pdf moments with field
  -> identify threshold

- At specific conditions, probe time dependencies of the QS pdf:
  Identify large fluctuations time dependency
  -> identify time constants
  -> mechanism

$$P_c(s,r,t) = \int_0^t P(s, r' > r, t')\,dt'$$



PRE-breakdown

- As the system approaches the critical point. Fluctuation diverge.

- Observable through standard deviation of the time correlation

$$SD(t) = \frac{\int_{t-\Delta}^{t+\Delta} (I(t) - <I>)^2 dt}{(<I>)^2}$$

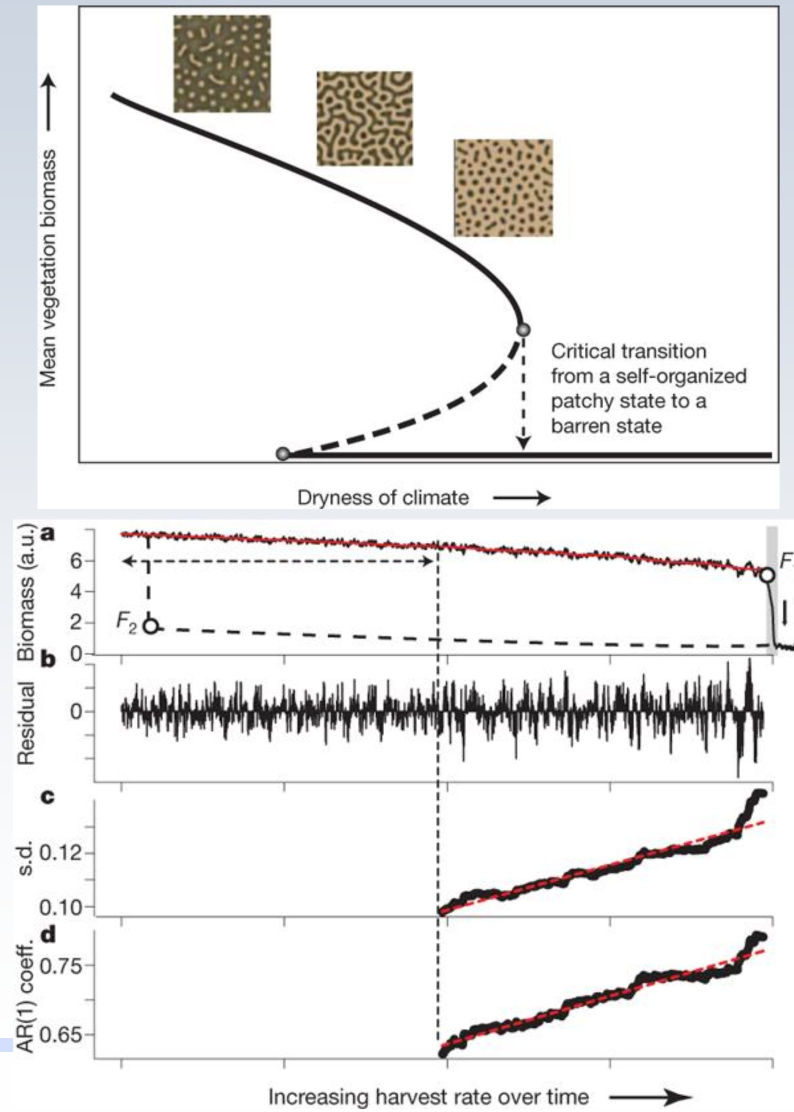
- Or, more generally, autocorrelation in the signal

$$R(k) = \frac{\int_0^{t-k} (I(t) - <I>)(I(t+k) - <I>) dt}{\int_0^{t-k} (I(t) - <I>)^2 dt}$$


