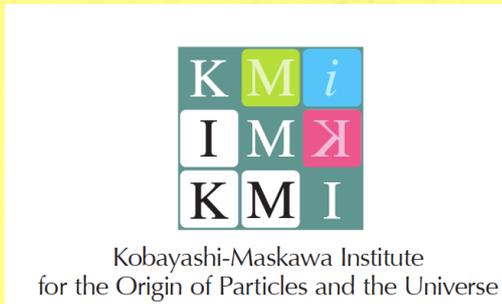


Discovering Walking Technicolor at LHC and on the Lattice



Koichi Yamawaki
KMI, Nagoya

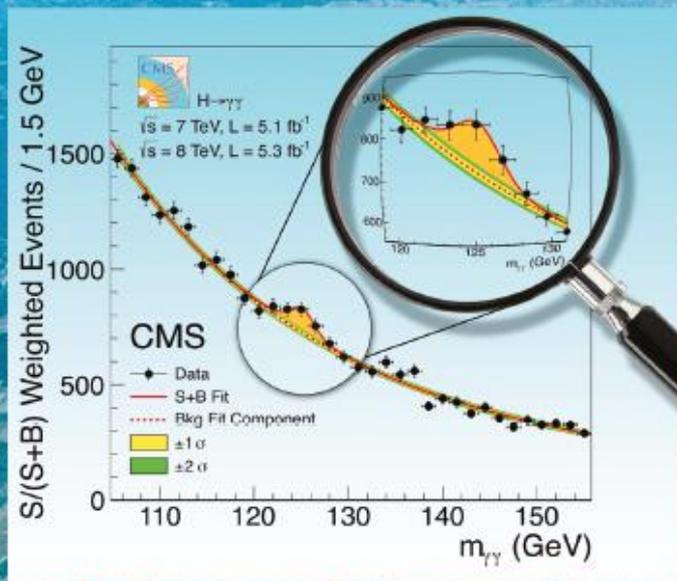
@Higgs and Beyond

June 7, 2013 Tohoku

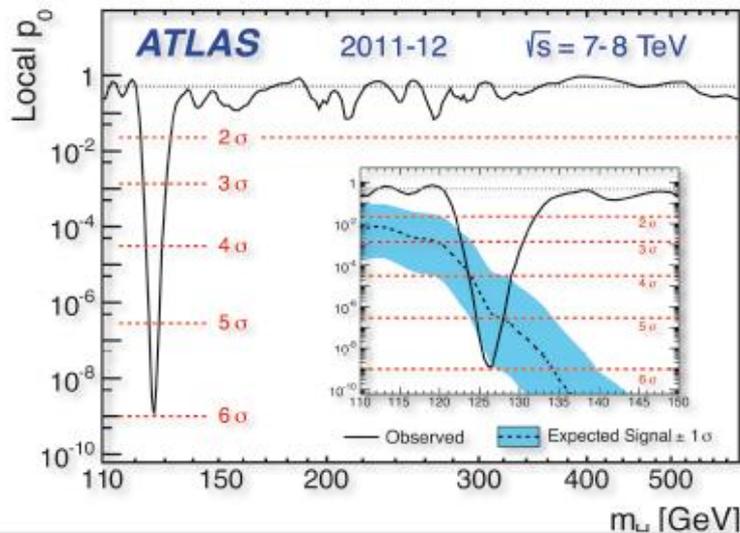
PHYSICS LETTERS B

Available online at www.sciencedirect.com

SciVerse ScienceDirect



Discovery of
125 GeV Higgs



Standard Model is incomplete

- No Dark matter candidates
- Baryogenesis: KM CP violation not enough,
No 1st order phase transition
- Strong CP Problem: neutron EDM
- ...

- Naturalness Problem \longleftrightarrow BSM on TeV

Technicolor (QCD — like Theory)

hierarchy & tachyon :

$$|\delta M_H^2| \sim \Lambda^2 \sim (10^{19} \text{ GeV})^2 \xrightarrow{\text{composite } \pi} m_{W,Z}$$
$$M_H^2 + \delta M_H^2 = -\mathcal{O}((10^2 \text{ GeV})^2)$$

TC was killed 3 times

- FCNC \implies Walking TC
 $m_{q,l} \ll m_{q,l}^{(\text{exp})}$
 $\gamma_m \simeq 1$
- S,T,U parameters \implies (Holographic) Walking TC
 $S/(N_{\text{TC}}N_D) \sim S_{\text{QCD}} \sim 0.3$
 $S^{(\text{exp})} < 0.1$
[or ETC effects]
- 125 GeV Higgs \implies Walking TC
 $125 \text{ GeV} \ll \Lambda_{\text{TC}} = \mathcal{O}(\text{TeV})$
scale inv.

~~Technicolor = Higgsless Model~~

S. Weinberg (1976)
L. Susskind (1979)

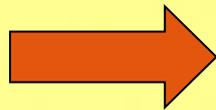
~~(No light scalar)~~

Walking Technicolor

KY-Bando-Matsumoto (1986)

= Composite Higgs Model

Approx. Scale Symmetry



Techni-dilaton



125 GeV Composite Higgs

Scale-Invariant Hypercolor Model and a Dilaton

Koichi Yamawaki, Masako Bando,^(a) and Ken-iti Matumoto^(b)

Department of Physics, Nagoya University, Nagoya 464, Japan

(Received 24 December 1985)

We propose a scale-invariant hypercolor model with a nontrivial ultraviolet fixed point having large anomalous dimension, which resolves the notorious flavor-changing neutral-current problem in hypercolor models, and at the same time predicts a $J^{PC} = 0^{++}$ Nambu-Goldstone boson (dilaton) associated with the spontaneous breakdown of the scale invariance.

INSPIRE

`%\cite{Yamawaki:1985zg}`

`\bibitem{Yamawaki:1985zg}`

K.~Yamawaki, M.~Bando and K.~-i.~Matumoto,

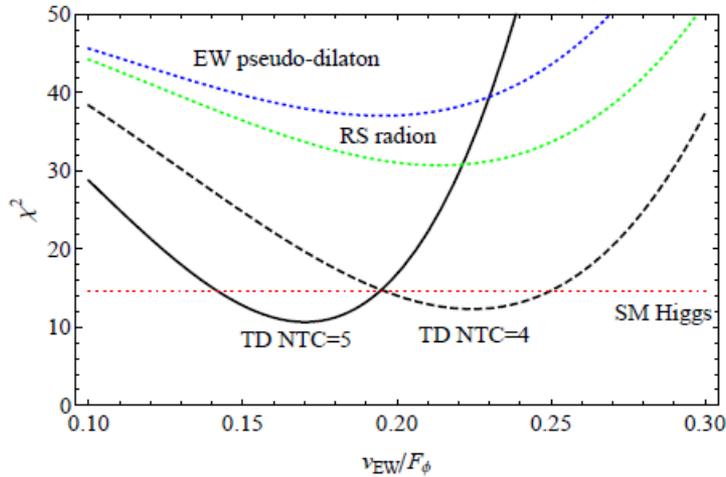
`%`Scale Invariant Technicolor Model and a Technidilaton,"`

`Phys.\ Rev.\ Lett.\ {\bf 56}, 1335 (1986).`

`%%CITATION = PRLTA,56,1335;%%`

125 GeV Techni-dilaton(TD) at LHC

S.Matsuzaki and K. Y.,
 PLB719 (2013) 378
 PRD86 (2012) 115004



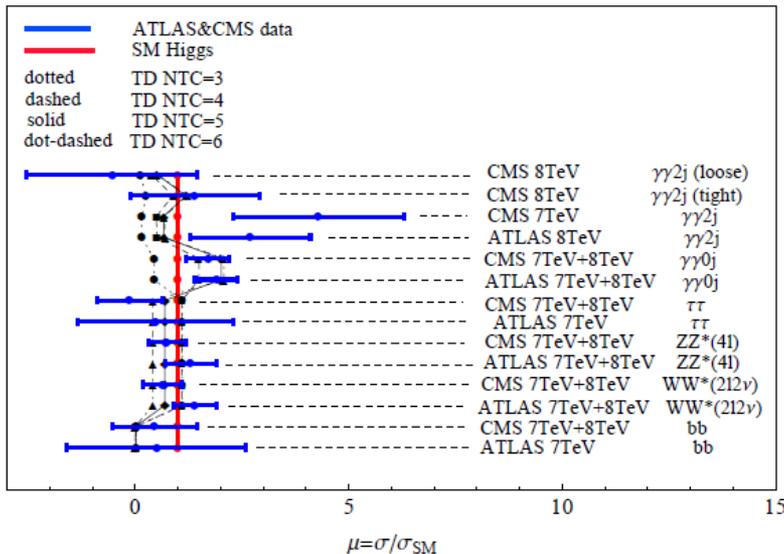
$$\chi^2 = \sum_{i \in \text{events}} \left(\frac{\mu_i - \mu_i^{\text{exp}}}{\sigma_i} \right)^2$$

TD (in 1FM) is favored by the current data !!

*** diphoton rate enhanced by techni-fermions (> W loop contribution)**

*** goodness-of-fit performed for each search category**

As of July 2012



Consistent with the updated after Moriond/Aspen in March 2013

CONTENTS

- Technicolor: QCD-Scale-up (**3 times R.I.P.**)
- Walking Technicolor and Techni-dilaton
- Discovering Walking Technicolor at LHC
Techni-dilaton at 125 GeV
- Discovering Walking Technicolor on the Lattice
KMI Lattice Project



Technicolor: a Scale-Up of QCD

S. Weinberg (1976)
L. Susskind (1979)

Composite $\pi \Rightarrow$ Composite π_{TC}

$\rightarrow m_{W,Z}$

$$H \sim \bar{F}F \quad F_\pi = 246 \text{ GeV}$$

$$\langle \bar{F}F \rangle \sim (700 \text{ GeV})^3$$

$$\frac{N_{TC}}{N_c}$$

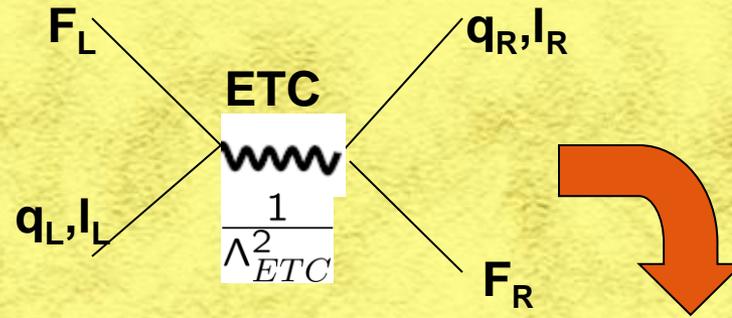
$$\sqrt{\frac{N_c}{N_{TC}N_D}}$$

X 2600

$$\sigma \sim \bar{q}q \quad f_\pi = 93 \text{ MeV}$$

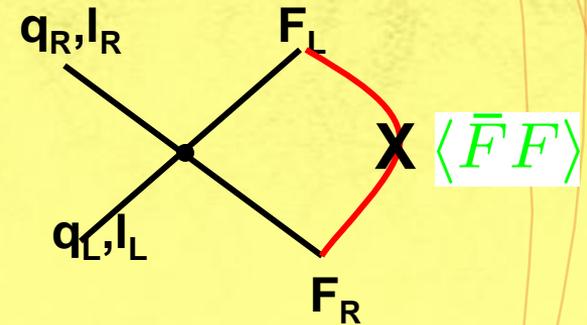
$$\langle \bar{q}q \rangle \sim (250 \text{ MeV})^3$$

FCNC Problems:



Mass of Quarks/Leptons

$$m_{q/l} \sim \frac{1}{\Lambda_{ETC}^2} \langle \bar{F} F \rangle$$



FCNC

$$\frac{1}{\Lambda_{ETC}^2} \bar{s} d \bar{s} d < (10^3 \text{ TeV})^{-2}$$

$$m_s < (10^3 \text{ TeV})^{-2} \times (0.7 \text{ TeV})^3 \sim 10^{-1} \text{ MeV}$$

Needs 10^3 enhancement

By Large Anomalous Dimension

 γ_m

Holdom (1981)

Pure Assumption of
Existence of Large γ_m
No Concrete Dynamics
No Concrete Value γ_m

$$m_{q/l} = \frac{1}{\Lambda_{ETC}^2} \langle (\bar{T}T)_{\Lambda_{ETC}} \rangle$$

$$\langle \bar{F}F \rangle |_{\Lambda_{ETC}} = Z_m^{-1} \cdot \langle \bar{F}F \rangle |_{\Lambda_{EW}}$$

$$Z_m^{-1} = (\Lambda_{ETC}/\Lambda_{EW})^{\gamma_m} \simeq (10^3)^{\gamma_m}$$

$$\gamma_m > 1 \quad \longrightarrow \quad > 10^3$$

Walking Technicolor

K.Y., Bando, Matumoto (Dec. 24, 1985)

Ladder Schwinger-Dyson Equation

Scale Invariance $\Leftrightarrow (\alpha(p) = \text{constant})$

$\gamma_m = 1$  FCNC Sol.

Techni-dilaton

Similar FCNC Sol. **without** notion of γ_m Scale Invariance, Techni-dilaton :

Akiba, Yanagida (Jan. 3, 1986)

Appelquist, Karabali, Wijewardhana (June 2, 1986)

(Holdom (Oct. 12, 1984), pure numerical)

Ladder SD

$$m_F \approx 4\Lambda \cdot \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{cr}} - 1}}\right)$$

Miransky Scaling

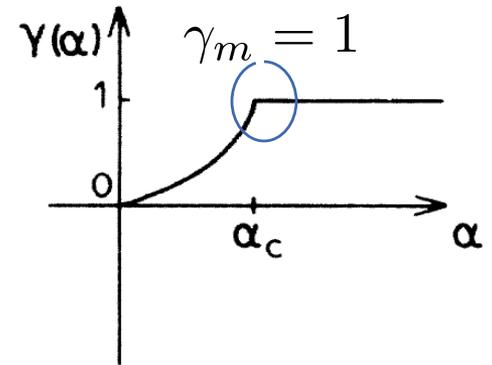
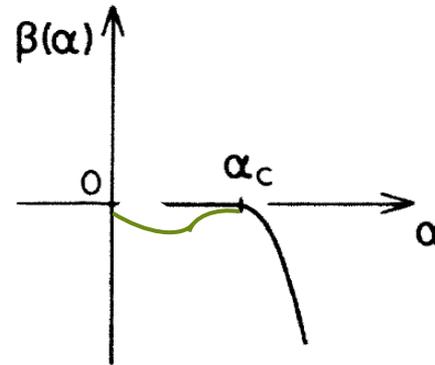
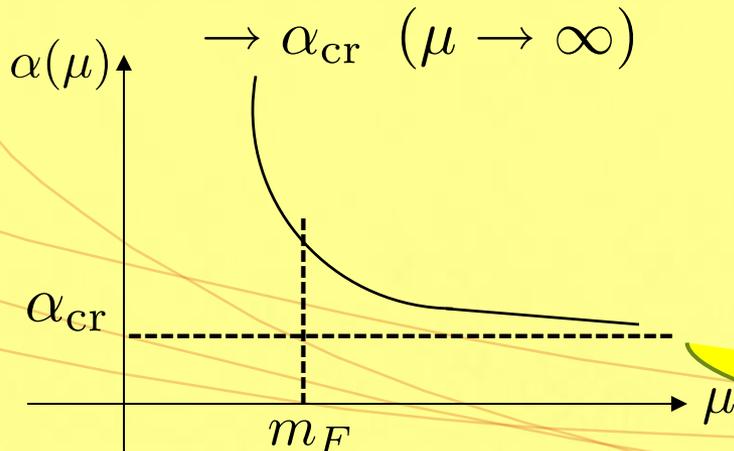
Essential singularity

Non-perturbative running ("Walking")

$$\beta(\alpha) = \Lambda \frac{\partial \alpha}{\partial \Lambda} = -\frac{2\alpha_{cr}}{\pi} \left(\frac{\alpha}{\alpha_{cr}} - 1\right)^{\frac{3}{2}}$$

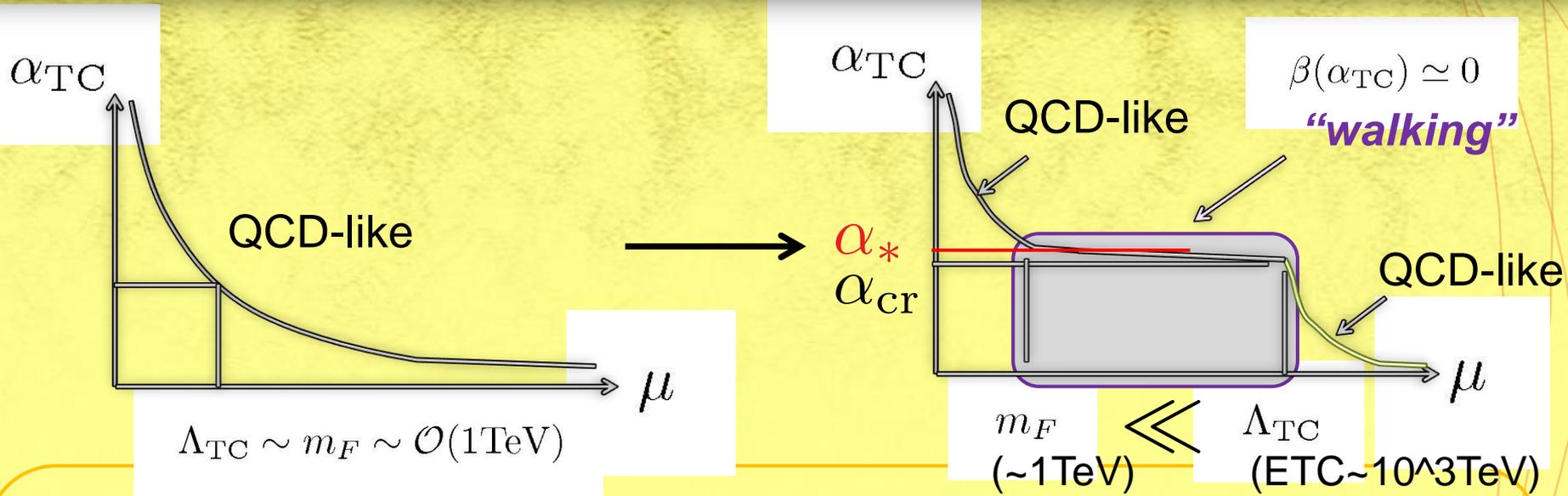
UVFP: not a linear zero = IRFP

$$\alpha(\mu) = \alpha_{cr} + \frac{\pi^2 \alpha_{cr}}{\ln^2\left(\frac{\mu}{\frac{1}{4}m_F}\right)}$$



KY-Bando-Matsumoto (1986)

A schematic view of Walking TC



**nonperturbative
scale anomaly
due to m_F**

$$\langle \partial_\mu D^\mu \rangle = \frac{\beta(\alpha)}{4\pi^2} \langle \alpha G_{\mu,\nu}^2 \rangle \ll m_F^4 (\ll \Lambda_{TC}^4)$$

Pseudo NG Boson: Techni-dilaton

Composite Higgs from technifermions having EW charges

Weakly Coupled Light Scalar Composite from Strongly Coupled Dynamics?

