Adding NLO corrections into the initial state parton shower Monte Carlo

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NLO MC activity in Kraków IFJPAN

- New method of NLO-correcting HARD process
  Phys.Rev. D87 (2013) 034029 (arxiv.org/abs/1103.5015)
  Acta Phys.Pol. B42 (2011) 1475 (arxiv.org/abs/1103.5015)
- Ladders of NLO evolution kernels in exclusive Monte Carlo. Since 2006 (arxiv.org/abs/1106.1787).
- Long term: NLO ladders + NNLO hard process:)
- The whole project still at the “feasibility study” stage:(

Relation to other works

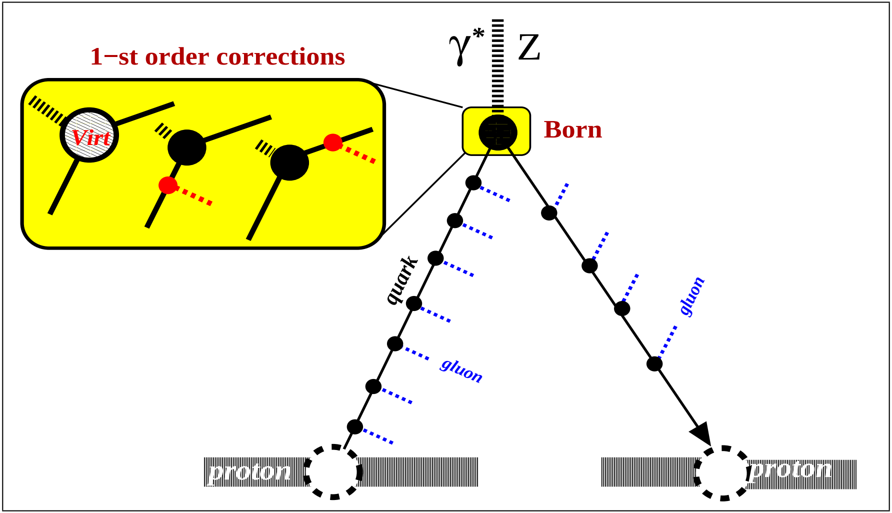
- The basic 2009 method (sum over gluons in NLO wt) is reminiscent to YFS method (worked out in QED by S.J. with Bennie Ward and Zbyszek Wąs)
- Departing from $\overline{MS}$ in order to simplify LO+NLO in the MC is independently advocated by Heidekazu Tanaka et.al. (2007). Who else???
- Unintegrated PDFs in the context of small x resummation (CCFM) and unitarity saturation...

Our alternative to MC@NLO and POWHEG
for NLO corrs. to hard process in MC parton shower

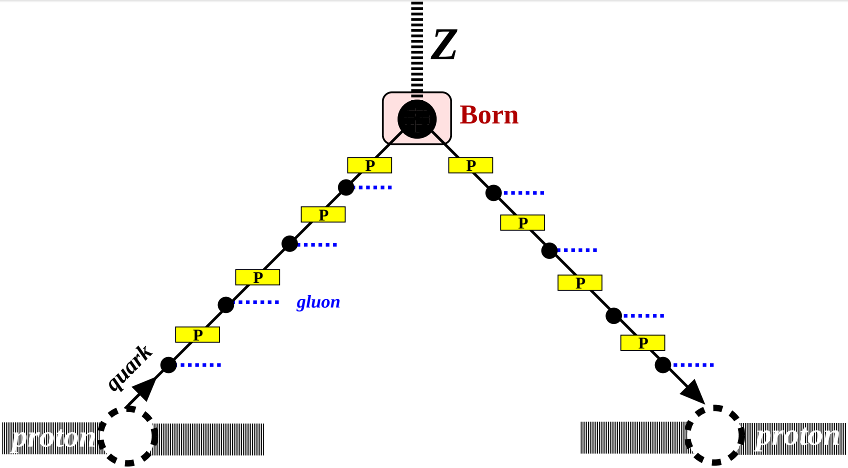
Advantages and costs:

- All $\left(\frac{f(x)}{1-x}\right)_+$ in NLO corrections eliminated,
  annoying $\Sigma^{c\pm}$ of MC@NLO are gone.
  Virtual+soft (unresolved) corrs x-independent.
- Positive weight adds NLO corrections, as in POWHEG,
  but without generating separately hardest gluon,
  no need of vetoed and/or truncated LO showers.
- The price:
  (a) LO psMC must cover entire NLO phase space (retuning?)
  (b) the use of non-MS-bar coll. factoriz. scheme (theory)
  (c) multiple sums in NLO weight (CPU expensive?).

