Review of Collective Effects for Low Emittance Rings

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Introduction

Recently there has been much development in the design of storage rings with very low emittances. A so-called “ultimate storage ring” is one with emittances that are diffraction limited at 1 angstrom wavelength

Among the collective effects that become important for such rings are: intrabeam scattering (IBS) that limits the emittances that are achievable and the Touschek effect that shortens the beam lifetime; there are impedance driven effects, like the microwave instability, the single-bunch, transverse mode coupling (TMCI) instability, and the multi-bunch transverse instability; one needs also to consider the fast ion instability and possibly space charge effects

Will review these effects, and use as example the PEP-X designs, a 4.5 GeV ring designed for the PEP tunnel:

A: 2010 (SLAC-PUB-13999), ε_x= 165 pm, ε_y= 8 pm, I= 1.5 A, lattice TME + DBA “Baseline”

B: 2012 (SLAC-PUB-14785), ε_x= ε_y= 12 pm (round beam), I= 0.2 A, lattice 7BA + damping wigglers “Ultimate”

Currently we are investigating a “Lasing” version that envisions much shorter bunches, and that contains a bypass that can act as a soft x-ray FEL

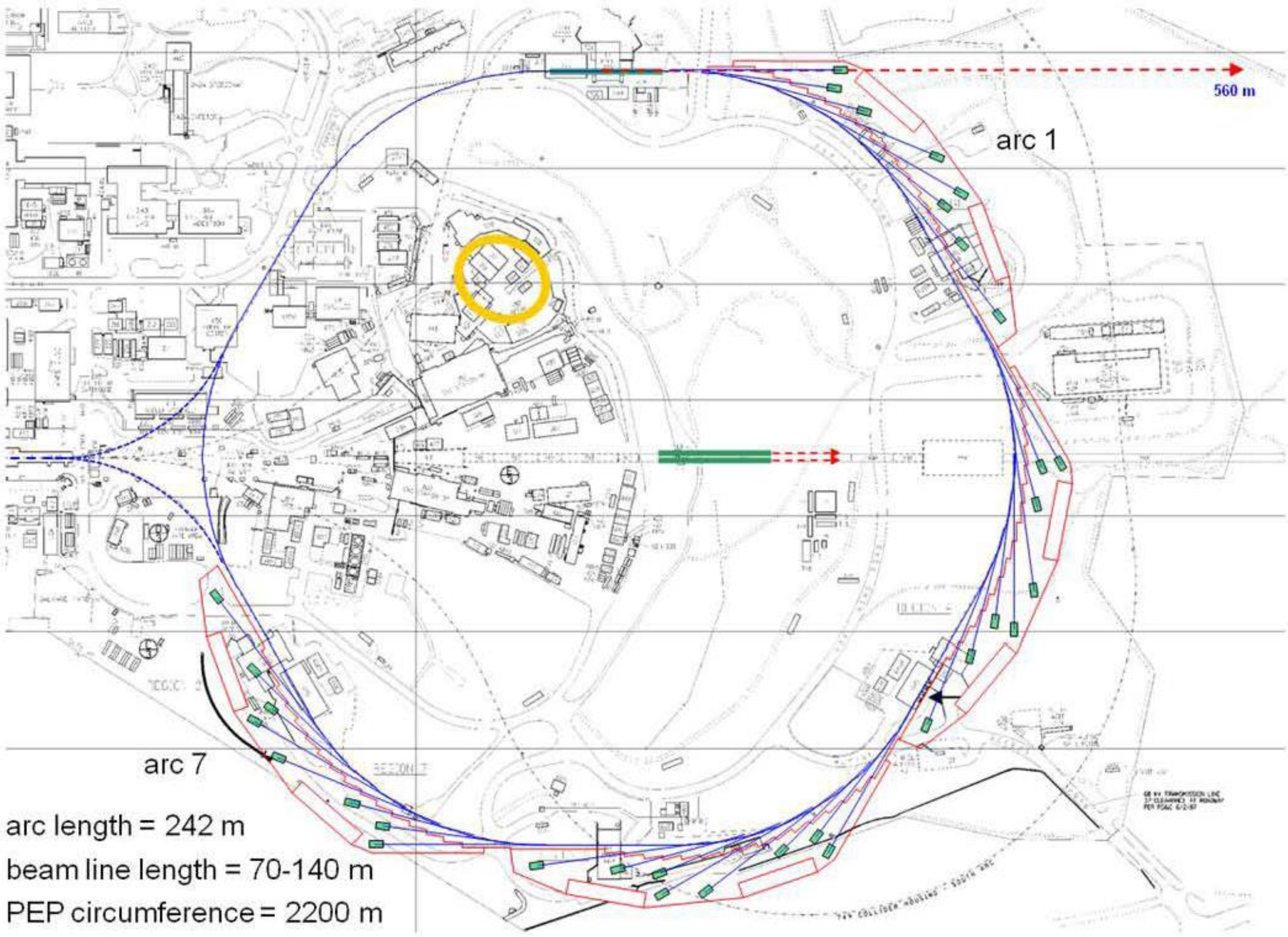
Outline

- Intra-beam scattering (IBS)
- Touschek lifetime
- Longitudinal microwave instability
  - --Coherent synchrotron radiation (CSR)
  - --Impedance budget
  - --Pseudo-Green function wake
- Other instabilities
- Shorter bunches
- Conclusions

As specific example illustrating the above topics I will use cases of PEP-X, a study effort of a group led by Y. Cai

Collective effect contributors include G.Stupakov, Y. Cai, L. Wang, M. Borland, …

PEP-X: An Ultimate Storage Ring



Yunhai Cai, SLAC

Selected Parameters for PEP-X

Parameter	A	B	Units
Energy, *E*	4.5	4.5	GeV
Circumference, *C*	2199	2199	m
Average current, *I*	1.5	0.2	A
Bunch population, N_b	2.18	0.28	10^{10}
Number of bunches, *M*	3154	3300	
Rel. rms energy spread, σ_p	1.14	1.2	10^{-3}
Rms bunch length, σ_z	3.0	3.0	mm
Nominal emittance, ε_0	85.7	11.0	pm
Momentum compaction, α	5.8	5.0	10^{-5}
Synchrotron tune, ν_s	7.7	6.9	10^{-3}
Horiz. rad. damping time, τ_x	13.5	19.	ms
Long. rad. damping time, τ_p	7.2	12.	ms

A—Baseline, B—Ultimate. Note that the nominal horizontal emittance $\varepsilon_{x0} = \varepsilon_0/(1+\kappa)$, with κ the x-y coupling parameter

Intra-Beam Scattering (IBS)

IBS describes multiple scattering that leads to an increase in all bunch dimensions and in energy spread. In low emittance e^- rings IBS increases the steady-state beam dimensions

Theory of IBS initially developed by Brueck and LeDuff (1965). More systematically developed by Piwinski (1974), Bjorken-Mtingwa (1983, using quantum mechanical scattering theory), Martini (1984, modification of Piwinski's formulation).

IBS theory was originally developed for proton machines. The predictions for growth rates seem to agree well for e.g. protons in the Tevatron (Lebedev 2005) and heavy ions in RHIC (Fedotov et al 2006)

Bjorken-Mtingwa (BM) Formulation

IBS (amplitude) growth rates (i= x, y, or p):

$$\boxed{\begin{aligned}\frac{1}{T_i} &= 4\pi A(\log)\left\langle \int_0^\infty \frac{d\lambda\, \lambda^{1/2}}{[\det(L+\lambda I)]^{1/2}} \left\{ \right.\right.\\ &\left.\left. TrL^{(i)}Tr\left(\frac{1}{L+\lambda I}\right) - 3TrL^{(i)}\left(\frac{1}{L+\lambda I}\right) \right\} \right\rangle\end{aligned}}$$

with

$$A = \frac{r_0^2 cN}{64\pi^2 \bar{\beta}^3 \gamma^4 \epsilon_x \epsilon_y \sigma_s \sigma_p}$$

(log) is Coulomb log

r_0 is classical radius of electron

$\bar{\beta}$= v/c

$\mathscr{H}_x = [\eta_x^2+(\beta_x\eta_x'-\beta_x'\eta_x/2)^2]/\beta_x$

$\phi_x = \eta_x' - \beta_x'\eta_x/2\beta_x$

$$L = L^{(p)} + L^{(x)} + L^{(y)} \quad ,$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ,$$

$$L^{(x)} = \frac{\beta_x}{\epsilon_x}\begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2\mathcal{H}_x/\beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L^{(y)} = \frac{\beta_y}{\epsilon_y}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2\mathcal{H}_y/\beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}$$

Finding T_i^{-1} means performing a type of elliptical integral at all lattice positions

Nagaitsev algorithm for evaluating elliptical integrals speeds up the calculation a factor of ~25 (in Mathematica)

Expected accuracy ~1/(log). For low emittance electron rings (log)~ 10

Vertical emittance in a ring is usually due to either η_y, produced by orbit errors, and/or by *x*-*y* coupling. Formulas so far have been without coupling. IBS with coupling is described in A. Piwinski (1991), B. Nash et al (2002), V. Lebedev (2005).

