Introduction to Accelerator Physics

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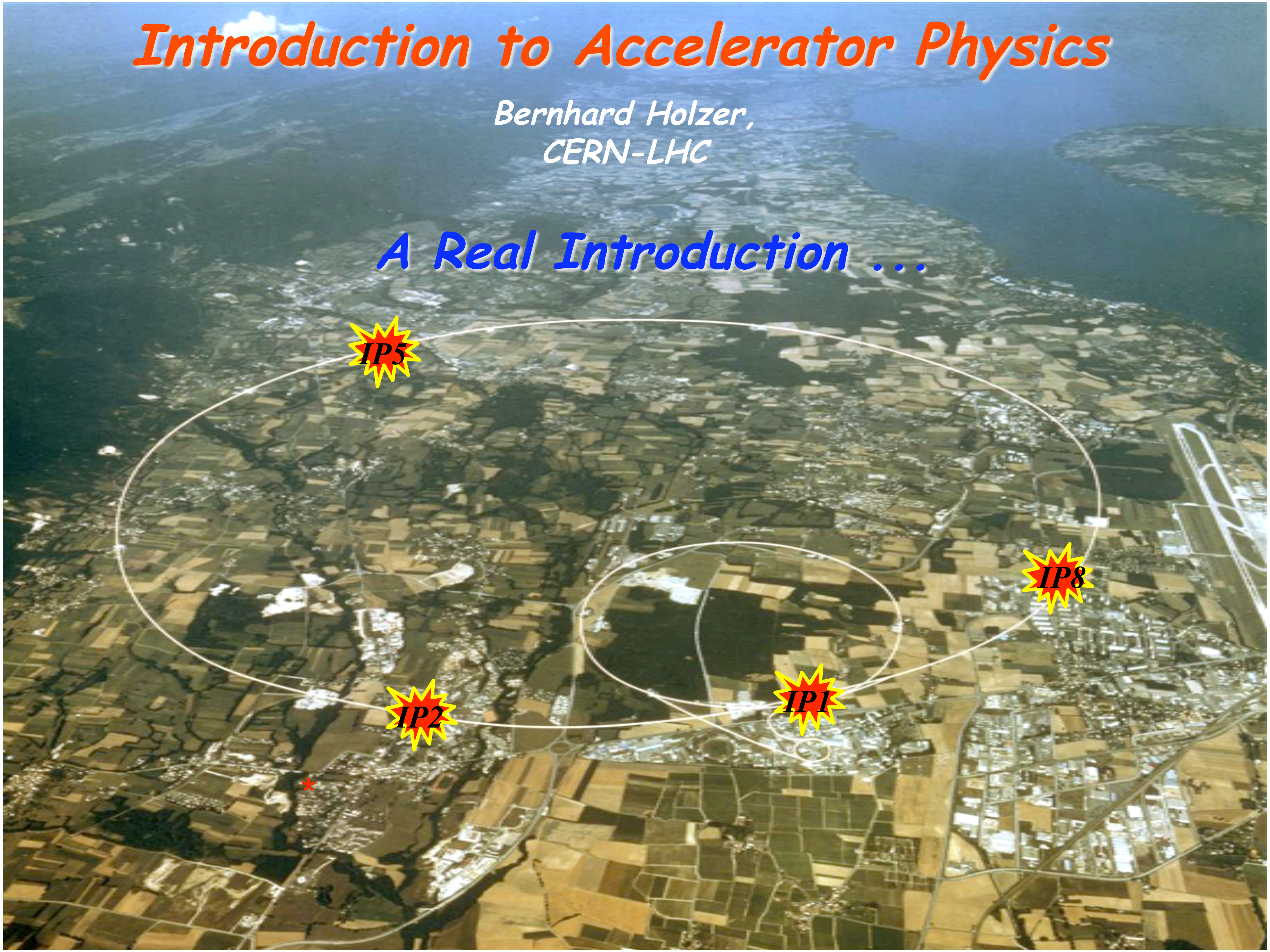
A Real Introduction ...



the beta function is usually obtained via the matrix element „m12", which is in Twiss form for the undistorted case

$$m_{12} = \beta_0 \sin 2\pi Q$$

$$(1) \quad m_{12}^* = \beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds$$

and including the error:

$$\underbrace{m_{12}^*}_{} = b_{11} a_{12} + b_{12} a_{22} - b_{12} a_{12} \Delta k ds$$

$$m_{12} = \beta_0 \sin 2\pi Q$$

As M is still a matrix for one complete turn we still can express the element m_{12} in twiss form:*

$$(2) \quad m_{12}^* = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

Equalising (1) and (2) and assuming a small error

$$-a_{12} b_{12} \Delta k ds = \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

$$dQ = \frac{\Delta k \beta_1 ds}{4\pi}$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

What we will NOT do .....

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi Q \underbrace{\cos 2\pi dQ}_{\approx 1} + \cos 2\pi Q \underbrace{\sin 2\pi dQ}_{\approx 2\pi dQ}$$

≈ 1

≈2 π dQ

$$-a_{12} b_{12} \Delta k ds = \frac{\beta_0 \Delta k \beta_1 ds}{2} \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = \beta_0 \sin 2\pi Q + \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q + d\beta_0 2\pi dQ \cos 2\pi Q$$

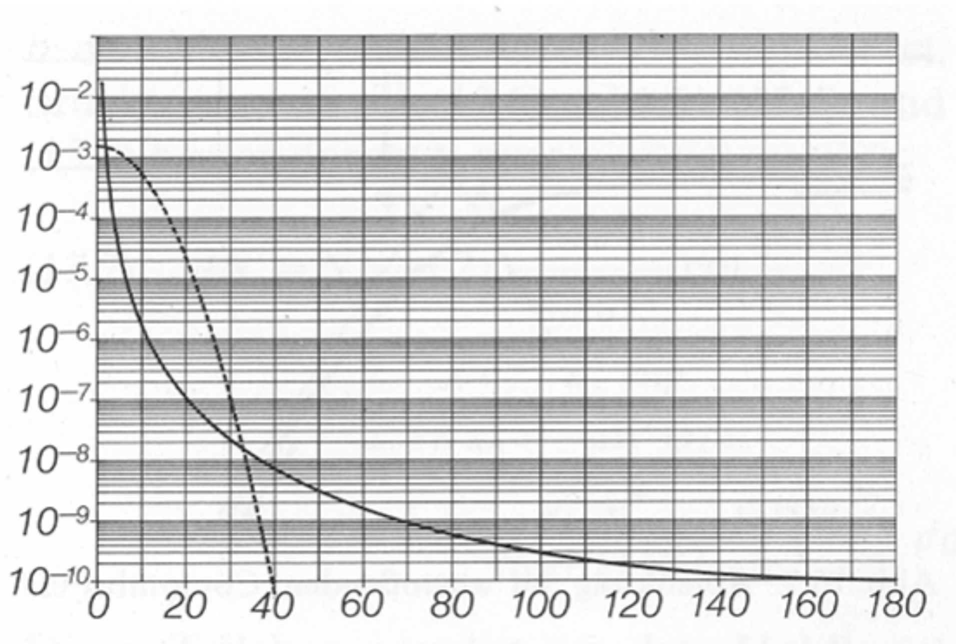
$$d\beta_0 = \frac{-1}{2 \sin 2\pi Q} \{2 a_{12} b_{12} + \beta_0 \beta_1 \cos 2\pi Q\} \Delta k ds$$

$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} (\cos\psi_s + \alpha_0 \sin\psi_s) & \sqrt{\beta_s \beta_0} \sin\psi_s \\ \frac{(\alpha_0 - \alpha_s)\cos\psi_s - (1 + \alpha_0 \alpha_s)\sin\psi_s}{\sqrt{\beta_s \beta_0}} & \sqrt{\frac{\beta_0}{\beta s}} (\cos\psi_s - \alpha_s \sin\psi_s) \end{pmatrix}$$

I.) A Bit of History

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$

N(θ)

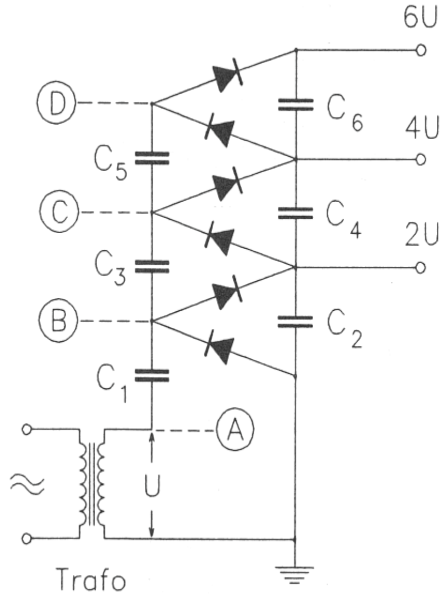


θ

Rutherford Scattering, 1911

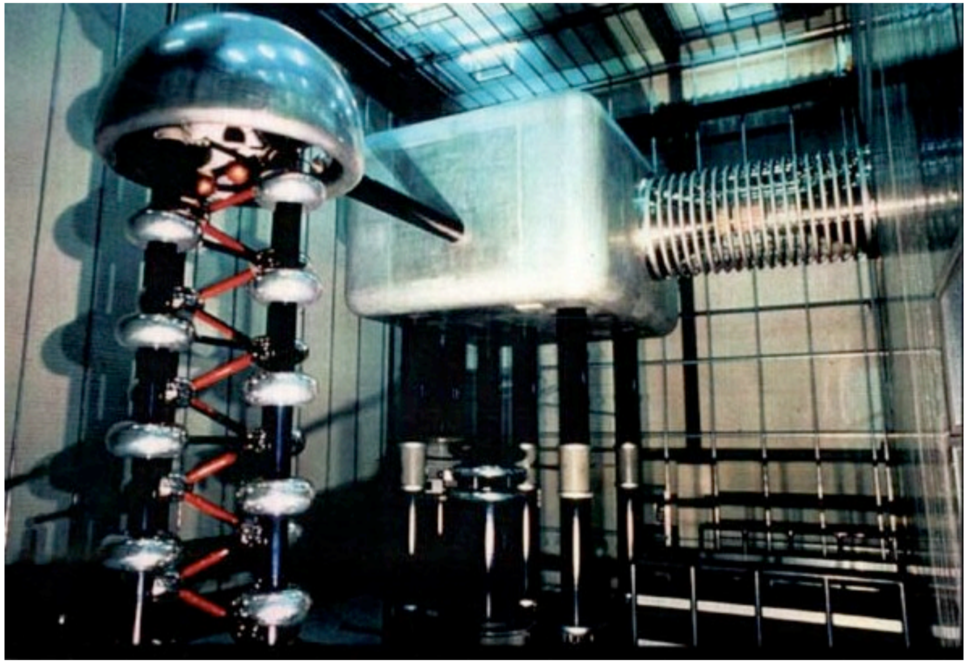
Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: ***Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam***

1932: ***First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV***



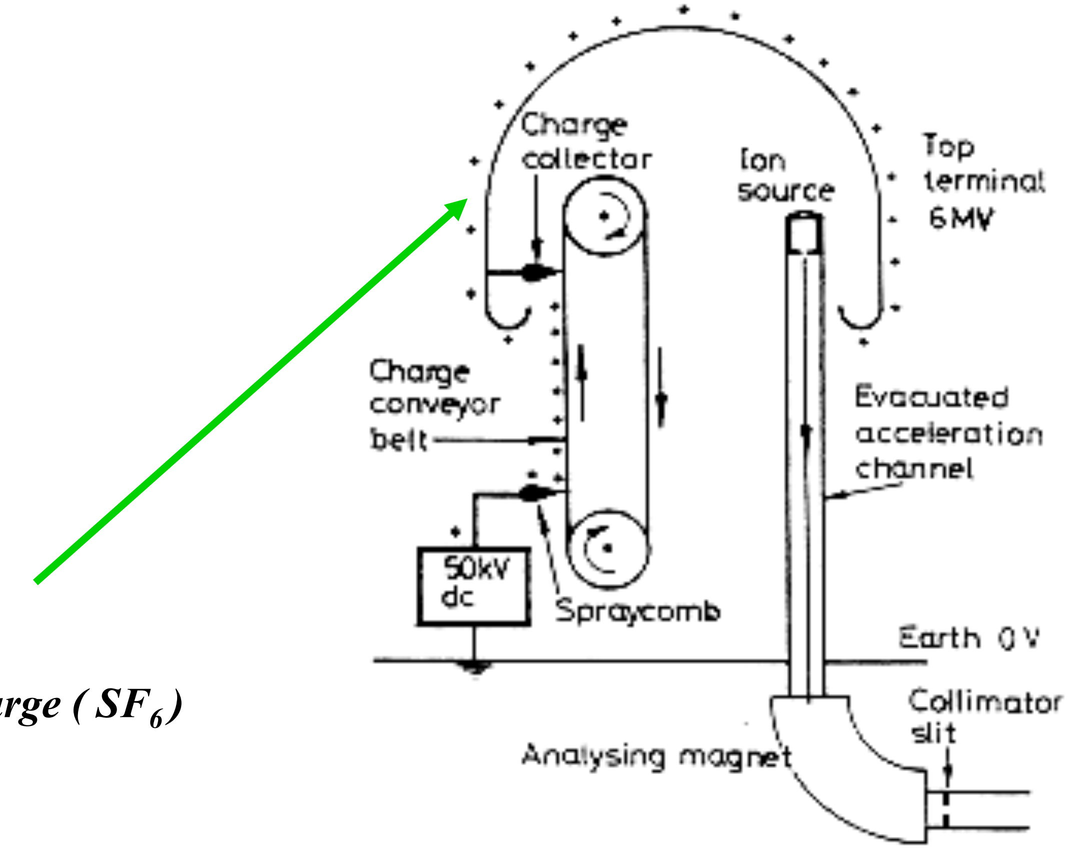
Particle source: ***Hydrogen discharge tube on 400 kV level***
Accelerator: ***evacuated glas tube***
Target: ***Li-Foil on earth potential***

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...