The aim: NLO correcting HARD process



LO parton shower MC according to standard Collinear Factorization Theorems:



$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\left\{\sigma\left[C_0^{(0)}(\mathbb{P}'K_{0F}^{(1)})^{n_1}(\mathbb{P}''K_{0B}^{(1)})^{n_2}\right]\right\}_{T.O.}$$

LO psMC for W/Z production: Details

$$
\begin{aligned}
\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = & \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\; d\hat{x}_B \\
& \times e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}\\
& \times e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}\\
& \times d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\;\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),
\end{aligned}
$$

S_F and S_B = Sudakov formfactors, $\bar{P}(z)=\frac{1}{2}(1+z^2)$,
Ξ = Rapidity of the division plane between F and B hemispheres.
θ = angle of decay products (leptons) in Z rest frame.
$\hat{s}=s\hat{x}_F\hat{x}_B$ = effective mass of Z boson.

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B$$

$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$

$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$

$$\times\, d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

Eikonal phase space for real gluon:
$d^3\mathcal{E}(k) = \frac{d^3k}{2k^0}\,\frac{1}{\mathbf{k}^2} = \pi\frac{d\phi}{2\pi}\frac{d\alpha}{\alpha}d\eta = \pi\frac{d\phi}{2\pi}\frac{d\beta}{\beta}d\eta,$
Lightcone variables: $\alpha = \frac{k^+}{2E},\ \beta = \frac{k^-}{2E}$; rapidity: $\eta = \frac{1}{2}\ln\frac{k^+}{k^-}$,
$d\tau_2(Q;q_1,q_2)$ = two-body phase space element.

LO psMC for W/Z production: Details

$$
\begin{aligned}
\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B \\
&\times e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j} \\
&\times e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j} \\
&\times d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),
\end{aligned}
$$

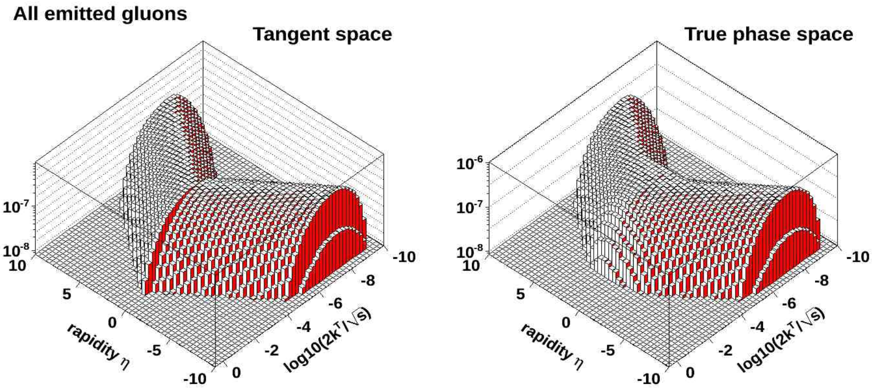
Variables in LO evolution kernels:

$z_{Fi}=\frac{\hat{x}_{Fi}}{\hat{x}_{F(i-1)}}$, $\hat{x}_{Fi}=1-\sum_{j=1}^{i}\hat{\alpha}_i=\prod_{j=1}^{i}z_{Fj}$,

$z_{Bi}=\frac{\hat{x}_{Bi}}{\hat{x}_{B(i-1)}}$ $\hat{x}_{Bi}=1-\sum_{j=1}^{i}\hat{\beta}_i=\prod_{j=1}^{i}z_{Bj}$,

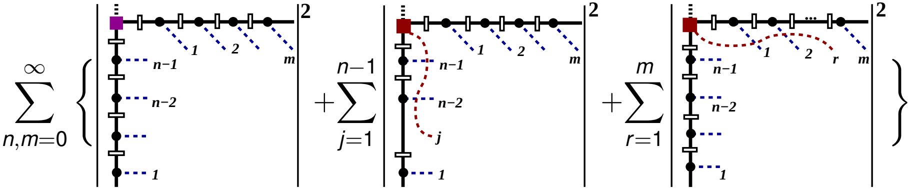
Lightcone variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ in the “tangent space”.

