Determination of X(3872) quantum numbers

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Tomasz Skwarnicki
(Bin Gui's thesis)

Outline

- Standard and exotic hadrons
- X(3872)
- Angular distribution in X(3872) decay
  - Helicity formalism
  - Applied to X(3872) decay
  - Lowest angular momentum approximation
- Previous results for J^{PC} of X(3872)
  - CDF (3D χ^2 tests)
  - Belle (1D χ^2 tests)
- LHCb analysis
  - LHCb detector and X(3872) data sample
  - Likelihood ratio test
  - Efficiency corrections in 5D likelihood
  - Background subtraction in 5D likelihood
  - Importance of angular correlations
  - Likelihood ratio results
- Physics implications of J^{PC} determination

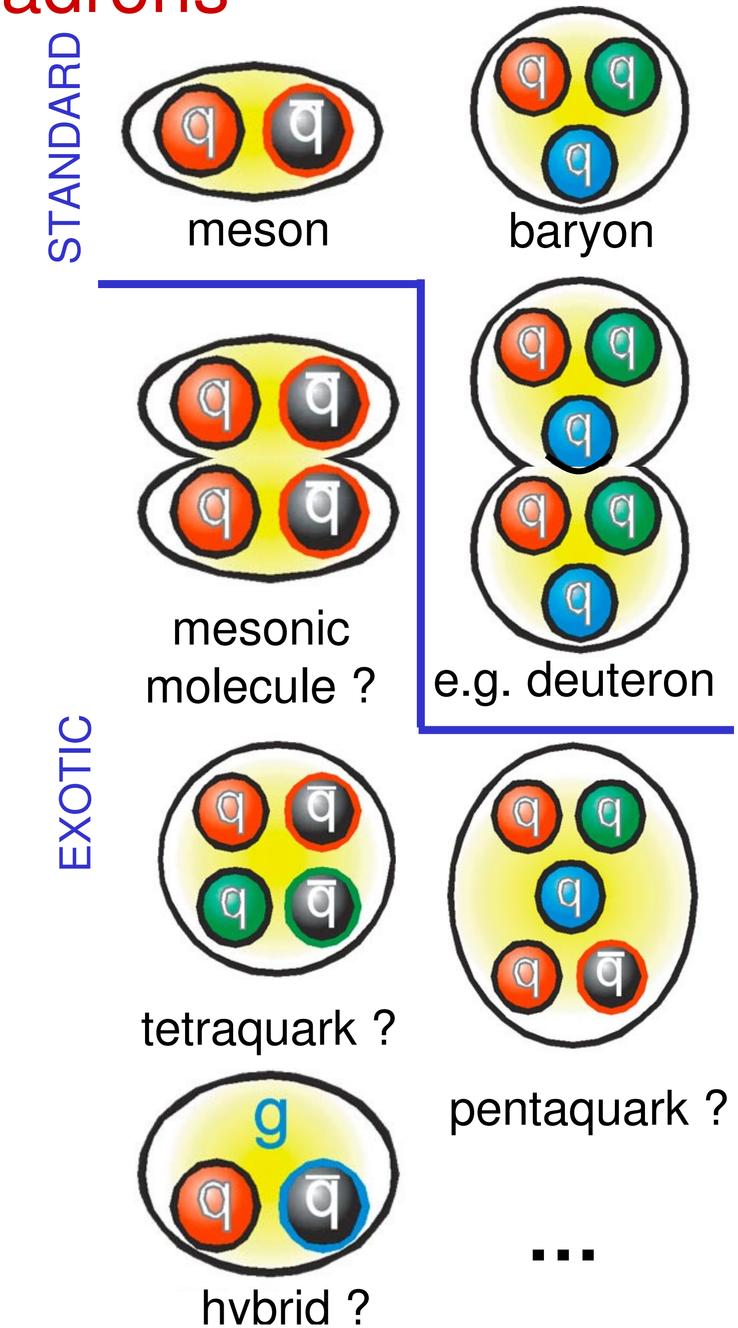
- Goals of the talk:
  - **Briefly** explain physics motivation and implications of the measurement
  - Illustrate helicity formalism without becoming too formal
  - Illustrate power of multi-dimensional likelihood-ratio test
  - Illustrate one of the ways to deal with background subtraction and efficiency corrections in many dimensions
- I will use blackboard when dealing with more technical points

Charmonium and its historical role

- Quark model was proposed by Gell-Man,Zweig in 1964 to explain "elementary" particle zoo via postulating they were not-so elementary but bound states of $q\bar{q}$ and qqq
- Spectroscopy of light-hadrons (made out of u,d,s) is complicated:
  - Highly relativistic constituents
  - Nearly equal u,d mass (isospin symmetry) mix $u\bar{u}$,$d\bar{d}$ states. Mass of s quark also similar $(SU(3)_{flavor}$ symmetry); adds $s\bar{s}$ component to the mix.
  - Except for the lightest ones they are broad; different excitations of the same J^{PC} mix
  - Makes quantitative spectroscopy difficult
- Many people remained skeptical until discovery of J/ψ in 1974 ("November Revolution") and other charmonium states ($c\bar{c}$). Mass of c quark much heavier:
  - Quarks are nearly non-relativistic
  - No mixing of $c\bar{c}$ with lighter $q\bar{q}$ states
  - 8 lowest excitations below $D\bar{D}$ threshold $D=c\bar{q}$ (q=u or d) are very narrow (most below mass resolution of the detectors). No mixing of excitations.
  - Makes quantitative spectroscopy easy. Simple non-relativistic potential models reproduced the observed mass spectrum with previously unheard precision – no doubt bound states of $c\bar{c}$!

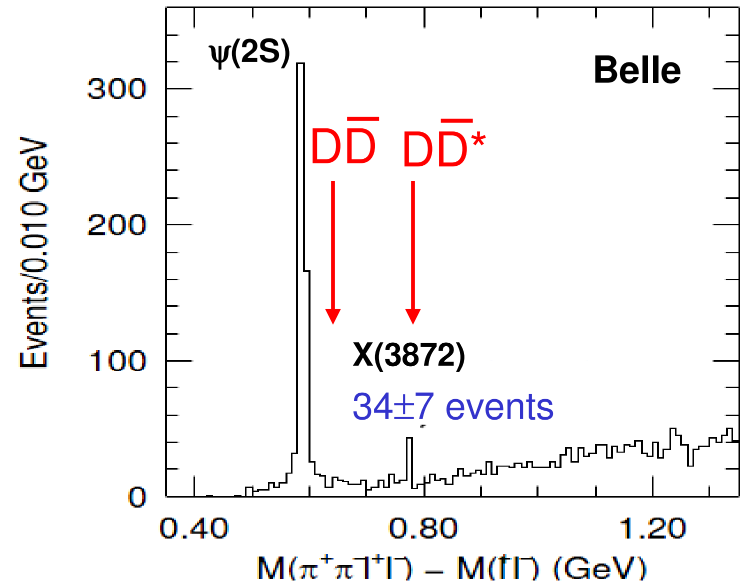
Standard and Exotic Hadrons

- Longstanding dispute in light meson spectroscopy if exotic states exist (too many scalar states?)
- No convincing experimental proofs for existence of elusive pentaquarks
- Recent discoveries in heavy quark states ($c\bar{c}$,$b\bar{b}$) have revived hopes for conclusive proofs for existence of exotic mesons – it all started from X(3872)

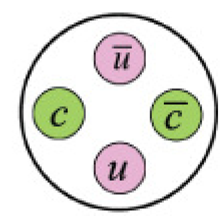


X(3872)

PRL 91, 262001 (2003) 152 M BB



- Discovered by Belle in 2003 at e^+e^- B-factory in $B^+ \to X(3872)K^+$, $X(3872) \to J/\psi\ \pi^+\pi^-$, $J/\psi \to l^+l^-$
- By far the most citied Belle's paper (797 citations) in spite of many fundamental measurements by Belle related to CPV in weak decays of B mesons – new "revolution"
- What makes it so special?
  - It is extremely narrow. Only upper limits on its width (<1.2 MeV) . None of the known $c\bar{c}$ states above DD threshold is so narrow
    - This automatically eliminates all $c\bar{c}$ excitations which can decay to $D\bar{D}$
  - Its mass is not near any of the predicted $c\bar{c}$ masses. Closest predicted $c\bar{c}$ states which could be narrow: $2^3P_1{}^{++}$, $1^1D_2{}^{-+}$
  - Its mass is nearly equal $m(D^0)+m(D^{0*})$:
    - It is loosely bound $D^0\bar{D}^{0*}=(c\bar{u})(\bar{c}u)$ molecule or $(c\bar{c}u\bar{u})$ tetraquark? Both models require $J^{PC}=1^{++}$



?