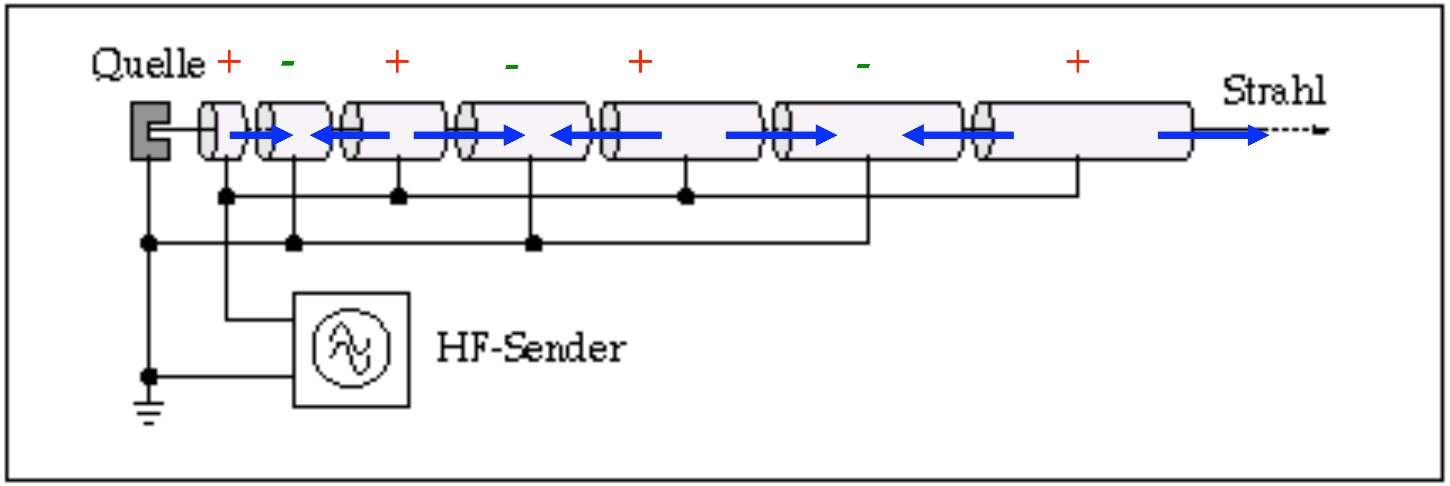
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

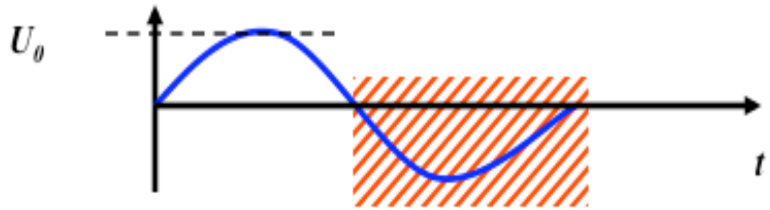
$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0 * \sin\psi_s}}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \ldots 0.6$, Particles: Protons/Ions

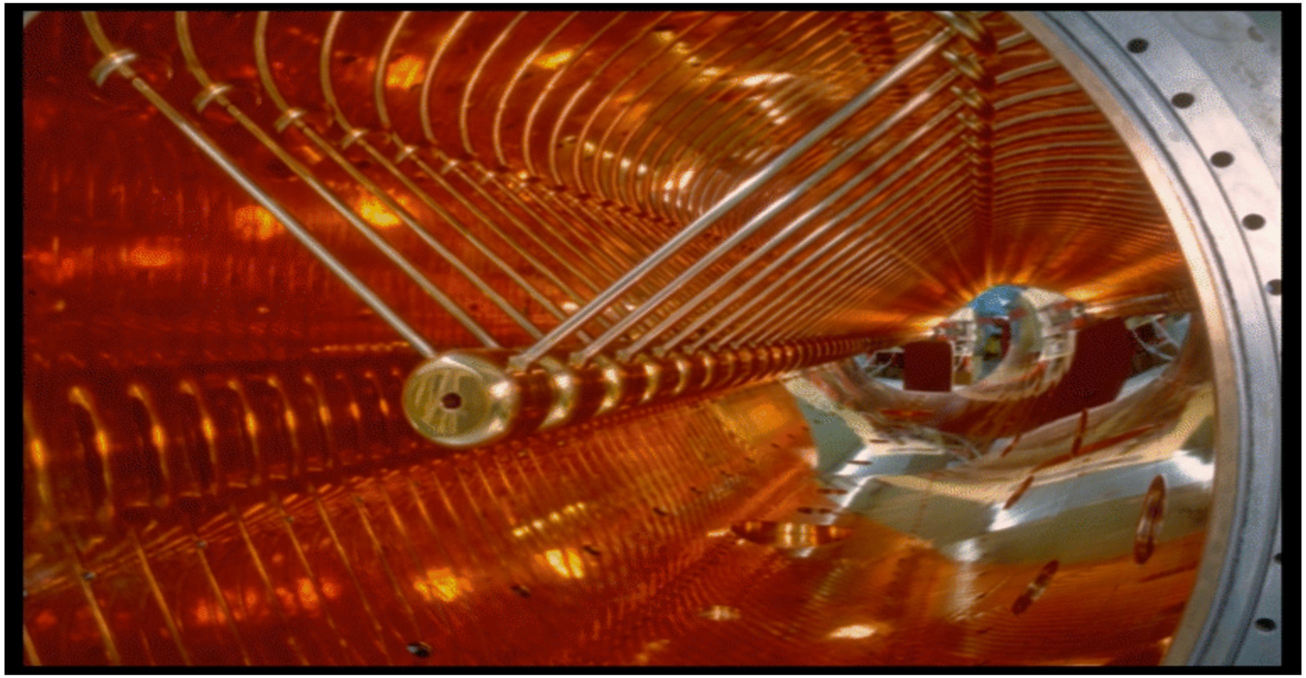
Accelerating structure of a Proton Linac (DESY Linac III)

$E_{total} = 988\ MeV$

$m_0 c^2 = 938\ MeV$

$p = 310\ MeV/c$

$E_{kin} = 50\ MeV$



Beam energies

Energy Gain per „Gap“: $W = q\,U_0 \sin \omega_{RF} t$

1.) reminder of some relativistic formula

rest energy $E_0 = m_0 c^2$

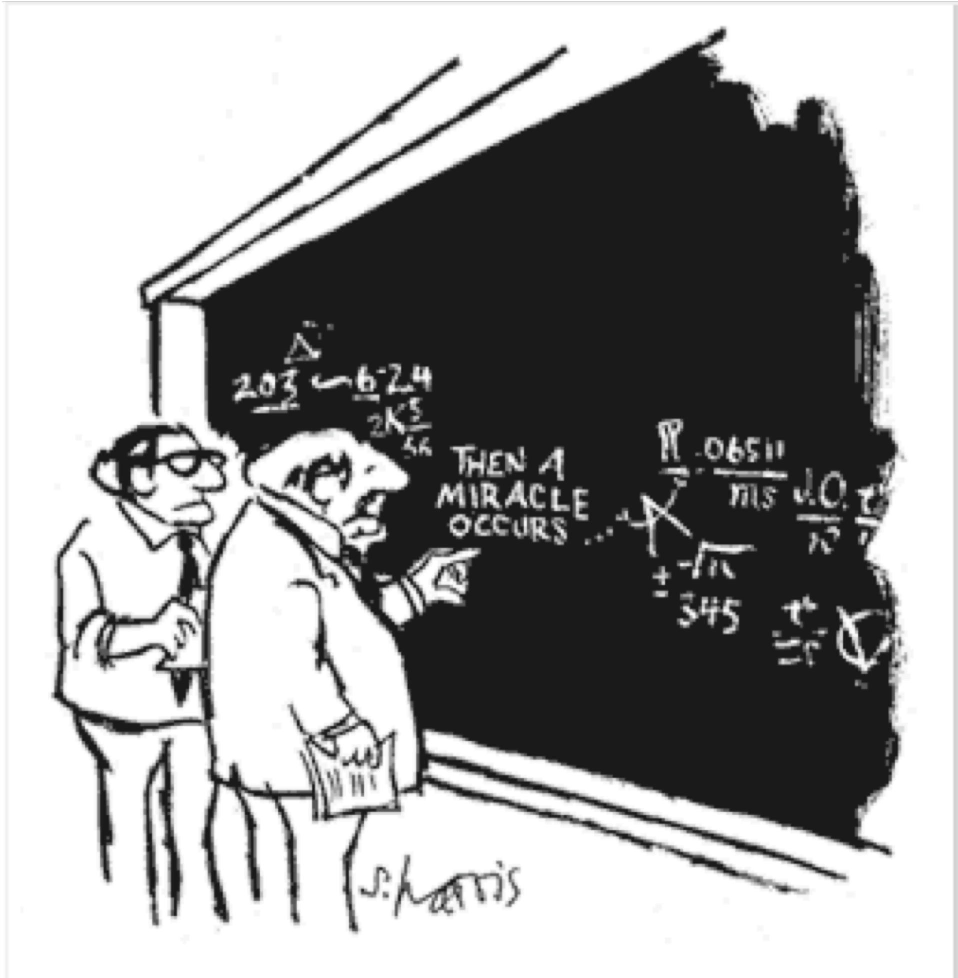
total energy $E = \gamma * E_0 = \gamma * m_0 c^2$

momentum $E^2 = c^2 p^2 + m_0^2 c^4$

kinetic energy $E_{kin} = E_{total} - m_0 c^2$

II.) A Bit of Theory

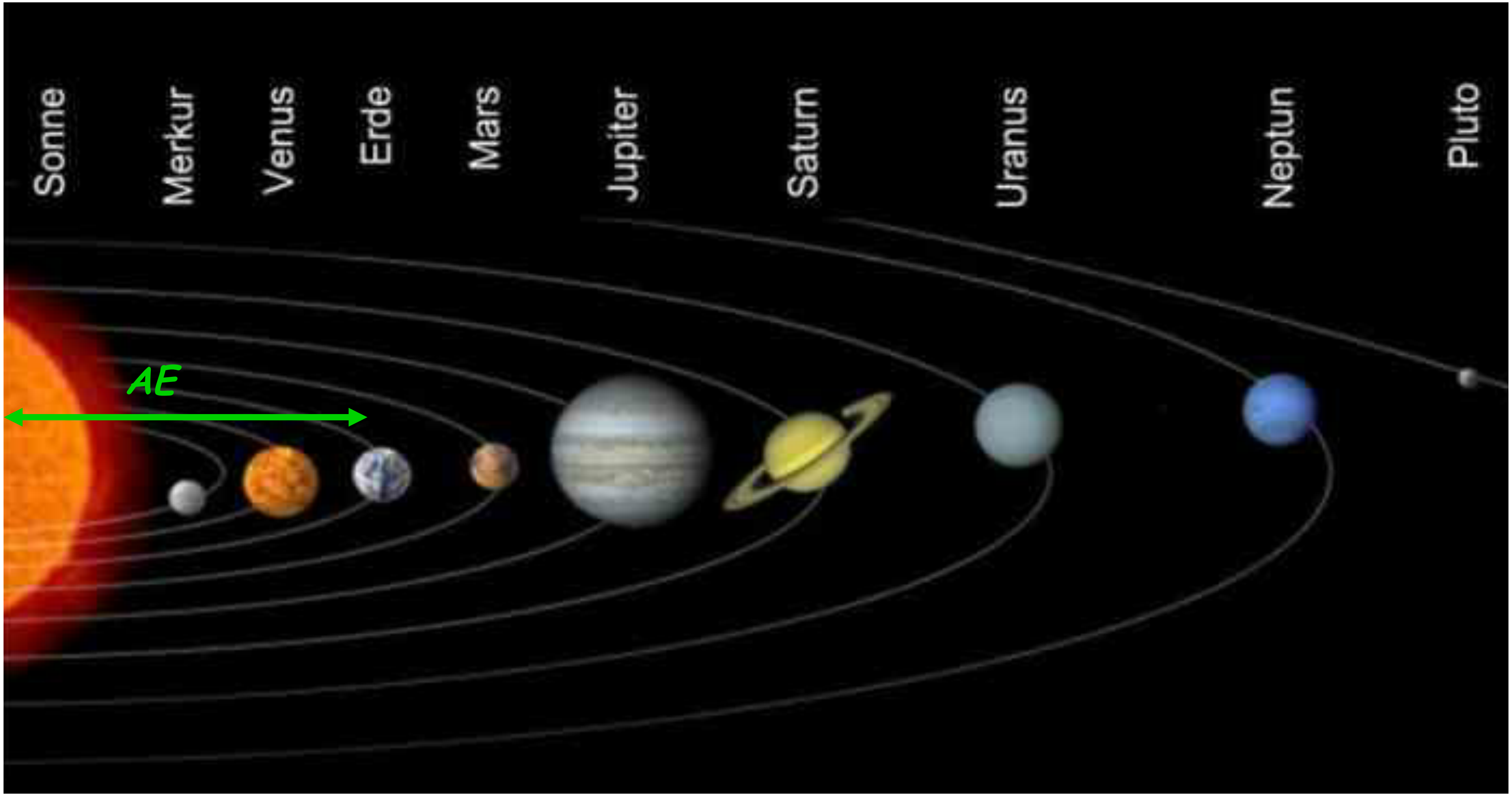
die grossen Speicherringe: „Synchrotrons“



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Largest storage ring: The Solar System

astronomical unit: average distance earth-sun
$1AE \approx 150 * 10^6$ km
Distance Pluto-Sun ≈ 40 AE



1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \, m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * 300 \frac{MV}{m}$$

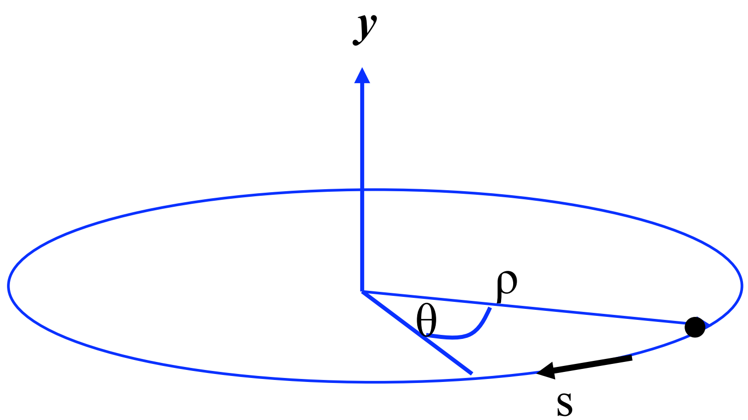
equivalent electrical field: E

technical limit for el. field:

$$E \leq 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $$F_L = e\, v\, B$$

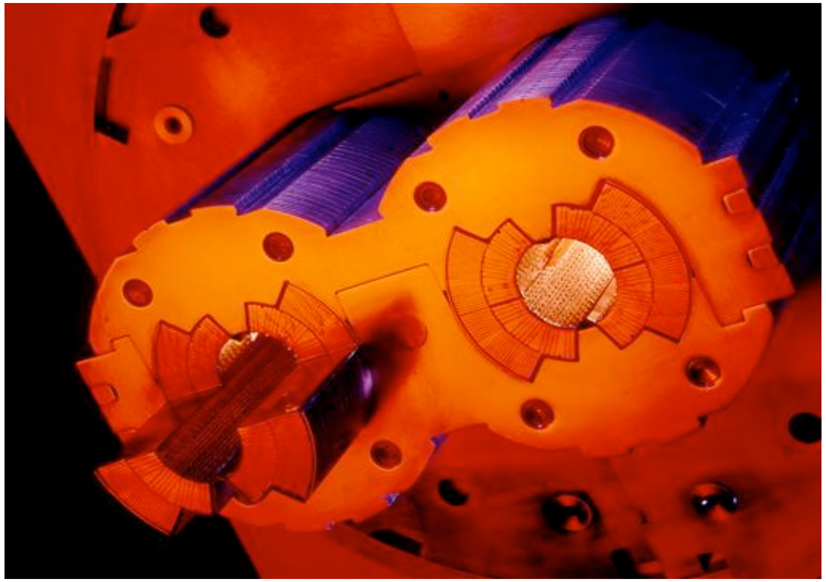
centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

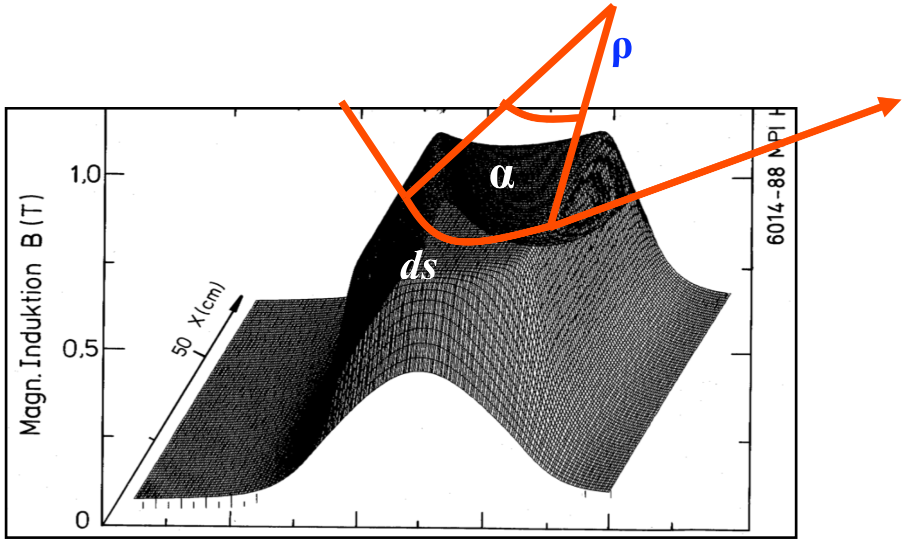
$$\frac{\gamma\, m_0\, v^2}{\rho} = e\, v\, B$$

$$\frac{p}{e} = B\,\rho$$

B ρ = "beam rigidity"

