A SUSY GUT of Flavour with $SU(5) \times S_4 \times U(1)$

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Outline

Introduction

❖ The SM & SUSY Flavour Problem.

❖ Solving it by imposing a Family symmetry.

The SU(5)xS_4xU(1) Model

❖ Construction of fermionic sector.

❖ Construction of SUSY breaking sector:

- SCKM basis
- **Mass Insertion (MI) parameters:** ***observables associated with FCNC processes***.

❖Predictions for low energy MIs Vs experimental constraints.

Summary

The Flavour Problem

Why are there 3 families of quarks & leptons?

Why are their masses so hierarchical?

Why is lepton mixing so large compared to quark mixing?

Why are neutrino masses so small?

More than 1 generations ⟶ Yukawa coupling terms become matrices

Understanding pattern of fermion masses & mixings = =Understanding structure of Yukawa matrices.

Here is an idea…

Family Symmetry

Extend symmetry group with a **Family Symmetry G_F**

- admits triplet reps

(*3 families in a triplet*)

Introduce extra heavy scalar fields Φ: **Flavons**

- couple to usual matter fields

Write down operators allowed by all symmetries

- typically non-renormalisable

$$O_Y = f^i \frac{\Phi_i \Phi_j}{M^2} f^{cj} H$$

M: heavy mass scale; UV cut-off

Spontaneously **break** G_F, as Φs develop **≠0 vevs**

- effective Yukawa couplings generated:

$$Y_{ij} = \frac{\langle\Phi_i\rangle\langle\Phi_j\rangle}{M^2} = f\left(\boxed{\lambda = \frac{\langle\Phi\rangle}{M}}\right)$$

expansion parameter

build up desired hierarchical Yukawa textures

Explain form of Yukawa matrices ⟺ Find appropriate symmetry G_F, field content & vacuum alignment for flavons

Extend to **SUSY GUTs**

- Fields become superfields.
- Yukawa operators arise from the superpotential W:

$$W = f w \left(\frac{\Phi^n}{M^n} \right) f^c H$$

- Kinetic terms & scalar masses arise from the Kähler potential K.
- **Spartner masses & mixings must also be explained.**
- **Control FC processes induced by loop diags involving sfermion masses** which are non-diagonal in the basis where Yukawa matrices are diagonal (SCKM basis).
- GUT models more constraining due to boundary conditions between hadronic & leptonic sectors.

$\theta^\nu_{13} << \theta^\nu_{12}, \theta^\nu_{23}$

- An interesting Family symmetry G_F would predict **TB-mixing** in the neutrino sector.

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \boxed{0} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

- Neutrino mass matrix:

✓ diagonalised by U_{TB}.

✓invariant under Klein symmetry: $Z_2^S \otimes Z_2^U$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\theta^\nu_{13} \approx 9^o \neq 0$

- Need deviations from TB.

Neutrino flavour symmetry arising from G_F

- G_F would contain the S & U generators
- preserved in the neutrino sector (m^ν_{eff} *invariant under S & U*).

A specific model :

$SU(5) \times S_4 \times U(1)$

permutations of 4 objects

Minimal GUT with smallest discrete group that contains S&U generators.

The SU(5) x S_4 x U(1) Model

$T = \mathbf{10} = (Q, u^c, e^c)$ $F = \bar{\mathbf{5}} = (L, d^c)$

Field	T_3	T	F	N	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$	Φ^u_2	$\widetilde{\Phi}^u_2$	Φ^d_3	$\widetilde{\Phi}^d_3$	Φ^d_2	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	0	x	y	$-y$	0	0	z	$-2x$	0	$-y$	$-x-y-2z$	z	$2y$	$2y$	$2y$

- **U(1) symmetry**: different flavons couple to distinct sectors at LO (according to their f label);
  charges depend on three integers: x, y, z.

- For an operator to be a singlet under U(1), the charges of the participating superfields must add up to zero for every x, y, z.

The fermion sector

C. Hagedorn, S.F. King, C. Luhn:

arXiv:1003.4249
arXiv:1205.3114v1

Constructing Y^u

Write down all possible **operators** that form a **singlet** under all symmetries (*using the S_4 Kronecker products*)

combinations of TTH_5 for the first two families & $T_3T_3H_5$ for the 3rd family with a number of flavons

$T = \mathbf{10} = (Q, u^c, e^c)$

$$y_3^u T_3 T_3 H_5 + y_2^u \frac{1}{M} T T \Phi_2^u H_5 + y_1^u \frac{1}{M^2} T T \Phi_2^u \tilde{\Phi}_2^u H_5 .$$

y^u_i: O(1) coefficients

Expand out all possible **contractions** (*using the Clebsh-Gordan coefficients of S_4*).

Putting all terms together **in terms of components**..

$$y_3^u T_3 T_3 H_5 + \frac{y_2^u}{M}\left(T_1 T_1 \Phi^u_{2,1} + T_2 T_2 \Phi^u_{2,2}\right) H_5 + \frac{y_1^u}{M^2}\left(T_1 T_1 \Phi^u_{2,2}\tilde{\Phi}^u_{2,2} + T_2 T_2 \Phi^u_{2,1}\tilde{\Phi}^u_{2,1}\right) H_5$$

Break family symmetry with non-zero flavon vevs

$$\langle \Phi_2^u \rangle = \varphi_2^u \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \tilde{\Phi}_2^u \rangle = \tilde{\varphi}_2^u \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\frac{\varphi_2^u}{M} \approx \lambda^4 \quad \text{and} \quad \frac{\tilde{\varphi}_2^u}{M} \approx \lambda^4$$

$\lambda \approx 0.22$: Wolfstein parameter

Generate Yukawa matrix

$$Y^u = \begin{pmatrix} y_1^u \lambda^8 & 0 & 0 \\ 0 & y_2^u \lambda^4 & 0 \\ 0 & 0 & y_3^u \end{pmatrix}$$

with the well-known mass hierarchy for the up-quarks:

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1$$

❖ Similarly, write down operators consisting of T , F & $\Phi^d{}_\rho$

$$Y^d = \begin{pmatrix} 0 & y_3^d\lambda^5 & -y_3^d\lambda^5 \\ -y_3^d\lambda^5 & y_1^d\lambda^4 & (y_3^d - yd_1)\lambda^4 \\ 0 & 0 & y_2^d\lambda^2 \end{pmatrix}$$

$$Y^e = \begin{pmatrix} 0 & -y_3^e\lambda^5 & 0 \\ y_3^e\lambda^5 & -3y_1^e\lambda^4 & 0 \\ -y_3^e\lambda^5 & (3y_1^e + y_3^e)\lambda^4 & y_2^e\lambda^2 \end{pmatrix}$$

$T = \mathbf{10} = (Q, u^c, e^c)$ $F = \bar{\mathbf{5}} = (L, d^c)$

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1$$
$$m_e : m_\mu : m_\tau \approx (1/3)\lambda^4 : 3\lambda^2 : 1$$

$$\theta_{12}^d \approx \lambda, \quad \theta_{13}^d \approx \lambda^3, \quad \theta_{23}^d, \approx \lambda^2$$
$$\theta_{12}^e \approx (1/3)\lambda, \quad \theta_{13}^e \approx 0, \quad \theta_{23}^e, \approx 0$$

❖ Y^u is diagonal ($U^u{}_L = \mathbf{1}$).

❖ All quark mixing ($\mathbf{V = U^u{}_L U^{d\dagger}{}_L}$) is coming from Y^d.

❖ Georgi-Jarlskog (GJ) relations: $\mathbf{m_b \approx m_\tau}$, $\mathbf{m_\mu \approx 3m_s}$, $\mathbf{m_d \approx 3m_e}$
and GST relation: $\boldsymbol{\theta_{12} \approx \sqrt{(m_d/m_s)}}$ incorporated
such that after RG-running we get the appropriate masses at M_W.

Neutrino sector

Operators:

$NFH_5 \rightarrow M_D$, $NN\Phi^{\nu}_{\rho} \rightarrow M_R$

Type I see-saw formula:

$\mathbf{m^{\nu}_{eff}} = M_D\, \mathbf{M_R}^{-1} M_D^{\,T}\, \upsilon_u^{\,2}$

$<\Phi_\rho^{\nu}>$: eigenvectors of **S&U**

$Z_2^S x Z_2^U$ **Klein** subgroup of S_4 preserved

TB-mixing in the neutrino sector

$U_{PMNS} = U^e_L\, U^{\nu\dagger}_L = U^e_L U_{TB}$

θ^l_{12} , θ^l_{23} of the correct order & $\theta^l_{13} \sim 3^o$

Deviation from TB due to charged-lepton sector not enough as $\theta^l_{13}{}^{exp} \approx 9^o$

Further deviation from **TB:** new flavon η (S_4 singlet).

$$\frac{1}{M}\eta\Phi_2^d NN$$

breaks Z_2^U as $<\Phi_2^d>$

Not eigenvector of U

θ^ν_{13} , θ^ν_{23} receive corrections $O(\lambda)$ → agreement with exp.

