The Dark Universe: Dark Matter and Dark Energy

Rocky I: The Universe Observed  Monday
Rocky II: Inflation  Tuesday
Rocky III: Dark Matter  Wednesday
Rocky IV: Dark Energy  Thursday

CERN Academic Training Lectures  January 2008
Rocky Kolb  The University of Chicago
Cosmology 1000 Years Ago
(and parts of present-day USA)
Cold Dark Matter: (CDM) 25%

Dark Energy ($\Lambda$): 70%

Chemical Elements: (other than H & He) 0.025%

Neutrinos: 0.17%

Stars: 0.8%

H & He: gas 4%

Cold Dark Matter: (CDM) 25%

Radiation: 0.005%

Chemical Elements: (other than H & He) 0.025%

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Cold Dark Matter: (CDM) 25%

Radiation: 0.005%
The construction of a model ... consists of snatching from the enormous and complex mass of facts called reality a few simple, easily managed key points which, when put together in some cunning way, becomes for certain purposes a substitute for reality itself.

Evsey Domar

*Essays on the Theory of Economic Growth*
Ptolemaic System

From The Almagest
From Book III of De Revolutionibus

quod epicyclum hoc modo. Sit mundo ac Soli homocentrum AB, & ACB diameter, in qua summa absis contingat. Et facto in centro epicyclus descriptur DE, ac rursus in centro epicyclium FG, in quo terra ueretur, omnium in eodem plano zodiaci.

Sic epicycli primi motus in succedentia, ac annuus sereretur, secuti quod hoc est, simili liter annuus, sed in praeecedentia, ambo rumque ad ac lineam pares sint revolutiones. Rursus ceterum terrae ex E in praeecedentia addat parumque ipsi. Ex hoc manifestum est quod cum terra fuerit in E, maximum efficiet Solis apogaeum, in Ω minimum; in medias autem circumferentia ipsius & epicycli faciet ipsum apogaeum praeedere ut sequi autum dimi
Precision Cosmology

$$\Omega_{\text{tot}} = 1.02^{+0.02}_{-0.02}$$
$$w < -0.78 \text{ (95\% CL)}$$
$$\Omega^{\Lambda} = 0.73^{+0.04}_{-0.04}$$
$$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$$
$$\Omega_b = 0.044^{+0.004}_{-0.004}$$
$$n_b = 2.5 \times 10^{-7}^{+0.1 \times 10^{-7}}_{-0.1 \times 10^{-7}} \text{ cm}^{-3}$$
$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$
$$\Omega_m = 0.27^{+0.04}_{-0.04}$$
$$\Omega_v h^2 < 0.0076 \text{ (95\% CL)}$$
$$m_v < 0.23 \text{ eV (95\% CL)}$$
$$T_{\text{cmb}} = 2.725^{+0.002}_{-0.002} \text{ K}$$
$$n = 410.4^{+0.9}_{-0.9} \text{ cm}^{-3}$$
$$\eta = 6.1 \times 10^{-10}^{+0.3 \times 10^{-10}}_{-0.2 \times 10^{-10}}$$
$$\Omega_b \Omega_m^1 = 0.17^{+0.04}_{-0.01}$$
$$\sigma_8 = 0.84^{+0.04}_{-0.04} \text{ Mpc}$$
$$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$$
$$A = 0.833^{+0.086}_{-0.083}$$

$$n_s = 0.93^{+0.03}_{-0.03}$$
$$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$$
$$r < 0.71 \text{ (95\% CL)}$$
$$z_{\text{dec}} = 1089^{+1}_{-1}$$
$$\Delta z_{\text{dec}} = 195^{+2}_{-2}$$
$$h = 0.71^{+0.04}_{-0.03}$$
$$t_0 = 13.7^{+0.2}_{-0.2} \text{ Gyr}$$
$$t_{\text{dec}} = 379^{+8}_{-7} \text{ kyr}$$
$$t = 180^{+220}_{-80} \text{ Myr (95\% CL)}$$
$$\Delta t_{\text{dec}} = 118^{+3}_{-2} \text{ kyr}$$
$$z_{\text{eq}} = 3233^{+194}_{-210}$$
$$\tau = 0.17^{+0.04}_{-0.04}$$
$$z_f = 20^{+10}_{-9} \text{ (95\% CL)}$$
$$\theta_A = 0.598^{+0.002}_{-0.002}$$
$$d_A = 14.0^{+0.2}_{-0.3} \text{ Gpc}$$
$$l_A = 301^{+1}_{-1}$$
$$r_s = 147^{+2}_{-2} \text{ Mpc}$$
The Universe seems to demand **NEW PHYSICS**!

