Fundamentals of Particle Detectors and Developments in Detector Technologies for Future Experiments

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Lecture 2/5

Abstract:

This lecture series will first review the elementary processes and techniques on which particle detectors are based. These must always be kept in mind when discussing the limits of existing technologies and motivations for novel developments. Using the examples of LHC detectors, the limits of state of the art detectors will be outlined and the current detector R&D trends for the LHC upgrade and other future experiments will be discussed. This discussion will include micro-pattern gas detectors, novel solid state detector technologies and trends in microelectronics.

Outline

1) History of Instrumentation
Cloud Chambers/Bubble Chambers/Geiger Counters/Scintillators/Electronics/Wire Chambers

2) Electro-Magnetic Interaction of Charged Particles with Matter
Excitation/ Ionization/ Bethe Bloch Formula/ Range of Particles/ PAI model/ Ionization Fluctuation/ Bremsstrahlung/ Pair Production/ Showers/ Multiple Scattering

3) Signals/Gas Detectors
Detector Signals/ Signal Theorems/
Gaseous Detectors/ Wire Chambers/ Drift Chamber/ TPCs/ RPCs/ Limits of Gaseous Detectors/ Current Trends in Gaseous Detector Development

4) Solid State Detectors
Principles of Solid State Detectors/ Diamond Detectors/ Silicon Detectors/ Limits of Solid State Detectors/ Current Trends in Solid State Detectors

5) Calorimetry & Selected Topics
EM showers/ Hadronic Showers/ Crystal Calorimeters/ Noble Liquid Calirimeters/ Current Trends in Calorimetry


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"POWERPOINT" POISONING.

Electro-Magnetic Interaction of Particles with Matter

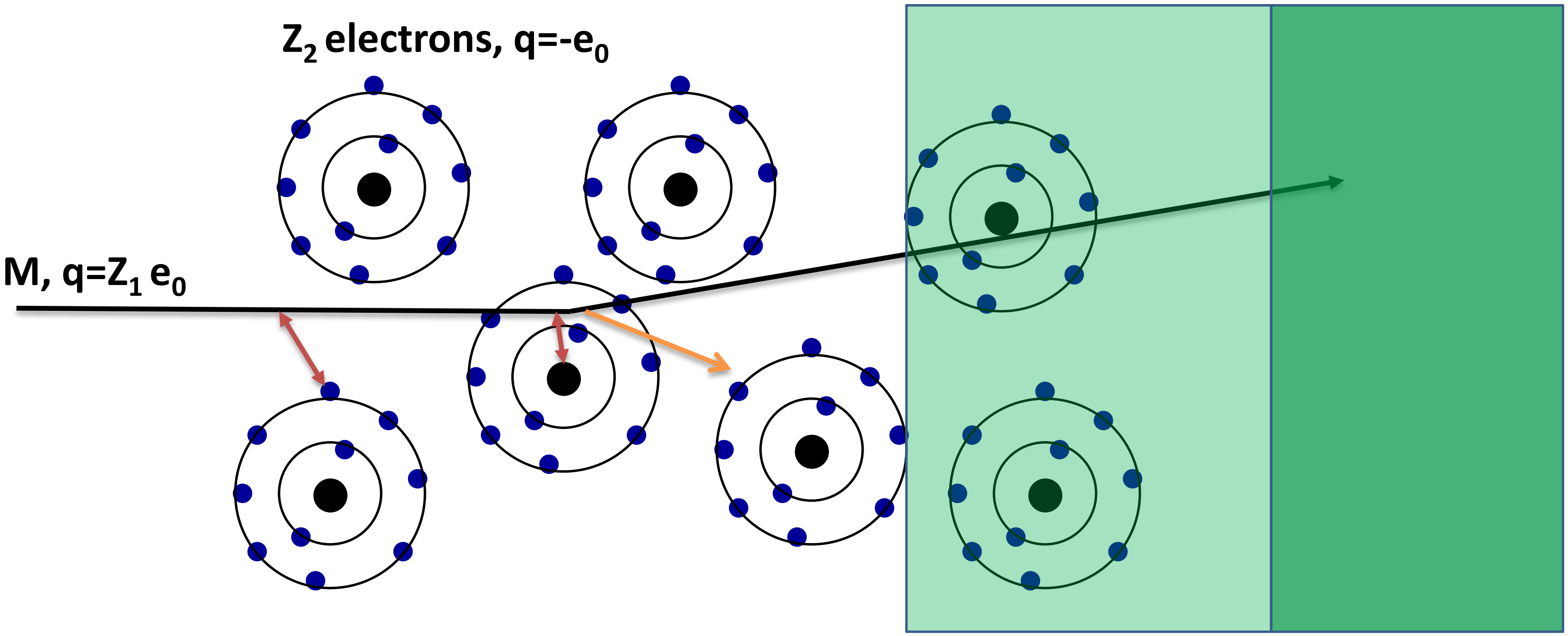
Various aspects of the penetration of charged particles in matter have occupied the thoughts of some of the finest physicists of the last century.
E.g. Thomson 1903, Rutherford 1911, Bohr 1913, 1915, 1948, Bethe 1930, 1932, Mott, 1931, Bloch 1933, Fermi 1940, Landau 1944

In the first half of the 20th century, the energy loss of the charged particles and the related stopping power of materials was the prime issue.

Nowadays, the actual amount of scintillation light and/or charge produced by the passing particle, and the fluctuations of these quantities, are the important quantity because these are the quantities produce the signals in particle detectors and their fluctuations are responsible for the resolution limits of the detectors.

We will therefore summarize the basic mechanisms that are responsible for the creation of excitation and ionization and will explain the models that are implemented in modern simulation programs like GEANT and HEED.

Electromagnetic Interaction of Particles with Matter

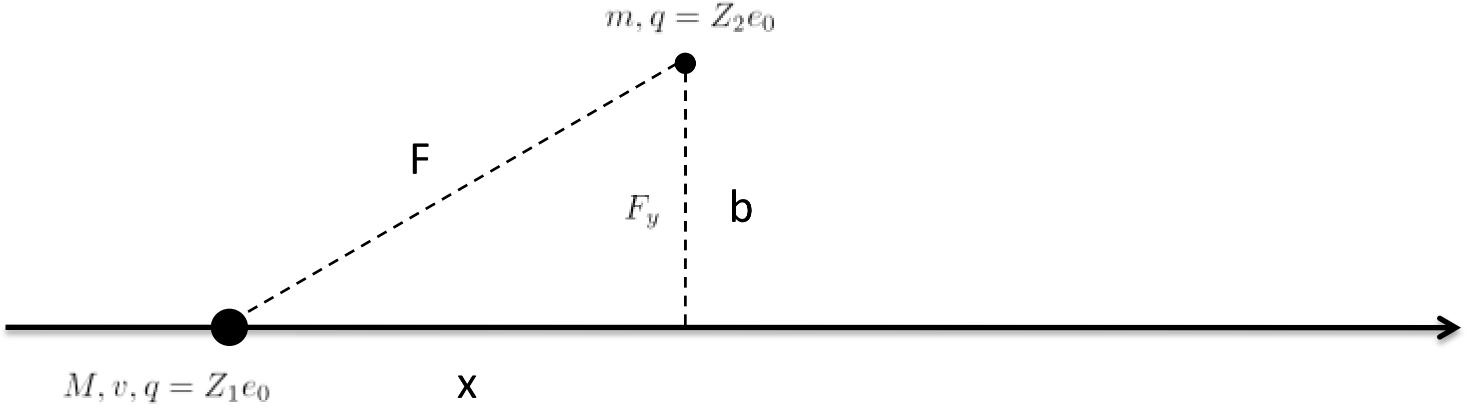


Interaction with the atomic electrons. The incoming particle loses energy and the atoms are <u>excited</u> or <u>ionized.</u>

Interaction with the atomic nucleus. The particle is deflected (scattered) causing <u>multiple scattering</u> of the particle in the material. During this scattering a <u>Bremsstrahlung</u> photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as <u>Cherenkov Radiation</u>. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called <u>Transition radiation</u>.

Interaction of Particles with Matter



While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer

$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 (b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}} \qquad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter

$$F_y = \frac{\gamma Z_1 Z_2 e_0^2 b}{4\pi\varepsilon_0 (b^2 + \gamma^2 v^2 t^2)^{3/2}} \qquad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The transferred energy is then

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2}$$

$$\Delta E(electrons) = Z_2 \frac{1}{m_e} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \Delta E(nucleus) = \frac{Z_2^2}{2 Z_2 m_p} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \frac{\Delta E(electrons)}{\Delta E(nucleus)} = \frac{2 m_p}{m_e} \approx 4000$$

→ The incoming particle transfer energy only to the atomic electrons !

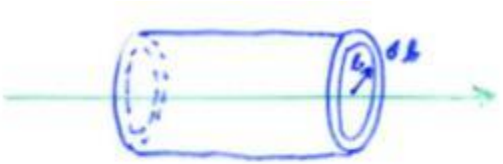
Interaction of Particles with Matter

Target material: mass A, Z_2, density ρ [g/cm^3], Avogadro number N_A

A gramm → N_A Atoms: **Number of atoms/cm^3** $n_a = N_A \rho / A$ [1/cm^3]

Number of electrons/cm^3 $n_e = N_A \rho Z_2 / A$ [1/cm^3]

$$\Delta E(electrons) = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} \frac{e_0^4}{(4\pi\varepsilon_0 m_e c^2)^2} = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$



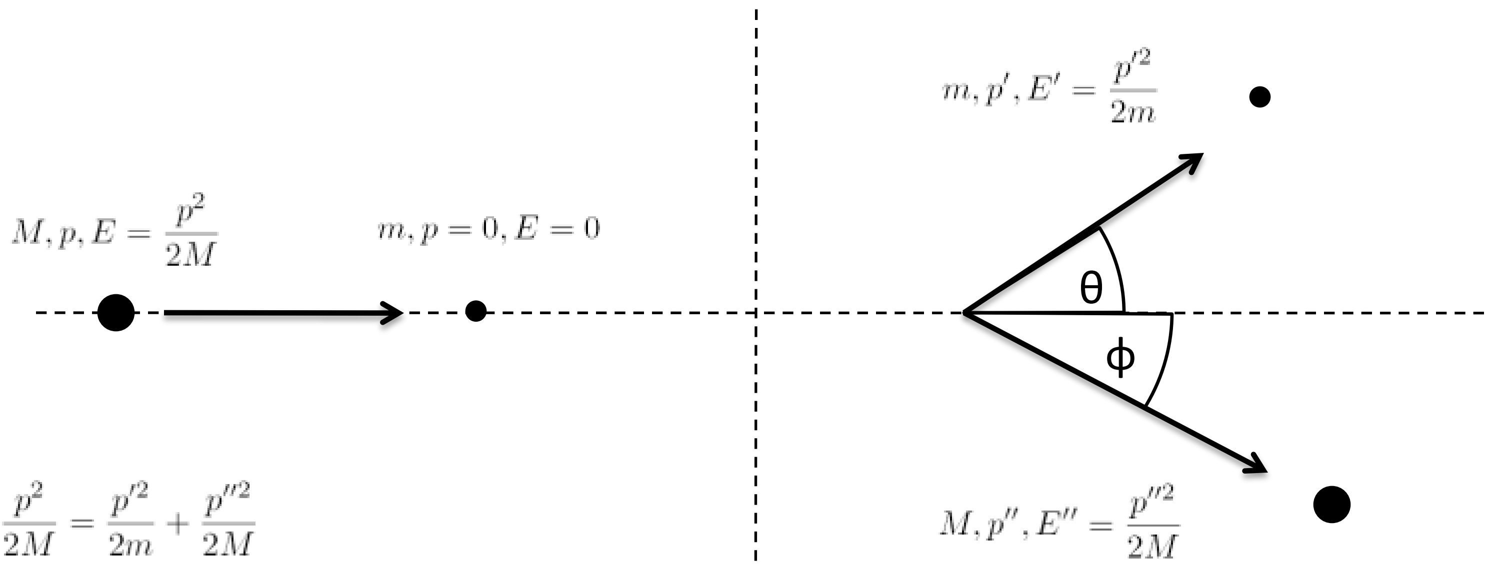
$$dE = -\int_{b_{min}}^{b_{max}} n_e \Delta E dx 2b\pi db = -\frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2} \frac{N_A \rho}{A} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

With $\Delta E(b)$ → $db/b = -1/2\, dE/E$ → $E_{max} = \Delta E(b_{min})$ $E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \int_{E_{min}}^{E_{max}} \frac{dE}{E} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

$E_{min} \approx I$ (Ionization Energy)

Nonrelativistic Collision Kinematics, E_{max}



1) $$\frac{p^2}{2M} = \frac{p'^2}{2m} + \frac{p''^2}{2M}$$

2) $$p = p' \cos\theta + p'' \cos\phi$$
$$0 = p' \sin\theta + p'' \sin\phi$$
$$p''^2 = p'^2 + p^2 - 2pp' \cos\theta$$

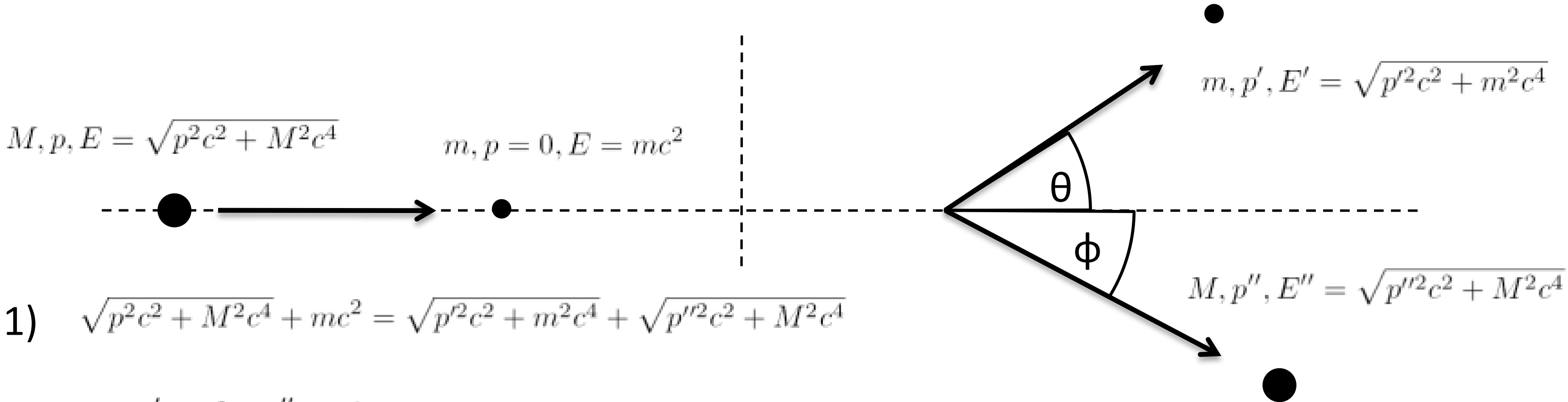
1+2) $$p' = \frac{2mp\cos\theta}{M+m} \quad \rightarrow \quad E' = \frac{p'^2}{2m} = \frac{2mp^2\cos^2\theta}{(m+M)^2}$$

$$E'_{max} = \frac{2mp^2}{(m+M)^2} \quad \rightarrow \quad \frac{E'_{max}}{E} = \frac{4mM}{(m+M)^2} \leq 1$$

$$M = m \quad \rightarrow \quad \frac{E'_{max}}{E} = 1$$

$$M = 200m \quad \rightarrow \quad \frac{E'_{max}}{E} = 0.02$$

Relativistic Collision Kinematics, E_{max}



1) $$\sqrt{p^2c^2 + M^2c^4} + mc^2 = \sqrt{p'^2c^2 + m^2c^4} + \sqrt{p''^2c^2 + M^2c^4}$$

2) $$p = p'\cos\theta + p''\cos\phi \qquad 0 = p'\sin\theta + p''\sin\phi$$

$$p''^2 = p'^2 + p^2 - 2pp'\cos\theta$$

1+2) $$E^{k'} = \sqrt{p'^2c^2 + m^2c^4} - mc^2 = \frac{2mc^2\, p^2c^2\cos^2\theta}{\left[mc^2 + \sqrt{p^2c^2 + M^2c^4}\right]^2 - p^2c^2\cos^2\theta}$$

$$E^{k'}_{max} = \frac{2mc^2p^2c^2}{(m^2+M^2)c^4 + 2m\sqrt{p^2c^2 + M^2c^4}} = 2mc^2\beta^2\gamma^2F \qquad F = \left(1 + \frac{2m}{M}\sqrt{1+\beta^2\gamma^2} + \frac{m^2}{M^2}\right)^{-1}$$

$$\frac{E^{k'}_{max}}{E^k} = \frac{E^{k'}_{max}}{\sqrt{p^2c^2 + M^2c^4} - Mc^2} = \frac{2mc^2p^2c^2}{Mc^2\left(\sqrt{1+\frac{p^2c^2}{M^2c^4}} - 1\right)\left(m^2c^4 + M^2c^4 + 2mMc^4\sqrt{1+\frac{p^2c^2}{M^2c^4}}\right)}$$

$$pc \ll Mc^2 \quad \rightarrow \quad \frac{E^{k'}_{max}}{E^k} = \frac{4mM}{(m+M)^2} \qquad\qquad pc \gg Mc^2 \quad \rightarrow \quad \frac{E^{k'}_{max}}{E^k} = 1$$

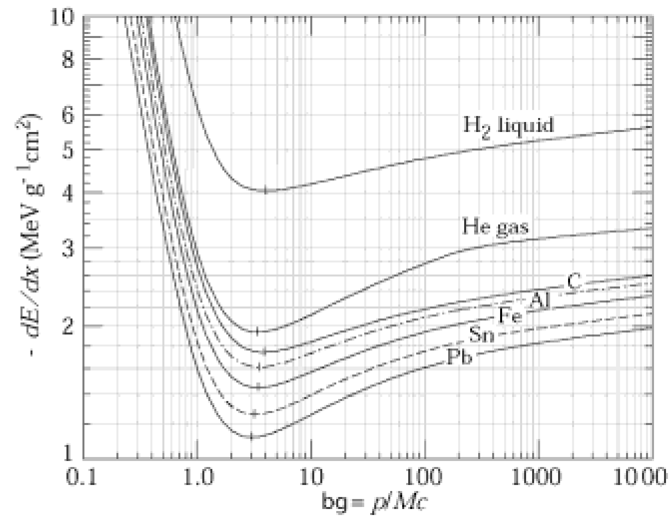
Specific Energy Loss

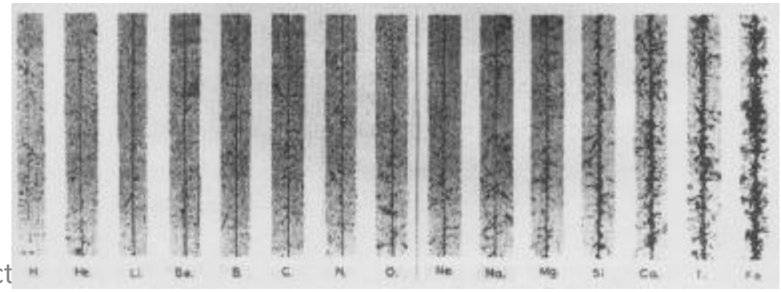
$$\frac{1}{\rho}\frac{dE}{dx} = -2\pi r_e^2 m_e c^2 N_A \frac{Z_1^2}{\beta^2}\frac{Z_2}{A}\ln\frac{2m_e c^2\beta^2\gamma^2 F}{I} \qquad F = \left(1 + \frac{2m_e}{M}\sqrt{1+\beta^2\gamma^2} + \frac{m_e^2}{M^2}\right)^{-1}$$

This formula is up to a factor two identical with the correct QM formula.