Cf: N. Seiberg, Aspen 2013



Naïve Arguments (base on LINEAR sigma model)

$$\mathcal{L}_{\text{SMH}} = |\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}$$

$$\lambda = \frac{1}{2} \frac{m_H^2}{v_{\text{EW}}^2} \simeq \frac{1}{2} \left(\frac{125}{250} \right)^2 = \frac{1}{8} \ll 1 !!$$

$$\mathcal{L}_{\text{GL}} = \frac{1}{2} \left((\partial_\mu \pi_a)^2 + (\partial_\mu \sigma)^2 - \mu^2 (\pi_a^2 + \sigma^2) \right)$$

$$\frac{m_\sigma}{4\pi F_\pi} = \mathcal{O}(1)$$

$$- \frac{1}{4} \lambda (\pi_a^2 + \sigma^2)^2$$

$$\lambda = \frac{1}{2} \frac{m_\sigma^2}{v_{\text{QCD}}^2} = 8\pi^2 \cdot \left(\frac{m_\sigma}{4\pi F_\pi} \right)^2 = \mathcal{O}(8\pi^2) \gg 1$$

GL(linear sigma model) \neq QCD

$\lambda \rightarrow \infty$ (Nonlinear σ model)

$$\mathcal{L}_{\text{NL}\sigma} = \frac{F_\pi^2}{4} \text{Tr} |\partial_\mu U|^2$$

$$U(x) = \exp\left(i \frac{2\pi(x)}{F_\pi}\right) \rightarrow g_L U(x) g_R^\dagger$$

WTC: Scale-Inv.

$$\mathcal{L}_{\text{NL}\sigma} = \frac{F_\pi^2}{4} \cdot \chi^2 \cdot \text{Tr} |\partial_\mu U|^2 + \frac{1}{2} (\partial_\mu \phi)^2$$

ϕ : dilaton

$$\chi(x) = \exp\left(\frac{\phi(x)}{F_\phi}\right) \rightarrow e^c \chi((e^{-c} x))$$

$M_\phi \Leftarrow$ Scale anomaly

$$\frac{M_\phi}{4\pi F_\pi} \sim \frac{F_\pi}{F_\phi} \ll 1$$

$$F_\phi^2 M_\phi^2 = -4 \langle \theta_\mu^\mu \rangle = \mathcal{O} \left((4\pi F_\pi)^2 F_\pi^2 \right)$$

eff. TD Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + \mathcal{L}_S - V_\chi$$

$$g_{\phi WW/ZZ}$$

i) The scale anomaly-free part:

$$\mathcal{L}_{\text{inv}} = \frac{F_\pi^2}{4} \chi^2 \text{Tr}[D_\mu U^\dagger D^\mu U] + \frac{F_\phi^2}{2} \partial_\mu \chi \partial^\mu \chi$$

$$\chi^2 = 1 + \frac{2}{F_\phi} \phi + \dots$$

ii) The expl. br. due to SM (invariant by including spurion field "S"):

$$\mathcal{L}_S = -m_f \left(\left(\frac{\chi}{S} \right)^{2-\gamma_m} \cdot \chi \right) \bar{f} f$$

reflecting ETC-induced TF 4-fermi w/ (3- γ_m)

$$+ \log \left(\frac{\chi}{S} \right) \left\{ \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2 + \frac{\beta_F(e)}{2e} F_{\mu\nu}^2 \right\} + \dots$$

iii) The WTC scale anomaly part:

$$V_\chi = \frac{F_\phi^2 M_\phi^2}{4} \chi^4 \left(\log \chi - \frac{1}{4} \right)$$

β_F : TF-loop contribution to SM beta function

which correctly reproduces the PCDC relation:

$$\langle \theta_\mu^\mu \rangle = -\delta_D V_\chi \Big|_{\text{vacuum}} = -\frac{F_\phi^2 M_\phi^2}{4} \langle \chi^4 \rangle \Big|_{\text{vacuum}} = -\frac{F_\phi^2 M_\phi^2}{4}$$

TD couplings to the SM particles $\Leftarrow \mathcal{L}_S$

- * TD couplings to W/Z boson (from \mathcal{L}_{inv})

$$g_{\phi WW/ZZ} = \frac{2m_{W/Z}^2}{F_\phi}$$

- * TD couplings to $\gamma\gamma$ and gg (from \mathcal{L}_S)

$$g_{\phi\gamma\gamma} = \frac{\beta_F(e)}{e} \frac{1}{F_\phi}$$

$$g_{\phi gg} = \frac{\beta_F(g_s)}{g_s} \frac{1}{F_\phi}$$

The same form as SM Higgs couplings except F_ϕ and betas

β_F : TF-loop contribution to beta function

Weakly Coupled Light Scalar Composite from Strongly Coupled Dynamics?

Cf: N. Seiberg, Aspen 2013

Lightness

$$m_\sigma \ll m_{a_0}, m_\rho, m_{a_1}, \dots?$$

Scale inv.

LHC confirmed

Yes !

Weakness

~~$$g_{\sigma N\bar{N}}, g_{\sigma\pi\pi}, g_{a_1\rho\pi} \ll 1?$$~~

Coupling to SM sector

$$g_{HW W/HZZ} \ll 1, g_{Hf\bar{f}} \ll 1?$$

Not a linear sigma model !

SM sector

TC sector
(Strongly coupled)

Weak !

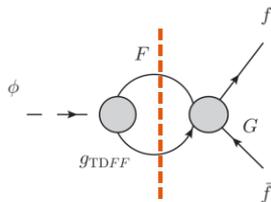
$$g_{\phi WW/ZZ} = \frac{2M_{W/Z}^2}{F_\phi}$$

$$g_{MMM}, g_{MB\bar{B}} \gg \mathcal{O}(1)$$

$$g_1, g_2 (\ll 1)$$

$$g_{\phi FF} = \frac{m_F}{F_\phi} > 1 \left(\gg \frac{F_\pi}{F_\phi} \right)$$

$$g_{\phi f \bar{f}} = (3 - \gamma_m) \frac{m_f}{F_\phi}$$



$$\frac{m_F}{F_\phi} \cdot \frac{N_{TC} m_F^2}{4\pi^2} \cdot \frac{1}{\Lambda_{ETC}^2} (\ll 1)$$

Even needs enhancement !

$$\left(\frac{\Lambda_{ETC}}{m_F} \right)^{\gamma_m}$$

Ladder estimate of TD mass

$$\Lambda_{TC} (\sim \Lambda_{ETC})$$

$$N_f (\rightarrow N_f^{cr})$$

* **LSD + BS** in large N_f QCD

Harada-Kurachi-K.Y. (1989)

* **LSD via gauged NJL**

*Shuto-Tanabashi-K.Y. (1990);
Carena-Wagner (1992); Hashimoto (1998)*

A composite Higgs mass

$$M_\phi \sim 4F_\pi \ll M_\rho, M_{a_1}$$

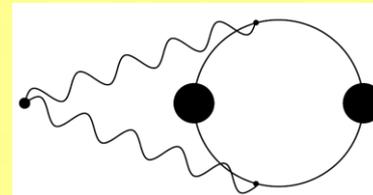
~ 500 GeV

for one-family model (1FM)
still larger than ~ 125 GeV

* Using only **PCDC** still accommodates 125 GeV

Lightness=Weak Coupling

$$F_\phi^2 M_\phi^2 = -4 \langle \theta_\mu^\mu \rangle = \frac{\beta(\alpha)}{\alpha} \langle G_{\mu\nu}^2 \rangle \simeq 3\eta m_F^4$$



Miransky-Gusynin (1989):

Hashimoto-K.Y. (2011):

where $\eta \simeq \frac{N_{TC} N_{TF}}{2\pi^2} = \mathcal{O}(1)$

$$\frac{F_\phi^2}{m_F^2} \cdot \frac{M_\phi^2}{m_F^2} = \text{finite}$$

No exactly massless NGB limit:

$$M_\phi/m_F \rightarrow 0.$$

only when $F_\phi/m_F \rightarrow \infty$, i.e., a decoupled limit.

Ladder Estimate of

$$\frac{v_{EW}}{F_\phi}$$

Ladder approximation is subject to **about 30% uncertainty** for estimate of critical coupling and QCD hadron spectrum

critical coupling : T. Appelquist et al (1988);

Hadron spectrum : K. -I. Aoki et al (1991); M. Harada et al (2004).

$$\frac{N_{TF}}{4N_{TC}} \simeq 1 \pm \underline{0.3}$$

$$\langle \theta_\mu^\mu \rangle = 4\mathcal{E}_{vac} = \underline{-\kappa_V} \left(\frac{N_{TC} N_{TF}}{2\pi^2} \right) m_F^4$$

30%

$$F_\pi^2 = \underline{\kappa_F^2} \frac{N_{TC}}{4\pi^2} m_F^2$$

30%

Estimate
w/ uncertainty included

$$\frac{v_{EW}}{F_\phi} \simeq \underline{(0.1 - 0.3)} \times \left(\frac{N_D}{4} \right) \left(\frac{M_\phi}{125 \text{ GeV}} \right)$$

Weaker than SMH

Holographic estimate w/ techni-gluonic effects

z_m, ξ, G

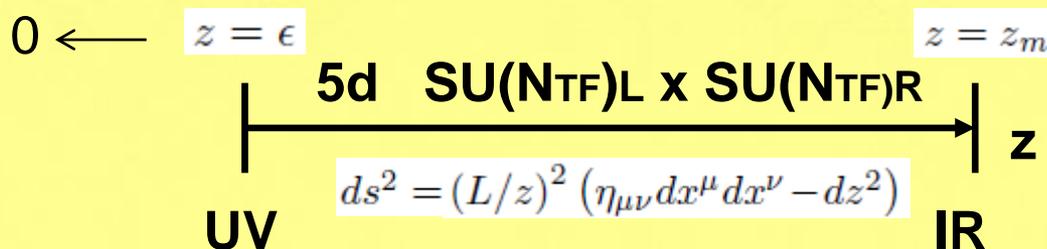
Haba-Matsuzaki-KY, PRD82 (2010) 055007

Matsuzaki- K.Y., PRD86 (2012) 115004 PPLB719 (2013) 115004

- * **Ladder approximation** : gluonic dynamics is neglected
- * **Deformation of successful AdS/QCD model (Bottom-up approach)**

Da Rold and Pomarol (2005); Erlich, Katz, Son and Stephanov (2005)

incorporates **nonperturbative gluonic effects**



$$S_5 = \int d^4x \int_{\epsilon}^{z_m} dz \sqrt{-g} \frac{1}{g_5^2} e^{cg_5^2 \Phi_X(z)} \left(-\frac{1}{4} \text{Tr} [L_{MN} L^{MN} + R_{MN} R^{MN}] \right. \\ \left. + \text{Tr} [D_M \Phi^\dagger D^M \Phi - \underline{m_\Phi^2} \Phi^\dagger \Phi] + \frac{1}{2} \partial_M \Phi_X \partial^M \Phi_X \right)$$

$$\underline{m_\Phi^2} = -\underline{(3 - \gamma_m)(1 + \gamma_m)} / \underline{\tilde{L}^2} \begin{cases} \text{QCD} & \gamma_m = 0 \\ \text{WTC} & \gamma_m = 1 \end{cases}$$

* QCD-fit $w/\gamma_m \simeq 0$

input

$$\begin{aligned}
 f_\pi &= 92.4 \text{ MeV} \\
 M_\rho &= 775 \text{ MeV} \\
 \langle \alpha G \mu^2 \rangle / \pi &= 0.012 \text{ GeV}^4
 \end{aligned}$$

fix



model parameters

$$\begin{aligned}
 \xi &= 3.1 \\
 G &= 0.25 \\
 z m^{-1} &= 347 \text{ MeV}
 \end{aligned}$$

Model predictions

Model predictions		measured
M_{a1}	[a1 meson] : 1.3 GeV	1.2 --- 1.3 GeV
$M_{f_0(1370)}$	[qqbar bound state] : 1.2 GeV	1.1 --- 1.2 GeV
M_G	[glueball] : 1.3 GeV	1.4 --- 1.7 GeV (lat.)
$S = -16 \pi L_{10}$	[S parameter] : 0.31	0.29 --- 0.37
$[-\langle \bar{q} q \rangle]^{1/3}$	[chiral condensate] : 277 MeV	200 --- 250 MeV

Monitoring QCD works well!



*WTC-case with $\gamma_m = 1$

$$G \sim \frac{\langle \alpha G_{\mu\nu}^2 \rangle}{F_\pi^4}$$

--- TD mass (lowest pole of dilatation current correlator)

$$\frac{M_\phi}{4\pi F_\pi} \simeq \sqrt{\frac{3}{N_{\text{TC}}}} \frac{\sqrt{3}/2}{1+G} \longrightarrow 0 \text{ as } G \longrightarrow \infty$$

125 GeV TD is realized by a large gluonic effect: $G \sim 10$
for one-family model w/ $F_\pi = 123 \text{ GeV}$ (c.f. QCD case, $G \sim 0.25$)

--- TD decay constant (pole residue)

$$\begin{aligned} \frac{F_\phi}{F_\pi} &\simeq \sqrt{2N_{\text{TF}}} \cdot \sqrt{J_0^2(x) + J_1^2(x)} \Big|_{x=(M_\phi z_m) \ll 1} \\ &\simeq \sqrt{2N_{\text{TF}}} \end{aligned}$$

free from holographic-parameters !!