Vol 461|3 September 2009|doi:10.1038/nature08227 nature
REVIEWS

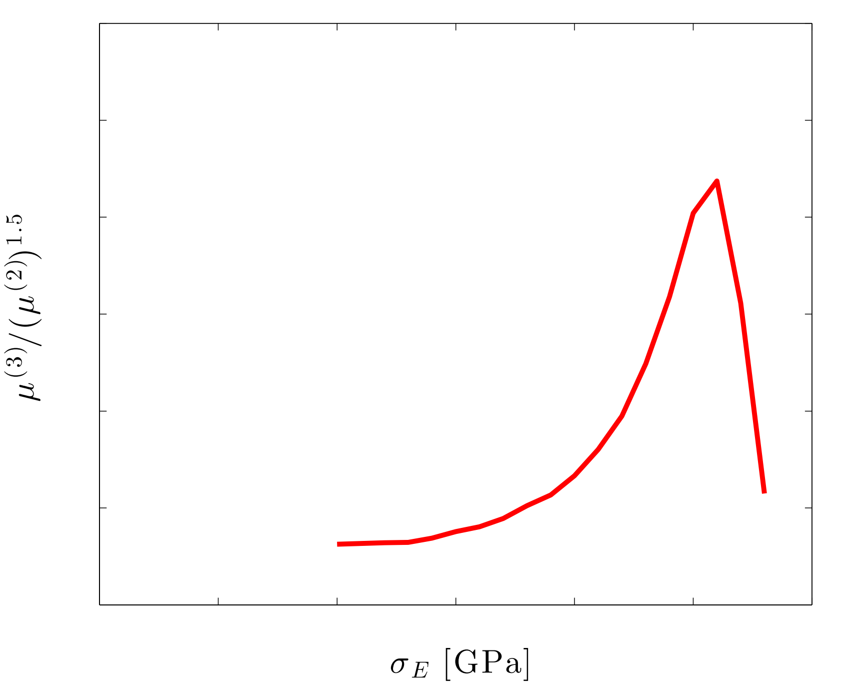
Early-warning signals for critical transitions

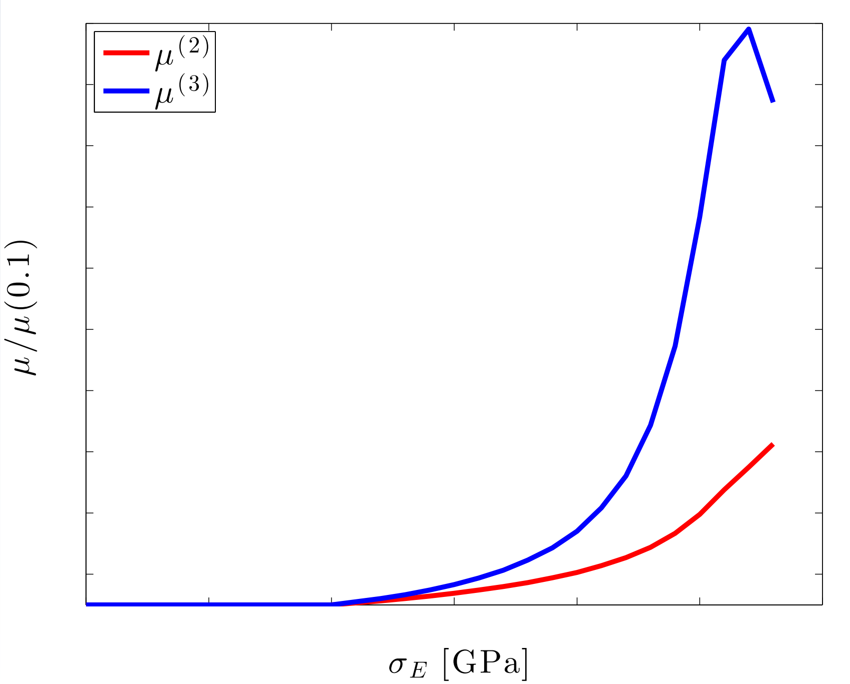
Marten Scheffer[1], Jordi Bascompte[2], William A. Brock[3], Victor Brovkin[5], Stephen R. Carpenter[4], Vasilis Dakos[1], Hermann Held[6], Egbert H. van Nes[1], Max Rietkerk[7] & George Sugihara[8]