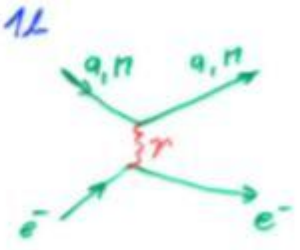
The specific Energy Loss $1/\rho\ dE/dx$

- **decreases at $1/\beta^2$ of the incoming particle**
- **increases like ln γ for β =1**
- **is $\approx$ independent of M ($M >> m_e$)**
- **is proportional to Z_1^2 of the incoming particle**
- **is $\approx$ independent of the Material (Z/A $\approx$ const.)**
- **dE/dx $\approx$ 1-2 x ρ [g/cm^3] MeV/cm**



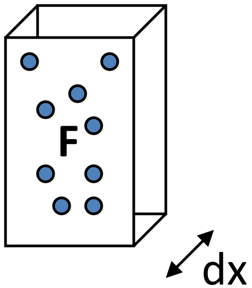


Crossection



Crossection σ: Material with Atomic Mass A and density ρ contains n Atoms/cm^3

$$n[\mathrm{cm}^{-3}] = \frac{N_A[\mathrm{mol}^{-1}]\,\rho[\mathrm{g/cm}^3]}{A[\mathrm{g/mol}]} \qquad N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$$



E.g. Atom (Sphere) with Radius R: Atomic Crossection $\sigma = R^2\pi$

A volume with surface F and thickness dx contains N=nFdx Atoms.
The total 'surface' of atoms in this volume is N σ.
The relative area is $p = N\sigma/F = N_A \rho\, \sigma / A\, dx$ =
Probability that an incoming particle hits an atom in dx.

What is the probability P that a particle hits an atom between distance x and x+dx ?
P = probability that the particle does NOT hit an atom in the m=x/dx material layers and that the particle DOES hit an atom in the mth layer

$$P(x)dx = (1-p)^m p \approx e^{-m} p = \exp\left(-\frac{N_A \rho \sigma}{A} x\right) \frac{N_A \rho \sigma}{A} dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

Mean free path $= \int_0^\infty x P(x) dx = \int_0^\infty \frac{x}{\lambda} e^{-\frac{x}{\lambda}} dx = \lambda$

Average number of collisions/cm $= \frac{1}{\lambda} = \frac{N_A \rho \sigma}{A}$

Differential Crossection

Differential Crossection: $\frac{d\sigma(E,E')}{dE'}$

→ Crossection for an incoming particle of energy E to lose an energy between E' and E'+dE'

Total Crossection: $\sigma(E) = \int \frac{d\sigma(E,E')}{dE'} dE'$

Probability P(E) that an incoming particle of Energy E loses an energy between E' and E'+dE' in a collision:

$$P(E,E')dE' = \frac{1}{\sigma(E)} \frac{d\sigma(E,E')}{dE'} dE'$$

Average number of collisions/cm causing an energy loss between E' and E'+dE' $= \frac{N_A \rho}{A} \frac{d\sigma(E,E')}{dE'}$

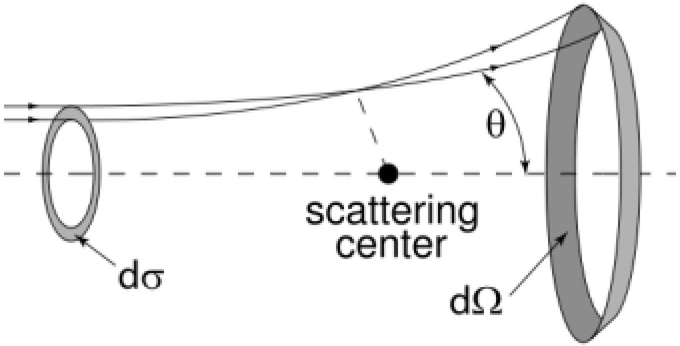
Average energy loss/cm: $\frac{dE}{dx} = -\frac{N_A \rho}{A} \int E' \frac{d\sigma(E,E')}{dE'} dE'$

Rutherford Scattering

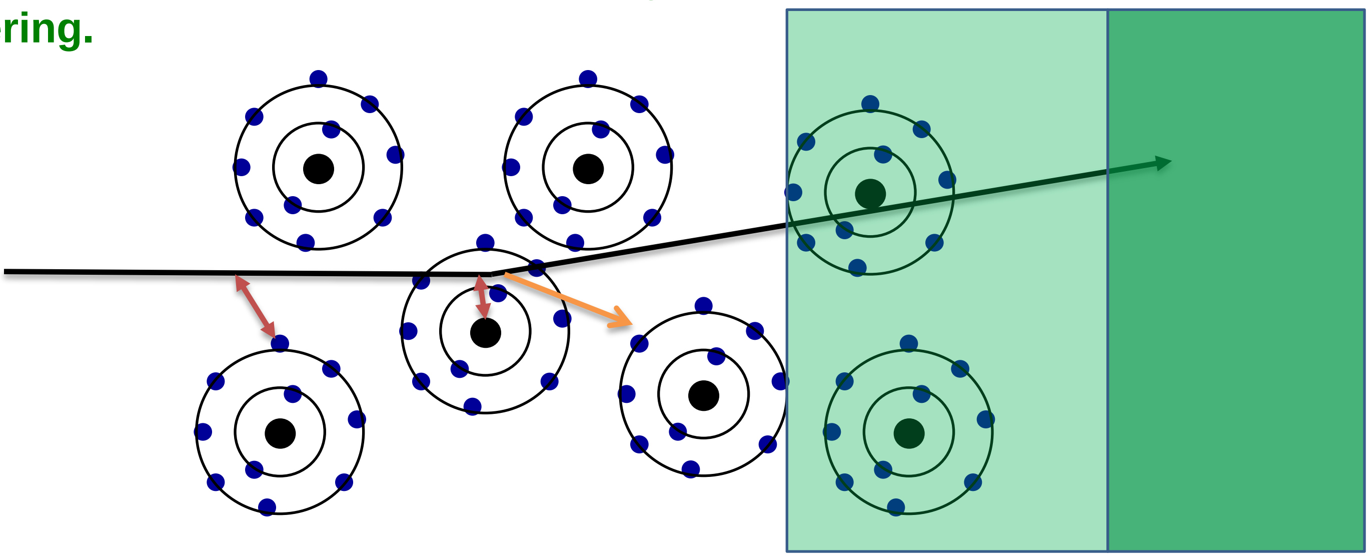
A particle of mass M and charge Z_1e_0 is scattering off a scattering center of charge Z_2e_0 by interaction through the Coulomb force.

The differential crossection for finding the particle scattered into a solid angle dΩ is given by (p=γβ Mc and v = βc).

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e_0^2}{2pv}\right)^2 \frac{1}{\sin^4\theta/2}$$



The incoming particle is typically much heavier than the atomic electrons → excitation and ionization, and much lighter than the nucleus → Multiple scattering.



Rutherford Scattering off Electrons

Jackson, 13.1: Assuming that the incoming particle is much heavier than the atomic electrons ($M >> m_e$), the collision is best viewed as elastic Coulomb scattering in the rest frame of the incident particle. Expressing the scattering angle in terms of energy transfer E' of the incoming particle to the electron we find

$$\frac{d\sigma}{dE'} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi Z_1^2 e_0^4}{m_e c^2 \beta^2} \frac{1}{E'^2} \quad = \quad Z_1^2 r_e^2 \frac{2\pi m_e c^2}{\beta^2} \frac{1}{E'^2}$$

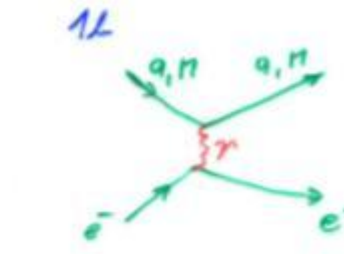
The energy loss is therefore

$$\frac{dE}{dx} = -\frac{N_A Z_2 \rho}{A} \int E' \frac{d\sigma}{dE'} dE' \quad = \quad -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

This is exactly the same result that we found before. In order to shed light on the question of atomic binding we have to treat the problem with QM.

QM Treatment of Energy Loss

For large energy transfers E'>η ~ 50keV the electrons can be considered quasi free and the relativistic formula for the scattering of a spin 0 particle of mass M and Charge $Z_1 e_0$ on a free electron can be used (Bhabha 1938)



$$\frac{d\sigma(E,E')}{dE'} = \frac{2\pi Z_1^2 r_e^2 m_e c^2}{\beta^2}\frac{1}{E'^2}\left(1-\beta^2\frac{E'}{E_{max}}\right)$$

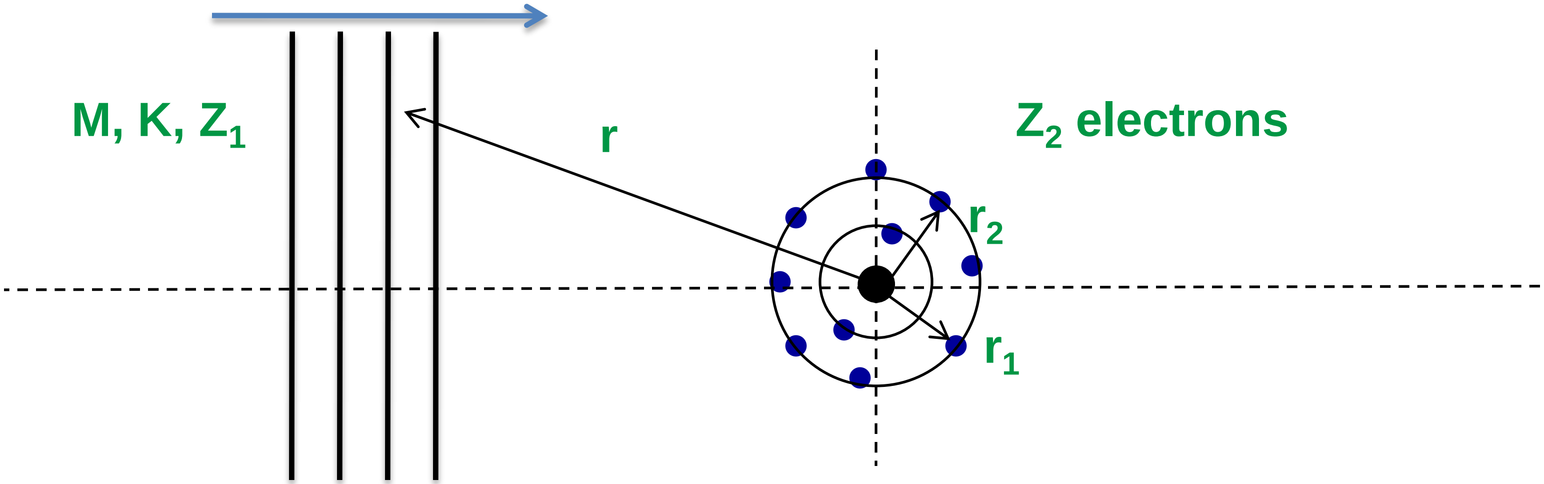
For small energy transfers E'<<E_{max} this is equal to the Rutherford crossection. For large energy transfers i.e. very small impact parameters (close encounters), the electron spin gives rise to a correction.

The energy loss due to these high energy collisions is then

$$\frac{dE}{dx}|_{E'>\eta} = -\frac{N_A Z_2 \rho}{A}\int_\eta^{E_{max}}\frac{d\sigma}{dE'}dE' = -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2}\frac{N_A Z_2\rho}{A}\left(\ln\frac{E_{max}}{\eta}-\beta^2\right)$$

For small energy transfers E' the atomic excitations have to be taken into account. Bethe 1930, 1932 gave a fully QM treatment of the problem. The main steps of the derivation are outlined in the following:

QM Treatment of Energy Loss, Bethe 1930



The QM system consists of the incoming particle and the atom. The interaction between the two is defined by V_{int}

$$H = H^0_{Atom} + H^0_{Part} + V_{int} \qquad i\hbar \frac{d}{dt}\psi = H\psi$$

$$H^0_{Atom}\psi_n(\vec{r}_1, \vec{r}_2, ..., \vec{r}_{Z_2}) = E_n \psi_n(\vec{r}_1, \vec{r}_2, ..., \vec{r}_{Z_2}) \qquad H^0_{Part}\Phi_K(\vec{r}) = \frac{\hbar^2 K^2}{2M}\Phi_K(\vec{r}) \qquad \Phi_K(\vec{r}) = e^{i\vec{K}\vec{r}}$$

$$V_{int} = \frac{Z_2}{r} - \sum_{i=1}^{Z_2} \frac{1}{|\vec{r} - \vec{r}_i|}$$

QM Treatment of Energy Loss

The probability, and associated crossection, that an incoming particle with momentum hK excites the atom to state n and is leaving with momentum hK' can be calculated to first order by applying Born's approximation.

$$u_i = e^{i\vec{K}\vec{r}}\psi_0(\vec{r}_i)e^{-i/\hbar W t} \qquad W = \frac{\hbar^2 K^2}{2M} + E_0 \qquad u_o = e^{i\vec{K}'\vec{r}}\psi_n(\vec{r}_i)e^{-i/\hbar W' t} \qquad W' = \frac{\hbar^2 K'^2}{2M} + E_n$$

$$V_{0n}(K,K') = \langle u_o|V_{int}|u_i\rangle = \int\left(\frac{Z_2}{r} - \sum_{i=1}^{Z_2}\frac{1}{|\vec{r}-\vec{r}_i|}\right)\psi_0(\vec{r}_i)\overline{\psi}_n(r_i)e^{(\vec{K}-\vec{K}')\vec{r}}d^3r d^3r_1 ... d^3r_{Z_2}$$

$$\frac{d\sigma_n(K,K')}{d\theta} = \frac{\alpha^2}{2\pi}\left(\frac{M}{m}Z_1\right)^2 |V_{0n}|^2 \frac{K'}{K}\sin\theta \qquad \vec{q} = \vec{K} - \vec{K}' \qquad q = |\vec{q}| \qquad \Delta p = \hbar q$$

This expression can be transformed into

$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2}\left(\frac{M}{m}Z_1\right)^2 \frac{1}{q^3}|\epsilon_n(q)|^2 \qquad \epsilon_n(q) = \int\left(Z_2 - \sum_{j=1}^{Z_2} e^{i(\vec{q}\vec{r}_j)}\right)\psi_0(\vec{r}_j)\overline{\psi}_n(r_j)d^3r_1 ... d^3r_{Z_2}$$

QM Treatment of Energy Loss

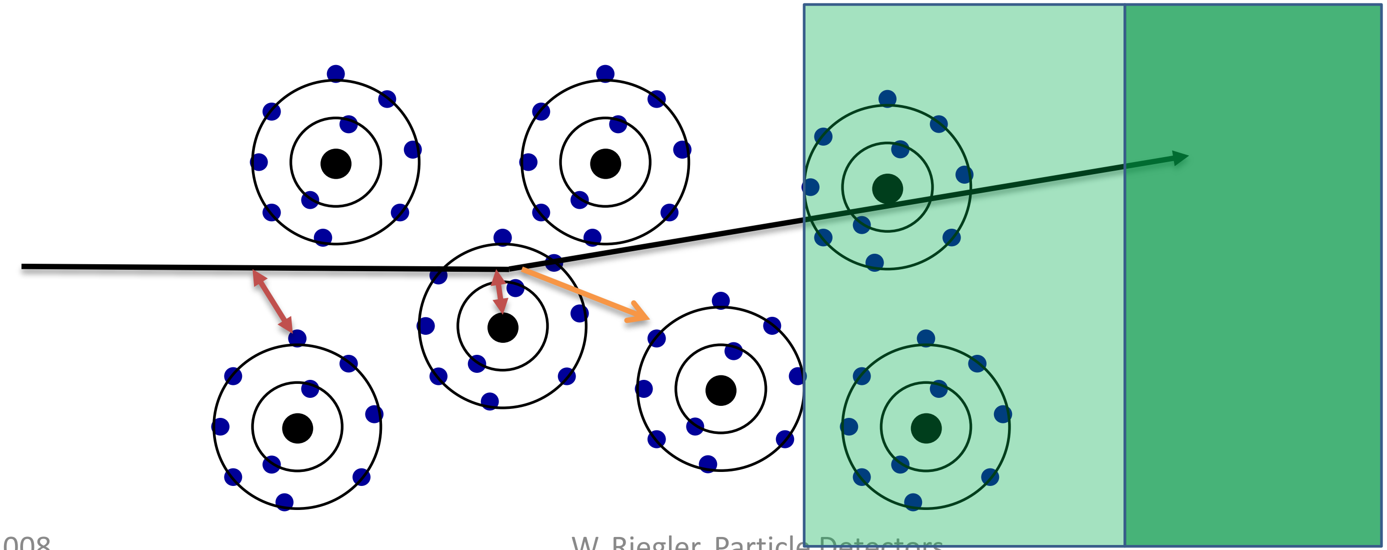
For elastic scattering, where the incoming particle doesn't lose energy, we set n=0 and have

$$\epsilon_0(q) = Z_2 - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0^2(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2} = Z_2 - F$$

where F is the atomic form factor (dependent on q) from the theory of X-ray scattering. Expressing the elastic crossection by the scattering angle we find

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1(Z_2 - F)e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$

This is again the Rutherford crossection for scattering off the Nucleus that is partially 'shielded' by the electrons. We will use it later for calculation of multiple scattering.