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.53\ km$ → $2\pi\rho = 17.6\ km$

$\approx 66\%$

$B \approx 1 \ldots 8\ T$

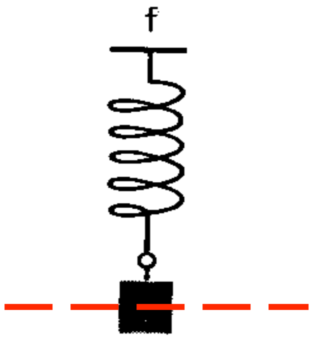
rule of thumb:

$$\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

„normalised bending strength"

2.) Focusing Properties – Kurzer Ausflug in die klassische Mechanik

classical mechanics: pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

***Storage Ring:* we need a *Lorentz force* that rises as a function of the distance to .... ?**

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

required: *focusing forces to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

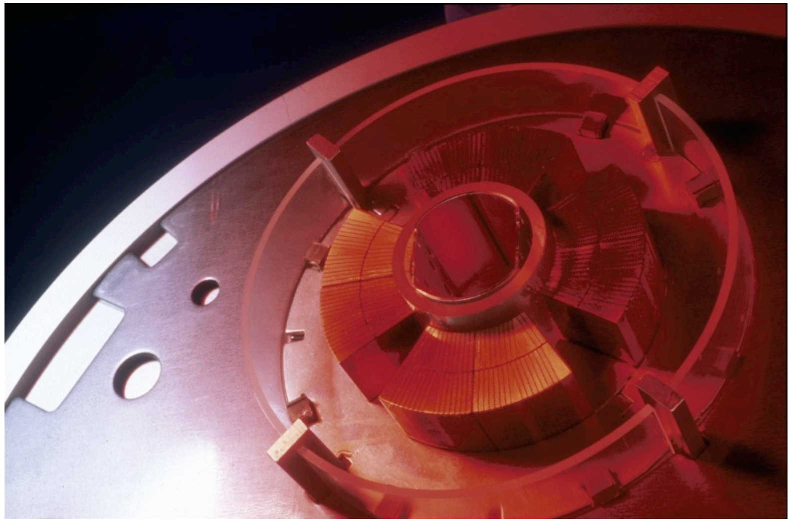
linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$

simple rule:

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220 \;\; T/m$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

Focusing forces and particle trajectories:

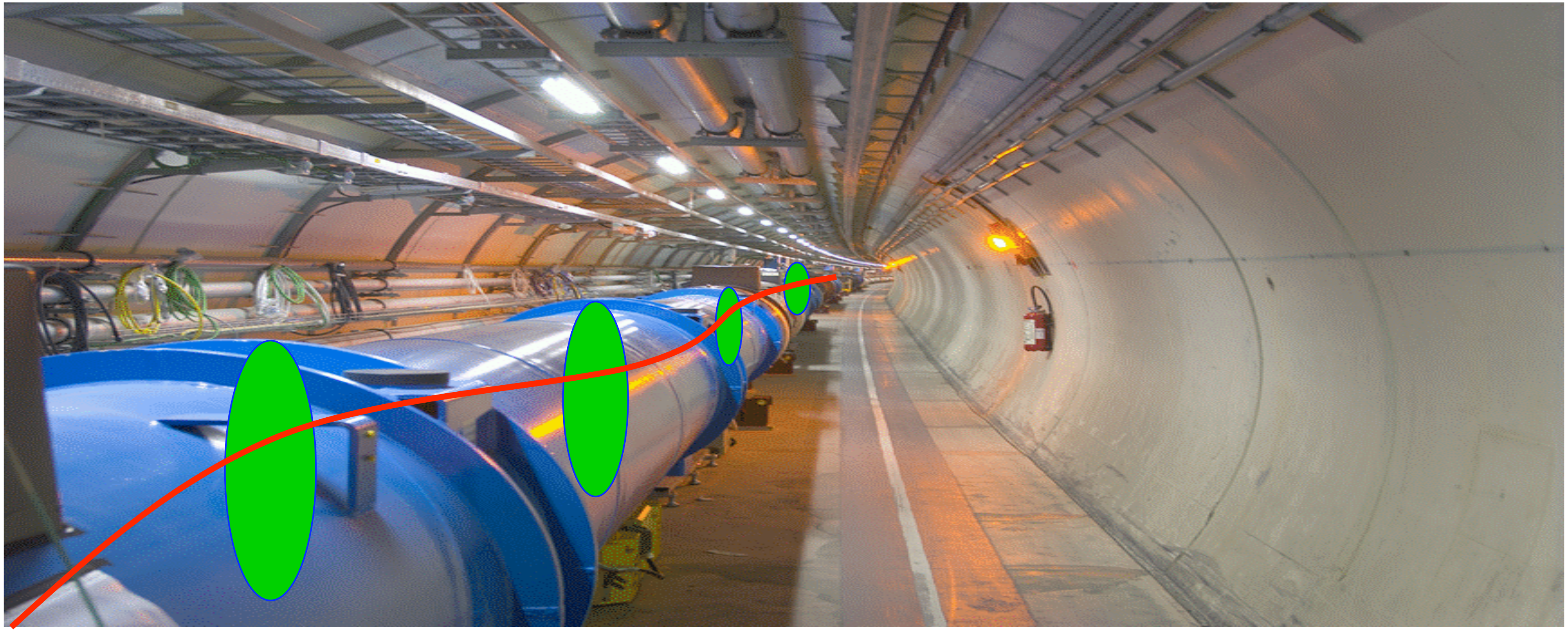
normalise magnet fields to momentum
(remember: ***B*ρ = p / q*** *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

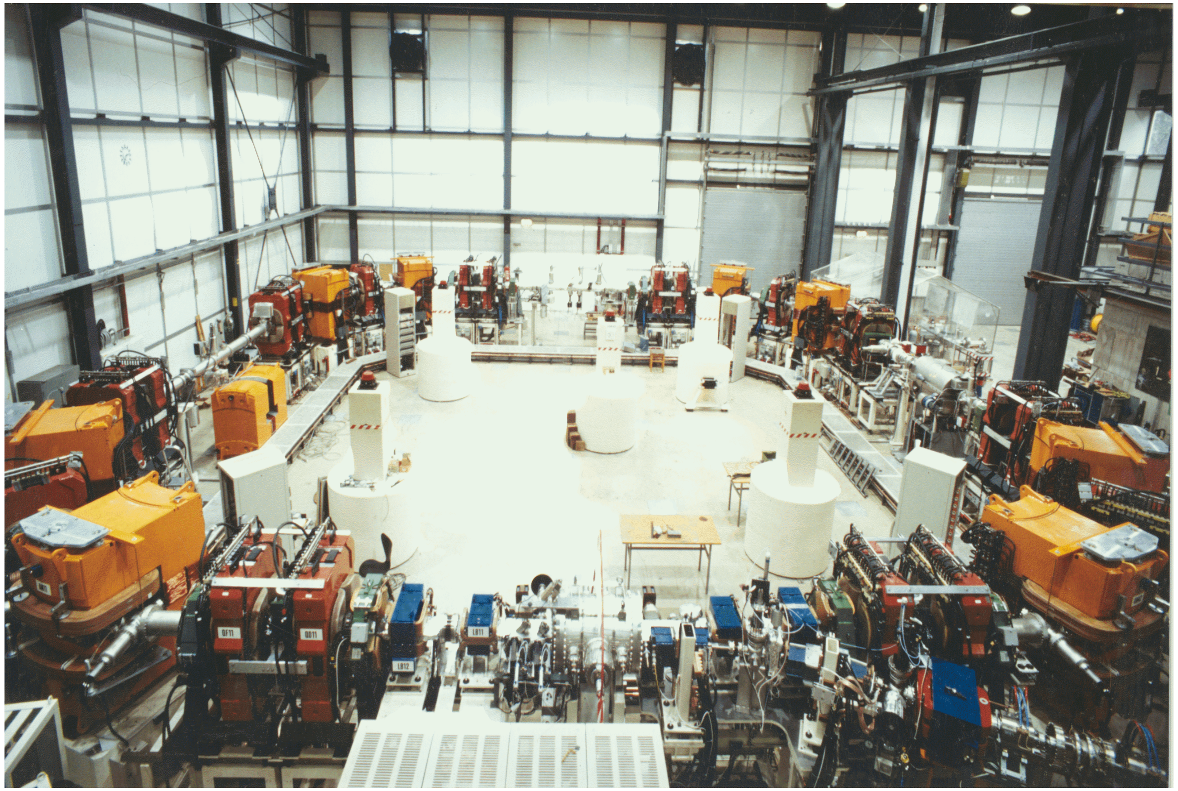
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \dots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

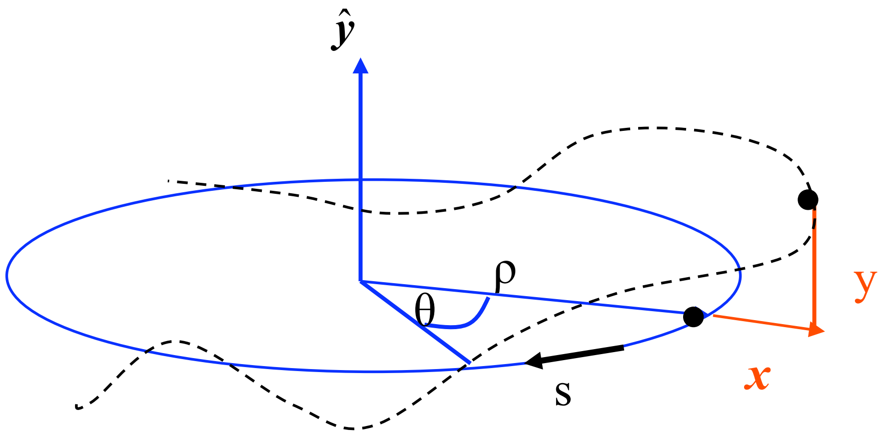
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* *man sieht nur dipole und quads → linear*

The Equation of Motion:



* *Equation for the horizontal motion:*

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

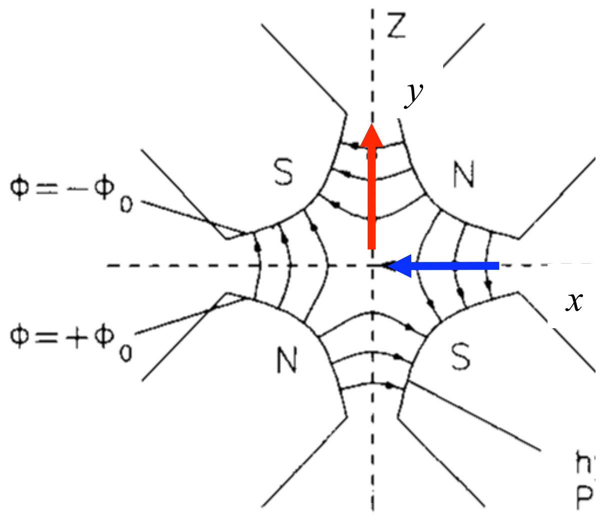
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* *Equation for the vertical motion:*

$\frac{1}{\rho^2} = 0$ *no dipoles … in general …*

$k \leftrightarrow -k$ *quadrupole field changes sign*

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

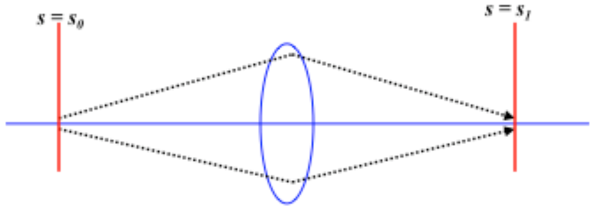
$$x'' + K\, x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



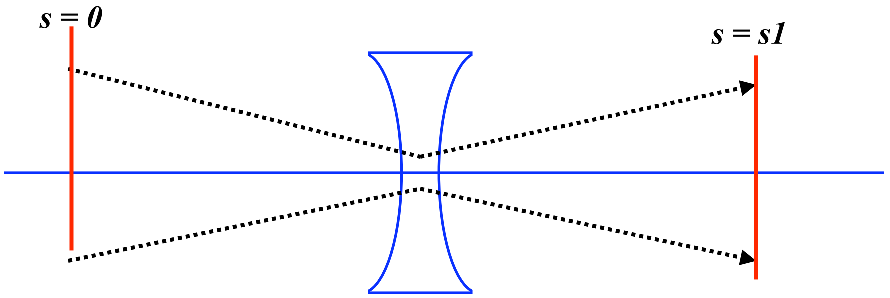
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



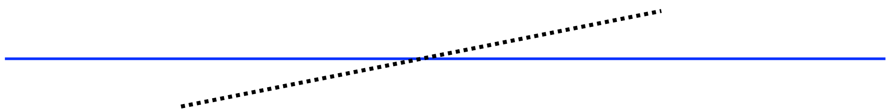
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x_0' * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

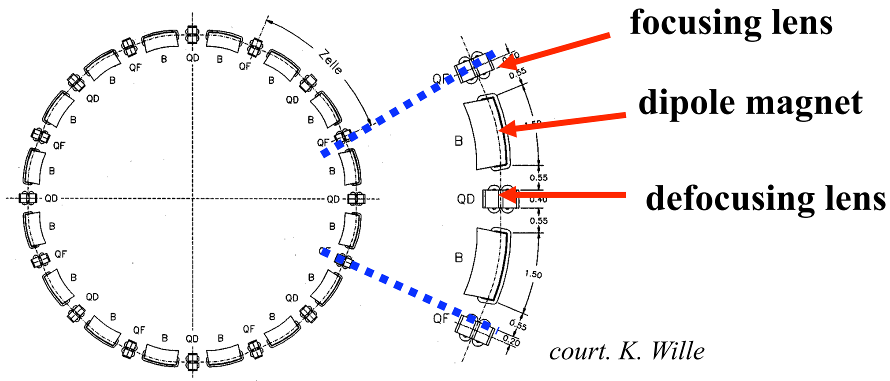
! ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“***

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

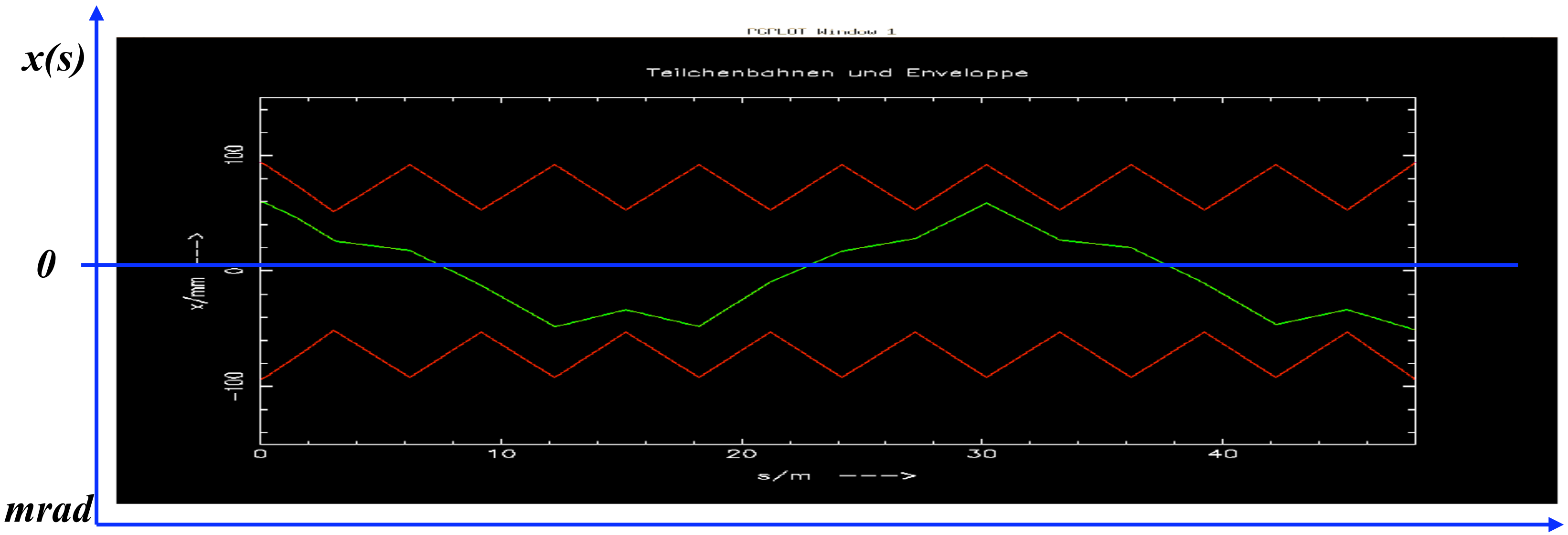
$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