Steady-State Emittances

In electron machines the IBS growth is counteracted by synchrotron radiation damping (with $\tau_i^{-1} >> T_i^{-1}$), leading to increased stead-state emittances

Steady-state IBS emittance and energy spread (no *x*-*y* coupling):

$$\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x/T_x}, \quad \epsilon_y = \frac{\epsilon_{y0}}{1 - \tau_y/T_y}, \quad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p/T_p}$$

Solution involves (i) integration at every lattice element to obtain T_i^{-1}, (ii) averaging around the ring, (iii) solving the above three equations simultaneously (e.g. using Newton's method)

A heuristic approach often used for coupling dominated machine, with κ small, is to solve only ε_x, σ_p^2, equations and take $\varepsilon_y = \kappa\varepsilon_x$

Programs that solve IBS (mostly BM formulation) are ZAP, SAD, MAD-X, Elegant, …

SAD treats the three axes equally and includes coupling (e.g. *x*-*y*, *x*-*p*) in a general way by diagonalizing to normal modes

Simplified Model of IBS

(K. Bane, EPAC02)

Longitudinal growth rate:

$$\frac{1}{T_p} \approx \frac{r_e^2 c N_b (\log)}{16\gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_z \sigma_p^3} \left\langle \sigma_H \, g(a/b) \; (\beta_x \beta_y)^{-1/4} \right\rangle \quad = \langle \delta(1/T_p) \rangle$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x}, \quad a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}}, \quad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}$$

$$g(\alpha) = \alpha^{(0.021 - 0.044 \ln \alpha)}$$

Transverse growth rate:

$$\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \rangle \frac{1}{T_p} \quad \Rightarrow \quad \frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \delta(1/T_p) \rangle$$

Valid for a, b<< 1, “high energy approximation”

Solution for PEP-X

- For PEP-X consider round beam, κ= 1

I [mA]	ε_x [pm]	ε_y [pm]	σ_p [10^{-3}]	σ_z [mm]
0	5.5	5.5	1.20	3.0
200	11.5	11.5	1.25	3.1

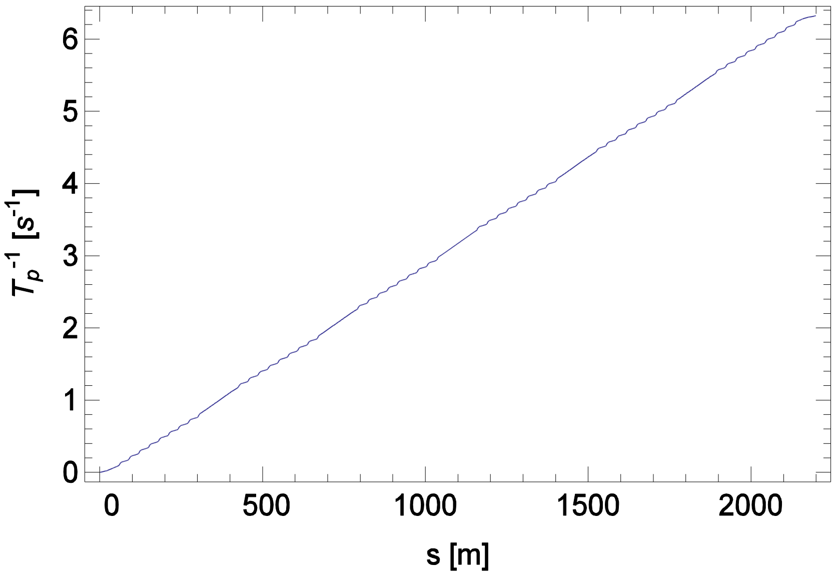
Table. Steady-state beam properties in PEP-X at zero current and nominal current. Results were obtained using the Bjorken-Mtingwa (B-M) formalism.

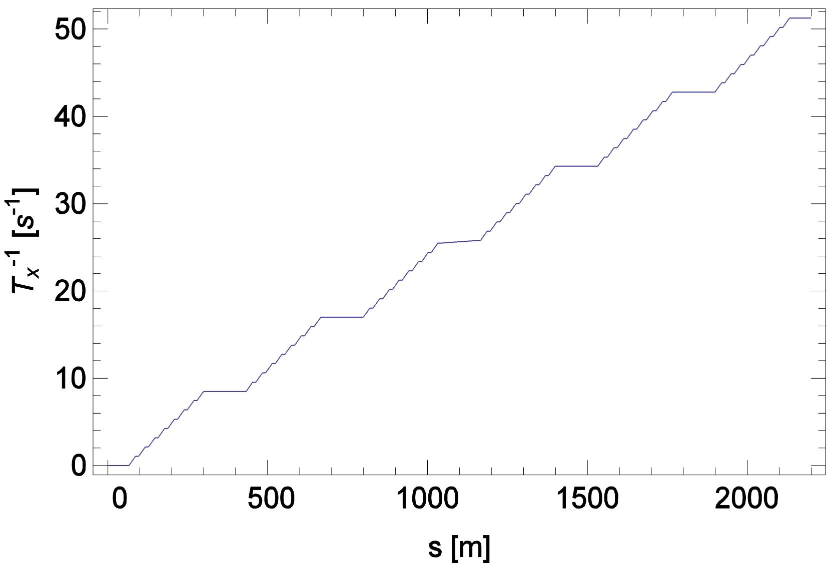
- Note: almost no growth in p or z

- In nominal configuration T_x^{-1}= 52. s^{-1}, T_p^{-1}= 7.4 s^{-1} (simplified model gets T_x^{-1}= 53.7 s^{-1}, T_p^{-1}= 8.9 s^{-1})

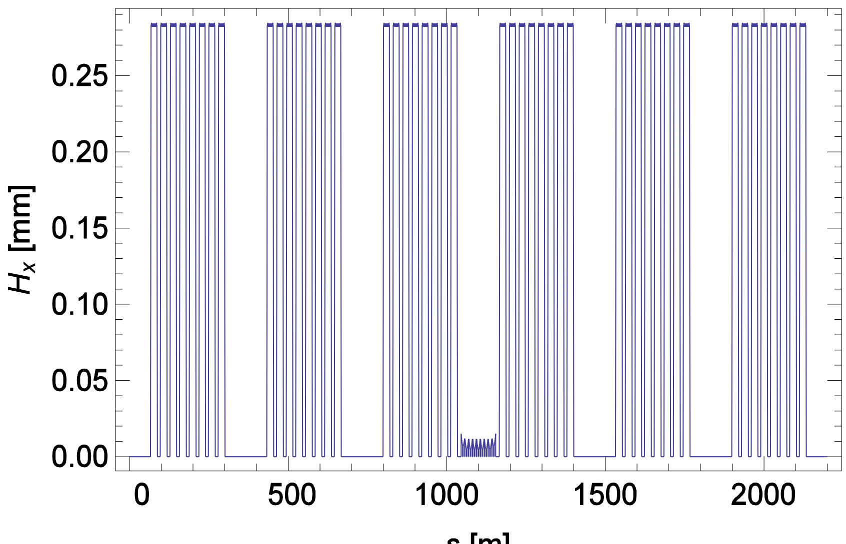
- Checked with SAD, an optics program that treats coupling without simplifying assumptions; results agree quite well

