Introduction to Accelerator Physics

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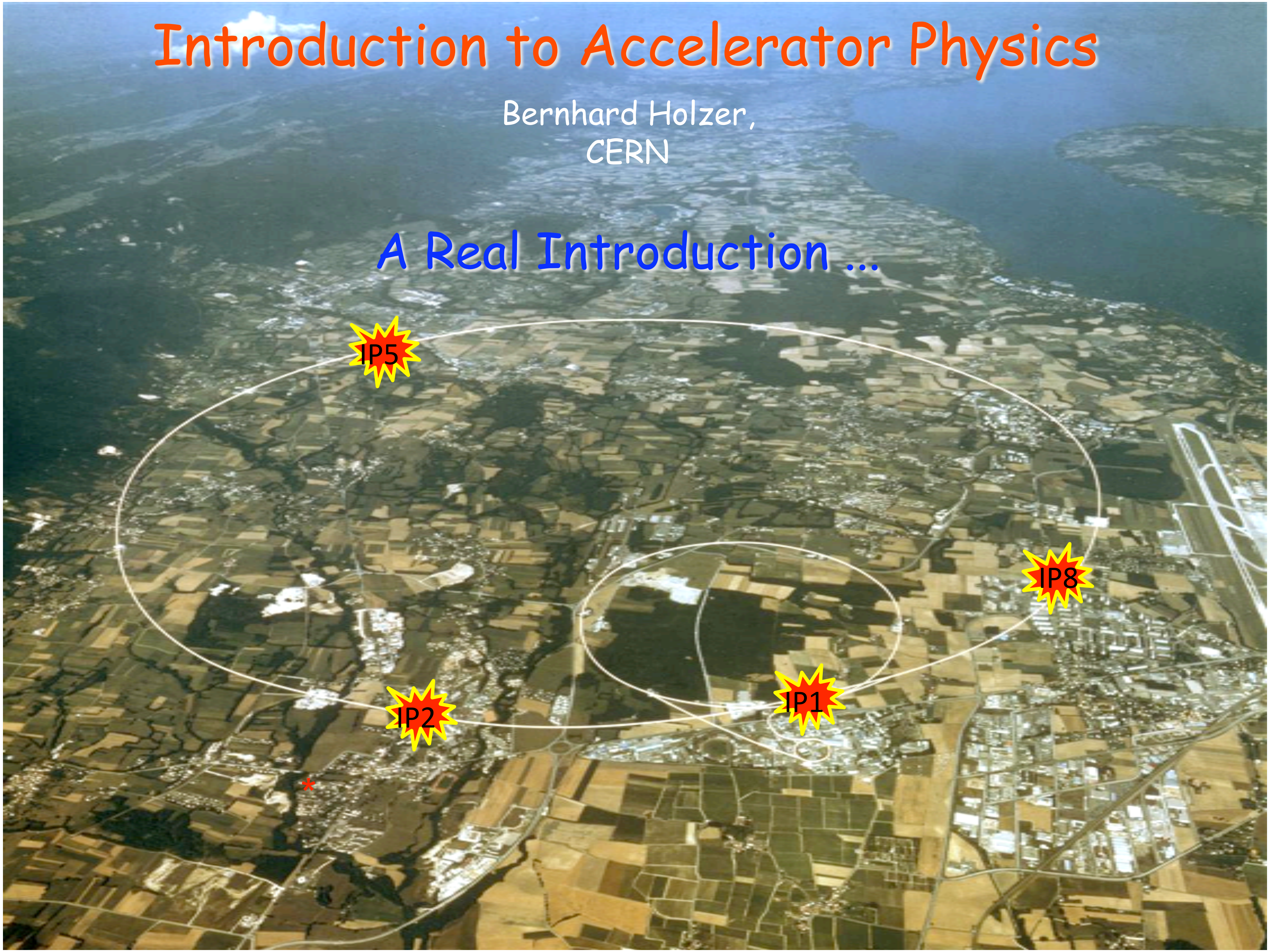
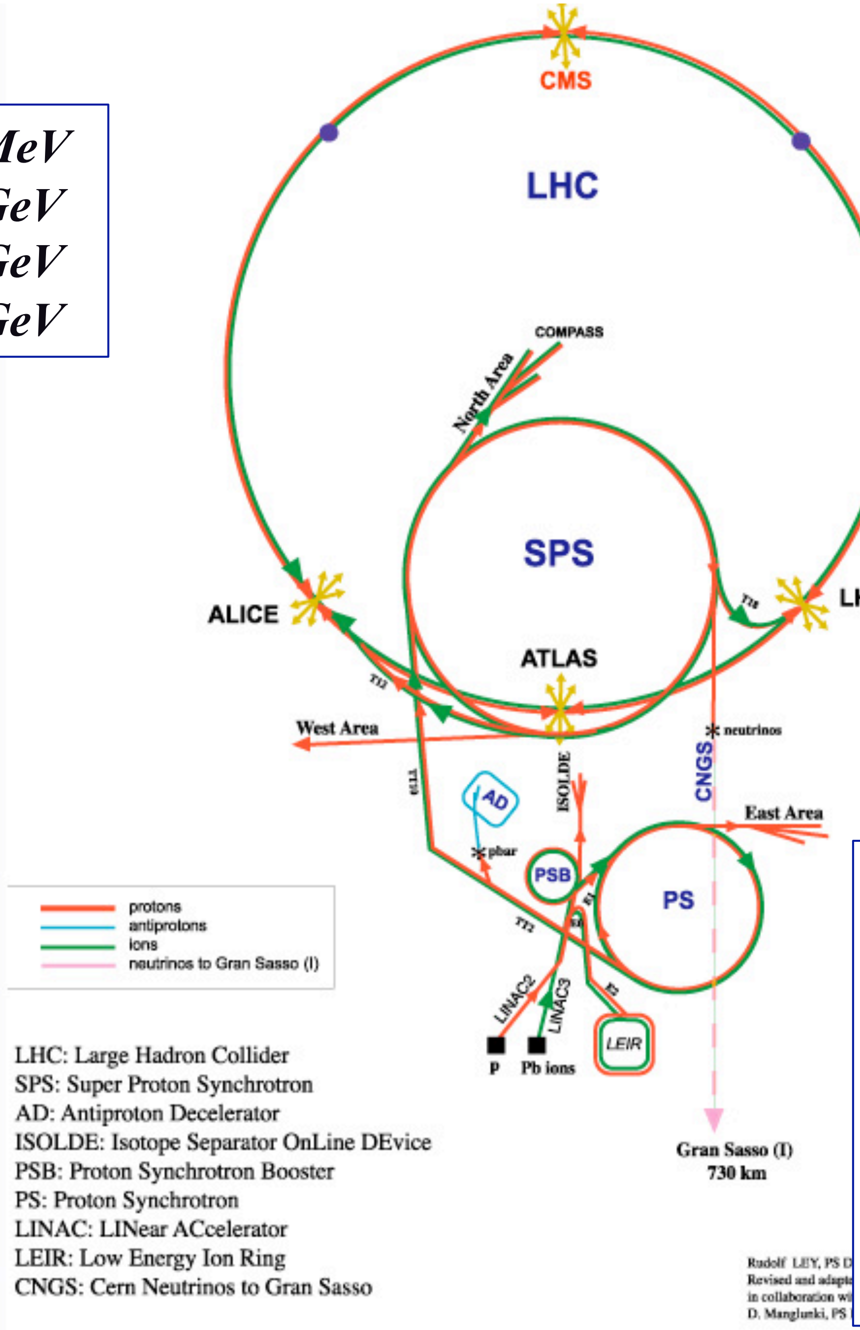
A Real Introduction ...



Teilchen Energien & Beschleuniger

Linac 2: 50 MeV
Booster: 1.4 GeV
PS: 26 GeV
SPS: 450 GeV



LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Separator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso

$E = mc^2$

Ruhemassen:

Proton: 938 MeV
Electron: 511 keV
Myon: 105 MeV

klassischer / relativistischer Bereich

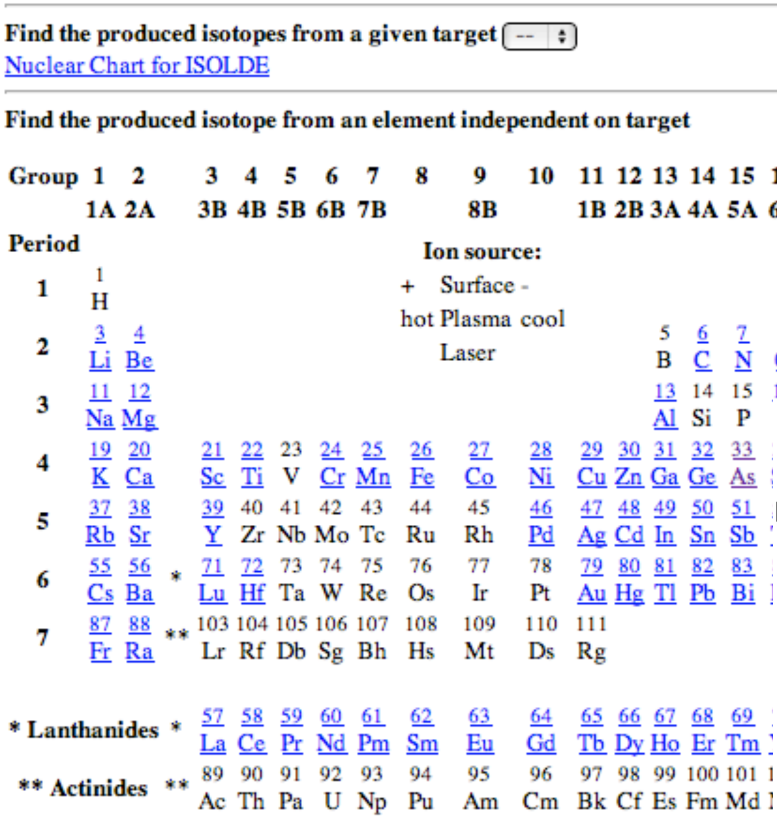
$$\gamma = \frac{E}{mc^2} = \sqrt{\frac{1}{1-\left(\frac{v}{c}\right)^2}}$$

Linac 2:

$\gamma = 988 / 938$

$\beta = v/c = 0.3$

Example: Kernphysik & Isotope : ISOLDE



Find the produced isotopes from a given target [--]

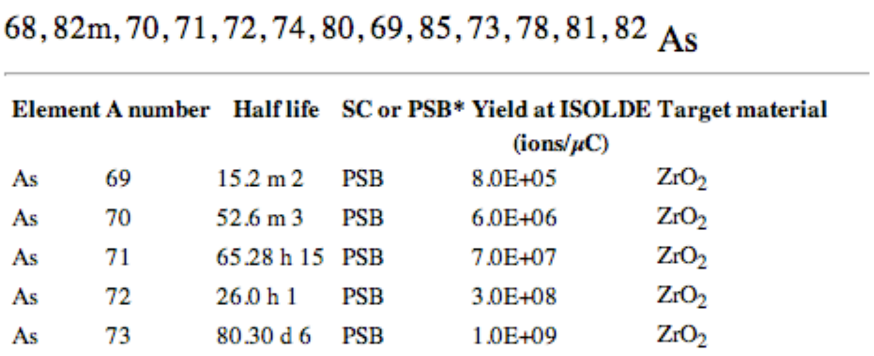
Nuclear Chart for ISOLDE

Find the produced isotope from an element independent on target

Group	**1** **1A**	**2** **2A**		**3** **3B**	**4** **4B**	**5** **5B**	**6** **6B**	**7** **7B**	**8**	**9** **8B**	**10**	**11** **1B**	**12** **2B**	**13** **3A**	**14** **4A**	**15** **5A**	**16** **6A**	**17** **7A**	**18** **8A**
Period																			
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16	17	18
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As			
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb			
6	55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi			
7	87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg							

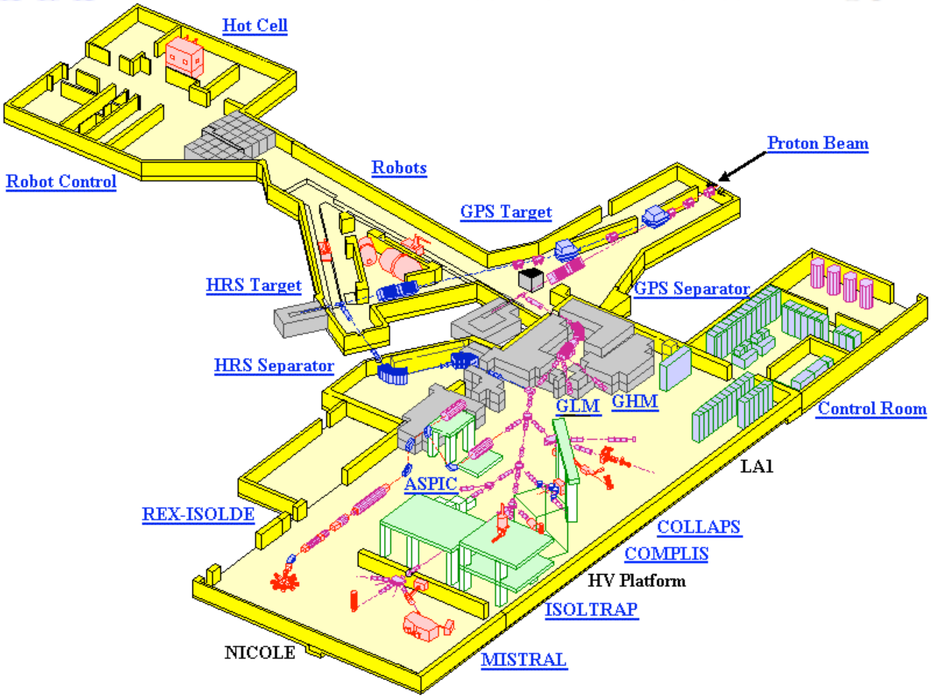
Ion source:
+ Surface -
hot Plasma cool
Laser

*** Lanthanides ***	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm
**** Actinides ****	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md

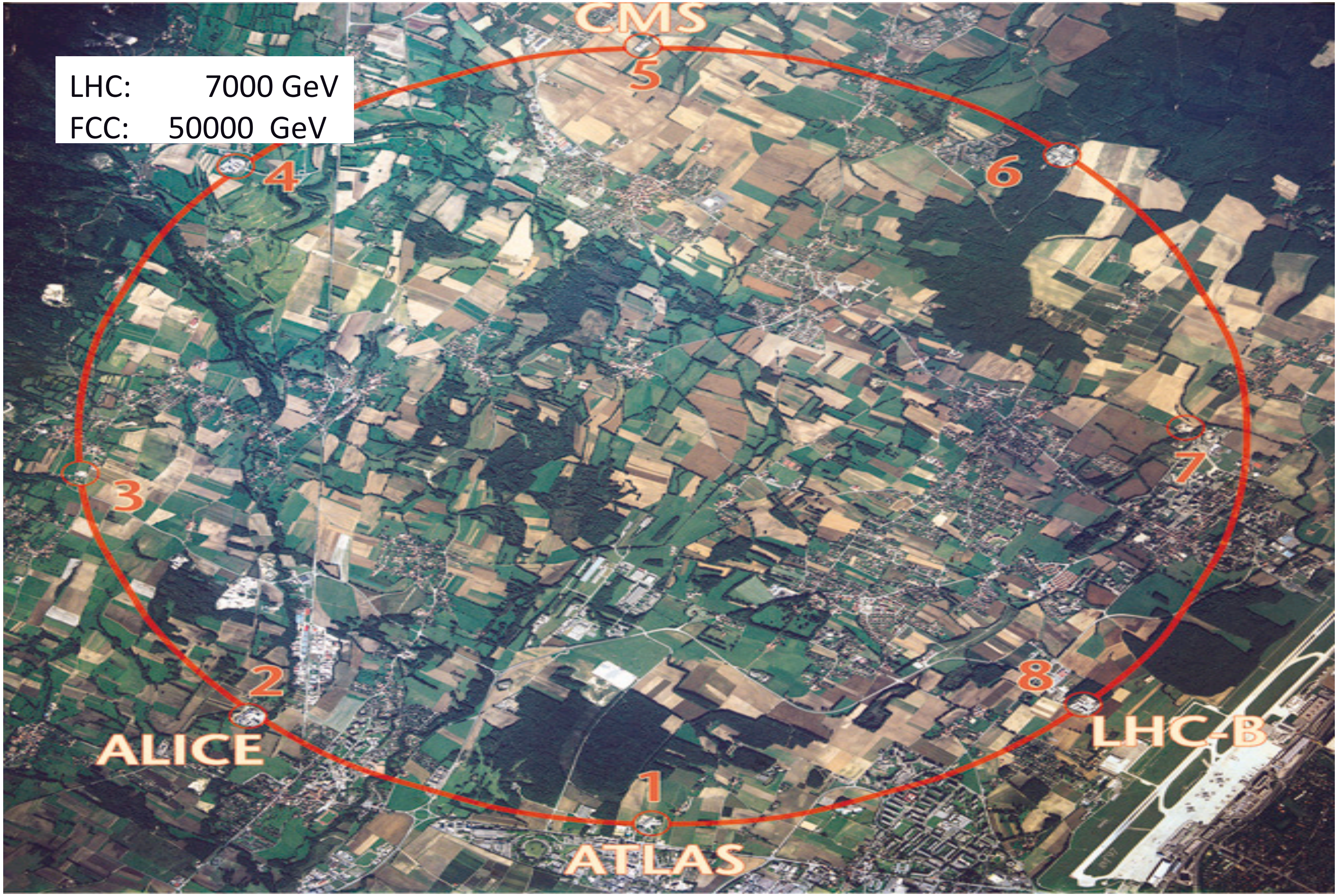


$^{68,82m,70,71,72,74,80,69,85,73,78,81,82}\mathrm{As}$

Element	**A number**	**Half life**	**SC or PSB***	**Yield at ISOLDE (ions/μC)**	**Target material**
As	69	15.2 m 2	PSB	8.0E+05	ZrO_2
As	70	52.6 m 3	PSB	6.0E+06	ZrO_2
As	71	65.28 h 15	PSB	7.0E+07	ZrO_2
As	72	26.0 h 1	PSB	3.0E+08	ZrO_2
As	73	80.30 d 6	PSB	1.0E+09	ZrO_2