The SUSY breaking sector

A-trilinear terms

$$\mathcal{L}_{soft} = \boxed{\epsilon_{\alpha\beta}\left(-H_u^\alpha \tilde{Q}^{\beta i} A^u_{ij} \tilde{u}^{cj} - H_d^\alpha \tilde{Q}^{\beta i} A^d_{ij} \tilde{d}^{cj} - H_u^\alpha \tilde{L}^{\beta i} A^\nu_{ij} \tilde{\nu}^{cj} - H_d^\alpha \tilde{L}^{\beta j} A^e_{ij} \tilde{e}^{cj} + h.c\right)}$$

$$+ \tilde{Q}^*_{i\alpha}(m^2_Q)^i_j \tilde{Q}^{\alpha j} + \tilde{u}^{c*}_i (m^2_{u^c})^i_j \tilde{u}^{cj} + \tilde{d}^{c*}_i (m^2_{d^c})^i_j \tilde{d}^{cj} + \tilde{L}^*_{i\alpha}(m^2_L)^i_j \tilde{L}^{\alpha j}$$

$$+ \tilde{e}^{c*}_i (m^2_{e^c})^i_j \tilde{e}^{cj} + \tilde{\nu}^{c*}(m^2_{\nu^c})^i_j \tilde{\nu}^{cj}$$

Scalar mass terms

A-terms

- Common origin with the Yukawas
- Write down the A-matrices by looking at the Yukawas & just changing the O(1) coefs.

soft scalar masses

- Look for combinations of a number of flavons and $T^\dagger T$ (*for the (12) block*), $\underbrace{T_3^\dagger T_3 \text{ (for the (33) entry)}}_{\text{absorbed into } b_{02}}$ & $T^\dagger T_3$ (*for the (i3) entries*)

$\Phi_2^d \Phi_2^{d\,\dagger}/M^2$

$$\frac{M_T^2}{m_0^2} = \begin{pmatrix} b_{01} & 0 & 0 \\ 0 & b_{01} & 0 \\ 0 & 0 & b_{02} \end{pmatrix} + \begin{pmatrix} \boxed{b_1 \lambda^2 [\varphi_2^d \varphi_2^{d*}]} & b_2 \lambda^4 [\tilde{\varphi}_2^u] & b_4 \lambda^7 [\varphi_2^d \varphi_3^d \varphi_{3'}^\nu] \\ b_2^* \lambda^4 [\tilde{\varphi}_2^{\tilde{u}*}] & -b_1 \lambda^2 [\varphi_2^d \varphi_2^{d*}] & c_3 \lambda^5 [(\varphi_2^d)^5] \\ b_4^* \lambda^7 [\varphi_2^{d*} \varphi_3^{d*} \varphi_{3'}^{\nu*}] & c_3^* \lambda^5 [(\varphi_2^{d*})^5] & 0 \end{pmatrix}$$

soft scalar masses

- In a similar way, we get the mass matrix M^2_F for the Fs.

- Note that in the **exact family symmetry limit**:
  - **Y, A : zero**
  - **M^2_T , M^2_F: diagonal**

- Violation of this property & production of **off-diagonal** entries occurs due to the **non-zero flavon vevs** that break the family symmetry.

It is these off-diagonalities that have to be controlled in order to lead to predictions that agree with FCNC bounds.

Super-CKM basis

Change to the basis where **Yukawas** are **diagonal**

$(U^f_L)^\dagger Y^f_C U^f_R = Y^f$: diag

Rotate matter fields:

$f_L \to U^f_L\, f_L,\ \ f_R \to U^f_R\, f_R$

- $V_{SCKM} = U^u_L (U^d_L)^\dagger$.
- m^2_Q corresponds to 2 physical masses: $(\tilde{m}^2 u)_{LL}$, $(\tilde{m}^2 d)_{LL}$.
- obtained by: $(\tilde{m}^2_u)_{LL} = (U^u_L)^\dagger m^2_Q U^u_L$
- Also have the RR-matrices: $(\tilde{m}^2_u)_{RR} = (U^u_R)^\dagger m^2_{u^c} U^u_R$
- In the SU(5) context:

$$m^2_Q = m^2_{u^c} = m^2_{e^c} = M^2_T$$

$$m^2_L = m^2_{d^c} = M^2_F$$

Mass Insertion (MI) Parameters

- Full fermion & sfermion 3x3 mass matrices defined as:

$$m^2_{\tilde{f}_{LL}} = (\tilde{m}^2_f)_{LL} + \tilde{Y}_f \tilde{Y}_f^\dagger v^2_{u,d} \quad , \quad m^2_{\tilde{f}_{RR}} = (\tilde{m}^2_f)_{RR} + \tilde{Y}_f^\dagger \tilde{Y}_f v^2_{u,d}$$

$$m^2_{\tilde{f}_{LR}} = \tilde{A}_f v_{u,d} - \mu \tilde{Y}_f v_{d,u} \quad , \quad m^2_{\tilde{f}_{RL}} = \tilde{A}_f^\dagger v_{u,d} - \mu \tilde{Y}_f^\dagger v_{d,u}$$

- Theoretical predictions in terms of the dim/less parameters:

$$(\delta^f_{LL})_{ij} = \frac{(m^2_{\tilde{f}_{LL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LL}}, \quad (\delta^f_{RR})_{ij} = \frac{(m^2_{\tilde{f}_{RR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RR}}$$

$$(\delta^f_{LR})_{ij} = \frac{(m^2_{\tilde{f}_{LR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LR}}, \quad (\delta^f_{RL})_{ij} = \frac{(m^2_{\tilde{f}_{RL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RL}}$$

with average masses : $\langle m_{\tilde{f}} \rangle^2_{AB} = \sqrt{(m^2_{\tilde{f}_{AA}})_{ii} (m^2_{\tilde{f}_{BB}})_{jj}}$

Low Energy MIs

- Calculations so far hold just below M_{GUT}.
- Need RGE evolution to M_W where experiments are performed.

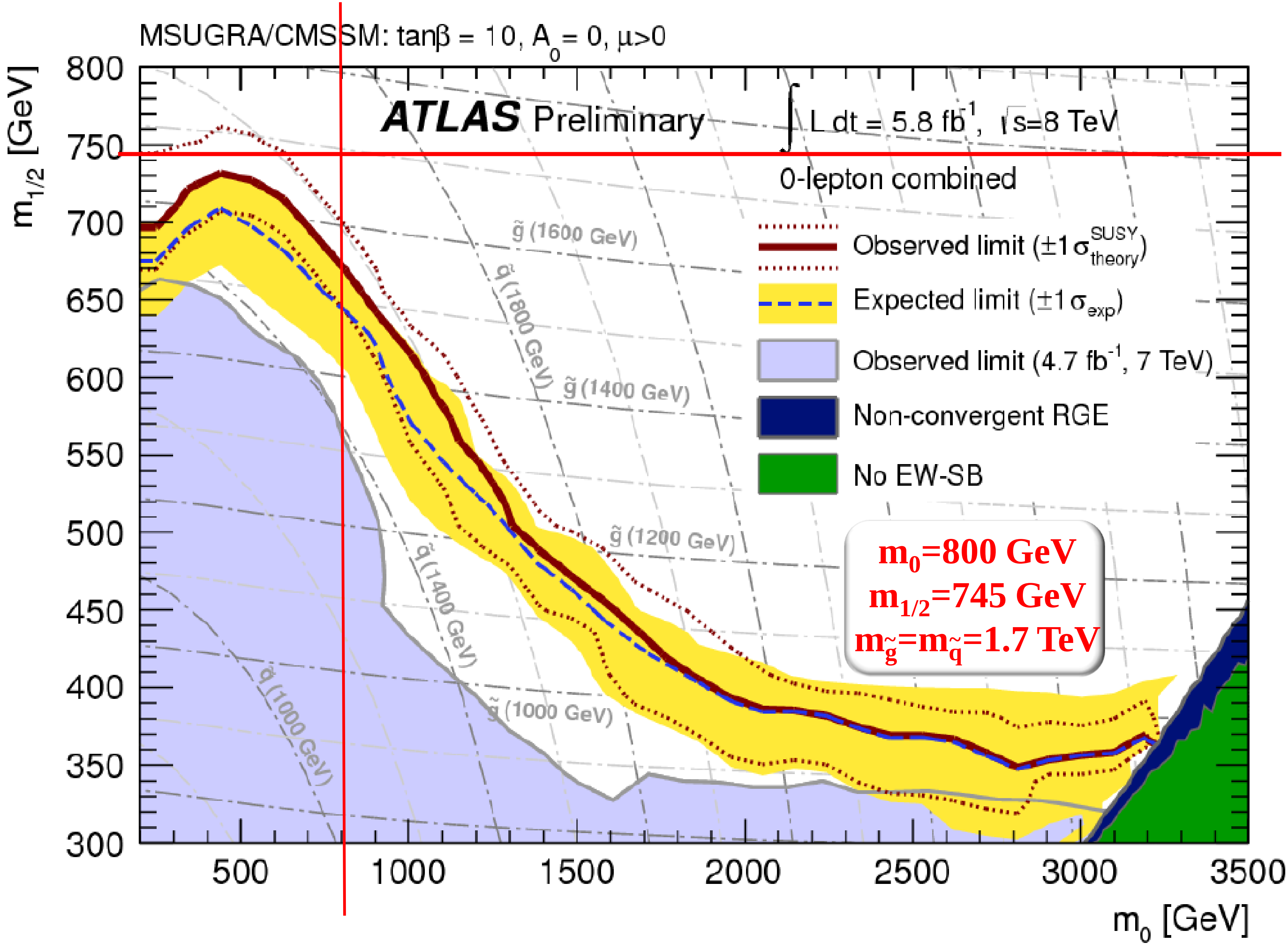
- Recast our parameters in a form:

$$(\delta^f)^{ij}_{AB}|_{M_W} = S^{f(ij)}_{AB} \times (\delta^f)^{ij}_{AB}|_{M_{GUT}}, \quad i,j = 1,2,3, \quad A,B \in L,R$$

- where $S^f < 1$ accounts for the effects of RG-running.