Tangent and real phase space of emitted gluons.

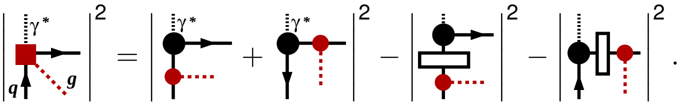


The inclusive distribution of gluons emitted from quark and antiquark in the "tangent space" and in the true phase space (after rescaling) on the Sudakov plane of rapidity and $ln(k^T)$.

The essence of our NLO methodology:



NLO real/virtual distributions (subtracted) from Feynman diags



free of any double/single collinear/soft singularities.

MC weight introducing NLO corrs. to DY hr.proc.

Once LO MC in the above FS scheme is in place, the NLO correction is introduced using simple positive MC weight:

$$W_{MC}^{NLO} = 1+\Delta_{S+V}+\sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s},\hat{\theta})/d\Omega}+\sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

where $\bar{P}(z) \equiv \frac{1+z^2}{2}$, the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F,\hat{p}_B;q_1,q_2,k) = \Big[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{B2})\Big]$$
$$-\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\frac{1}{2}$$

Terms like $\left(\frac{f(z)}{1-z}\right)_+$ completely absent! ($d\sigma_B$ cancels exactly with LO.)

EXACT analytical integration of the LO+NLO MC distributions over the multigluon phase space:

$$\sigma(C_0^{(1)}\Gamma_F\Gamma_B) = \int_0^1 d\hat{x}_F\, d\hat{x}_B\, dz\, D_F(t,\hat{x}_F)\, D_B(t,\hat{x}_B)\, \sigma_B(sz\hat{x}_F\hat{x}_B)$$

$$\times \left\{ \delta_{z=1}(1+\Delta_{S+V}) + C_{2r}^{psMC}(z) \right\}$$

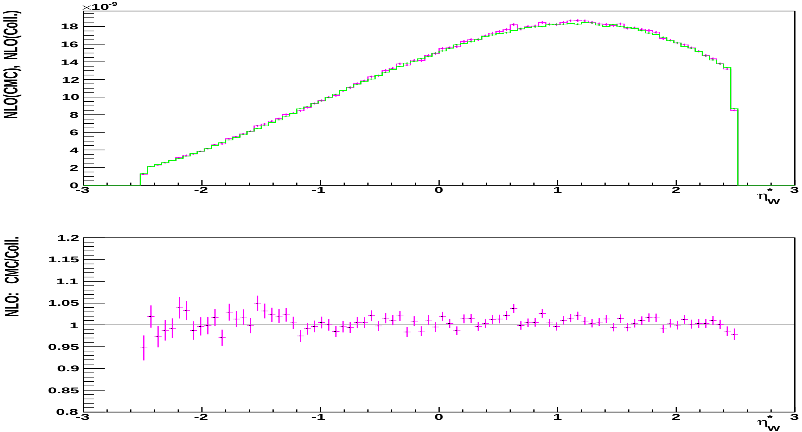
where t= rapidity diff. hadron – hard process,

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi} \left[-\frac{1}{2}(1-z) \right].$$

It differs from $\overline{MS}$ result (Altarelli-Ellis-Martinelli 1979)

$$C_{2r}^{\overline{MS}}(z) = \frac{C_F\alpha_s}{\pi} \left(\frac{1+z^2}{2(1-z)} [2\ln(1-z) - \ln z] \right)_+$$

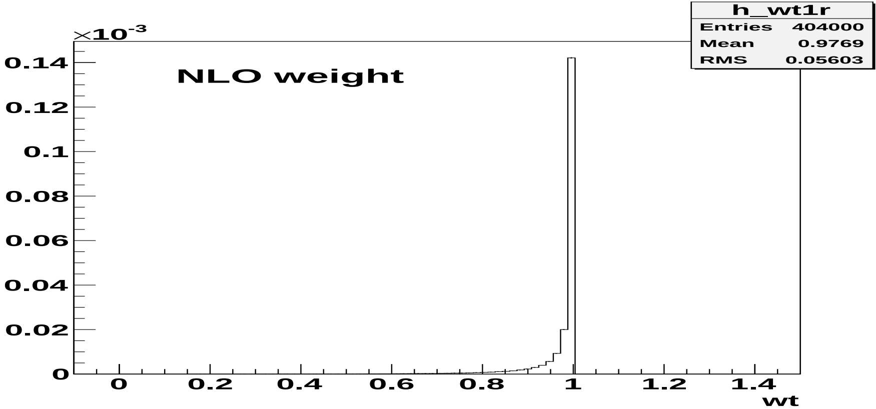
Precision numerical x-check of NLO corr. (Feb. 2012)