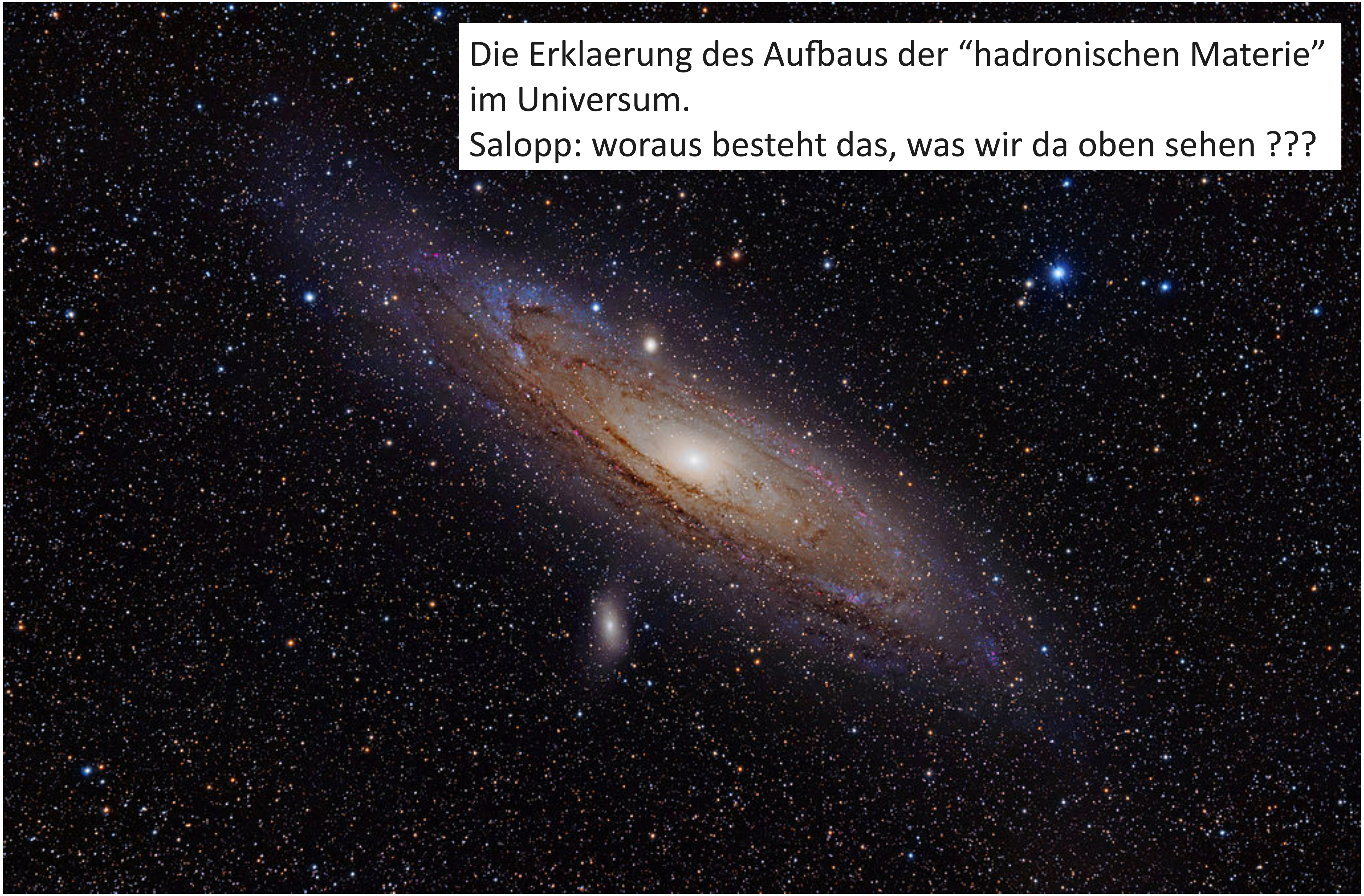


Und schlussendlich die Teilchen Physik



LHC: 7000 GeV
FCC: 50000 GeV

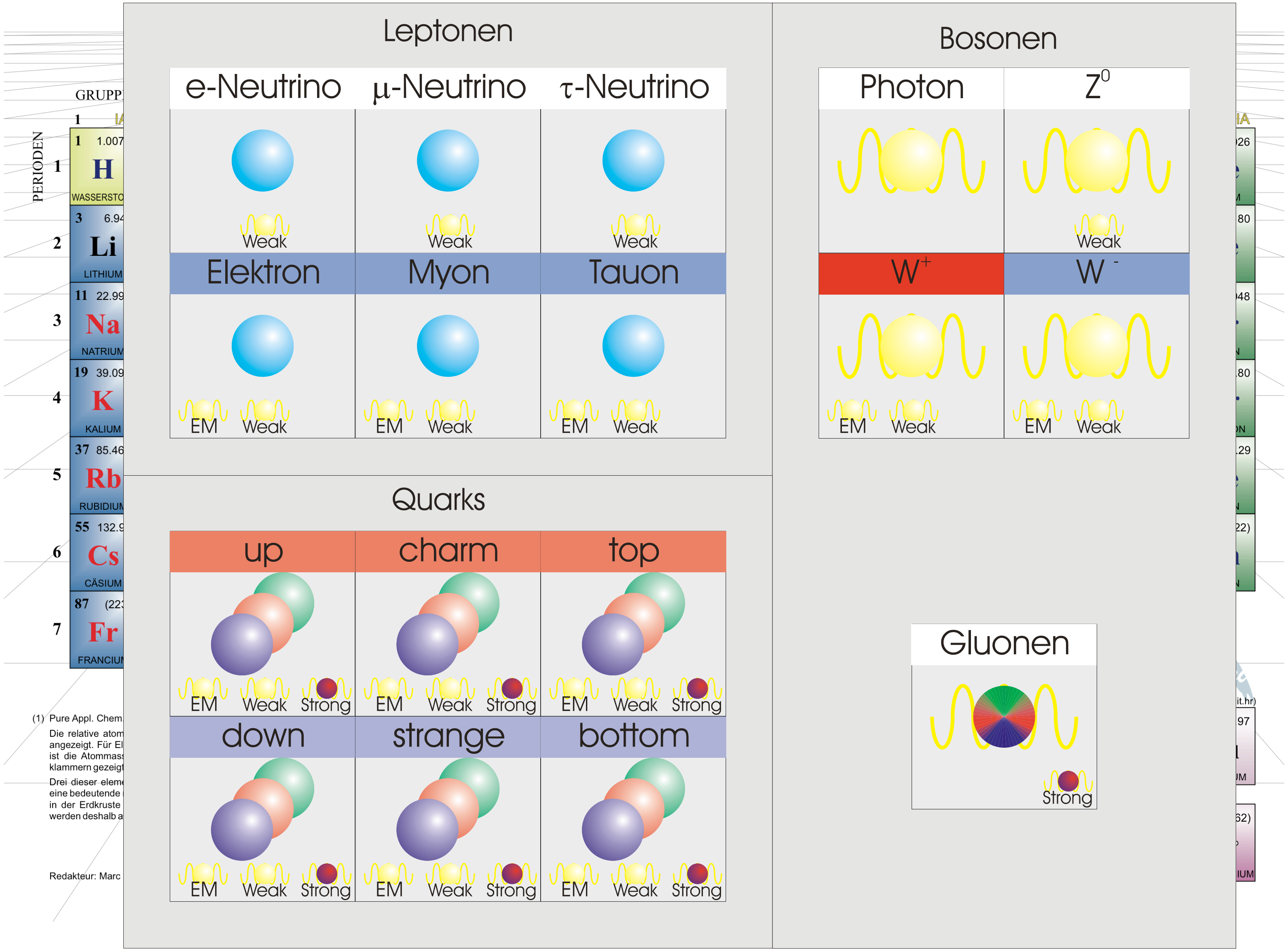
Standard Model: Worum gehts eigentlich ??



Die Erklaerung des Aufbaus der “hadronischen Materie” im Universum.

Salopp: woraus besteht das, was wir da oben sehen ???

Standard Model: Worum gehts eigentlich ?? um den Versuch Ordnung & Systematik zu erkennen



Leptonen

e-Neutrino | μ-Neutrino | τ-Neutrino

Elektron | Myon | Tauon

Quarks

up | charm | top

down | strange | bottom

Bosonen

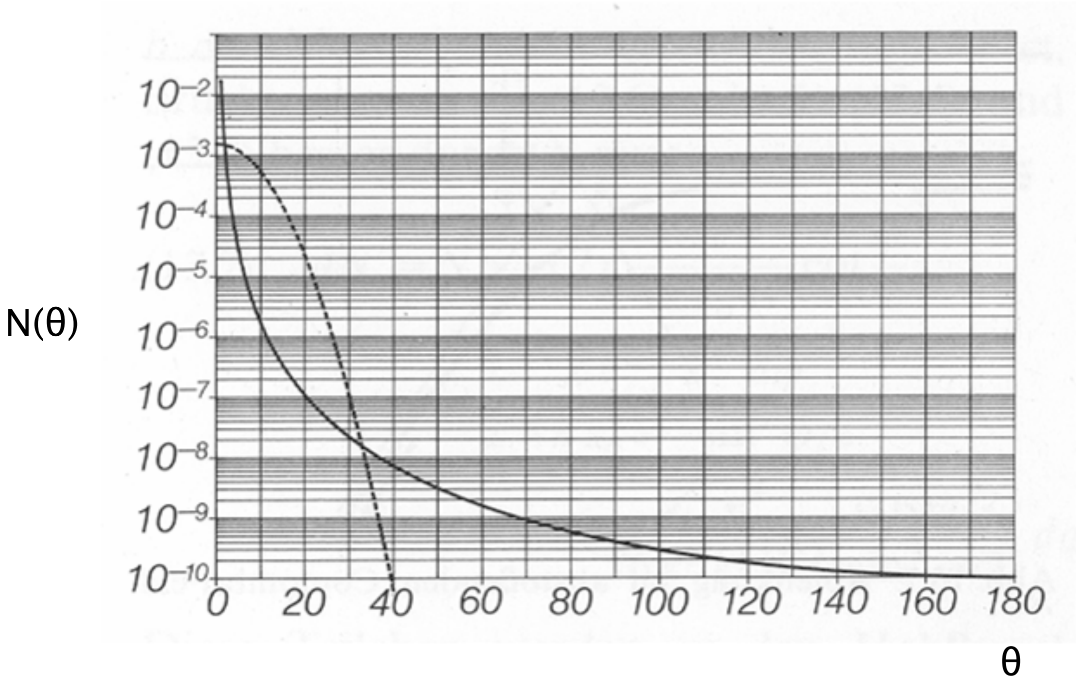
Photon | Z^0

W^+ | W^-

Gluonen

I.) A Bit of History

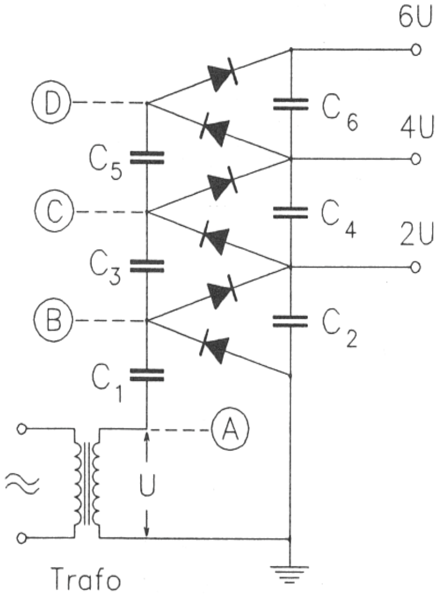
$$N(\theta) = \frac{N_i ntZ^2e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



Rutherford Scattering, 1911

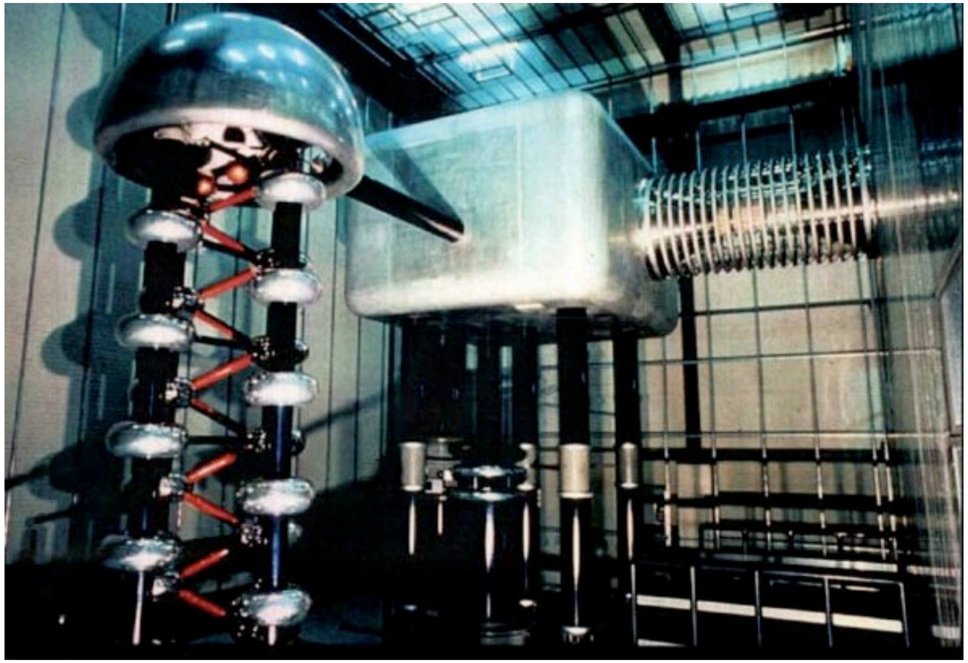
Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



Particle source: Hydrogen discharge tube
on 400 kV level

Accelerator: evacuated glas tube

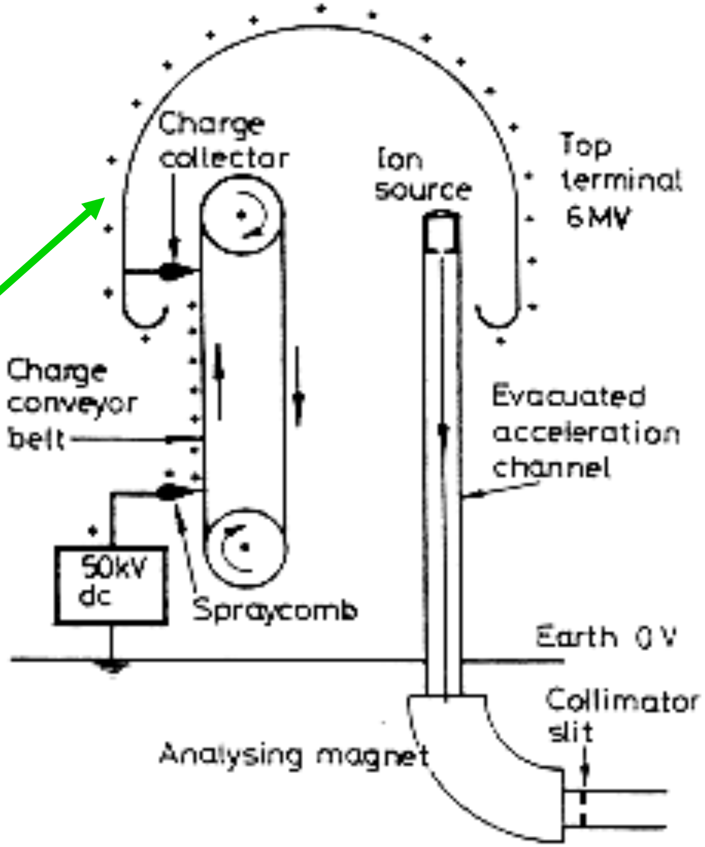
Target: Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



* Terminal Potential: U ≈ 12 ...28 MV
using high pressure gas to suppress discharge (SF_6)

Problems: * Particle energy limited by high voltage discharges
* high voltage can only be applied once per particle ...
... or twice ?