Accumulated IBS Growth Rates

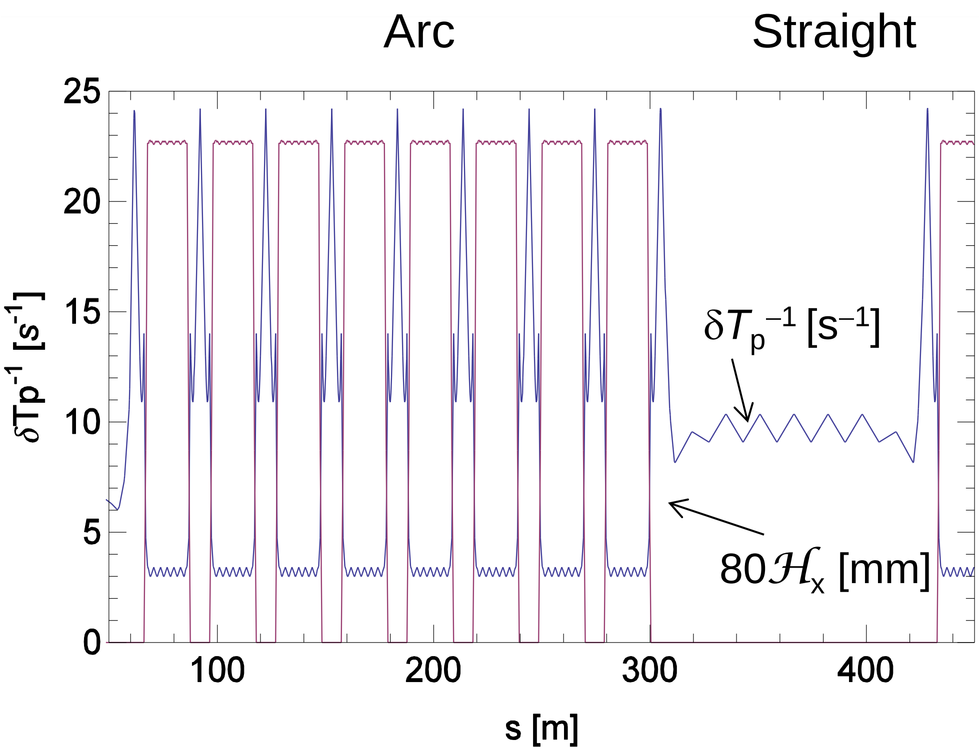




Accumulated growth rates in p, x;
$\mathcal{H}_x$ optics function



Correlation between $\mathcal{H}_x$ and $\delta(1/T_p)$ in PEP-X



$\mathcal{H}_x$ and $\delta(1/T_p)$ over one arc and one straight of PEP-X

Note the anti-correlation of the two functions in arc

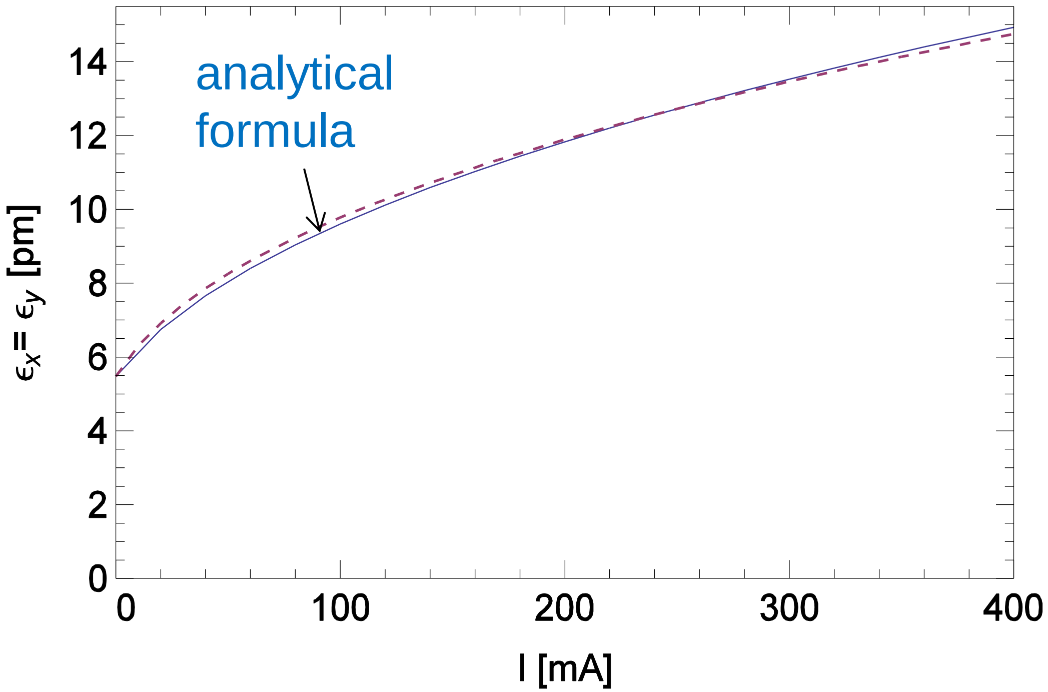
With no correlation but "same" lattice parameters, $1/T_x$ would be twice as large

Emittance Dependence on Current

• With T_p^{-1} small, from simplified model can show that steady-state emittances can be approximated by

$$\left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{5/2} - \left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{3/2} = \alpha\left(\frac{I}{I_A}\right)$$

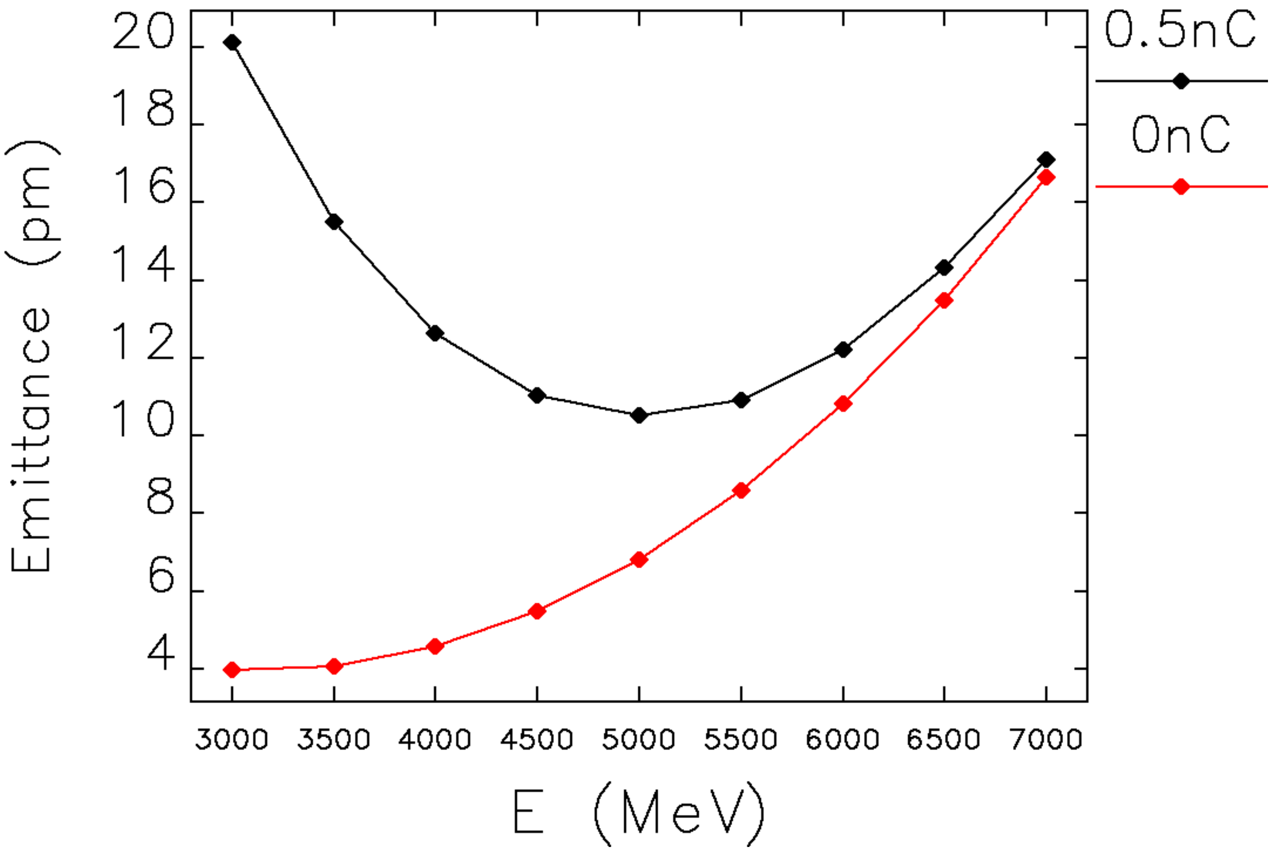
with I_A= 17 kA and α a constant



Steady-state emittances as function of bunch current in PEP-X. The dashed curve gives the analytical approximation.

Dependence on Energy

(M. Borland)



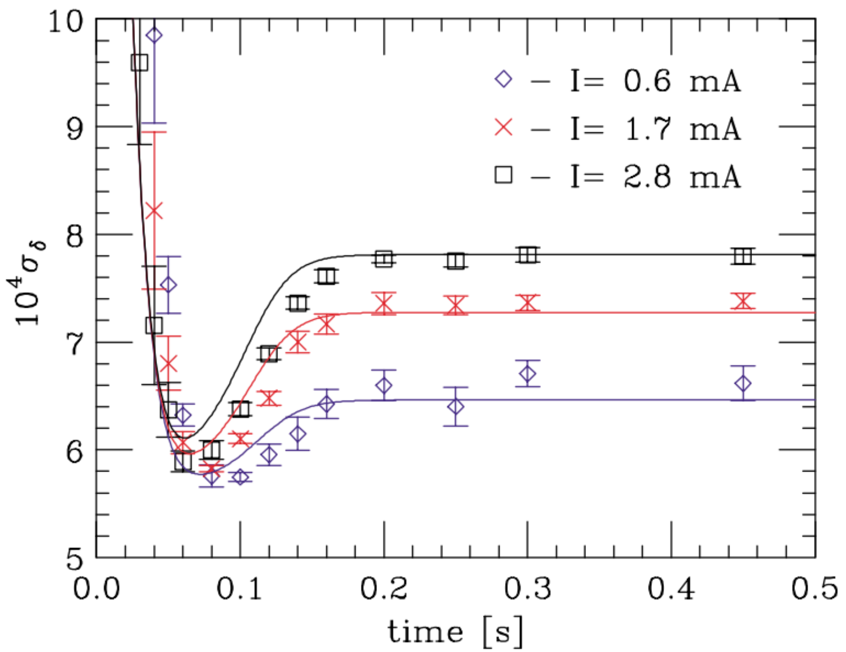
Emittance $\varepsilon_x = \varepsilon_y$ vs. energy for a round beam at nominal current (black) and at zero current (red).