→ **Massless NGB limit (“conformal limit”) is realized:**

$$\frac{M_\phi}{F_\pi} \rightarrow 0 \quad \text{and} \quad \frac{F_\phi}{F_\pi} \rightarrow \text{finite}, \quad \text{as } G \rightarrow \infty. \quad (\langle \Phi(z_m) \rangle \sim \xi \rightarrow 0)$$

in contrast to ladder approximation

Estimate of $\frac{v_{EW}}{F_\phi}$ -- Holographic approach

Matsuzak- K.Y., PRD86 (2012) 115004

* TD decay constant for the light TD case w/ $G \sim 10$:

$$\frac{F_\phi}{F_\pi} \simeq \sqrt{2N_{TF}} \cdot \sqrt{J_0^2(x) + J_1^2(x)} \Big|_{x=(M_\phi z_m) \ll 1}$$

Indep. of S
(S < 0.1 tunable)

$\sqrt{2N_{TF}}$ **holographic-parameter free !!**

Theoretical Uncertainties: $1/N_{TC}$ corr. (20% ~ 30%)

$$\left. \frac{v_{EW}}{F_\phi} \right|_{\text{holo}}^{+1/N_{TC}} \sim 0.2 - 0.4$$

Weaker than SMH

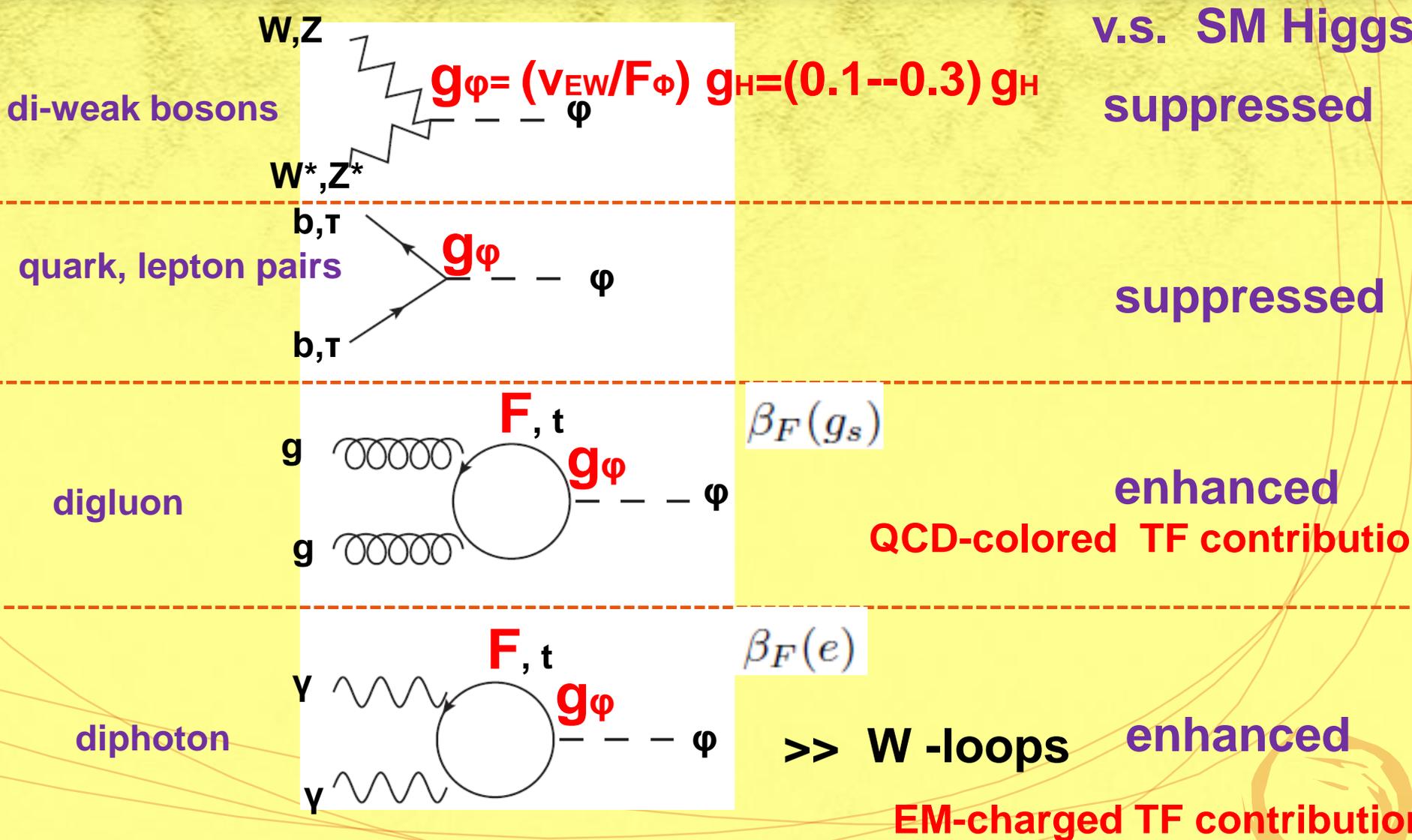
This is consistent with ladder estimate:

LHC best fit (before Moriond '13)

$$\begin{aligned} \frac{v_{EW}}{F_\phi} &\simeq 0.22 (N_{TC} = 4) \\ &\simeq 0.17 (N_{TC} = 5) \end{aligned}$$

$$\frac{v_{EW}}{F_\phi} \stackrel{\text{ladder}}{\simeq} \underline{(0.1 - 0.3)} \times \left(\frac{N_D}{4} \right) \left(\frac{M_\phi}{125 \text{ GeV}} \right)$$

Characteristic features of 125 GeV TD in 1FM (w/ $N_{TC}=4,5$) at LHC



Technifermion loop contributions to $g_{\phi gg}$ $g_{\phi\gamma\gamma}$

$N_{\text{TC}}=4$

1/3

10

$$\frac{g_{\phi gg}}{g_{h_{\text{SM}} gg}} \simeq \frac{v_{\text{EW}}}{F_{\phi}} \cdot ((3 - \gamma_m) + 2N_{\text{TC}}) ,$$

$$\frac{g_{\phi\gamma\gamma}}{g_{h_{\text{SM}}\gamma\gamma}} \simeq \frac{v_{\text{EW}}}{F_{\phi}} \cdot \left(\frac{63 - 16(3 - \gamma_m)}{47} - \frac{32}{47} N_{\text{TC}} \right)$$

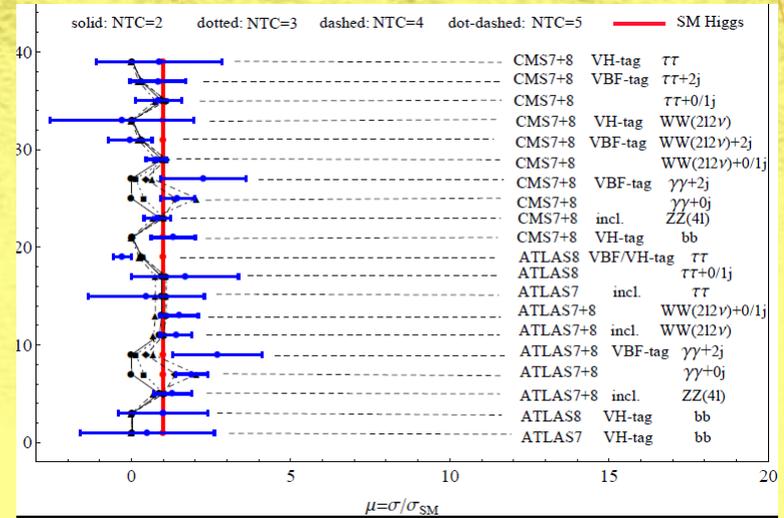
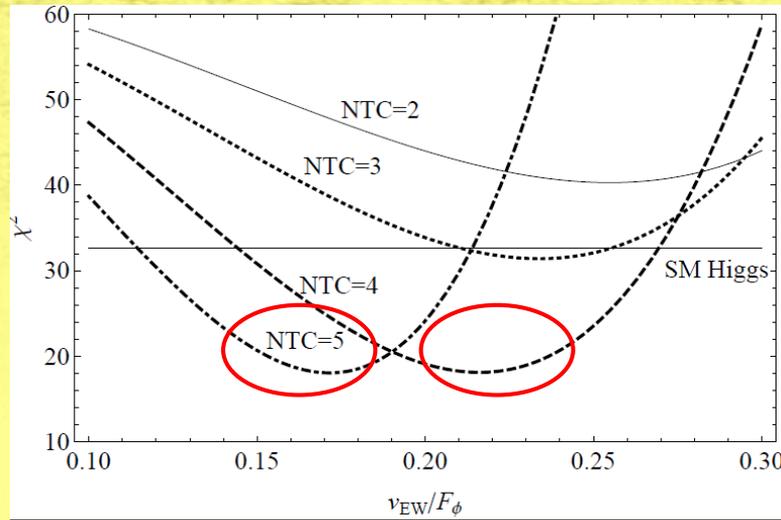
1 $(\gamma_m^t = 2)$

3

<1

The 125 GeV TD signal fitting *updated after HCP2012 to the current Higgs search data

S. Matsuzaki, 1304.4882



$$\chi^2 = \sum_{i \in \text{events}} \left(\frac{\mu_i - \mu_i^{\text{exp}}}{\sigma_i} \right)^2$$

NTC	[v_{EW}/F_ϕ] best	χ^2 min /d.o.f.
4	0.22	18/19 = 0.95
5	0.17	18/19 = 0.95

*** TD can be better than the SM Scalar ($\chi^2/\text{d.o.f.} = 33/20 = 1.6$), due to the enhanced diphoton rate, by extra BSM (TF) contributions!**

TD signal strengths ($\mu = \sigma \times BR/SM \text{ Higgs}$) w/ NTC=4, $v_{EW}/F\phi = 0.2$ vs the data *Moriond EW&QCD (ASPEN) March, 2013*

(i) ggF-tag

TD signal strength	ATLAS (significance)	CMS (significance)
$\mu_{\gamma\gamma}^{ggF} \simeq 1.4$	$\simeq 1.6_{-0.4}^{+0.4}$ ($\sim 6\sigma$)	$\simeq 0.5_{-0.5}^{+0.5}$ ($\sim 3\sigma$)
$\mu_{WW}^{ggF} \simeq 1.0$	$\simeq 0.8_{-0.4}^{+0.4}$ ($\sim 4\sigma$)	$\simeq 0.8_{-0.2}^{+0.2}$ ($\sim 4\sigma$)
$\mu_{ZZ}^{ggF} \simeq 1.0$	$\simeq 1.8_{-0.5}^{+0.8}$ ($\sim 6\sigma$)	$\simeq 0.9_{-0.4}^{+0.5}$ ($\sim 6\sigma$)
$\mu_{\tau\tau}^{ggF} \simeq 1.0$	$\simeq 2.1_{-1.0}^{+1.0}$ ($< 2\sigma$)	$\simeq 0.7_{-0.5}^{+1.5}$ ($< 2\sigma$)

Distinguished from SM Higgs

(ii) VBF-tag

TD signal strength	ATLAS (significance)	CMS (significance)
$\mu_{\gamma\gamma}^{VBF} \simeq 0.4$	$\simeq 1.7_{-0.9}^{+0.9}$ ($< 2\sigma$) w/ VH	$\simeq 1.5_{-1.1}^{+1.5}$ ($< 2\sigma$) w/ VH
$\mu_{WW}^{VBF} \simeq 0.3$	$\simeq 1.7_{-0.8}^{+0.8}$ ($\sim 2\sigma$)	$\simeq 0.04_{-0.57}^{+0.77}$ ($< 2\sigma$)
$\mu_{ZZ}^{VBF} \simeq 0.3$	$\simeq 1.2_{-1.4}^{+3.8}$ ($< 2\sigma$) w/ VH	$\simeq 1.2_{-5.6}^{+5.6}$ ($< 2\sigma$)
$\mu_{\tau\tau}^{VBF} \simeq 0.3$	$\simeq -0.4_{-2.0}^{+3.0}$ ($< 2\sigma$) w/ VH	$\simeq 1.4_{-0.6}^{+0.6}$ ($< 2\sigma$)

(iii) VH-tag

TD signal strength	ATLAS (significance)	CMS (significance)
$\mu_{\gamma\gamma}^{VH} \simeq 0.02$	$\simeq 1.8_{-1.3}^{+1.5}$ ($< 2\sigma$) w/ VBF	$\simeq 1.5_{-1.1}^{+1.5}$ ($< 2\sigma$) w/ VBF
$\mu_{WW}^{VH} \simeq 0.01$	N/A	$\simeq -0.3_{-2.0}^{+2.3}$ ($< 2\sigma$)
$\mu_{ZZ}^{VH} \simeq 0.01$	$\simeq 1.2_{-1.4}^{+3.8}$ ($< 2\sigma$) w/ VBF	N/A
$\mu_{\tau\tau}^{VH} \simeq 0.01$	$\simeq -0.4_{-2.0}^{+3.0}$ ($< 2\sigma$) w/ VBF	$\simeq 0.8_{-1.4}^{+1.5}$ ($< 2\sigma$)
$\mu_{bb}^{VH} \simeq 0.01$	$\simeq -0.4_{-1.0}^{+1.0}$ ($< 2\sigma$)	$\simeq 1.3_{-0.6}^{+0.7}$ ($\sim 2\sigma$)

Theoretical Issues

- Walking Dynamics beyond Ladder/Holography ?
- More Precise Quantitative Predictions?

$F_\pi, F_\phi, M_\phi, M_\rho, M_{a_1}, M_{\text{baryon}}, \text{etc.}$

S, T, U – Parameters

Lattice !

Discovering Walking Technicolor on the Lattice

KMI Lattice Project (LatKMI Collaboration)

- Finding a **candidate** for WTC on the Lattice
- Finding a **light scalar** composite on the Lattice
- Calculating the **composite spectra** on the Lattice



LatKMI collaboration members



M. Kurachi T. Maskawa K. Nagai K. Yamawaki



Y. Aoki



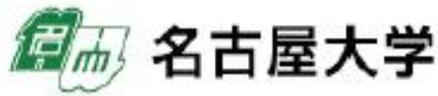
T. Aoyama



T. Yamazaki



H. Ohki



E. Rinaldi

A. Shibata



KMI Computer



(March 02, 2011~)

Only for Beyond SM Physics

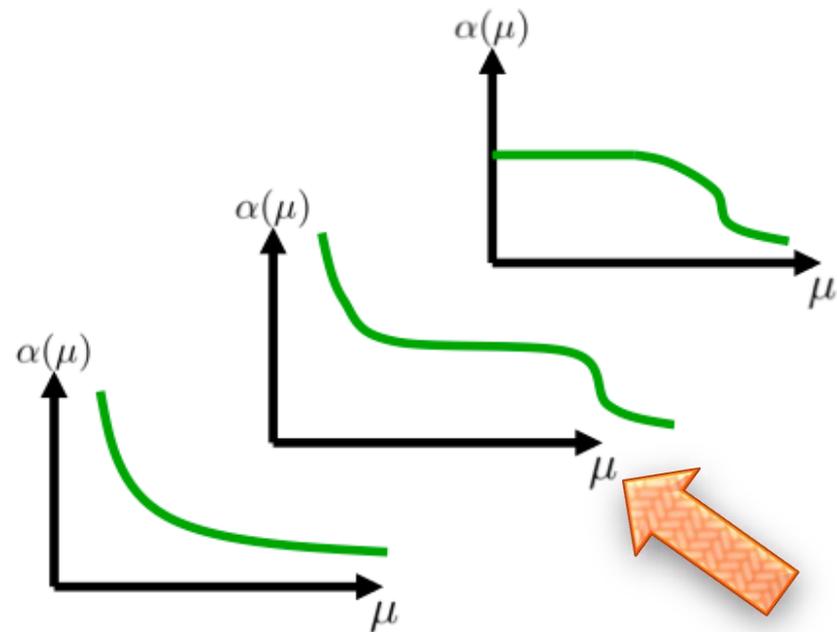
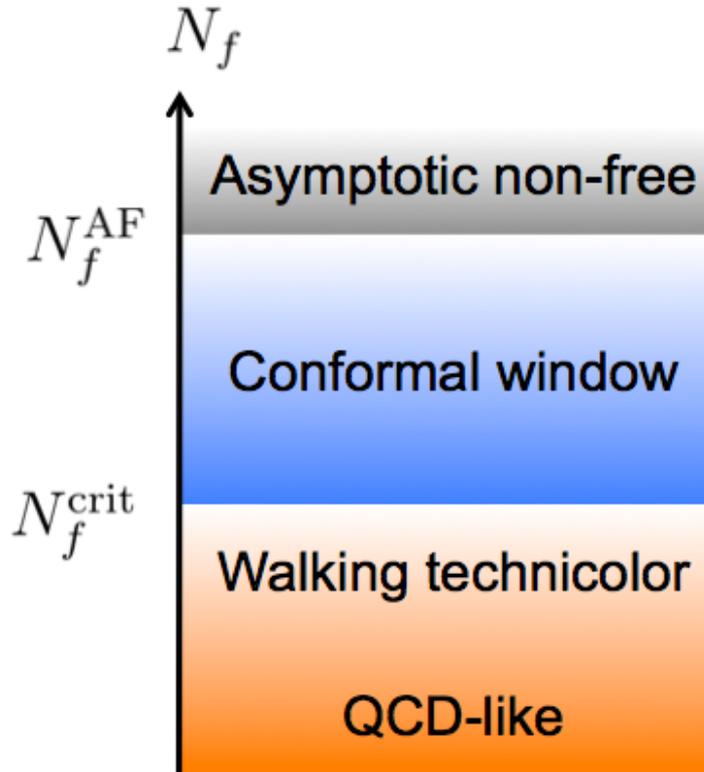
62.4 TFLOPS

26.88 TFLOPS (128 nodes)

35.53 TFLOPS (23 nodes /w GPGPU)

$SU(3); N_c = 3$

$N_f = 4, 8, 12, 16 (< N_f^{\text{AF}} = 11N_c/2 = 16.5)$



2-loop : $N_f^{\text{crit}} = 8.05$

2-loop + ladder SD equation : $N_f^{\text{crit}} = 11.9$

Recent study of LatKMI Collaboration

LatKMI Collaboration, PRD86, 054506 (2012); D87, 094511 (2013); arXiv:1305.6006

Unique setup for all N_f : Improved staggered action (HISQ/Tree)

Cheapest calculation cost in lattice fermion actions

+ small a systematic error

Simulation parameters

- $\beta \equiv 6/g^2 \rightarrow$ lattice spacing a
- $L, T \sim O(10)$
- $m_f \neq 0 \rightarrow$ IR scales $m_f \gg 1/L$

Large enough L at each m_f : $m_\pi L \gtrsim 6$ ($\gtrsim 4$ in $N_f = 4$)

N_f	β	$L^3 \times T$	m_f
4	3.7	$12^3 \times 18 - 20^3 \times 30$	0.005–0.05
8	3.8	$18^3 \times 24 - 36^3 \times 48$	0.015–0.16
12	3.7	$18^3 \times 24 - 30^3 \times 40$	0.04–0.2
12	4.0	$18^3 \times 24 - 30^3 \times 40$	0.05–0.2

Machines: φ at KMI, CX400 at Kyushu Univ.