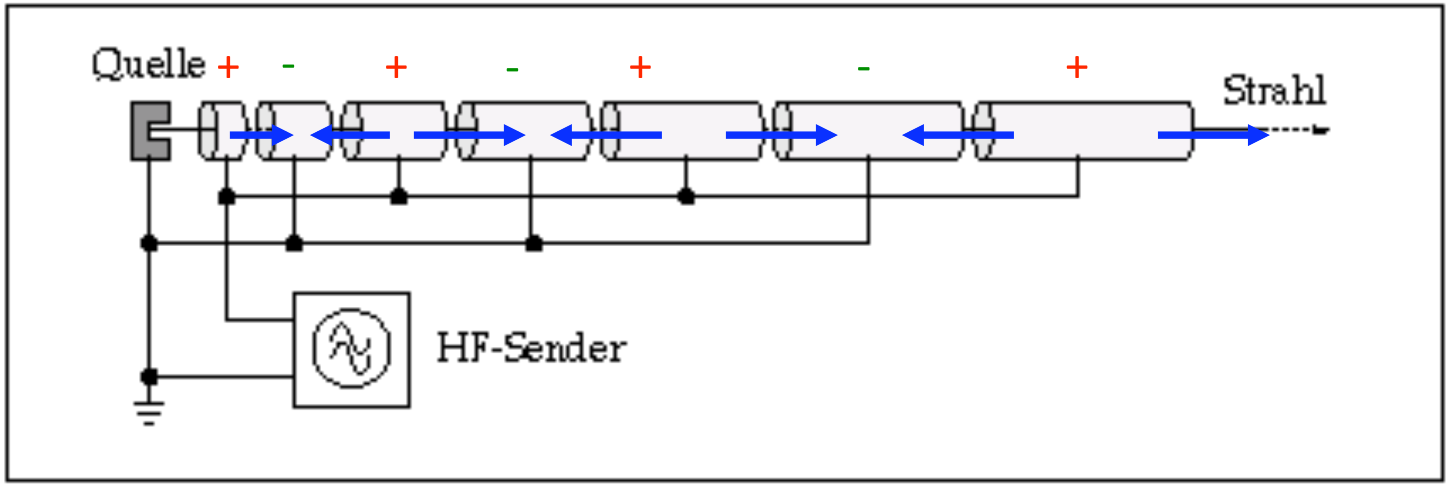
The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the
structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

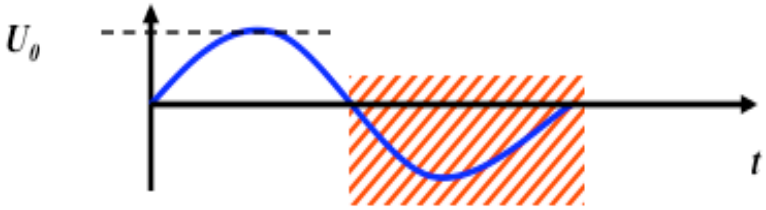
$$E_n = n * q * U_0 * \sin\psi_s$$

n number of gaps between the drift tubes
q charge of the particle
U_0 Peak voltage of the RF System
Ψ_S synchronous phase of the particle

* acceleration of the proton in the first gap
* voltage has to be „flipped“ to get the right sign in the second gap → RF voltage
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0*\sin\psi_s}}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \dots 0.6$, Particles: Protons/Ions

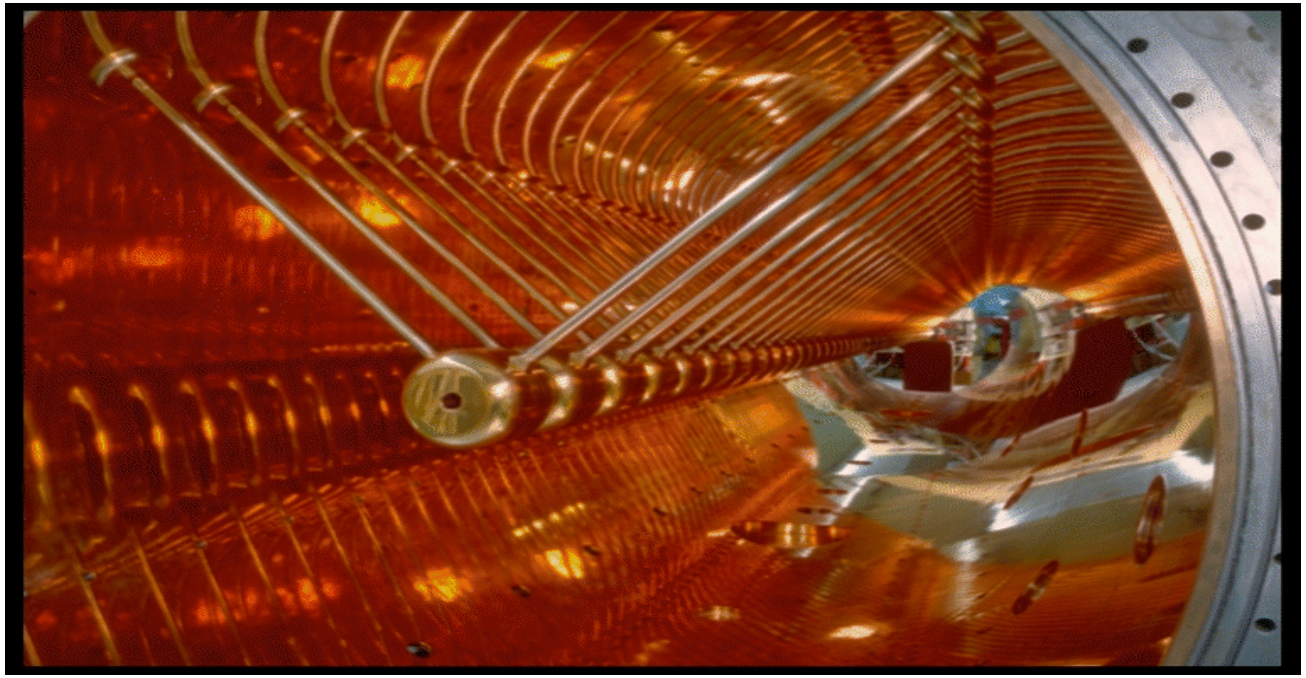
Accelerating structure of a Proton Linac (DESY Linac III)

$E_{total} = 988\ MeV$

$m_0c^2 = 938\ MeV$

$p = 310\ MeV\ /\ c$

$E_{kin} = 50\ MeV$



Beam energies

Energy Gain per „Gap“: $\boxed{W = q\, U_0 \sin \omega_{RF} t}$

1.) reminder of some relativistic formula

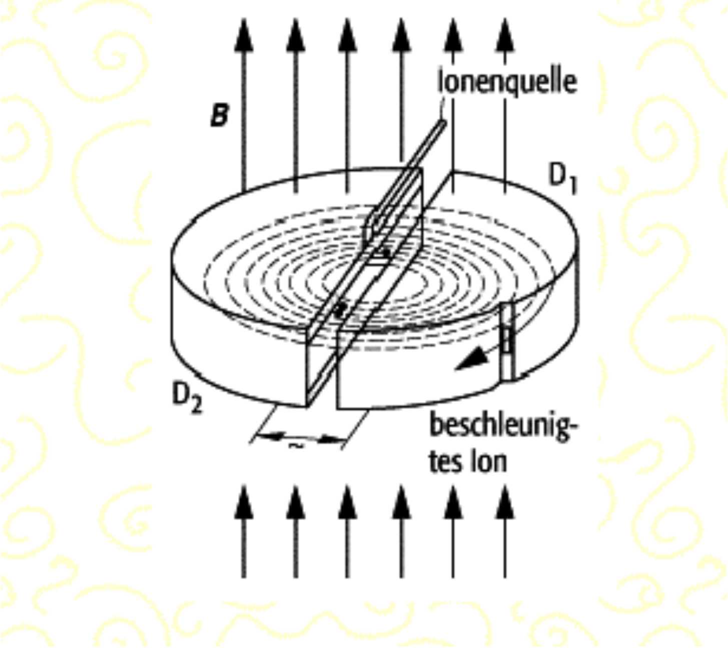
rest energy $E_0 = m_0c^2$

total energy $E = \gamma * E_0 = \gamma * m_0c^2$

momentum $E^2 = c^2p^2 + m_0{}^2c^4$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

4.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: Bend a Linac on a Spiral
Application of a constant magnetic field
keep B = const, RF = const

➔ Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p / q$$

increasing radius for
increasing momentum
➔ Spiral Trajectory

revolution frequency

$$\omega_z = \frac{q}{m} * B_z$$

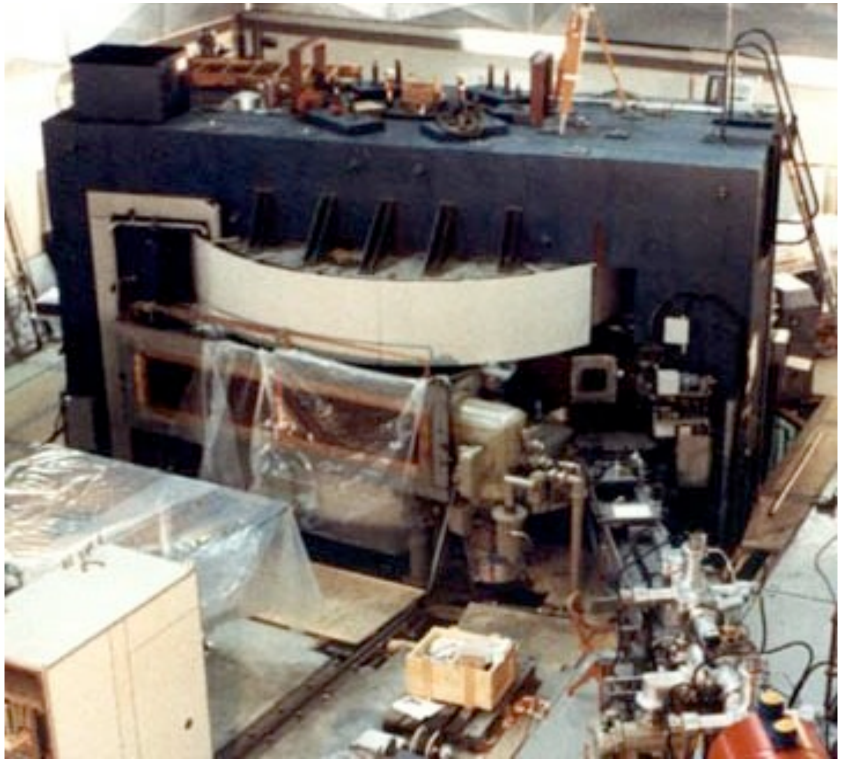
the cyclotron (rf-) frequency
is independent of the momentum

Cyclotron:

! ω is constant for a given q & B

!! B*R = p/q
large momentum → huge magnet

!!!! ω ~ 1/m ≠ const works properly only for non relativistic particles



PSI Zurich

Application:
Work horses for medium energy protons
Proton / Ion Acceleration up to ≈ 60 MeV (proton energy)
nuclear physics
radio isotope production, proton / ion therapy

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine“
→ need transverse deflecting force

Lorentz force $$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines: $$v \approx c \approx 3*10^8 \, {}^{m}\!/\!{}_{s}$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q*3*10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * \underbrace{300 \frac{MV}{m}}$$

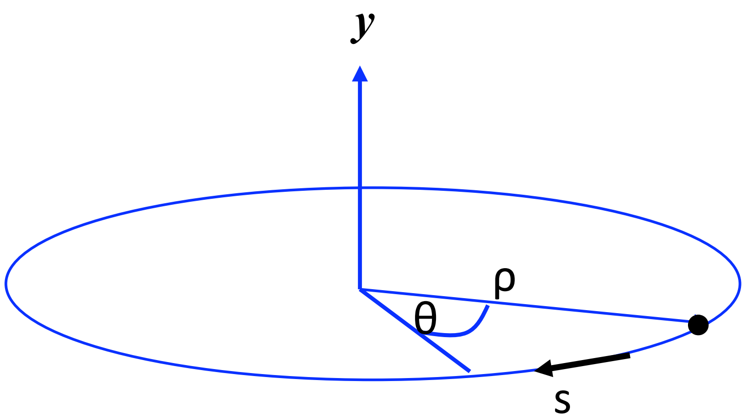
equivalent E
electrical field:

technical limit for el. field:♪

$$E \leq 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

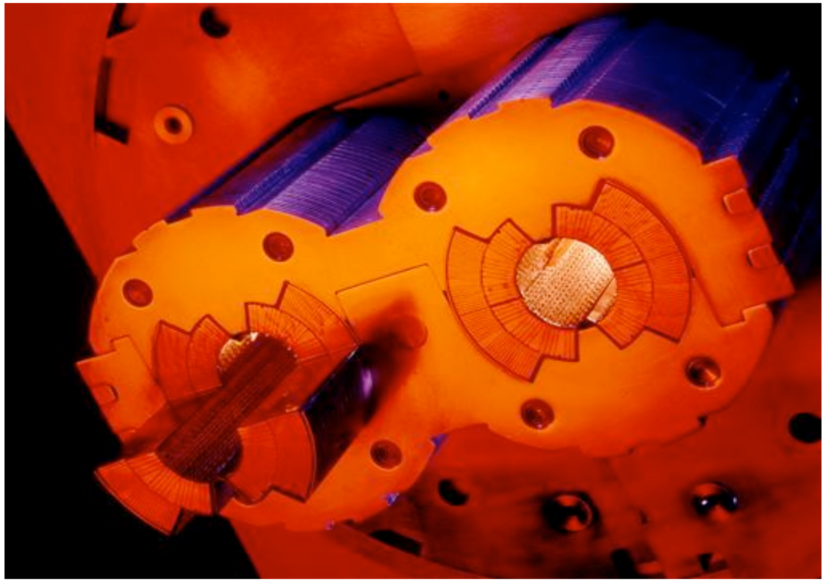
Lorentz force $$F_L = e\,v\,B$$

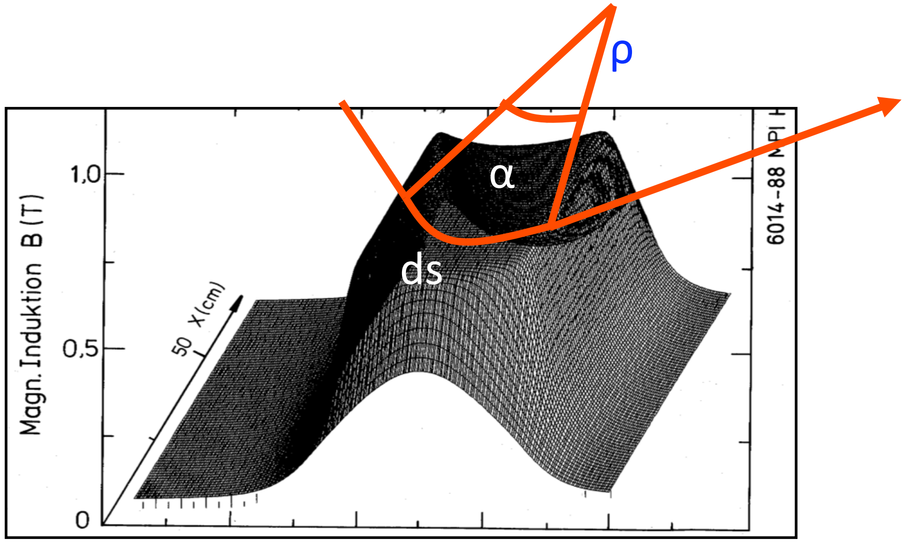
centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

$$\left.\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B \right\} \quad \boxed{\frac{p}{e} = B\,\rho}$$

B ρ = "beam rigidity"

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.53 \; km$ → 2πρ = 17.6 km

≈ 66%

$B \approx 1 \ldots 8 \;\; T$

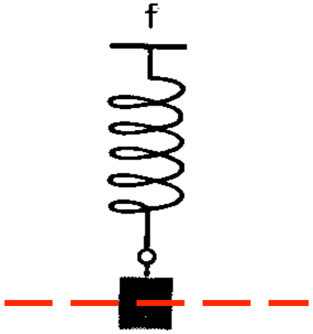
rule of thumb:

$$\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

„normalised bending strength“

2.) Focusing Properties – Kurzer Ausflug in die klassische Mechanik

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

Storage Ring: we need a Lorentz force that rises as a function of the distance to .... ?