KEK's Accelerator Test Facility (ATF)

ATF is an electron storage ring with C= 138 m, E= 1.28 GeV, ε_{x0}~ 1 nm, ε_{y0}~ 10 pm, maximum N~ 10^{10}. Complete IBS measurements performed in short time in April 2000

At ATF all bunch dimensions can be measured. Unique is that σ_p can be measured to a few percent (at high dispersion point after extraction). λ_z measured by streak camera.

Measured energy spread as function of time after injection, for three different currents (the plotting symbols). The curves give BM simulations.



K. Kubo

Other Measurements in Electron Machines

A more complete set of measurements was also performed at the ATF (simultaneous measurement of σ_p, σ_z, ε_x, ε_y) [K. Bane et al, PRST-AB 2002]. Agreement with theory was good but not perfect. Now it is believed that discrepancy was largely due to ε_y measurement errors

Recently IBS measurements are being performed and planned at CESR-TA [Ehrlichmann et al, IPAC 12] and SLS [Antoniou et al, IPAC 12]

Touschek Lifetime

- Touschek effect concerns large, single Coulomb scattering events where energy transfer from transverse to longitudinal leads to immediate particle loss

- Number of particles in bunch decays as:

$$N_b = \frac{N_{b0}}{1 + t/\mathcal{T}}$$

- Normally, for flat beams, use formula of Brueck

- Otherwise use general formula due to Piwinski. Inverse of Touschek lifetime:

$$\frac{1}{\mathcal{T}} = \frac{r_e^2 c N_b}{8\sqrt{\pi}\beta^2\gamma^4\sigma_z\sigma_p\epsilon_x\epsilon_y}\left\langle \sigma_H \mathcal{F}(\delta_m)\right\rangle$$

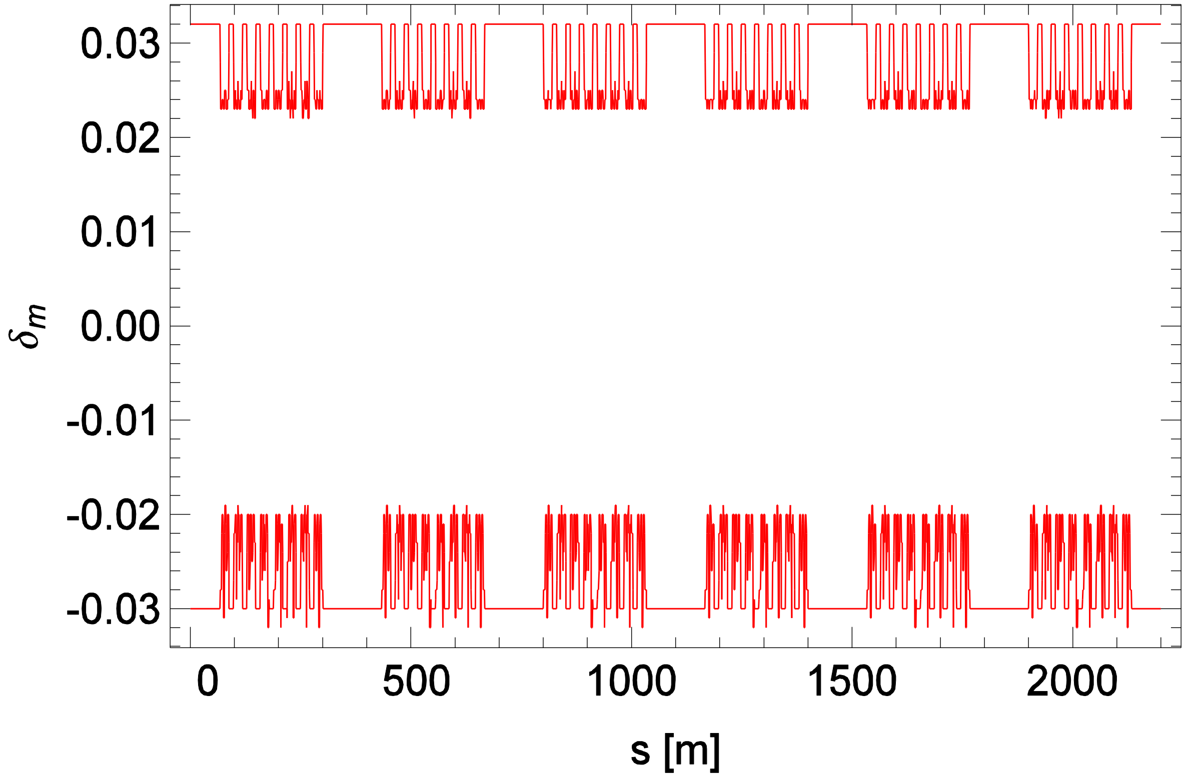
$$\mathcal{F}(\delta_m) = \int_{\delta_m^2}^{\infty} \frac{d\tau}{\tau^{3/2}} \boxed{e^{-\tau B_1}} I_0(\tau B_2)\left[\frac{\tau}{\delta_m^2} - 1 - \frac{1}{2}\ln\left(\frac{\tau}{\delta_m^2}\right)\right]$$

$$B_{1,2} = \frac{1}{2\beta^2\gamma^2}\left|\frac{\beta_x\sigma_x^2}{\epsilon_x\tilde{\sigma}_x^2} \pm \frac{\beta_y}{\epsilon_y}\right|$$

- $B_1 \sim \beta_{x,y}/\varepsilon_{x,y}$ => where $\sigma_{x,y}$ is large, $1/T$ is small because of $\exp(-\tau B_1)$ factor in integral. This factor is also reason $1/T$ becomes small at very small $\varepsilon_{x,y}$.

Momentum Acceptance in PEP-X

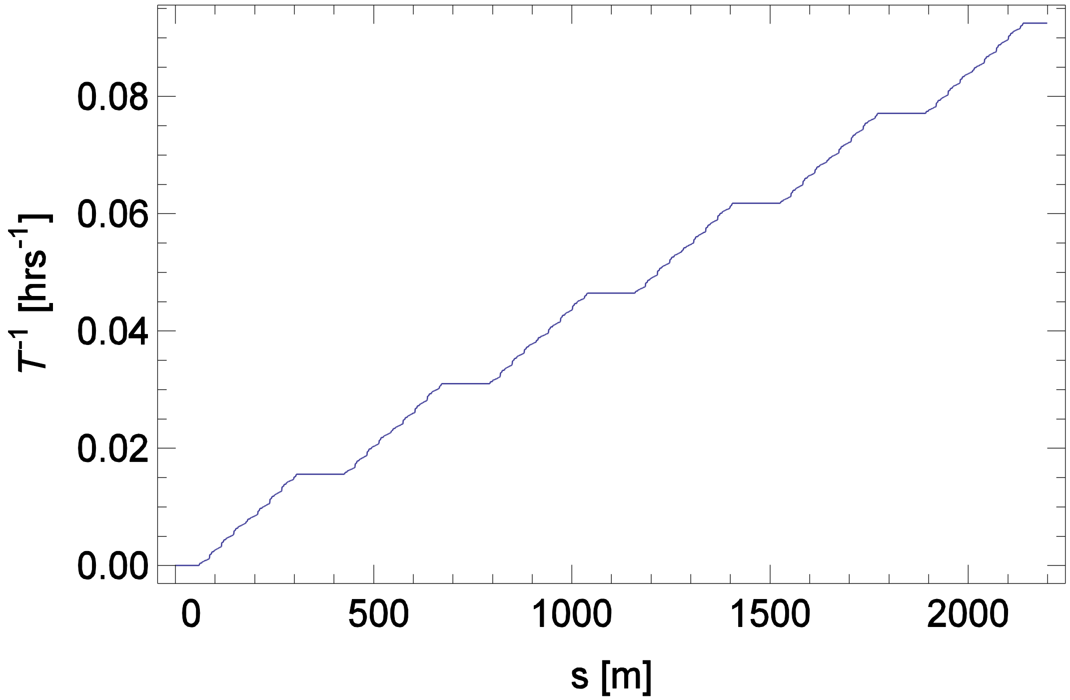
(Min-Huey Wang)



Momentum acceptance due to linear optics for PEP-X.
The average value is $\delta_m = \pm 2.8\%$.