Walking candidate & Scalar

- $N_f=8$: Walking, $\gamma_m = 0.62 - 0.97$
LatKMI Collaboration, Phys. Rev. D 87, 094511 (2013).
- $N_f=12$: Conformal $\gamma_m = 0.4 - 0.5$
LatKMI Collaboration, Phys. Rev. D86, 054506 (2012)
- Light flavor-singlet scalar (& scalar glueball)
in $N_f=12$
LatKMI Collaboration, (arXiv: 1302.4577), arXiv:1305.6006
- Light flavor-singlet scalar (& scalar glueball)
in $N_f=8$ (**Very Preliminary**)

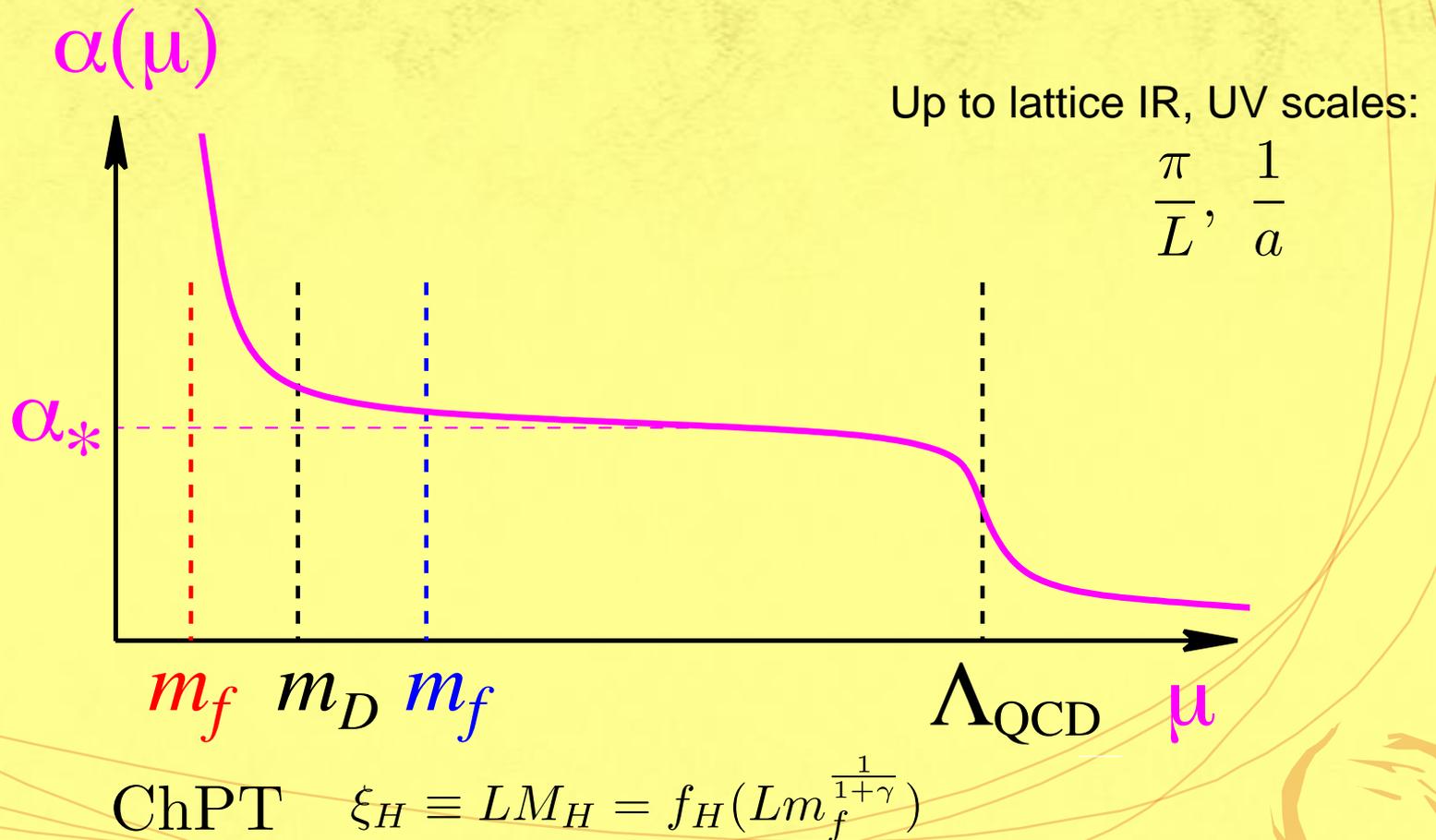
$$N_f = 8$$

PHYSICAL REVIEW D **87**, 094511 (2013)

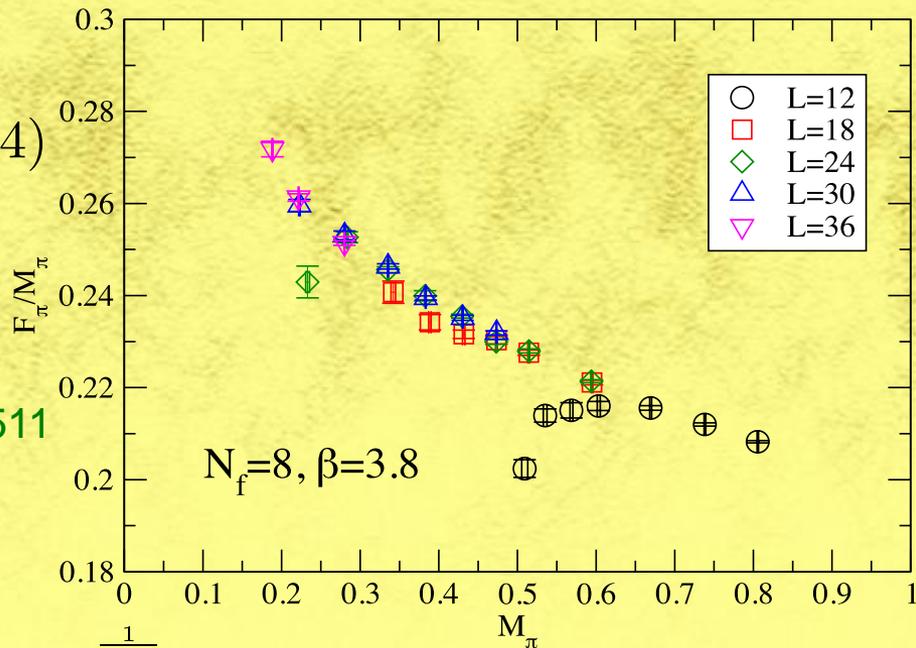
Walking signals in $N_f = 8$ QCD on the lattice

Yasumichi Aoki,¹ Tatsumi Aoyama,¹ Masafumi Kurachi,¹ Toshihide Maskawa,¹ Kei-ichi Nagai,¹ Hiroshi Ohki,¹
Akihiro Shibata,² Koichi Yamawaki,¹ and Takeshi Yamazaki¹

(LatKMI Collaboration)

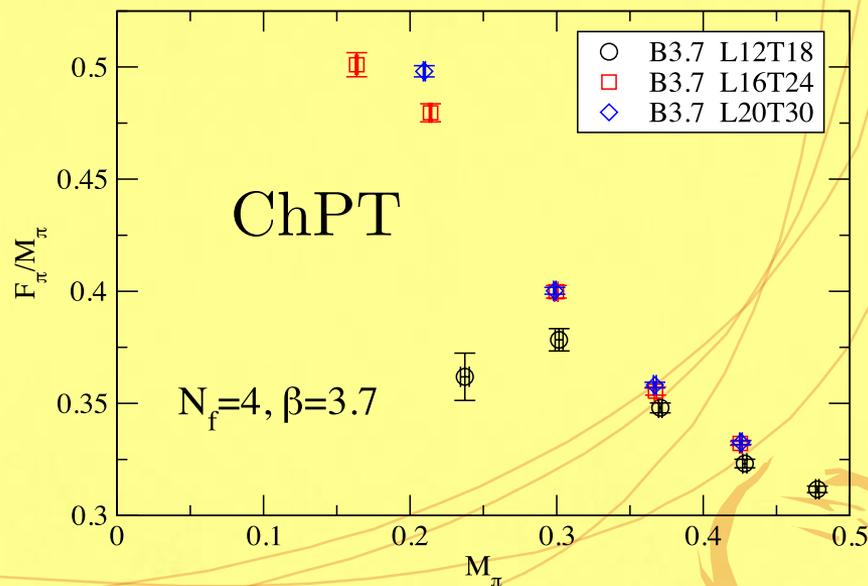
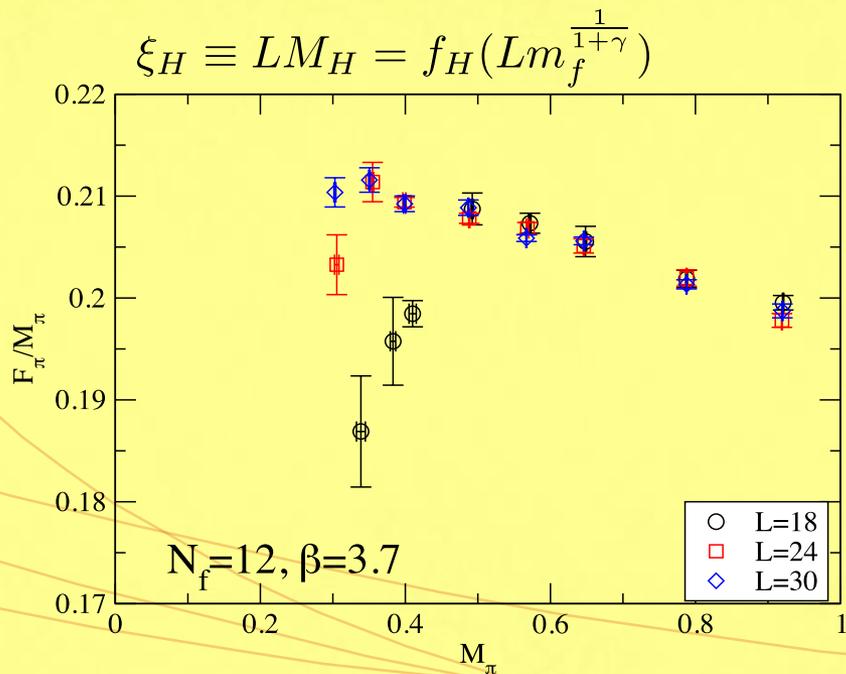


ChPT
 ($m_f = 0.015 - 0.04$)



$\xi_H \equiv LM_H = f_H(Lm_f^{\frac{1}{1+\gamma}})$
 ($m_f = 0.05 - 0.16$)

LatKMI Coll, PRD 87, 094511
 (2013)



LatKMI Coll, PRD86 (2012)054506

LatKMI Coll, PRD 87, 094511 (2013)

$$N_f = 8$$

Walking signals in $N_f = 8$ QCD on the lattice

Yasumichi Aoki,¹ Tatsumi Aoyama,¹ Masafumi Kurachi,¹ Toshihide Maskawa,¹ Kei-ichi Nagai,¹ Hiroshi Ohki,¹
 Akihiro Shibata,² Koichi Yamawaki,¹ and Takeshi Yamazaki¹

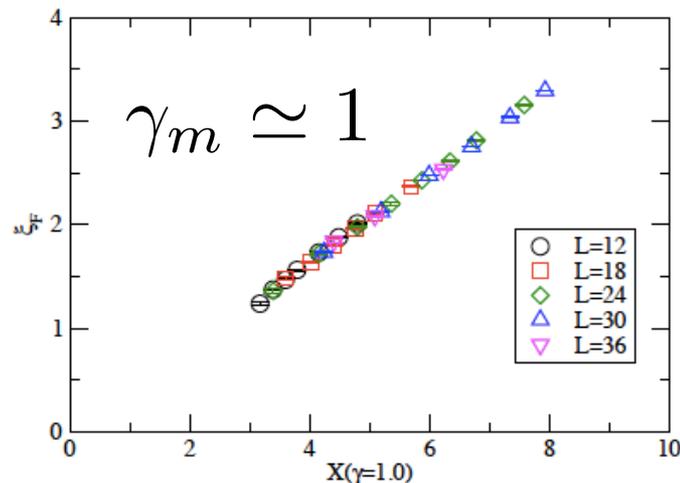
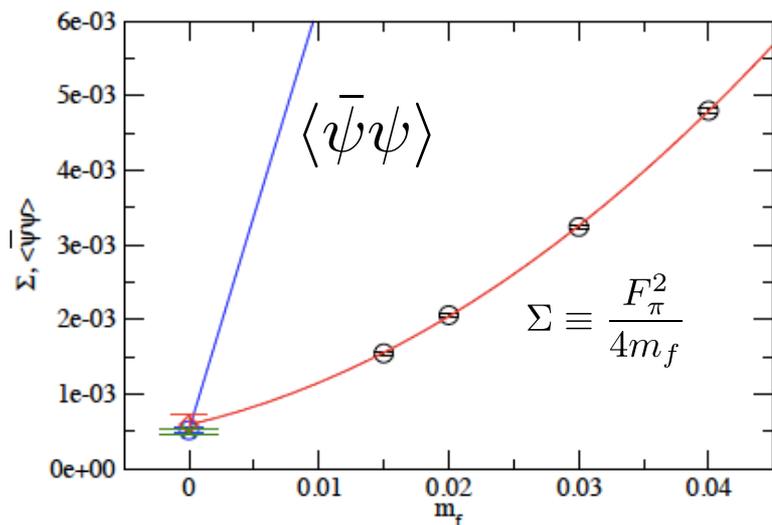
(LatKMI Collaboration)

HISQ $\beta = 3.8$

$L = 12, 18, 24, 30$ ($T/L = 4/3$)

SχSB $m_f = 0.015 - 0.04$

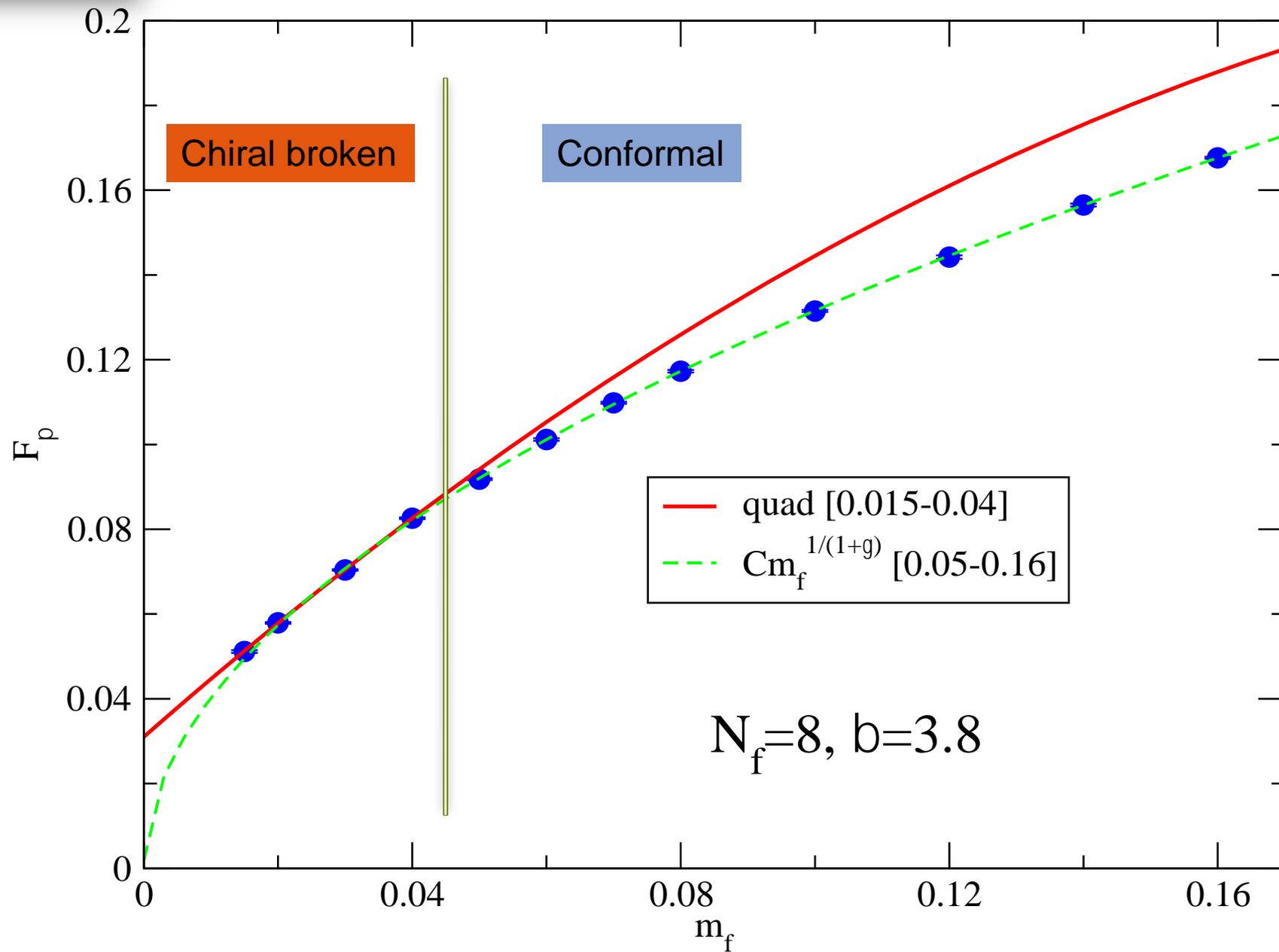
“Conformal” $m_f = 0.05 - 0.16$



$F_\pi \rightarrow \neq 0, M_\pi \rightarrow 0, M_\rho \rightarrow \neq 0$
 at $m_f \rightarrow 0$

$$\xi_H \equiv LM_H = f_H(Lm_f^{\frac{1}{1+\gamma}})$$

$$N_f = 8$$



$$N_f = 8$$

Nf=8 data

Hyperscaling relation is **not** for a **universal** γ_m

$$\gamma(M_\pi) \simeq 0.57, \gamma(F_\pi) \simeq 0.93, \gamma(M_\rho) \simeq 0.80$$