Pure NLO correction (real part) of the NLO weight $1 - W^{NLO}_{MC}$ compared with collinear formula $-C_{2r} \otimes D_{q\in p}(t) \otimes D_{\bar{q}\in p}(t)$ in the distribution of $\eta^*_W = \frac{1}{2}\ln(\hat{x}_F/\hat{x}_B)$.

Coll. LO PDFs $D(t,x)$ from special Markov MC run.

How good is NLO weight distribution?



The distribution of the MC weight W_{MC}^{NLO} implementing NLO corrections in gluonstrahlung FROM quark and antiquark annihilating into W boson is just perfect.

Coeff. functions in $\overline{MS}$ and MC factoriz. scheme

Our "coeff. function"

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi}\left[-\frac{1}{2}(1-z)\right]_+$$

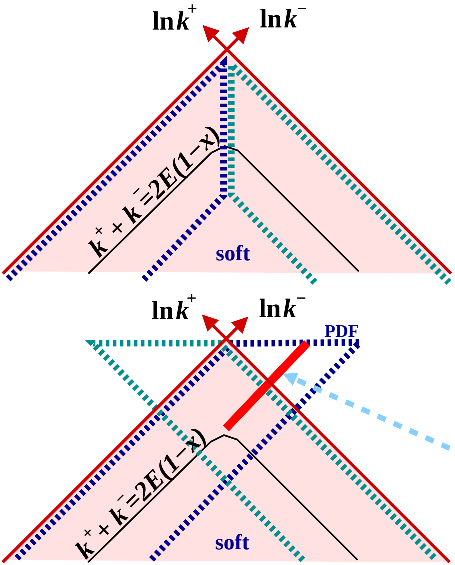
differs from $\overline{MS}$ result (Altarelli-Ellis-Martinelli 1979)

$$C_{2r}^{\overline{MS}}(z) = \frac{C_F\alpha_s}{\pi}\left(\frac{1+z^2}{2(1-z)}[2\ln(1-z)-\ln z]\right)_+$$

by the difference of collinear counterterms in $\overline{MS}$ and MC schemes.

In MC@NLO and POWHEG this convolution is part of NLO corr. to hard proc. – absent in KRKMC NLO scheme.
In KRKMC PS the use of MC FS is a built-in feature anyway.
No problems with universality, see arxiv.org/abs/1103.5015.

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



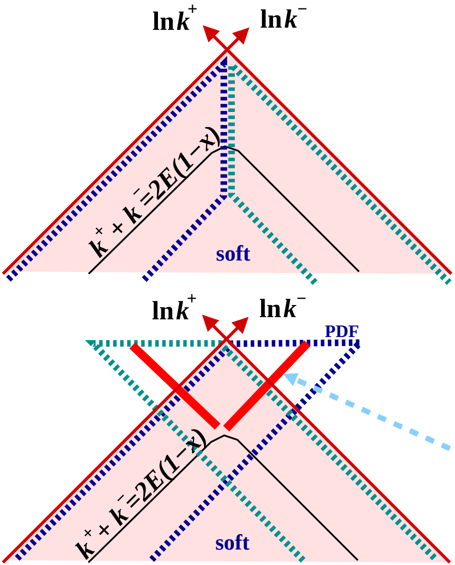
psMC fact. scheme:

$$0 \; \frac{|\ln(1-x)|}{1-x}$$

$\overline{MS}$ fact. scheme:

$$\frac{1}{1-x}\int_{1-x}^{1/(1-x)} \frac{d\beta}{\beta} = 2\frac{|\ln(1-x)|}{1-x}$$

Difference between $\overline{MS}$ and MC fact. schemes
Simple kinematics explains $4\ln(1-x)/(1-x)_+$



psMC fact. scheme:

$$0\ \frac{|\ln(1-x)|}{1-x}$$

$\overline{MS}$ fact. scheme:

$$\frac{2}{1-x}\int_{1-x}^{1/(1-x)}\frac{d\beta}{\beta} = 4\frac{|\ln(1-x)|}{1-x}$$