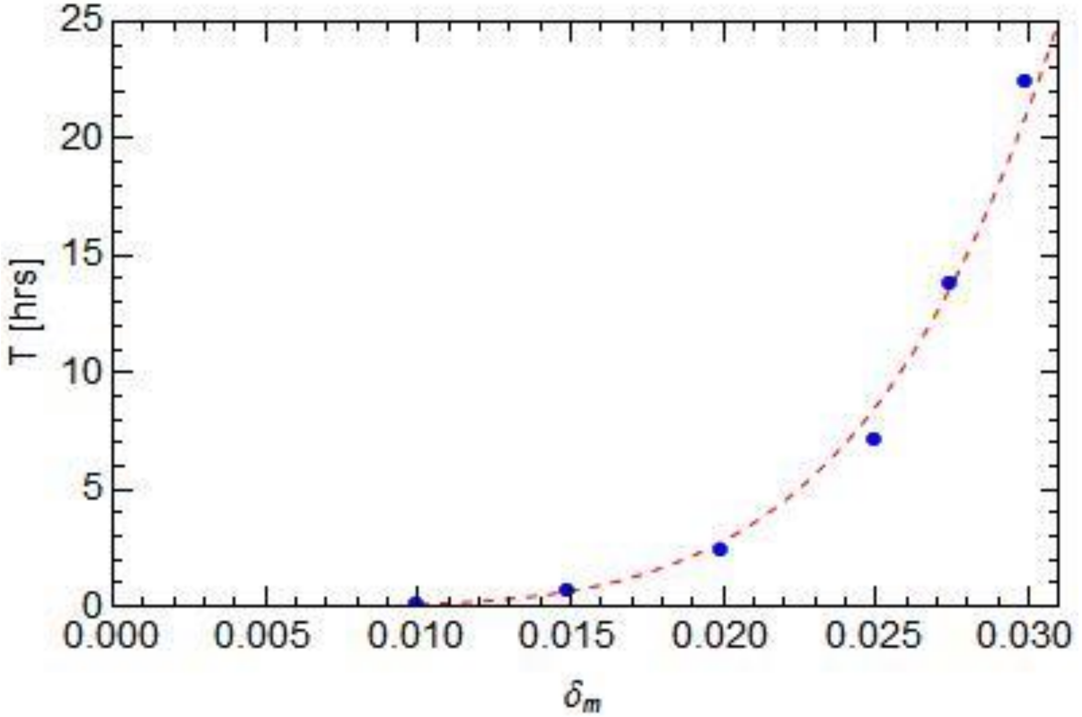
Touschek Lifetime Results

- Result for the IBS-determined steady-state beam sizes is: T= 11 hrs



Accumulation around the ring of the Touschek growth rate in PEP-X. The growth is significant only in the arcs, where $\sigma_{x,y}$ are small.

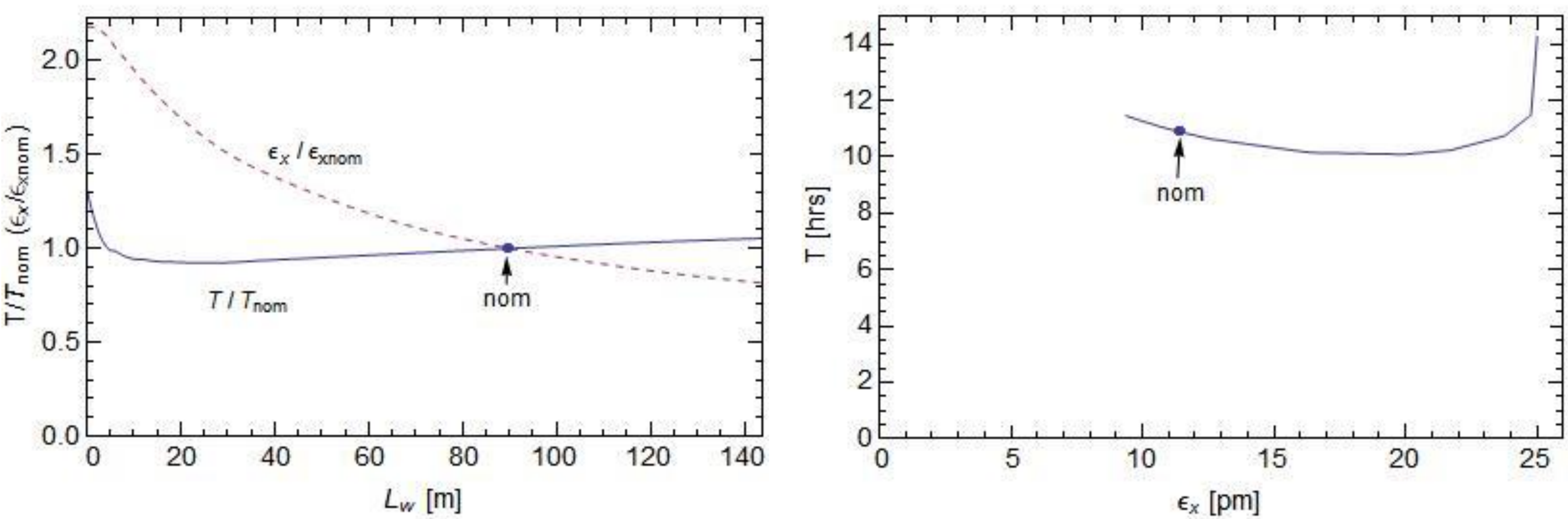
Dependence on Momentum Acceptance



Touschek lifetime T *vs. (global) momentum acceptance parameter,* δ_m *(blue symbols). The dashed curve gives the fit:* T= $0.088(\delta_m/0.01)^5$ *hrs.*

Touschek Lifetime vs Emittance

As the length of wiggler L_w increases, the emittance decreases. In PEP-X design L_w= 90 m, ε_{xnom}= 11 pm (including IBS), T_{nom}= 11 hrs



Emittance ε_x (= ε_y) and Touschek lifetime T *vs wiggler length* L_w *(left plot), and* T *vs* ε_x *(right). These are results of self-consistent calculations including IBS.*

Longitudinal Impedance Calculations for PEP-X

- For PEP-X, without an actual vacuum chamber design available, we developed a straw man design, inspired by objects in other machines, such as PEP-II

Sources include: RF cavities, BPM's, wiggler transitions, undulator transitions, resistive wall, coherent synchrotron radiation (CSR)

For the microwave instability, generate:

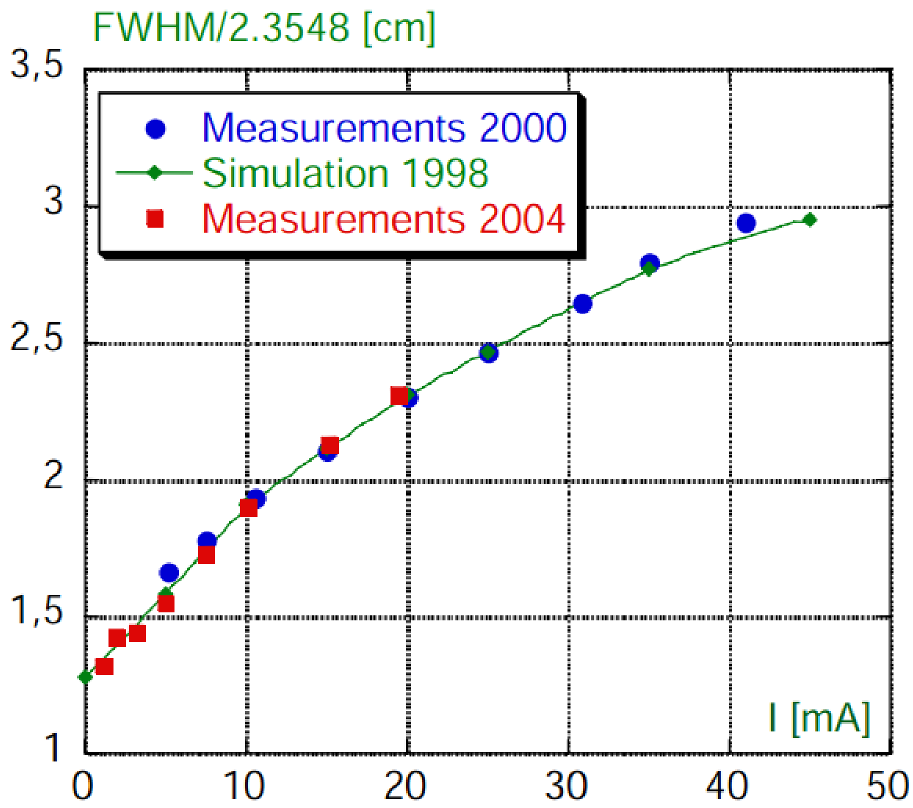
(i) a pseudo-Green function wake representing the ring—to be used in simulations (σ_z= .5 mm; nominal is 3 mm)

(ii) an impedance budget—to assess relative importance of contributors

People involved in 3D code development and impedance calculation include L.-Q. Lee, C.-K. Ng, L. Wang, L. Xiao

This approach was successfully applied many years ago to the SLC damping rings and DaΦne, where drawings of the vacuum chamber components were available [see e.g. Bane et al, HHH-2004 and references therein]