Measuring J^{PC}

- C = + since decay to $\gamma J/\psi$ was observed
  BaBar PRD74(2006)071101; Belle PRL107(2011)091803
- Best sensitivity to J^P via **angular correlations** among decay products in the most copiously observed decay mode $X(3872) \to (J/\psi \to \mu^+\mu^-)(\rho \to \pi^+\pi^-)$
- Need to predict decay amplitude for given J^P and test against the data:
  - Different theoretical formalisms can be used
  - Helicity formalism relatively "easy"

Helicity formalism - 1

- Helicity (λ) – projection of $\vec{J}$ onto $\vec{p}$ (λ= –J, … ,+J)
- Angular distribution in two-body decay: a→b c assuming all helicities are known (not realistic, see next slide):

$$\text{Probability} \propto \left| A_{\lambda_b,\lambda_c} d^{J_a}_{\lambda_a,\lambda_b-\lambda_c}(\theta) e^{i(\lambda_a-(\lambda_b-\lambda_c))\phi} \right|^2$$

- $A_{\lambda b, \lambda c}$ – helicity couplings:
  - complex QM amplitudes driven by dynamics of the decay
  - in case of strong decay constrained by conservation of parity
  - for J=0 (often the case - K,π !) λ=0 thus the index drops out
  - in practice, often free parameters to be determined by the fit to the data
- $d^{Ja}_{\lambda,\lambda'}(\theta)$ – small-d Wigner functions (tables and simple formula exists)
- θ – helicity angle (polar angle between the mother's spin quantization axis and the daughter's spin quantization axis) e.g.: $\cos\theta = \hat{n}_a \cdot \hat{p}_b$

$\hat{p}_b$ unit vector along daughter's momentum in the rest frame of the mother

$\hat{n}_a$ in cascade decay unit vector along grand-mother's momentum in the rest frame of the mother

- ϕ – azimuthal angle defined by the decay plane:
  - dependence on it drops out unless study cascading decays; enters as an angle between the decay planes of mother and daughter

Helicity formalism - 2

- For the cascade decays a→b c, b→d e need to sum up coherently over helicity of intermediate particle, λ_b (interference):

$$\text{Probability} \propto \left| \sum_{\lambda_b} A^a{}_{\lambda_b,\lambda_c} A^b{}_{\lambda_d,\lambda_e} \ldots \right|^2$$

- For the final state particles with spin need to sum up incoherently over their λs (no interference):

$$\text{Probability} \propto \sum_{\lambda_c} \sum_{\lambda_d} \sum_{\lambda_e} \left| \sum_{\lambda_b} A^a{}_{\lambda_b,\lambda_c} A^b{}_{\lambda_d,\lambda_e} \ldots \right|^2$$

- In principle need to know polarization of the initial state particle, unless can assume lack of it in which case:

$$\text{Probability} \propto \sum_{\lambda_a} \sum_{\lambda_c} \sum_{\lambda_d} \sum_{\lambda_e} \left| \sum_{\lambda_b} A^a{}_{\lambda_b,\lambda_c} A^b{}_{\lambda_d,\lambda_e} d^{J_a}_{\lambda_a,\lambda_b-\lambda_c} \ldots \right|^2$$

Helicity structure of amplitude for $B^+ \to X(3872) K^+$, $X(3872) \to J/\psi\ \rho$, $J/\psi \to l^+ l^-$, $\rho \to \pi^+ \pi^-$

- $B^+ \to X(3872) K^+$:
  - $J_B=0$, $\lambda_B=0$; fortunate:
    - No worry about its polarization or its "mother"
    - Also implies $\lambda_X - \lambda_K = 0$
  - $J_K=0$, $\lambda_K=0$; fortunate:
    - Produces fully polarized X(3872): $\lambda_X = \lambda_K = 0$
    - Only one helicity coupling independetly of J_X
- $J/\psi \to l^+ l^-$:
  - electromagnetic decay: $\Delta\lambda_\mu = -1, +1$
  - Only one helicity coupling via P,C-conservation
- $\rho \to \pi^+ \pi^-$:
  - $J_\pi=0$, $\lambda_\pi=0$; fortunate: only one helicity coupling
- $X(3872) \to J/\psi\ \rho$:
  - $J_\psi = J_\rho = 1$; $\lambda_\psi = -1,0,+1$; $\lambda_\rho = -1,0,+1$
  - Potentially large number of $A_{\lambda\psi,\lambda\rho}$ reduced by half by P-conservation (strong decay)

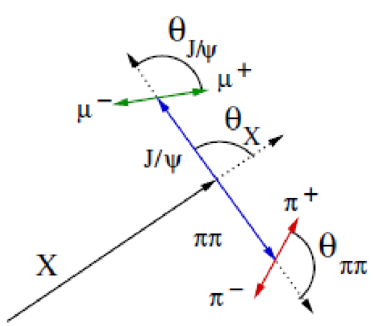
Helicity amplitude for $B^+ \to X(3872)K^+$, $X(3872) \to J/\psi\ \rho$, $J/\psi \to l^+l^-$, $\rho \to \pi^+\pi^-$

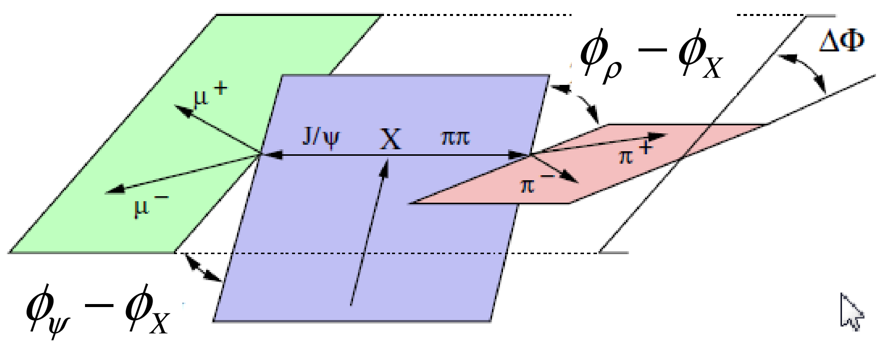
$$P(\Omega \mid J_X, A^{J_X}_{\lambda_\psi,\lambda_\rho}) \propto \sum_{\Delta\lambda_\mu=-1,1} \left| \sum_{\lambda_\psi=-1,0,1} \sum_{\lambda_\rho=-1,0,1} A^{J_X}_{\lambda_\psi,\lambda_\rho} d^{J_X}_{0,\lambda_\psi-\lambda_\rho}(\theta_X) d^1_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi) e^{i\lambda_\psi(\phi_\psi-\phi_X)} d^1_{-\lambda_\rho,0}(\theta_\rho) e^{i\lambda_\rho(\phi_X-\phi_\rho)} \right|^2$$

$$\Omega \equiv (\theta_X, \theta_\psi, \theta_\rho, \phi_\psi - \phi_X, \phi_\rho - \phi_X)$$