Astronomy is helpful to physics!

The cosmological standard model $\Lambda$CDM:

- Dark Matter
- Dark Energy
- Seed Perturbations (Inflation)
- Baryo/Leptogenesis
"How helpful to us is astronomy's pedantic accuracy, which I used to secretly ridicule!"

Einstein's statement to Arnold Sommerfeld on December 9, 1915 (regarding measurements of the advance of the perihelion of Mercury)
• Big-bang primer (just theory)

• Cosmological parameters (just numbers)

  \[ H_0 \rightarrow \text{the present expansion rate (Hubble’s constant)} \]

  \[ \Omega_i \rightarrow \text{the present cosmic food chain} \]

  \( \left( \Omega_{\text{TOTAL}}, \Omega_M, \Omega_B, \Omega_\Lambda, \Omega_\gamma, \Omega_\nu, \ldots \right) \)

  \[ T_0 \rightarrow \text{the present temperature of the Universe} \]

  \[ t_0 \rightarrow \text{the present age of the Universe} \]

• Power spectra—characterization of perturbations:

  Galaxies: \( P(k) \)  \quad \text{Radiation: } C_\ell \)

• “Standard model”: \( \Lambda \text{CDM} \)  \text{ Dark Energy and Dark Matter}
Big-Bang (Just A Theory)

Big Bang Theory,
You've Got To Be Kidding.

-God
Meet two brainiacs with a lot to learn. Leonard and Sheldon can tell their quarks from their quantum physics, but have no clue how women add up. Leave it to their pretty new neighbor, just off a messy breakup, to teach them a thing or two in THE BIG BANG THEORY.
Modern Commandments of Genesis

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8\pi G T_{\mu \nu} \]

(10 nonlinear partial differential equations)
Robertson-Walker metric

\[a(t) = \text{cosmic scale factor}\]

\[k = 0, \pm 1\]

\[ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)\]

Perfect-fluid stress tensor

\[\rho = \text{energy density}\]

\[p = \text{pressure}\]

\[T^\mu_\nu = \text{diag}(\rho, p, p, p)\]
**Robertson-Walker Metric**

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$k = 0$ (spatially flat)

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right) = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

(comoving coordinates: $\vec{x}, \vec{y}, \vec{z}$)

(physical distance: $d\vec{l}^2 = a^2(t)\left(dx^2 + dy^2 + dz^2\right)$)
**Perfect Fluid Stress-Energy Tensor**

- Must specify energy and pressure content of the Universe

- Assume pressure is related to energy density:  \( p_i = w_i \rho_i \)

- Conservation of stress-energy tensor,  \( T^{\mu \nu}_{\,; \nu} = 0 \rightarrow \rho_i \propto a^{-3(1+w_i)} \)

- If \( w = w(a) \) then  \( a^{-3(1+w)} \rightarrow \exp \left( -3 \int_a^{a'} \frac{da'}{a'} \left[ 1 + w(a') \right] \right) \)

\[ T^{\mu \nu} : \text{fluids with different } w \]

**Radiation:**  
\( p_R = \frac{\rho_R}{3} \quad w = 1/3 \quad \rho_R \propto a^{-4} \)

**Matter:**  
\( p_M = 0 \quad w = 0 \quad \rho_M \propto a^{-3} \)

**Curvature:**  
\( p_k = -\frac{\rho_k}{3} \quad w = -1/3 \quad \rho_k \propto a^{-2} \)

**Vacuum:**  
\( p_V = -\rho_V \quad w = -1 \quad \rho_V \propto a^0 \)
\[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho\] \text{ Friedmann Equation}

\[\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\] \text{ Deceleration equation}

\[H \equiv \frac{\dot{a}}{a} = \text{ expansion rate}\]

\[q \equiv -\frac{\ddot{a}}{a} \frac{1}{H^2} = \text{ deceleration parameter}\]
\[
\left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \\
\rho = \rho_M(a) + \rho_R(a) + \rho_\Lambda + \ldots
\]

- \(a(t)\) & \(H(t)\) depend on matter/energy content
- \(a(t)\) measurable via redshift
- Redshift \(z\) is a proxy for time or scale factor: \(1 + z = a_0/a(t)\)
Expansion Rate Is A Key Quantity

Friedmann equation \((G_{00} = 8\pi GT_{00})\): Expansion rate \(H(z)\)

\[
\Omega_i \equiv \frac{\rho_i}{\rho_C} \quad \rho_C \equiv \frac{3H_0^2}{8\pi G}
\]