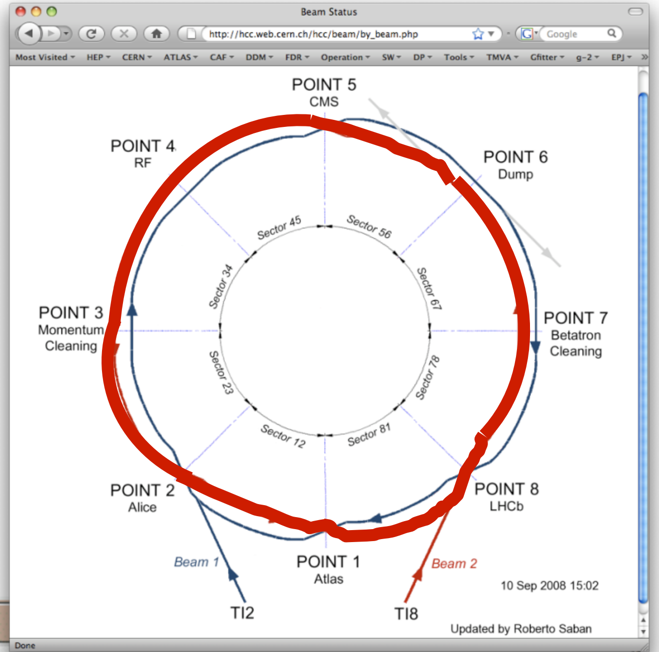
court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



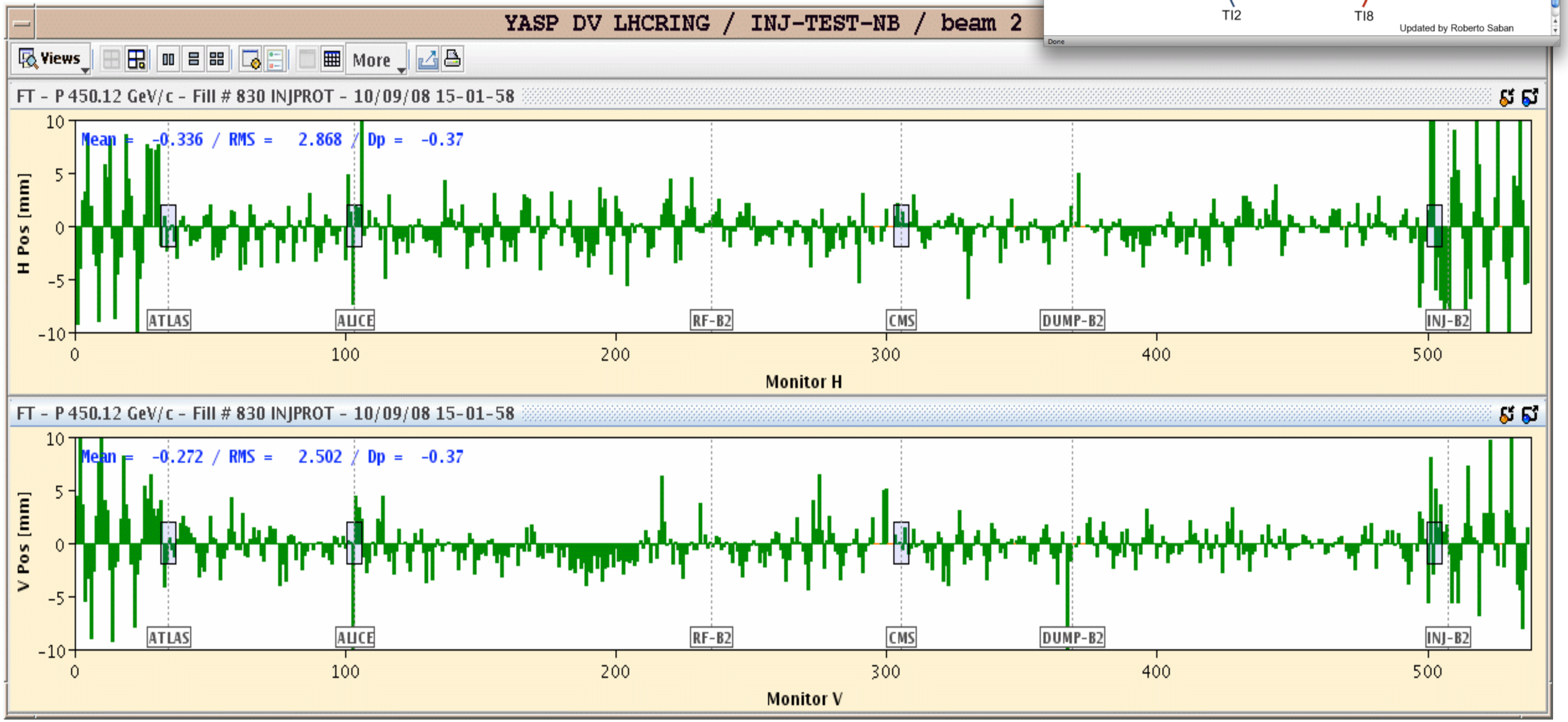
typical values
in a strong
foc. machine:
x ≈ mm, x´ ≤ mrad

LHC Operation: Beam Commissioning



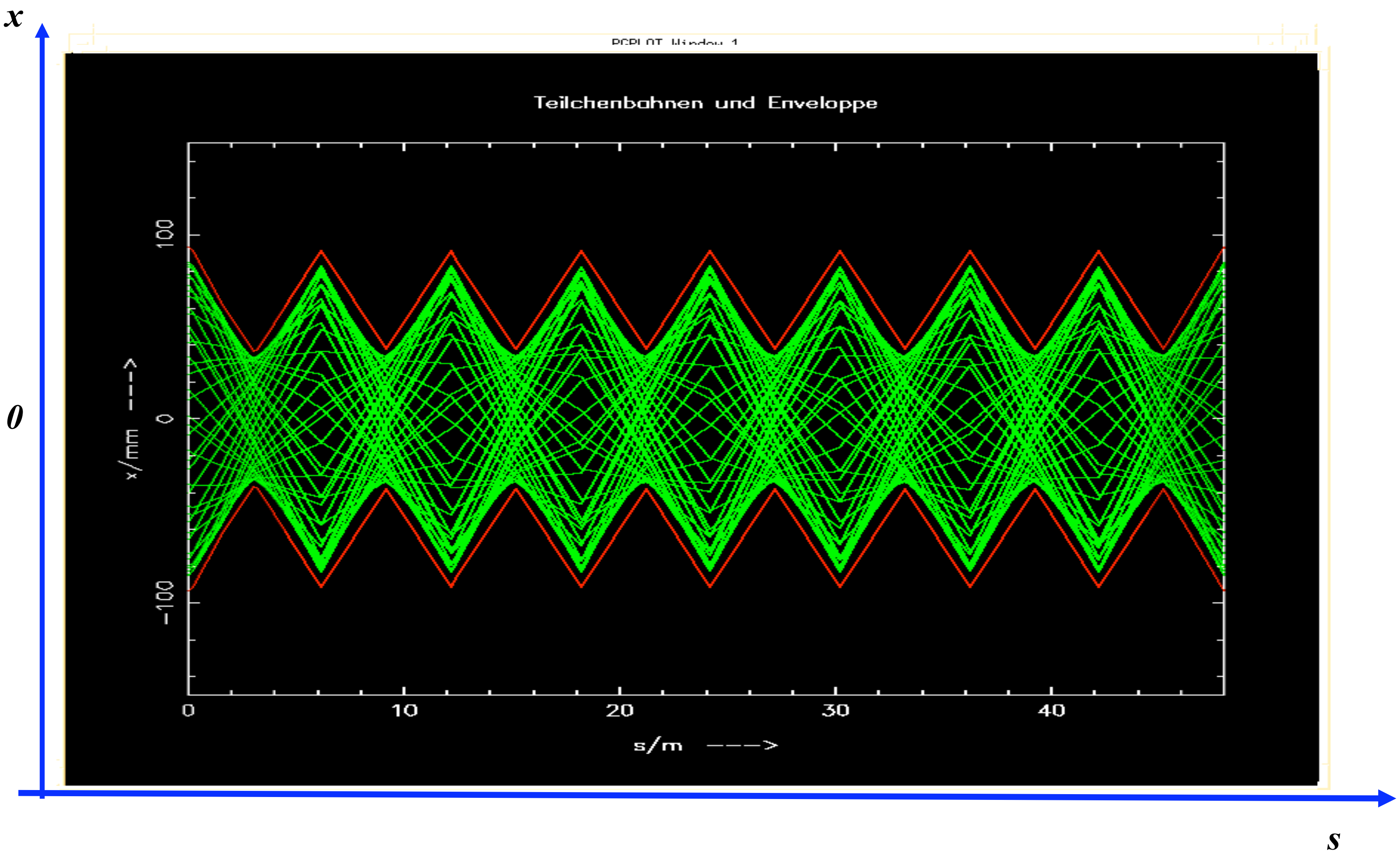
The transverse focusing fields create a harmonic oscillation of the particles with a well defined "Eigenfrequency" which is called tune

First turn steering "by sector:"



Question: ***what will happen, if the particle performs a second turn ?***

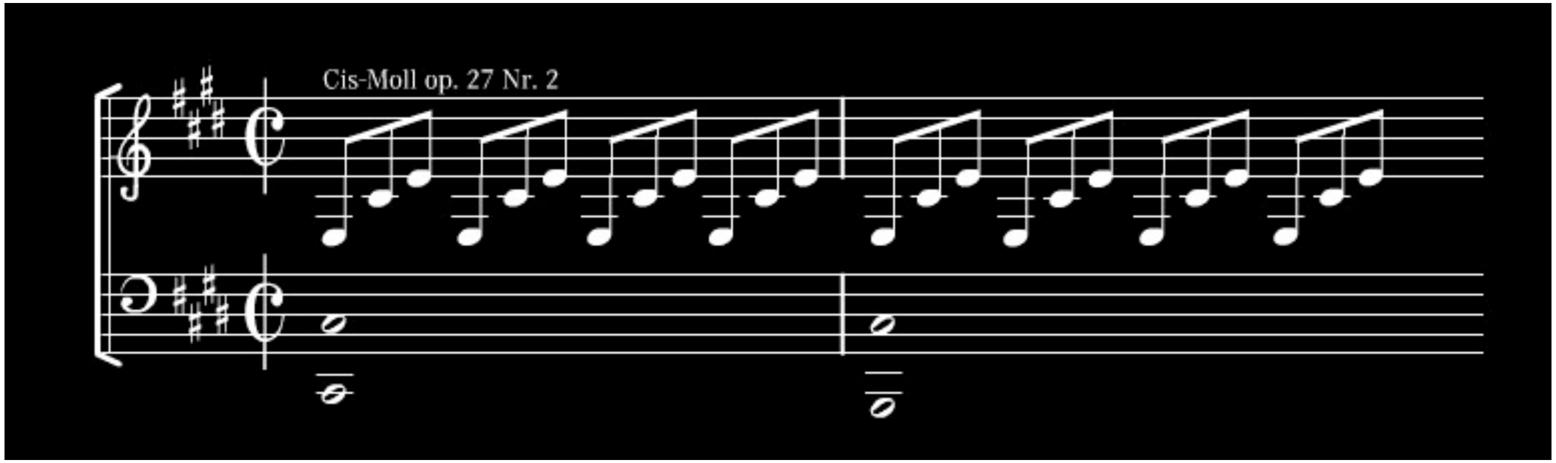
... or a third one or ... 10^{10} turns



19th century:

Ludwig van Beethoven: „Mondschein Sonate“

Sonate Nr. 14 in cis-Moll (op. 27/II, 1801)



Astronomer Hill:

differential equation for motions with periodic focusing properties „Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

Maximum size of a particle amplitude

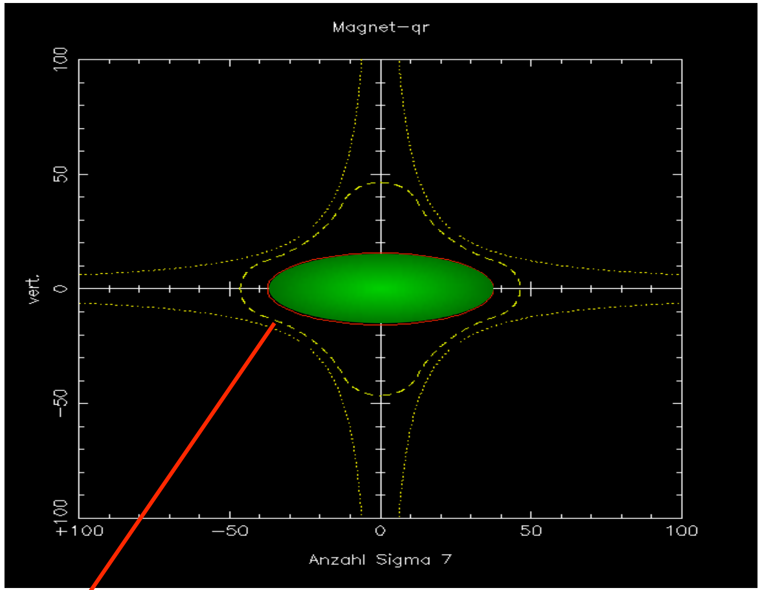
$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$

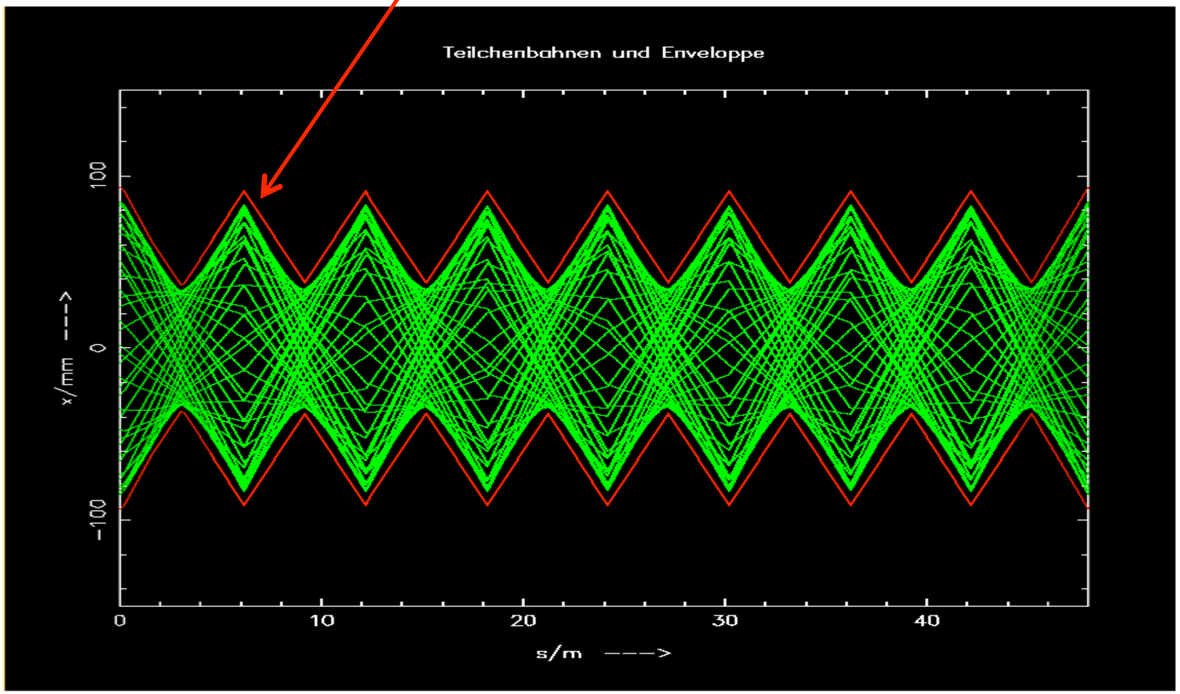
The Beta Function

β determines the beam size

... the envelope of all particle trajectories at a given position "s" in the storage ring under the influence of all (!) focusing fields.