- In general, $$S^{f\,(ij)}_{AB} \sim \frac{\langle \tilde{m}_f \rangle^2_{AB\,(ij)}|_{M_{GUT}}}{\langle \tilde{m}_f \rangle^2_{AB\,(ij)}|_{M_W}}$$

$$\langle m_{\tilde{f}} \rangle^2_{AB} = \sqrt{(m^2_{\tilde{f}_{AA}})_{ii}(m^2_{\tilde{f}_{BB}})_{jj}}$$


MSUGRA/CMSSM: tanβ = 10, A0 = 0, μ>0
ATLAS Preliminary
∫ L dt = 5.8 fb-1, √s=8 TeV
0-lepton combined
Observed limit (±1 σ SUSY theory)
Expected limit (±1 σexp)
Observed limit (4.7 fb-1, 7 TeV)
Non-convergent RGE
No EW-SB
g̃ (1600 GeV)
g̃ (1800 GeV)
g̃ (1400 GeV)
g̃ (1200 GeV)
q̃ (1400 GeV)
g̃ (1000 GeV)
q̃ (1000 GeV)
m0=800 GeV
m1/2=745 GeV
mg̃=mq̃=1.7 TeV
m1/2 [GeV]
m0 [GeV]
800
750
700
650
600
550
500
450
400
350
300
500
1000
1500
2000
2500
3000
3500

Low Energy Mis Vs Exp. Constraints

$\mathbf{A_0 = 0(2m_0),\ m_{1/2} = 745,\ m_0 = 800,\ x = 1}$

$m_{\tilde{l}} \sim 600$ GeV

Our model predicts:

δ^e	LL	LR/RL	RR
$\lvert\mathrm{Re}\delta_{12}\rvert$	4.4×10^{-4}	$0(2.3 \times 10^{-6})$	1.5×10^{-3}
$\lvert\mathrm{Re}\delta_{13}\rvert$	4.4×10^{-4}	$0(9.0 \times 10^{-8})$	1.6×10^{-6}
$\lvert\mathrm{Re}\delta_{23}\rvert$	4.2×10^{-5}	$0(1.6 \times 10^{-5})$	1.7×10^{-5}

Experiment constraints:

δ^ℓ	LL	LR/RL	RR	source
$\lvert\delta_{12}\rvert$	$_{LL}: 6.8 \times 10^{-2}$	$_{LR}: 2.5 \times 10^{-6}$	$-$	$\mu \to e\gamma$
$\lvert\delta_{13}\rvert$	$_{LL}: 18$	$_{LR}: 1.2 \times 10^{-2}$	$-$	$\tau \to e\gamma$
$\lvert\delta_{23}\rvert$	$_{LL}: 22$	$_{LR}: 1.4 \times 10^{-2}$	$-$	$\tau \to \mu\gamma$

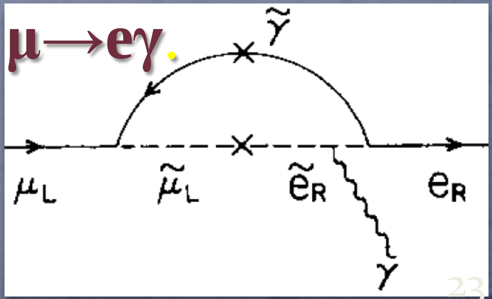
Summary & work in progress

- SU(5) x S_4xU(1) Flavour model successfully predicts the fermionic masses and mixing angles.

- Considering canonical normalisation effects that stem from the soft SUSY sector do not spoil the original features of the fermionic sector.

- Predicted off-diagonalities of soft scalar mass matrices (and MIs) small at even at M_{GUT}. Easy to overcome experimental bounds.

- $(\delta^{e}_{LR})^{12}$ the only constrained parameter through $\mu \to e\gamma$.



- Consider CP-violation.
  More constraining bounds from experiment..

Thank you for your attention

On the vacuum alignment…

- Introduce **driving fields** that couple to the flavons.

- Require their F-terms to vanish: ($F^i=\partial W/\partial \phi^i=0$)

e.g. couple the driving field $\mathbf{X_1^d}$ (S_4 singlet) Φ_2^d (S_4 doublet):

$$X_1^d(\Phi_2^d)^2 = 2X_1^d\Phi_{2,1}^d\Phi_{2,2}^d$$

require: $$\frac{\partial}{\partial X_1^d}2X_1^d\Phi_{2,1}^d\Phi_{2,2}^d = 0 \Rightarrow \langle\Phi_2^d\rangle \sim \begin{pmatrix}1\\0\end{pmatrix} \quad \text{or} \quad \langle\Phi_2^d\rangle \sim \begin{pmatrix}0\\1\end{pmatrix}$$

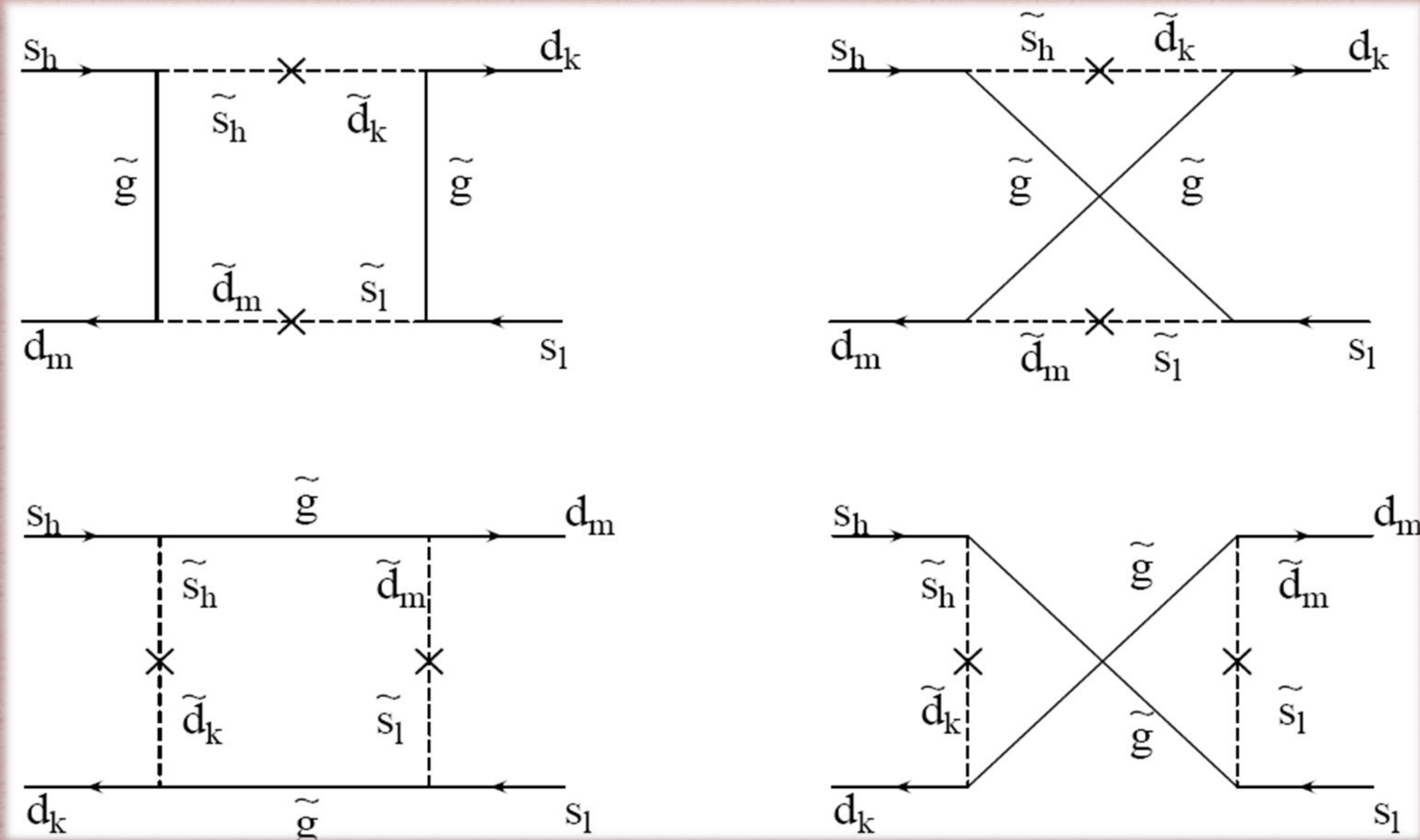
Without loss of generality, pick $\Phi_{2,1}^d\neq 0$.

- In a similar way, all flavons are aligned through vanishing F-terms of driving fields. For the neutrino sector in particular, this process not only fixes $<\Phi_i^\nu>$ but also requires that: $\varphi_1^\nu \sim \varphi_1^\nu \sim \varphi_{3'}^\nu$

Experimental Constraints

e.g.