Hubble const. radiation  matter  curvature  dark energy

\[
H^2(z) = H_0^2 \left[ \Omega_R (1+z)^4 + \Omega_M (1+z)^3 + (1-\Omega_{\text{TOTAL}})(1+z)^2 + \Omega_w (1+z)^{3(1+w)} \right]
\]

Equation of state parameter: \(w = p/\rho\)

\[
w = \begin{cases} 
1/3 & \text{radiation} \\
0 & \text{matter} \\
-1/3 & \text{curvature} \\
-1 & \Lambda
\end{cases}
\]
Newton
Einstein
\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \]

scale factor $a$

decelerated
\[ \ddot{a} < 0 \]
\[ \rho + 3p > 0 \]

accelerated
\[ \ddot{a} > 0 \]
\[ \rho + 3p < 0 \]

fluid with $w = \frac{p}{\rho} < -\frac{1}{3}$

scale factor $a$

time
Evolution of $a(t)$ For Flat Models

$$
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ \rho_{M0} \left(\frac{a_0}{a}\right)^3 + \rho_{\Lambda} \right]
$$

![Graph showing the evolution of $a(t)/a_0$ with time in billions of years from today.]
"... physically, he is a splendid specimen…"
"... magnificent physique ….”
"... manly …”
"... loveable character …”
University of Chicago  1909 National Champions

Hubble
$v = H_0 d$  \hspace{1cm} (H_0 = \text{Hubble's constant})$

$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$

$h = 5$
\[ h = 0.72 \pm 0.03 \pm 0.07 \text{ Freedman et al. (Hubble Key Project)} \]

\[ h = 0.71^{+0.04}_{-0.03} \text{ (WMAP 2003)} \]
\[ h^2 = \frac{1}{2} \]
Distance–Redshift Relation

\[ F = \frac{L}{4\pi d_L^2} \]

defines *luminosity distance* – “know” \( L \), measure \( F \)

\[ 4\pi d_L^2 = \text{area of } ^2S \text{ centered on source at time of detection } t_0 \]

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \rightarrow \text{area} = 4\pi a_0^2 r^2 \]

Energy redshifted: \((1 + z)\)

Time interval redshifted: \((1 + z)\)

Flux redshifted: \((1 + z)^2\)

\[ d_L^2 = a_0^2 r^2 (1 + z)^2 \]

Light emitted from comoving coordinate \( r \) at time \( t \) reaches us now \( (t_0) \) redshifted by an amount \( 1 + z = a_0 / a(t) \)
\[ d_L = a(t_0) r (1 + z) \]

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \]

Light travels on geodesics
\[ ds^2 = 0 \]

\[ \int \frac{dr}{\sqrt{1 - kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2} \]

\[ \int_{0}^{r} \frac{dr'}{\sqrt{1 - kr'^2}} = \int_{0}^{z} \frac{a^{-1}(t_0)H_0^{-1}dz'}{\sqrt{(1 - \Omega_0)(1 + z')^2 + \Omega_M (1 + z')^3 + \Omega_w (1 + z')^{3(1+w)}} + ...} \]
\[ d_L = a_0 r (1 + z) \] 

Program:

• measure \( d_L \) (via \( d_L^2 = L / 4\pi F \)) and \( z \)

• input a model cosmology (\( \Omega_i \)) and calculate \( a_0 r \)

• compare to data

• need bright “standard candle”
Supernovae
Monastic Chronicles re: Supernova 1006:

“in 1006 there was a very great famine and a comet appeared for a long time ….”

“at the same time a comet, which always announces human shame, appeared in the southern regions, which was followed by a great pestilence…”

“three years after the king was raised to the throne, a comet with a horrible appearance was seen in the southern part of the sky, emitting flames this way and that…”

Georg Busch (German painter) in 1572:

“It is a sign that we will be visited by all sorts of calamities such as inclement weather, pestilence, and Frenchmen.”
Supernova Taxonomy

Early Spectra:
- No Hydrogen / Hydrogen

SN I
  - Si/ No Si
    - SN Ia
      - 1985A
      - 1989B
    - He poor/He rich
      - SN Ic
        - 1983I
      - SN Ib
        - 1983N 1984L

SN II
  - ~3 mos. spectra
    - He dominant/H dominant
      - SN IIb
        - 1993J 1987K

“Normal” SN II
  - Light Curve decay after maximum:
    - Linear / Plateau
      - SN IIL
        - 1980K 1979C
      - SN IIP

Believed to originate from **deflagration** or **detonation** of an **accreting white dwarf**.