QM Treatment of Energy Loss

For inelastic scattering, where the incoming particle excites the Atom, we set n>0 and have

$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2}\left(\frac{M}{m}Z_1\right)^2 \frac{1}{q^3}|\epsilon_n(q)|^2 \qquad \epsilon_n(q) = -\sum_{j=1}^{Z_2}\int e^{i(\vec{q}\vec{r}_j)}\psi_0(\vec{r}_j)\overline{\psi}_n(\vec{r}_j)d^3r_1 ... d^3r_{Z_2}$$

The (lengthy) evaluation of this expression for the nonrelativistic case (Bethe 1930) and the relativistic case (Bethe 1932), using exact expressions for the Hydrogen atom and sum rules for the higher Z atoms, leads to the energy loss assuming energy transfers E' < η

$$\frac{dE}{dx}|_{E'<\eta} = -\sum_n (E_n - E_0)\int \frac{d\sigma_n(q)}{dq}dq = \sum_n E'_n \int \frac{d\sigma_n(q)}{dq}dq \qquad E'_n < \eta$$

$$\frac{dE}{dx}|_{E'<\eta} = -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2}\frac{N_A Z_2 \rho}{A}\left(\ln\frac{E_{max}\eta}{I^2} - \beta^2\right)$$

Adding the expressions for E'< η and E'> η leads to the final expression for the Energy loss

$$\frac{dE}{dx} = \frac{dE}{dx}|_{E'<\eta} + \frac{dE}{dx}|_{E'>\eta} = -4\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2}\frac{N_A Z_2 \rho}{A}\left(\ln\frac{E_{max}}{I} - \beta^2\right)$$

The entire complexity of the atomic physics ends up in the average ionization potential I.

Bethe Bloch Formula

$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2\, m_e c^2\, \frac{Z_1^2}{\beta^2}\, N_A \frac{Z}{A}\left[\ln\frac{2m_e c^2\beta^2\gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Density effect

$$\delta(\beta\gamma) = \ln h\omega_p/I + \ln\beta\gamma - \frac{1}{2}$$

For very high momenta the polarization of the medium by the strong transverse field, which reduces the Engery loss, must be taken into account.

At large Energy transfer (delta electrons) the produced electron can leave the material. In reality, E_{max} must therefore be replaced by T_{cut} and the energy loss reaches a plateau.

We distinguish three distinct regions of energy loss as a function of the particles momentum

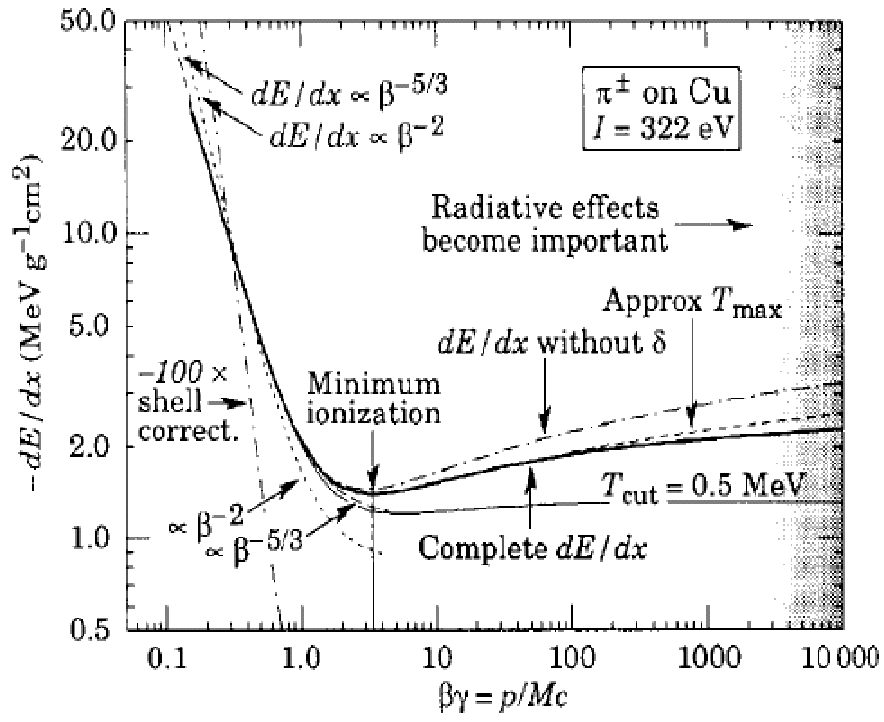
1) $1/\beta^2$ region with Minimum at $\beta\gamma \approx 3$

2) Relativistic Rise

3) Density Effect und Saturation

The Energy loss depends on the particle's velocity and is independent of it's Mass

For Z>1, $I \approx 16 Z^{0.9}$ eV



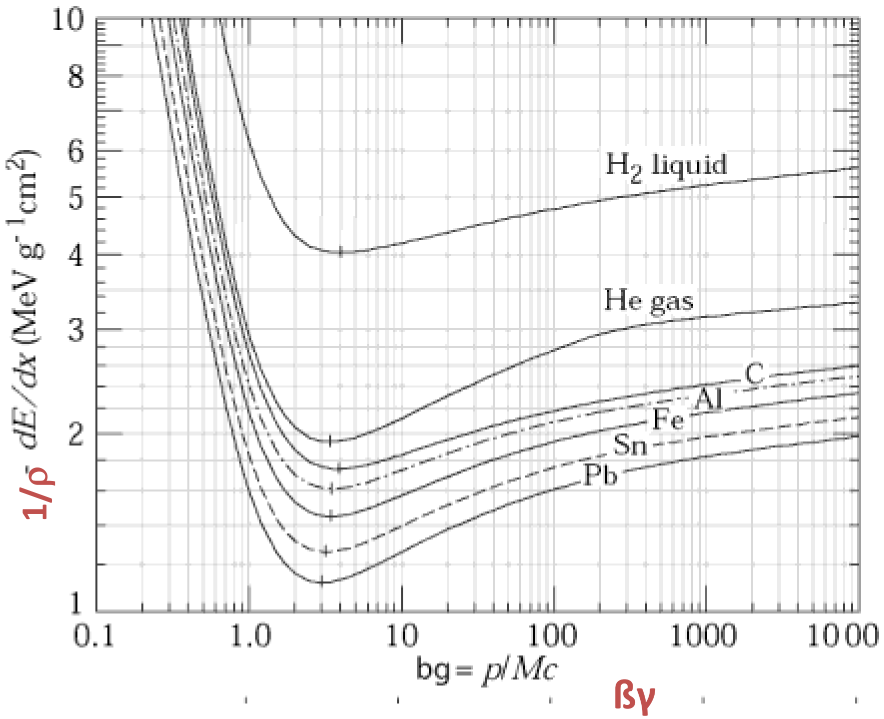
Bethe Bloch Formula

Für Z ≈ 0.5 A
1/ρ dE/dx ≈ 1.4 MeV cm ^{2}/g fur ßγ ≈ 3

Example 1:
Scintillator: Thickness = 2 cm; ρ = 1.05 g/cm^3
Particle with βγ = 3 and Z=1
1/ρ dE / dx ≈ 1.4 MeV
dE ≈ 1.4 * 2 * 1.05 = 2.94 MeV

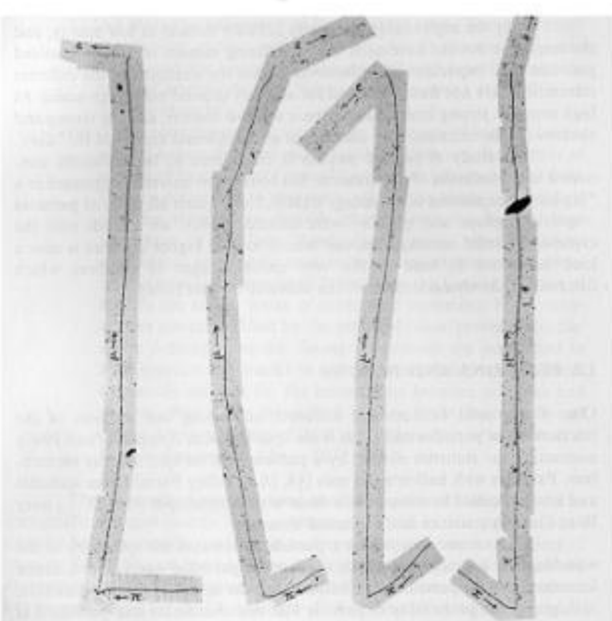
Example 2:
Iron: Thickness = 100 cm; ρ = 7.87 g/cm^3
dE ≈ 1.4 * 100* 7.87 = 1102 MeV = 1.1GeV

Example 3:
Energy Loss of Carbon – Ions with Z=6 and Momentum of 330 MeV/Nucleon
in Water, i.e. βγ = p/m = 330/940 ≈ .35→
β ≈ .33
dE/dx ≈ 1.4 Z^2 /β^2 ≈ 460 MeV/cm→
Cancer Therapy !

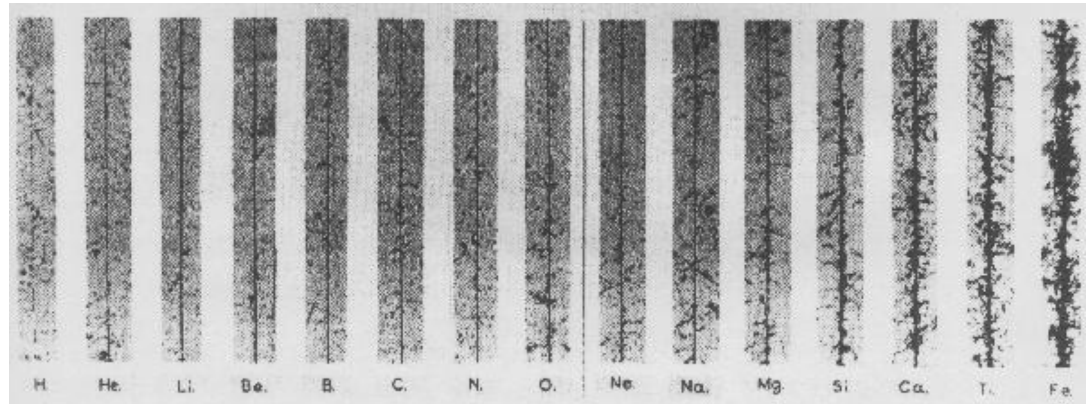


This number must be multiplied with the material density ρ [g/cm^3]
→ dE/dx [MeV/cm]

Small energy loss
→ Fast Particle

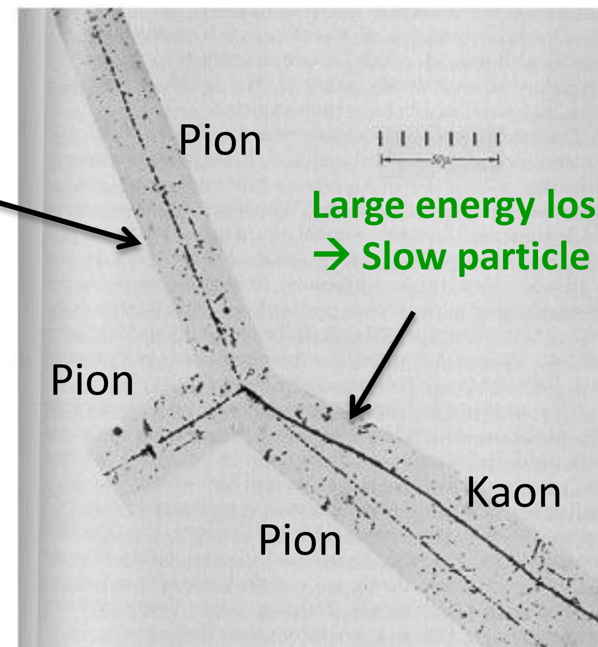


Discovery of muon and pion



Cosmis rays: dE/dx α Z^2

Small energy loss
→ Fast particle

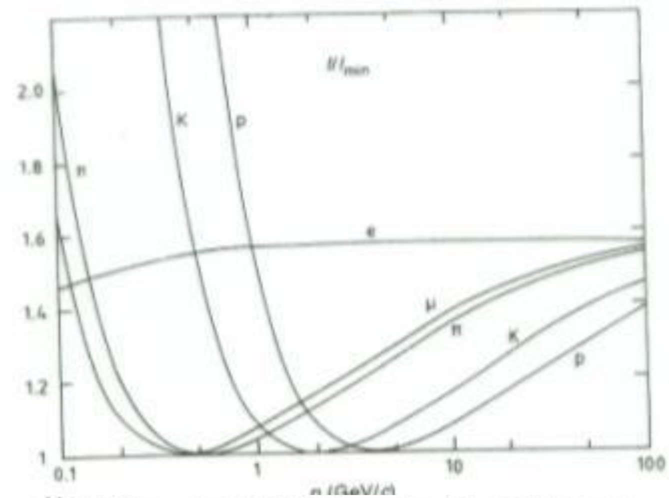


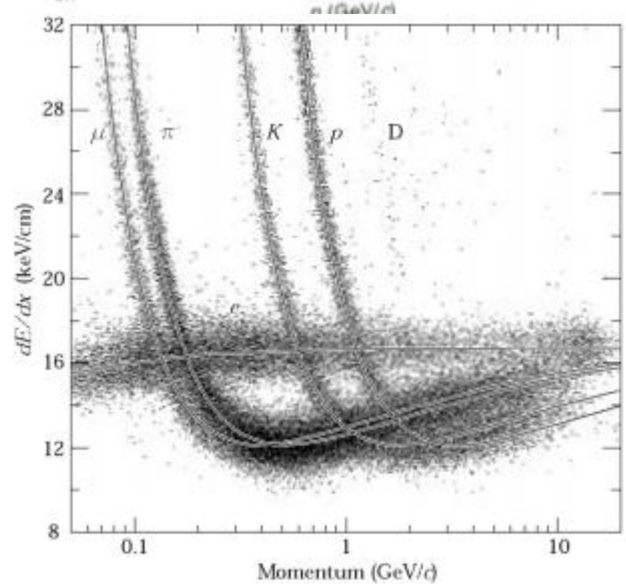
Bethe Bloch Formula, Particle ID

The energy loss as a function of the particle velocity is a universal function for particles of different masses.

The energy loss as a function of particle momentum P= Mcβγ IS depending on the particle's mass M.

By measuring the particle momentum (deflection in the magnetic field) and the energy loss of the particle, particles can be identified in certain momentum regions.





$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2\, m_e c^2\, Z_1^2 \frac{p^2 + M^2c^2}{p^2}\, N_A \frac{Z}{A}\left[\ln\frac{2m_e c^2 F}{I}\frac{p^2}{M^2c^2} - \frac{p^2}{p^2 + M^2c^2}\right]$$

Range of Particles

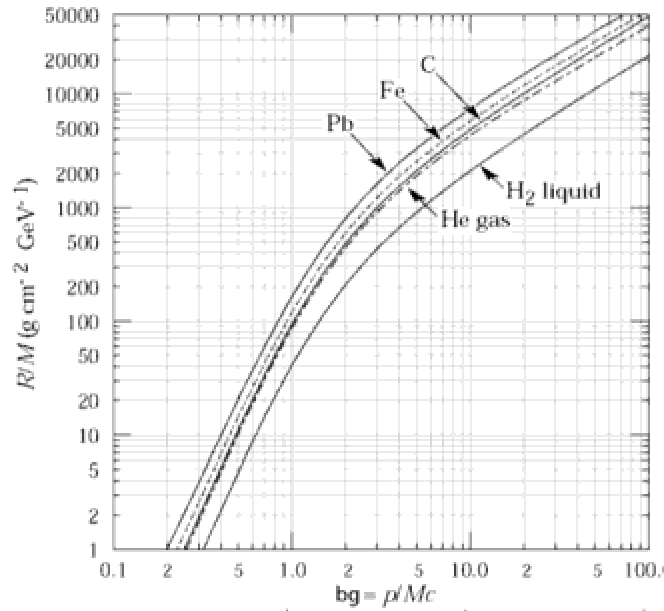
Particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{-1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0\gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

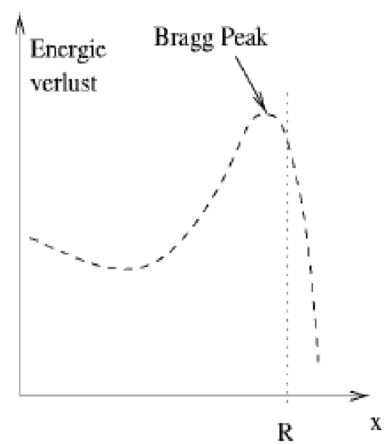
≈Independent of the material

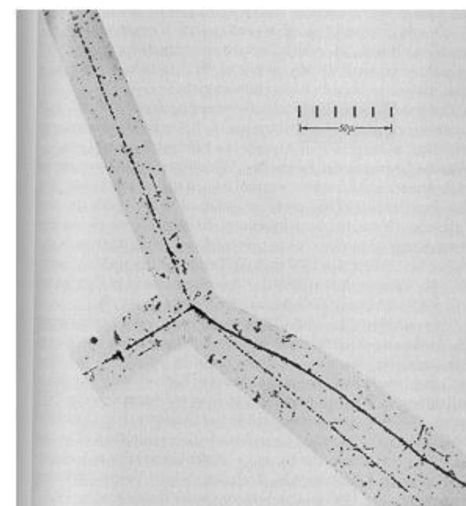


Bragg Peak: For βγ>3 the energy loss is ≈ constant (Fermi Plateau)

If the energy of the particle falls below βγ=3 the energy loss rises as $1/\beta^2$

Towards the end of the track the energy loss is largest → Cancer Therapy.