SUSY contributions to K^0-$\bar{K}^0$ mixing & δ^d_{12}



Experimental Constraints

$$\Delta m_K = 2\,\mathrm{Re}\langle K^0|\mathcal{H}_{\mathit{eff}}\,|\bar{K}^0\rangle$$

$$\begin{aligned}\Delta m_K = -\frac{\alpha_s^2}{216 m_{\tilde q}^2}\frac{2}{3} m_K f_K^2 \Bigg\{ & \left(\delta_{12}^d\right)^2_{LL}\left(24\,x\,f_6(x) + 66\,\tilde f_6(x)\right) + \left(\delta_{12}^d\right)^2_{RR}\left(24\,x\,f_6(x) + 66\,\tilde f_6(x)\right) \\ + \; & \left(\delta_{12}^d\right)_{LL}\left(\delta_{12}^d\right)_{RR}\left[\left(384\left(\frac{m_K}{m_s+m_d}\right)^2 + 72\right)x\,f_6(x) + \left(-24\left(\frac{m_K}{m_s+m_d}\right)^2 + 36\right)\tilde f_6(x)\right] \\ + \; & \left(\delta_{12}^d\right)^2_{LR}\left[-132\left(\frac{m_K}{m_s+m_d}\right)^2 x\,f_6(x)\right] + \left(\delta_{12}^d\right)^2_{RL}\left[-132\left(\frac{m_K}{m_s+m_d}\right)^2 x\,f_6(x)\right] \\ + \; & \left(\delta_{12}^d\right)_{LR}\left(\delta_{12}^d\right)_{RL}\left[-144\left(\frac{m_K}{m_s+m_d}\right)^2 - 84\right]\tilde f_6(x)\Bigg\}\end{aligned}$$

$$x = m_{\tilde g}^2/m_{\tilde q}^2$$

❖ Require that the abs. value of each term does not exceed the measured Δ_{mK}.

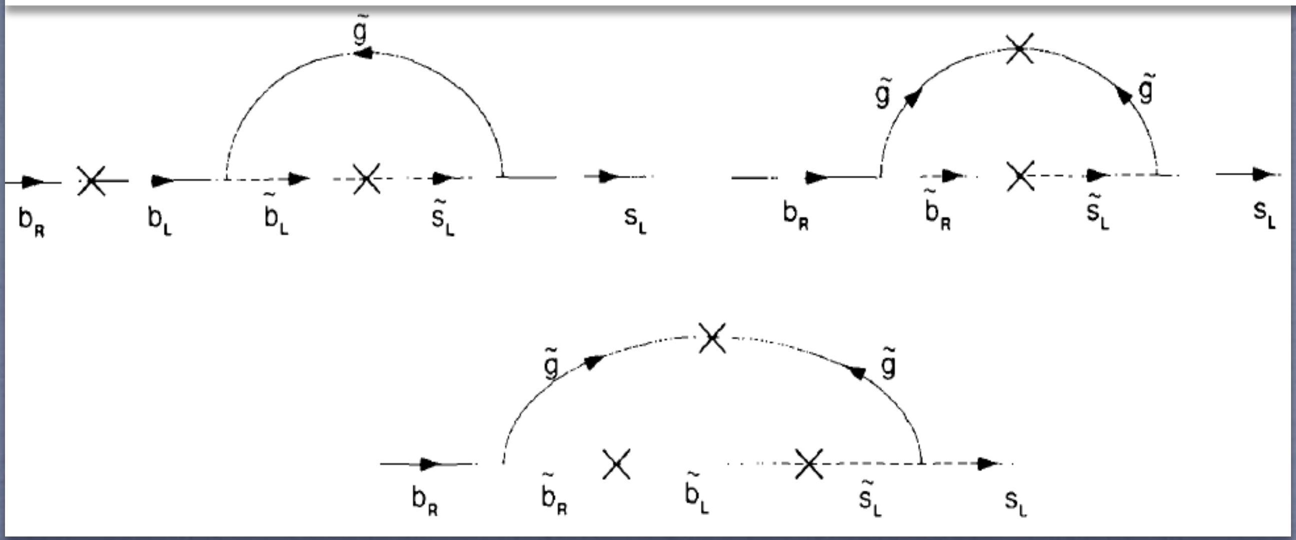
❖ Constraint $|(\delta^d_{12})_{AB}|$

Experimental Constraints

- Similar expressions for Δ_{mD}, Δ_{mB} and Δ_{mBs} constraint $|(\delta^u_{12})_{AB}|$, $|(\delta^d_{13})_{AB}|$ and $|(\delta^d_{23})_{AB}|$ respectively.

- $|(\delta^d_{23})_{AB}|$ is also constrained through:

$$\mathrm{BR}(b \to s\gamma) = \frac{\alpha_s^2 \alpha}{81\pi^2 m_{\tilde{q}}^4} m_b^3 \tau_B \left\{ \left| m_b M_3(x) \left(\delta^d_{23}\right)_{LL} + m_{\tilde{g}} M_1(x) \left(\delta^d_{23}\right)_{LR} \right|^2 + L \leftrightarrow R \right\}$$



- Similar constraints for lepton sector…

Low Energy Mis Vs Exp. Constraints

$\mathbf{A_0 = 0(2m_0),\ m_{1/2} = 745,\ m_0 = 800,\ x = 1}$

Our model predicts:

δ^d	LL	LR/RL	RR
$\lvert\mathrm{Re}\delta_{12}\rvert$	8.4×10^{-4}	$0(8.6 \times 10^{-7})$	9.2×10^{-5}
$\lvert\mathrm{Re}\delta_{13}\rvert$	$3.6 \times 10^{-4}(3.7 \times 10^{-3})$	$0(8.6 \times 10^{-7})$	9.2×10^{-5}
$\lvert\mathrm{Re}\delta_{23}\rvert$	$2.5 \times 10^{-5}(2.6 \times 10^{-5})$	$0(3.9 \times 10^{-6})$	9.2×10^{-5}

Experiment constraints:

δ^d	LL	LR/RL	RR	source
$\lvert\mathrm{Re}\delta^2_{12}\rvert^{\frac{1}{2}}$	$_{LL^2}: 1.4 \times 10^{-1}$	$_{LR^2}: 9.8 \times 10^{-3}$	$_{LLRR}: 6.3 \times 10^{-3}$	Δm_K
$\lvert\mathrm{Re}\delta^2_{13}\rvert^{\frac{1}{2}}$	$_{LL^2}: 3.5 \times 10^{-1}$	$_{LR^2}: 6.0 \times 10^{-2}$	$_{LLRR}: 1.2 \times 10^{-1}$	Δm_{B_d}
$\lvert\mathrm{Re}\delta^2_{23}\rvert^{\frac{1}{2}}$	$_{LL^2}: 1.7$	$_{LR^2}: 5.3 \times 10^{-1}$	$_{LLRR}: 2.9 \times 10^{-1}$	Δm_{B_s}
$\lvert\delta_{23}\rvert$	$-$	$_{LR}: 5.7 \times 10^{-2}$	$-$	$b \to s\gamma$

Neutrino sector

$$\boxed{y_D F N H_5} + \boxed{\alpha N N \Phi_1^\nu + \beta N N \Phi_2^\nu + \gamma N N \Phi_3^\nu}$$

$$Y_D^\nu = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Choose appropriate vacuum alignment for the flavons to get **TB-mixing**.

$\mathbf{M_R}$

- **Type I see-saw** formula: $\mathbf{m^\nu_{eff} = Y^\nu\, M_R^{-1}\, Y^{\nu T}\, \upsilon_u^2}$
- $\mathbf{m^\nu_{eff}}$ diagonalised by $\mathbf{U_{TB}}$, to give the light neutrino masses

flavon vevs have been chosen as: $\varphi_1^\nu \approx \lambda^4 M, \quad \varphi_2^\nu \approx \lambda^4 M, \quad \varphi_{3'}^\nu \approx \lambda^4 M$

such that: $\mathbf{m_1, m_2, m_3 \sim 0.1\ eV \sim y_D^2\, \upsilon_u^{\,2}/(\lambda^4 M)}$

Resulting predictions for leptonic mixing angles….

$U_{PMNS}=U^e_L\, U^{\nu\dagger}_L= U^e_L U_{TB}$ contains:

$$\sin\theta^l_{13} \approx \lambda/(3\sqrt{2}), \quad \sin^2\theta^l_{23} \approx 1/2, \quad \sin^2\theta^l_{12} \approx 1/3 + 2/9\lambda\cos\delta^l$$

2012 global fits for exp. data: $\sin^2\theta^l_{13} \approx 0.024$
$\sin^2\theta^l_{23} \approx 0.4$
$\sin^2\theta^l_{12} \approx 0.3$

For $\lambda \approx 0.22$

- predicted θ^l_{13} ,θ^l_{23} of the correct order
- predicted $\theta^l_{13} \sim 3^o$
- $\theta^{l\,exp}_{13} \approx 9^o$

Deviation from TB due to charged-lepton sector not enough

- **partly break** Klein symmetry

TB-mixing in the neutrino sector

alignments of flavon vevs **preserve** the $Z_2^S x Z_2^U$ subgroup of S_4.