Core Collapse:
Most (NOT all) H is removed during evolution by tidal stripping.

Core Collapse:
Outer Layers stripped by winds (Wolf-Rayet Stars) or binary interactions
Ib: H mantle removed
Ic: H & He removed

Core Collapse of a massive progenitor with plenty of H.
Type Ia Supernovae

Supernova Cosmology Project

Type Ia Supernovae (not calibrated)
**Type Ia Supernovae**

Supernova Cosmology Project

Luminosity / Solar Luminosity

-20                0                  20                 40      60  

Type Ia Supernovae (calibrated)
Evolution of $H(z)$ Is A Key Quantity

Robertson–Walker metric
$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

Many observables based on $H(z)$ through coordinate distance $r(z)$

- Luminosity distance
  Flux = (Luminosity / 4$\pi$ $d_L^2$)

- Angular diameter distance
  $\alpha = \text{Physical size} / d_A$

- Volume (number counts)
  $N \propto V^{-1}(z)$

- Age of the universe
Evidence For Dark Energy

Astier et al. (2006)
SNLS
confusing astronomical notation related to supernova brightness
fainter →
brighter

Einstein-de Sitter: spatially flat matter-dominated model (maximum theoretical bliss)

\( \Lambda \) CDM

supernova redshift \( z \)
Evidence For
Dark Energy

1. Observe magnitude & redshift
2. Assume a cosmological model
   \[ \Lambda \]
   Assumes \( w = -1 \) (i.e., \( \Lambda \))
   Assumes priors on \( H_0 \), etc.
3. Compare observations & model
4. Fit needs cosmological constant
   \[ \rho \sim 10^{-30} \text{ g cm}^{-3} \]
   \[ \rho = \frac{\Lambda}{8\pi G} \]
   length scale \( 10^{-3} \text{ cm} \) \( 10^{+29} \text{ cm} \)
   mass scale \( 10^{-4} \text{ eV} \) \( 10^{-33} \text{ eV} \)

Einstein-de Sitter model
flat, matter-dominated
(maximum theoretical bliss)

Closed
Flat
Open
Accelerating
Decelerating

Astier et al. (2006)
SNLS
Evidence For Dark Energy

The case for $\Lambda$:
1) Hubble diagram (SNe)
2) Cosmic Subtraction
3) Baryon acoustic oscillations
4) Weak lensing
5) Galaxy clusters
6) Age of the universe
7) Structure formation
Theorist’s view of the universe (isotropic)
Temperature Of The Universe (COBE)

Thermal spectrum
\[ T = 2.728 \pm 0.004 \text{ K} \]

100σ error bars

\[ \Omega \gamma h^2 = 2.5 \times 10^{-5} \]
Observer’s view of the universe (fluctuations)
Angular Power Spectrum

\[ \delta T(\theta_1, \phi_1) \quad \delta T(\theta_2, \phi_2) \]

\[ \delta T(\theta, \phi) = \sum a_{lm} Y_{lm}(\theta, \phi) \]

\[ C_l \equiv \left\langle |a_{lm}|^2 \right\rangle \]
Baryon Acoustic Oscillations

Pre-recombination
- universe ionized
- photons provide enormous pressure and restoring force
- perturbations oscillate (acoustic waves)

Post-recombination
- universe neutral
- photons travel freely (decouple from baryons)
- perturbations grow (structure formation)

Big Bang

ionized

recombination
$z \sim 1100$
$T \sim 400,000 \text{ yr}$

neutral

Time

Today
Dark Matter, Gas, Photon, Neutrino

Density

Radius (Mpc)

110 yrs

z=82507

Eisenstein
**Acoustic Peaks**

Sound travel distance known

**Observed** $l_{\text{peak}} \sim \text{geometry}$

- Flat (Euclidean)
- Spherical (closed)
- Hyperbolic (open)

$$\Omega_0 = \pm 0.1$$

![Diagram](image_url)
$\Omega_{\text{TOTAL}} = 1.011 \pm 0.055$
ESA Mission Planck - Launch In 2008?
Cosmic Subtraction

\[ \Omega_{\text{TOTAL}} = 1 \]

\[ \Omega_M \sim 0.3 \]

\[ 1.0 - 0.3 = 0.7 \]

CMB

many methods

subtraction
Observer’s View of the Universe

lumpy (inhomogeneous and anisotropic)
full of stars, galaxies, clusters, ....
smooth (homogeneous and isotropic)
full of dark matter (and dark energy)
Power Spectrum