nuisance parameters

- Full angular space is 5 dimensional





Lowest L approximation

- Helicity basis is different for LS basis (L-angular momentum between decay products, S-total spin of the decay products). However there is a transformation between them via Clebsh-Gordan coefficients:

$$A_{\lambda_\psi,\lambda_\rho} = \sum_L \sum_S B_{L,S} \left(\begin{array}{cc|c} J_\psi & J_\rho & S \\ \lambda_\psi & -\lambda_\rho & \lambda_\psi - \lambda_\rho \end{array}\right) \left(\begin{array}{cc|c} L & S & J_X \\ 0 & \lambda_\psi - \lambda_\rho & \lambda_\psi - \lambda_\rho \end{array}\right)$$

$$|J_\psi - J_\rho| \le S \le J_\psi + J_\rho$$

$$S = 0,1,2$$

$$|J_X - S| \le L \le J_X + S$$

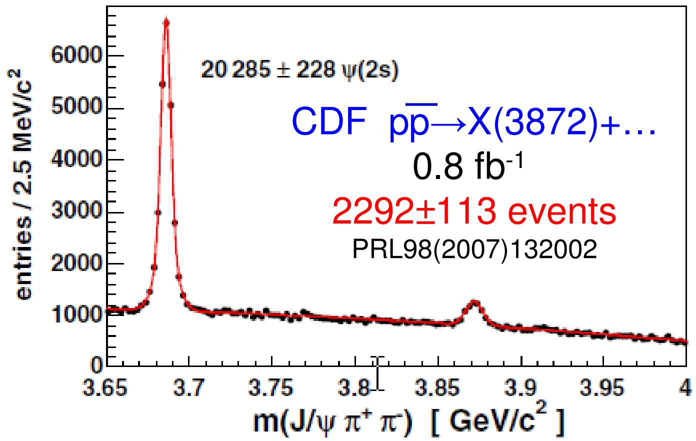
$$P_X = P_\psi P_\rho (-1)^L = (-1)^L$$

Number of B_{LS} amplitudes equals number of independent $A_{\lambda\psi,\lambda\rho}$ amplitudes – no gain

(P-conservation since strong decay)

- However, higher L values suppressed by "orbital momentum barrier"
  - L advances by units of 2 because of the P-conservation; makes the suppression more effective
  - X(3872) $\rightarrow$ J/ψ ρ , have has very low energy release; makes the suppression very effective
- Restricting L to its lowest possible value reduces number of B_{LS} amplitudes; less nuisance parameters; better J_X sensitivity
  - For example, for **J^P=1$^+$** L_{min}=0, S=1; only one B_{LS} amplitude – phase unobservable, magnitude absorbed into normalization to the data: **no nuisance parameters**
  - For **J^P=2$^-$** L_{min}=1, S=1 or 2; two B_{LS} amplitudes; the complex ratio is the nuisance parameter (our choice **$\alpha=B_{12}/(B_{11}+B_{12})$**).

Previous angular analysis - CDF



Inclusive production in $p\bar{p}$ at Tevatron
most from direct $p\bar{p}\rightarrow X(3872)$ + anything

- unknown **X(3872) polarization**
- **assume unpolarized**
- dilutes sensitivity
- J^{PC} info only in 3 angles:

$\cos\theta_\psi, \cos\theta_\rho, \Delta\Phi=\phi_\psi-\phi_\rho$

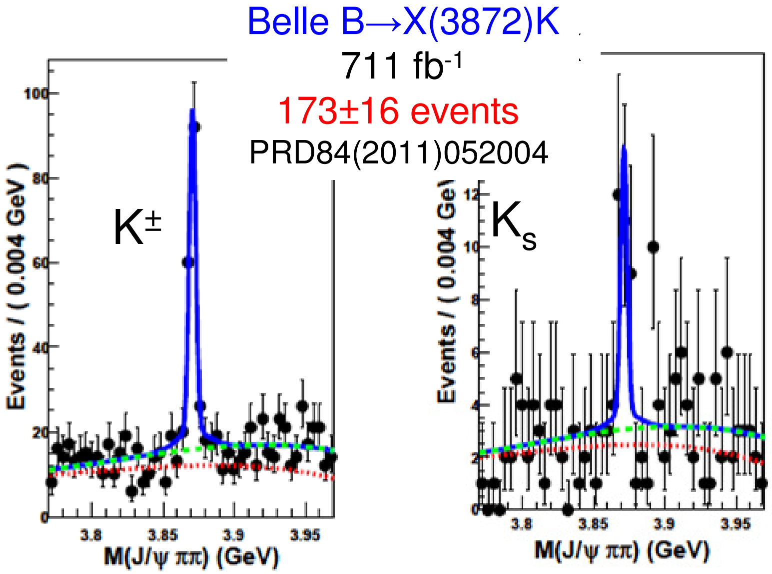
CDF's binned 3D angular χ^2 fit:

J^{PC}	decay	LS	χ^2 (11 d.o.f.)	χ^2 prob.
1^{++}	$J/\psi\rho^0$	01	13.2	0.28
2^{-+}	$J/\psi\rho^0$	11,12	13.6	0.26
1^{--}	$J/\psi(\pi\pi)_S$	01	35.1	2.4×10^{-4}
2^{+-}	$J/\psi(\pi\pi)_S$	11	38.9	5.5×10^{-5}
1^{+-}	$J/\psi(\pi\pi)_S$	11	39.8	3.8×10^{-5}
2^{--}	$J/\psi(\pi\pi)_S$	21	39.8	3.8×10^{-5}
3^{+-}	$J/\psi(\pi\pi)_S$	31	39.8	3.8×10^{-5}
3^{--}	$J/\psi(\pi\pi)_S$	21	41.0	2.4×10^{-5}
2^{++}	$J/\psi\rho^0$	02	43.0	1.1×10^{-5}
1^{-+}	$J/\psi\rho^0$	10,11,12	45.4	4.1×10^{-6}
0^{-+}	$J/\psi\rho^0$	11	104	3.5×10^{-17}
0^{+-}	$J/\psi(\pi\pi)_S$	11	129	$\leq1\times10^{-20}$
0^{++}	$J/\psi\rho^0$	00	163	$\leq1\times10^{-20}$

Cannot distinguish between 1^{++} and 2^{-+}
All other ruled out.

- CDF narrowed J^{PC} choices down to 1^{++} and 2^{-+}

Previous angular analysis - Belle



Exclusive production in B→X(3872)K:

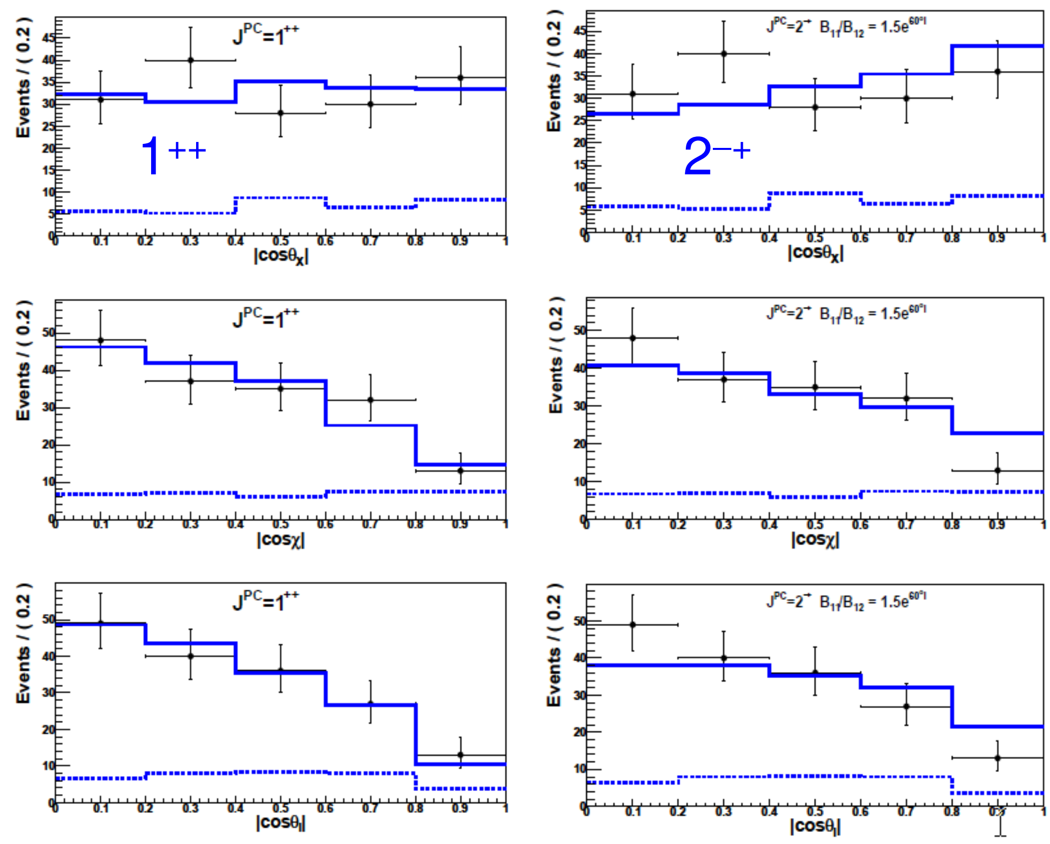
J^{PC} info in all 5 angles

Not enough signal events to bin in 5D and expect χ^2–statistics to work.

Belle chose to inspect 1D distributions in 3 different angles

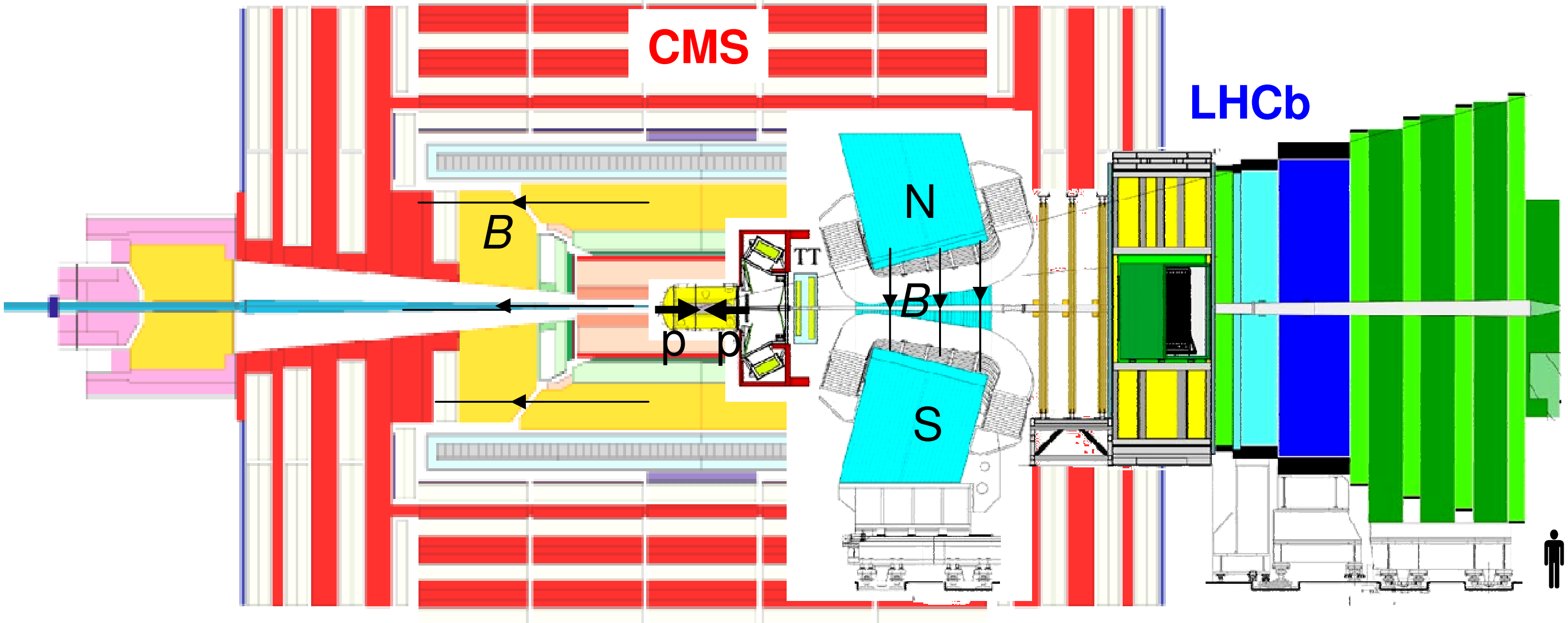
Belle's 1D analysis:

$\alpha = B_{12}/(B_{11}+B_{12}) = (0.64, 0.27)$



Could not distinguish between 1^{++} and 2^{-+} !

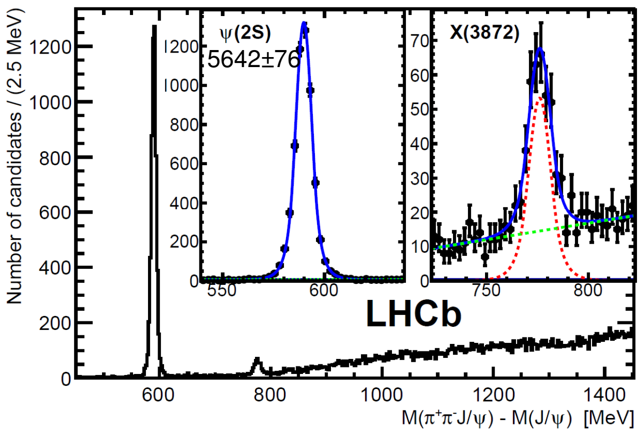
Advantages of LHCb experiment



- Advantages of LHC vs e^+e^-:
  - Orders of magnitude larger B-meson production rates (also large prompt production of X(3872))
- Advantages of LHC vs Tevatron:
  - Higher cross-section thanks to higher energy
- Advantages of LHCb vs central detectors:
  - Large trigger bandwidth totally devoted to heavy flavor physics; higher trigger efficiencies. Compensates for the lower luminosity (4×10^{32} $cm^{-2}s^{-1}$)
  - Can identify and trigger on lower p_T (di)muons
  - K/π separation (RICH detectors) helps background suppression

LHCb sample of $B^+ \to X(3872)K^+$, $X(3872) \to J/\psi\ \rho$, $J/\psi \to \mu^+\mu^-$, $\rho \to \pi^+\pi^-$

LHCb $B^+ \to X(3872)K^+$
1 fb^{-1} (2011 data)
313±26 events
arXiv:1302.6269
Accepted by PRL



Quality of the sample as good as at e^+e^-!