$<\Phi_i^\nu>$: eigenvectors of **S&U**

Deviation from **TB-mixing:** new flavon **η** (S_4 singlet).

$$\frac{1}{M}\eta\Phi_2^d N N$$

break Z_2^U as $<\Phi_2^d>$: Not eigenvector of U

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\varphi_2^d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\varphi_2^d$$

with $<\eta> \approx \lambda^4 M$, $\varphi_2^d \approx \lambda M$

$$m_\nu^{eff'} = \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} -A_\nu + B_\nu + C_\nu & A_\nu & A_\nu \\ A_\nu & B_\nu & C_\nu \\ A_\nu & C_\nu & B_\nu \end{pmatrix} + \boxed{\frac{v_u^2}{\lambda^4 M}\begin{pmatrix} 0 & 0 & \lambda D_\nu \\ 0 & \lambda D_\nu & 0 \\ \lambda D_\nu & 0 & 0 \end{pmatrix}}$$

θ^ν_{12} remains tri-maximal, **θ^ν_{13} & θ^ν_{23} receive corrections O(λ).**

The SU(5) x S_4 x U(1) Model

- Matter fields fall into $\bar{\mathbf{5}}$ and **10** representations of SU(5):

$$F = \bar{\mathbf{5}} = \begin{pmatrix} d_R^c & d_B^c & d_G^c & e & -\nu \end{pmatrix}$$

$$T = \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_G^c & u_B^c & -u_R & -d_R \\ u_G^c & 0 & -u_R^c & -u_B & -d_B \\ -u_B^c & u_R^c & 0 & -u_G & -d_G \\ u_R & u_B & u_G & 0 & -e^c \\ d_R & d_B & d_G & e^c & 0 \end{pmatrix}$$

- RH-neutrinos : SU(5) singlets.
- SU(5) Higgs fields: H_5, $H_{\bar{5}}$, $H_{\overline{45}}$.
- $H_u=\alpha_1 H_5+\alpha_2 H_{\bar{5}}$, $H_d=\beta_1 H_5+\beta_2 H_{\overline{45}}$.
- Flavon fields: Φ^f_ρ $\begin{cases} f\text{: fermionic sector} \\ \rho\text{: rep of } S_4 \end{cases}$

Mass Insertion (MI) Parameters

❖ LO forms at the GUT scale:

$$(\delta^u)_{LL} = \begin{pmatrix} 1 & \lambda^4 & \lambda^7 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^u)_{RR} = \begin{pmatrix} 1 & \lambda^4 & \lambda^7 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^u)_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\delta^d)_{LL} = \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^d)_{RR} = \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^d)_{LR} = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^7 & \lambda^6 & \lambda^2 \end{pmatrix}$$

$$(\delta^e)_{LL} = \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^6 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^e)_{RR} = \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^e)_{LR} = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^7 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}$$

- **SUSY** : approximate symmetry at low energies.

- **Broken** in a different, ***hidden*** **sector** of the theory by some sfields **X**.

- SUSY breaking communicated to MSSM sfields through interactions between hidden & visible sectors.

- **Integrate out X** $\longrightarrow$ effective L_{MSSM}

- Coupling constants become functions of X.

  e.g. $$Y \to Y\left(\frac{X}{M^X}\right)$$

  M: characteristic scale of interactions.

Superpotential W

Gives rise to Yukawa and A-trilinear terms

$$\langle X \rangle = x + F_X \theta^2$$

$$Y \to Y\left(\frac{X}{M^X}\right)$$

Allow X to develop **≠0 vevs** on its scalar & **F-components**.

$$Y \sim \frac{x}{M^X}$$

$$A^u \sim \frac{F_X}{M^X} = A_0^u$$

Kähler potential K

Gives rise to kinetic and scalar mass terms

generic sfields

$$K = \left(1 + \frac{X^\dagger X}{M^{X^2}}\right) \Phi_i^\dagger \Phi_i \Rightarrow \tilde{m}_\phi^2 = \frac{|F_X|^2}{M^{X^2}}$$

In general, the kinetic part of the Lagrangian is:

$$\mathcal{L}_K \supset K^{ij} \left(\partial_\mu \phi_i^* \partial^\mu \phi_j + i\eta_{i*} \partial_\mu \bar{\sigma}^\mu \eta_j + F_i^* F_j\right)$$

where: $K^{ij} = \dfrac{\partial^2 K}{(\partial \Phi_i^\dagger \partial \Phi_j)|_{\Phi=\varphi}}$ is the Kähler metric.

Work in a basis where **K^{ij}=1** $\longrightarrow$ **Canonical Normalisation**

A-terms

- Common origin with the Yukawas

Operator Expansion in terms of flavons:

$$\frac{X}{M^X} f \sum_{\Phi,\Phi'} \frac{\Phi \otimes \Phi'}{M^2} f^c H$$

$$\langle X \rangle = x + F_X \theta^2$$

$$\frac{x}{M^X} f^i \sum_{\Phi,\Phi'} y^X_{\Phi,\Phi'} \frac{\langle\Phi\rangle_i \langle\Phi'\rangle_j}{M^2} f^{cj} v$$

Yukawas

$$\frac{F_X}{M^X} f^i \sum_{\Phi,\Phi'} a^X_{\Phi,\Phi'} \frac{\langle\Phi\rangle_i \langle\Phi'\rangle_j}{M^2} f^{cj} v$$

A_0 A-terms

A-terms

❖ Therefore, in order to write down the A-matrices, we look at the Yukawas and just change the O(1) coefs.

e.g.

$$Y^u = \begin{pmatrix} y_1^u \lambda^8 & 0 & 0 \\ 0 & y_2^u \lambda^4 & 0 \\ 0 & 0 & y_3^u \end{pmatrix}$$

$$\frac{A^u}{A_0} = \begin{pmatrix} a_1^u \lambda^8 & 0 & 0 \\ 0 & a_2^u \lambda^4 & 0 \\ 0 & 0 & a_3^u \end{pmatrix}$$

soft scalar masses

Operator Expansion in terms of flavons:

$$M_F^2 = \frac{X^\dagger X}{M^{X^2}} B_0 \left(1 + \sum_{\Phi,\Phi'} c_{\Phi,\Phi'}^{M_F^2} \frac{\Phi \otimes \Phi'}{M^2} \right) F^\dagger F$$

$|F_X|^2/M_X^2$

m_0^2

and similarly for the Ts :

$$M_T^2 = \frac{X^\dagger X}{M^{X^2}} b_{01} \left(1 + \sum_{\Phi,\Phi'} c_{\Phi,\Phi'}^{M_T^2} \frac{\Phi \otimes \Phi'}{M^2} \right) T^\dagger T + \frac{X^\dagger X}{M^{X^2}} b_{02} \left(1 + \sum_{\Phi,\Phi'} c_{\Phi,\Phi'}^{M_{T_3}^2} \frac{\Phi \otimes \Phi'}{M^2} \right) T_3^\dagger T_3$$

- Once we couple the matter to the flavon fields, we get off-diagonalities.

soft scalar masses

$$T = \mathbf{10} = (Q, u^c, e^c)$$
$$F = \bar{\mathbf{5}} = (L, d^c)$$

❖ **SU(5)** symmetry at the scale where the soft masses are generated.

$$m_Q^2 = m_{e^c}^2 = m_{u^c}^2 = M_T^2 \qquad m_L^2 = m_{d^c}^2 = M_F^2$$

$$K_T = \left(k_{01} + b_{01}\frac{X^\dagger X}{M_{X^2}}\right) T^\dagger T + \left(k_{02} + b_{02}\frac{X^\dagger X}{M_{X^2}}\right) T_3^\dagger T_3$$

$$= \boxed{(k_{01} T^\dagger T + k_{02} T_3^\dagger T_3)} + \boxed{\frac{X^\dagger X}{M_{X^2}}(b_{01} T^\dagger T + b_{02} T_3^\dagger T_3)}$$

$$K_F = \boxed{K_0 F^\dagger F} + \boxed{\frac{X^\dagger X}{M_{X^2}} B_0 F^\dagger F}$$

Kähler metrics scalar masses

DIAGONAL

As an example, let's see the contractions of a simple term..

$$y_3^u T_3 T_3 H_5 + y_2^u \frac{1}{M} T T \Phi_2^u H_5 + y_1^u \frac{1}{M^2} T T \Phi_2^u \tilde{\Phi}_2^u H_5$$

Remember: T ~ **2**, Φ_2^u ~ **2**, H_5 ~ **1**

We need:

- *Kronecker product:* $\boxed{\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}}$

- *Clebsh-Gordan coefs:*

$$\boxed{\begin{array}{c}\mathbf{2} \times \mathbf{2} : \quad b_1\tilde{b}_2 + b_2\tilde{b}_1 \sim \mathbf{1}\ , \quad b_1\tilde{b}_2 - b_2\tilde{b}_1 \sim \mathbf{1}'\ , \quad (b_2\tilde{b}_2, b_1\tilde{b}_1)^t \sim \mathbf{2} \\ (b_1, b_2)^t, (\tilde{b}_1, \tilde{b}_2)^t \sim \ \mathbf{2}\end{array}}$$

Constructing Y^d & Y^e

LO operators responsible for down-quark & charged -lepton masses:

$$T = \mathbf{10} = (Q, u^c, e^c)$$
$$F = \bar{\mathbf{5}} = (L, d^c)$$

$$y_2^d \frac{1}{M} F T_3 \Phi_3^d H_{\bar{5}} + y_1^d \frac{1}{M^2} (F \tilde{\Phi}_3^d)_{\mathbf{1}} (T \Phi_2^d)_{\mathbf{1}} H_{\overline{45}} + y_3^d \frac{1}{M^3} (F \Phi_2^d \Phi_2^d)_{\mathbf{3}} (T \tilde{\Phi}_3^d)_{\mathbf{3}} H_{\bar{5}}$$

using the following vevs…

$$\langle \Phi_2^d \rangle = \varphi_2^d \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \langle \Phi_3^d \rangle = \varphi_3^d \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \tilde{\Phi}_3^d \rangle = \tilde{\varphi}_3^d \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

$$v_d \frac{\phi_3^d}{M} y_2^d T_3 F_3$$

$$v_d \frac{\tilde{\phi}_3^d \phi_2^d}{M^2} y_1^d \left(T_2 F_2 - T_2 F_3 \right)$$

$$v_d \frac{\tilde{\phi}_3^d (\phi_2^d)^2}{M^3} y_3^d \left(-T_1 F_3 + T_2 F_3 - T_2 F_1 + T_1 F_2 \right)$$

$$\frac{\varphi_2^d}{M} \approx \lambda, \quad \frac{\varphi_3^d}{M} \approx \lambda^2, \quad \frac{\tilde{\varphi}_3^d}{M} \approx \lambda^3$$

$$Y^d = \begin{pmatrix} 0 & y_3^d\lambda^5 & -y_3^d\lambda^5 \\ -y_3^d\lambda^5 & y_1^d\lambda^4 & (y_3^d - yd_1)\lambda^4 \\ 0 & 0 & y_2^d\lambda^2 \end{pmatrix}$$

Transpose Y^d & multiply the terms that involve H_{45} by "-3"

$$Y^e = \begin{pmatrix} 0 & -y_3^e\lambda^5 & 0 \\ y_3^e\lambda^5 & -3y_1^e\lambda^4 & 0 \\ -y_3^e\lambda^5 & (3y_1^e + y_3^e)\lambda^4 & y_2^e\lambda^2 \end{pmatrix}$$

Georgi-Jarlskog (GJ) relations:

$$m_b \approx m_\tau, \quad m_\mu \approx 3m_s, \quad m_d \approx 3m_e$$

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1,$$

$$m_e : m_\mu : m_\tau \approx (1/3)\lambda^4 : 3\lambda^2 : 1$$

Gatto-Santori-Tonin (GST) relation:

$$\theta_{12} \approx \sqrt{m_d/m_s}$$

$$\theta_{12}^d \approx \lambda, \quad \theta_{13}^d \approx \lambda^3, \quad \theta_{23}^d, \approx \lambda^2,$$

$$\theta_{12}^e \approx (1/3)\lambda, \quad \theta_{13}^e \approx 0, \quad \theta_{23}^e, \approx 0$$

Comments…

- Y^u is diagonal.