Range of Particles

Average Range: Example

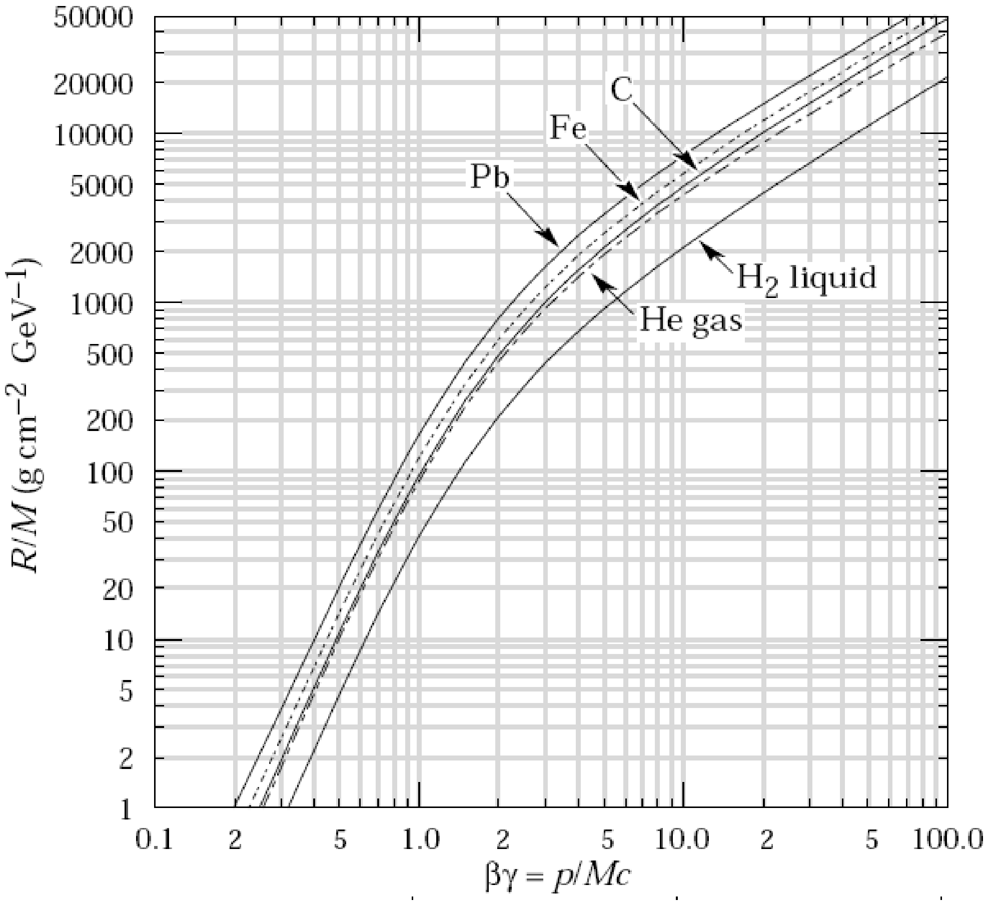
Kaon, p=700 MeV/c in Water:

ρ=1 [g/cm³], Mc^2=494 MeV →

$\beta\gamma$=1.42 → ρ/Mc^2 R = 396g/cm² GeV

→R=195cm

In Pb: ρ =11.35 → R=17cm



Search for Hidden Chambers in the Pyramids

The structure of the Second Pyramid of Giza is determined by cosmic-ray absorption.

Luis W. Alvarez, Jared A. Anderson, F. El Bedwei, James Burkhard, Ahmed Fakhry, Adib Girgis, Amr Goneid, Fikhry Hassan, Dennis Iverson, Gerald Lynch, Zenab Miligy, Ali Hilmy Moussa, Mohammed-Sharkawi, Lauren Yazolino

Fig. 2 (bottom right). Cross sections of (a) the Great Pyramid of Cheops and (b) the Pyramid of Chephren, showing the known chambers: (A) Smooth limestone cap, (B) the Belzoni Chamber, (C) Belzoni's entrance, (D) Howard-Vyse's entrance, (E) descending passageway, (F) ascending passageway, (G) underground chamber, (H) Grand Gallery, (I) King's Chamber, (J) Queen's Chamber, (K) center line of the pyramid.

6 FEBRUARY 1970

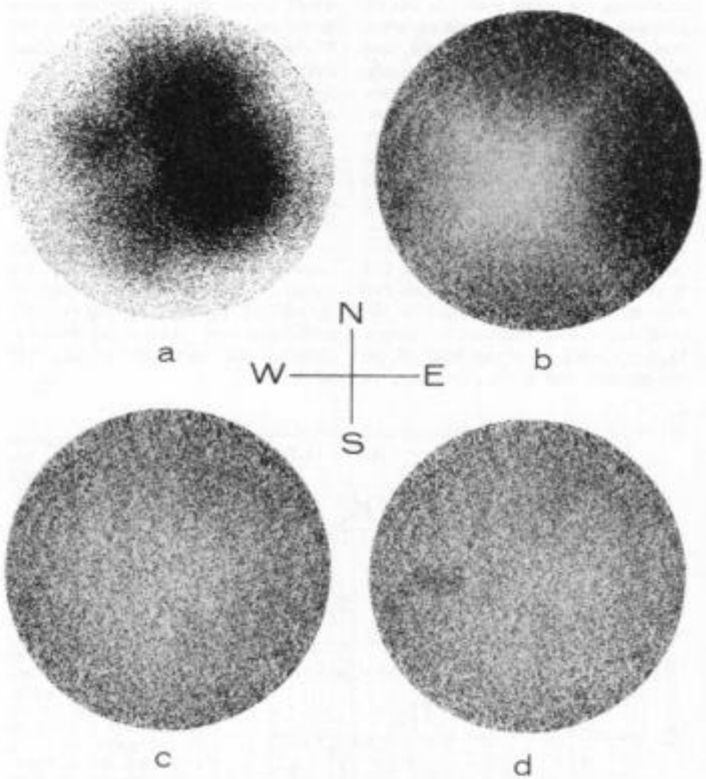
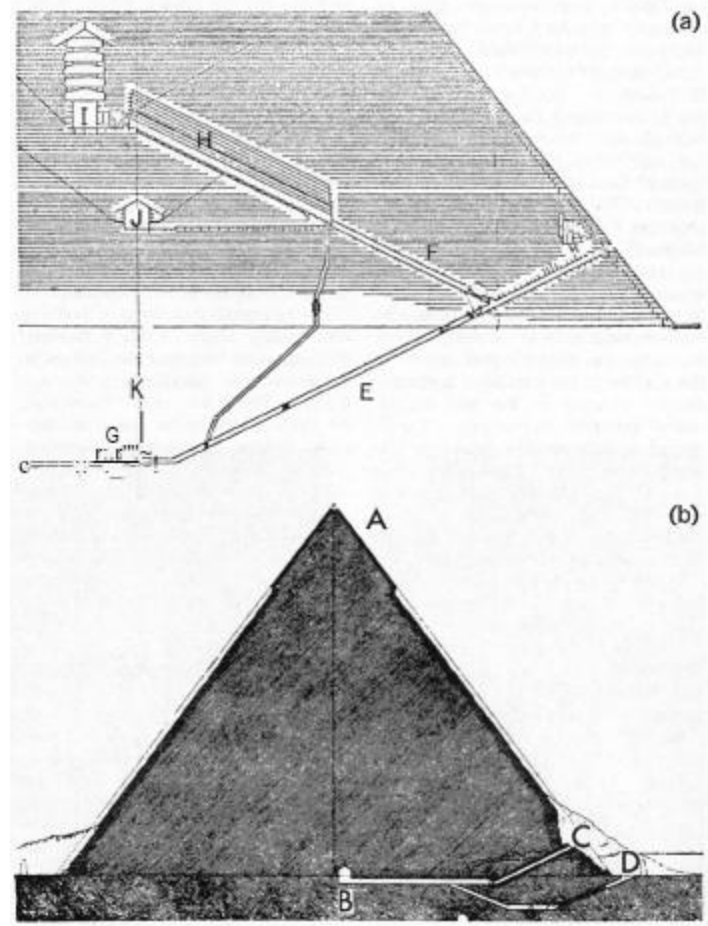


Fig. 13. Scatter plots showing the three stages in the combined analytic and visual analysis of the data and a plot with a simulated chamber, (a) Simulated "x-ray photograph" of uncorrected data. (b) Data corrected for the geometrical acceptance of the apparatus. (c) Data corrected for pyramid structure as well as geometrical acceptance. (d) Same as (c) but with simulated chamber, as in Fig. 12.

Luis Alvarez used the attenuation of muons to look for chambers in the Second Giza Pyramid → Muon Tomography

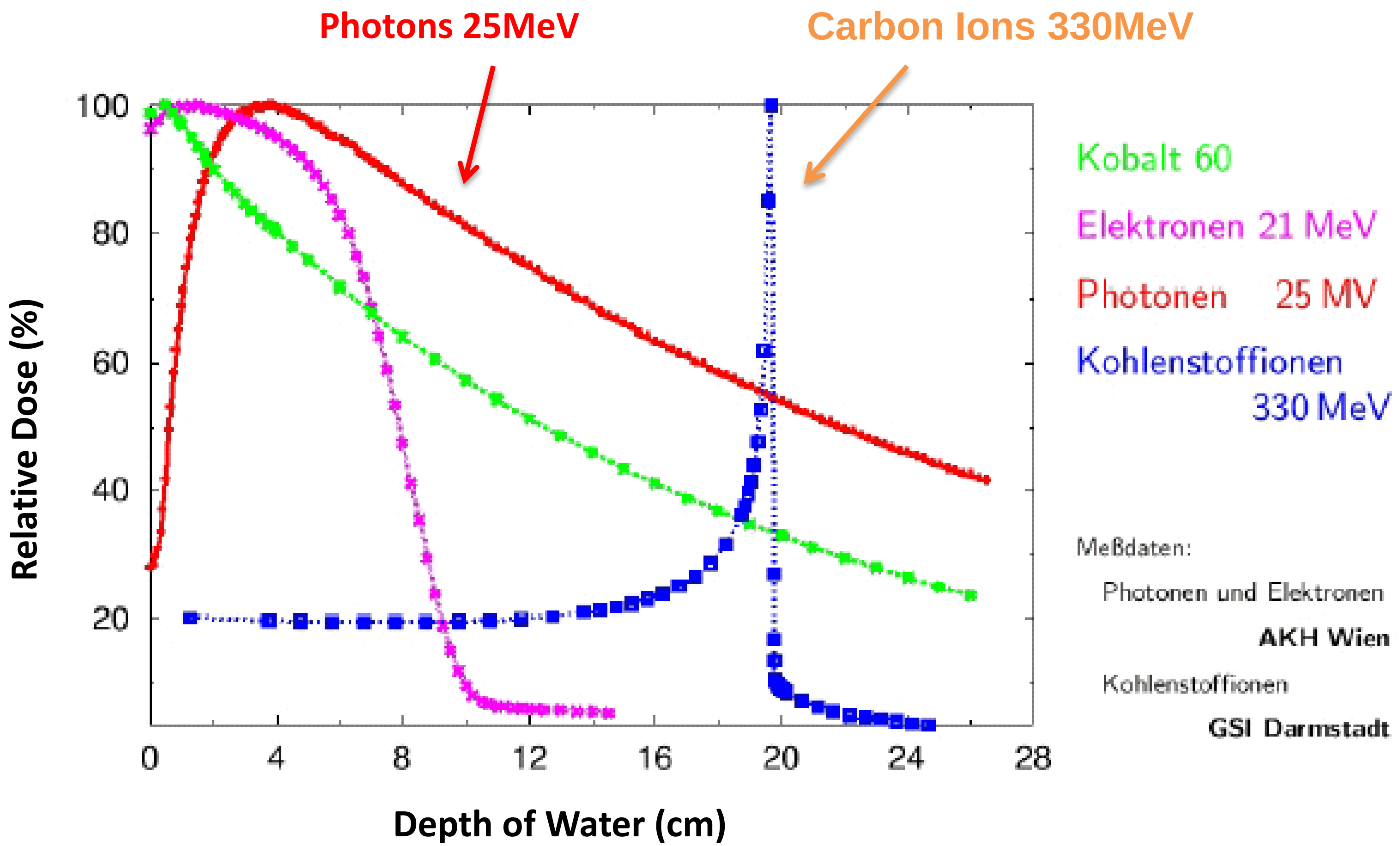
He proved that there are no chambers present.



Range of Particles

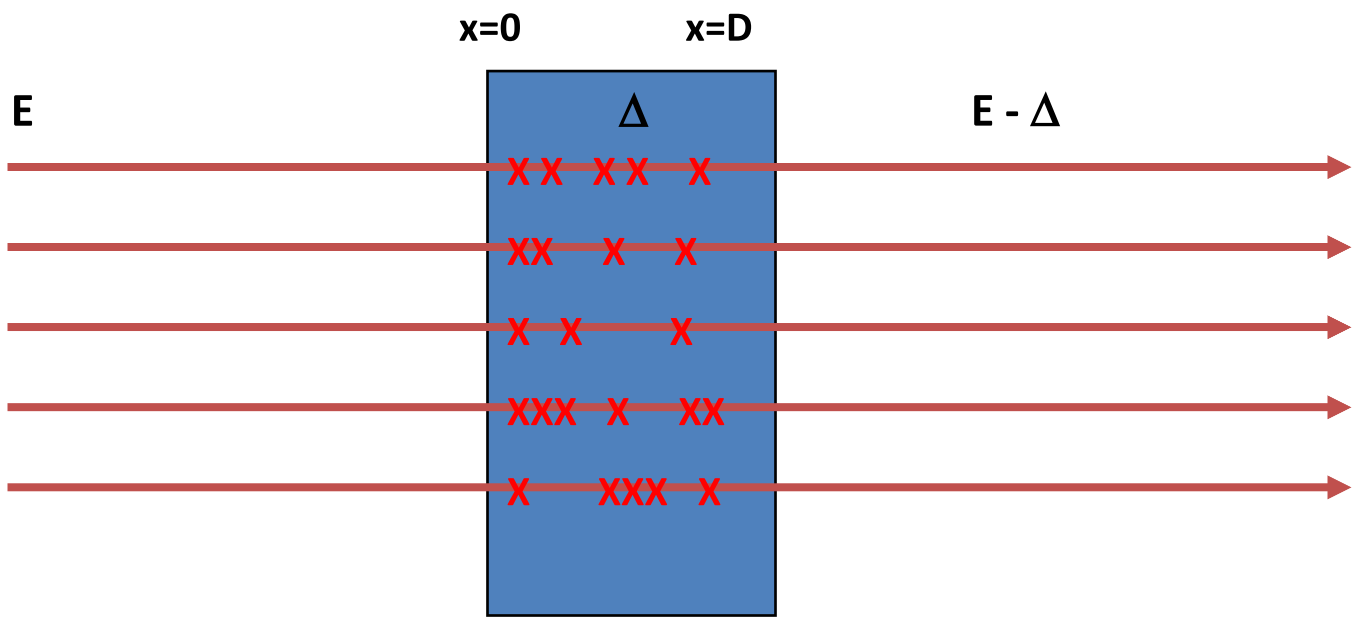
Average Range:

Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



Fluctuation of Energy Loss

Up to now we have calculated the average energy loss. The energy loss is however a statistical process and will therefore fluctuate from event to event.



P(Δ) = ? Probability that a particle loses an energy Δ when traversing a material of thickness D

We have see earlier that the probability of an interaction ocuring between distance x and x+dx is exponentially distributed

$$P(x)dx = \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right)dx \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

Probability for n Interactions in D

We first calculate the probability to find n interactions in D, knowing that the probability to find a distance x between two interactions is P(x)dx = 1/ λ exp(-x/λ) dx with λ = A/ N_Aρ σ

Probability to have no interaction between 0 und D:

$$P(x > D) = \int_D^\infty P(x_1)dx_1 = e^{-\frac{D}{\lambda}}$$

Probability to have one interaction at x_1 and no other interaction:

$$P(x_1, x_2 > D) = \int_D^\infty P(x_1)P(x_2 - x_1)dx_2 = \frac{1}{\lambda}e^{-\frac{D}{\lambda}}$$

Probability to have one interaction independently of x_1:

$$\int_0^D P(x_1, x_2 > D) = \frac{D}{\lambda}e^{-\frac{D}{\lambda}}$$

Probability to have the first interaction at x_1, the second at x_2 .... the n^{th} a
x_n and no other interaction:

$$P(x_1, x_2 ... x_n > D) = \int_D^\infty P(x_1)P(x_2 - x_1)...P(x_n - x_{n-1})dx_n = \frac{1}{\lambda^n}e^{-\frac{D}{\lambda}}$$

Probability for n interactions independently of $x_1, x_2 ... x_n$

$$\int_0^D \int_0^{x_{n-1}} \int_0^{x_{n-1}} ... \int_0^{x_1} P(x_1, x_2 ..., x_n > D)dx_1 ... dx_{n-1} = \frac{1}{n!}\left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}}$$

Probability for n Interactions in D

For an interaction with a mean free path of λ , the probability for n interactions on a distance D is given by

$$P(n) = \frac{1}{n!}\left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}} = \frac{\overline{n}^n}{n!} e^{-\overline{n}} \qquad \overline{n} = \frac{D}{\lambda} \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

→Poisson Distribution !