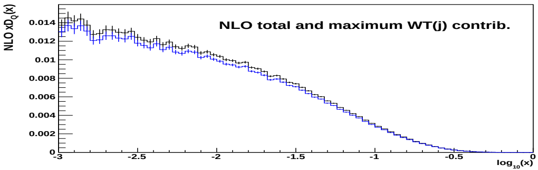
Summation over spectator LO gluons in W_{MC}^{NLO}

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

Summation over ladder gluons are the landmark of our scheme.
In POWHEG gluon entering NLO corr. to hard proc. is handled separately:
→ vetoed/truncated gluons (ang.ord.) In MC@NLO non-positive *H*-events.

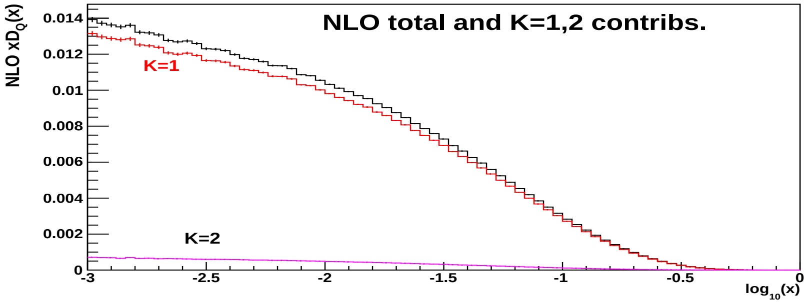
IS THE SUM OVER LADDER GLUONS REALLY ESENTIAL?

It can be checked that *only one single term dominates the sum:*



which one?

Only one single term dominates the sum in W_{MC}^{NLO}



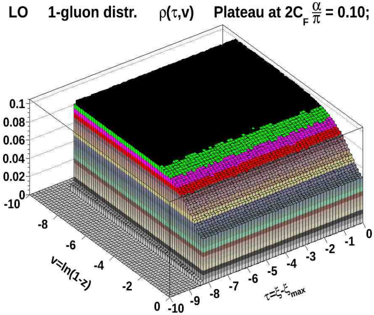
The (-)NLO contributions from $K = 1, 2$ single gluons, the one with maximum and another one with next-to-max. kT, in the x-distribution of quark entering W boson.

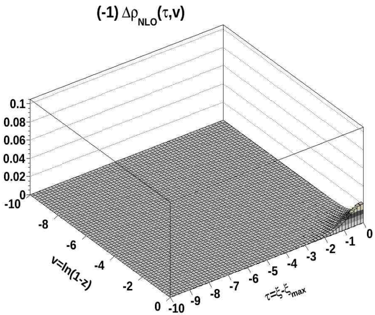
POWHEG exploits the above. We can do it differently, without vetoed/truncated gluons, see next slides...

The location and size of the (real) NLO correction on the Sudakov plane (rapidity, ln(1-z))