- All quark mixing is coming from Y^d.

- The specific contractions of the down-quark/charged-lepton operators have been assumed to be dominant, such that Y^d & Y^e have the desired form, after the insertion of the appropriate flavon vevs.

$$y_2^d \frac{1}{M} F T_3 \Phi_3^d H_{\bar{5}} + \boxed{y_1^d \frac{1}{M^2} (F\tilde{\Phi}_3^d)_{\mathbf{1}} (T\Phi_2^d)_{\mathbf{1}} H_{\overline{45}}} + \boxed{y_3^d \frac{1}{M^3} (F\Phi_2^d\Phi_2^d)_{\mathbf{3}} (T\tilde{\Phi}_3^d)_{\mathbf{3}} H_{\bar{5}}}$$

$$\upsilon_d y_1^d \left(T_2 F_2 - T_2 F_3\right) \lambda^4$$

"-3"

$m_b \approx m_\tau$, $m_\mu \approx 3m_s$, $m_d \approx 3m_e$

$$\upsilon_d y_3^d \left(-T_1 F_3 + T_2 F_3 - T_2 F_1 + T_1 F_2\right) \lambda^5$$

(12)=(21) , (11)=0

$\theta_{12} \approx \sqrt{(m_d/m_s)}$

❖ Applying the **Type I see-saw** formula: $\mathbf{m^{\nu}_{eff} = Y^{\nu} M_R^{-1} Y^{\nu T} \upsilon_u^2}$ we get the effective neutrino mass matrix,

❖ diagonalised by the TB-mixing matrix $\mathbf{U_{TB}}$, to give the light neutrino masses (*we have reparametrised*…):

$$U_{TB}^T m^{\nu}_{eff} U_{TB} = \begin{pmatrix} -2A_\nu + B_\nu + C_\nu & 0 & 0 \\ 0 & A_\nu + B_\nu + C_\nu & 0 \\ 0 & 0 & B_\nu - C_\nu \end{pmatrix} \frac{v_u^2}{\lambda^4 M}$$

❖ The flavon vevs have been chosen as:

$$\varphi_1^\nu \approx \lambda^4 M, \quad \varphi_2^\nu \approx \lambda^4 M, \quad \varphi_{3'}^\nu \approx \lambda^4 M$$

such that: $\mathbf{m_1, m_2, m_3 \approx 0.1\ eV.}$

$$m_\nu^{eff'} = \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} -A_\nu + B_\nu + C_\nu & A_\nu & A_\nu \\ A_\nu & B_\nu & C_\nu \\ A_\nu & C_\nu & B_\nu \end{pmatrix} + \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} 0 & 0 & \lambda D_\nu \\ 0 & \lambda D_\nu & 0 \\ \lambda D_\nu & 0 & 0 \end{pmatrix}$$

Introducing the parameter:

$$n = \frac{(-2A_\nu^* + B_\nu^* + C_\nu^*)D_\nu + (B_\nu - C_\nu)D_\nu^*}{4(\mathbb{R}e[B_\nu C_\nu^* - A_\nu(B_\nu^* + C_\nu^*)] + |A_\nu|^2])}$$

we can express the angles of the matrix that diagonalises $m_\nu^{eff\prime}$ as:

$$\sin\theta_{13}^\nu \approx \frac{|n|}{\sqrt{2}}\lambda, \quad \sin\theta_{23}^\nu \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\mathrm{Re}(n)}{2}\lambda\right)$$

Multiplying with U^e_L, we get the PMNS matrix with the corrected mixing angles,

$$\sin\theta_{13}^l \approx \frac{\lambda}{\sqrt{2}}\left(\frac{1}{3} + |n|\right), \quad \sin\theta_{23}^l \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\mathrm{Re}(n)}{2}\lambda\right)$$

that can account for a θ^l_{13} as large as measured by experiment.

Forbidding “unwanted ” operators using the U(1) symmetry…

❖ Want to forbid any additional operators that would <u>disturb</u> the forms of the fermion masses & mixings that have been successfully found.

❖ We refer to the operators that have been used so far as LO operators. They are the ones in which the U(1) charges of the participating superfields add up to zero for every x,y,z. e.g. $\mathbf{TT\Phi_2^u\tilde{\Phi}_2^u H_5}$

❖ If we start fixing the values of x,y,z, the segregation of different flavons associated with a particular type of fermion breaks down.

❖ The resulting operators are "unwanted" if their contribution to the mass matrices at an order of λ that spoils their structure.

e.g. An operator like $\mathbf{TT\Phi_2^d H_5}$ would contribute to $\mathbf{Y_u^{(11)}}$ a term of order λ , IF **2x+z=0**, spoiling the desired structure!

Require: $\boxed{\mathbf{2x+z \neq 0}}$

Canonical Normalisation

- Kähler metrics $\mathcal{L}_K \supset K^{ij}(\partial_\mu \phi_i^* \partial^\mu \phi_j + i\eta_{i*}\partial_\mu \bar{\sigma}^\mu \eta_j + F_i^* F_j)$

  result from an **expansion in terms of flavons**
  ($K_{T,F}$ same as $M^2_{T,F}/m_0^2$ with different O(1) coefs.)

- Leads to non-canonical metrics
- Redefine matter fields $F \to P_F^{-1} F, \quad T \to P_T^{-1} T$

  such that Kähler metrics are identified with the identity matrix.

$$(P_F^\dagger)^{-1} K_F P_F^{-1} = \mathbb{I} \implies K_F = P_F^\dagger P_F$$
$$(P_T^\dagger)^{-1} K_T P_T^{-1} = \mathbb{I} \implies K_T = P_T^\dagger P_T$$

- All quantities are rotated into that basis.
- **Pattern of fermion masses & mixings survives.**

Canonical Normalisation

e.g.

$$Y^u_C = (P_T^{-1})^T Y^u P_T^{-1}$$

$$Y^u = \begin{pmatrix} y^u_1\lambda^8 & 0 & 0 \\ 0 & y^u_2\lambda^4 & 0 \\ 0 & 0 & y^u_3 \end{pmatrix}$$

$$Y^u_C = \begin{pmatrix} y^u_{11}\lambda^8 & y^u_{12}\lambda^6 & y^u_{13}\lambda^7 \\ y^u_{12}\lambda^6 & y^u_{224}\lambda^4 + y^u_{226}\lambda^6 + y^u_{228}\lambda^8 & y^u_{235}\lambda^5 + y^u_{237}\lambda^7 \\ y^u_{13}\lambda^7 & y^u_{235}\lambda^5 + y^u_{237}\lambda^7 & y^u_{33} \end{pmatrix}$$

- ❖ Canonical normalisation effects have left θ^u_{13}, θ^u_{23} small but have produced a rather large $\theta^u_{12} \sim O(\lambda^2)$. Rest of the mixing still comes from Y^d_C.

- ❖ Hierarchy of masses survives.

Canonical Normalisation

❖ Canonically normalise the soft sector as well.

$$\frac{A^u_C}{A_0} = (P_T^{-1})^T \frac{A^u}{A_0} P_T^{-1}$$

$$\frac{A^d_C}{A_0} = (P_T^{-1})^T \frac{A^d}{A_0} P_F^{-1}$$

$$\frac{M^2_{T_C}}{m_0^2} = (P_T^{-1})^\dagger \frac{M^2_T}{m_0^2} P_T^{-1}$$

$$\frac{M^2_{FC}}{m_0^2} = (P_F^{-1})^\dagger \frac{M^2_F}{m_0^2} P_F^{-1}$$

❖ In the special case where the $K_{T,F}$ & $M^2{}_{T,F}/m_0{}^2$ coefs are proportional to each other, the off-diagonal entries would vanish.