<u>If the distance between interactions is exponentially distributed with an mean free path of λ→ the number of interactions on a distance D is Poisson distributed with an average of n̄=D/λ.</u>

How do we find the energy loss distribution ?
If f(E) is the probability to lose the energy E' in an interaction, the probability p(E) to lose an energy E over the distance D ?

$$f(E) = \frac{1}{\sigma}\frac{d\sigma}{dE}$$

$$p(E) = P(1)f(E) + P(2)\int_0^E f(E-E')f(E')dE' + P(3)\int_0^E \int_0^{E'} f(E-E'-E'')f(E'')f(E')dE''dE' + \dots$$

$$F(s) = \mathcal{L}\left[f(E)\right] = \int_0^\infty f(E)e^{-sE}dE$$

$$\mathcal{L}\left[p(E)\right] = P(1)F(s) + P(2)F(s)^2 + P(3)F(s)^3 + \dots = \sum_{n=1}^{\infty} P(n)F(s)^n = \sum_{n=1}^{\infty}\frac{\overline{n}^n F^n}{n!}e^{-\overline{n}} = e^{\overline{n}(F(s)-1)} - 1 \approx e^{\overline{n}(F(s)-1)}$$

$$p(E) = \mathcal{L}^{-1}\left[e^{\overline{n}(F(s)-1)}\right] = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} e^{\overline{n}(F(s)-1)+sE}ds$$

Probability for Energy Loss E in D

If n is the average number of interactions in D and F(s) is the Laplace transform for f(E), giving the probability to lose the energy E in a collision, the probability p(E) to lose the energy E in D is given by

$$p(E) = \mathcal{L}^{-1}\left[e^{\overline{n}(F(s)-1)}\right] = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} e^{\overline{n}(F(s)-1)+sE}\,ds$$

Landau used the Rutherford scattering crossection

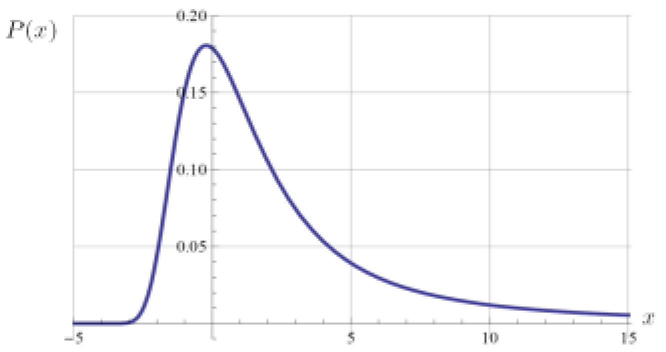
$$\frac{d\sigma}{dE} = Z_1^2 r_e^2 \frac{2\pi m_e c^2}{\beta^2}\frac{1}{E^2} = \frac{k}{E^2} \qquad \epsilon < E < E_{max}$$

The total crossection, using a lower cutoff energy is then

$$\sigma = \int_{\epsilon}^{E_{max}} \frac{d\sigma}{dE}dE = k\left(\frac{1}{\epsilon} - \frac{1}{E_{max}}\right) \approx \frac{k}{\epsilon} \qquad \overline{n} = \frac{N_A \rho Z_2 \sigma D}{A} = \frac{N_A \rho Z_2 k D}{A\epsilon}$$

The probability to lose and Energy E in a collision is then

$$f(E) = \frac{1}{\sigma}\frac{d\sigma}{dE} = \frac{\epsilon}{E^2} \qquad \epsilon < E < E_{max}$$



The Laplace transform of this probability density is

$$F(s) = \mathcal{L}\left[\frac{\epsilon}{E^2}\right] = \int_{\epsilon}^{\infty} \frac{\epsilon\, e^{-sE}}{E^2}dE \approx 1 + s\epsilon(1 - C_\gamma + \ln s\epsilon)$$

Which results in the energy loss distribution

$$p(E) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \exp\left[\overline{n} s\epsilon\left(C_\gamma - 1 + \ln s\epsilon\right) + sE\right] ds$$

$$C_\gamma = 0.577218... = \text{Euler constant}$$

Landau Distribution

The previous expression can be rewritten as

$$P(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{s\ln s + xs}ds = \frac{1}{\pi}\int_0^\infty e^{-t\ln t - xt}dt \qquad x = \frac{E}{\overline{n}\epsilon} + C_\gamma - 1 - \ln\overline{n} \qquad \overline{n} = \frac{N_A\rho Z_2 kD}{A\epsilon}$$

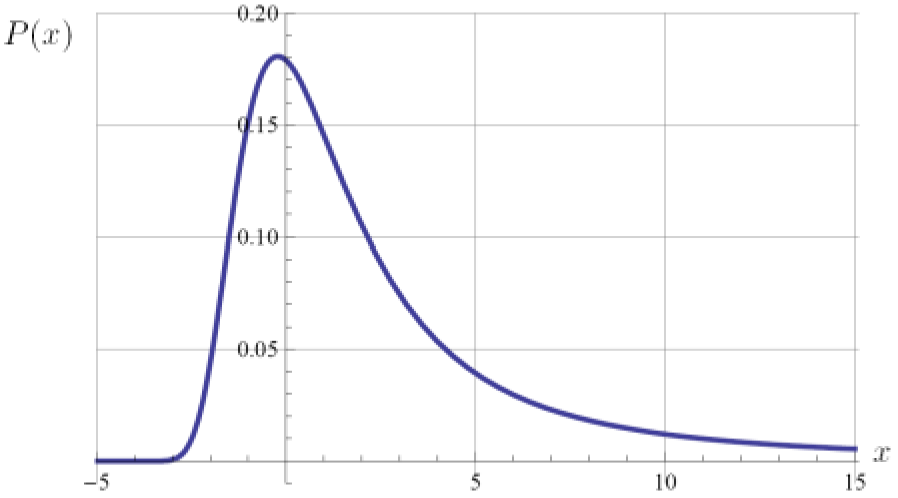
Landau determined the lower cutoff energy by requiring the average energy loss to be equal to the Bethe Bloch theory

$$\frac{dE}{dx} = -\frac{N_A\rho Z_2}{A}\int_\epsilon^{E_{max}} E\frac{d\sigma}{dE}dE = -\frac{N_A\rho Z_2 k}{A}\ln\frac{E_{max}}{\epsilon} = \text{Bethe-Bloch} = -2\frac{N_A\rho Z_2 k}{A}\left(\ln\frac{E_{max}}{\epsilon} - \beta^2\right)$$

which gives

$$\ln\epsilon = \ln\frac{I^2}{E_{max}} + 2\beta^2$$

The Landau Distribution P(x) has a peak at
x_0= -0.2228 ~ $C\gamma$-0.8
and a full width of half maximum of
Δx = 4.02



$$E_{MP} = \overline{n}\epsilon(x_0 + 1 - C_\gamma + \ln\overline{n}) \approx \overline{n}\epsilon(0.2 + \ln\overline{n})$$

$$\Delta E_{FWHM} = 4.02\,\overline{n}\epsilon = \frac{4.02\,N_A\rho Z_2 kD}{A}$$

$$\frac{\Delta E_{FWHM}}{E_{MP}} = \frac{4.02}{0.2 + \ln\overline{n}}$$

Landau Distribution

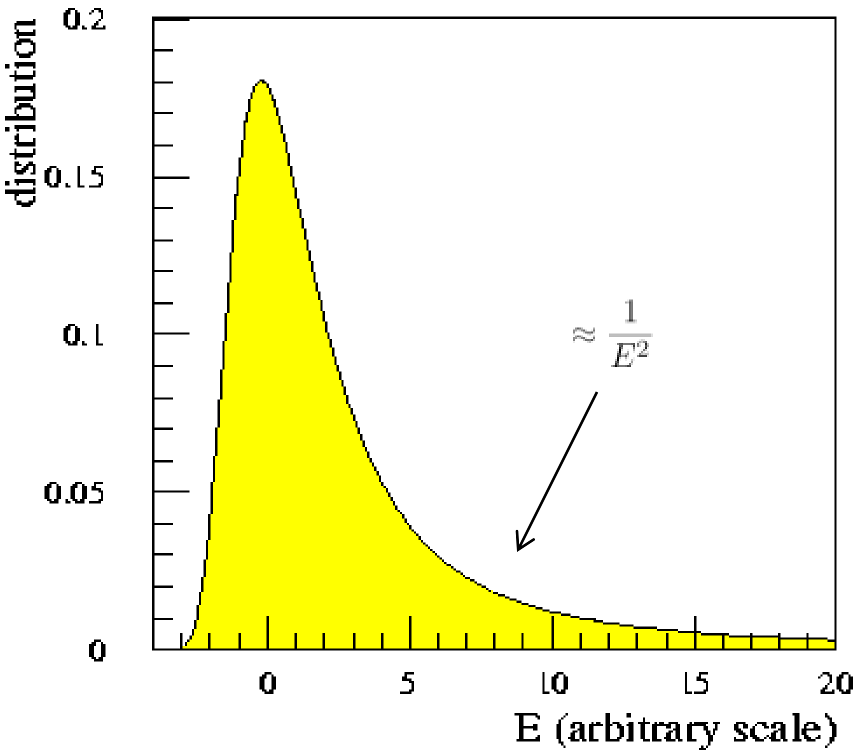
p(E): Probability for energy loss E in matter of thickness D.

Landau distribution is very asymmetric.

Average and most probable energy loss must be distinguished !

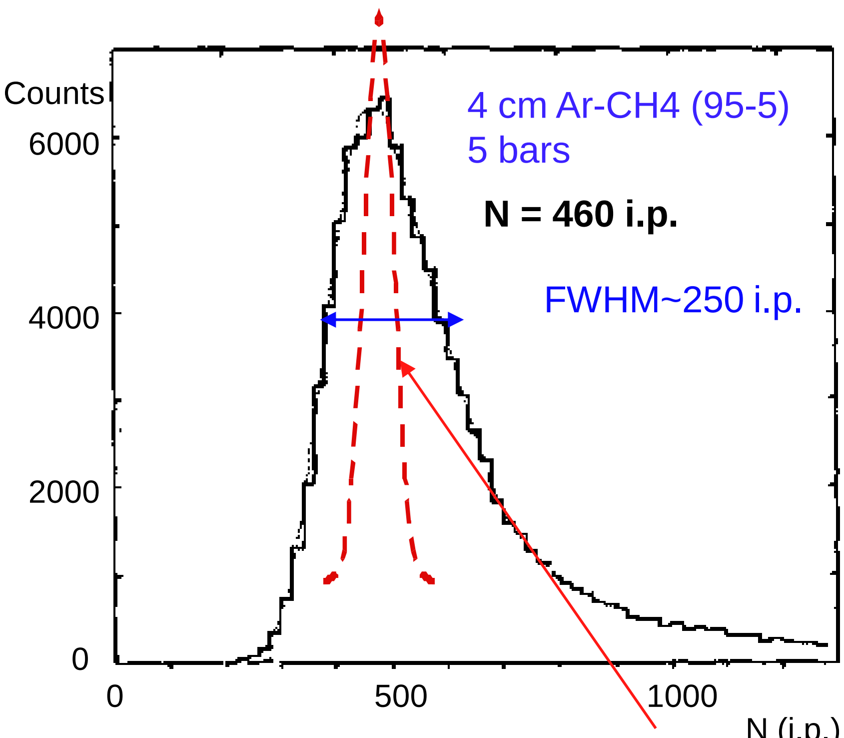
Measured Energy Loss is usually smaller that the real energy loss:

3 GeV Pion: E'_{max} = 450MeV → A 450 MeV Electron usually leaves the detector.



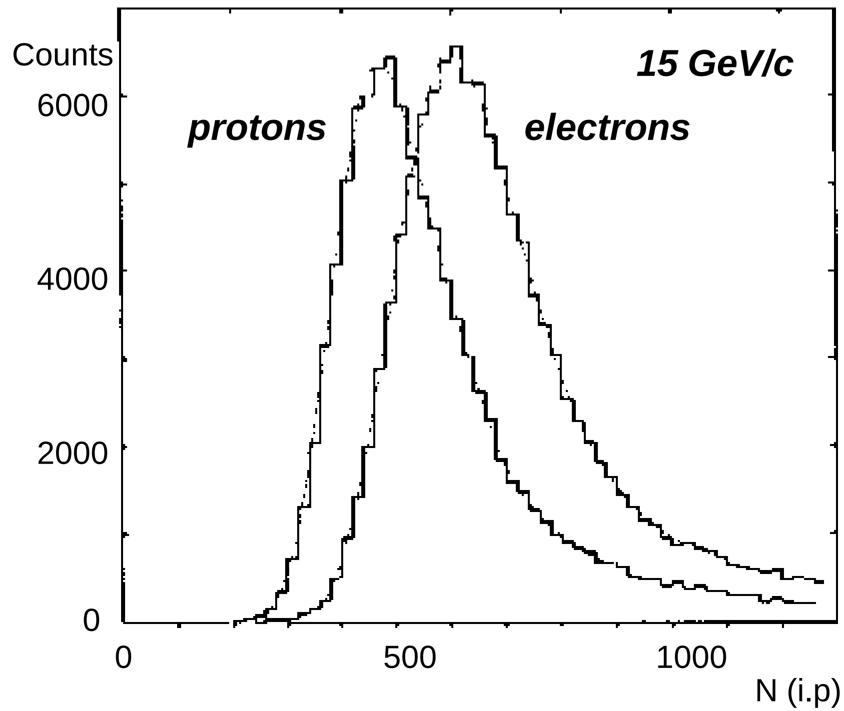
Landau Distribution

LANDAU DISTRIBUTION OF ENERGY LOSS:



For a Gaussian distribution: **σ_N ~ 21 i.p.**
FWHM ~ 50 i.p.

PARTICLE IDENTIFICATION
Requires statistical analysis of hundreds of samples



I. Lehraus et al, Phys. Scripta 23(1981)727

Energy Loss Fluctuation

For calculation of the <u>average</u> energy loss, the integration

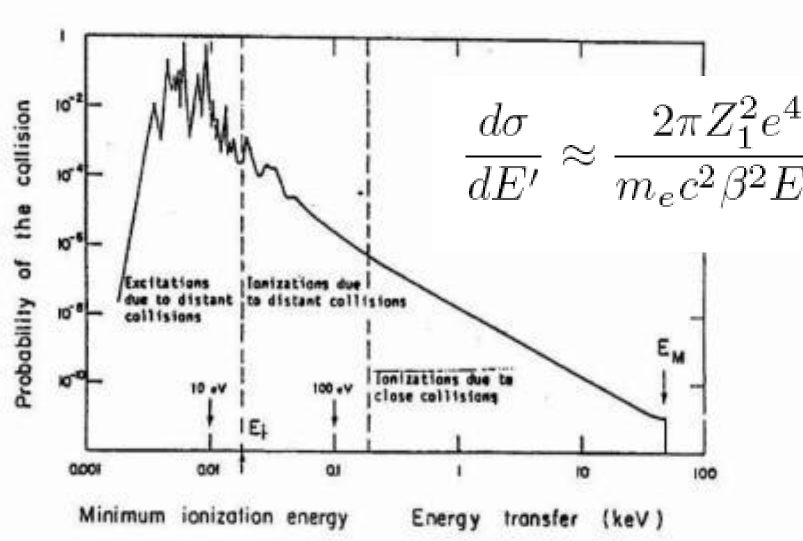
$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2}\left(\frac{M}{m}Z_1\right)^2 \frac{1}{q^3}|\epsilon_n(q)|^2 \qquad \epsilon_n(q) = -\sum_{j=1}^{Z_2}\int e^{i(\vec{q}\vec{r}_j)}\psi_0(\vec{r}_j)\overline{\psi}_n(\vec{r}_j)d^3r_1...d^3r_{Z_2}$$

$$\frac{dE}{dx}|_{E'<\eta} = -\sum_n (E_n - E_0)\int \frac{d\sigma_n(q)}{dq}dq = \sum_n E_n' \int \frac{d\sigma_n(q)}{dq}dq \qquad E_n' < \eta$$

can be performed without the explicit calculation of ψ due to sum rules that must be satisfied.

For calculation of the differential energy loss and therefore the energy loss fluctuations, the explicit knowledge of ψ is necessary, which is however only explicitly possible for atomic Hydrogen

dσ (E,E')/dE'/ σ(E)



$$\frac{d\sigma}{dE'} \approx \frac{2\pi Z_1^2 e^4}{m_e c^2 \beta^2 E'^2}$$

How do we find P(E) for different Atoms and Molecules ?

Fermi Virtual Photon Method (FVP) or Photo Absorbtion Ionization Models (PAI, Allison & Cobb)

Fermi (Z. f. Physik 20. Oct 1924 !):
Über die Theorie des Stosses zwischen Atomen und elektrisch geladenen Teilchen
(On the theory of atomic collisions of charged particles).

Abstract: **The electric field of a charged particle, which is flying by an atom, is harmonically decomposed and compared to light of the corresponding frequency distribution. We assume that the probability of atomic excitation and ionization by the passing particle is equal to atomic excitation and ionization of the equivalent radiation …**

Bethe 1930:

$$\epsilon_n(q) = -\sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0(\vec{r}_j) \overline{\psi}_n(\vec{r}_j) d^3r_1 ... d^3r_{Z_2}$$

$$e^{i\vec{q}\vec{r}_j} \approx 1 + i\vec{q}\vec{r}_j + ... \qquad \epsilon_n(q) = i\vec{q} \int \sum_{j=1}^{Z_2} \vec{r}_j \psi_0(r_i) \overline{\psi}_n(r_i) d^3r_1 ... r^3 r_{Z_2} = i\vec{q}\vec{x}_{0n}$$

"…Thus, for small collision vectors q (i.e. small changes in momentum of the colliding particle), the collision probability is proportional to the square of the coordinate matrix, i.e., proportional to the optical transition probability for the respective excitation of the atom …" (Bethe, 1930)

Knowing the optical photoabsorbtion crossection, if which detailed measurements are available !

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

Allison & Cobb, Ann. Rev. Nucl. Part. Sci. 1980, 253-298:

First they calculate the energy loss of a particle passing a homogenous medium with dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$. The result will therefore contain the density effect, i.e. the reduction of the energy loss due to polarization of the medium. Then they make a model for ϵ_2 by using the atomic photo absorption crossection.

Particle of charge e_0 and velocity v (E.g. Landau and Lifshitz):

$$\mathrm{div} H = 0 \qquad \mathrm{rot} E = -\frac{1}{c}\frac{\partial H}{\partial t} \qquad \mathrm{div} \epsilon E = 4\pi\rho \qquad \mathrm{rot} H = \frac{1}{c}\frac{\partial \epsilon E}{\partial t} + \frac{4\pi}{c} j$$

$$\rho = e_0 \delta^3 (r - \beta c t) \qquad j = \beta c \rho \qquad \epsilon = \epsilon_1 + i\epsilon_2$$

$$\frac{dE}{dx} = -\frac{2e_0^2}{\beta^2 \pi} \int_0^\infty d\omega \int_{\omega/v}^\infty dk \left[\omega k \left(\beta^2 - \frac{\omega^2}{k^2 c^2} \right) \mathrm{Im} \left(\frac{1}{-k^2 c^2 + \epsilon \omega^2} \right) - \frac{\omega}{k c^2} \mathrm{Im} \left(\frac{1}{\epsilon} \right) \right]$$

A plane light wave travelling along x is attenuated in in the medium if the imaginary part of ϵ is different from zero.

$$E(x) \sim e^{ikx} \qquad I(x) \sim E(x)^2 = e^{2ikx} \qquad k = \frac{\omega}{v} = \frac{\omega}{c}\sqrt{\epsilon} \approx \frac{\omega}{c}(1 + i\frac{\epsilon_2}{2}) \qquad \epsilon_1 - 1, \epsilon_2 \ll q \quad I(x) \sim e^{-\frac{\omega}{c}\epsilon_2(\omega) x}$$

A photon interacting with material with an atomic crossection σ has a mean free path of $\lambda = A/N_A \rho \sigma$. The probability of interacting after travelling a distance x is

$$P(x) = \frac{1}{\lambda} e^{-x/\lambda} \sim e^{-N_A \rho \sigma(\omega) x/A} \qquad \rightarrow \qquad \frac{N_A \rho}{A} \sigma(\omega) = \frac{\omega}{c} \epsilon_2(\omega)$$

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

This defines the imaginary part of the dielectric constant in terms of the atomic photoabsorption crossection for small k. For large k and expression is introduced in order to satisfy the Bethe Sum rule:

Small k: $\epsilon_2(\omega) = \frac{N_A \rho}{A} \frac{c}{\omega} \sigma(\omega)$ **Large k:** $\epsilon_2(k,\omega) = C\,\delta(\omega - k^2/2m_e)$ $\int \omega \epsilon_2(k,\omega) d\omega = \frac{2\pi^2 e_0^2 N_A \rho Z_2}{A m_e}$

The real part is uniquely related to the imaginary part by the Kramers-Kronig relations

$$\epsilon_1(\omega) - 1 = \frac{2}{\pi} \mathrm{P} \int_0^\infty \frac{x \epsilon_2(x)}{x^2 - \omega^2} dx$$

We are therefore in a position to integrate the expression over k and have

$$\frac{dE}{dx} = -\int_0^\infty d\omega \frac{e_0^2}{\beta^2 c^2 \pi^2} \left[\frac{N_A \rho}{A} \sigma(\omega) \ln[(1-\beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2]^{-1/2} + \omega \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right) \Theta + \frac{N_A \rho}{A} \sigma(\omega) \ln \left(\frac{2 m_e \beta^2 c^2}{\hbar \omega} \right) + \frac{1}{\omega} \int_0^\omega \frac{\sigma(\omega')}{Z_2} d\omega' \right]$$

$$\Theta = \arg(1 - \epsilon_1 \beta^2 + i \epsilon_2 \beta^2) = \frac{\pi}{2} - \arctan \frac{1 - \epsilon_1 \beta^2}{\epsilon_2 \beta^2}$$

We can then reinterprete this equation in terms of a number of discrete collisions with energy transfer E=hω and find the differential crossection for losing an energy E in a single collision

$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2 \pi} \frac{\sigma(E)}{EZ} \ln[(1-\beta^2\epsilon_1)^2 + \beta^4 \epsilon_2^2]^{-1/2} + \frac{\alpha}{\beta^2 \pi} \frac{A}{N_A \rho Z_2 \hbar c} \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right) \Theta + \frac{\alpha}{\beta^2 \pi} \frac{\sigma(E)}{E Z_2} \ln \left(\frac{2 m_e \beta^2 c^2}{E} \right) + \frac{\alpha}{\beta^2 \pi} \frac{1}{E^2} \int_0^E \frac{\sigma(E)}{Z_2} dE'$$

This is the model included in GEANT and HEED.