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

required: focusing forces to keep trajectories in vicinity of the ideal orbit

linear increasing Lorentz force

linear increasing magnetic field

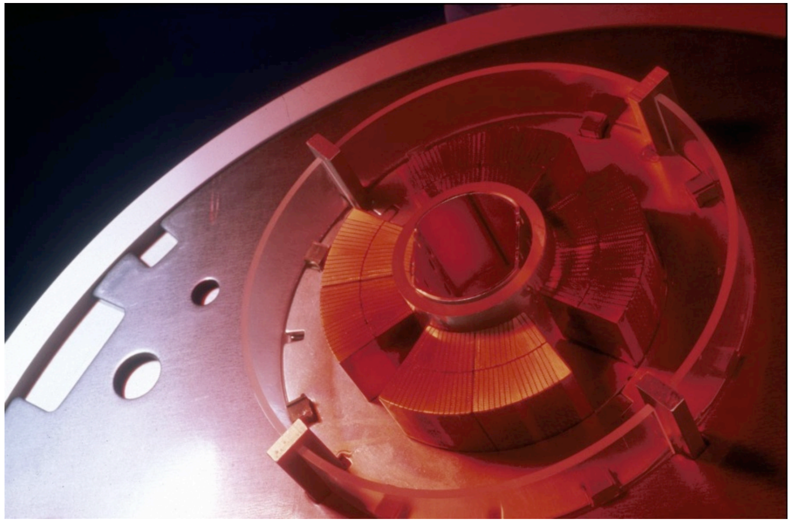
$$B_y = g\,x \qquad B_x = g\,y$$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$

simple rule:

$$\boxed{k = 0.3\,\frac{g(T/m)}{p(GeV/c)}}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220 \;\; T/m$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

Focusing forces and particle trajectories:

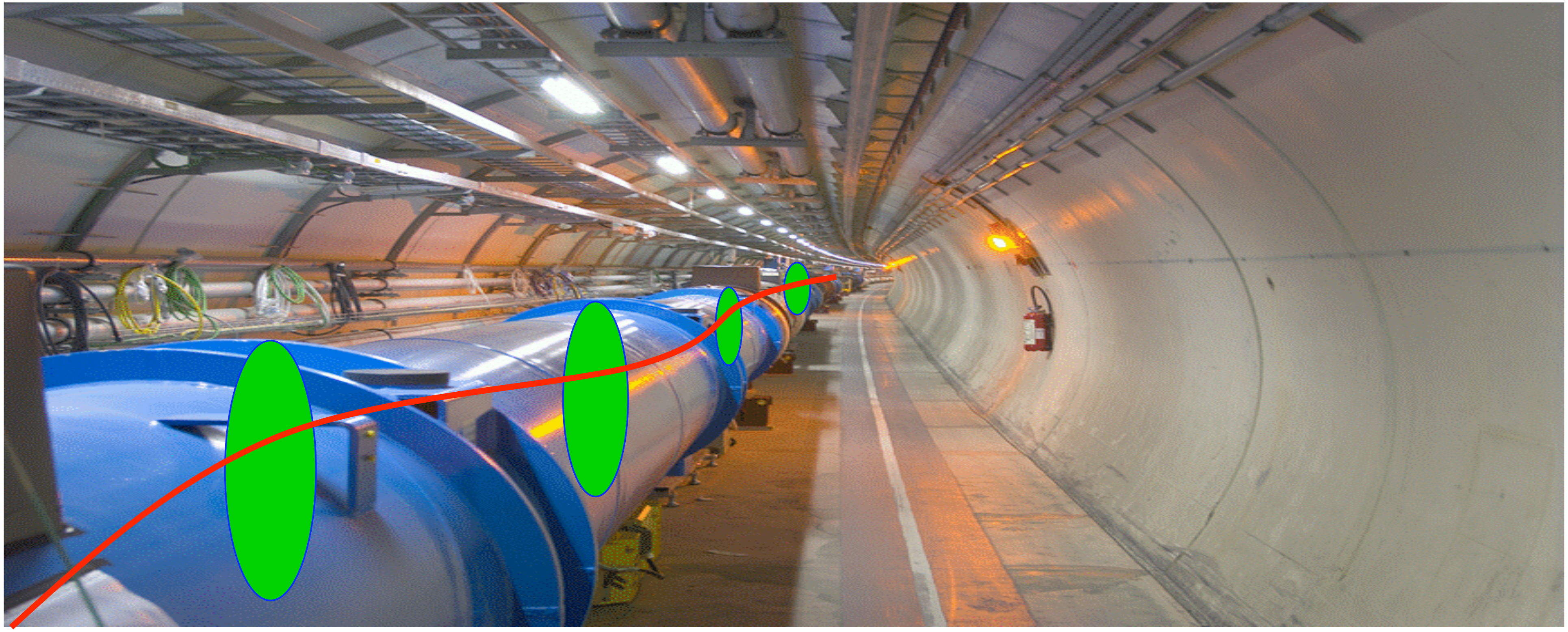
normalise magnet fields to momentum
(remember: B*ρ = p / q)

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

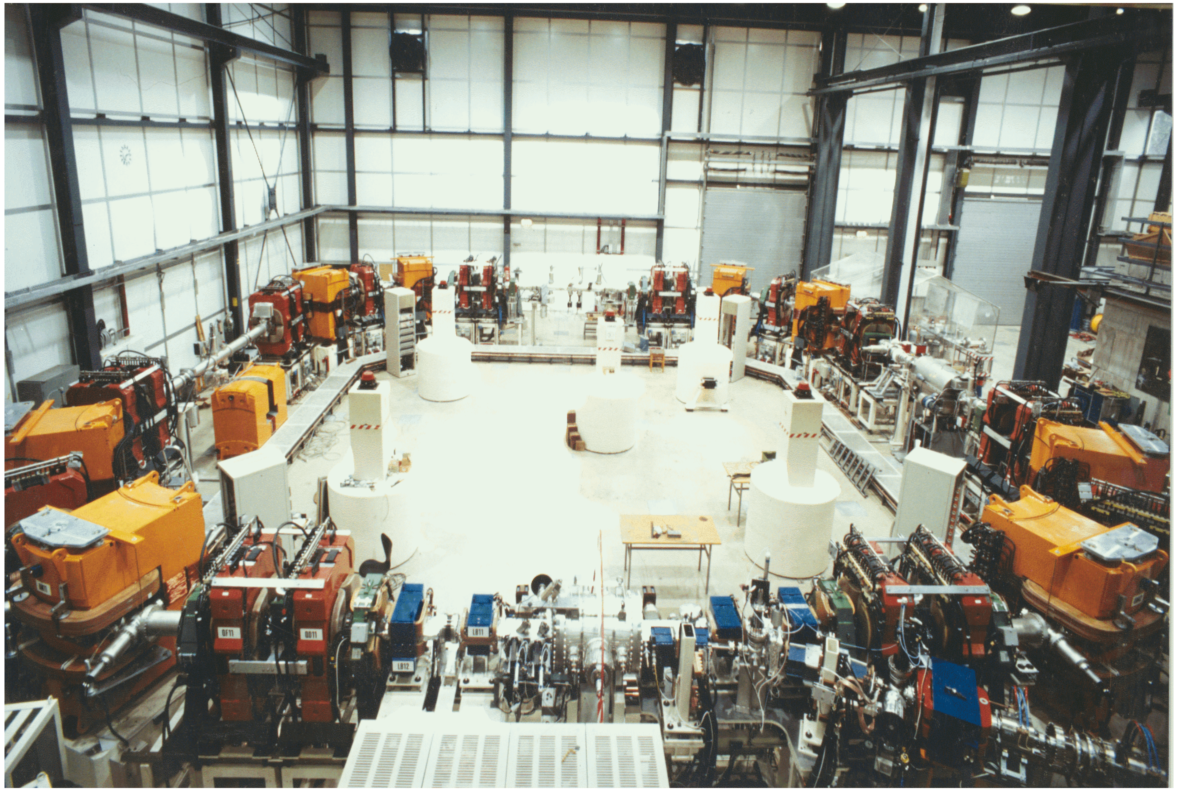
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}m\,x^2} + \cancel{\frac{1}{3!}n\,x^3} + \ldots$$

only terms linear in x, y taken into account dipole fields
quadrupole fields



Separate Function Machines:

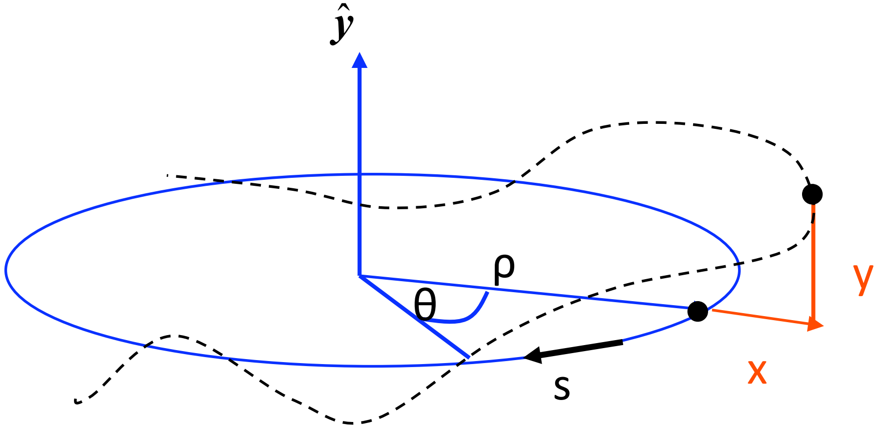
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* man sieht nur
dipole und quads → linear

The Equation of Motion:



* Equation for the horizontal motion:

$$x'' + x\,(\frac{1}{\rho^2} + k) = 0$$

x = particle amplitude

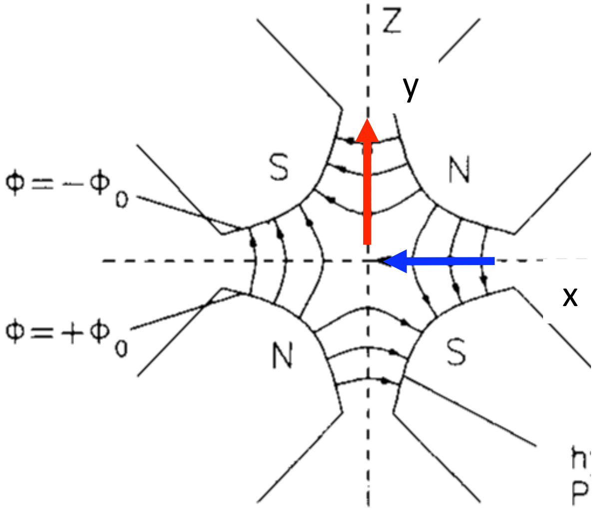
x’ = angle of particle trajectory (wrt ideal path line)

* Equation for the vertical motion:

$$\frac{1}{\rho^2} = 0$$ no dipoles … in general …

$$\boldsymbol{k} \;\leftrightarrow\; -\boldsymbol{k}$$ quadrupole field changes sign

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

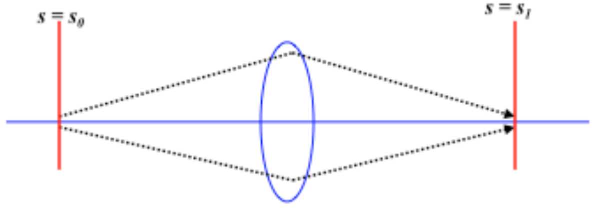
$$\boldsymbol{x}'' + \boldsymbol{K}\ \boldsymbol{x} = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: Hor. Focusing Quadrupole K > 0:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



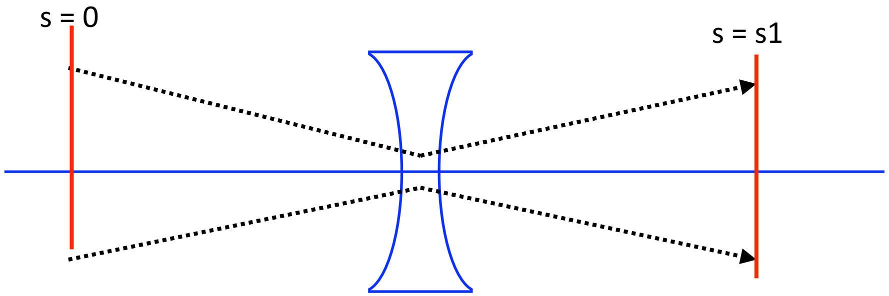
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boldsymbol{M}_{foc} = \begin{pmatrix} \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \frac{1}{\sqrt{|\boldsymbol{K}|}} \sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \\ -\sqrt{|\boldsymbol{K}|} \sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$\boldsymbol{x}'' - \boldsymbol{K}\ \boldsymbol{x} = 0$$



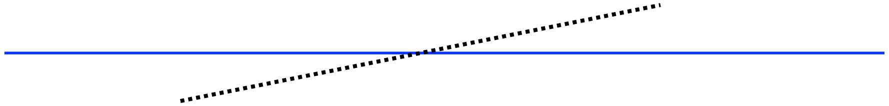
Ansatz: Remember from school

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

K = 0



$$x(s) = x_0' * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

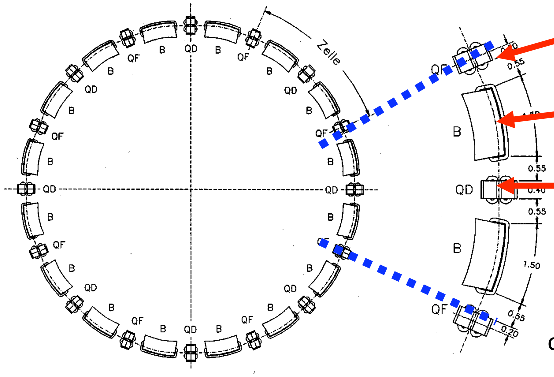
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2,s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



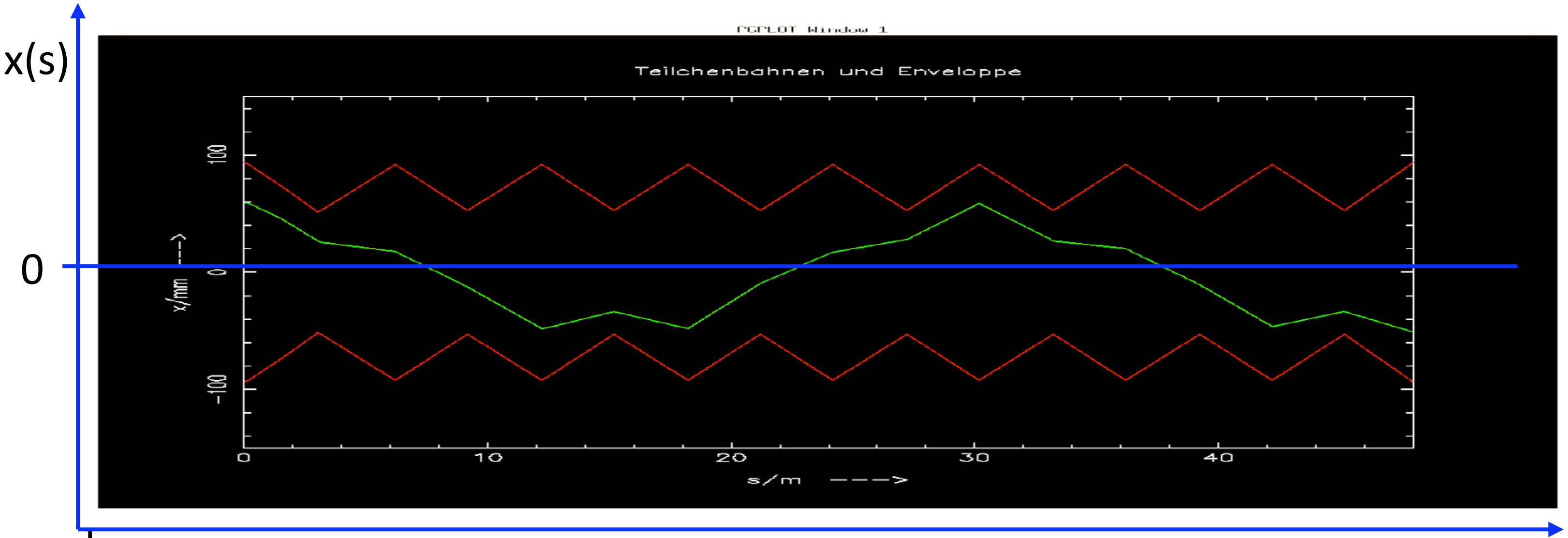
focusing lens

dipole magnet

defocusing lens

court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



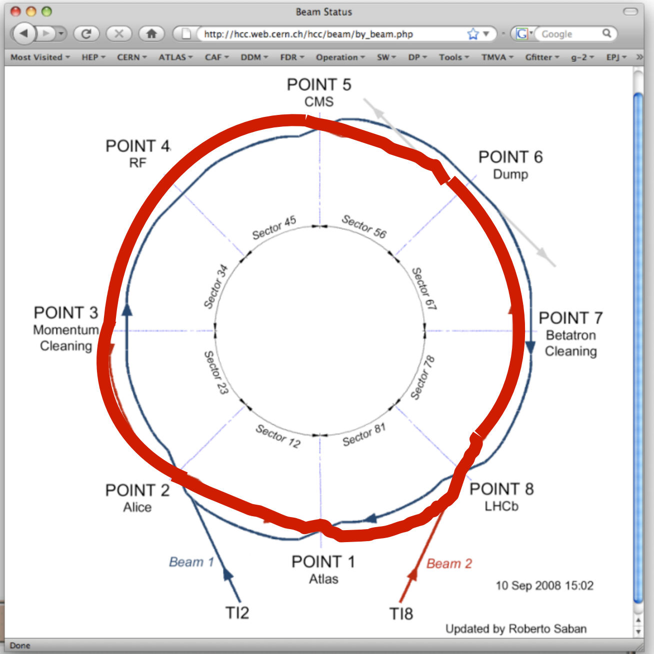
typical values
in a strong
foc. machine:

x ≈ mm, x´ ≤ mrad

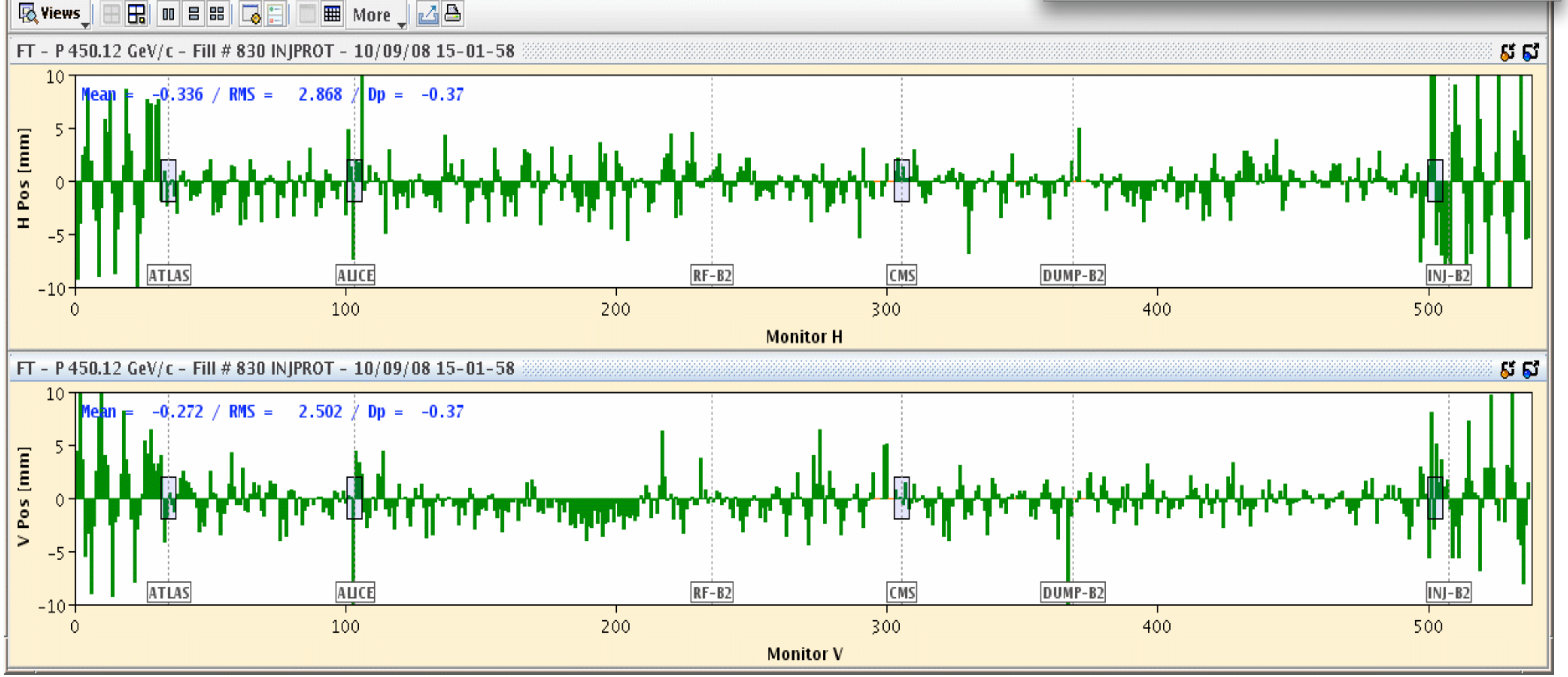
LHC Operation: Beam Commissioning

The transverse focusing fields create a harmonic oscillation of the particles with a well defined “Eigenfrequency” which is called tune