• Assume there is an average density \( \bar{\rho} \)

• Expand density contrast \( \delta(\bar{x}) \) in Fourier modes

\[
\delta(\bar{x}) \equiv \frac{\rho(\bar{x}) - \bar{\rho}}{\bar{\rho}} = \int \delta_k \exp(-ik \cdot \bar{x}) \, d^3k
\]

• Autocorrelation function defines power spectrum

\[
\left\langle \frac{\delta \rho(\bar{x})}{\rho} \right\rangle^2 = \left\langle \delta(\bar{x}) \delta(\bar{x}) \right\rangle = \int_0^\infty \frac{dk}{k} \frac{k^3 |\delta_k^2|}{2\pi^2}
\]

\[
\Delta^2(k) \equiv \frac{k^3 |\delta_k^2|}{2\pi^2} \quad P(k) \equiv |\delta_k^2|
\]
• Power spectrum related to \textit{rms} fluctuations

\[
\left\langle \frac{\delta \rho(\vec{x})}{\rho} \right\rangle_R \approx \Delta(k = R^{-1})
\]

sphere of radius \( R \)

\begin{array}{ccc}
\text{trial} & N & \text{variance} \\
1 & N_1 & N_1 - \bar{N} \\
2 & N_2 & N_2 - \bar{N} \\
3 & N_3 & N_3 - \bar{N} \\
\vdots & \vdots & \vdots \\
\bar{N} & \sqrt{\sum (N_i - \bar{N})^2} \\
\end{array}
Power Spectrum

![Power Spectrum Graph](image-url)
Baryons $\Omega_B h^2 \sim 0.02$

Burles et al.

QSO 1937-1009

WMAP: $\Omega_B h^2 = 0.0229 \pm 0.00073$

Tytler

Ly-α
**Neutrinos** $\Omega_\nu h^2 \sim 0.001$

- Neutrinos decouple earlier than photons (about 1 second AB).
- Their number density relative to photons is reduced because of $e^\pm$ annihilation.
- Today $T_\nu = 1.9$ K (rather than 2.7 K for photons).
- $\Omega_\nu h^2 = m_\nu / 93$ eV
- Of course have never detected directly background $\nu$s, ... but strong circumstantial evidence that they exist.
**Age Of The Universe: $t_0$**

- white dwarf star cooling $\pm 11 \pm 2$ Gyr
- nucleocosmochronology $12.6 \pm 3$ Gyr
- globular cluster evolution $13.5 \pm 2$ Gyr

### Table

<table>
<thead>
<tr>
<th>$H_0 = 70$</th>
<th>$\Omega_M$</th>
<th>$\Omega_\Lambda$</th>
<th>$t_0$ (Gyr)</th>
</tr>
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<tbody>
<tr>
<td>Flat</td>
<td>1.0</td>
<td>0</td>
<td>9.3</td>
</tr>
<tr>
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<td>0</td>
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<tr>
<td>Flat</td>
<td>0.2</td>
<td>0.8</td>
<td>15</td>
</tr>
</tbody>
</table>
• Big-bang primer (just a little theory)

• Cosmological parameters (after all, they are just numbers):
  
  \( H_0 \) → the present expansion rate (Hubble’s constant)
  
  \( \Omega_i \) → the present cosmic food chain
  
  \( (\Omega_{\text{TOTAL}}, \Omega_M, \Omega_B, \Omega_{\Lambda}, \Omega_\gamma, \Omega_\nu, …) \)

  \( T_0 \) → the present temperature of the Universe

  \( t_0 \) → the present age of the Universe

• Power spectra—characterization of perturbations:
  
  Galaxies: \( P(k) \)          Radiation: \( C_\ell \)

• “Standard model”: \( \Lambda \text{CDM} \)  Dark Energy and Dark Matter
Cold Dark Matter (CDM): 25%
Dark Energy ($\Lambda$): 70%

Chemical Elements: (other than H & He) 0.025%
Neutrinos: 0.17%
Stars: 0.8%
H & He: gas 4%

Radiation: 0.005%

+ inflationary perturbations
+ baryo/lepto genesis
Suggested reading:

Particle orientation:
*The Early Universe*, Kolb & Turner

Structure formation, closer to astronomy:
*Cosmological Physics*, Peacock

Inflation and structure formation:
*Cosmological Inflation and large-Scale Structure*, Liddle & Lyth

Cosmic Background Radiation:
*Modern Cosmology*, Dodelson
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