After corrections



$$\gamma \simeq 0.62 - 0.97$$

Universal value (up to correction ansatz)

For large m_f Corrections such as higher power of m_f

Cf: SD equation in the conformal phase $\alpha < \alpha_{\text{cr}} (N_f > N_f^{\text{cr}})$

$$\frac{m_f}{\Lambda} = \xi \left[\frac{\Gamma(1 - \gamma_m^*)}{\Gamma(\frac{2 - \gamma_m^*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1 + \gamma_m^*} + \frac{\Gamma(-1 + \gamma_m^*)}{\Gamma(\frac{\gamma_m^*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3 - \gamma_m^*} \right].$$

LatKMI Collaboration, Phys. Rev. D85, 074502 (2012)

Light composite scalar in twelve-flavor QCD on the lattice

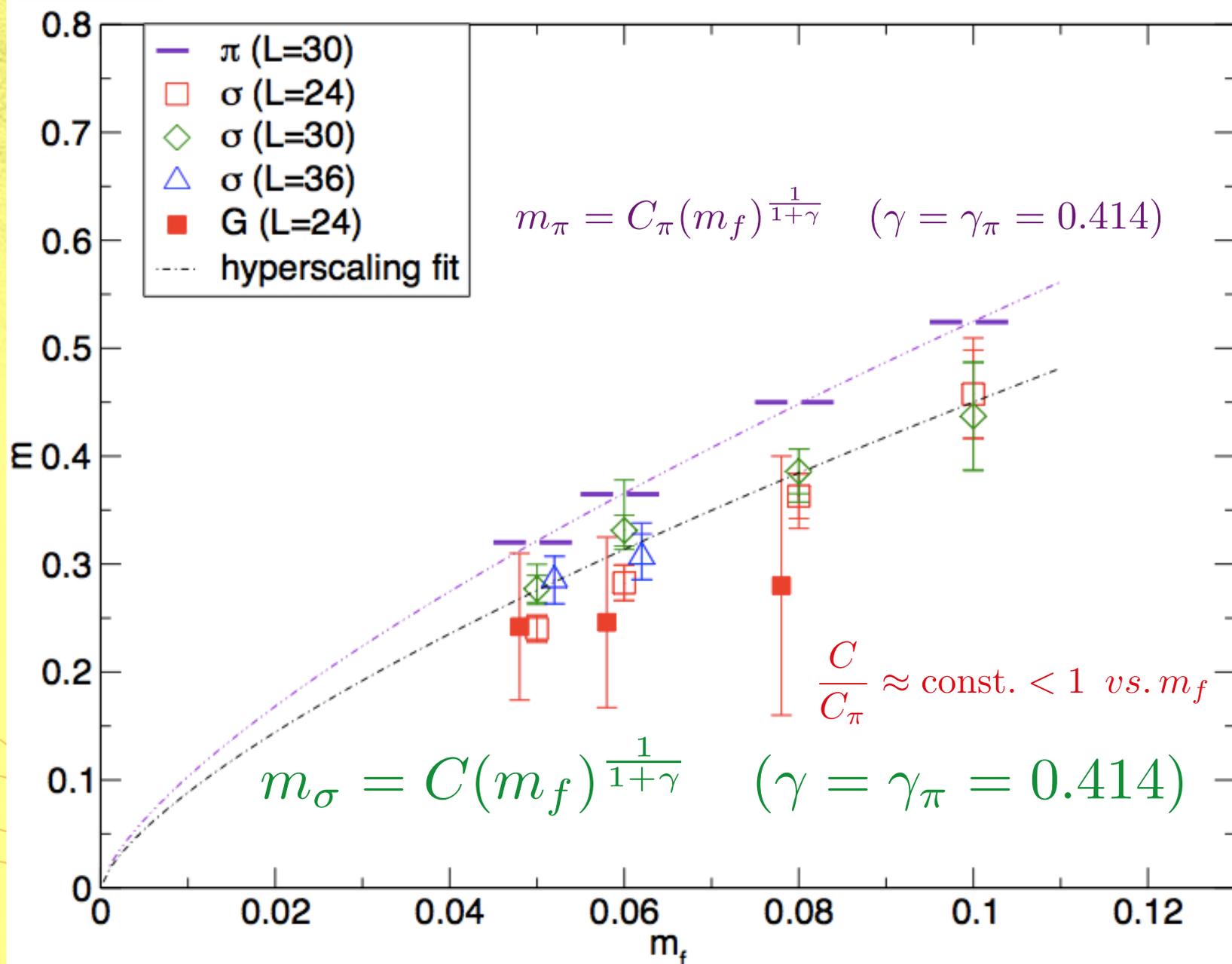
Yasumichi Aoki,¹ Tatsumi Aoyama,¹ Masafumi Kurachi,¹ Toshihide Maskawa,¹ Kei-ichi Nagai,¹
 Hiroshi Ohki,¹ Enrico Rinaldi,^{1,2} Akihiro Shibata,³ Koichi Yamawaki,¹ and Takeshi Yamazaki¹
 (LatKMI collaboration)

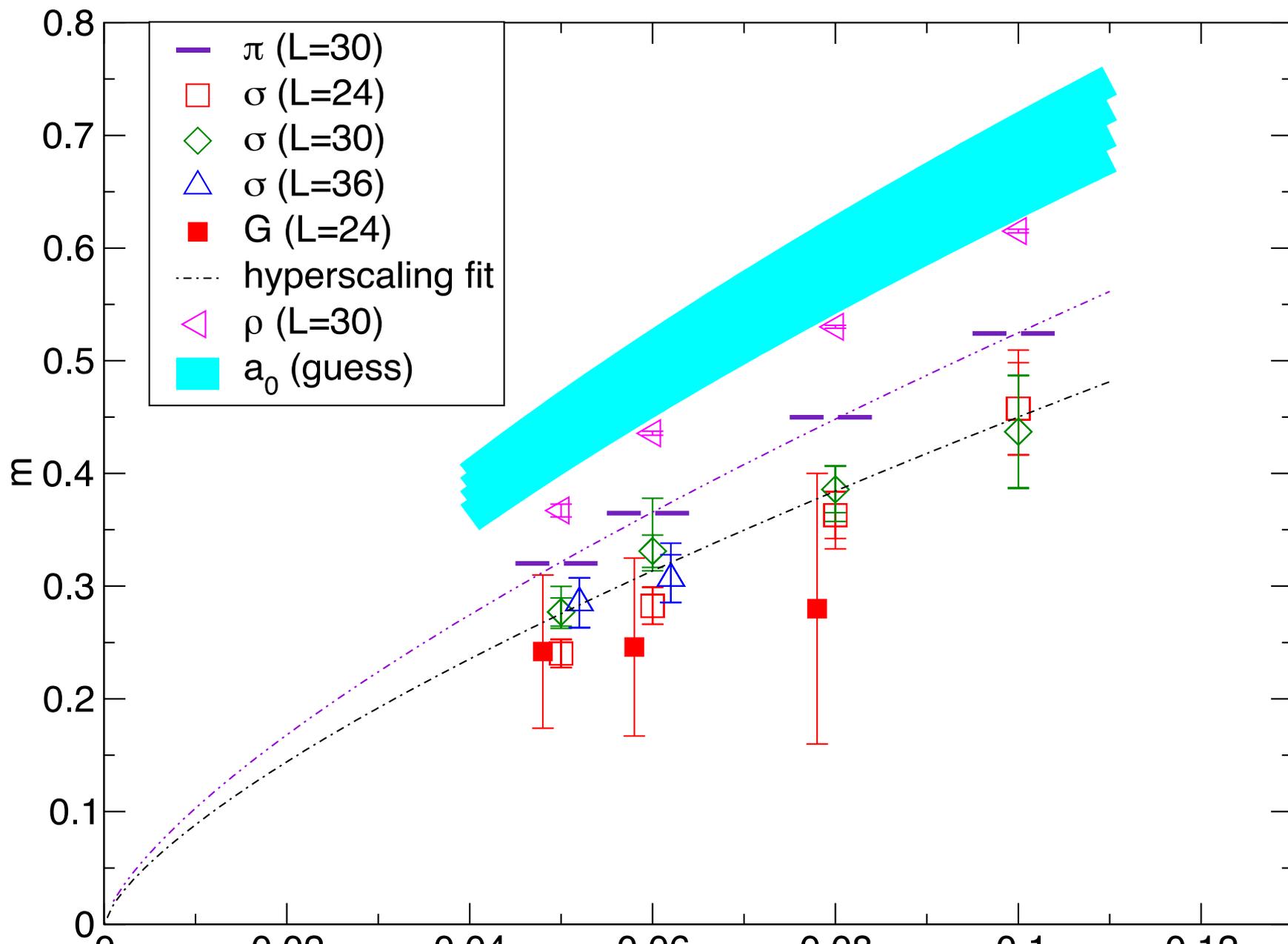
(arXiv: 1302.4577), arXiv:1305.6006

Nf=12,
 $\beta=4.0$

Noise
 reduction
 method
 with Nr=64

$L^3 \times T$	m_f	N_{cfgs}	m_σ	m_π	m_σ/m_π
$24^3 \times 32$	0.05	8800	0.250(15) $_{(01)}^{(00)}$	0.3273(19)*	0.76(5) $_{(0)}^{(0)}$
$24^3 \times 32$	0.06	14000	0.283(16) $_{(01)}^{(04)}$	0.3646(16)*	0.78(4) $_{(0)}^{(1)}$
$24^3 \times 32$	0.08	15000	0.363(21) $_{(22)}^{(02)}$	0.4459(11)	0.81(5) $_{(5)}^{(0)}$
$24^3 \times 32$	0.10	9000	0.458(41) $_{(06)}^{(32)}$	0.5210(7)	0.88(8) $_{(1)}^{(6)}$
$30^3 \times 40$	0.05	8000	0.284(15) $_{(09)}^{(24)}$	0.3201(16)*	0.89(5) $_{(3)}^{(7)}$
$30^3 \times 40$	0.06	14000	0.337(15) $_{(12)}^{(51)}$	0.3648(9)*	0.92(4) $_{(3)}^{(14)}$
$30^3 \times 40$	0.08	15000	0.386(21) $_{(20)}^{(00)}$	0.4499(8)	0.86(5) $_{(4)}^{(0)}$
$30^3 \times 40$	0.10	4000	0.437(50) $_{(09)}^{(07)}$	0.5243(7)	0.83(9) $_{(2)}^{(1)}$
$36^3 \times 48$	0.05	5000	0.285(22) $_{(03)}^{(00)}$	0.3204(7)*	0.89(7) $_{(1)}^{(0)}$
$36^3 \times 48$	0.06	6000	0.307(21) $_{(04)}^{(23)}$	0.3636(9)*	0.84(6) $_{(1)}^{(6)}$

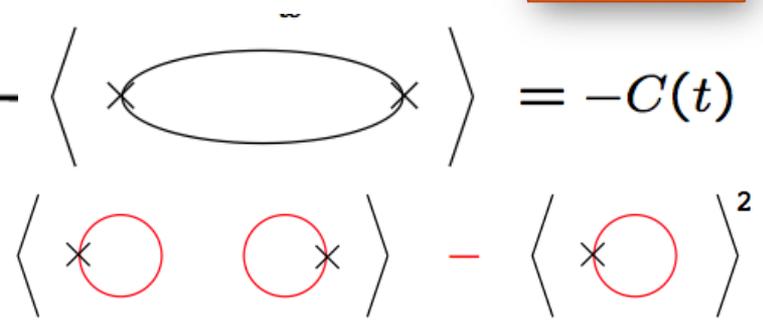
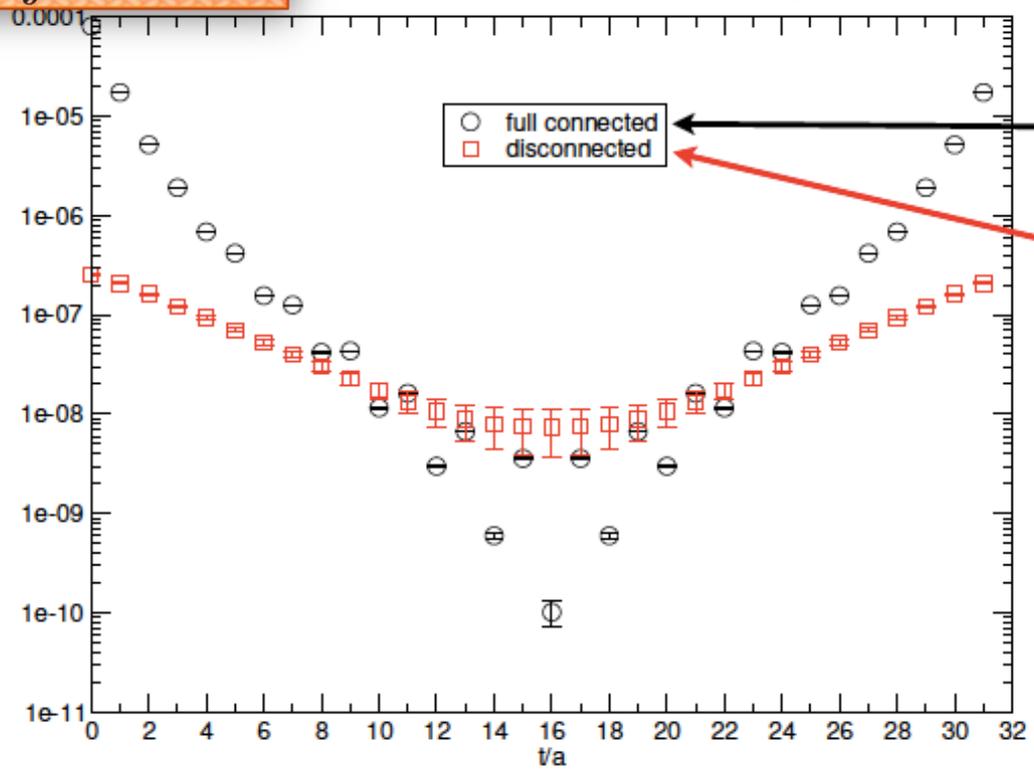