First turn steering "by sector:”



YASP DV LHCRING / INJ-TEST-NB / beam 2



FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

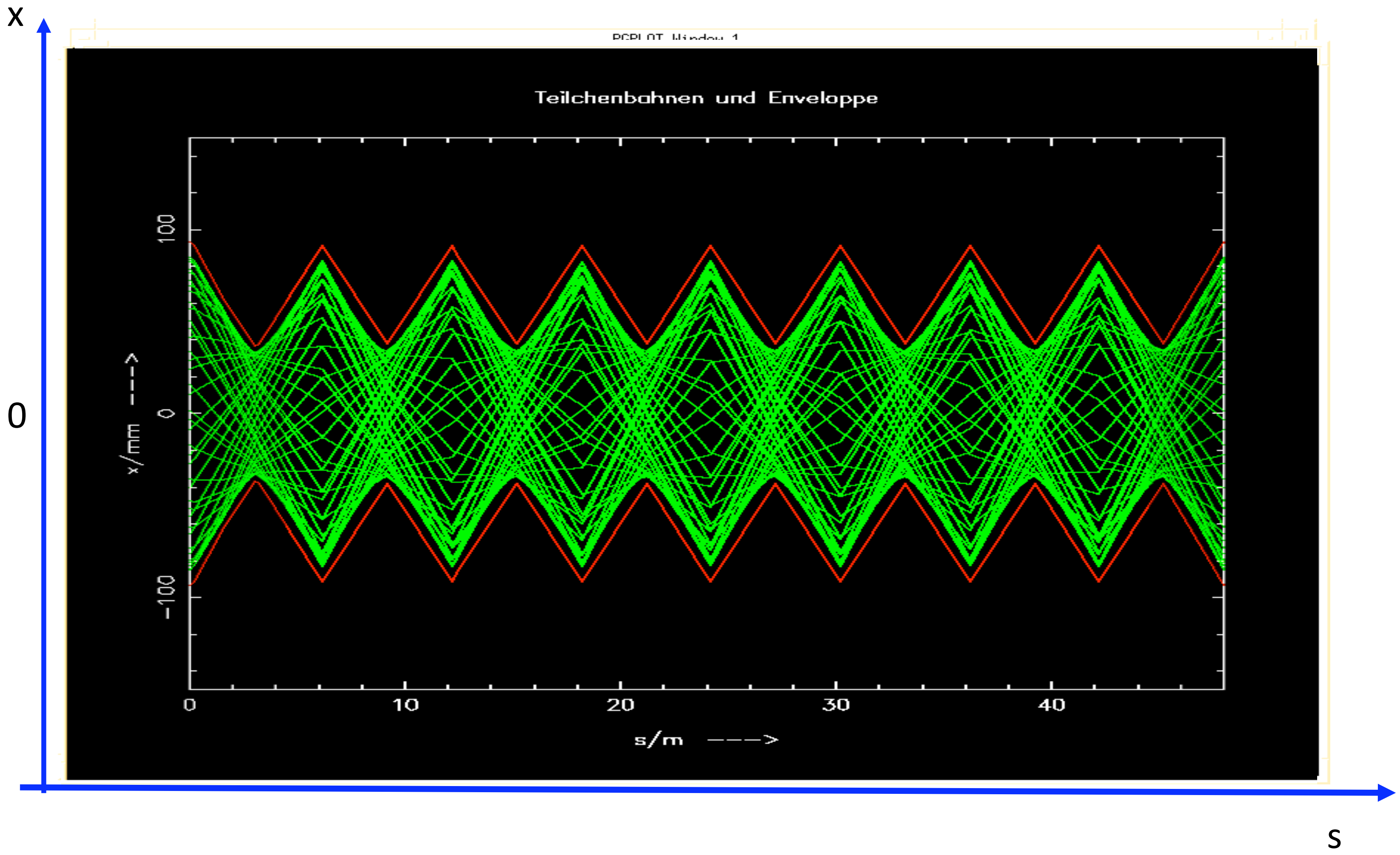
Mean = -0.336 / RMS = 2.868 / Dp = -0.37

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.272 / RMS = 2.502 / Dp = -0.37

Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill‘s equation“

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

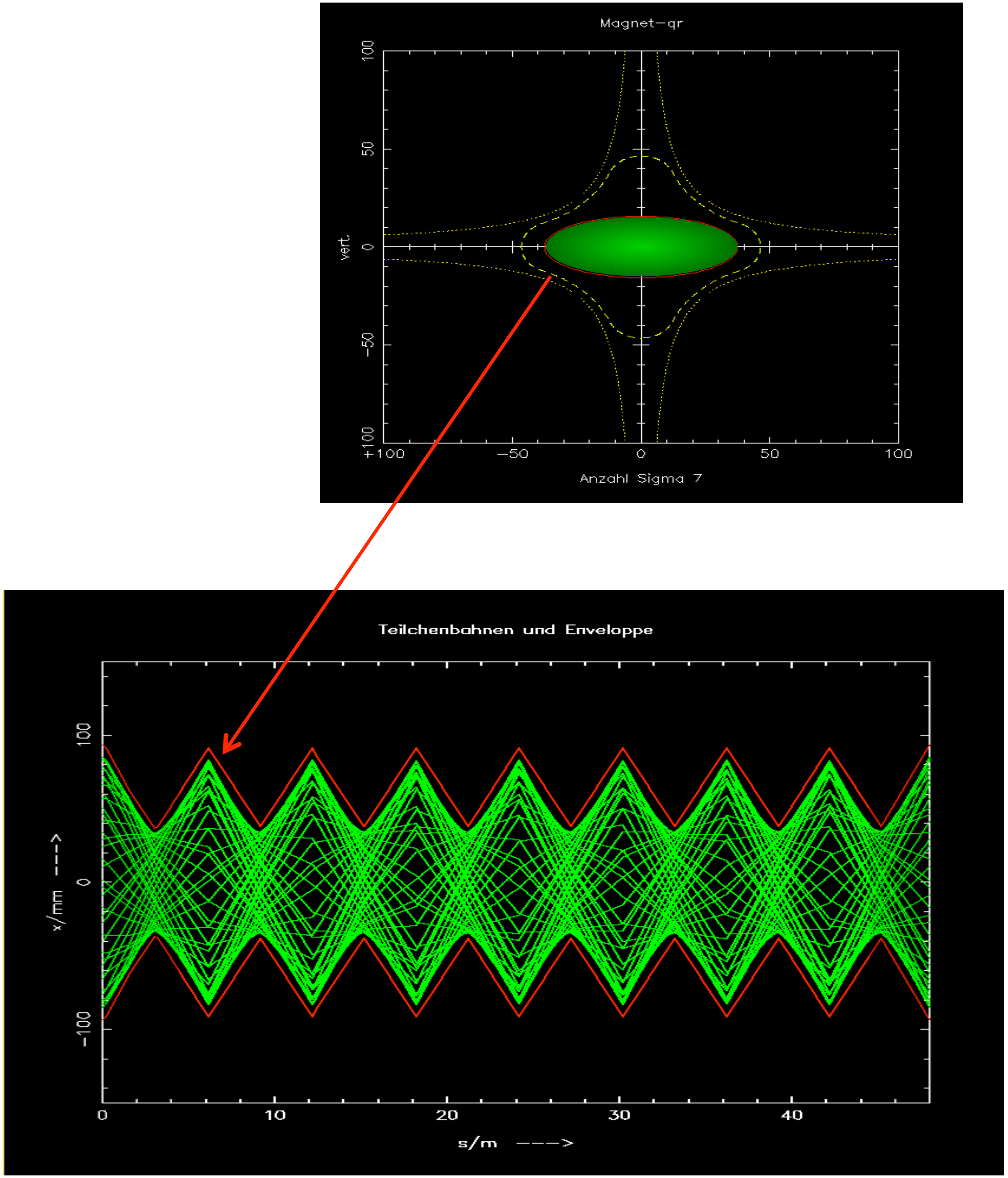
Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$

The Beta Function

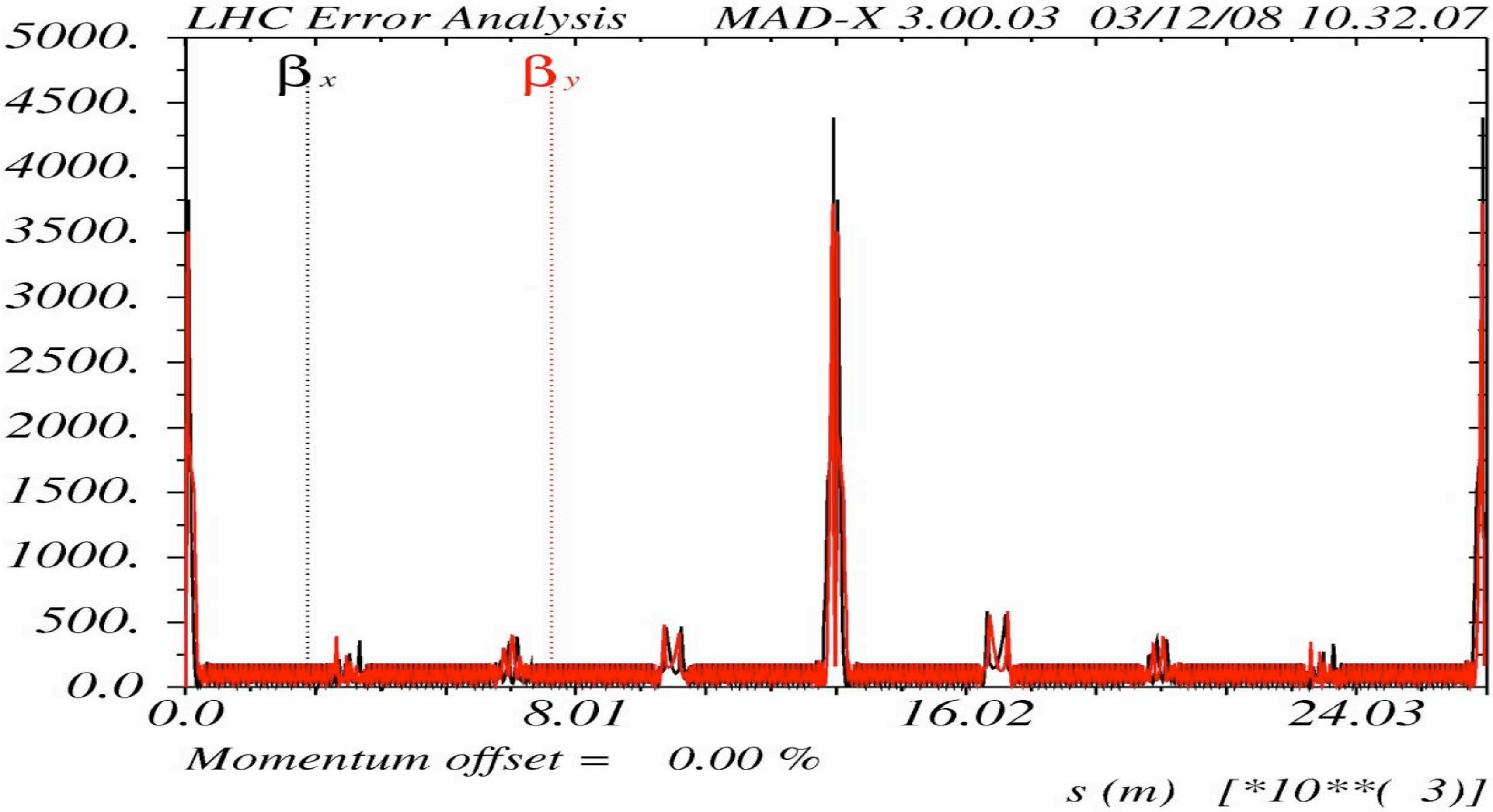
β determines the beam size
... the envelope of all particle trajectories at a given position "s" in the storage ring under the influence of all (!) focusing fields.

It reflects the periodicity of the magnet structure.



The Beta Function: Lattice Design & Beam Optics

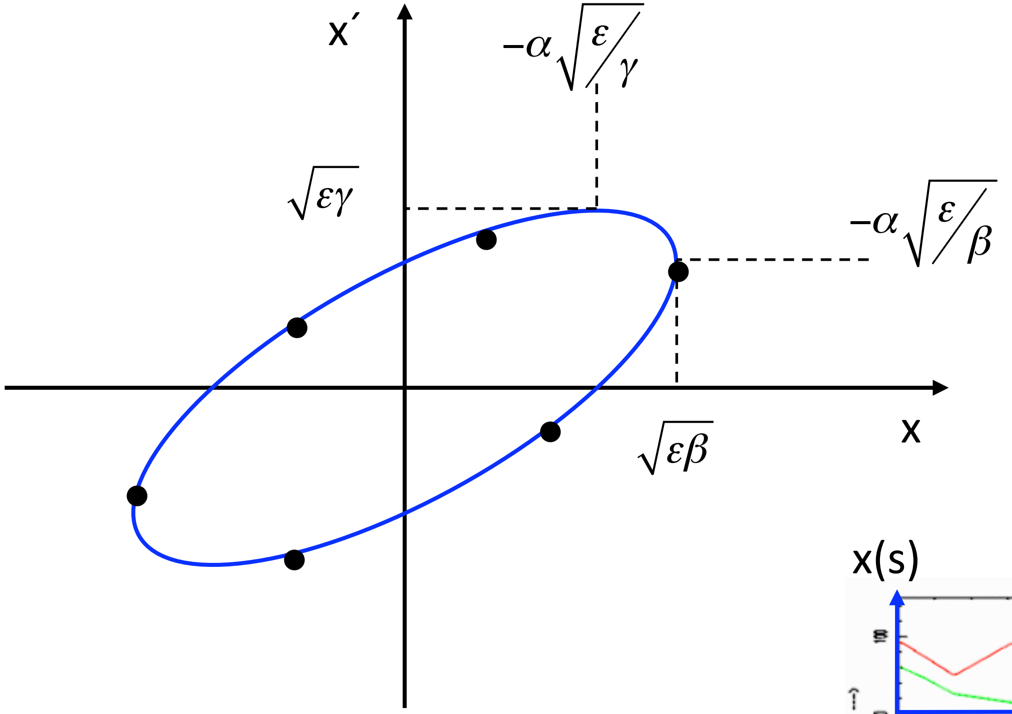
The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.
It is determined by the focusing properties of the lattice and follows the periodicity of the machine.



optics at 7000 GeV

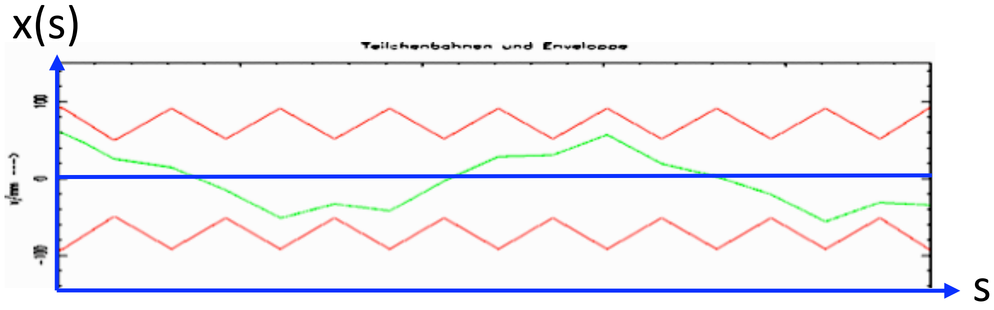
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



Liouville: in reasonable storage rings area in phase space is constant.