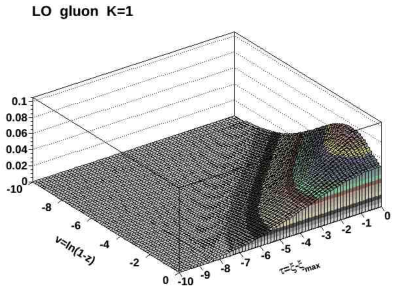
LO inclusive

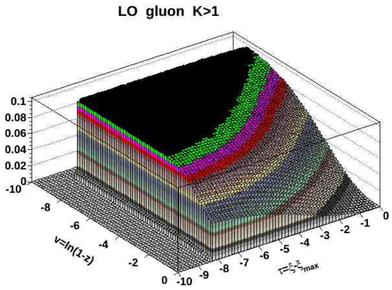
pure NLO





Gluons generated in ang.ord. and re-ordered in kT





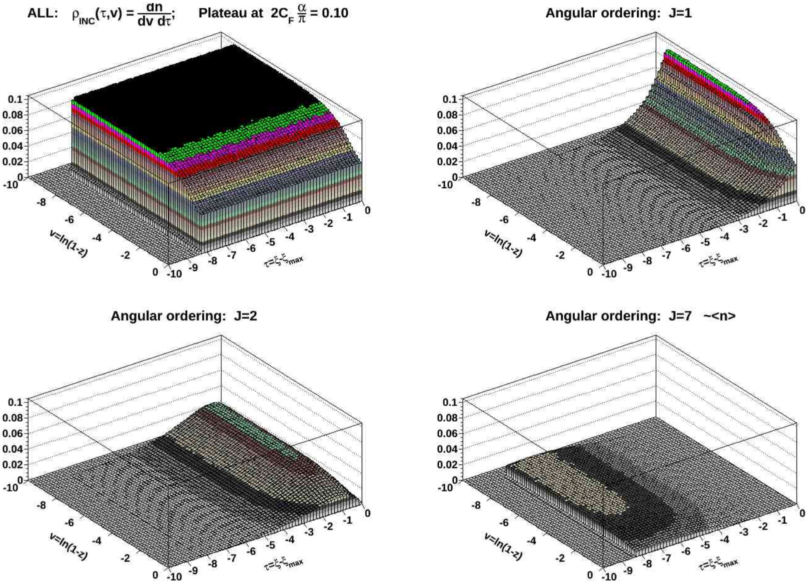
Sudakov suppression for the highest kT gluon (K=1)!

Our NLO weight with summation is ignorant about the above kT (re-)ordering.

The summation (or max. kT-selection) takes care of picking up correctly the hardest gluon.

No need of truncated/vetoed gluon showers.

Gluons ordered in rapidity (in our LO MC)

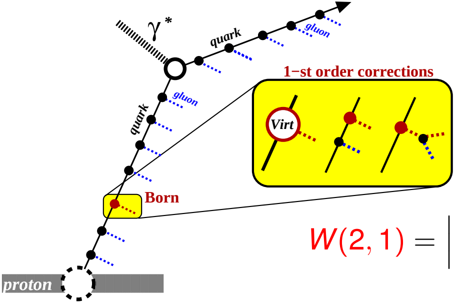


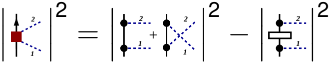
Rapidity *labeling* used in our LO MC (and in HERWIG).

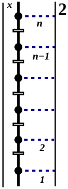
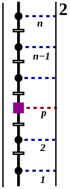
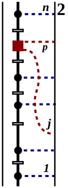
NLO in the ladder

Sudakov suppression exploited to reduce multiple sums over spectator gluons, while introducing NLO corrections in the ladder!

NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$



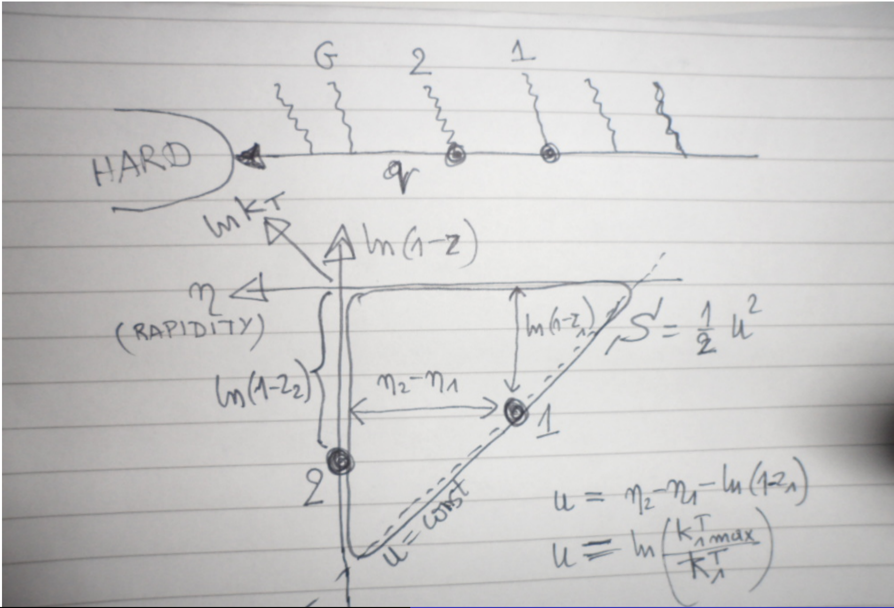
$W(2,1) =$ 

$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$  $$+ \sum_{p=1}^{n}$$  $$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$  $$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

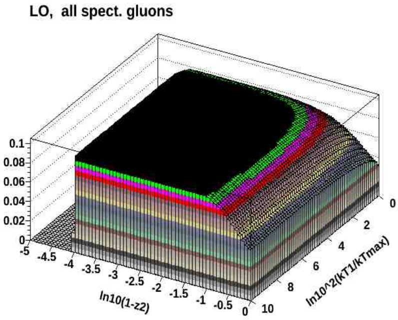
$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