Early warning signals

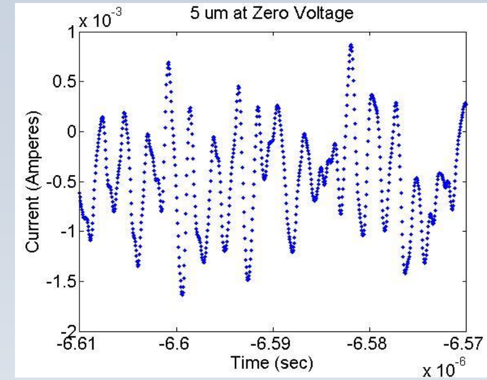
- Using moments $\mu^{(n)} = \sum (r - \langle r \rangle)^n$
- Variance and third moment indicate to Strong variation of quasi-stationary PDF - in breakdown states



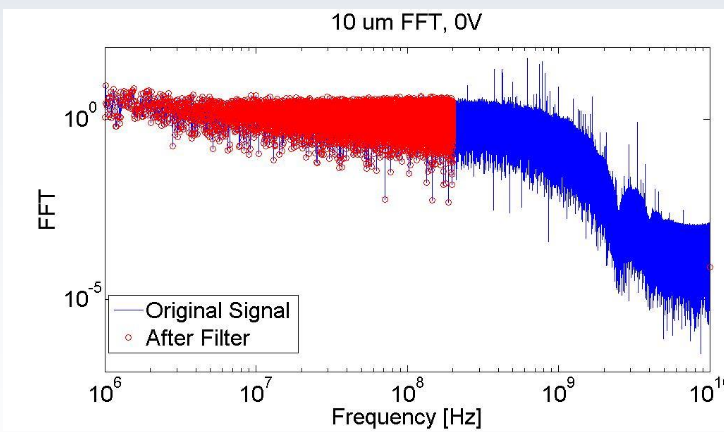


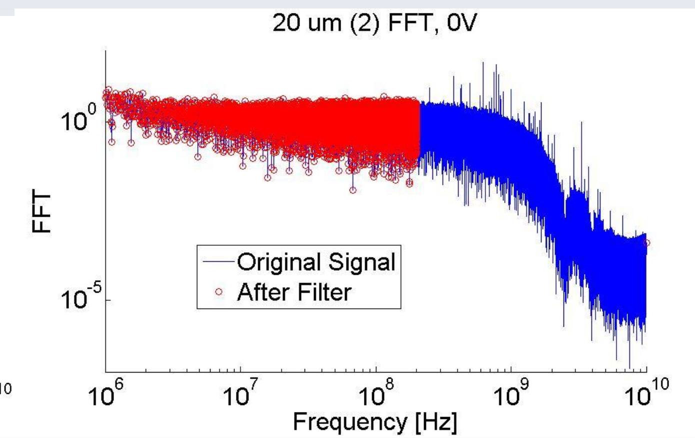
Initial try – DC measurements

- Dark current measurements (Varying field and gap distance)



- Low pass filter is clearly needed (applied 0.2 GhZ)