Canonical form of A-terms

e.g.

$$\boxed{\frac{A^u}{A_0}_C = (P_T^{-1})^T \frac{A^u}{A_0} P_T^{-1}}$$

$$\frac{A^u}{A_0} = \begin{pmatrix} a_1^u \lambda^8 & 0 & 0 \\ 0 & a_2^u \lambda^4 & 0 \\ 0 & 0 & a_3^u \end{pmatrix}$$

$$\frac{A_C^u}{A_0} = \begin{pmatrix} a_{118}^u \lambda^8 & a_{126}^u \lambda^6 & a_{137}^u \lambda^7 \\ a_{126}^u \lambda^6 & a_{224}^u \lambda^4 + a_{226}^u \lambda^6 + a_{228}^u \lambda^8 & a_{235}^u \lambda^5 + a_{237}^u \lambda^7 \\ a_{13}^u \lambda^7 & a_{235}^u \lambda^5 + a_{237}^u \lambda^7 & a_{33}^u \end{pmatrix}$$

Canonical form of masses

$$\frac{M_T^2}{m_0^2} = \begin{pmatrix} b_{01} & 0 & 0 \\ 0 & b_{01} & 0 \\ 0 & 0 & b_{02} \end{pmatrix} + \begin{pmatrix} b_1\lambda^2 & b_2\lambda^4 & b_4\lambda^7 \\ b_2^*\lambda^4 & -b_1\lambda^2 & c_3\lambda^5 \\ b_4^*\lambda^7 & c_3^*\lambda^5 & 0 \end{pmatrix}$$

e.g.

$$\frac{M_{T_C}^2}{m_0^2} = (P_T^{-1})^\dagger \frac{M_T^2}{m_0^2} P_T^{-1}$$

$$\frac{M_{TC}^2}{m_0^2} = \begin{pmatrix} \frac{b_{01}}{k_{01}} & 0 & 0 \\ 0 & \frac{b_{01}}{k_{01}} & 0 \\ 0 & 0 & \frac{b_{02}}{k_{02}} \end{pmatrix}$$

$$+ \begin{pmatrix} b_{112}\lambda^2 + b_{114}\lambda^4 + b_{116}\lambda^6 & b_{124}\lambda^4 & b_{137}\lambda^7 \\ b_{124}^*\lambda^4 & -b_{112}\lambda^2 + b_{114}\lambda^4 - b_{116}\lambda^6 & b_{235}\lambda^5 + b_{237}\lambda^7 \\ b_{137}^*\lambda^7 & b_{235}^*\lambda^5 + b_{237}^*\lambda^7 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} b_{118}\lambda^8 & b_{128}\lambda^8 & 0 \\ b_{128}^*\lambda^8 & b_{118}\lambda^8 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- ❖ In the special case where the K_T & M^2_T coefs are proportional to each other, the off-diagonal entries would vanish.

Super-CKM basis

e.g.

$$Y^u = \begin{pmatrix} y_1^u \lambda^8 & 0 & 0 \\ 0 & y_2^u \lambda^4 & 0 \\ 0 & 0 & y_3^u \end{pmatrix}$$

$$\tilde{Y}_u = (U_L^u)^\dagger Y_{uC} U_R^u = \begin{pmatrix} \tilde{y}_{118}^u \lambda^8 & 0 & 0 \\ 0 & \tilde{y}_{224}^u \lambda^4 + \tilde{y}_{226}^u \lambda^6 + \tilde{y}_{228}^u \lambda^8 & 0 \\ 0 & 0 & \tilde{y}_{33}^u \end{pmatrix}$$

$$\tilde{y}_{118}^u = \frac{y_1^u}{k_{01}}, \quad \tilde{y}_{224}^u = \frac{y_2^u}{k_{01}}, \quad \tilde{y}_{226}^u = \frac{k_1 y_2^u}{k_{01}^2}, \quad \tilde{y}_{228}^u = \frac{k_1^2 y_2^u}{k_{01}^3}, \quad \tilde{y}_{33}^u = \frac{y_3^u}{k_{02}}$$

- ❖ Similarly, the rest of the Yukawa matrices are diagonalised.
- ❖ At this point, it can be checked whether canonical normalisation has spoiled the original features of a model.
- ❖ In our model, the structure of the fermionic masses & mixings survives, with the SCKM matrix being given by $\mathbf{U^u_L (U^d_L)^\dagger}$.

A-terms in SCKM basis

❖ Rotate the canonically normalised A-terms, into the SCKM basis.

$$\frac{\tilde{A}^u}{A_0} = (U_L^u)^\dagger \frac{A_C^u}{A_0} U_R^u = \begin{pmatrix} \tilde{a}^u_{118}\lambda^8 & 0 & 0 \\ 0 & \tilde{a}^u_{224}\lambda^4 + \tilde{a}^u_{226}\lambda^6 + \tilde{a}^u_{228}\lambda^8 & 0 \\ 0 & 0 & \tilde{a}^u_{33} \end{pmatrix}$$

$$\tilde{a}^u_{118} = \frac{a^u_1}{k_{01}}, \quad \tilde{a}^u_{224} = \frac{a^u_2}{k_{01}}, \quad \tilde{a}^u_{226} = \frac{a^u_2 k_1}{k_{01}^2}, \quad \tilde{a}^u_{228} = \frac{a^u_2 k_1^2}{k_{01}^3}, \quad \tilde{a}^u_{33} = \frac{a^u_3}{k_{02}}$$

❖ A-matrices are ONLY diagonal in the SCKM basis in the special case where $\alpha^f_i \sim y^f_i$.

$$Y^u = \begin{pmatrix} y^u_1\lambda^8 & 0 & 0 \\ 0 & y^u_2\lambda^4 & 0 \\ 0 & 0 & y^u_3 \end{pmatrix} \qquad \frac{A^u}{A_0} = \begin{pmatrix} a^u_1\lambda^8 & 0 & 0 \\ 0 & a^u_2\lambda^4 & 0 \\ 0 & 0 & a^u_3 \end{pmatrix}$$

❖ The reason our $\tilde{A}^u$ is diagonal, is not due to the above point though.

❖ A-matrices in the other sectors are not diagonal.

Effects of RG-running

- Calculations so far hold just below M_{GUT}.
- Need RGE evolution to M_W where experiments are performed.

e.g.

$$\frac{d(\tilde{m}_u^2)^{ij}_{RR}}{dt} = -\frac{1}{4\pi^2}\boxed{G_u}\delta^{ij} + \frac{1}{4\pi^2}\boxed{(\tilde{F}_u)^{ij}}$$

Yukawa & soft driven matrix

gaugino driven scalar

$$\tilde{F}_u \sim \begin{pmatrix} \lambda^{16} & \lambda^{12} & \lambda^{7} \\ \lambda^{12} & \lambda^{8} & \lambda^{5} \\ \lambda^{7} & \lambda^{5} & 1 \end{pmatrix}$$

Diagonal Elements

$$(\tilde{m}_u^2)^{ii}_{RR}|_{M_W} \approx (\tilde{m}_u^2)^{ii}_{RR}|_{M_{GUT}} + 6.15 m^2_{1/2}\,, \quad i = 1, 2$$

$$(\tilde{m}_u^2)^{33}_{RR}|_{M_W} \approx (\tilde{m}_u^2)^{33}_{RR}|_{M_{GUT}} + 6.15 m^2_{1/2} - \\ -\frac{1}{4\pi^2}\left(|\tilde{Y}_u^{33}|^2\left((\tilde{m}_u^2)^{33}_{RR} + (\tilde{m}_u^2)^{33}_{LL} + (\tilde{m}^2_{Hu})\right) + |\tilde{A}_u^{33}|^2\right)\log\left(\frac{M_{GUT}}{M_W}\right)$$

Effects of RG-running

Off-diagonal Elements

$$t = \log\left(\frac{\mu}{M}\right)$$

$$\frac{d(\tilde{m}_u^2)_{RR}^{ij}}{dt} = -\frac{1}{4\pi^2} G_u \delta^{ij} + \frac{1}{4\pi^2} \boxed{(\tilde{F}_u)^{ij}}$$

Yukawa & soft driven matrix

$$\tilde{F}_u \sim \begin{pmatrix} \lambda^{16} & \lambda^{12} & \lambda^7 \\ \lambda^{12} & \lambda^8 & \lambda^5 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix}$$

- Stem purely from the $\tilde{F}_u$-term.
- Small compared to the O(1) terms that appear in the diagonal elements.

$$\boxed{(\tilde{m}_u^2)_{RR}^{ij}|_{M_{GUT}} \approx (\tilde{m}_u^2)_{RR}^{ij}|_{M_W}\,,\, i \neq j}$$

- Similarly, we run down $(\tilde{m}^2_d)_{RR}$, $(\tilde{m}^2_e)_{RR}$ *(with different G_f & F_f).*
- Worth commenting on the LL parameters though…

Effects of RG-running

arXiv:0103324v2

❖ Numerical estimates for the low energy masses in the literature do NOT take into account the effects of going to the SCKM basis.

❖ No big difference for RR-parameters because:

❖ G_f is independent of basis. So are $(\tilde{F}_f)^{ii}$ but only to LO in λ, while $(\tilde{F}_f)^{ij}$ are usually small and ignored anyway.