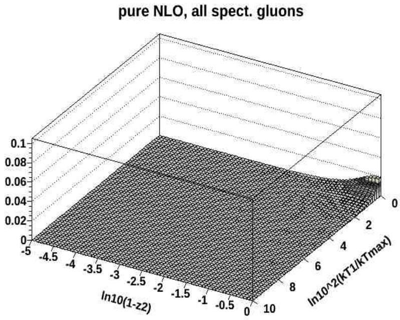
Kinematics of for 2 gluons in the ladder

NLO nonzero ONLY if both 1 and 2 in the upper left corner



Location and size of the (real) NLO correction in the ladder on the Sudakov log space

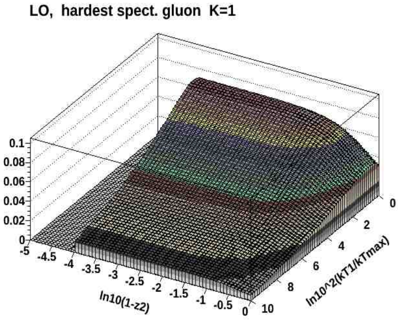


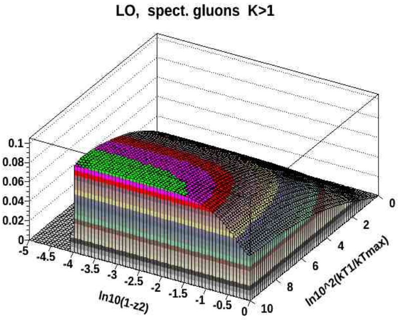


LO inclusive distribution features triple-log IR/coll. singularity, seen as a plateau in 2-dim. projection.

NLO correction IR/coll. finite, nonzero in the corner of the size ~ 1.

Split of inclusive LO distribution of gluons into contr. from the hardest one and the rest

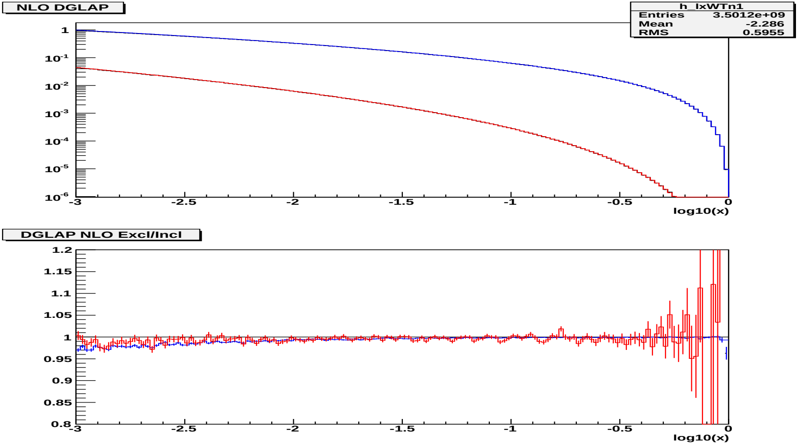




Distribution of the hardest LO spectator gluon approximates the total distribution in small corner where NLO is non-zero.

Repetition of 2009 test for NLO-corrected ladder

RADCOR 2009: NLO MC vs. analyt. NLO kernels. Perfect agreement

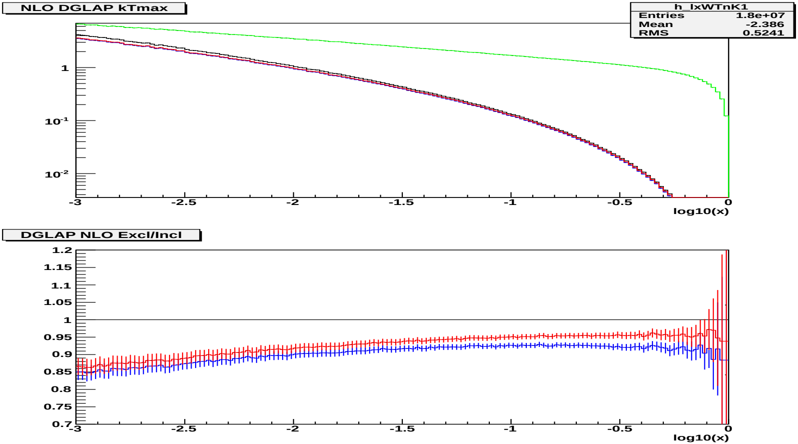


Single ladder, 1GeV-1TeV, 1 or 2 kernels NLO-corrected (3G ev.)