A = π*ε=const



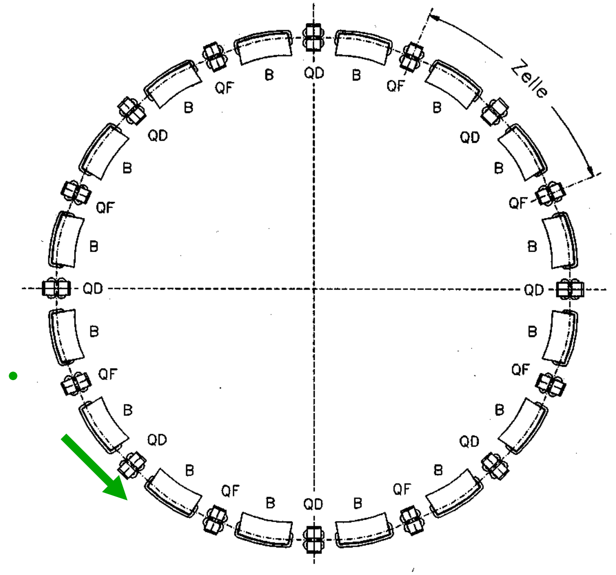
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter, cannot be changed by the foc. properties.

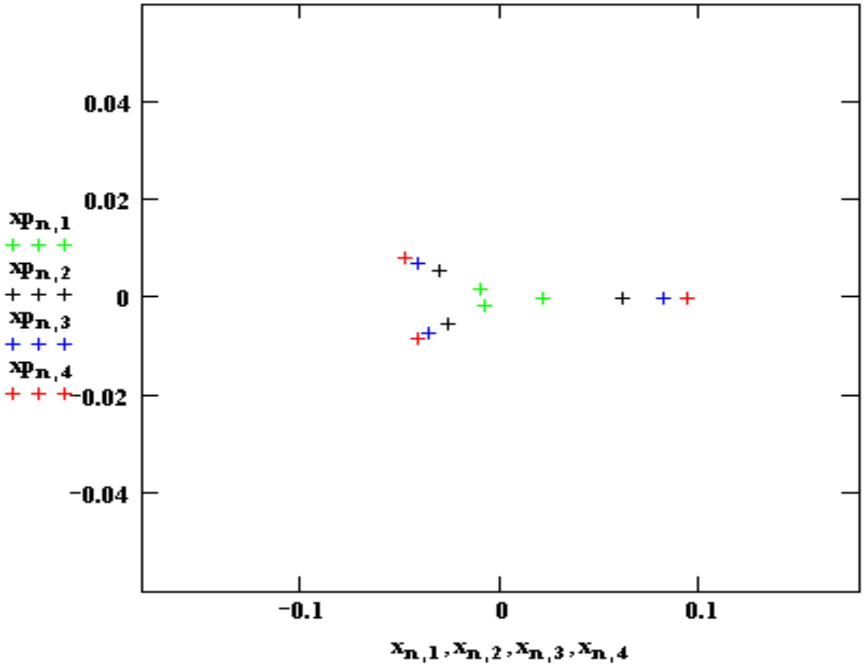
Scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!

Particle Tracking in a Storage Ring

Calculate x, x´ for each accelerator element according to matrix formalism and plot x, x´at a given position „s“ in the phase space diagram

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



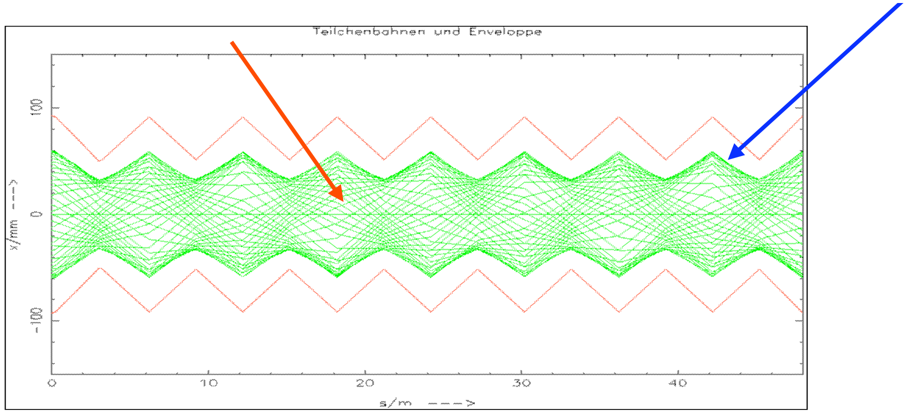


A beam of 4 particles

– each having a slightly different emittance:

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 ^{11} per bunch

Gauß
Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x{}^2}}$$

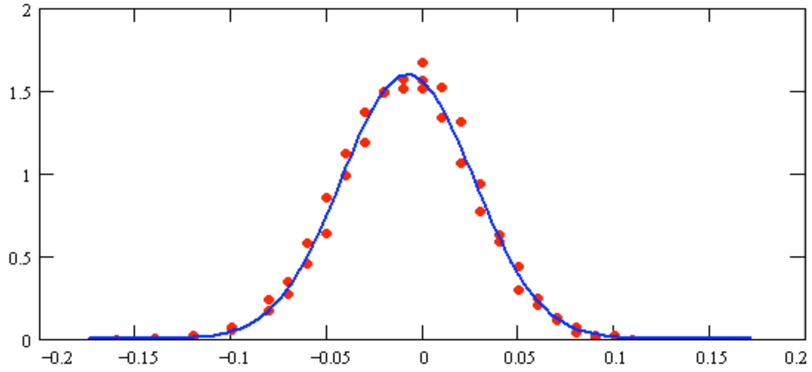
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

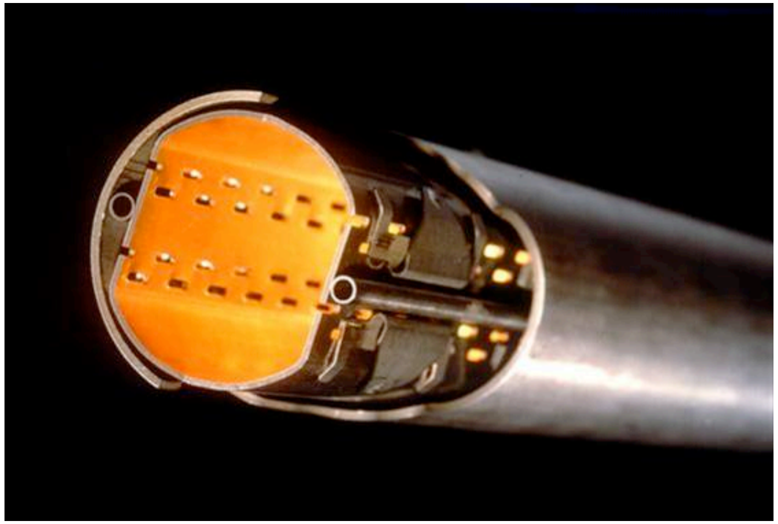
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5 * 10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\,m * 180\,m} = 0.3\,mm$$

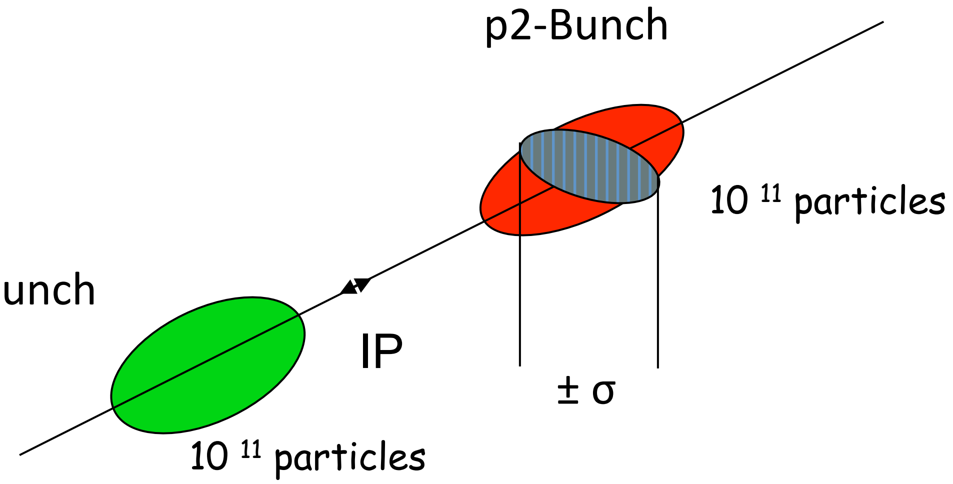




aperture requirements: r $_0$ = 17 * σ

5.) Luminosity

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55\ m \qquad f_0 = 11.245\ kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

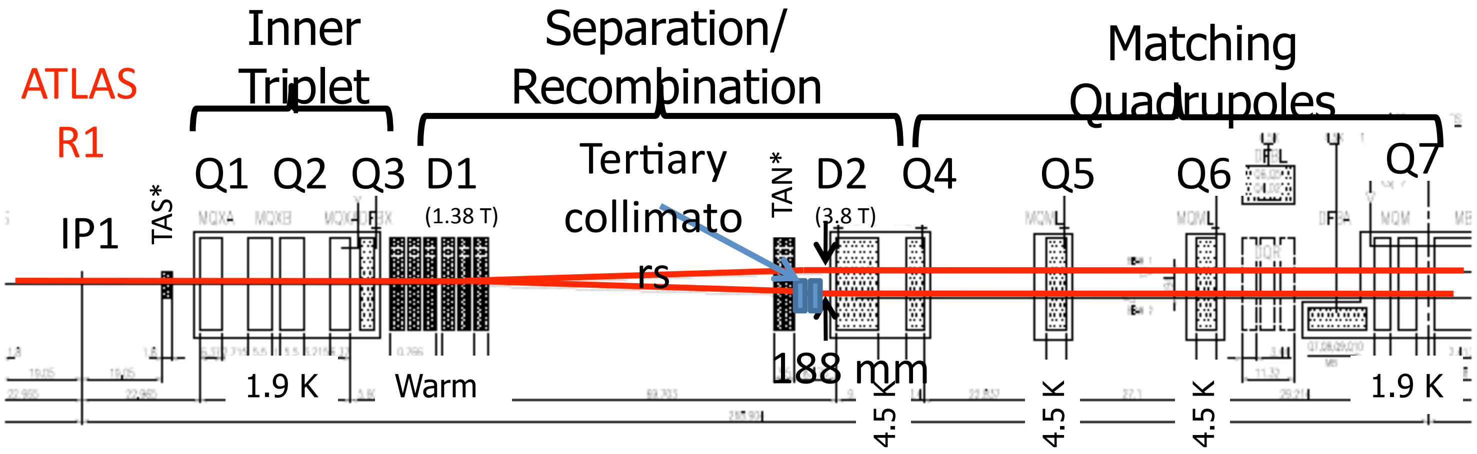
$$\sigma_{x,y} = 17\ \mu m$$

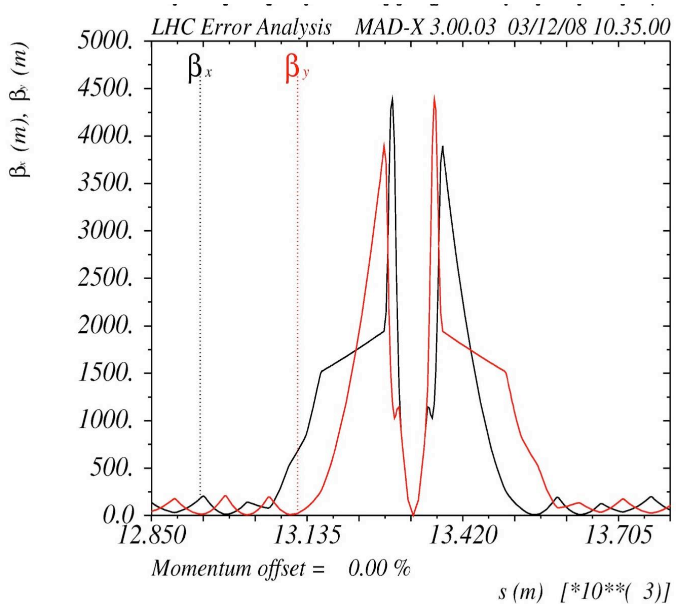
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$I_p = 584\ mA$$

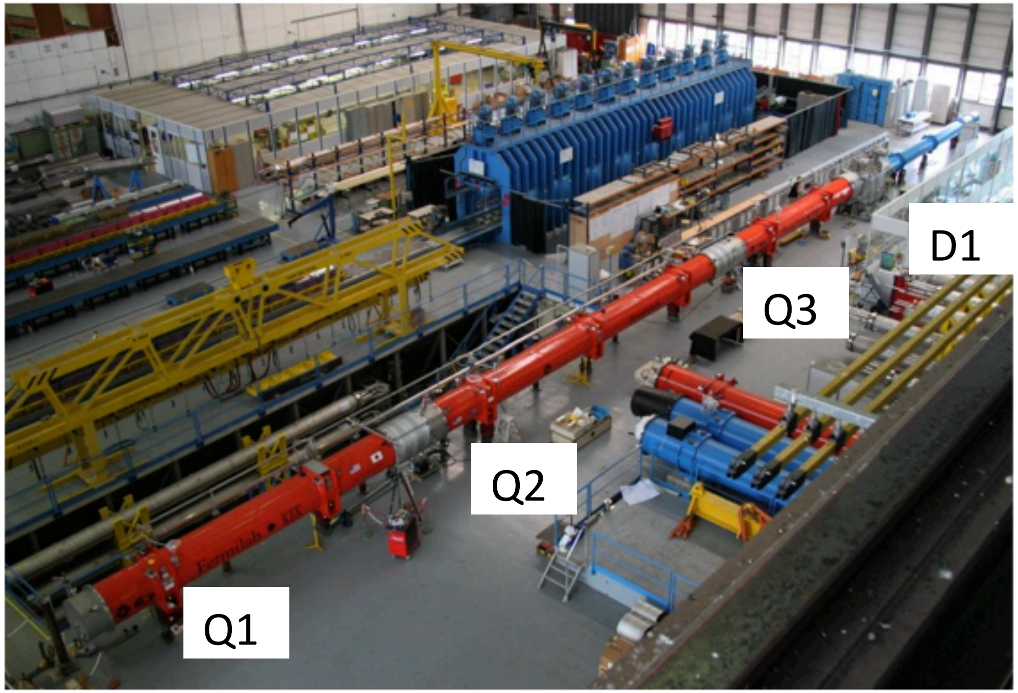
$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