$N_f = 12$

Scalar mesonic correlators L=24 T=32 $\beta=4.0$ $am_f=0.06$

Scalar



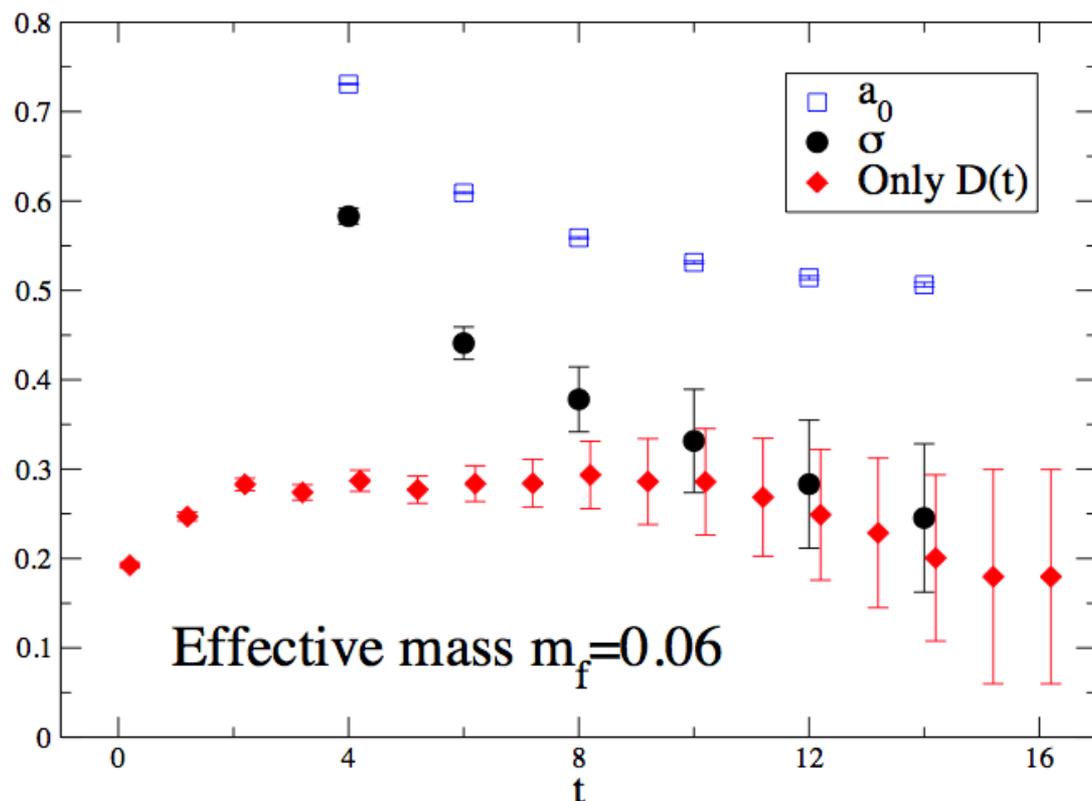
- connected and disconnected correlator measured on 14000 configurations
- 64 stochastic gaussian sources used for the disconnected piece on each configuration
- 2 stochastic gaussian sources used for the connected piece on each configuration

$$C_\sigma(t) = \langle \sum_i^{N_f} \bar{\psi}_i \psi_i(t) \sum_j^{N_f} \bar{\psi}_j \psi_j(0) \rangle = N_f(-C(t) + \underline{N_f}D(t))$$

$$\mathcal{O}_F(t) \equiv \bar{\psi}_i \psi_i(t), \quad D(t) = \langle \mathcal{O}_F(t) \mathcal{O}_F(0) \rangle - \langle \mathcal{O}_F(t) \rangle \langle \mathcal{O}_F(0) \rangle$$

Effective mass in $N_f = 12$ ($m_f = 0.06, 24^3 \times 32$ with $N_{\text{conf}} = 14000$, Preliminary)

$$m_{\text{eff}}(t) = \log(C_H(t)/C_H(t+1)) \xrightarrow{t \gg 1} m_H$$



Nonsinglet scalar

$$a_0: -C_+(2t)$$

Singlet scalar

$$\sigma: 3D_+(2t) - C_+(2t)$$

$$m_\sigma < m_{a_0}$$

Only $D(t)$

Consistent m_σ

Smaller error

$$X_+(2t) = 2X(2t) + X(2t+1) + X(2t-1)$$

Good signal of m_σ from $D(t)$

$N_f = 8$

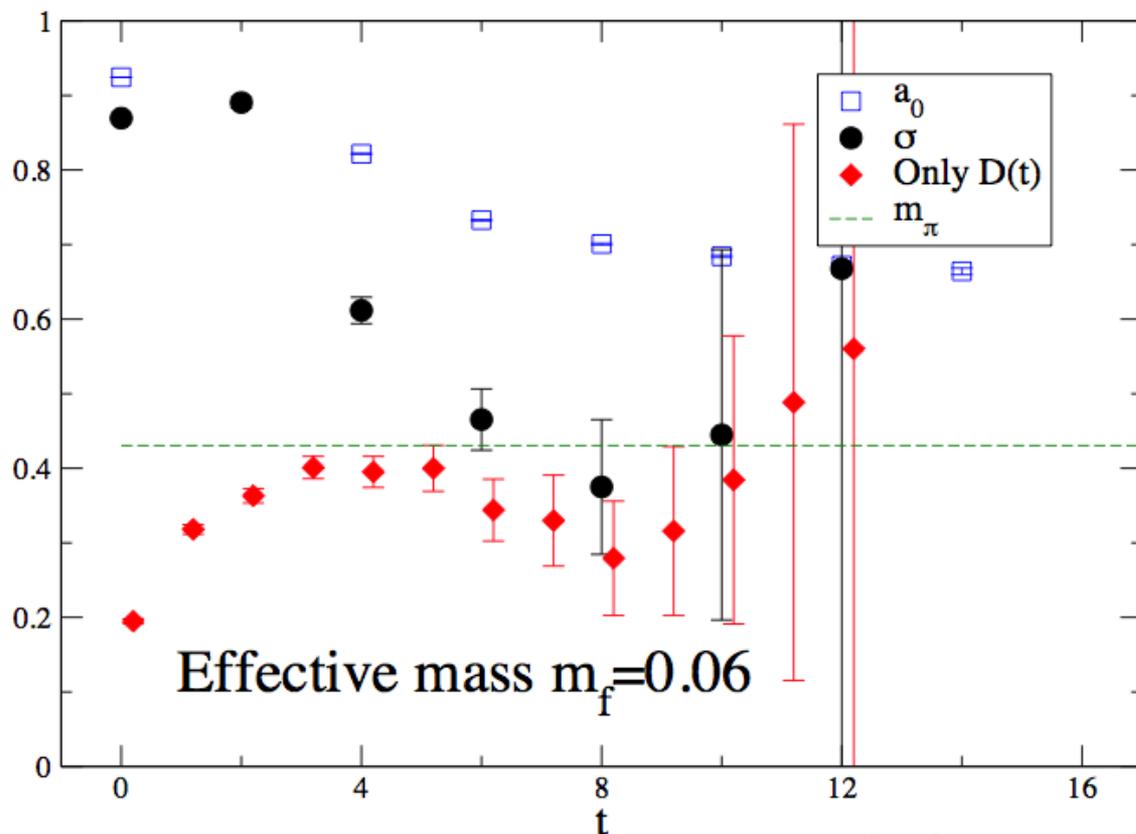
Very Preliminary

Scalar

Nf=8

 $\beta=3.8$

L, T	m_f	confs
24,32	0.06	7600

Noise reduction
with Nr=64

Nonsinglet scalar

$$a_0: -C_+(2t)$$

Singlet scalar

$$\sigma : 3D_+(2t) - C_+(2t)$$

$$m_\sigma \lesssim m_\pi < m_{a_0}$$

$$F_\pi \sim 0.1 \text{ at } m_f = 0.06$$

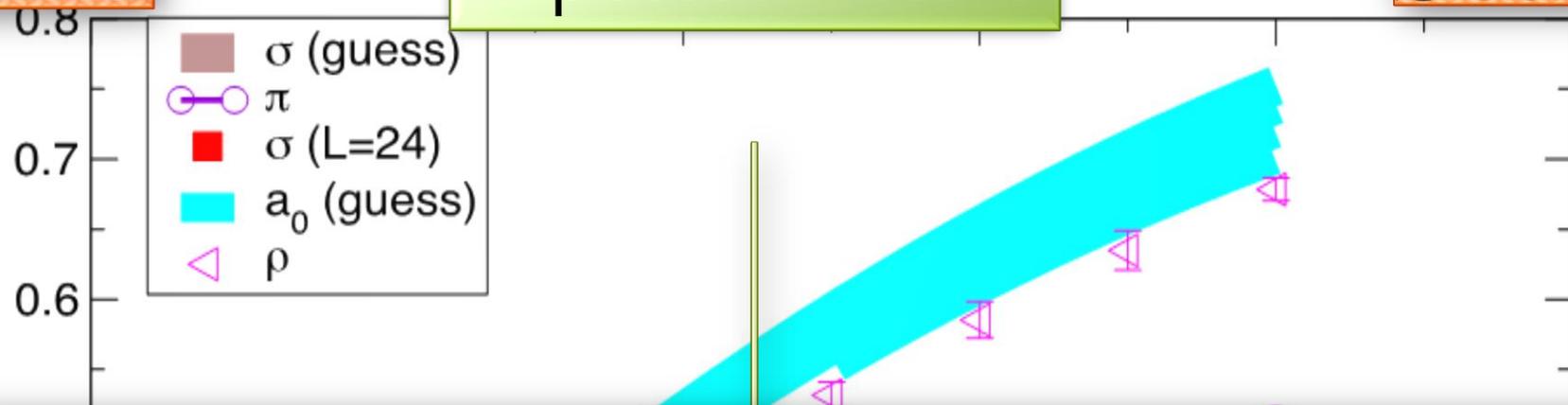
$$X_+(2t) = 2X(2t) + X(2t+1) + X(2t-1)$$

$$m_\sigma \lesssim m_\pi \text{ at } m_f = 0.06$$

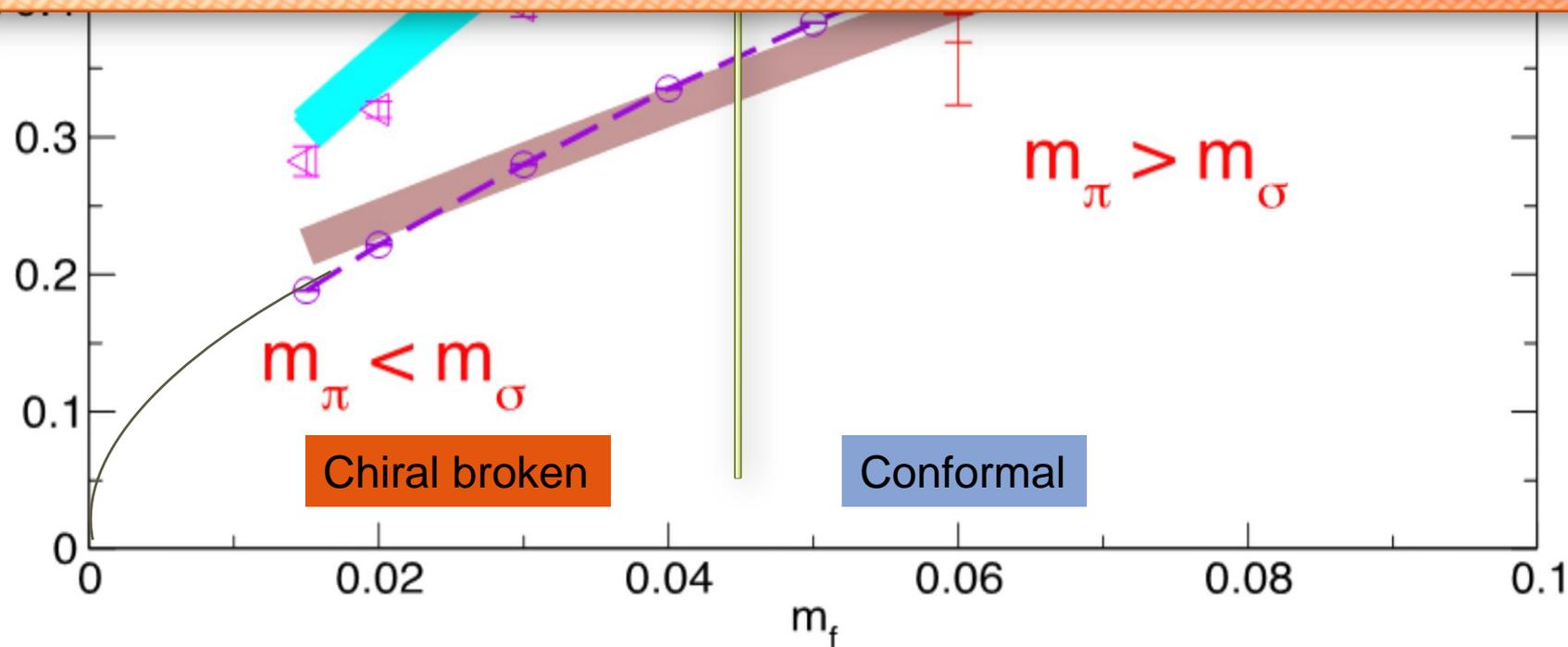
$$N_f = 8$$

Expected Picture

Scalar



$\sigma = \text{Technidilaton} = \text{Composite Higgs}$



Conclusion

- A **light composite Higgs** can be generated in the **Walking Technicolor** (Strongly coupled theory) as a Pseudo-NG boson of Scale Symmetry (Techni-dilaton), which is **Weakly coupled to the SM particles**.
 - Techni-dilaton is **consistently identified with the 125 GeV Higgs**
 - Lattice results of LatKMI Collaboration are consistent with
 - Nf=12 QCD: conformal behavior
 - Nf=8 QCD : walking behavior; chiral broken ($m_f=0.015-0.04$),
(approx.) conformal ($m_f=0.05-0.16$)
 - Lattice results of LatKMI Collaboration observed
 - Nf=12: clean signal of a **scalar lighter than pion**
 - Nf=8: indication of a **scalar slightly lighter than pion** (just for one parameter $m_f=0.06$) (**Very preliminary**)
- Both **reflecting (near) conformality for a wide IR region** below the asymptotically free UV region

Hope to give the lattice answer to the theoretical issues
before 13/14 TeV LHC

Backup Slides

★ Other characteristic spectrum in one-family WTC

Chiral symmetry breaking: $SU(8)_L \times SU(8)_R \rightarrow SU(8)_V$

3 “would-be” NGBs: eaten by W, Z
60 (pseudo) NGB = techni-pions (TPs)

$\theta_a^{\pm,3,0}$: color-octet scalars (# 32)

$$\sim \bar{Q} \gamma_5 \lambda_a \tau^{\pm,3,0} Q$$

$T_c^{\pm,3,0} (\bar{T}_c^{\pm,3,0})$: color-triplet scalars (“leptoquark”) (#24)

$$\sim \bar{Q} \gamma_5 \tau^{\pm,3,0} L \text{ (h.c.)}$$

$P^{\pm,3,0}$: color-singlet scalars (# 4)

$$\sim \bar{Q} \gamma_5 \tau^{\pm,3,0} Q - 3 \bar{L} \gamma_5 L$$

Expected masses :

300 ~ 500 GeV

J.Junji, S.Matsuzaki, K.Y.,
PRD87.016006. (2012)

Techni-pions: **couplings to WW and ZZ highly suppressed** (NO NGB-NBG-NGB)

→ narrow resonances (tot.width ~ 5--10GeV)

J.Junji, S.Matsuzaki, K.Y., PRD87.016006. (2012)

Discovering isospin singlet TPs: P^0 and θ_a^0

- * produced ONLY from ggF :
~~VBF, VH,~~
- * Predominantly decaying to SM fermions

$$\sigma_{ggF}(pp \rightarrow P^0 \rightarrow \tau^+\tau^-) \sim 0.1\text{pb} \quad @ \sim 300\text{GeV}$$

$$\sigma_{ggF}(pp \rightarrow P^0 \rightarrow t\bar{t}) \sim 10\text{pb} \quad @ \sim 400\text{GeV}$$