Very slow in CPU time.

Repetition of 2009 test for NLO-corrected ladder

NEW 2012: NLO contrib. to 1 kernel, 1 and 2 gluons with max. kT



This difference $\sim$ 15% is formally the NNLO/NLO class. (15M evts).

Very fast in CPU time!

Summary

- **Alternative (to MC@NLO or POWHEG) scenario for NLO-corrected hard proc. and LO PSMC is proposed.**
- **Parton shower MC implementing complete NLO DGLAP in the ladders in exclusive way is feasible.**
- Long term: NLO ladder + NNLO hard process, but (LO ladder + NLO hard proc. to be optimized first!)
- Most likely application: high quality QCD+EW+QED MC with hard process like $W/Z/H$ boson production.
- Potential gains from new QCD methods are:
  – reducing uncertainties due to distributions of partons in hadrons (PDFs, parton luminosities etc.)
  – easier implementation of NLO and NNLO corrections to hard process due to elimination of "trivial" (albeit numerically sizeable) soft gluon corrections
  – better environment for low x resummation (BFKL, CCFM) and heavy quark masses.

Description of LO parton shower Monte Carlo

for W/Z production (Drell-Yan)

Define hat-variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ of tangent space

Mappings entering definition of $\mathbb{P}'$ and $\mathbb{P}''$ proj. operators of CFT

Order ALL gluons according to rapidity distance from Ξ, rapidity position of the hard process (W/Z+G):
Permutation $\pi = \{\pi_1, \pi_2, ..., \pi_{n_1+n_2}\}$ defined such that
$|\eta_{\pi_i} - \Xi| > |\eta_{\pi_{i-1}} - \Xi|, \quad i = 1, ..., n_1 + n_2$

Recursively defined dilatations transform from tangent space to true phsp:

$$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}, \quad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2(P - \sum_{j=1}^{i-1} k_{\pi_j}) \cdot \bar{k}_{\pi_i}}, \quad i = 1, 2, ..., n_1 + n_2.$$

Rescaling factor λ_i obey:

$$\bar{s}_i = s\bar{x}_i = s \prod_{j=1}^{i} \hat{z}_{(F,B)\pi_j} = (P - \sum_{j=1}^{i} k_{\pi_j})^2 = (P - \sum_{j=1}^{i} \lambda_j \bar{k}_{\pi_j})^2.$$

In F hemisphere $\hat{\alpha}_i = \lambda_i \alpha_i$ and in B hemisphere $\hat{\beta}_i = \lambda_i \beta_i$.
(An improvement over 1st such scenario publ. in 2007 in APP, Stephes at.al.)

Overview of the LO Monte Carlo algorithm:

- Variables $\hat{z}_F$ and $\hat{z}_B$ are generated by FOAM. $\Xi = 0$ is used.
- Four momenta $\bar{k}_i^\mu$ are generated separately in F and B tangets spaces using CMC module, with the constraints $\sum_{j\in F} \hat{\alpha}_j = 1 - \hat{z}_F$ and $\sum_{j\in B} \hat{\beta}_j = 1 - \hat{z}_B$.
- Double ordering permutation π is established.
- Using P and $\bar{k}_{\pi_1}$ rescaling parameter λ_1 is calculated, $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set. At this stage $(P - k_{\pi_1})^2 = sx_1$, where $x_1 = z_{\pi_1} = 1 - \hat{\alpha}_{\pi_1}$ or $x_1 = z_{\pi_1} = 1 - \hat{\beta}_{\pi_1}$, depending whether k_{π_1} was in F or B part of LIPS.
- Using $P - k_{\pi_1}$ and $\bar{k}_{\pi_2}$ parameter λ_2 is found and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set. At this stage we enforce $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$. This recursive procedure continues until the last gluon.
- In the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ 4-momenta of q_1^μ and q_2^μ are generated according to Born angular distribution.
- Get rapidity η_h of the hard process (W/Z + hard G), take $\Xi = \eta_h$ and repeat mapping from tangent space to true phsp.