The LHC Mini-Beta-Insertions





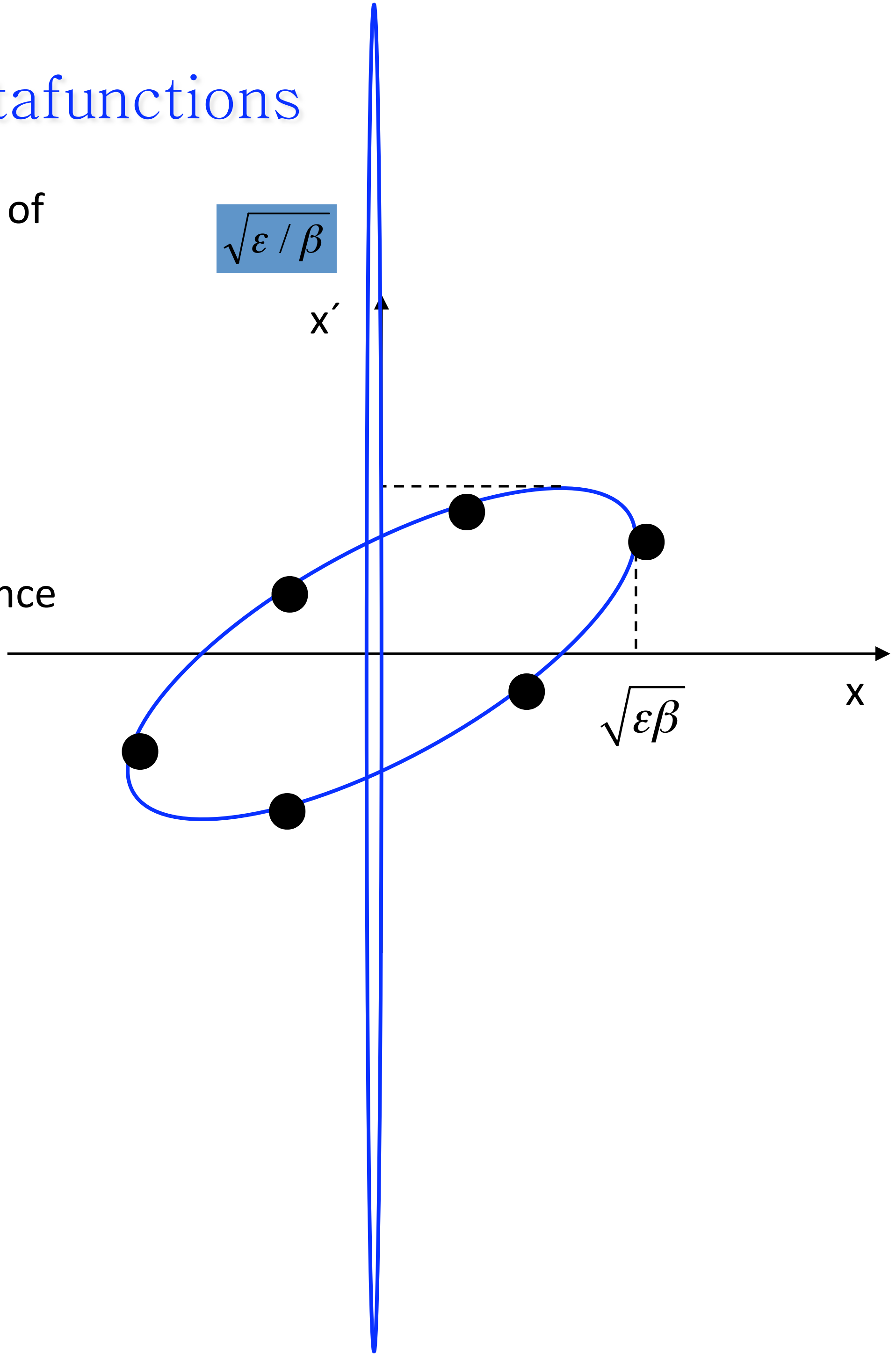
mini β optics



Mini-β Insertions: Betafunctions

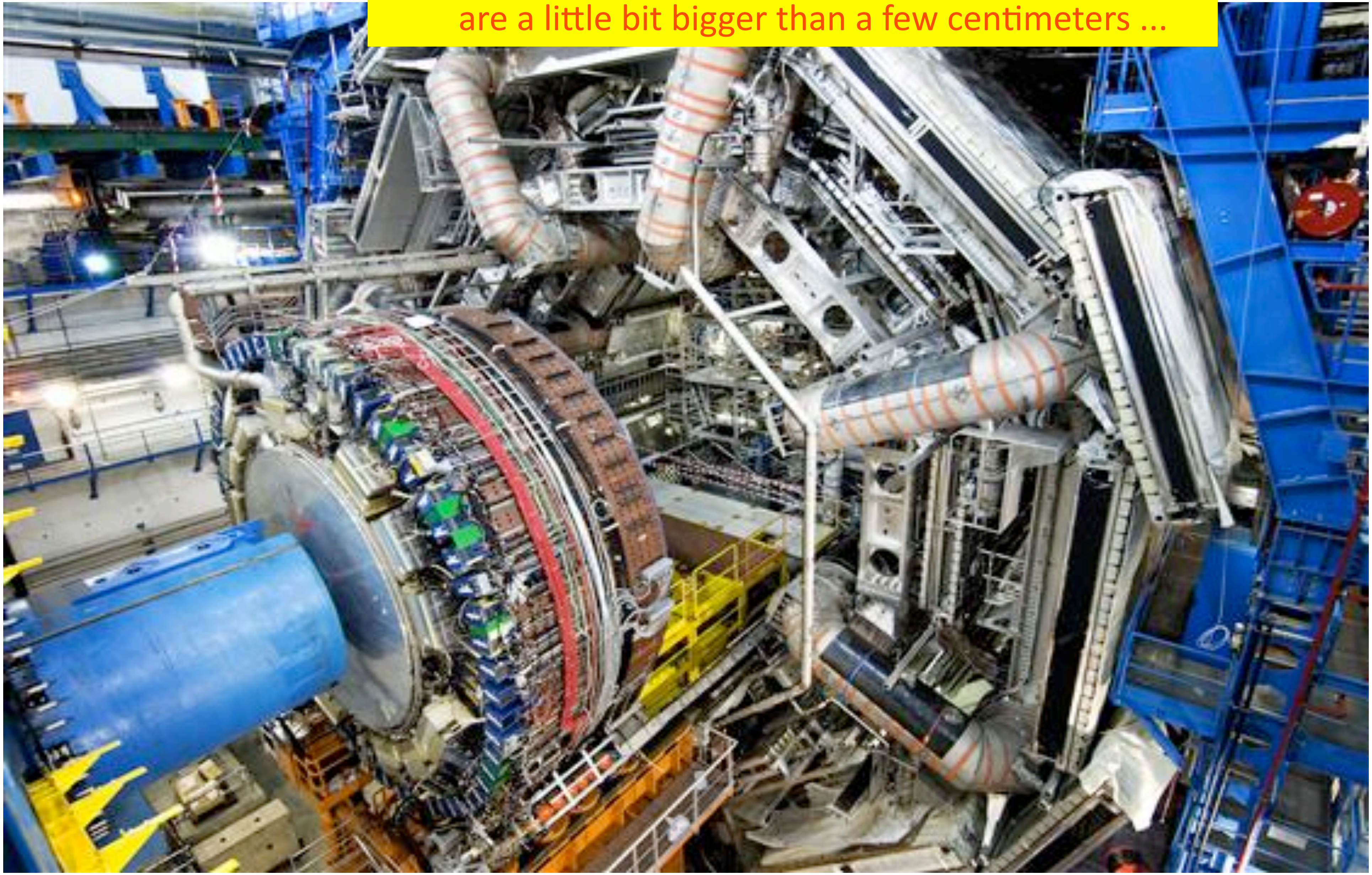
A mini-β insertion is always a kind of
special symmetric drift space.
→greetings from Liouville

the smaller the beam size
the larger the bam divergence



... clearly there is ano

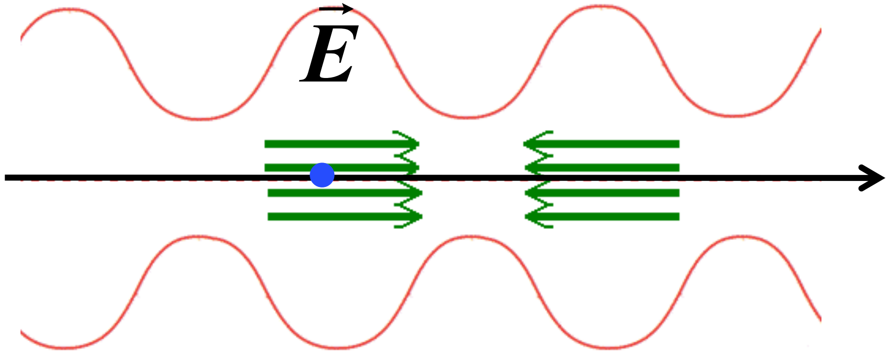
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

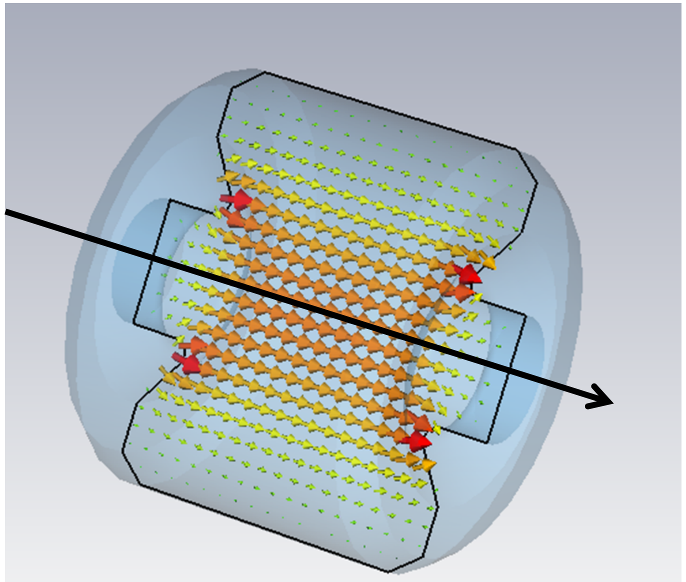


III. The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:





B. Salvant
N. Biancacci

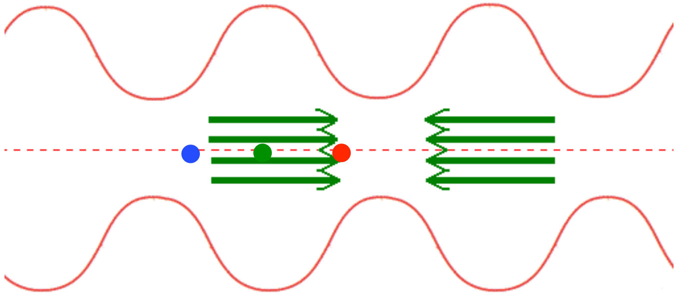
The Acceleration & “Phase Focusing”

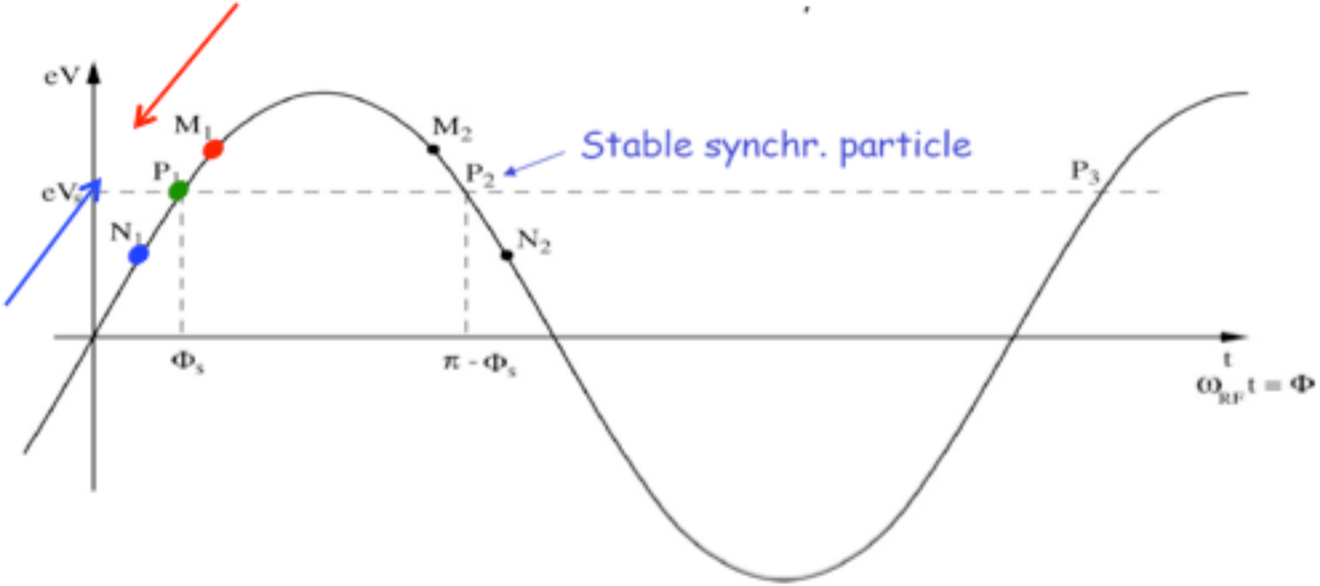
Δp/p≠0 below transition

ideal particle ●

particle with Δp/p > 0 ● faster

particle with Δp/p < 0 ● slower





Focussing effect in the longitudinal direction keeping the particles close together
... forming a “bunch”

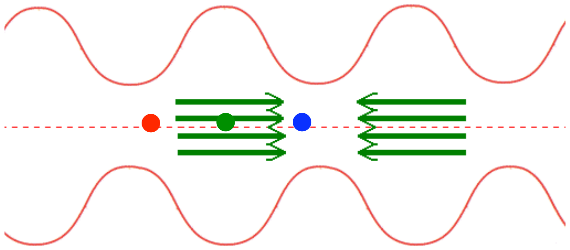
oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ ≈ some Hz

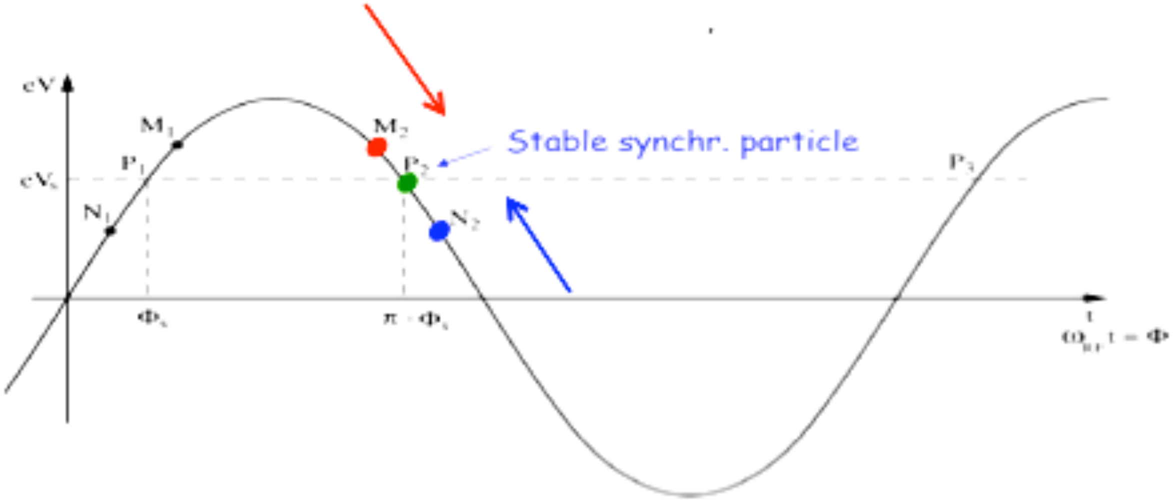
The Acceleration above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● heavier

particle with $\Delta p/p < 0$ ● lighter





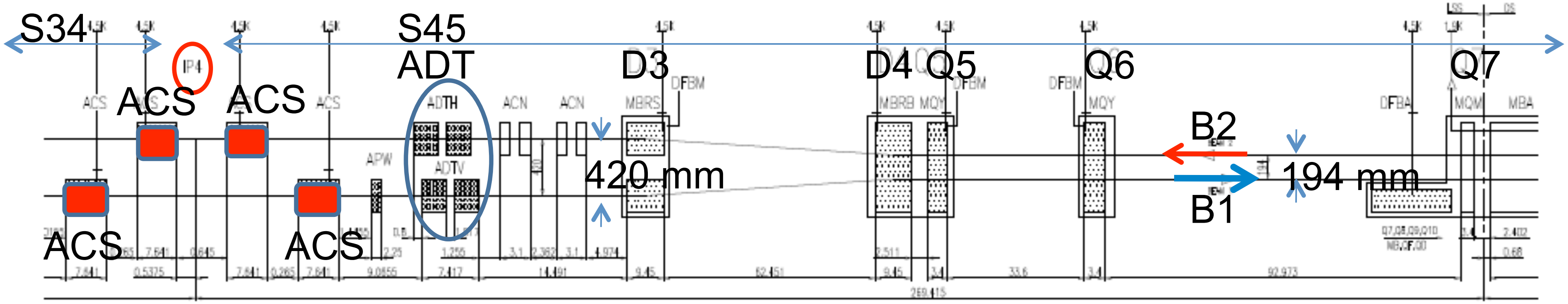
Focussing effect in the longitudinal direction

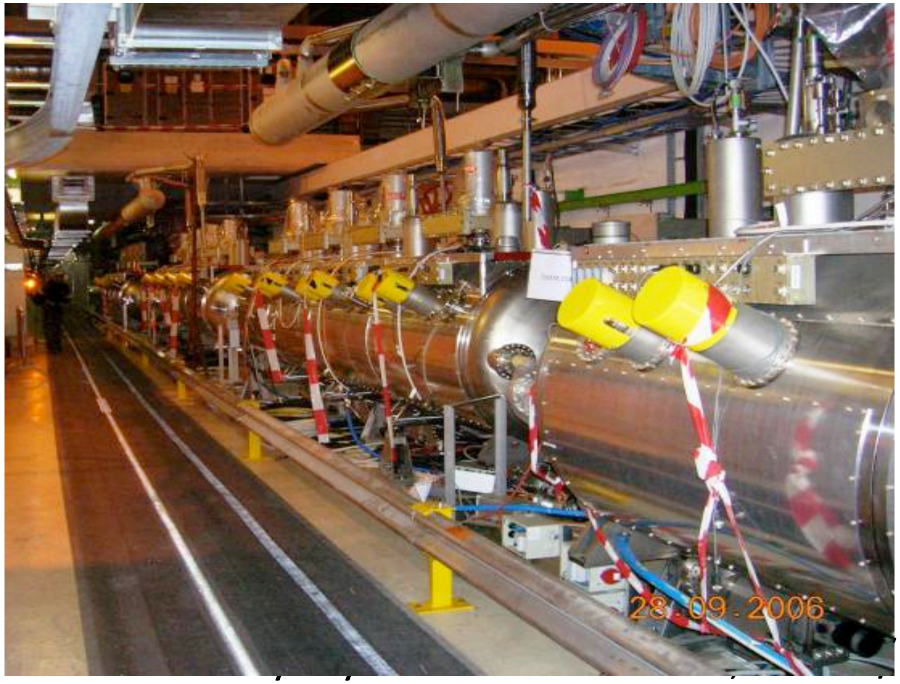
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





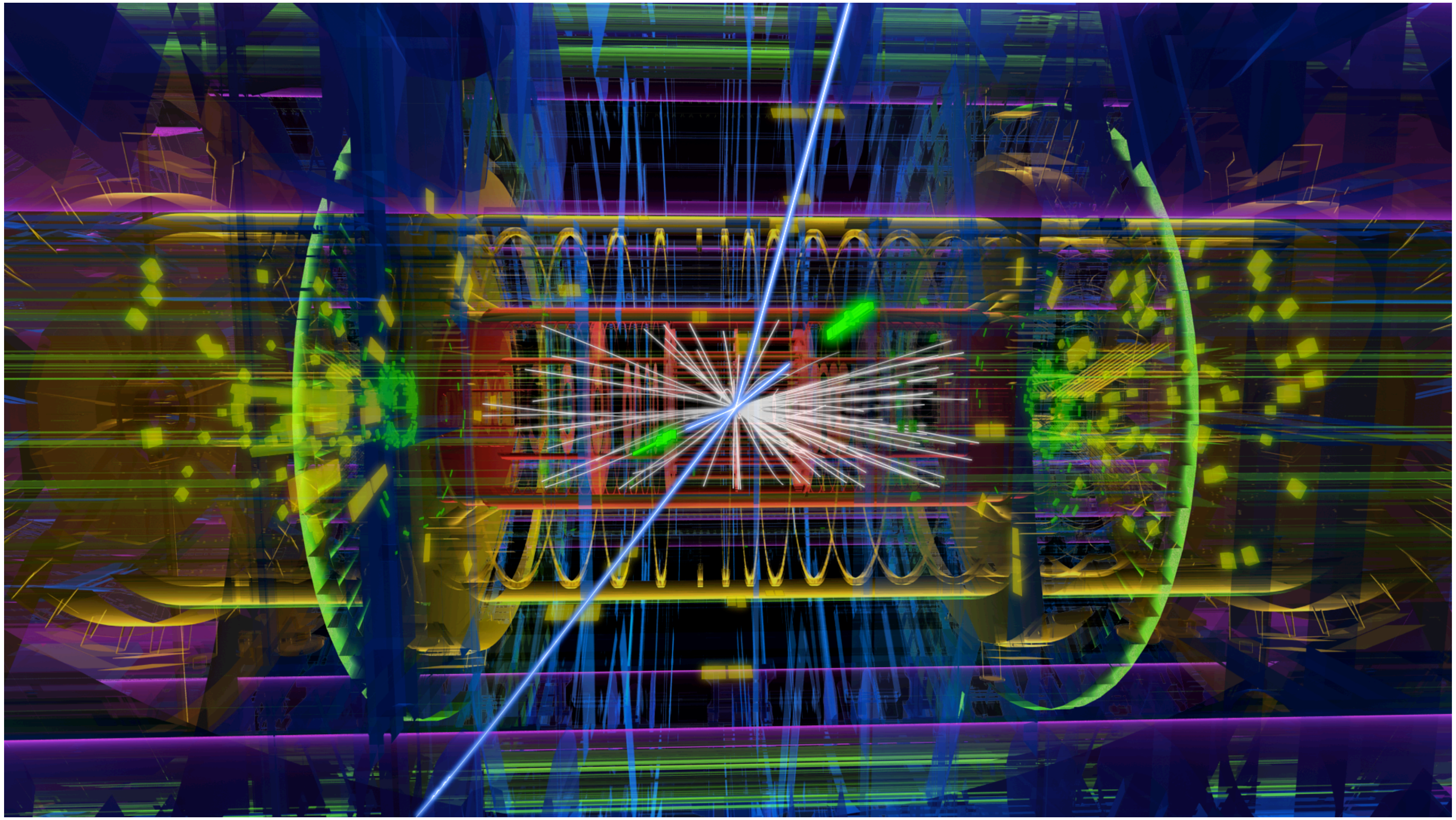
bear

Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*MHz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

... und wozu das alles ??