It** reflects the periodicity **of the magnet structure.

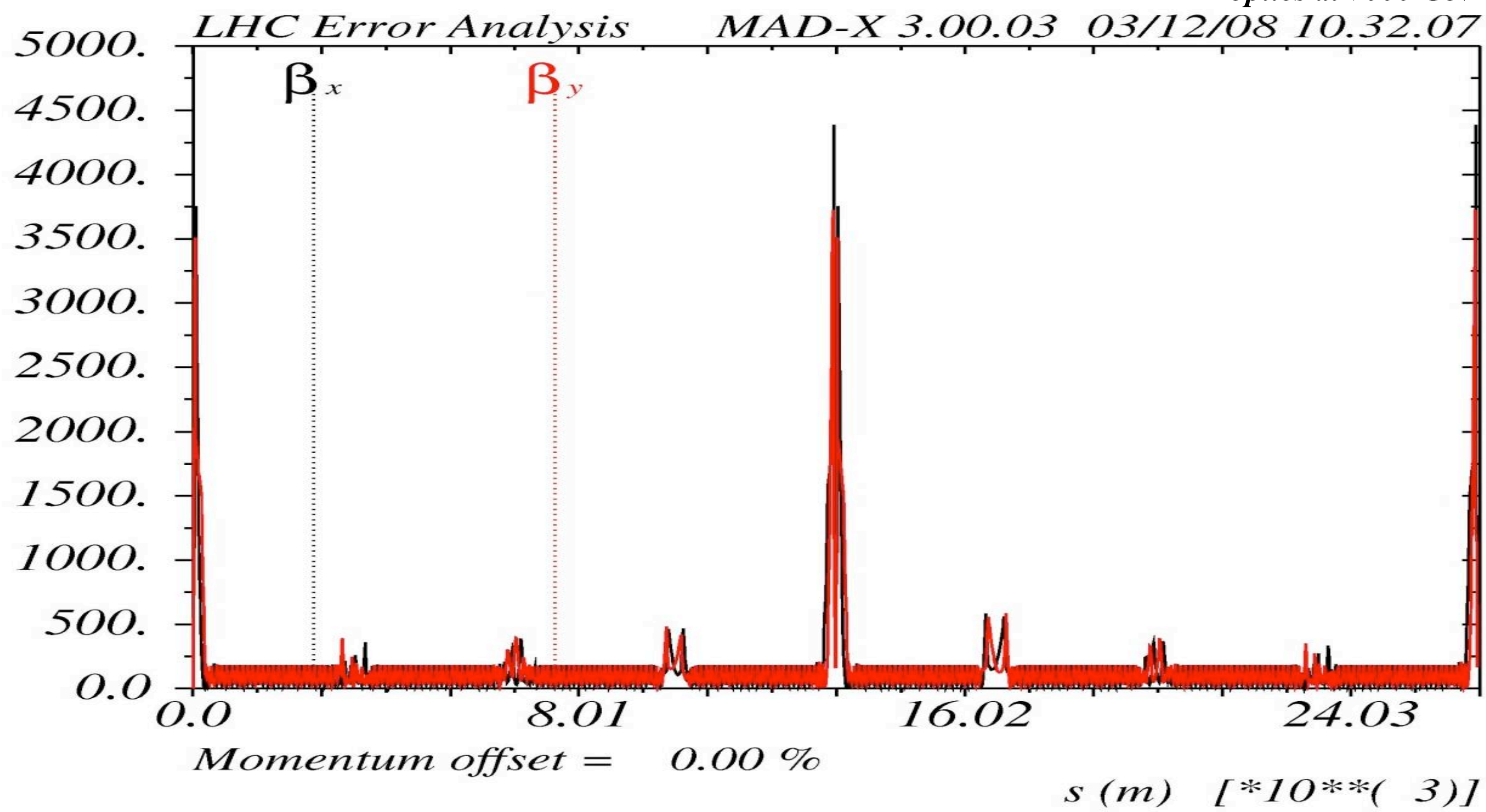




The Beta Function: Lattice Design & Beam Optics

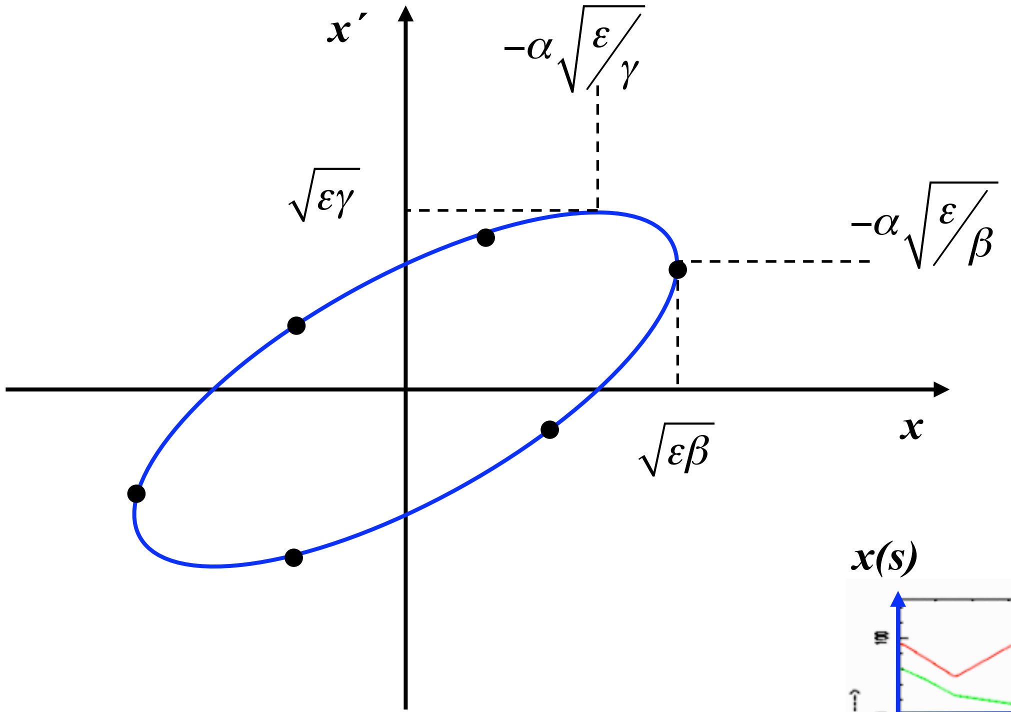
The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.
It is determined by the focusing properties of the lattice and follows the periodicity of the machine.

optics at 7000 GeV



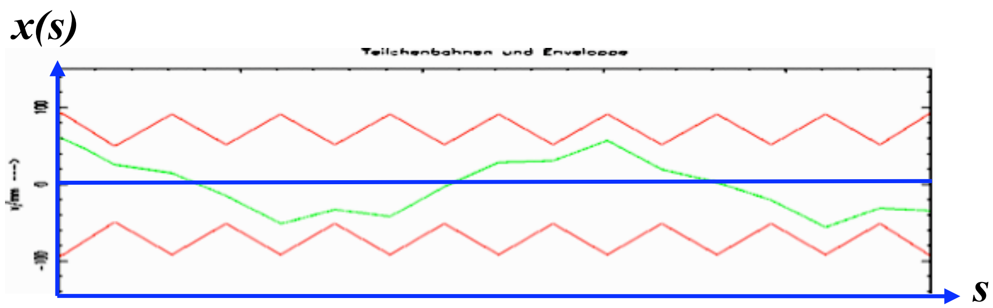
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



Liouville: *in reasonable storage rings area in phase space is constant.*

*A = π*ε=const*

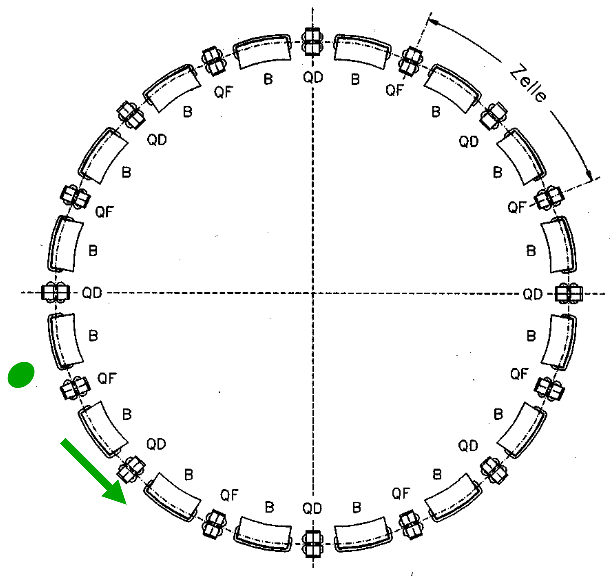


ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter, cannot be changed by the foc. properties.

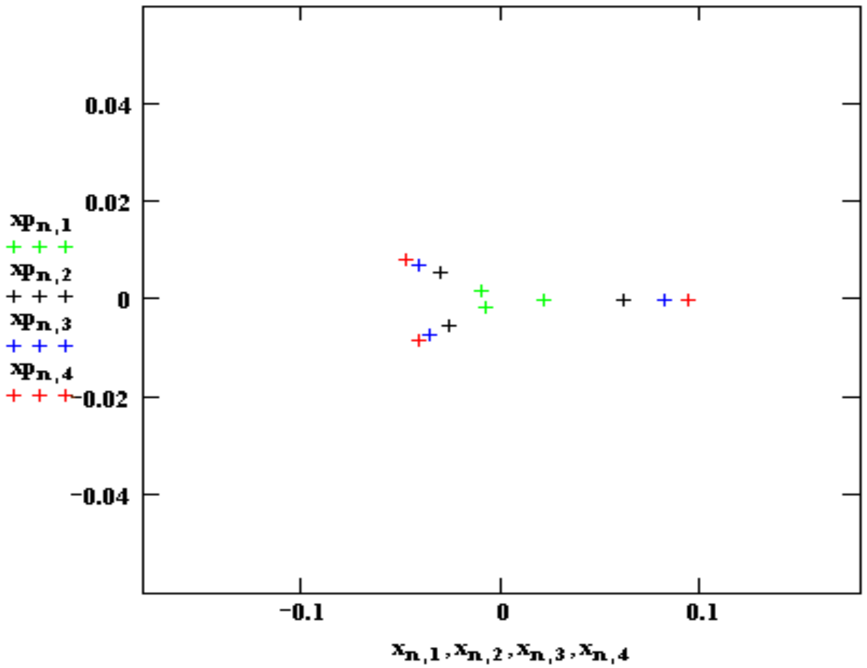
Scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!

Particle Tracking in a Storage Ring

Calculate x, x´ for each accelerator element according to matrix formalism and plot x, x´at a given position „s" in the phase space diagram



$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



A beam of 4 particles

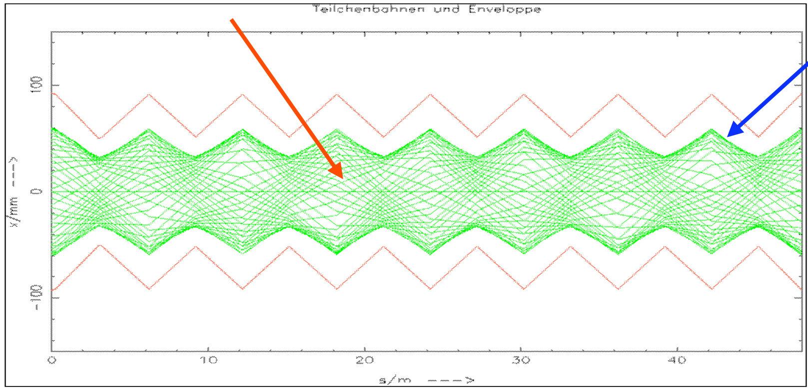
– each having a slightly different emittance:

… just as Big Ben

… and just as any harmonic pendulum

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

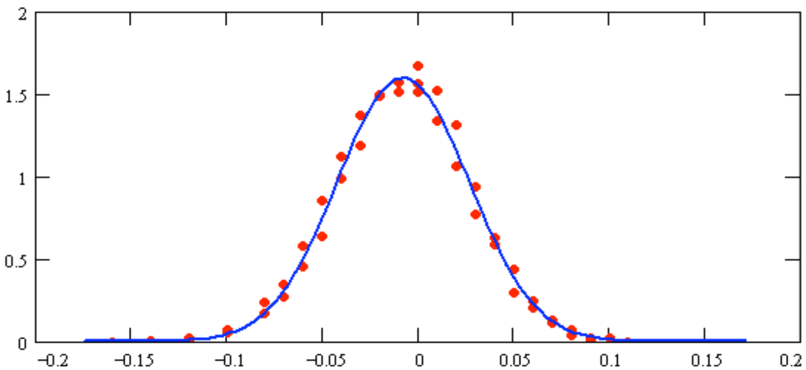
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

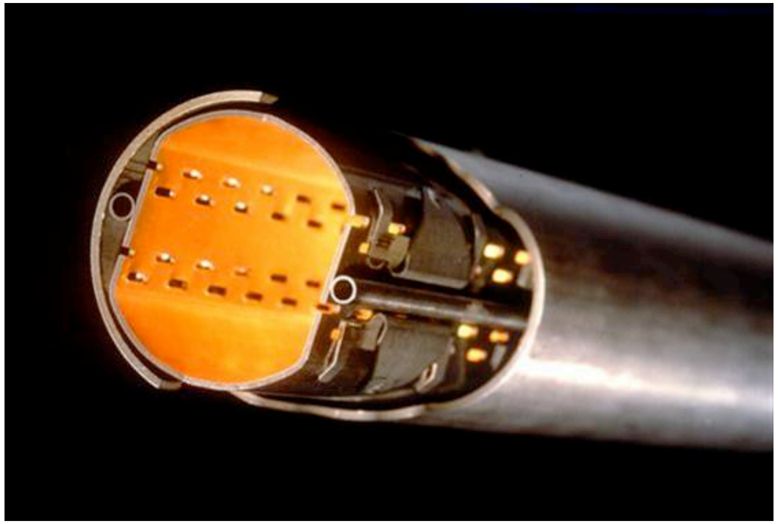
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5 * 10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\,m * 180\,m} = 0.3\,mm$$