Background can be further reduced by a factor of 4 at 15% loss of efficiency by requiring high $m(\pi^+\pi^-)$

- Signal statistics 1.8 higher than in Belle measurement
- sqrt(1.8)=1.3; statistical errors only slightly better
- Try to squeeze all sensitivity from the data:
  - **5D analysis**; use power of full angular correlations
  - **do not bin data**; binning looses information and quite impossible to handle in 5D
  - use the most effective statistical method to discriminate between 1^{++} and 2^{-+}; a **likelihood ratio test**

Likelihood ratio test

- If you pick any book on statistics and read about hypothesis testing, this is the first test you will learn

e.g. Frodesen, Skjeggestad, Tofte

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The 1^{++} hypothesis is simple
The 2^{-+} hypothesis is composite since it
. has 2 nuisance parameter (Reα, Imα)

- For simple hypotheses guaranteed to be the most effective test (Neyman-Pearson test)
- Even for composite hypotheses, generally the most effective test except for unusual cases
- Often used in data selection:
  - e.g. LHCb uses it for PID cuts
  - in this analysis we use signal/background likelihood-ratio for the final background suppression
- Amazingly, rarely used in spin determinations.
  - One exception, I am aware of:
    - T. Skwarnicki, Ph.D. thesis, DESY-F31-86-02 (98 citations … because of the appendix on normalization of Crystal Ball shape); spin analysis of $\chi_b(1^3P)$ states published by Crystal Ball in PRL 58(1987)972

Likelihood ratio test applied to X(3872) spin analysis

- Test variable ("test statistic"):

$$t = -2 \sum_{i=1}^{N_{events}} \ln \frac{P(\Omega_i \,|\, 2^{-+}, \hat{\alpha})}{P(\Omega_i \,|\, 1^{++})}$$

where $\hat{\alpha}$ is the value which maximizes 2^{-+} likelihood (L) i.e. minimizes -2lnL (χ^2-like variable)

$$\chi^2(2^{-+}, \alpha) = -2 \sum_{i=1}^{N_{events}} \ln P(\Omega_i \,|\, 2^{-+}, \alpha)$$

- Naïve interpretation of value of t is that it behaves as $\chi^2(2^{-+}) - \chi^2(1^{++})$:
  - However, no statistical theorems to back it up
- Proper statistical analysis requires numerical simulations (**generation of many toy experiments**) to obtain probability density distributions of t under each hypothesis:

$$P(t \,|\, 1^{++}), P(t \,|\, 2^{-+}, \hat{\alpha})$$

Implementing efficiency corrections

- The most effective test variable must be based on probabilities that know about detection efficiency:

$$P(\Omega\,|\,J) = \frac{\varepsilon(\Omega\,|\,J)P_{theory}(\Omega\,|\,J)}{\int \varepsilon(\Omega\,|\,J)P_{theory}(\Omega\,|\,J)d\Omega}$$

Actually could neglect efficiency here (we don't) and loose the hypotheses separation, but can't neglect it in simulations of $P(t|J)$.

- Because we are using almost all kinematic variables describing the final state (benefit of using all angles!):

$$\varepsilon(\Omega\,|\,J) \approx \varepsilon(\Omega)$$

Remnant dependence via averaging over m($\pi^+\pi^-$): ρ has substantial width and its resonant shape is slightly J dependent.

We checked with MC that this effect is negligible.

- Efficiency in the numerator of $P(\Omega\,|J)$ drops out in $P(\Omega\,|2^{-+})/P(\Omega\,|1^{++})$ and contributes only constant term in $\chi^2(2^{-+},\alpha)$:
  - don't need to determine efficiency on event-by-event basis; no need for $\varepsilon(\Omega)$ parametrization; no need for approximations!
  - only integrated efficiency needs to be known: $I(J) \equiv \int \varepsilon(\Omega\,|\,J)P_{theory}(\Omega\,|\,J)d\Omega$
    - the latter easy to do with MC generated with uniform angular distributions and passed through the data selection:

$$I(J) \propto \sum_{i=1}^{N_{eve}^{MC}} P_{theory}(\Omega_i\,|\,J)$$

Implementing background subtraction

- The data sample in the X(3872) peak region (±2.5σ) contains 30% of the background entries
  - mostly $B^+ \to J/\psi K_1(1270)^+$
  - can make it smaller but can't make it zero
- The background either needs to be represented in the likelihoods (complicated but doable in 5D) or "subtracted"
- We opted for the background subtraction using **sPlot technique**
  - Think about it as a fancy sideband-subtraction method in $\Delta M = M(\pi^+\pi^- J/\psi) - M(J/\psi)$ using X(3872) sidebands
  - Assign events weights, $w_i(\Delta M)$; positive in the signal region; negative in sidebands:

$$N_{signal} \approx \sum_{i=1}^{N_{events}} w_i \qquad \sum_{j=1}^{N_{events}^{signal}} f(X_j) \approx \sum_{i=1}^{N_{eve}} w_i f(X_i)$$

as long as X not correlated with ΔM

OK if X is multidimensional (X=Ω) !

$$t = -2\, s_w \sum_{i=1}^{N_{events}} w_i \ln \frac{P(\Omega_i \,|\, 2^{-+}, \hat{\alpha})}{P(\Omega_i \,|\, 1^{++})}; \qquad s_w = \frac{\sum_{i=1}^{N_{events}} w_i}{\sum_{i=1}^{N_{events}} w_i^{\,2}}$$

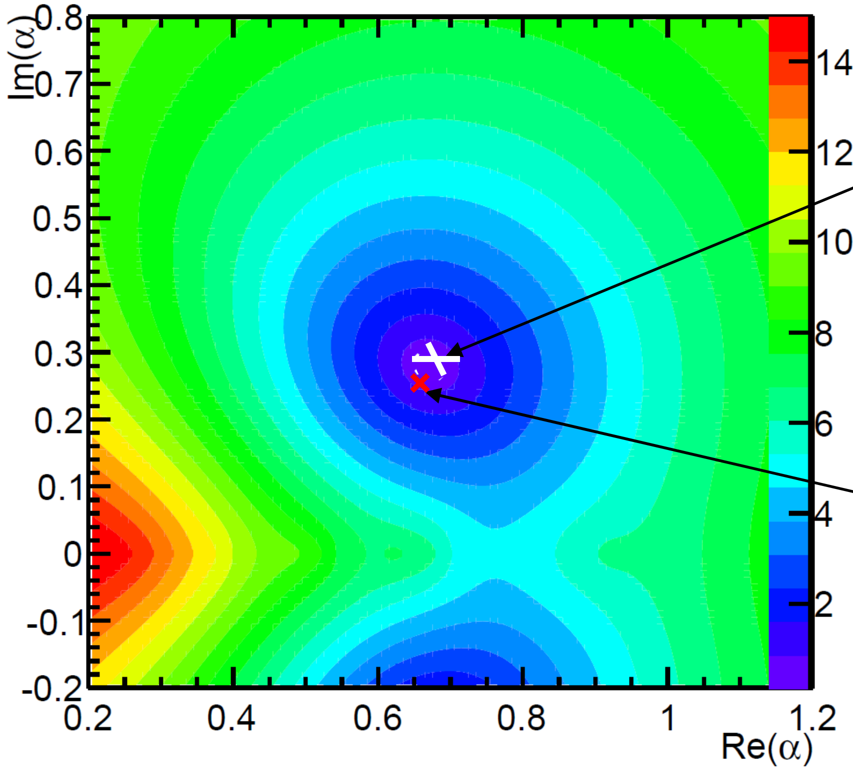
  - background and the event weighting technique included in the toy experiments used to predict $P(t|J)$

s_w is constant for given data sample.
It corrects the likelihoods for the statistical error in the background subtraction.