DaΦne Bunch Length Measurements

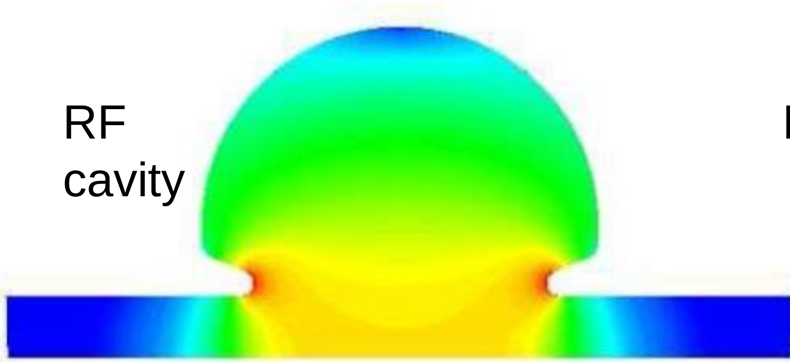


M. Zobov

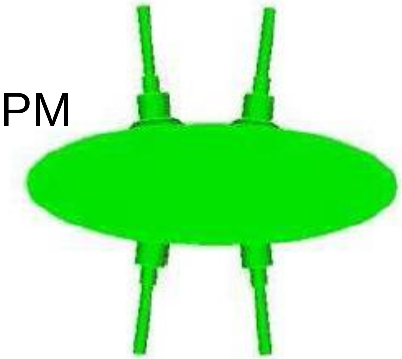
Comparison of bunch lengthening simulations (green line) with measurements (symbols) for DaΦne. The wake used in the simulations was obtained from the drawings of vacuum chamber components

Selected PEP-X Impedance Sources

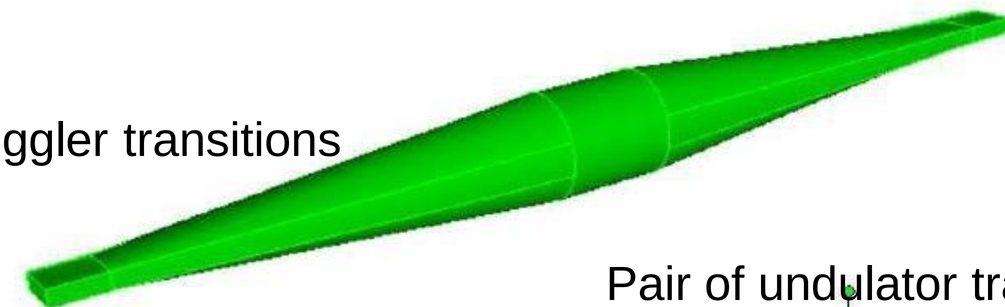
RF cavity



BPM



Pair of wiggler transitions



Pair of undulator transitions

Selected impedance objects included in our straw man PEP-X design.
Note: the fundamental mode fields are shown in the RF cavity.

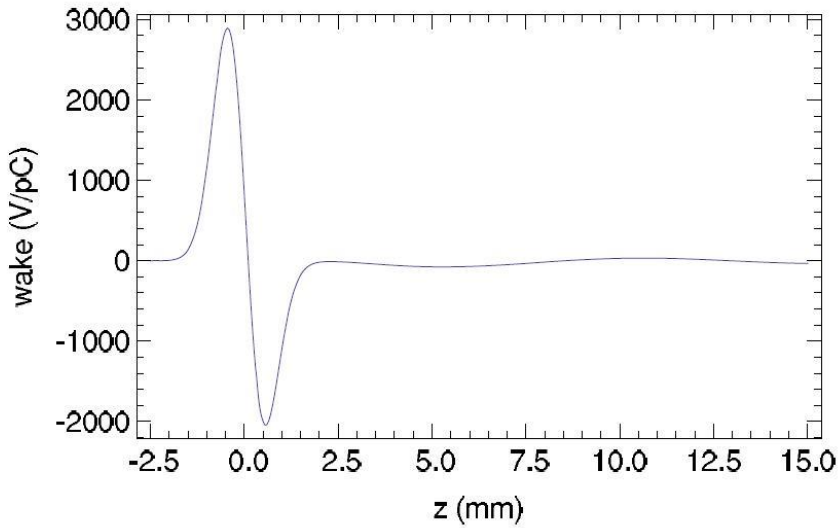
Impedance Budget

Object	Single Contribution			Total Contribution			
	k_{loss}[V/pC]	R [Ω]	L [nH]	N_{obj}	k_{loss}[V/pC]	R [Ω]	L [nH]
RF cavity	.92	30.4	–	16	14.7	487	–
Undulator taper (pair)	.06	3.2	.32	30	1.9	95	9.6
Wiggler taper (pair)	.43	21.4	.72	16	6.8	340	11.5
BPMs	.013	.6	.005	839	11.3	465	4.1
Bellows slots	.00	.0	4e-4	720	.0	.0	.3
Bellows masks	.005	.2	.004	720	3.7	142	2.7
Resistive wall wake					21.3	880	11.3
Total					59.7	2409	39.5

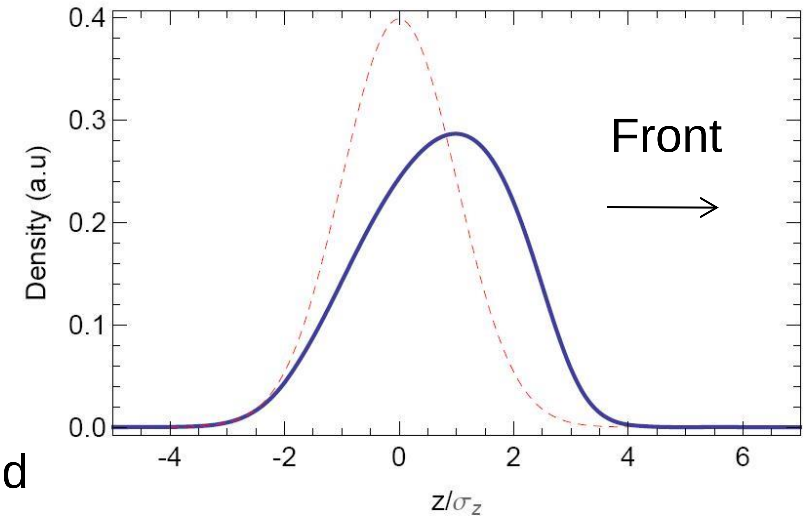
Impedance budget for PEP-X, giving the loss factor, and the effective resistance and inductance of the various objects in the ring. The results are at nominal bunch length σ_z= 3 mm.

Pseudo-Green Function Wake

Pseudo-Green function wake representing the PEP-X ring: wake of a σ_z= .5 mm bunch



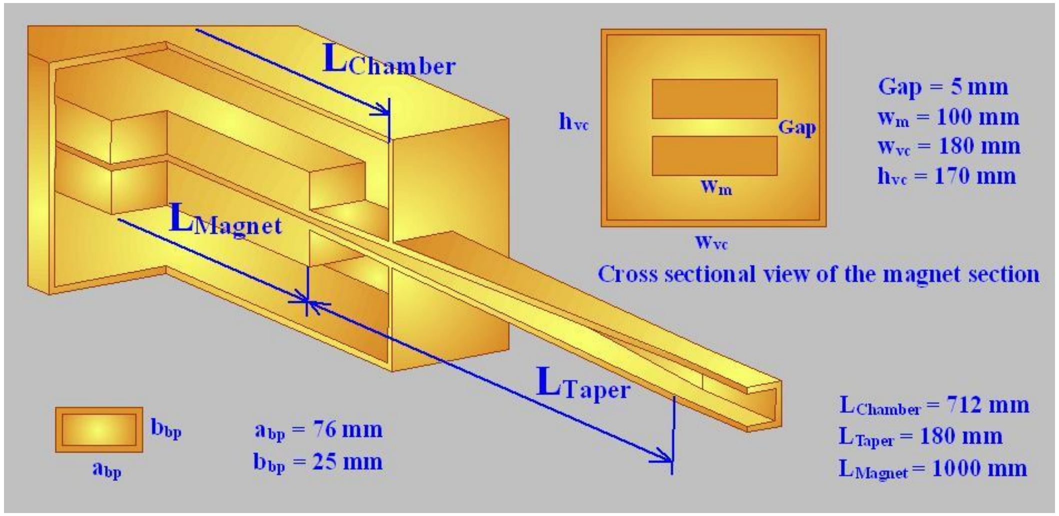
Haissinski solution, giving the steady-state bunch shape. Bunch length is 25% above nominal length.