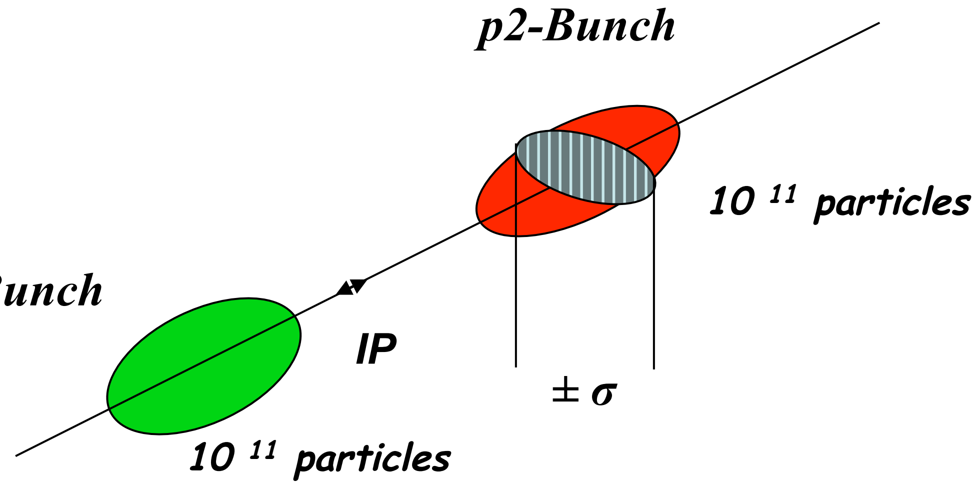




aperture requirements: $r_0 = 17 * \sigma$

5.) Luminosity

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$f_0 = 11.245\ kHz$

$n_b = 2808$

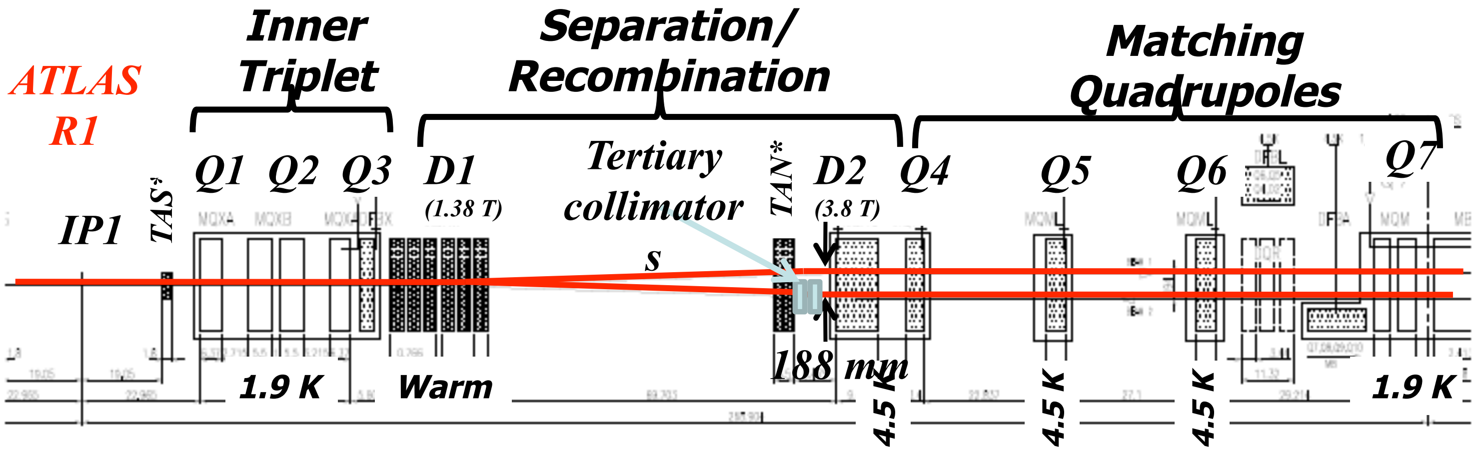
$I_p = 584\ mA$

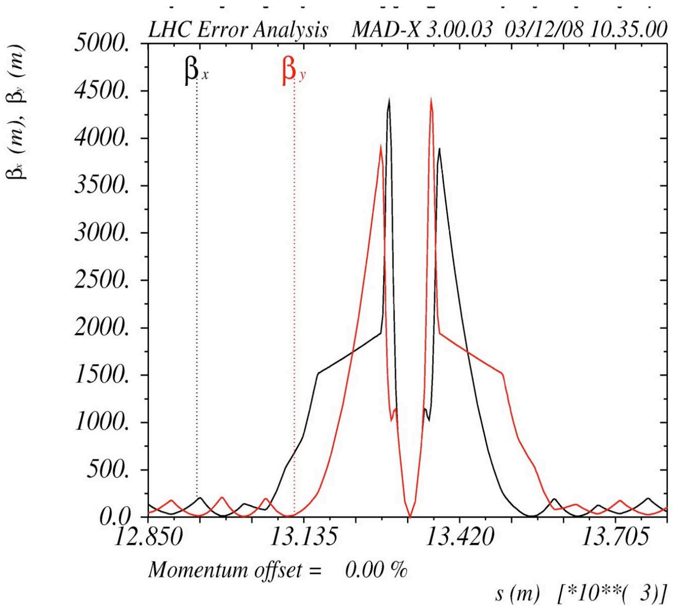
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

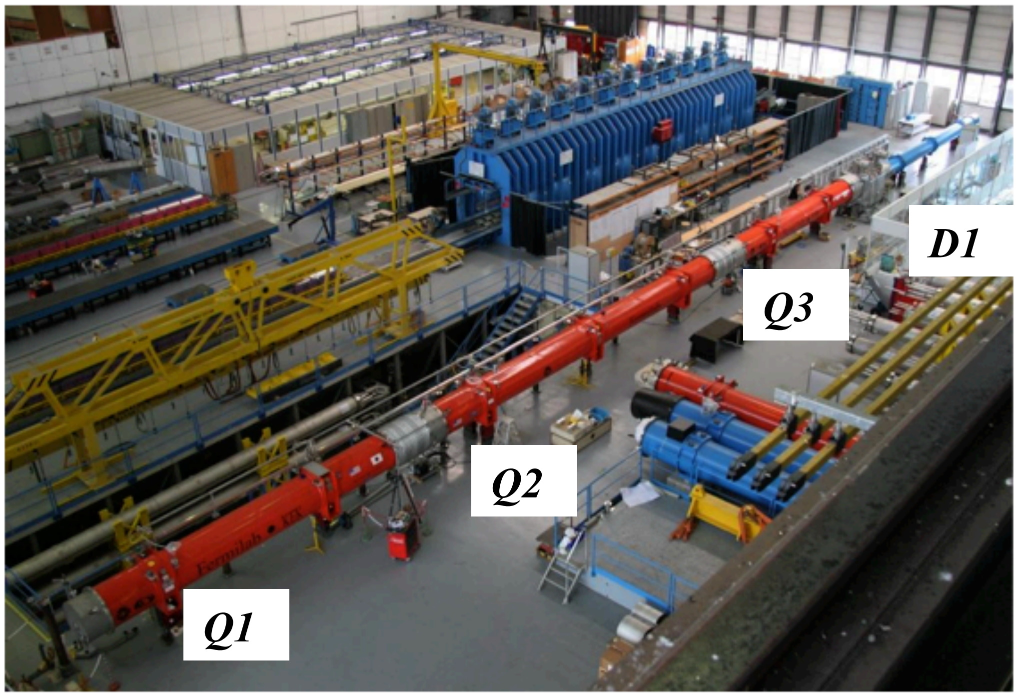
beam sizes in the order of my cat’s hair !!

The LHC Mini-Beta-Insertions





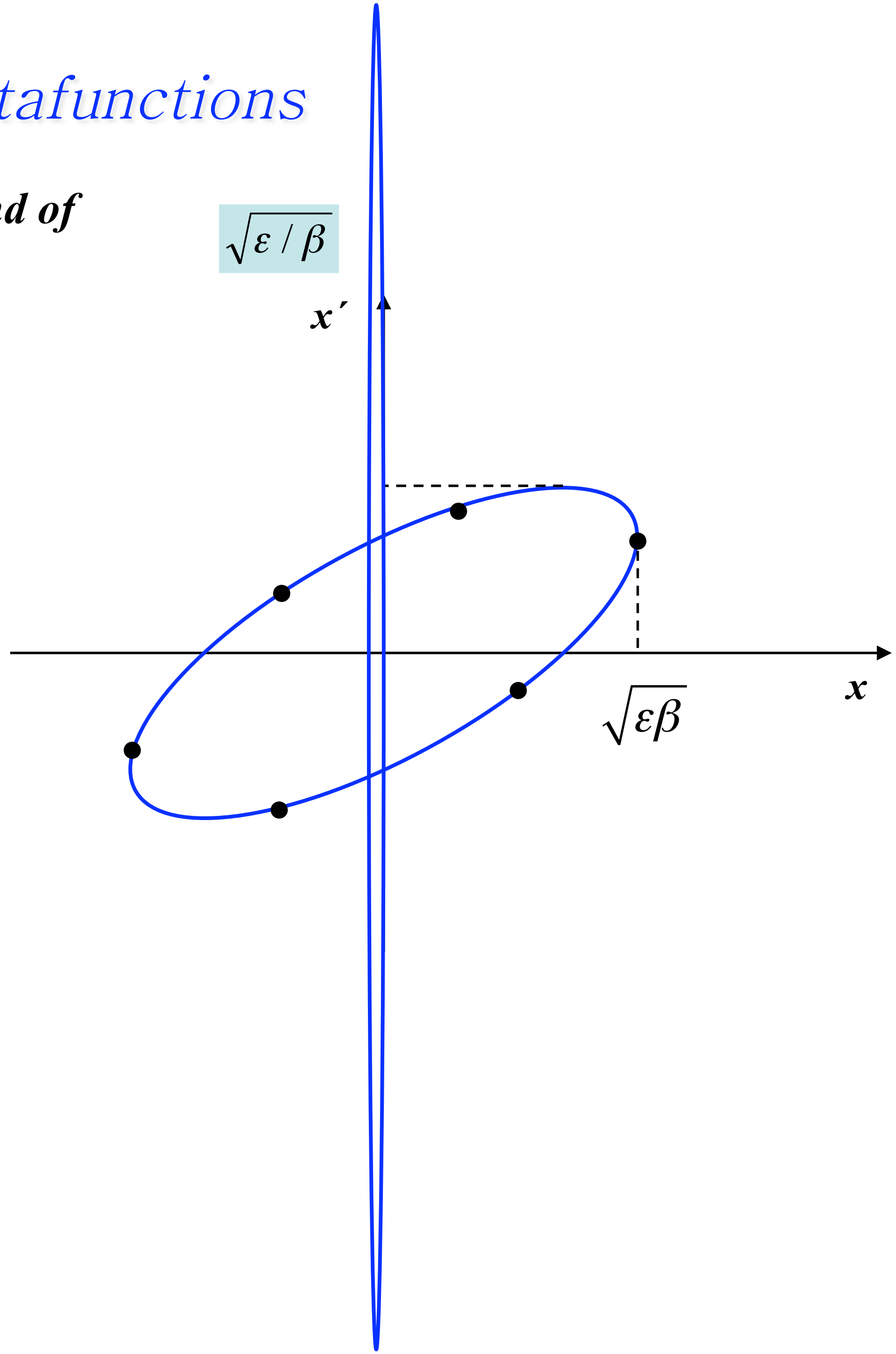
mini β optics



Mini-β Insertions: Betafunctions

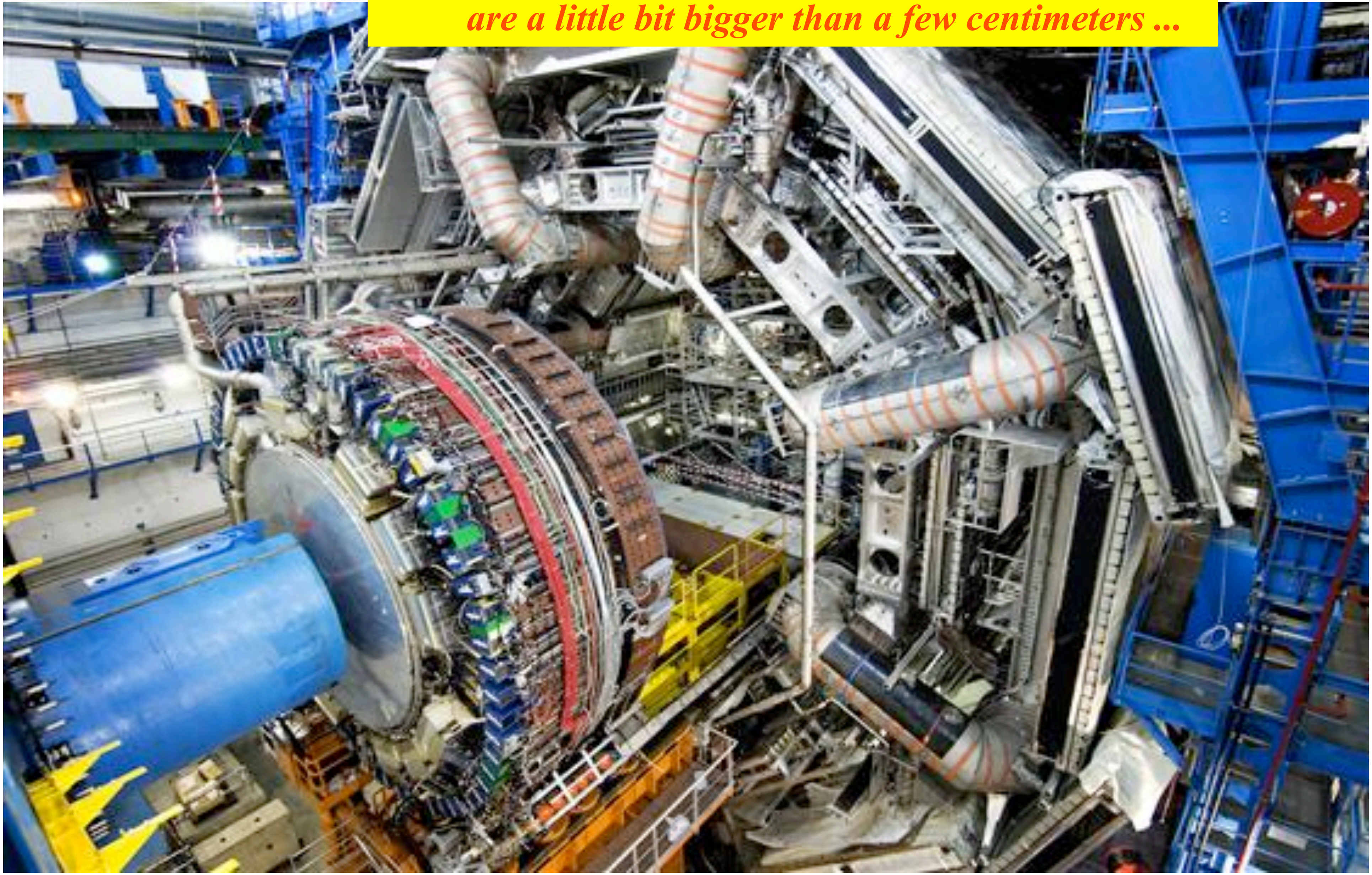
A mini-β insertion is always a kind of
special symmetric drift space.
→greetings from Liouville

the smaller the beam size
the larger the bam divergence



... clearly there is an

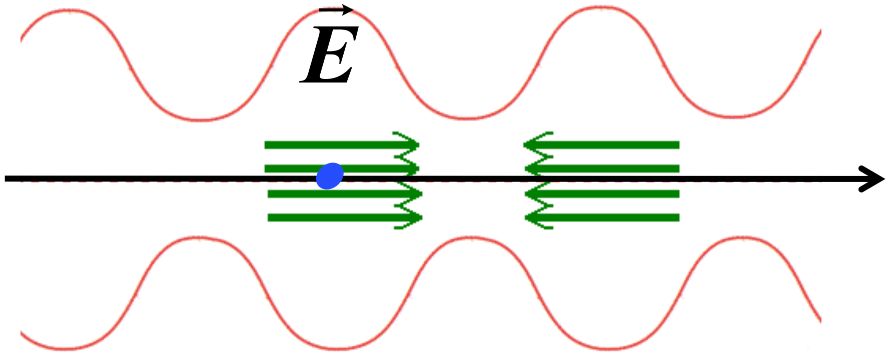
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