Results for $\alpha = B_{12}/(B_{11}+B_{12})$ under 2^{-+} hypothesis

- Minimize:

$$\chi^2(2^{-+},\alpha) = -2\, s_w \sum_{i=1}^{N_{events}} w_i \ln P(\Omega_i \mid 2^{-+},\alpha)$$



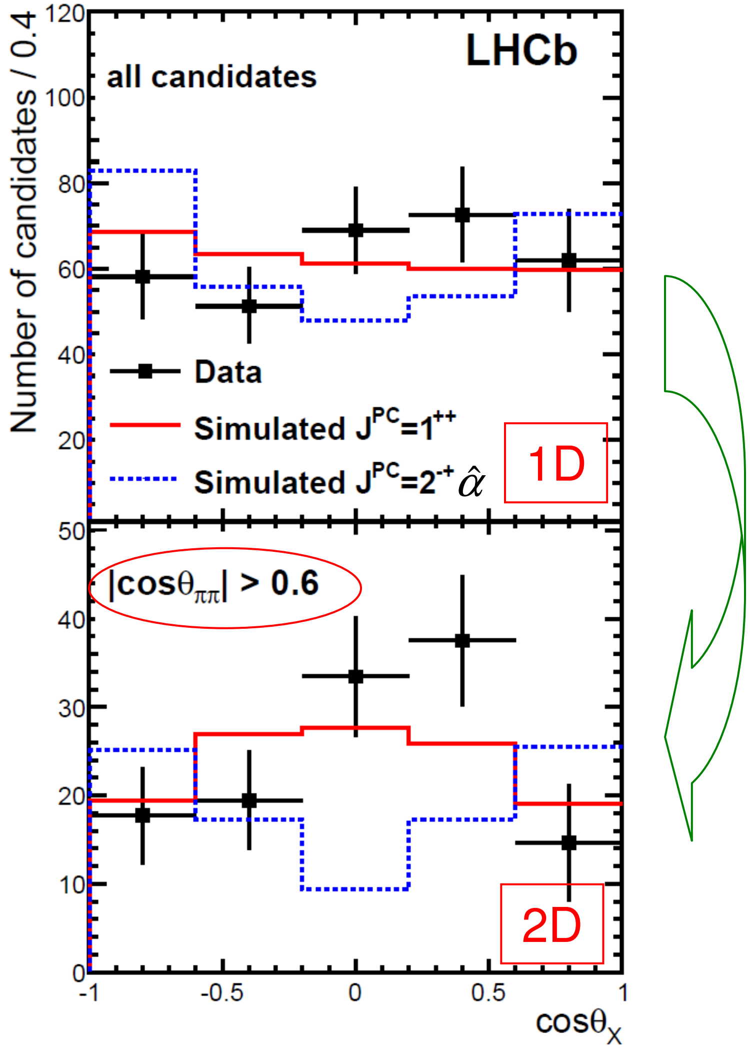
Contours of $\sqrt{\chi^2(2^{-+},\alpha) - \chi^2(2^{-+},\hat{\alpha})}$

$\hat{\alpha}_{LHCb} = (0.671\pm0.046,\ 0.280\pm0.046)$

$\alpha_{Belle} = (0.64,\ 0.27)$

Extracted using different techniques from different data samples but consistent!

Power of angular correlations



With present data sample cannot distinguish between 1^{++} and 2^{-+} using 1D distributions (same conclusion as in the Belle measurement)

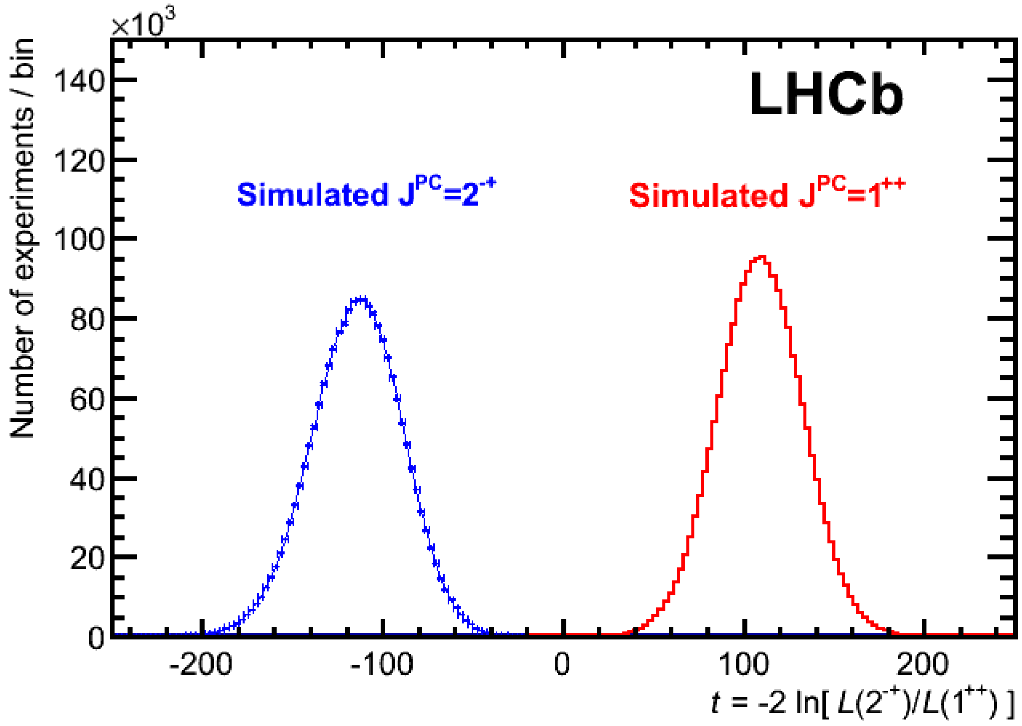
However, the angular correlations magnify the differences between the spin hypotheses

Let us see what 5D analysis can do…

Results for $P(t\,|\,1^{++}), P(t\,|\,2^{-+}, \hat{\alpha})$

- Results of numerical simulations (2M experiments for each hypothesis) for the distribution of likelihood-ratio test statistic:

$P(t|J_X^P)$

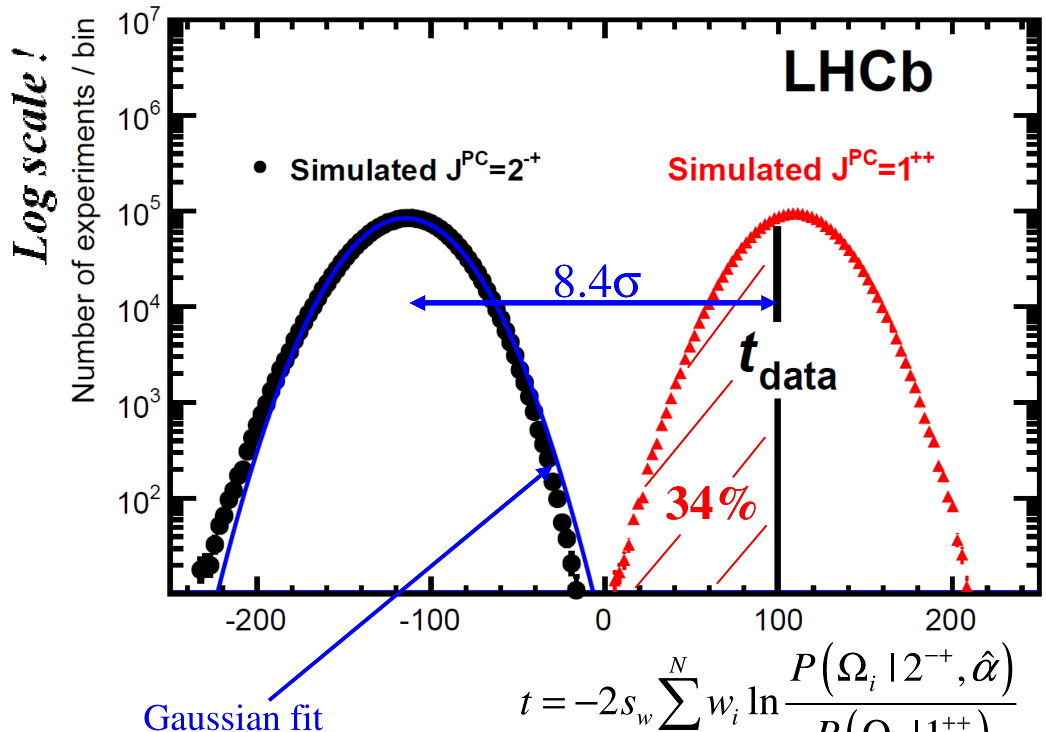


The two spin hypotheses are expected to be completely separated!

What is the value of the test statistics on the real data sample - t_{data}?