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

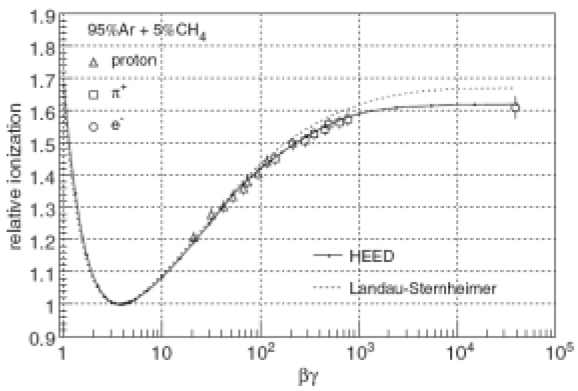


Fig. 7. The simulations of experiment [62] by HEED (truncated means by PAIR) and by the Landau–Sternheimer theory (the most probable ionization) compared with the normalized experimental data.

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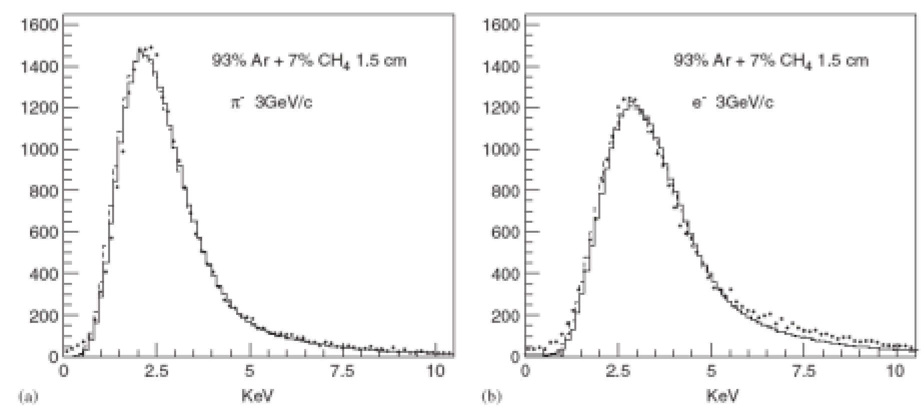
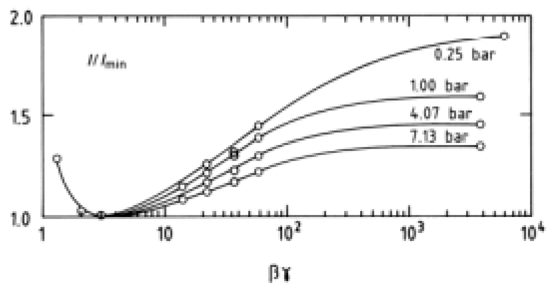


Fig. 3. The experimental (points) and theoretical "ionization loss" distributions expressed in energy units. Histograms drawn by solid lines are obtained by the PAIR model, dashed lines (they are practically coincide with solid ones) are by the PAI model.



Density Effect

The PAI model together with the good knowledge of the Photoabsorption Crossections allows the accurate calculation of primary ionization & the associated fluctuations (Straggling functions)

Cherenkov Radiation

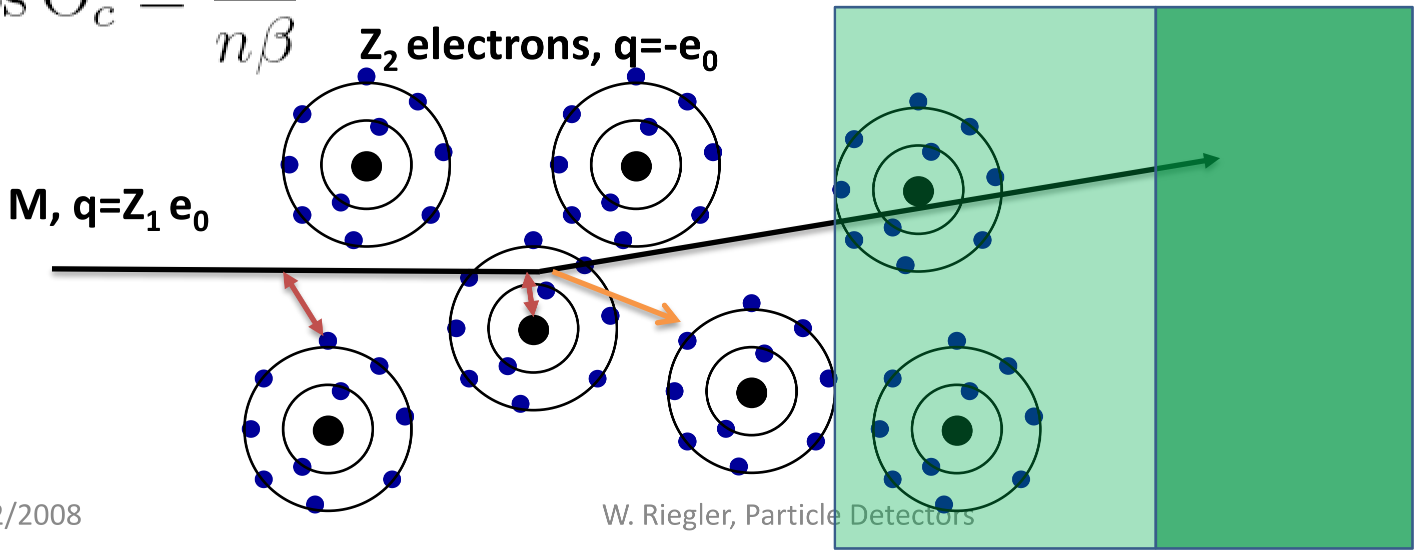
If we set the imaginary part of the dielectric permittivity to zero, the energy loss in the PAI expression doesn't become zero – there is a term left, which describes the Cherenkov Radiation → This is a classic effect described by Maxwell's equations

$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2\pi}\frac{A}{N_A\rho Z_2\hbar c}\left(\beta^2 - \frac{1}{\epsilon_1}\right) \quad \rightarrow \quad \frac{N_A\rho Z_2}{A}\frac{d\sigma}{d\omega}\frac{d\omega}{dE} = \frac{\alpha}{c}\left(1 - \frac{1}{\beta^2 n^2}\right) \qquad n = \sqrt{\epsilon_1} \qquad E = \hbar\omega$$

$$\frac{dE}{dx d\omega}\frac{1}{\hbar} = \frac{\alpha}{c}\left(1 - \frac{1}{\beta^2 n^2}\right) \quad \rightarrow \quad \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2}\right) \qquad \omega = \frac{2\pi c}{\lambda}$$

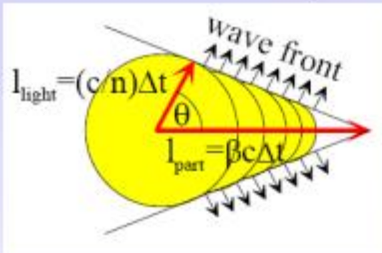
N is the number of Cherenkov Photons emitted per cm of material. The expression is in addition proportional to Z_1^2 of the incoming particle.
This radiation is emitted if the velocity of the particle is larger than the velocity of light in the medium v=c/n (shock wave). It is emitted at the characteristic angle θ_c .

$$\cos\Theta_c = \frac{1}{n\beta}$$



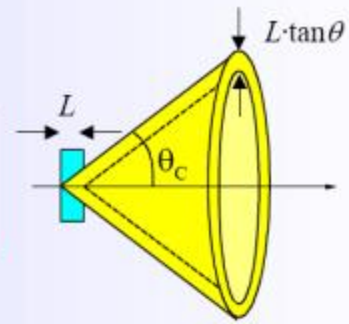
Cherenkov Radiation

with velocity $\beta \geq \beta_{thr} = \frac{1}{n}$ n: refractive index



$$\cos\theta_C = \frac{1}{n\beta}$$

with $n = n(\lambda) \geq 1$



- $\beta_{thr} = \frac{1}{n} \rightarrow \theta_C \approx 0$ Cherenkov threshold
- $\theta_{max} = \arccos\frac{1}{n}$ 'saturated' angle (β=1)

If the velocity of a charged particle is larger than the velocity of light in the medium $v > \frac{c}{n}$ (n... Refractive Index of Material) it emits 'Cerenkov' radiation at a characteristic angle of $\cos\theta_c = \frac{1}{n\beta}$ $(\beta = \frac{v}{c})$

$$\frac{dN}{dx} \approx 2\pi\alpha Z_1^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{\lambda_2 - \lambda_1}{\lambda_2 \cdot \lambda_1}$$

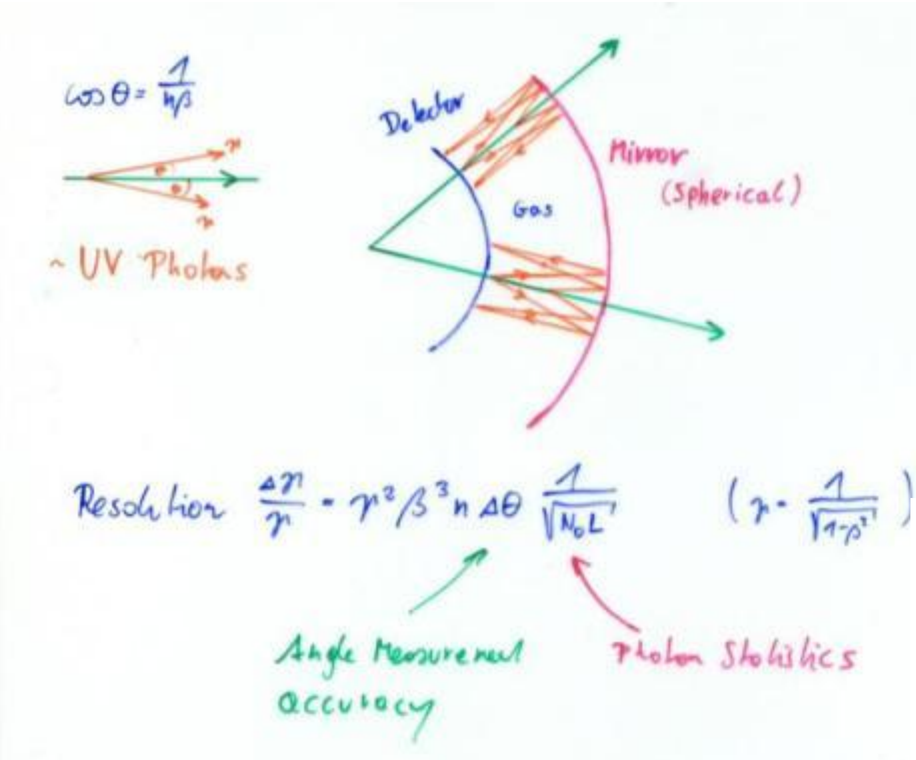
- Number of emitted Photons/length with λ between λ_1 and λ_2

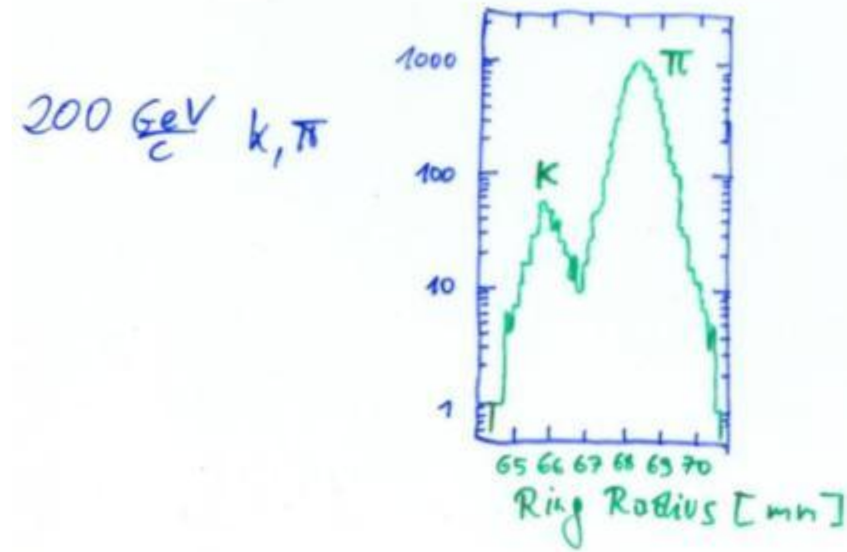
With $\lambda_1 = 400nm$ $\lambda_2 = 700nm$

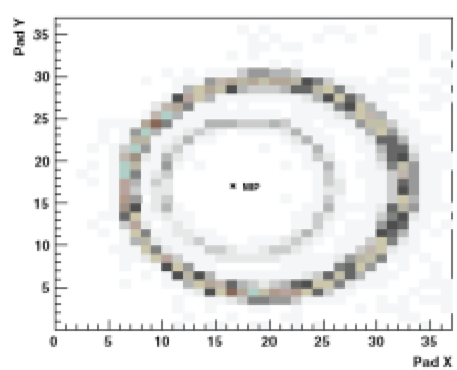
$$\frac{dN}{dx} = 490 \left(1 - \frac{1}{\beta^2 n^2}\right) \left[\frac{1}{cm}\right]$$

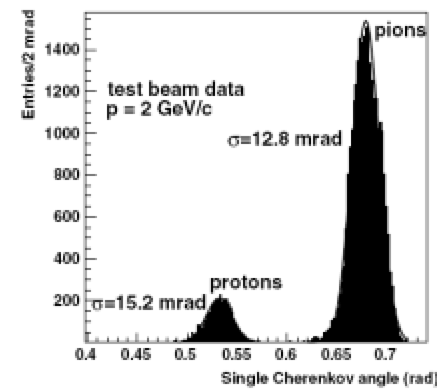
Material	n-1	β threshold	γ threshold
solid Sodium	3.22	0.24	1.029
lead glass	0.67	0.60	1.25
water	0.33	0.75	1.52
silica aerogel	0.025-0.075	0.93-0.976	2.7 - 4.6
air	$2.93 \cdot 10^{-4}$	0.9997	41.2
He	$3.3 \cdot 10^{-5}$	0.99997	123

Ring Imaging Cherenkov Detector









medium	n	θ_{max} (deg.)	N_{ph} (eV^{-1} cm^{-1})
air*	1.000283	1.36	0.208
isobutane*	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

There are only ‘a few’ photons per event →one needs highly sensitive photon detectors to measure the rings !