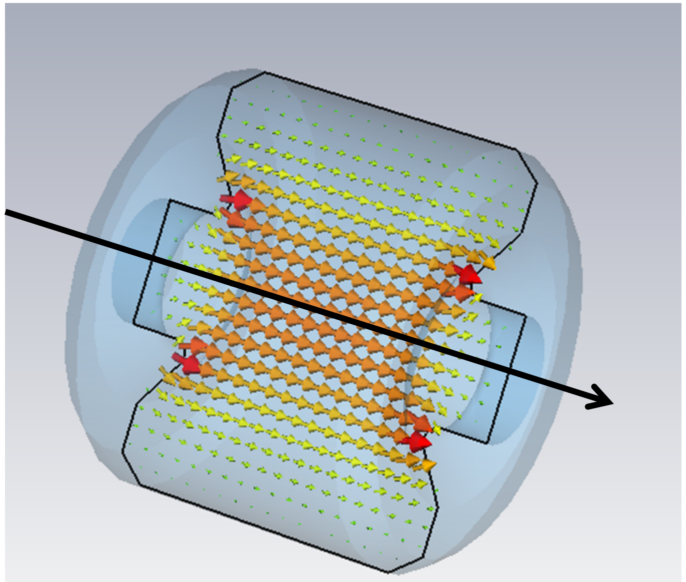


III. The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:





B. Salvant
N. Biancacci

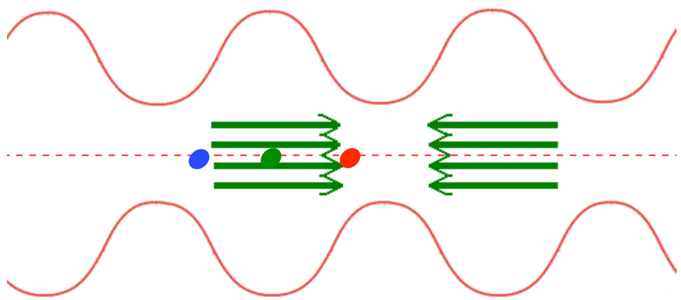
The Acceleration & "Phase Focusing"

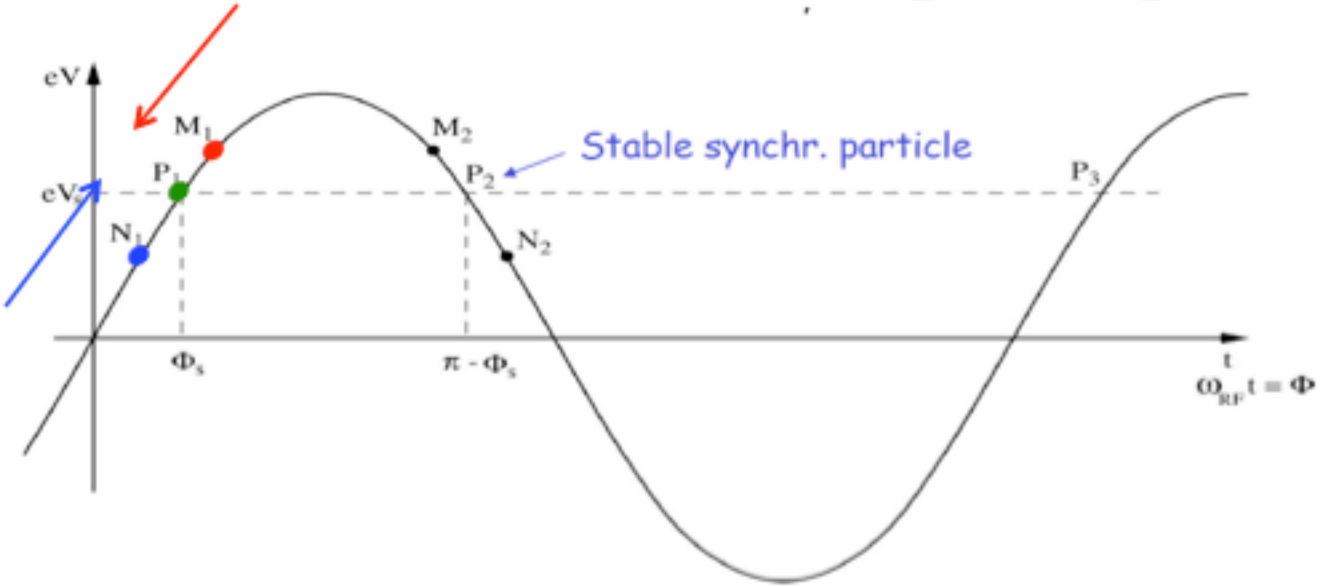
$\Delta p/p \neq 0$ below transition

ideal particle •

particle with $\Delta p/p > 0$ • *faster*

particle with $\Delta p/p < 0$ • *slower*



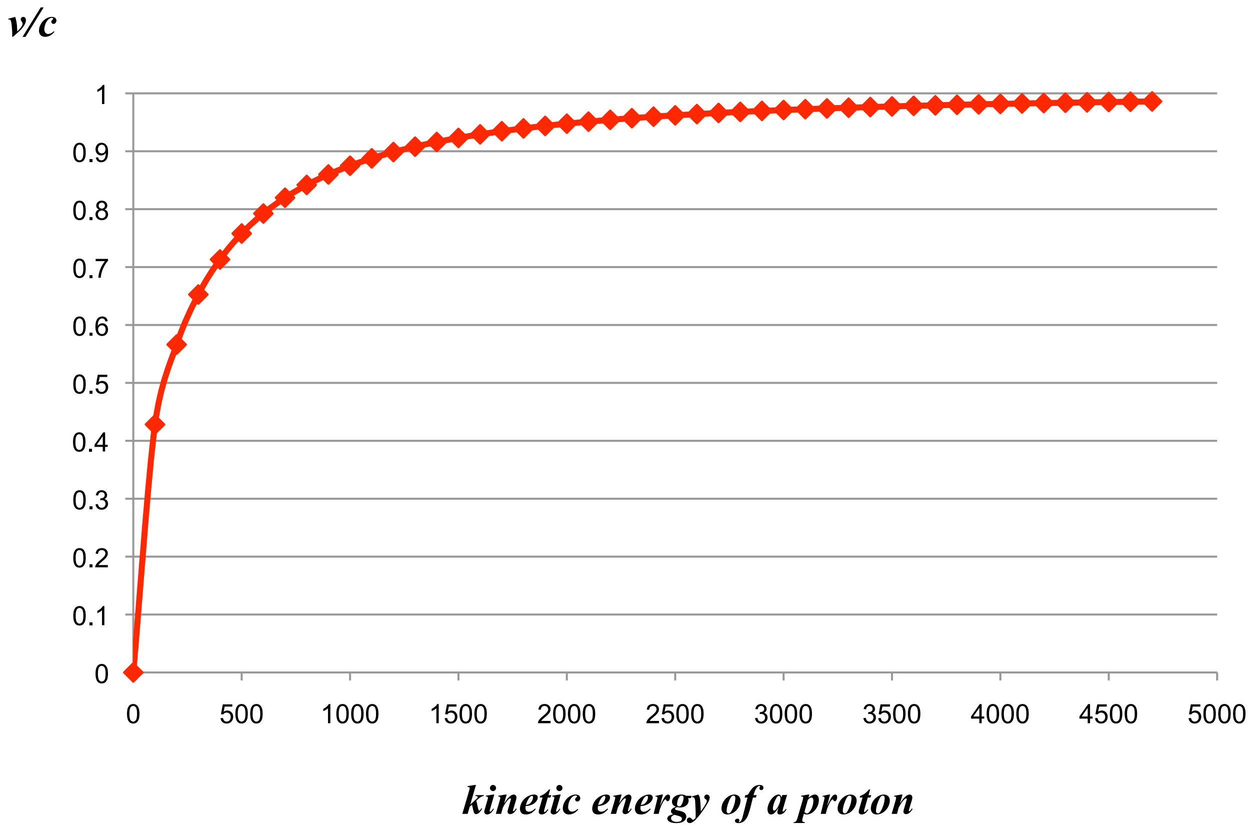


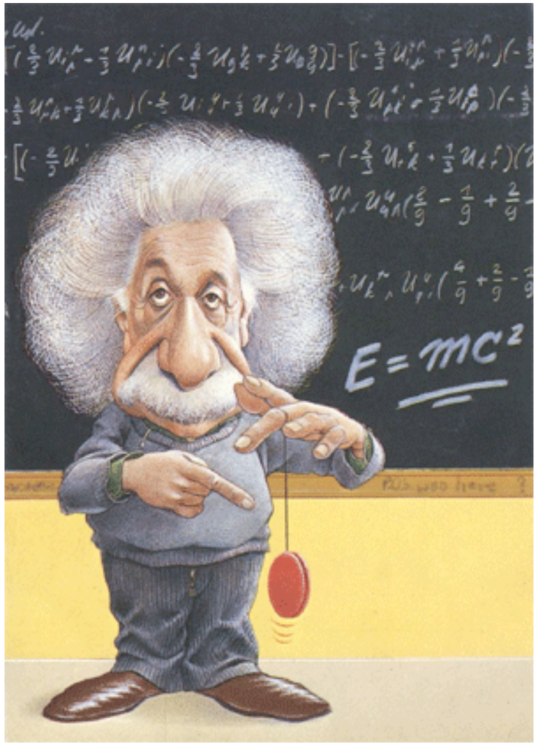
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

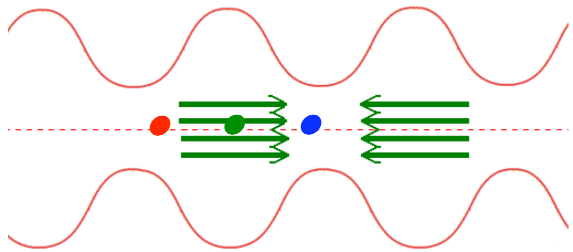
.... but heavier !

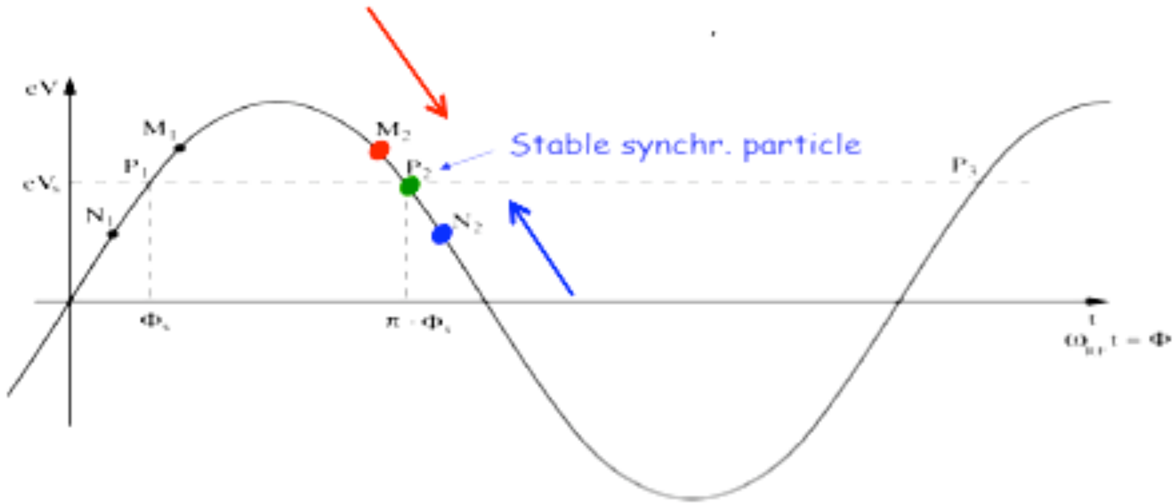
The Acceleration ***above transition***

ideal particle •

particle with $\Delta p/p > 0$ • *heavier*

particle with $\Delta p/p < 0$ • *lighter*



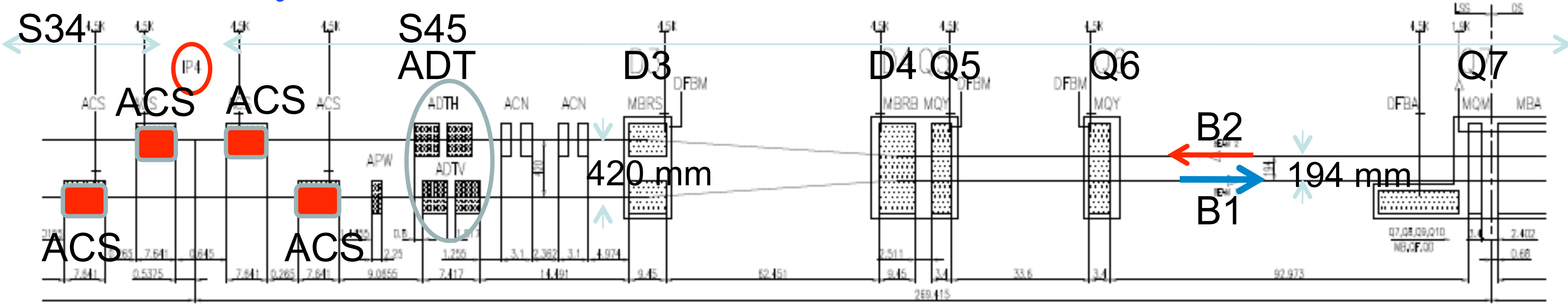


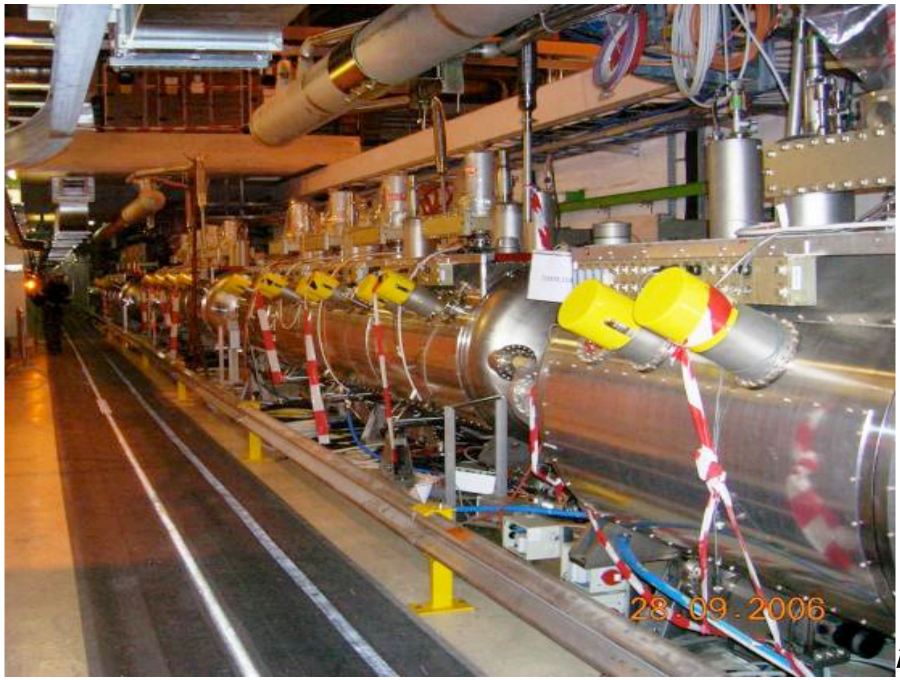
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