Likelihood-ratio test result

$P(t|J_X^P)$



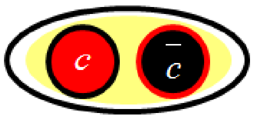
$$t = -2 s_w \sum_{i=1}^{N} w_i \ln \frac{P(\Omega_i \,|\, 2^{-+}, \hat{\alpha})}{P(\Omega_i \,|\, 1^{++})}$$

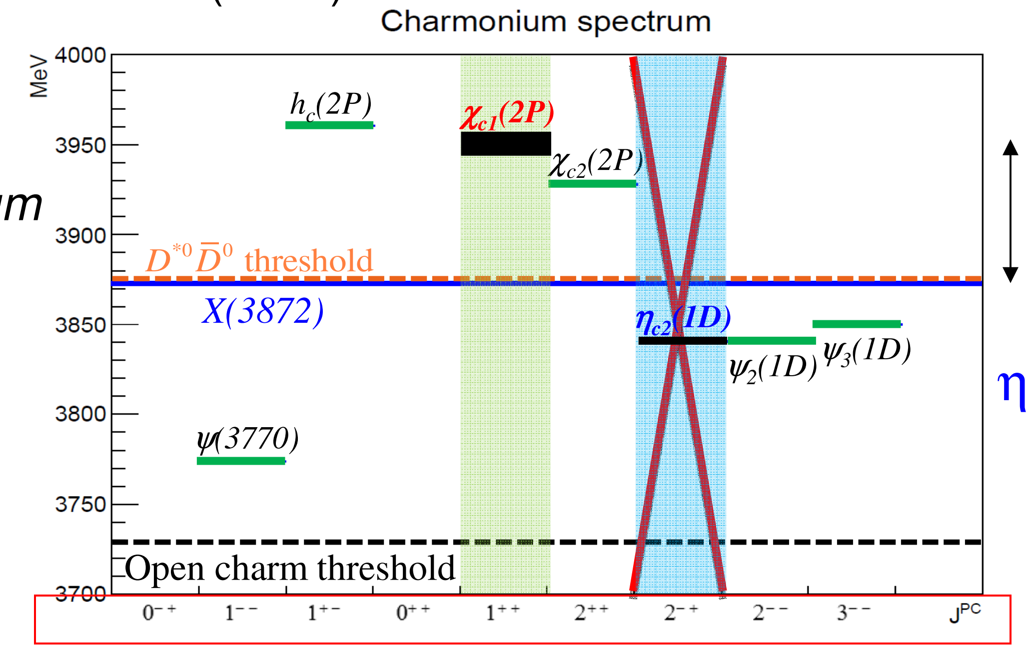
- The Gaussian approximation conservative since the actual distribution to the left of the Gaussian fit.
  - The 2^{-+} hypothesis is ruled out at 8.4σ (>8 after systematics)
- 1^{++} C.L. is high (34%).

X(3872) interpretations

J^{PC} of X(3872) has been determined to be 1^{++}

Charmonium





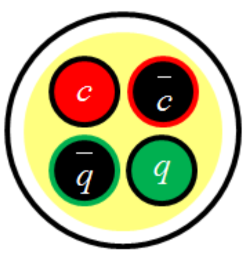
$\chi_{c1}(2^3P_{1++})$ possible but disfavored by mass

$\eta_{c2}(1^1D_{2-+})$ is now ruled out!

1^{++} was expected in both tetra-quark and molecular models

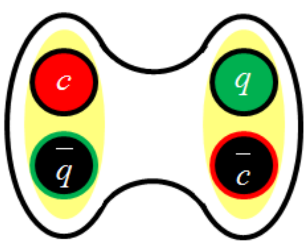
The four-quark models also favored by the coincidence of X(3872) mass with the $D^{*0}\bar{D}^0$ threshold

Tetra-quark



Nearly degenerate charged partners expected but not observed.

$D^{*0}\bar{D}^0$ molecule



Favored interpretations.

Mixed state of 2^3P_1 charmonium and $D^{*0}\bar{D}^0$ molecule also a possibility

None of the models describes **all** X(3872) properties well. Theoretical and experimental work on this state will continue.

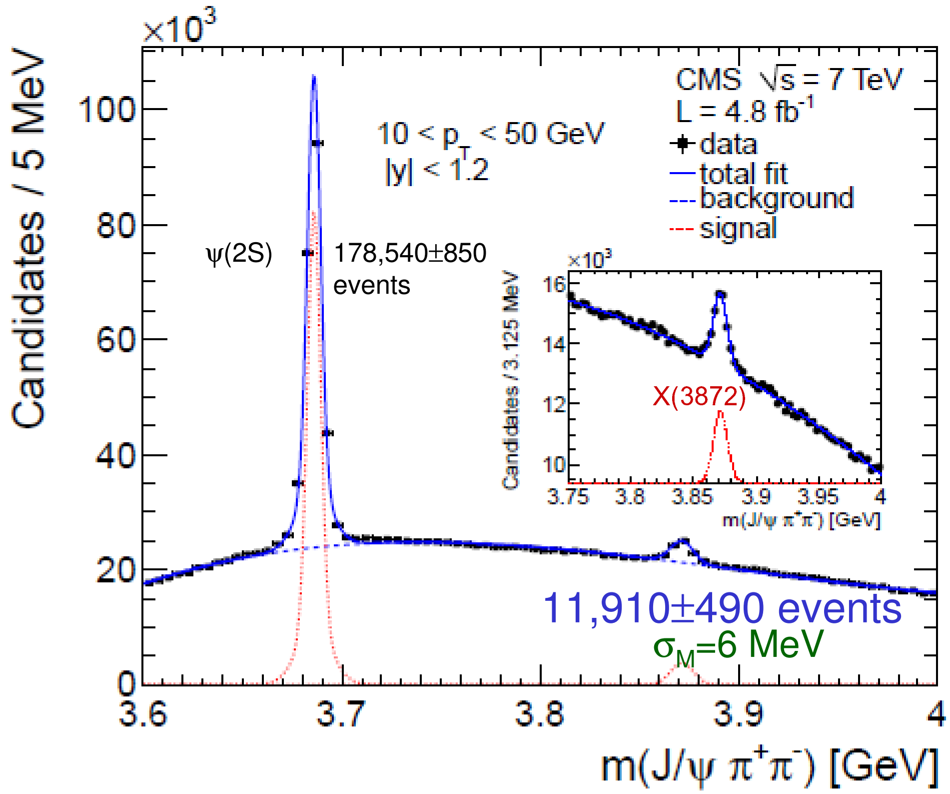
Summary

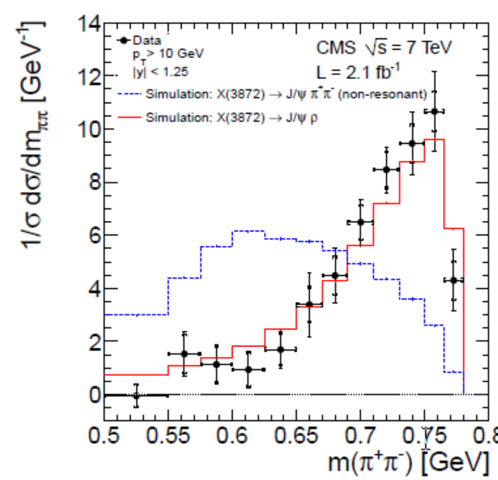
- Physics:
  - Quantum numbers of X(3872) are now settled to be $J^{PC}=1^{++}$
  - This eliminates popular explanation of this state, $\eta_{c2}(1^1D_{2-+})$,and together with X(3872) mass, favors the exotic four-quark models
- Techniques:
  - Multidimensional angular correlations provide much improved spin sensitivity relative to analysis in reduced dimensions
  - Multidimensional analysis can be simple if using unbinned likelihoods and likelihood ratio test
  - Efficiency corrections and background subtractions in multidimensional analysis don't need to be complicated