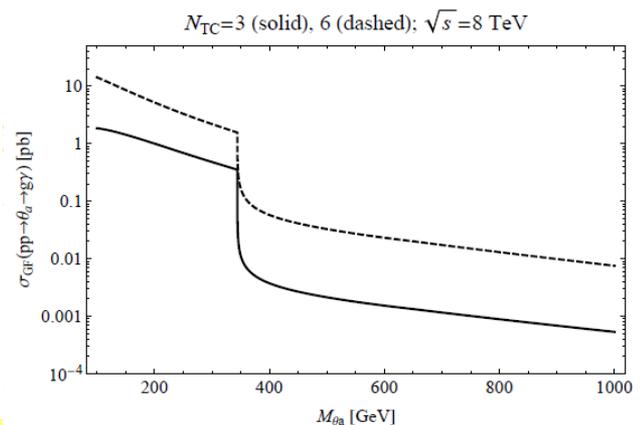
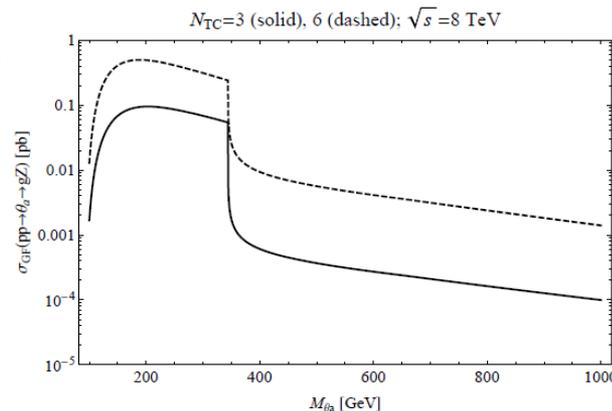
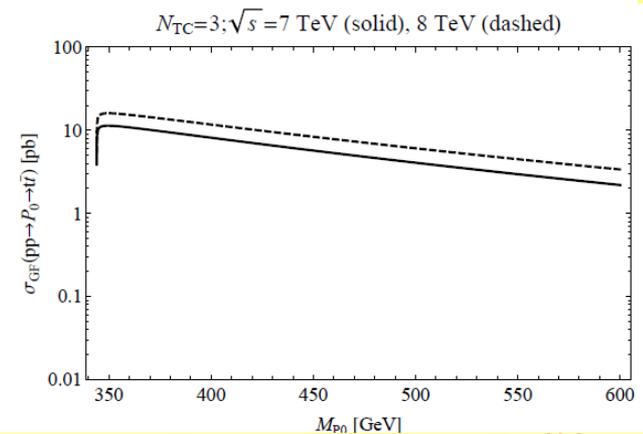
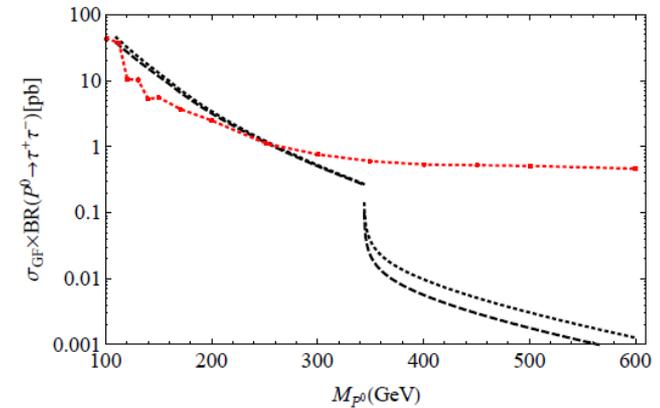
$$\sigma_{GF}(pp \rightarrow \theta_a) \times \text{BR}(\theta_a \rightarrow t\bar{t}) \simeq 60\text{pb} \quad @ \sim 500\text{GeV}$$

- * decays to glue-gamma/Z

$$\sigma_{GF}(pp \rightarrow \theta_a \rightarrow gZ/\gamma) :$$

$$\sim 1\text{fb}$$

$$@ \sim 500\text{GeV}$$



* One-family walking techni-rho mesons (#63)

$$\rho_{\theta_a}^{\pm,3,0}$$

: color-octet vectors (# 32)

$$\rho_{T_c}^{\pm,3,0} \quad (\bar{\rho}_{\bar{T}_c}^{\pm,3,0})$$

: color-triplet vectors (# 24)

$$\rho_P^{\pm,3,0}$$

: color-singlet vectors (# 4)

$$\rho_{\pi}^{\pm,3}$$

: color-singlet vectors (# 3)
[corresponding to vector states for eaten NGBs]

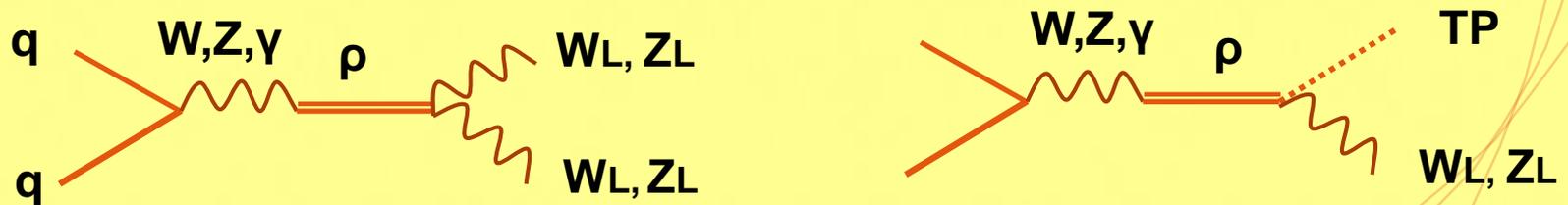
Expected masses :

1 ~ 4 TeV

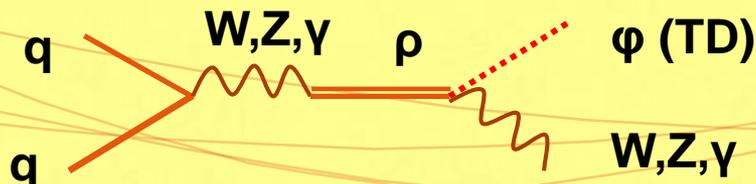
consistent w/
EW precision tests

S.Matsuzaki and K.Y.,
PRD86.115004. (2012)

- Typical discovery channels: decays to $W_L W_L$ ($Z_L Z_L$) or TP and W_L

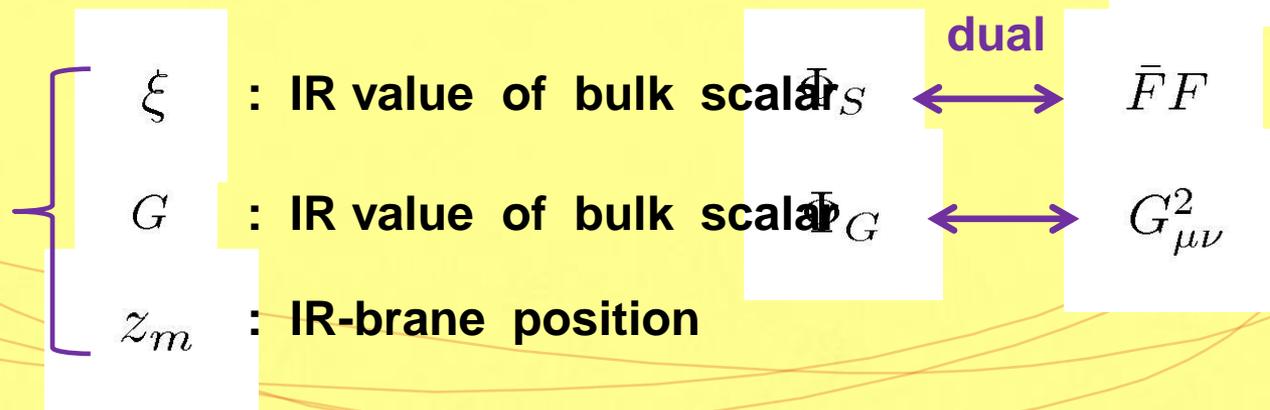
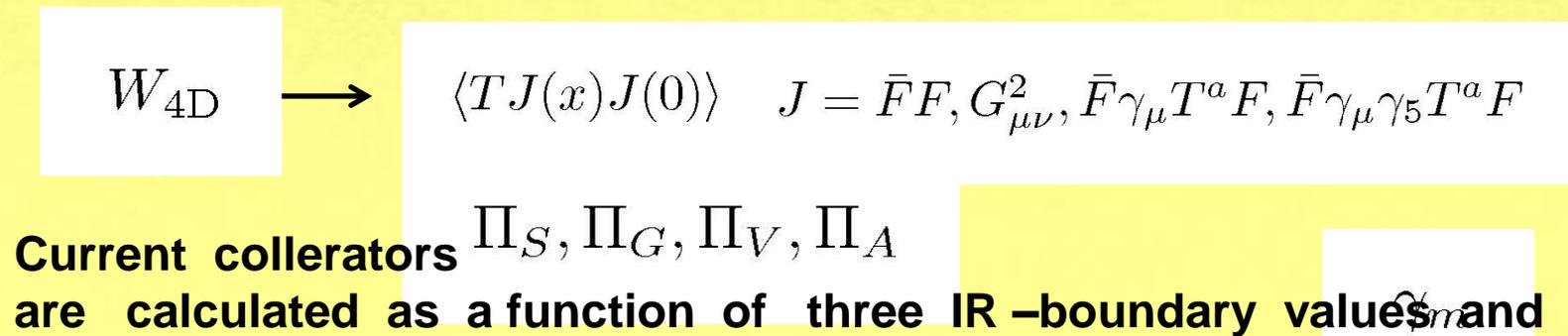
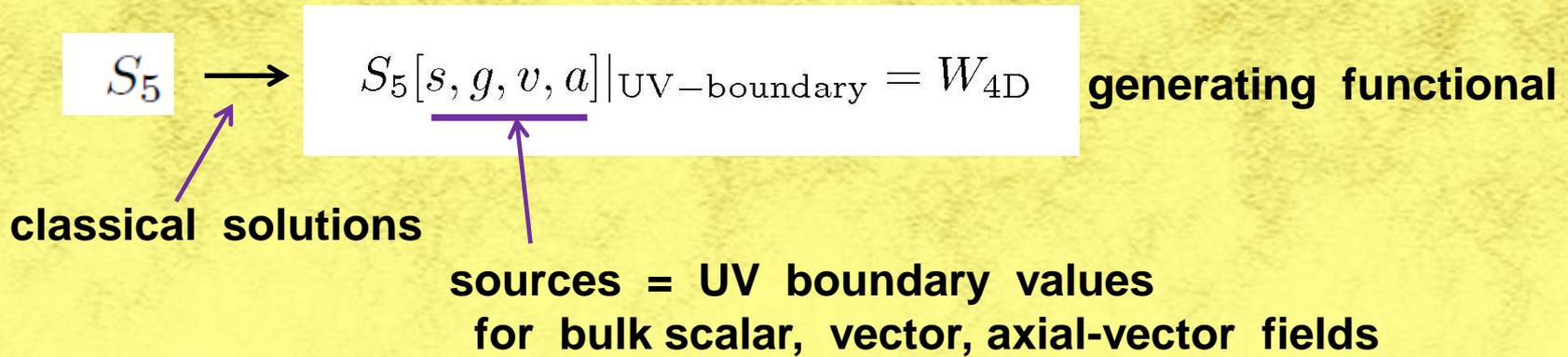


- Novel discovery channel: decays to $W(Z,\gamma)$ and TD!



S.Matsuzaki and K.Y., in progress

* **AdS/CFT recipe:**



$$S_5 = \int d^4x \int_{\epsilon}^{z_m} dz \sqrt{-g} \frac{1}{g_5^2} e^{cg_5^2 \Phi_X(z)} \left(-\frac{1}{4} \text{Tr} [L_{MN} L^{MN} + R_{MN} R^{MN}] \right. \\ \left. + \text{Tr} [D_M \Phi^\dagger D^M \Phi - m_\Phi^2 \Phi^\dagger \Phi] + \frac{1}{2} \partial_M \Phi_X \partial^M \Phi_X \right)$$

$$\Phi(x, z) = \frac{1}{\sqrt{2}} (v(z) + \sigma(x, z)) \exp[i\pi(x, z)/v(z)] \\ \Phi_X(z) = v_X(z),$$

AdS/CFT dictionary:

* UV boundary values = sources

$$\alpha M = \lim_{\epsilon \rightarrow 0} Z_m \left(\frac{L}{z} v(z) \right) \Big|_{z=\epsilon}, \quad Z_m = Z_m(L/z) = \left(\frac{L}{z} \right)^{\gamma_m}$$

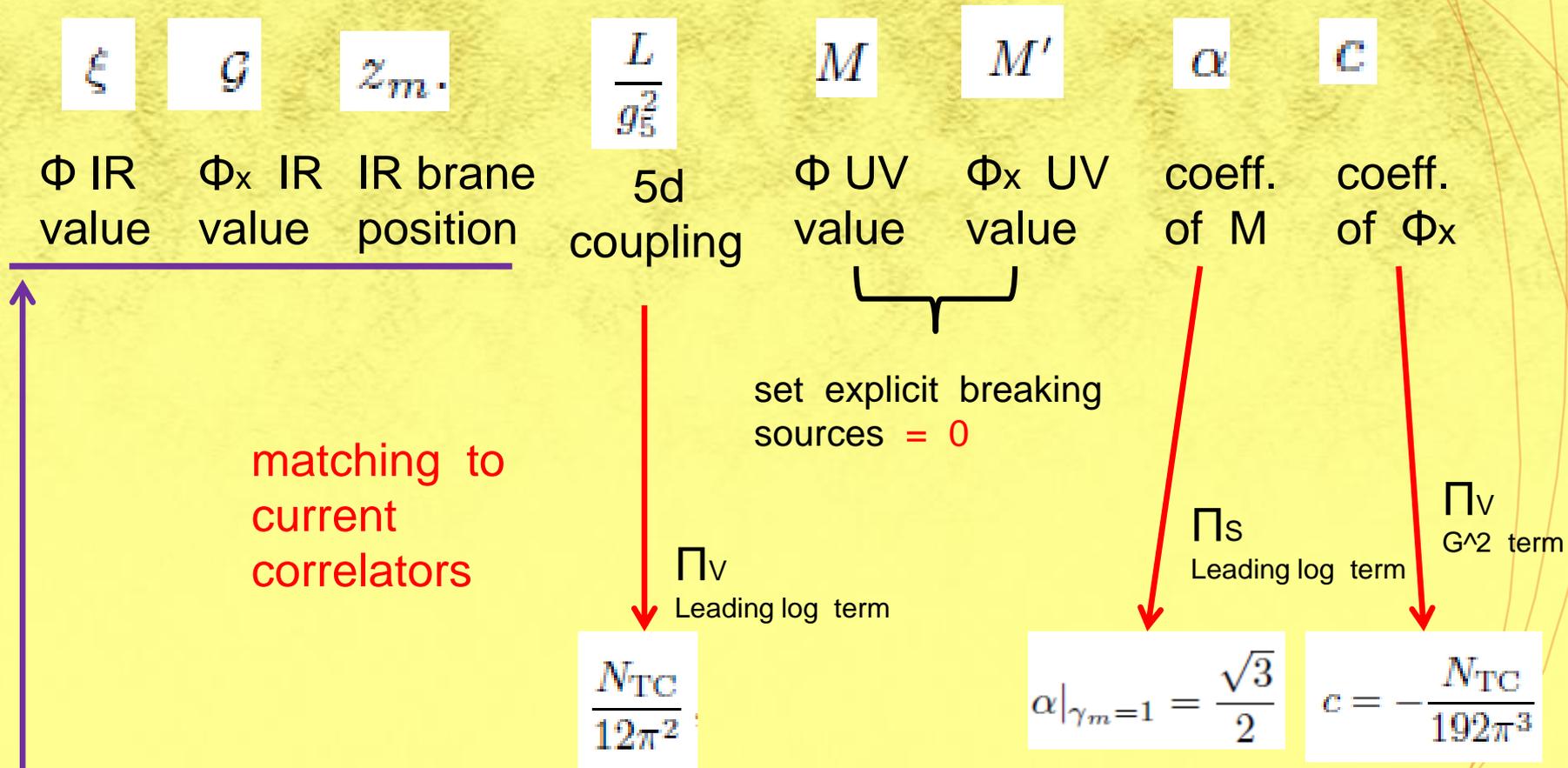
$$M' = \lim_{\epsilon \rightarrow 0} L v_X(z) \Big|_{z=\epsilon}$$

* IR boundary values:

$$\xi = L v(z) \Big|_{z=z_m} \longleftrightarrow \text{chiral condensate } \langle \bar{T} T \rangle$$

$$\mathcal{G} = L v_X(z) \Big|_{z=z_m} \longleftrightarrow \text{gluon condensate } \langle \alpha G_{\mu\nu}^2 \rangle$$

The model parameters:



Fix

3 phenomenological input values

$$\begin{aligned}
 F_\pi &= 246 \text{ GeV} / \sqrt{N_D} = 123 \text{ GeV} \quad (1\text{FM}) \\
 M_\Phi &= 125 \text{ GeV} \\
 S &= 0.1
 \end{aligned}$$



Other holographic predictions (1FM w/ S=0.1)

NTC = 3

Techni- ρ , a1 masses	:	$M_\rho = M_{a1} = 3.5$ TeV
Techni-glueball (TG) mass	:	$M_G = 19$ TeV
TG decay constant	:	$F_G = 135$ TeV
dynamical TF mass m_F	:	$m_F = 1.0$ TeV

NTC = 4

Techni- ρ , a1 masses	:	$M_\rho = M_{a1} = 3.6$ TeV
Techni-glueball (TG) mass	:	$M_G = 18$ TeV
TG decay constant	:	$F_G = 156$ TeV
dynamical TF mass m_F	:	$m_F = 0.95$ TeV

NTC = 5

Techni- ρ , a1 masses	:	$M_\rho = M_{a1} = 3.9$ TeV
Techni-glueball (TG) mass	:	$M_G = 18$ TeV
TG decay constant	:	$F_G = 174$ TeV
dynamical TF mass m_F	:	$m_F = 0.85$ TeV

Other pheno. issues in TC scenarios

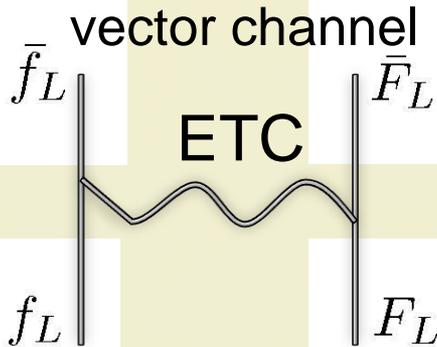
S parameter

$$S \approx N_D \cdot \frac{8\pi F^2}{M_\rho^2} \simeq \underline{0.3 \cdot N_D} \quad (\text{for QCD-like})$$