❖ However, the effects of the SCKM matrix $V=(U^u{}_L)^\dagger\, U^d{}_L$ can potentially be important for the LL-parameters.

$$\tilde{F}^u_Q = V\,\tilde{F}^d_Q V^\dagger$$

$$\tilde{F}^u_Q \sim \begin{pmatrix} \lambda^{10} & \lambda^9 & \lambda^7 \\ \lambda^9 & \lambda^8 & \lambda^5 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix} \quad , \quad \tilde{F}^d_Q \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Effects of RG-running

❖ Finally, we remark on the dominance of the **right-handed neutrinos** in the RGE for **$(\tilde{m}^2_e)_{LL}$**.

$$\tilde{F}^\nu_L = \frac{1}{2}\left(\frac{1}{2}\tilde{Y}_\nu\tilde{Y}^\dagger_\nu(\tilde{m}^2_e)_{LL} + \frac{1}{2}(\tilde{m}^2_e)_{LL}\tilde{Y}_\nu\tilde{Y}^\dagger_\nu + \tilde{Y}_\nu(\tilde{m}^2_N)_{RR}\tilde{Y}^\dagger_\nu + (m^2_{H_u})\tilde{Y}_\nu\tilde{Y}^\dagger_\nu + \tilde{A}_\nu\tilde{A}^\dagger_\nu\right)$$

$$Y_\nu = y_D\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad A_\nu = \alpha y_D\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad M_N = \begin{pmatrix} M_{N_1} & 0 & 0 \\ 0 & M_{N_2} & 0 \\ 0 & 0 & M_{N_3} \end{pmatrix}$$

$$\boxed{M_N \sim 10^{14}\mathrm{GeV}}$$

$$\boxed{\tilde{F}^\nu_L \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}}$$

$$(\tilde{m}^2_e)^{ii}_{LL}|_{M_W} \approx (\tilde{m}^2_e)^{ii}_{LL}|_{M_{GUT}} + 0.5\ m^2_{1/2} - \frac{1}{4\pi^2}\tilde{F}^\nu_L \log\left(\frac{M_{GUT}}{M_N}\right), \quad i = 1, 2$$

Low Energy MIs

$$(\delta^f)^{ij}_{AB}|_{M_W} = S^{f(ij)}_{AB} \times (\delta^f)^{ij}_{AB}|_{M_{GUT}}, \ \ i,j = 1,2,3, \quad A, B \in L, R,$$

e.g.

$$S^{u(12)}_{RR} = \frac{\langle \tilde{m}_u \rangle^2_{RR(12)}|_{M_{GUT}}}{\langle \tilde{m}_u \rangle^2_{RR(12)}|_{M_W}}$$

$$\approx \frac{\sqrt{(\tilde{m}^2_u)^{11}_{RR}|_{M_{GUT}} (\tilde{m}^2_u)^{22}_{RR}|_{M_{GUT}}}}{\sqrt{\left((\tilde{m}^2_u)^{11}_{RR}|_{M_{GUT}} + 6.15\ m^2_{1/2}\right)\left((\tilde{m}^2_u)^{22}_{RR}|_{M_{GUT}} + 6.15\ m^2_{1/2}\right)}}$$

$$\boxed{S^{u(12)}_{RR} \approx \frac{\frac{b_{01}}{k_{01}}}{\frac{b_{01}}{k_{01}} + 6.15\ \frac{m^2_{1/2}}{m^2_0}}}$$

$$\boxed{S^{u(12)}_{RR} \approx \frac{\frac{b_{01}}{k_{01}}}{\frac{b_{01}}{k_{01}} + 6.15\ \frac{m^2_{1/2}}{m^2_0}}}$$

$$\boxed{S^{u(12)}_{LL} \approx S^{d(12)}_{LL} \frac{\frac{b_{01}}{k_{01}}}{\frac{b_{01}}{k_{01}} + 6.5\ \frac{m^2_{1/2}}{m^2_0}}}$$

$$\boxed{S^{d(12)}_{RR} \approx S^{d(13)}_{RR} \approx S^{d(23)}_{RR} \approx \frac{\frac{B_0}{K_0}}{\frac{B_0}{K_0} + 6.15\ \frac{m^2_{1/2}}{m^2_0}}}$$

$$\boxed{S^{u(13)}_{RR} = S^{u(23)}_{RR} \approx \frac{\sqrt{\frac{b_{01}}{k_{01}}\ m^2_0 \left(\frac{b_{02}}{k_{02}}\ m^2_0 + \left(\upsilon \frac{t_\beta}{\bar{t}_\beta}\right)^2 \frac{|y^u_3|^2}{k^2_{02}}\right)}}{\sqrt{\left(\frac{b_{01}}{k_{01}}\ m^2_0 + 6.15\ m^2_{1/2}\right) \left((\tilde{m}^2_u)^{33}_{RR}|_{M_W} + m^2_t|_{M_W}\right)}}}$$

$$\boxed{S^{u(13)}_{LL} = S^{u(LL)}_{23} \approx \frac{\sqrt{\frac{b_{01}}{k_{01}}\ m^2_0 \left(\frac{b_{02}}{k_{02}}\ m^2_0 + \left(\upsilon \frac{t_\beta}{\bar{t}_\beta}\right)^2 \frac{|y^u_3|^2}{k^2_{02}}\right)}}{\sqrt{\left(\frac{b_{01}}{k_{01}}\ m^2_0 + 6.5\ m^2_{1/2}\right) \left((\tilde{m}^2_u)^{33}_{LL}|_{M_W} + m^2_t|_{M_W}\right)}}}$$

$$S_{LL}^{d(13)} \approx \frac{\langle (\tilde{m}_d^2)_{LL(13)} \rangle|_{M_{GUT}}}{\langle (\tilde{m}_d^2)_{LL(13)} \rangle|_{M_W}} \left(1 - \frac{1}{4\pi^2} \log\left(\frac{M_{GUT}}{M_W}\right) \frac{(\tilde{F}_Q^d)^{13}}{(\tilde{m}_d^2)_{LL}^{13}}|_{M_{GUT}} \right)$$

$$\boxed{S_{LL}^{d(13)} \approx \frac{m_0^2 \sqrt{\frac{b_{01}}{k_{01}} \ \frac{b_{02}}{k_{02}}}}{\sqrt{\left(\frac{b_{01}}{k_{01}} \ m_0^2 + 6.5 \ m_{1/2}^2\right) (\tilde{m}_d^2)_{LL}^{33}|_{M_W}}} \left(1 - \frac{L_1}{\tilde{B}_{13,3} m_0^2}\right)} \ ,$$

$$S_{LL}^{d(23)} \approx \frac{\langle (\tilde{m}_d^2)_{LL(23)} \rangle|_{M_{GUT}}}{\langle (\tilde{m}_d^2)_{LL(23)} \rangle|_{M_W}} \left(1 - \frac{1}{4\pi^2} \log\left(\frac{M_{GUT}}{M_W}\right) \frac{(\tilde{F}_Q^d)^{23}}{(\tilde{m}_d^2)_{LL}^{23}}|_{M_{GUT}} \right)$$

$$\boxed{S_{LL}^{d(23)} \approx \frac{m_0^2 \sqrt{\frac{b_{01}}{k_{01}} \ \frac{b_{02}}{k_{02}}}}{\sqrt{\left(\frac{b_{01}}{k_{01}} \ m_0^2 + 6.5 \ m_{1/2}^2\right) (\tilde{m}_d^2)_{LL}^{33}|_{M_W}}} \left(1 - \frac{L_2}{\tilde{B}_{23,2} m_0^2}\right)} \ ,$$

$$S_{LL}^{e(12)} \approx \frac{\frac{B_0}{K_0} m_0^2}{\sqrt{(\tilde{m}_e^2)_{LL}^{11}|_{M_W} (\tilde{m}_e^2)_{LL}^{22}|_{M_W}}} \left(1 - \frac{1}{8\pi^2} \log \left(\frac{M_{GUT}}{M_N} \right) \frac{(\tilde{F}_L^{\nu})^{12}}{(\tilde{m}_e^2)_{LL}^{12}} |_{M_{GUT}} \right)$$

Observable	Value
Δm_K	$< 3.484 \times 10^{-12}$ MeV [30]
Δm_D	$< 9.478 \times 10^{-12}$ MeV [30]
Δm_B	$< 3.337 \times 10^{-10}$ MeV [30]
Δm_{Bs}	$< 1.164 \times 10^{-8}$ MeV [30]
$\mathrm{BR}(b \to s\gamma)$	$\approx 3.37 \times 10^{-4}$ [31]
$\mathrm{BR}(b \to s\mu^+\mu^-)$	$\approx 1.60 \times 10^{-6}$ [31]

Constant	Value
m_K	494.6 MeV [28]
m_D	1864.86 MeV [30]
m_B	5297.58 MeV [30]
m_{Bs}	5366.77 MeV [30]
f_K	155.192 MeV [28]
f_B	186 MeV [29]
f_{Bs}	224 MeV [29]
τ_{Bs}	1.497×10^{-12} s [30]
τ_B	1.519×10^{-12} s [30]
m_u	2.3 MeV ($\mu = 2$MeV) [30]
m_d	4.8 MeV($\mu = 2$MeV) [30]
m_c	1.275 GeV ($\mu = m_c$) [30]
m_s	95 MeV ($\mu = 2$MeV) [30]
m_b	4.18 GeV ($\mu = m_b$) [30]

Fermion masses & mixings

- Fermion mass terms generated through Yukawa couplings to Higgs:

$$-y_u \bar{Q}_L \Phi^c u_R + \text{h.c.} \xrightarrow{\text{SSB}} -(y_u \upsilon)\bar{u}_L u_R + \text{h.c.}$$

- Additional generations $\Rightarrow$ coupling terms that mix them

$$-(Y_u)^{ij}\bar{Q}_L{}^i \Phi^c u_R{}^j + \text{h.c.} \qquad -\bar{u}_L{}^i (Y_u)^{ij} u_R{}^j\, \upsilon + \text{h.c.}$$

$$\Downarrow$$

$$-\bar{u}_L{}^i \underbrace{(U^{u\dagger}{}_L Y_u U^u{}_R)^{ij}}_{Y^u:\ \text{diag}} u_R{}^j \upsilon + \text{h.c.}$$

- Then, $m_u{}^i \sim (Y^u{}_{diag})^{ii}$,
- while off-diag. elements of $(Y_u)^{ij}$ are related to mixing between the three generations.

Understanding the pattern of fermion masses =
=Understanding Yukawa matrices.