BACKUP SLIDES

Kinematics of X(3872) → ρJ/ψ

CMS JHEP 1304(2013)154 arXiv:1302.3968



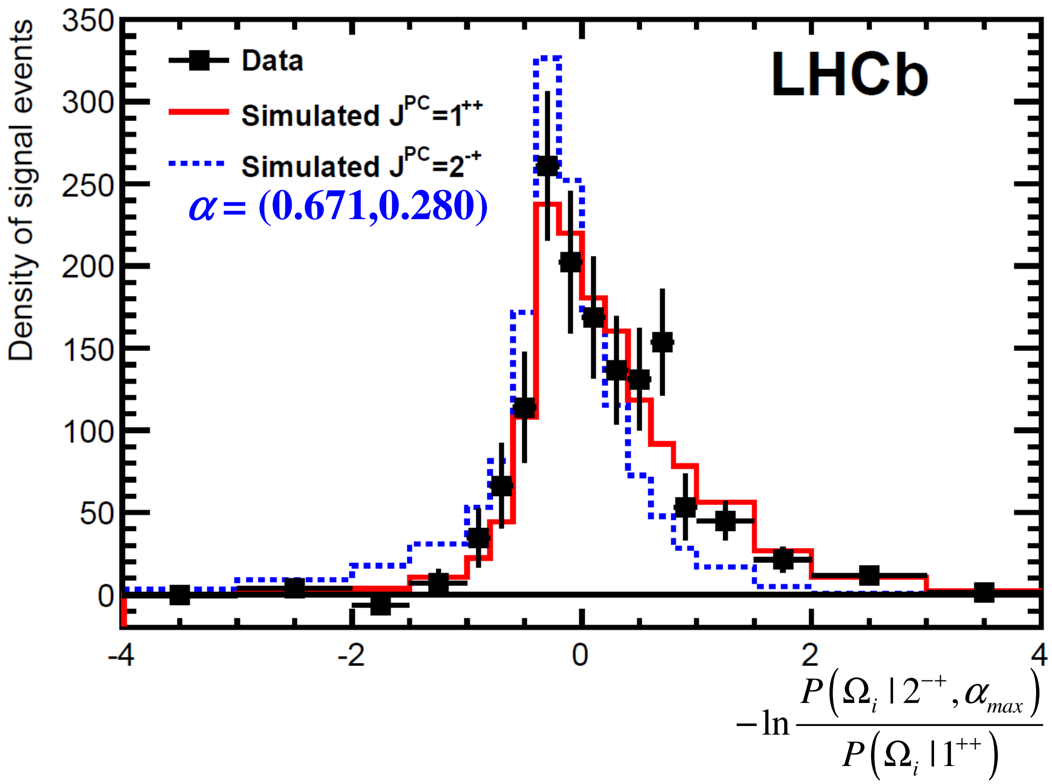


Data consistent with
X(3872) → ρJ/ψ
as previously observed
with smaller statistics
(also in LHCb data)

- The X(3872) → ρJ/ψ decays pile-up close to the upper phase-space limit in $m(\pi^+\pi^-)$, thus energy release is very small; daughter momenta are small in the X(3872) rest frame; making orbital angular momentum barrier very effective – lowest L should dominate these decays

Additional 1^{++} consistency check

- Likelihood ratio test based on *t* value uses the likelihood ratio **averaged** over all events in the experiment.
- Perform additional goodness-of-fit check using binned distribution of **single-event** log-likelihood ratio. The background is subtracted using sWeights.



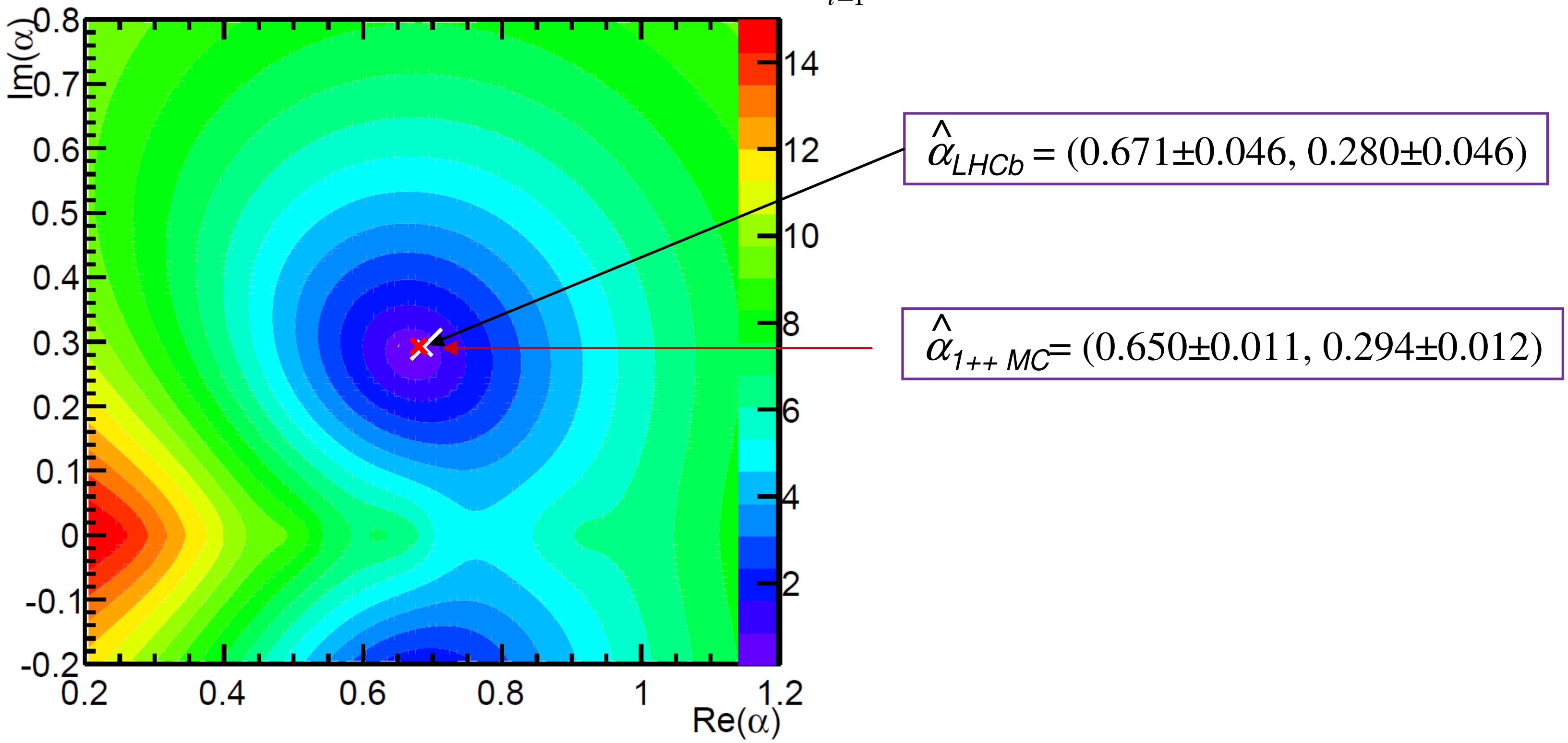
Consistent with the 1^{++} simulations!

Inconsistent with the 2^{-+} simulations!

Results for α=B$_{12}$/(B$_{11}$+B$_{12}$) under 2^{-+} hypothesis

- Minimize:

$$\chi^2(2^{-+},\alpha) = -2\, s_w \sum_{i=1}^{N_{events}} w_i \ln P(\Omega_i \mid 2^{-+},\alpha)$$



Contours of $\sqrt{\chi^2(2^{-+},\alpha) - \chi^2(2^{-+},\hat{\alpha})}$

The data behaves as expected from 1^{++} simulations in all respects.