LHCb RICH

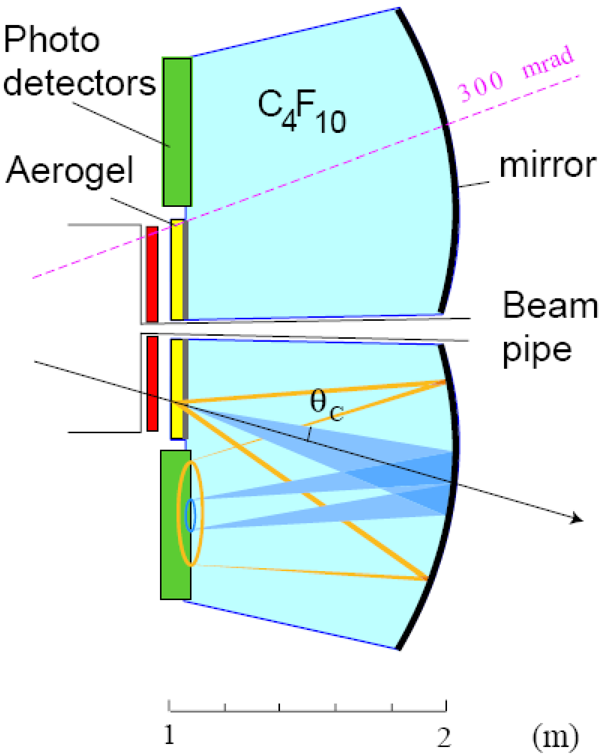
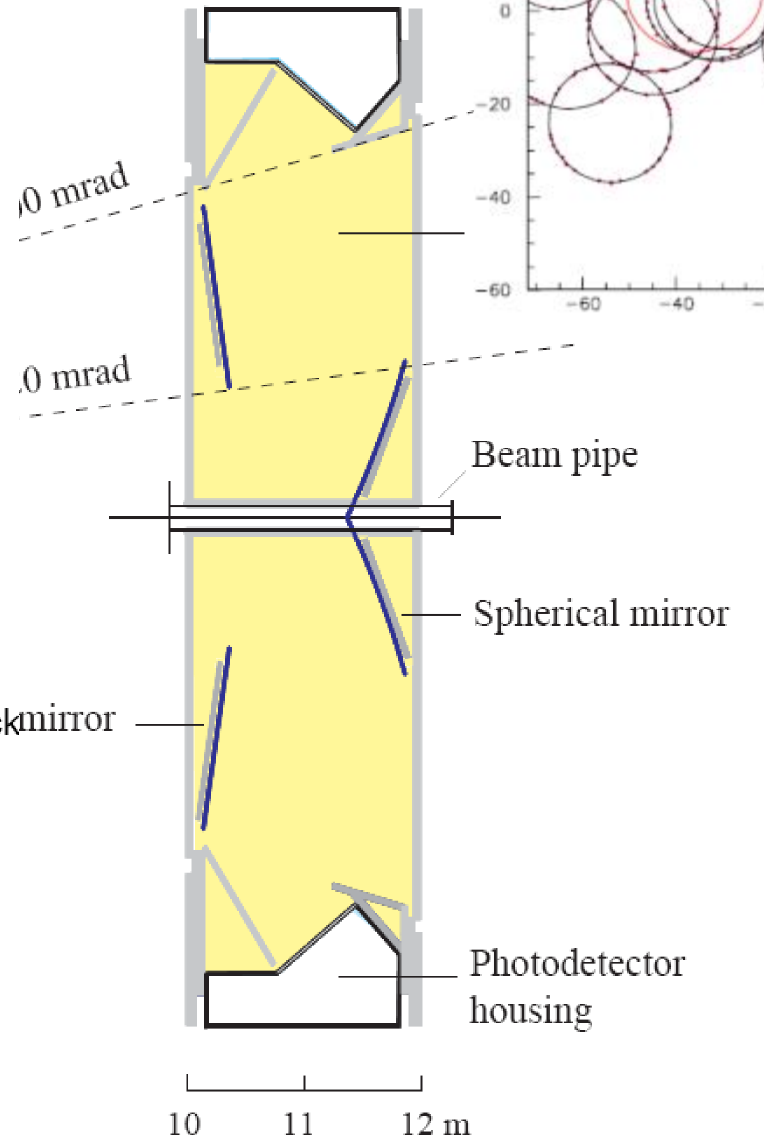
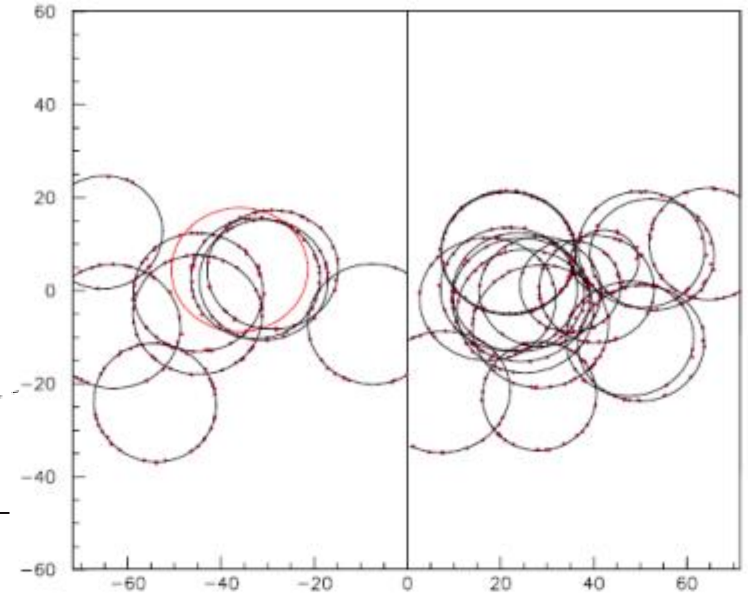


Photo detectors
C4F10
300 mrad
Aerogel
mirror
Beam pipe
θc
Track
1
2
(m)


Beam pipe
Spherical mirror
mirror
Photodetector housing
10
11
12 m


60
40
20
0
−20
−40
−60
−60
−40
−20
0
20
40
60

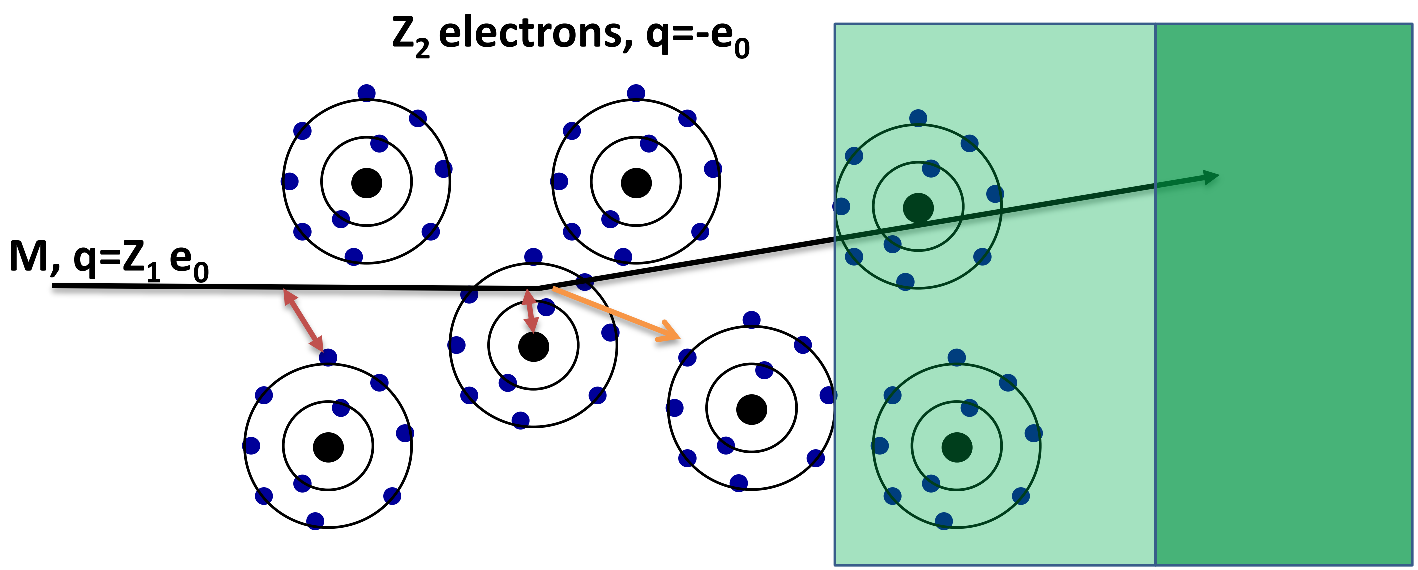
Bremsstrahlung

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated → Bremsstrahlung.

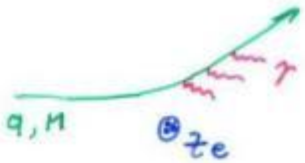
From Bethe's theory we have seen that the elastic scattering off the Nucleus is given by

$$\epsilon_0(q) = Z_2 - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0^2(\vec{r}_j) d^3r_1 ... d^3r_{Z_2} = Z_2 - F \qquad \frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Z_1(Z_2-F)e_0^2}{2pv}\right)^2 \frac{1}{\sin^4\theta/2}$$

Where F(q) describes the partial shielding of the nucleus by the electrons. Effective values for F are used in the following expressions.



Bremsstrahlung, Classical



$q = Z_1 \cdot e$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 Z_1 Z_2 e^2}{4\pi\varepsilon_0 \, p \cdot v}\right)^2 \frac{1}{\left(2 \sin \frac{\theta}{2}\right)^4} \qquad p = Mv\gamma$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer $Q^2 = 2p^2(1-\cos\theta)$

$$\frac{d\sigma}{dQ} = 8\pi \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \beta c}\right)^2 \cdot \frac{1}{Q^3}$$

$Q = |\vec{p} - \vec{p}\,'|$

→ From Maxwell's Eq (Jackson)

$$\lim_{\omega \to 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\varepsilon_0} Q^2$$, Radiated Energy between $\omega, \omega + d\omega$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \int_0^{\omega_{max}} d\omega \int_{Q_{min}}^{Q_{max}} dQ \, \frac{dI}{d\omega} \cdot \frac{d\sigma}{dQ}, \quad \omega_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \frac{16}{3} \alpha \cdot Z^2 \cdot \left(\frac{Z_1^2 e^2}{4\pi\varepsilon_0 M c^2}\right)^2 \cdot E \cdot \ln \frac{Q_{max}}{Q_{min}}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \sim \frac{1}{137}$$

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of Charge Ze.

Because of the acceleration the particle radiated EM waves → energy loss.

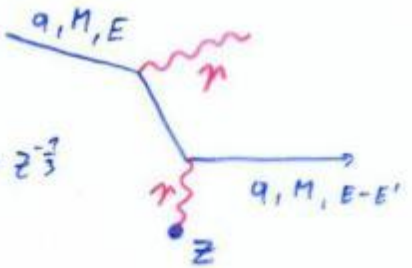
Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

→ dE/dx

Bremsstrahlung, QM

2.6 Bremsstrahlung QM



$q = Z_1 e, \quad E + Mc^2 \gg 137\, Mc^2 Z^{-\frac{1}{3}}$

→ highly Relativistic:

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 \frac{1}{E'} F(E,E')$$

$$F(E,E') = \left[1 + \left(1 - \frac{E'}{E+Mc^2}\right)^2 - \frac{2}{3}\left(1 - \frac{E'}{E+Mc^2}\right)\right] \ln 183 Z^{-\frac{1}{3}} + \frac{1}{9}\left(1 - \frac{E'}{E+Mc^2}\right)$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A}\int_0^E E' \frac{d\sigma}{dE'} dE' \approx 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 E \left[\ln 183 Z^{-\frac{1}{3}} + \frac{1}{18}\right]$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A} 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 E \ln\left(183 Z^{-\frac{1}{3}}\right)$$

$$E(x) = E_0 e^{-\frac{x}{X_0}} \qquad X_0 = \frac{A}{4\alpha N_A \rho Z^2 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 \ln 183 Z^{-\frac{1}{3}}}$$

X_0 ... Radiation length

Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional zu ρ of the particle.

Proportional $1/M^2$ of the incoming particle.

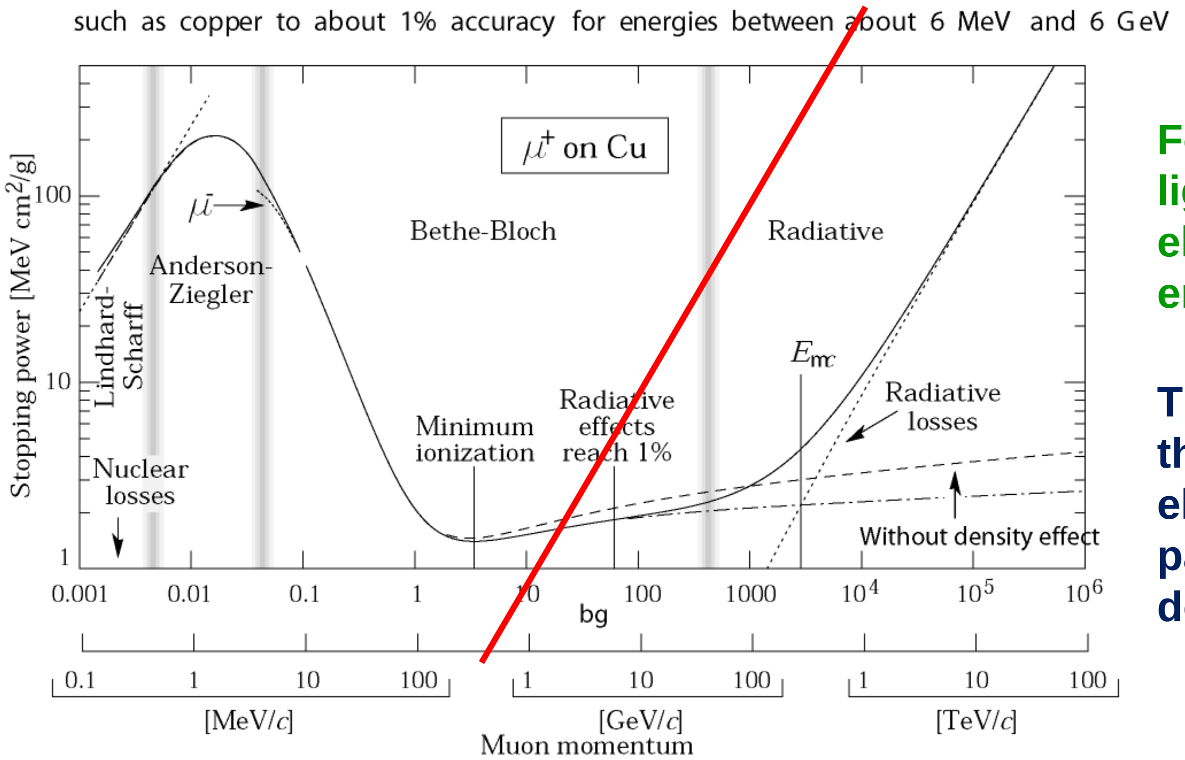
Proportional to the Energy of the Incoming particle →

$E(x)=Exp(-x/X_0)$ – 'Radiation Length'

$X_0 \propto M^2 A / (\rho\, Z_1^4\, Z^2)$

X_0: Distance where the Energy E_0 of the incoming particle decreases $E_0 Exp(-1)=0.37 E_0$.

Critical Energy



Electron Momentum 5 50 500 MeV/c

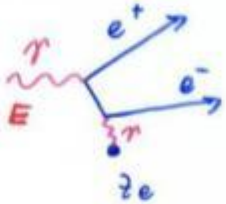
For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

The EM Bremsstrahlung is therefore only relevant for electrons at energies of past and present detectors.

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

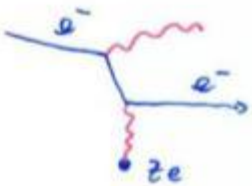
Myon in Copper: p ≈ 400GeV
Electron in Copper: p ≈ 20MeV

Pair Production, QM



$\gamma + \text{Nucl.} \rightarrow e^+ + e^- + \text{Nucl}$

The Diagram is very similar to Bremsstrahlung



$e^- + \text{Nucl} \rightarrow \gamma + e^- + \text{Nucl}$

'Crossing Symmetry: bring particle to the other side and make it the antiparticle' → 'some' correction ...

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 r_e^2 \frac{1}{E} \cdot G(E,E') \qquad E \gg 137 m_e c^2 Z^{-\frac{1}{3}}$$

$$G(E,E') = \left[\left(\left(\frac{E'+m_ec^2}{E}\right)^2 + \left(1-\frac{E'+m_ec^2}{E}\right)^2 + \frac{2}{3}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\right)\ln Z^{-\frac{1}{3}} - \frac{1}{9}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\right]$$

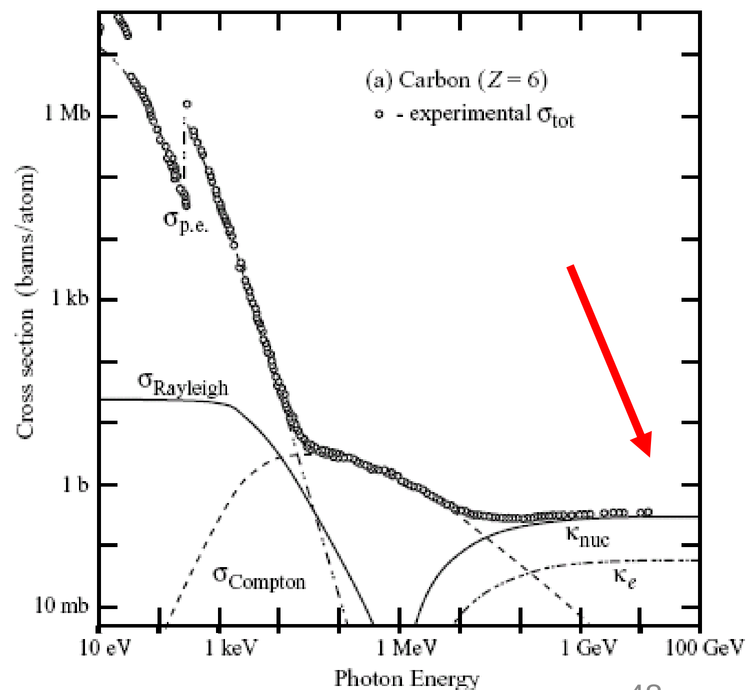
$$\sigma = \int_0^{E-2m_ec^2} \frac{d\sigma}{dE'}dE' = 4\alpha Z^2 r_e^2 \cdot \frac{7}{9}\ln 183 Z^{-\frac{1}{3}}$$

$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \qquad \lambda = \frac{A}{\rho N_A \sigma} = \frac{9}{7}X_0$$

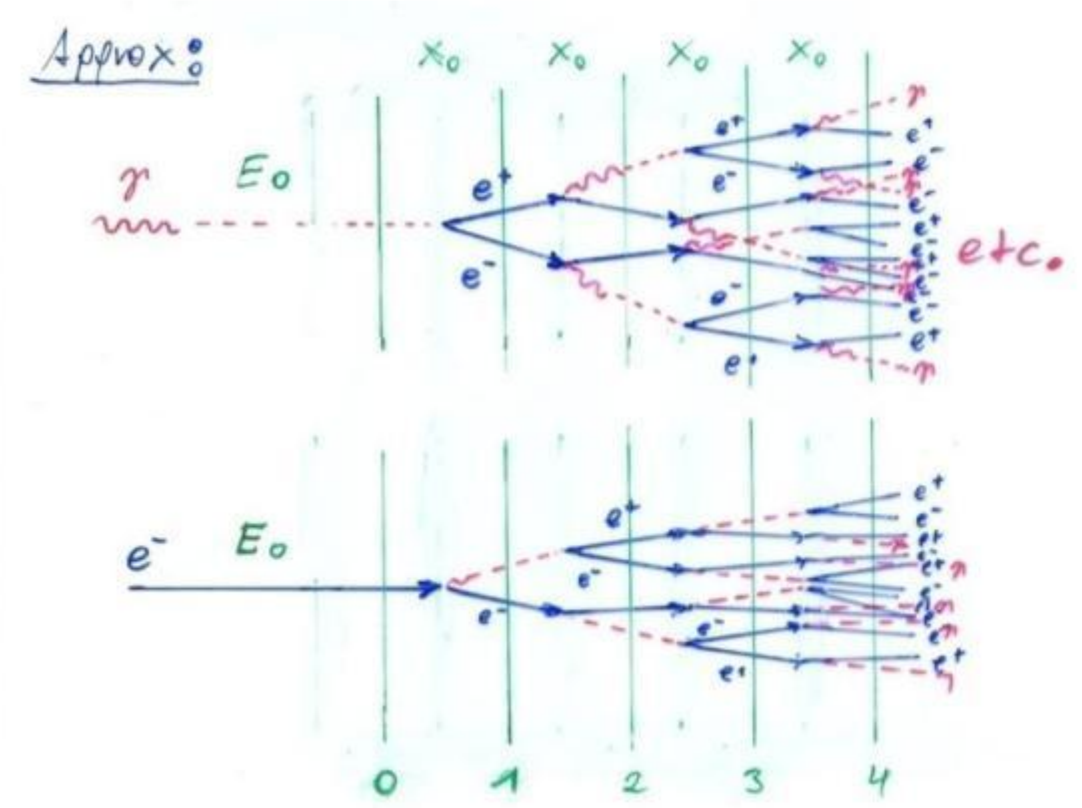
↳ Probability that Photon converts to e^+e^- after a distance x.