N_D : # EW doublets

too large! Cf: $S(\text{exp}) < 0.1$ around $T=0$

One resolution: **ETC-induced “delocalization” operator**



$$-\frac{1}{\Lambda_{\text{ETC}}^2} J_{\mu\text{SM}_L}^a J_{\text{TC}_L}^{\mu a}$$

Chivukula-Simmons-He-Kurachi-Tanabashi (2005)

in low-energy

$$J_{\text{TC}_L}^{\mu a} \rightarrow \text{Tr}[U^\dagger \frac{\sigma^a}{2} iD^\mu U]$$

$$\text{w/ } U = e^{2i\pi_{\text{eaten}}/v_{\text{EW}}}$$

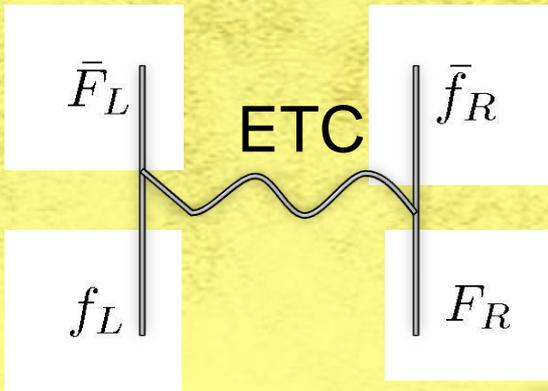
$\ni g_W W_\mu - g_Y B_\mu$ modifies SM f-couplings to W, Z

contributes to S “negatively”

$$\Delta S \sim -\frac{8\pi}{g_W^2} \left(\frac{v_{\text{EW}}}{\Lambda_{\text{ETC}}} \right)^2$$

$S_{\text{total}} \rightarrow 0$ (“ideal delocalization”)

Top quark mass generation



$$m_t \approx \frac{\langle \bar{U}U \rangle_{\text{ETC}}}{\Lambda_{\text{ETC}}^2} \approx \left(\frac{\Lambda_{\text{TC}}}{\Lambda_{\text{ETC}}} \right)^2 \Lambda_{\text{TC}}$$

ETC scale associated w/ top mass

$$\Lambda_{\text{ETC}}^{\text{top}} \approx 1\text{TeV} \left(\frac{\Lambda_{\text{TC}}}{1\text{TeV}} \right)^{3/2} \left(\frac{172\text{GeV}}{m_t} \right)^{1/2}$$

too small!

One resolution: *Strong ETC* Miransky-K.Y. (1989), Matumoto(1989), Appelquist-Einhorn-Takeuchi-Wijewardhana (1989)

--- makes induced 4-fermi (tt UU) coupling large enough to trigger chiral symm. breaking (almost by NJL dynamics)

$$\langle \bar{U}U \rangle_{\text{ETC}} \approx \left(\frac{\Lambda_{\text{ETC}}}{\Lambda_{\text{TC}}} \right)^{\gamma_m} \langle \bar{U}U \rangle_{\text{TC}} \quad 1 < \gamma_m \leq 2$$

boost-up



$$m_t \approx \left(\frac{\Lambda_{\text{TC}}}{\Lambda_{\text{ETC}}} \right)^{2-\gamma_m} \Lambda_{\text{TC}} \leq \Lambda_{\text{TC}} \sim 1\text{TeV}$$

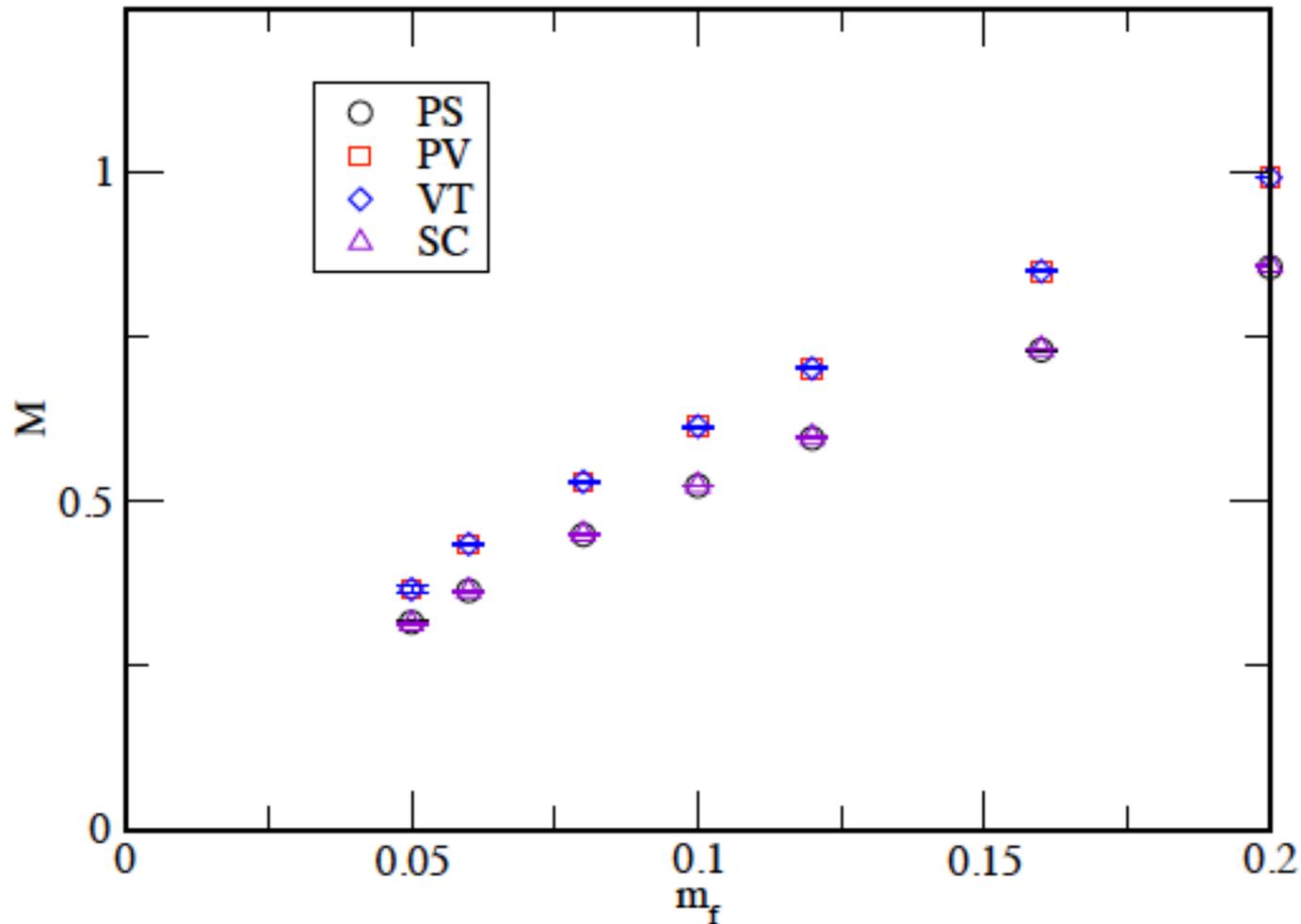
T parameter (Strong) ETC generates large isospin breaking
 → highly model-dependent issue

$N_f = 12$

Nf=12 Taste Symmetry (HISQ)

LatKMI Collaboration, PRD86 (2012)054506

PS, SC; 1^- : PV, VT



Difficulty Flavor – singlet scalar

$$\langle 0|S(t)S^\dagger(0)|0\rangle, \quad S(t) = \sum_{\vec{x}} \bar{\psi}_a(\vec{x}, t)\psi_a(\vec{x}, t)$$

$$D(t) = \left\langle \begin{array}{c} \times \bigcirc \quad \bigcirc \times \end{array} \right\rangle - \left\langle \begin{array}{c} \times \bigcirc \end{array} \right\rangle^2$$

1. $\times \bigcirc = \text{Tr}[\psi(x)\bar{\psi}(x)] = \text{Tr}[D^{-1}[U](x, x)]$ at each U

$O(L^3 \times T)$ $D^{-1}[U]$ in naive method

$O(1000)$ $D^{-1}[U]$ in simple method

$\rightarrow O(100)$ $D^{-1}[U]$ in noise reduction method

(Kilcup and Sharpe;NPB283(1987)493, Venkataraman and Kilcup;hep-lat/9711006)

2. $\langle \text{Large} + \text{small} \rangle - \langle \text{Large} \rangle = \langle \text{small} \rangle + (\text{stat. error})$

$\langle \text{small} \rangle: \exp(-m_\sigma t)$; stat. error: independent of t

$\rightarrow O(10000)$ configuration

Reduce calculation cost and use huge N_{conf}

Flavor-singlet state from Glueball operator

no EW charge

Flavor-singlet scalar (0^{++} glueball) operator from U

$$O_i = \text{a) } \begin{array}{|c|} \hline \square \\ \hline \end{array} \text{ b) } \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \text{ c) } \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \dots$$

$$\langle 0|O_i(t)O_j^\dagger(0)|0\rangle - \langle 0|O_i|0\rangle\langle 0|O_j^\dagger|0\rangle, \quad i, j = a, b, c$$

Same difficulty as meson operator \rightarrow Huge statistical noise

Noise reduction techniques (Lucini, Rago, Rinaldi; JHEP08(2010)119)

- Fattening link
- Large size operator
- Diagonalization of correlation function matrix

Same m_σ is obtained from meson and glueball correlators, in principle.

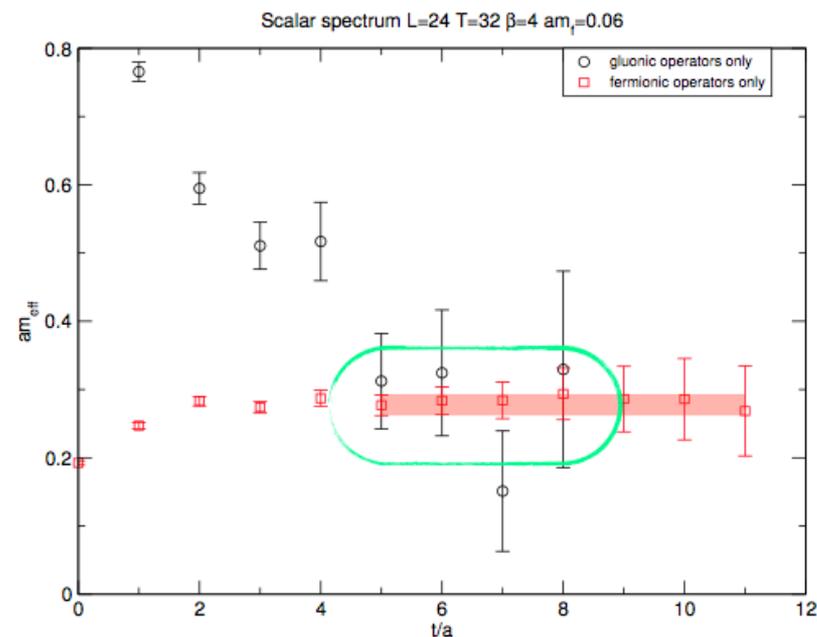
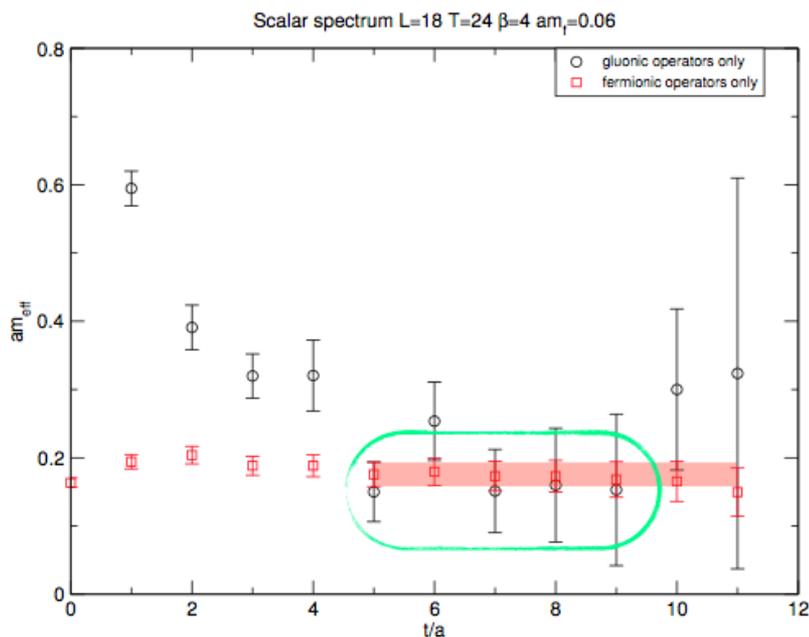
\rightarrow Reliability check of result, but never done before

Comparison of effective mass in $N_f = 12$

($m_f = 0.06$, $18^3 \times 24$ with $N_{\text{conf}} = 5000$, $24^3 \times 32$ with $N_{\text{conf}} = 14000$, Preliminary)

$$m_{\text{eff}}(t) = \log(C_H(t)/C_H(t+1)) \xrightarrow{t \gg 1} m_H$$

Glueball correlator and meson $D(t)$



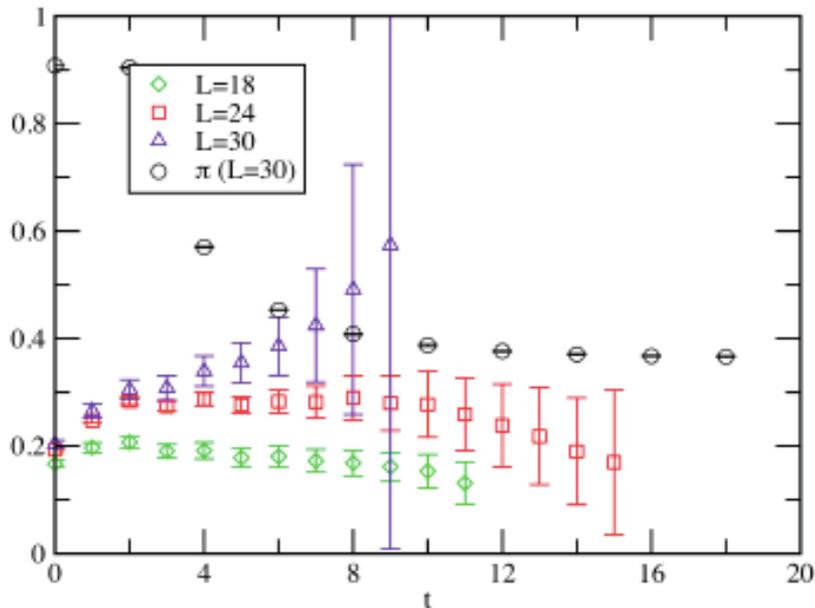
Larger error in glueball correlator

Reasonably consistent in large t

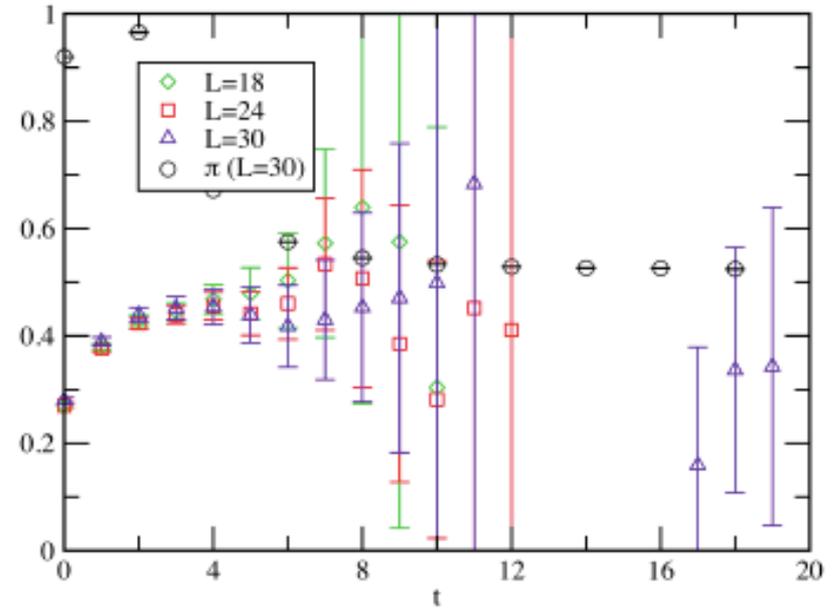
→ show only meson results

Volume dependence of effective mass from $D(t)$
 $L=18, 24, 32$ $\beta=4.0$ $mf=0.06$ & 0.10

$m=0.06$



$m=0.10$



- At $m=0.10$, all the results for $L=18, 24, 30$ are consistent.
- At $m=0.06$, $m\sigma(L=18) < m\sigma(L=24)$ and large statistical fluctuation in $L=30$.