High Light of the HEP-Year natuerlich das HIGGS

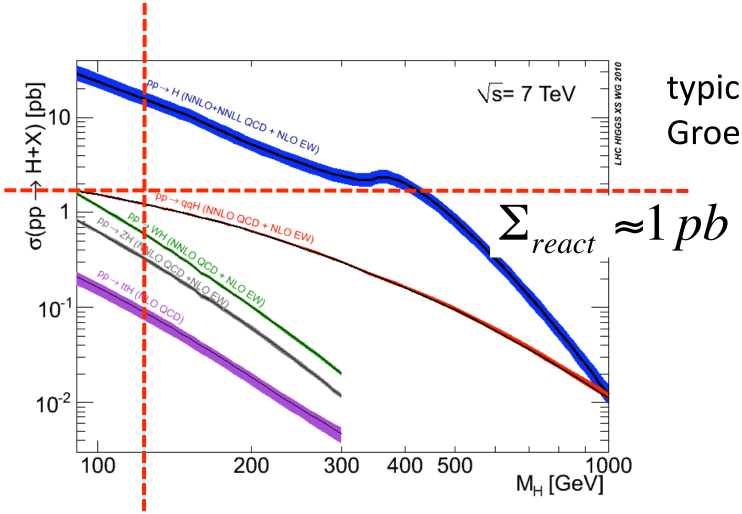


ATLAS event display: Higgs => two electrons & two muons

The High light of the year

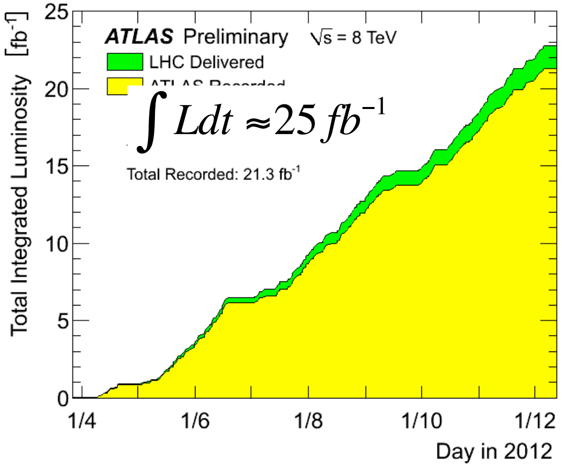
production rate of events is determined by the cross section Σ_{react}
and a parameter L that is given by the design of the accelerator:
… the luminosity

$$R = L * \Sigma_{react} \quad \approx 10^{-12} b \cdot 25 \frac{1}{10^{-15} b} = some\, 1000\, H$$



$$\Sigma_{react} \approx 1\, pb$$

typiche Teilchen
Groesse



$$\int L dt \approx 25\, fb^{-1}$$

akkumulierte
Kollisions Rate

$$1b = 10^{-24} cm^2 \quad = \quad 1mio * 1mio * 1mio * \frac{1}{10000} mm^2$$

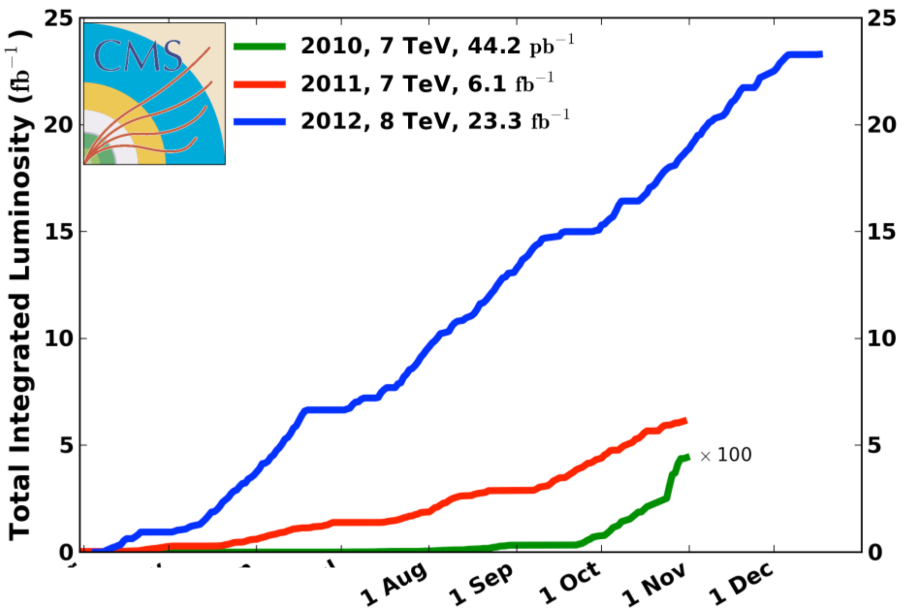
Die Teilchen sind “sehr klein”

The luminosity is a storage ring quality parameter and depends on beam size (β !!) and stored current

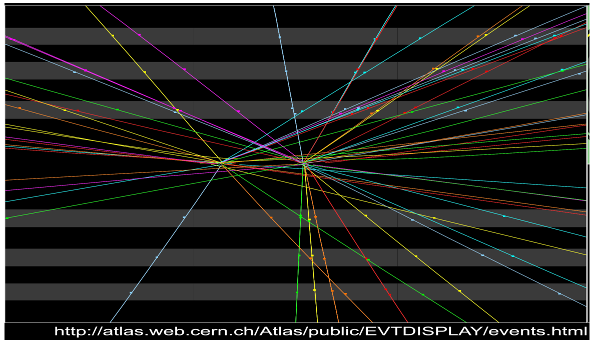
$$L = \frac{1}{4\pi e^2 f_0\, \mathrm{b}} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$

Δp/p = $5*10^{-4}$

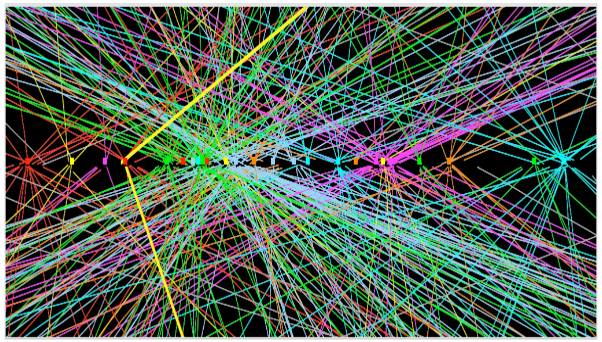
And still...
The LHC Performance in Run 1



	Design	2012
Momentum at collision	7 TeV /c	4 TeV/c
Luminosity	10^{34} cm $^{-2}$ s^{-1}	$7.7*10^{33}$ cm $^{-2}$ s^{-1}
Protons per bunch	1.15×10^{11}	1.50×10^{11}
Number of bunches/beam	2808	1380
Nominal bunch spacing	25 ns	50ns
Normalized emittance	3.75 µm	2.5µm
beta *	55 cm	60 cm
rms beam size (arc)	300 µm	350 µm
rms beam size IP	17 µm	20 µm

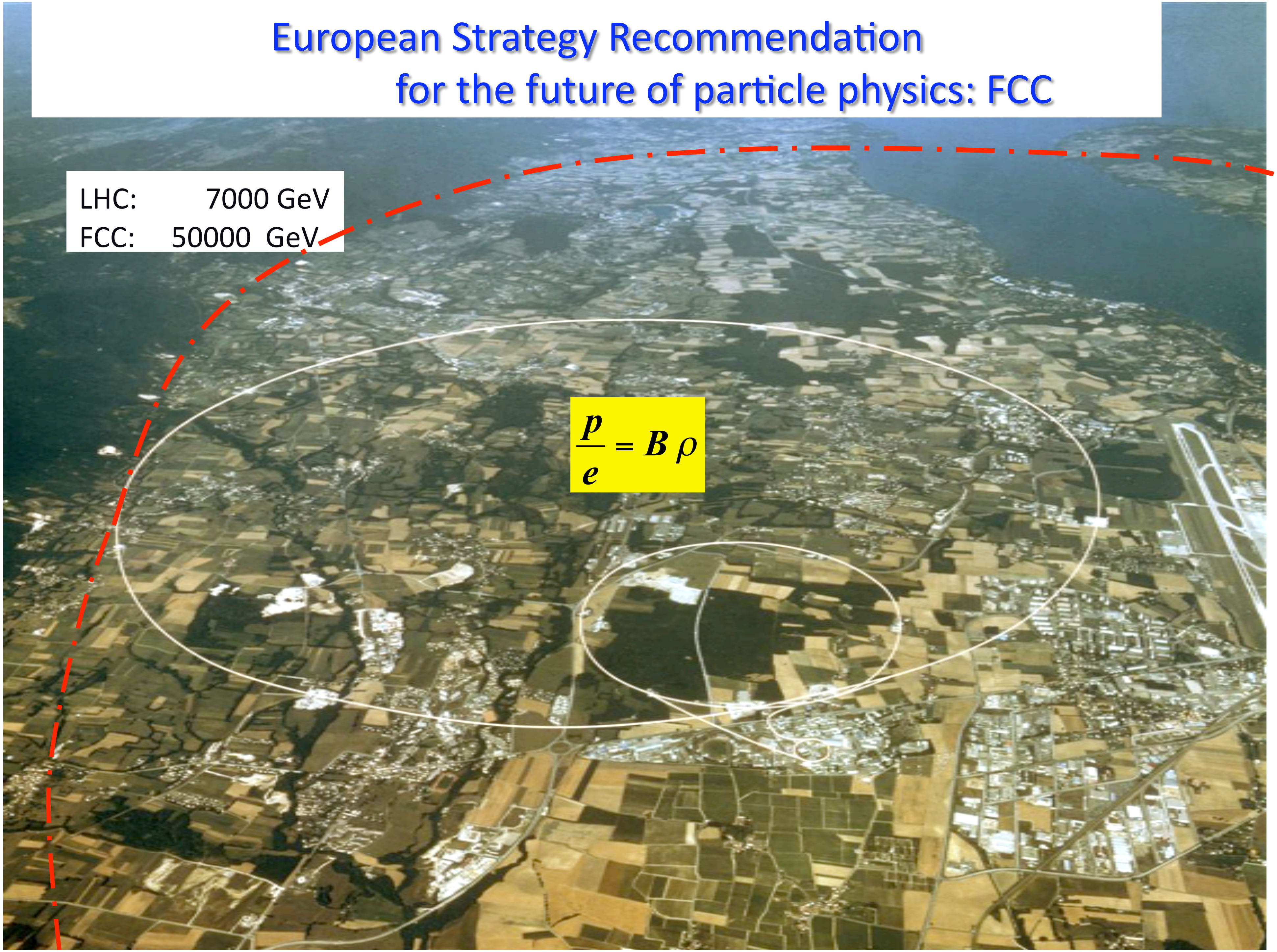


2 vertices



20 vertices

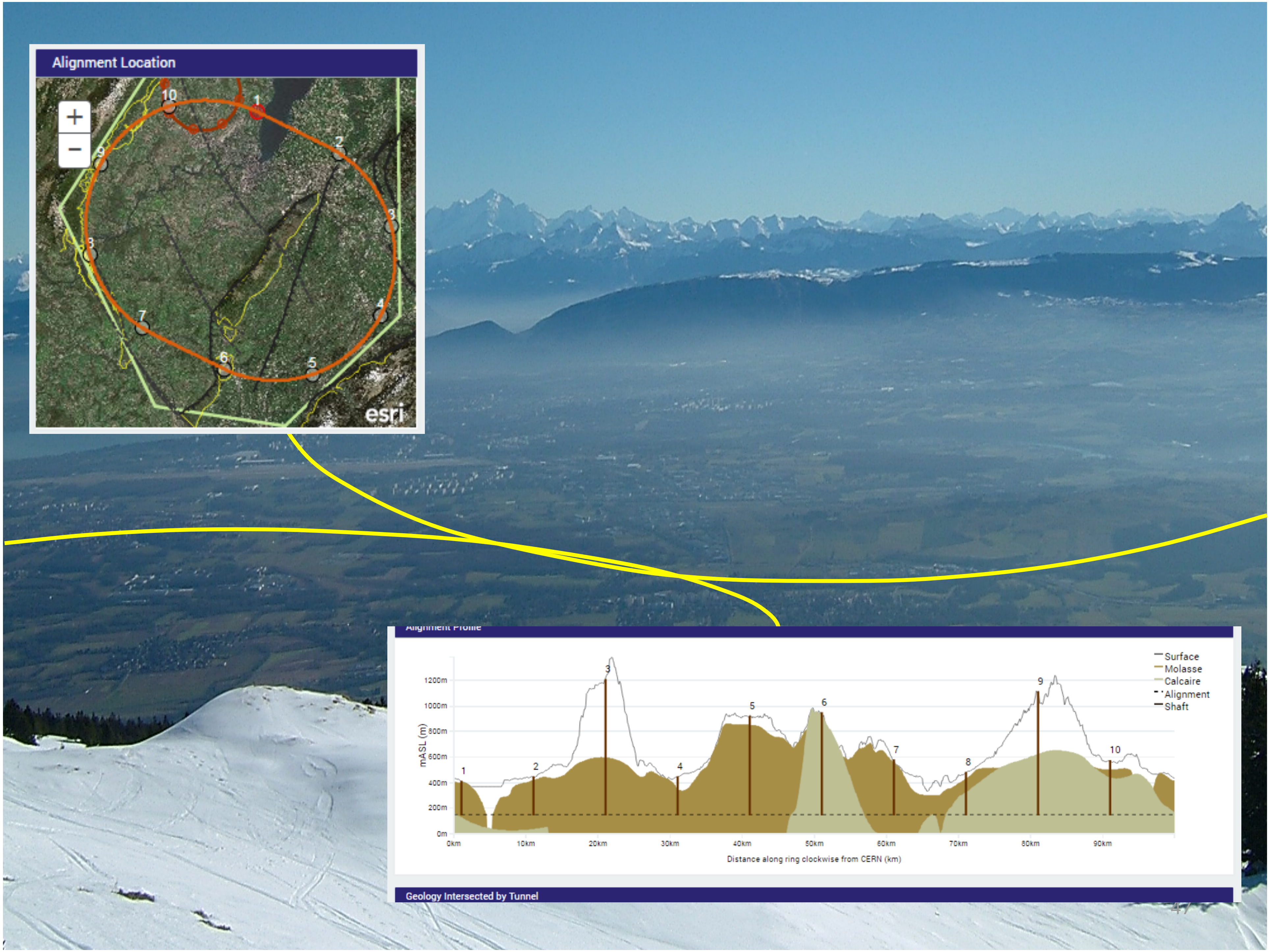
European Strategy Recommendation for the future of particle physics: FCC



LHC: 7000 GeV

FCC: 50000 GeV

$$\frac{p}{e} = B\,\rho$$


Alignment Location
+
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1
2
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esri
Alignment Profile
Surface
Molasse
Calcaire
Alignment
Shaft
1200m
1000m
800m
600m
400m
200m
0m
mASL (m)
1
2
3
4
5
6
7
8
9
10
0km
10km
20km
30km
40km
50km
60km
70km
80km
90km
Distance along ring clockwise from CERN (km)
Geology Intersected by Tunnel