For $E\gamma \gg m_ec^2 = 0.5\text{MeV}$: $\lambda = 9/7X_0$

Average distance a high energy photon has to travel before it converts into an $e^+ e^-$ pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing it's energy from E_0 to E_0*Exp(-1) by photon radiation.



Bremsstrahlung + Pair Production → EM Shower



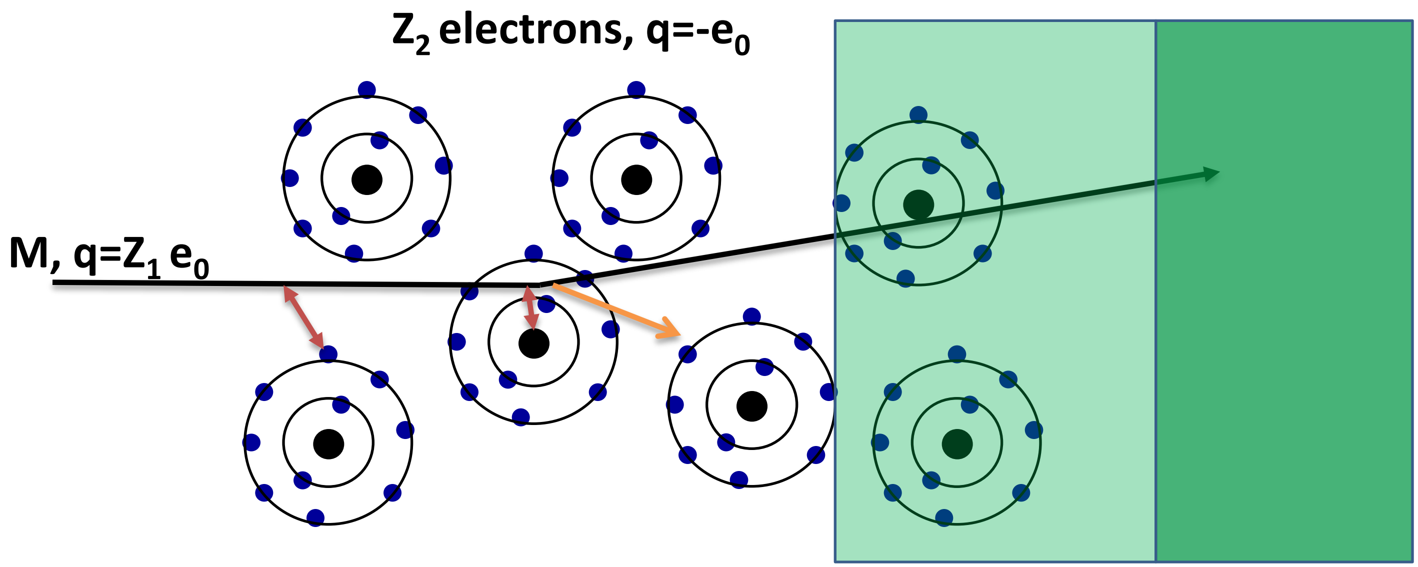
Electromagnetic Shower → EM Calorimeter

Multiple Scattering

The 'Rutherford scattering' of the incoming particle on the nuclei, that is also the reason for Bremsstrahlung, results in multiple small angle scattering scattering of the particles when traversing material.

The statistical analysis of the small angle scattering together with inclusion of the shielding effects by the electrons results in simple expressions for the multiple scattering angles of particles.

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1(Z_2 - F)e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$



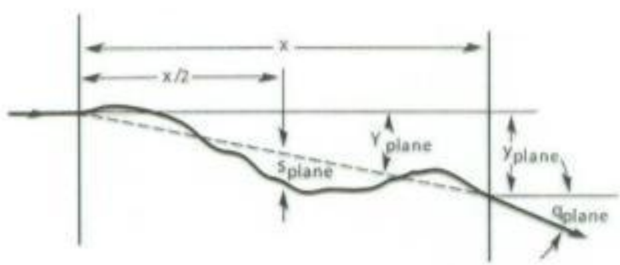
Multiple Scattering

Statistical (quite complex) analysis of multiple collisions gives:

Probability that a particle is defected by an angle θ after travelling a distance x in the material is given by a Gaussian distribution with sigma of:

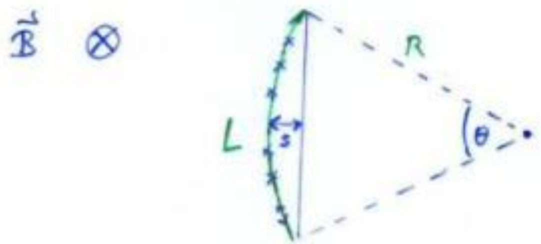
$$\Theta_0 = \frac{0.0136}{\beta c p[\mathrm{GeV/c}]} Z_1 \sqrt{\frac{x}{X_0}}$$

X_0 ... Radiation length of the materials
Z_1 ... Charge of the particle
p ... Momentum of the particle



Multiple Scattering

Magnetic Spectrometer: A charged particle describes a circle in a magnetic field:



$$p = q \cdot R \cdot B$$

$$p\left[\frac{GeV}{c}\right] = 0.3\, R[m]\, B[T]$$

$$L = R \cdot \theta$$

$$s = R\left(1 - \cos\frac{\theta}{2}\right) \sim R\frac{\theta^2}{8} = \frac{L^2}{8R} \rightarrow R = \frac{L^2}{8s}$$

$$\Delta p = 0.3\, B\, \Delta R = 0.3\, B\, \frac{L^2}{8s^2}\, \Delta s$$

$$\Delta s = \frac{\sigma_x}{\sqrt{N}}$$ σ_x ... point resolution, N ... Measurement Points

$$\frac{\Delta p}{p} = \frac{\Delta s}{s} = \frac{\sigma_x[m]}{\sqrt{N}} \cdot \frac{3.3 \cdot 8\, p\left[\frac{GeV}{c}\right]}{B[T] \cdot L^2[m^2]}$$

E.g.: $p = 10\,\frac{GeV}{c}$, $B = 1T$, $L = 1m$, $\sigma = 200\mu m$, $N = 25$

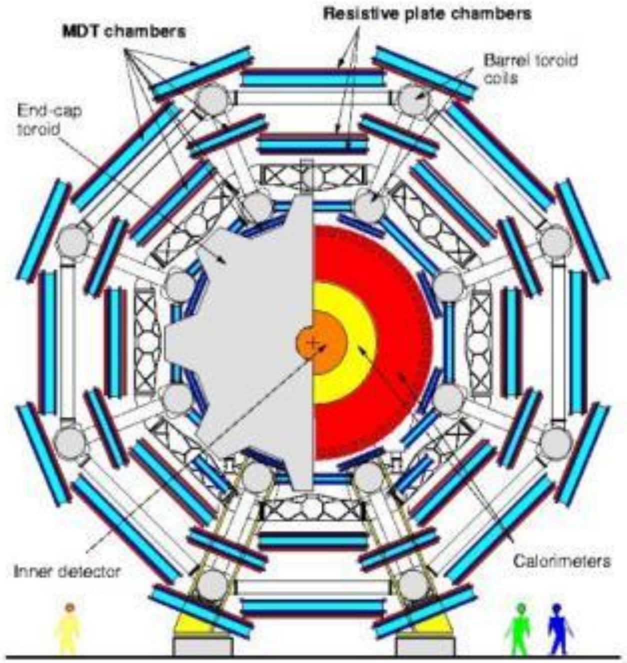
$$\frac{\Delta p}{p} = 0.01 \rightarrow 1\%$$

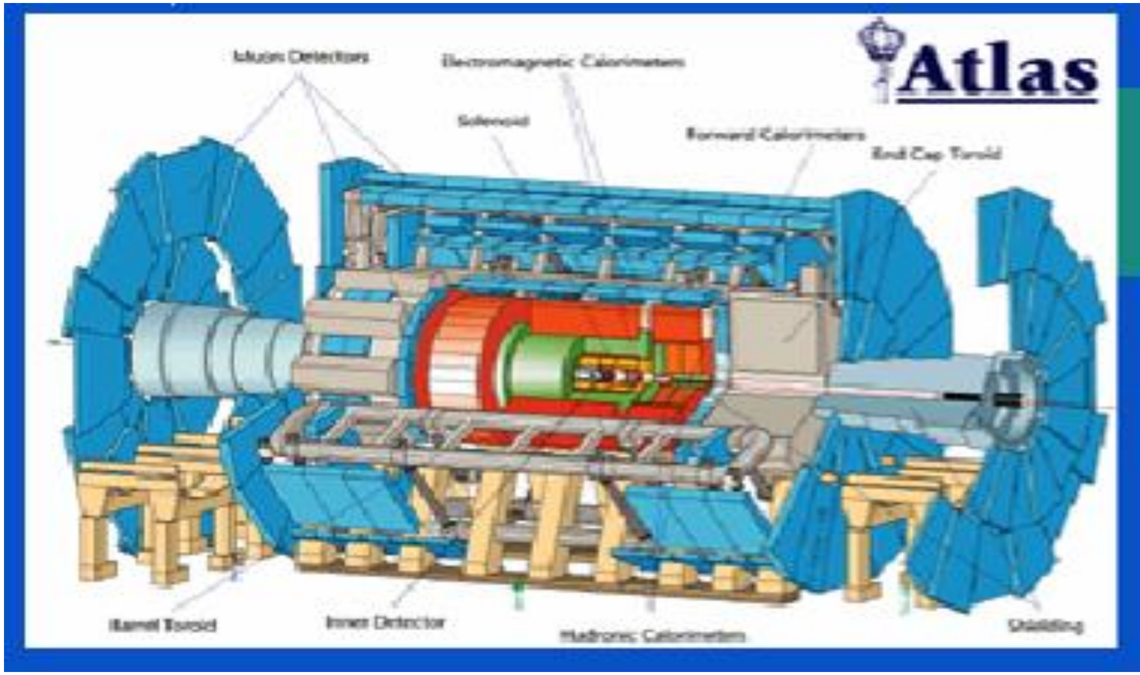
Limit → Multiple Scattering

Multiple Scattering

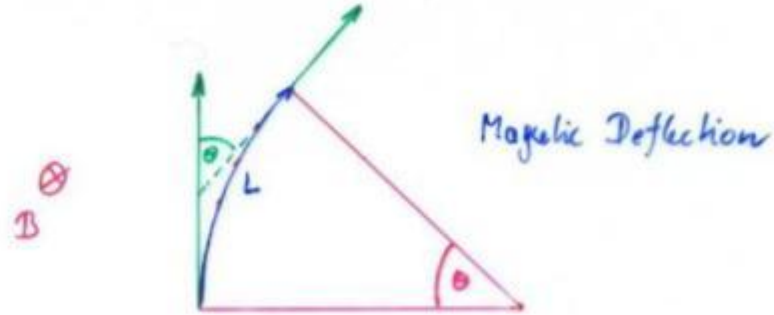
ATLAS Muon Spectrometer:
N=3, sig=50um, P=1TeV,
L=5m, B=0.4T

$\Delta p/p$ ~ 8% for the most energetic muons at LHC





Multiple Scattering



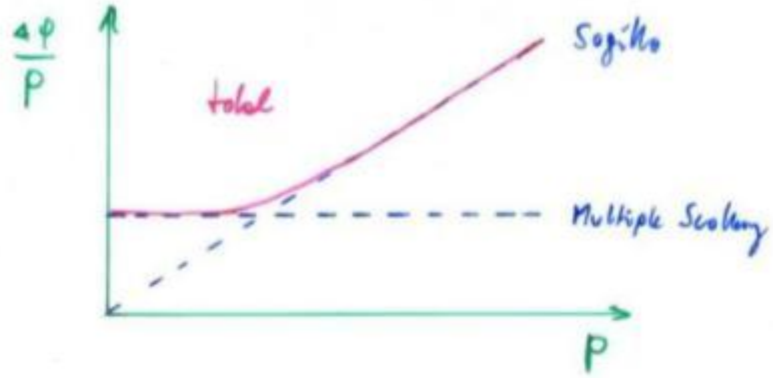
$$p\left[\frac{GeV}{c}\right] = 0.3\, R[m]\, B[T]$$

$$\theta = \frac{L}{R} = \frac{L}{p} \cdot 0.3\, B$$

$$\frac{\Delta p}{p} = \frac{\Delta \theta}{\theta} = \frac{\theta_0}{\theta} \sim \frac{0.05}{\beta B[T] L[m]} \cdot \sqrt{\frac{L}{X_0}}$$

→ Independent of p

$$\left.\frac{\Delta p}{p}\right|_{tot} = \sqrt{\left(\left.\frac{\Delta p}{p}\right|_{sag}\right)^2 + \left(\left.\frac{\Delta p}{p}\right|_{ms}\right)^2}$$



Transition Radiation

Radiation (~ keV) emitted by ultra-relativistic Particles when they traverse the boarder of 2 materials of different Dielectric Permittivity ($\varepsilon_1, \varepsilon_2$)

Classical Picture

$q = Z_1 e$

$$I = \frac{1}{3} \alpha Z_1^2 (\hbar\omega_p) \gamma$$ ... Radiated Energy per Transition

$\hbar\omega_p$ .... plasma Frequency of the Medium
.... ~20 eV for Styrene

About half the Energy is radiated between

$$0.1\,\hbar\omega_p\gamma < \hbar\omega < \hbar\omega_p\gamma$$

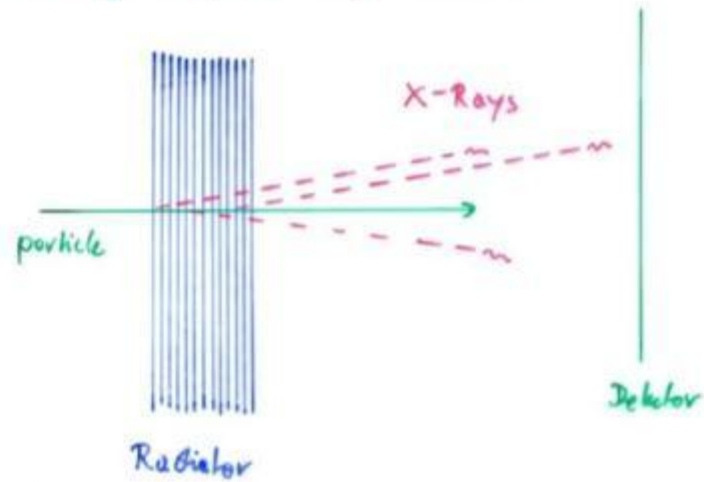
E.g. $\gamma = 1000$ 2-20 keV X-Rays

$$N_\gamma \sim \frac{2}{3}\alpha Z_1^2 \sim 5\cdot 10^{-3}\cdot Z_1^2$$

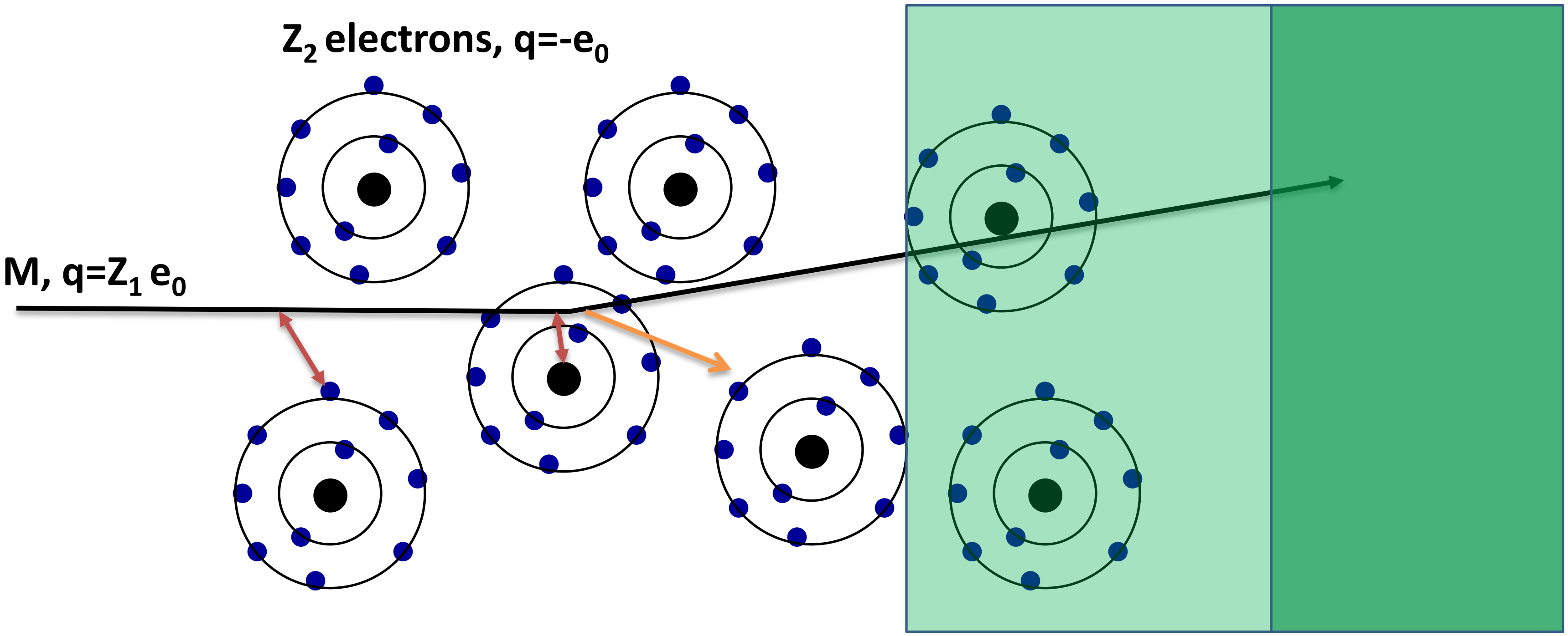
γ - Dependence from hardening rather than N_γ

Emission Angle ~ $\frac{1}{\gamma}$

The Number of Photons can be increased by placing many foils of Material.



Electromagnetic Interaction of Particles with Matter



Now that we know all the Interactions we can talk about Detectors !

Interaction with the atomic electrons. The incoming particle loses energy and the atoms are <u>excited</u> or <u>ionized.</u>

Interaction with the atomic nucleus. The particle is deflected (scattered) causing <u>multiple scattering</u> of the particle in the material. During this scattering a <u>Bremsstrahlung</u> photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as <u>Cherenkov Radiation</u>. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called <u>Transition radiation</u>.

Now that we know all the Interactions we can talk about Detectors !