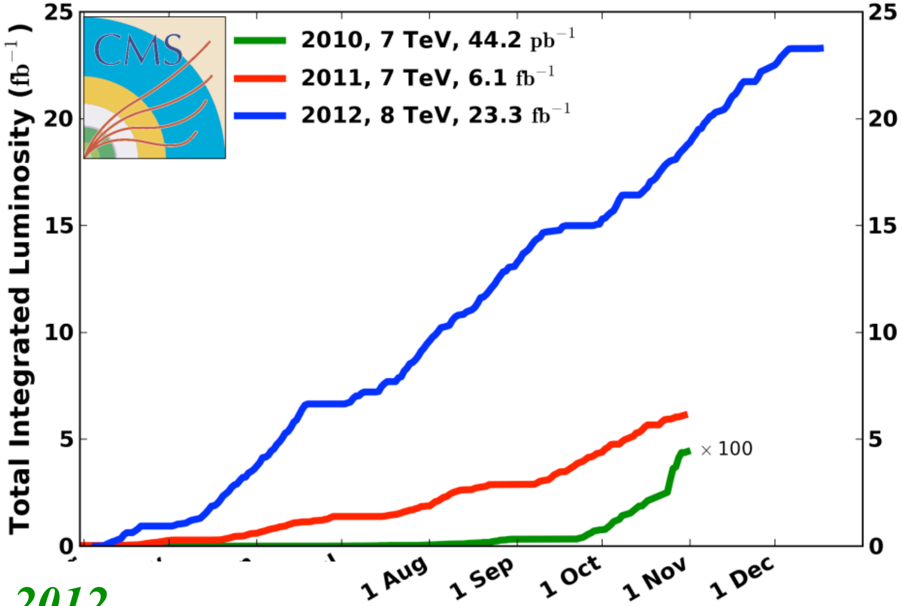


m

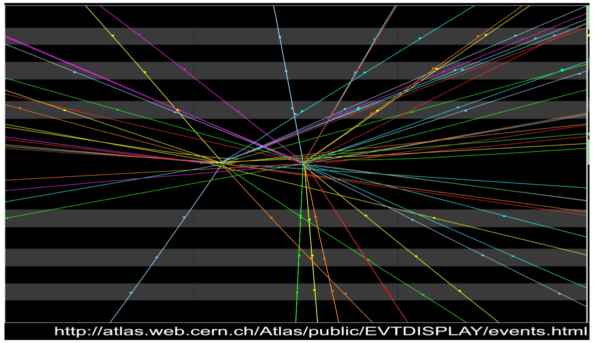
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	*10^{-3}*	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

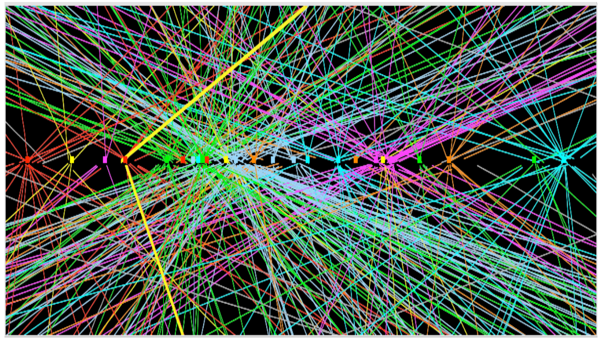
And still...
The LHC Performance in Run 1



	Design	2012
Momentum at collision	7 TeV /c	4 TeV/c
Luminosity	10^{34} cm $^{-2}$ s^{-1}	$7.7*10^{33}$ cm $^{-2}$ s^{-1}
Protons per bunch	1.15×10^{11}	1.50×10^{11}
Number of bunches/beam	2808	1380
Nominal bunch spacing	25 ns	50ns
Normalized emittance	3.75 μm	2.5μm
beta *	55 cm	60 cm
rms beam size (arc)	300 μm	350 μm
rms beam size IP	17 μm	20 μm



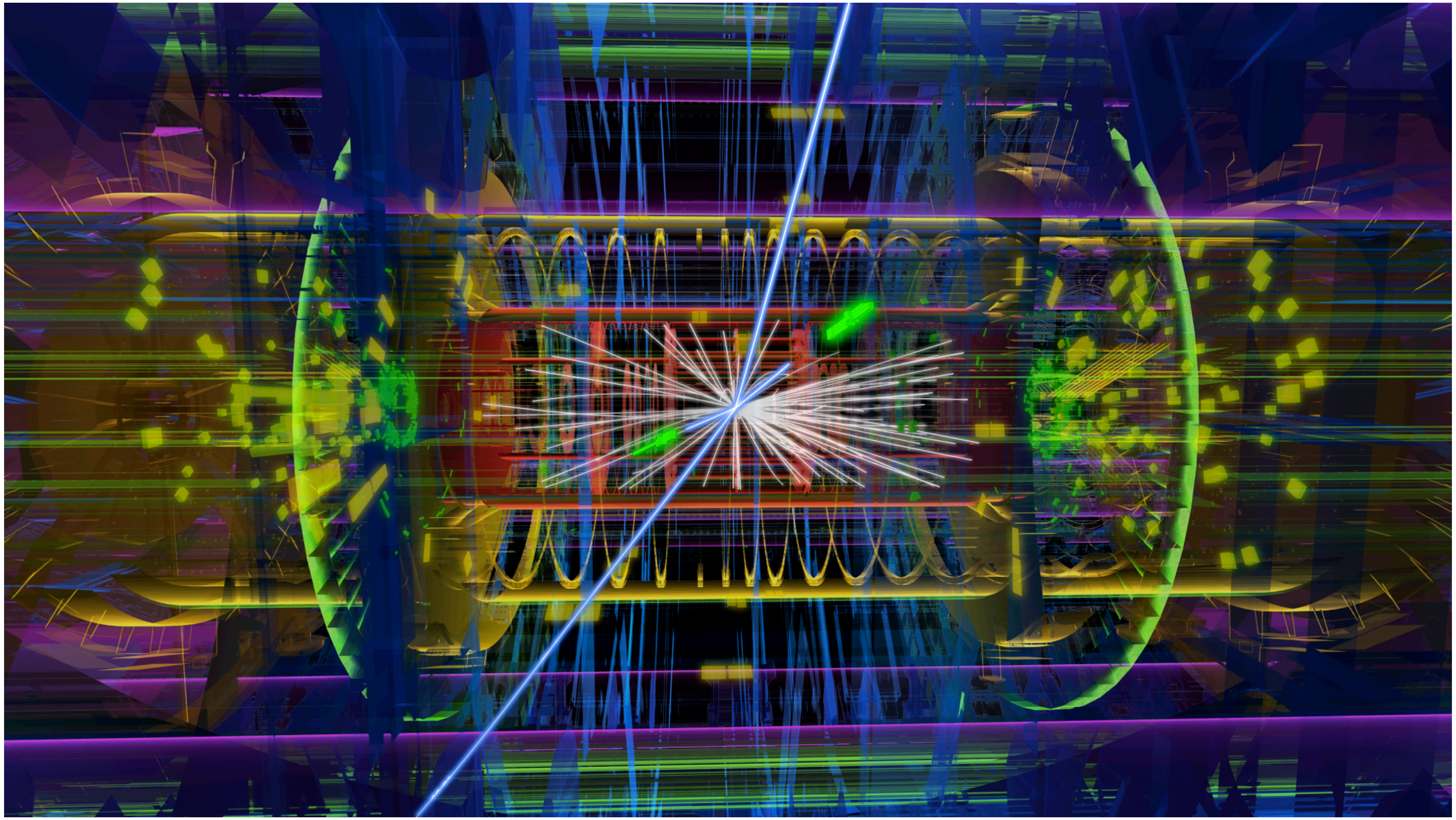
2 vertices



20 vertices

... und wozu das alles ??

High Light of the HEP-Year natuerlich das HIGGS

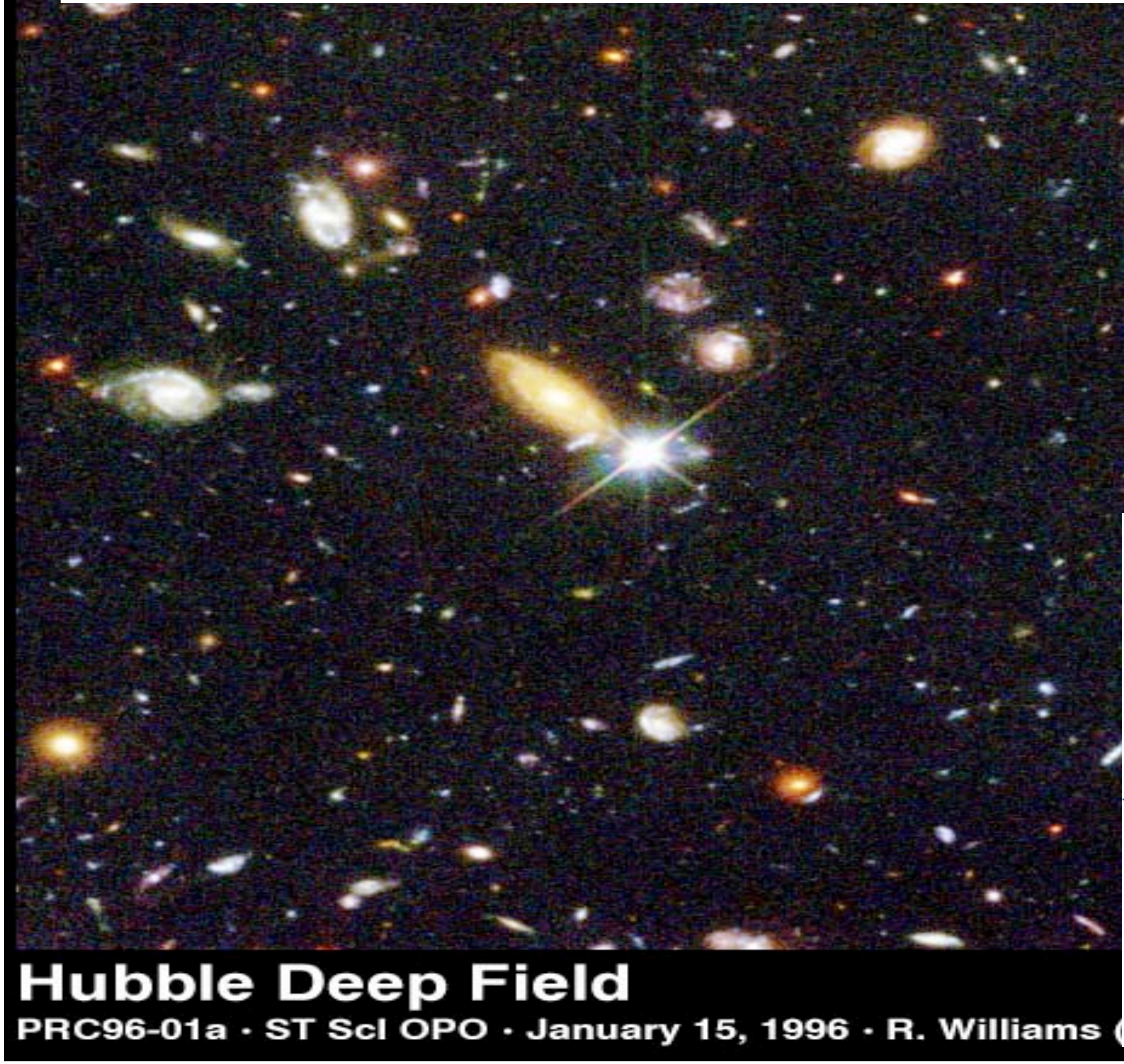


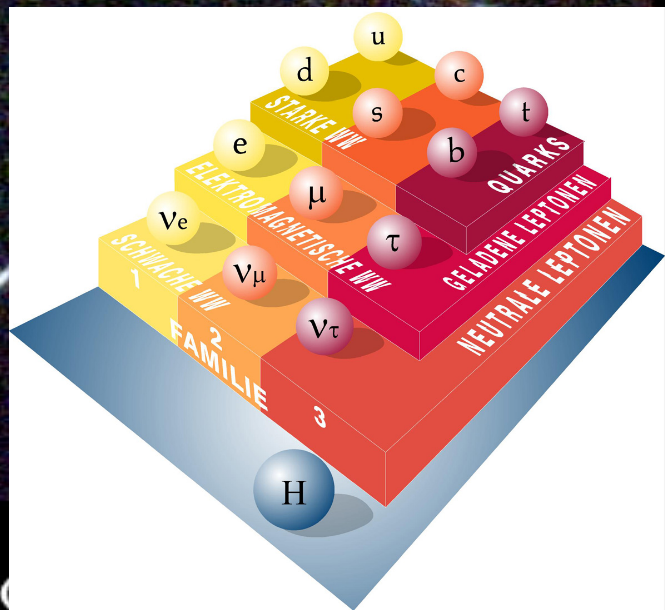
ATLAS event display: Higgs => two electrons & two muons

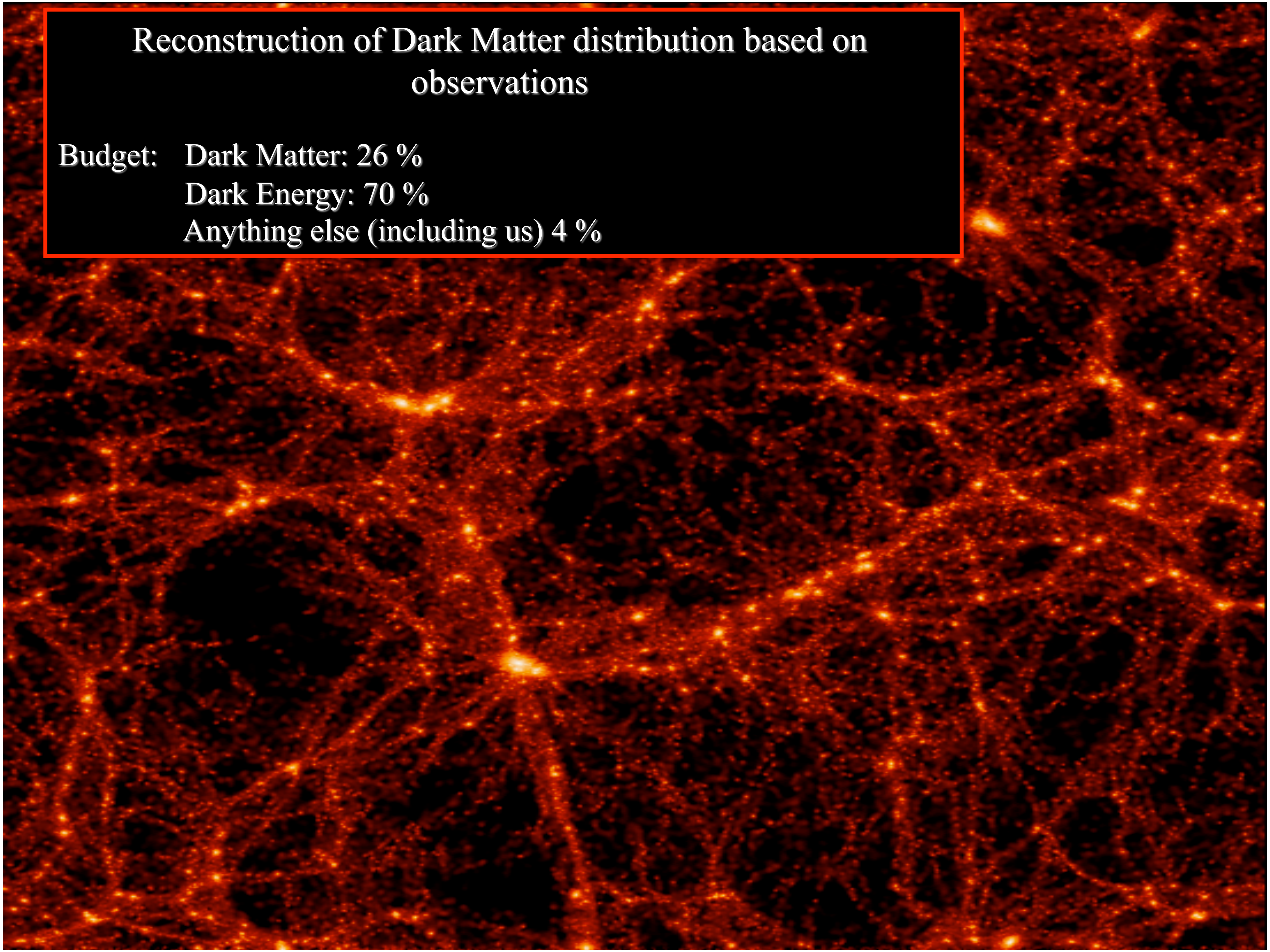
What‘s next ???

Dark Matter & Dark Energy

Physics beyond the Standard Model






Reconstruction of Dark Matter distribution based on observations
Budget: Dark Matter: 26 %
Dark Energy: 70 %
Anything else (including us) 4 %


I am still confused, but now I am confused at a much higher level!
Research Conference on
OH