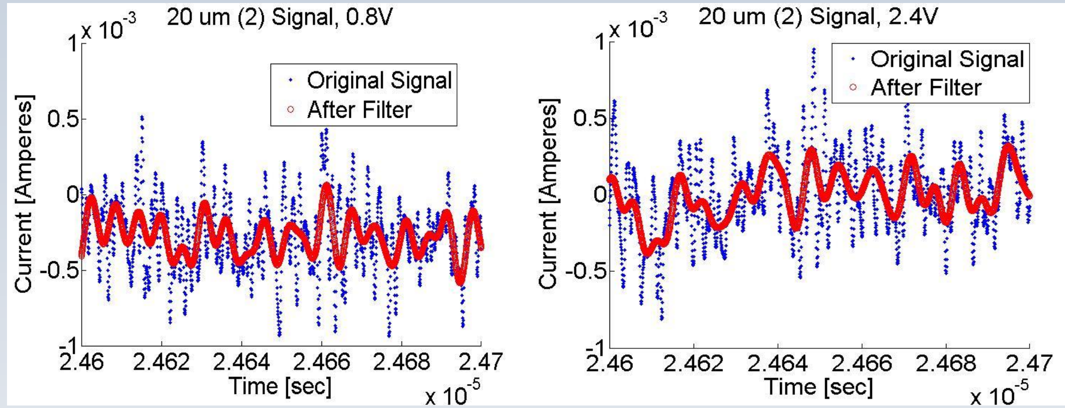




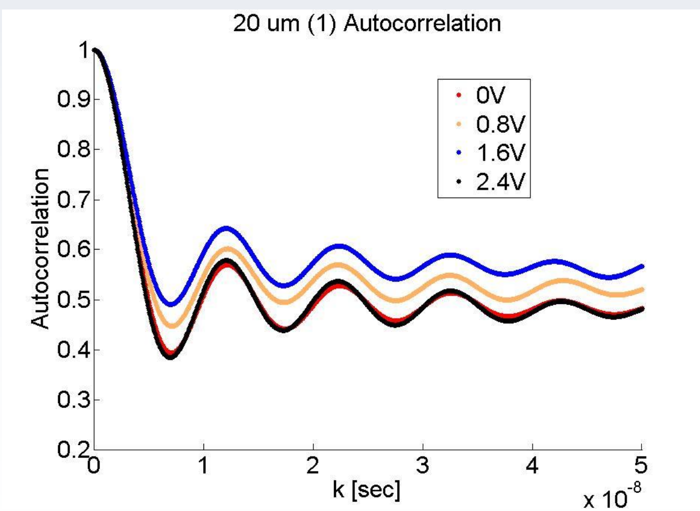
Nick Shipman, Adar Sharon

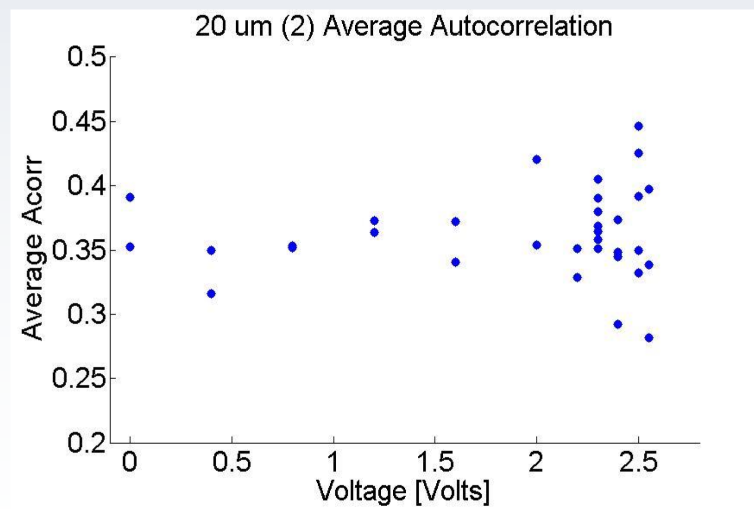
Initial try – DC measurmanets

- Filtered signal



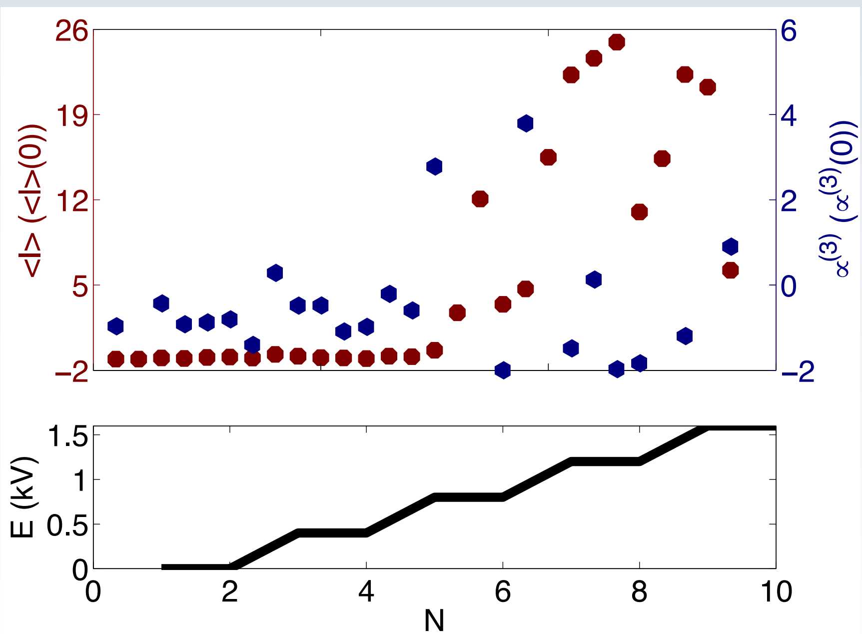
- Sadly … autocorrelation shows nothing…

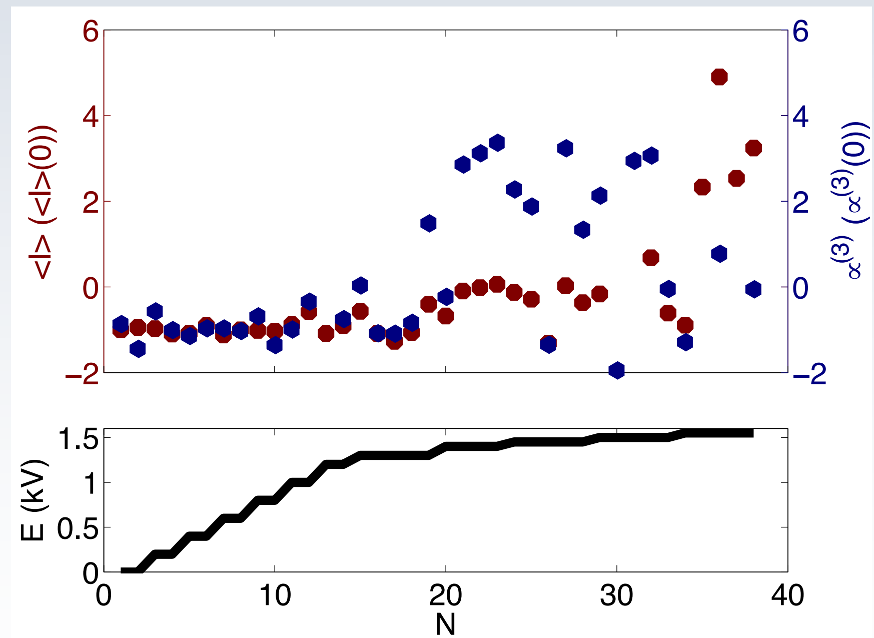




Normalized average and third moment

- Mean and third moments variatons with field.
- Need for better measurements ...





Summary

- Intermittency due to collective dislocation response is well established.
  - Experimental scenarios: acoustic emission, micro-compression.
  - Universal behavior, - earth quakes, other non local bifurcating systems.
- Proposed a simple stochastic model to describe breakdown phenomena
  - Using a minimal model – MANY simplifying assumption – demonstrate critical behavior, bifurcation and reproduce observed BDR (E) .
  - Analytically (or at least numerically) solvable
  - reproduce observed BCR exponent. Universality?
- Unique experimental scenarios:
  - PDF - pre breakdown analysis: using PDF and PDF tail.
  - Pre breakdown fluctuation.
- Future directions
  - Effects of intrinsic noise?
  - Join forces with experimental work to identify signs of criticality.
  - Suggest a modified conditioning scheme .

- Can serve to bridge microscopic mechanisms to experimental scenarios. – need for an explicit response function.
- New opportunity for stochastic analysis…