(G. Stupakov)

For PEP-X microwave threshold is very high (> 8 A)

A Difficult Example: an Insertion Device

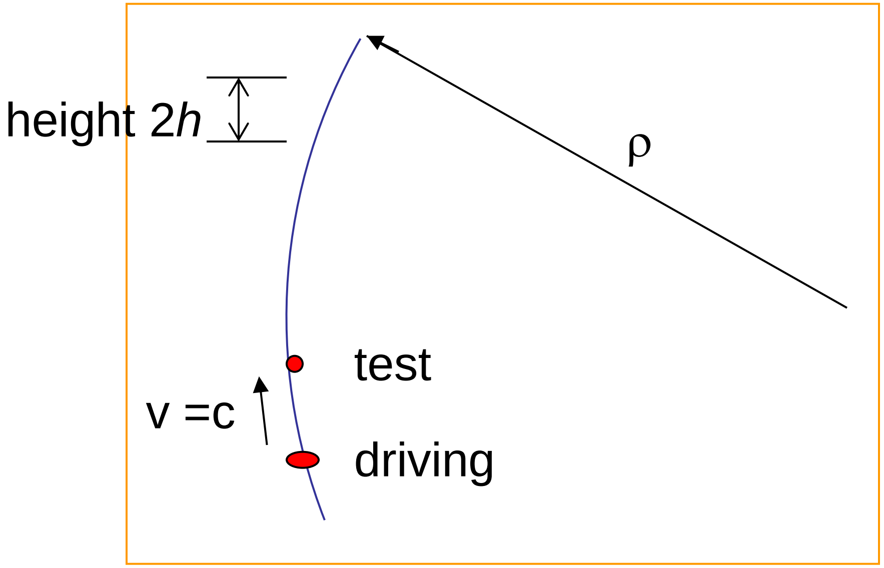


From "Impedance calculation for the NSLSII storage ring," A. Blednykh

With the insertion gap becoming ever smaller, the insertion region becomes a dominating part of the ring impedance

Insertion transitions tend to be long, gradually tapered, and 3D => it is very challenging to obtain the wakefield for a short bunch

Microwave Instability Due to Shielded CSR



K. Bane, Y. Cai, G. Stupakov, PRST-AB, 2010

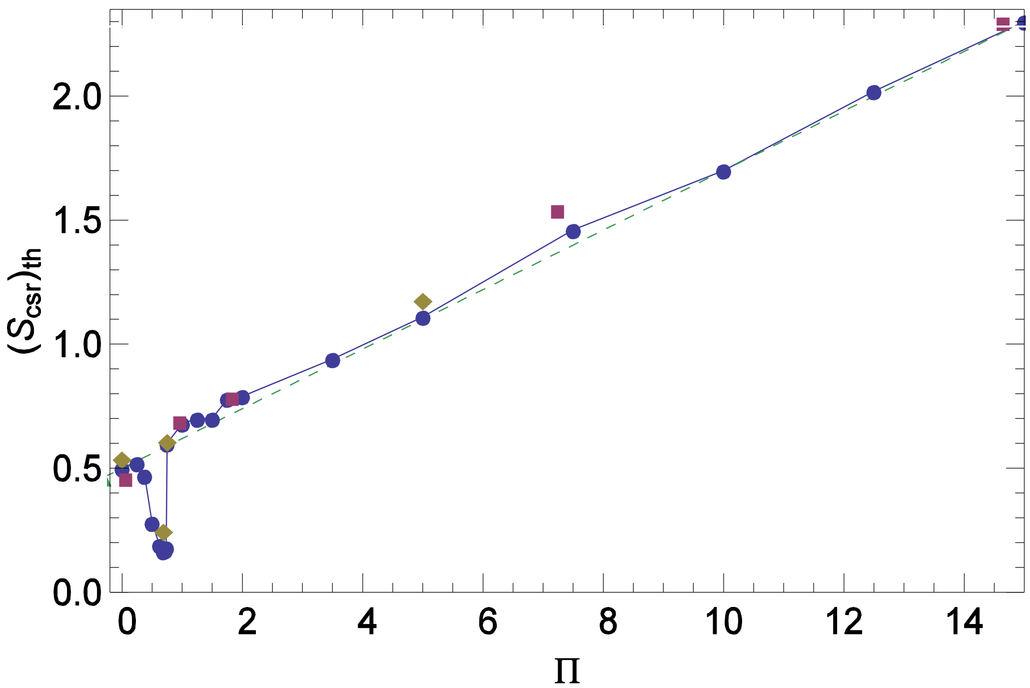
- Wake of particles moving at speed *c*, on circle of radius ρ, between two metallic plates located at $y = \pm h$ was found by J. Murphy, et al.

- For a bunch, the normalized threshold current *S* is a function of normalized shielding parameter Π, with

$$S = \frac{eN_b\rho^{1/3}}{2\pi\nu_s\gamma\sigma_\delta\sigma_z^{4/3}}\ , \qquad \Pi = \frac{\sigma_z\rho^{1/2}}{h^{3/2}}$$

- Used Vlasov-Fokker-Planck solver (a la Warnock) and Vlasov solver to find threshold current $(S_{csr})_{th}$ as function of Π

Threshold for Shielded CSR Impedance

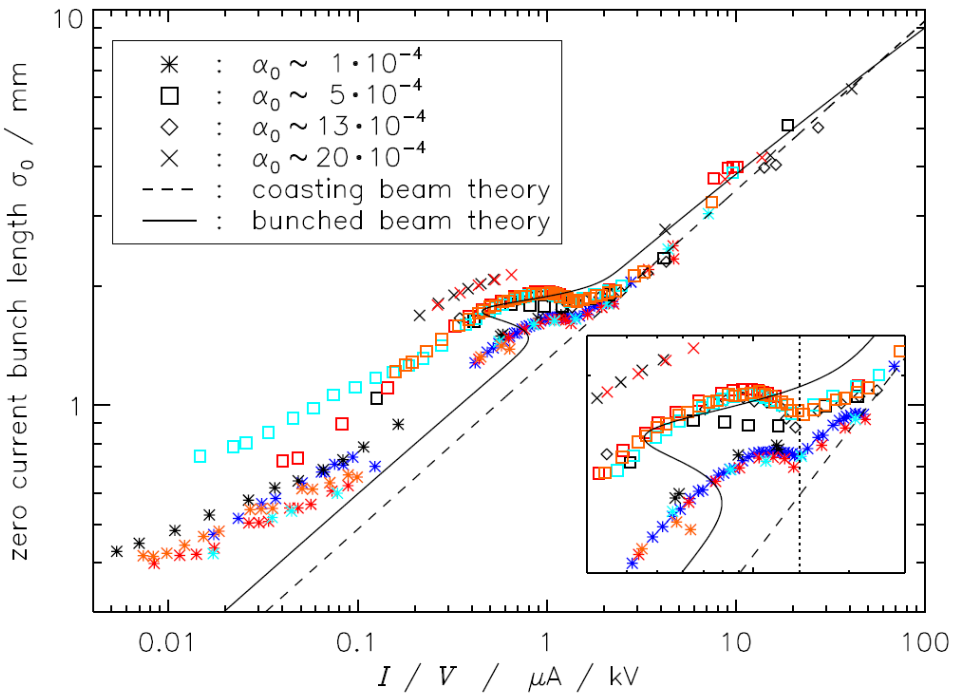


For the CSR wake, threshold values of current S vs. shielding parameter Π. The dashed curve is S_{th}= 0.50 + 0.12 Π.

- Note dip at Π~ 0.7

- For PEP-X, vacuum chamber is elliptical with axes (20.0, 12.5) mm and bending radius ρ= 100.8 m => Π= 22.7 and I_{th}= 3.6 A, far above design I.

Comparison with Measurement (see P. Kuske talk)



M. Ries, et al, IPAC 12

Scaled values of measured bursting threshold at Metrology Light Source. Colors indicate measurement series within one α_0 set.

- Reasonably good agreement at longer bunch lengths and for location of dip. Note that simulations assumed simple phase space (not low α)

- Similarly good agreement for BESSY-II (P. Kuske), ANKA (M. Klein). Shows that for short bunches CSR can dominate a ring impedance, and can be simply modeled

Other Instabilities

Transverse Single Bunch (TMCI):

In light sources with regions of small aperture vacuum chambers, the resistive wall impedance is normally the dominant contributor. For a Gaussian bunch the average kick in a round chamber of radius *b*

$$\kappa_y = (0.723) \frac{c}{\pi^{3/2} b^3} \sqrt{\frac{Z_0}{\sigma_z \sigma_c}}$$

Single-bunch threshold can be approximated by [S. Krinsky]

$$I_b^{th} \approx 0.7 \frac{4\pi c \, \nu_s (E/e)}{\mathcal{C}} \frac{1}{\sum_i \ell_i \beta_{y,i} \kappa_{y,i}}$$

For PEP-X, multi-bunch threshold $I^{th} = M I_b^{th} = 1.8$ A

PEP-X beam chamber types. The first three entries are Al, the last two Cu

Type	Length [m]	Shape	(b_x, b_y) [mm]	$\langle \beta_y \rangle$ [m]
Arcs	1318	E	(30.0, 12.5)	7.0
Straights r	510	C	(48.0, 48.0)	15.6
Straights i	123	C	(48.0, 48.0)	60.0
Undulators	158	E	(25.0, 3.0)	2.8
Wigglers	90	R	(22.5, 4.0)	12.0

Multi-bunch Transverse Instability:

Including only the resistive wall wake, which often dominates the multibunch transverse instability, the growth rate is given by [A. Wolski et al, 2006]

$$\Gamma = \frac{c\,(I/I_A)}{4\gamma\sqrt{\mathcal{C}(1-[\nu_y])}}\langle\beta A\rangle$$

with

$$\langle\beta A\rangle = \frac{4}{\sqrt{\pi Z_0}}\sum_i \frac{\ell_i\beta_{y,i}}{b_i^3\sqrt{\sigma_{c,i}}}$$

For PEP-X (ultimate) the growth rate is 1.4 ms^{-1} or 99 turns

Fast Ion Instability:

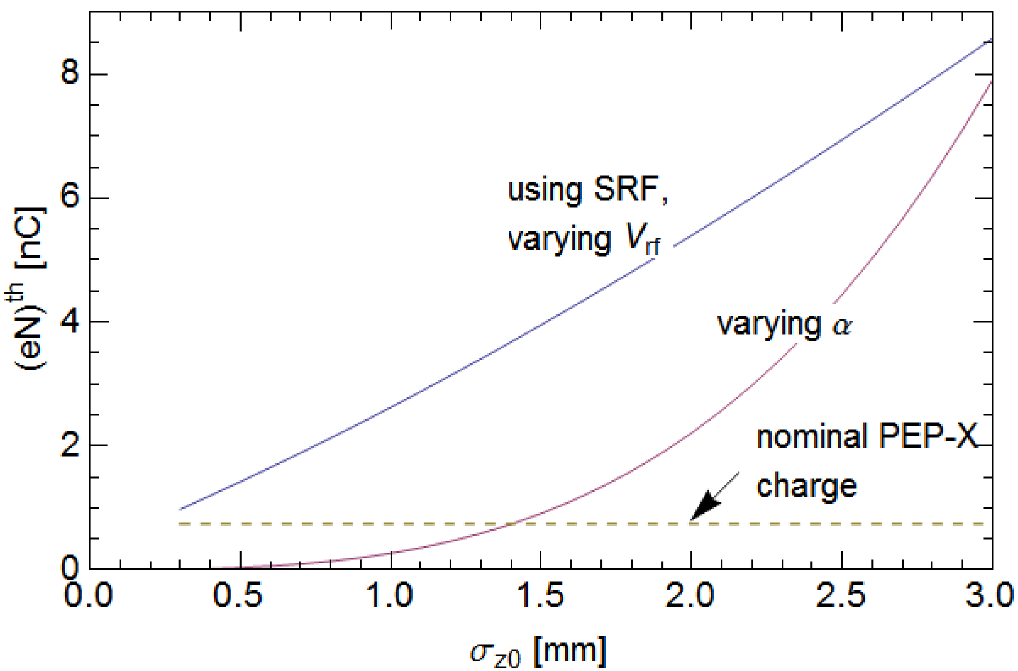
Places requirements on vacuum pressure, gap in bunch train. Multi-particle tracking shows that the instability is manageable (see L. Wang's talk)

Shorter Bunches

Superconducting RF is proposed for BESSY, three RF frequencies to allow for short and long bunches [G. Wuestefeld et al, IPAC 11]

Threshold can be written in terms of nominal bunch length as

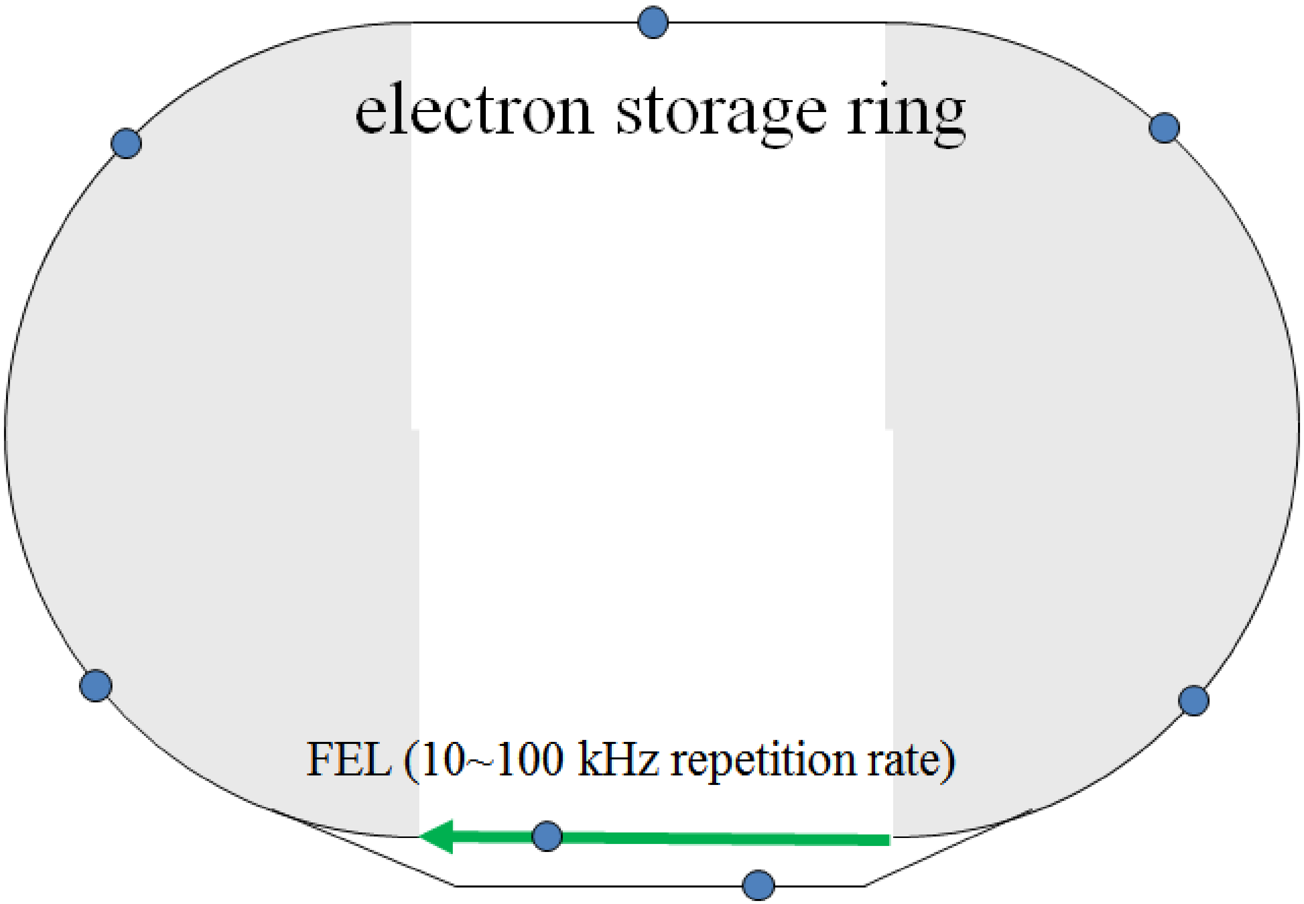
$$\sigma_{z0}^{7/3} = \frac{Z_0 c^2}{8\pi^2} \frac{(eN)^{th}}{S^{th}(\Pi)} \frac{\rho^{1/3}}{f_{rf} V_{rf} \cos\phi_s}$$



Y. Cai PEP-X talk

Microwave threshold in PEP-X due to shielded CSR, when using SRF and varying V_{rf} (blue), or normal RF and adjusting α (red). Note: SRF has factor 3 higher frequency (f_{rf}=1.4 GHz). Shielding parameter $\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$

FEL Lasing in a Long Switched Bypass



Y. Cai

Electron bunch is recycled after lasing in the bypass

PEP-X Lasing Version Parameters

Table: Selected lasing PEP-X parameters. Numbers in red indicate parameters that differ from the non-lasing version of PEP-X

Parameters	Value	Units
Circumference, C	2200	m
Energy, E	4.5	GeV
Bunch charge, eN	0.75	nC
Nominal peak current, $\hat{I}$	300	A
Nominal bunch length, σ_{z0}	0.3	mm
Nominal energy spread, $\sigma_{\delta 0}$	1.14	10^{-3}
Synchrotron tune, ν_s	0.077	
Momentum compaction, α	5.81	10^{-5}
Tunes, ν_x, ν_y	87.23, 36.14	
Average β function, β_y	9.7	m
Longitudinal damping time, τ_s	10.8	ms

Short Bunches

For PEP-X with σ_z= 0.3 mm, shielded CSR parameter Π= 2.2, $(eN)^{th}$= 1.1 nC

With round beams, IBS yields $\varepsilon_x = \varepsilon_y$= 27 pm, Touschek lifetime is 63 min

We are considering JLAB 7-cell 1.4 GHz SRF, 64 cavities, total voltage 300 MV

A serious problem is multi-bunch transverse instability driven by dipole modes of RF cavities

For high currents, short bunches HOM heating can be serious problem. Losses $\sim\sigma_z^{-4/3}$ (CSR), $\sim\sigma_z^{-3/2}$ (resistive wall wake) and these sources tend to dominate

Conclusions

• In low emittance rings such as PEP-X “ultimate”, impedance effects tend not to be important since the current is quite low (200 mA)

• IBS sets the limit of current that can be stored in an ultimate ring. In PEP-X with round beams, IBS doubles the emittance to 11 pm at the design current of 200 mA.

• The Touschek lifetime in ultimate PEP-X is quite large, 11 hrs, but it is a very sensitive function of the momentum acceptance

• How to run a machine with a round beam needs serious study. The choice will affect the IBS and Touschek effect

In a ring like PEP-X “lasing”, where the beam is shortened using lots of SRF, impedance effects will be stronger. One needs to carefully consider the microwave instability, multi-bunch transverse instability (due to cavity modes), HOM heating, …