Neutron and proton EDM from the lattice

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for RBC/UKQCD collaboration

MITP Workshop, 10/10/2013

Outline

- Introduction
  - Motivation of lattice calculation of EDM
- Strategy and method in lattice QCD
  - Lattice QCD
  - Extraction of EDM from correlation function
- Recent update (preliminary)
- Summary and future work

1. Introduction

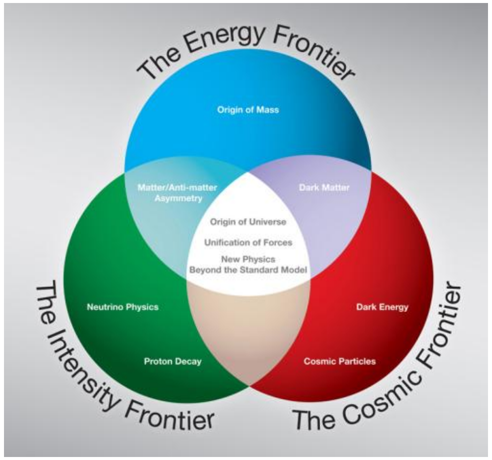
Neutron EDM

- To discover CPV in QCD and the new physics
- Since 1970's, sensitivity of experiment has been developed.
  - Current nEDM upper limit is $|d_N^{\mathrm{exp}}| < 2.9 \times 10^{-26}\,\mathrm{e} \cdot \mathrm{cm}$
- Important observable
  - Naturally QCD has CPV from θ term, but it seems to be unnaturally small. (strong CP problem)
  - Direct search of CPV from NP

    BSM (SUSY, etc) say the discover is coming soon…
  - Intensity frontier physics

    Alternative direction from high energy collision.

    Precision of the SM calculation is necessary.

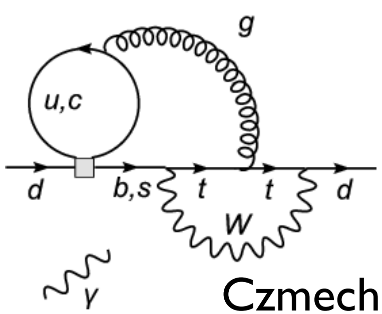


http://www.fnal.gov/pub/science/frontiers/

Nucleon EDM in EW

- Contribution to EDM in weak interaction is very small
  - CP phase in 1-loop ($|V_{dq}|^2$) and 2-loop diagram (cancelation) vanishes.
  - Three-loop order(short) and pion loop correction (long):

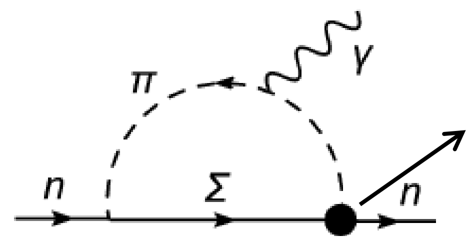
Short distance

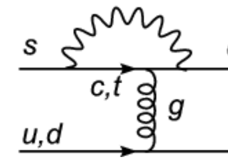


Czmechi, Krause (1997)

$$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G_F^2) \sim -10^{-34}\,\mathrm{e\cdot cm}$$

Long distance





Khriplovich, Zhitnitsky (1982)

$$d_N^{\mathrm{KM\,long}} \sim 10^{-30}\text{–}10^{-32}\,\mathrm{e\cdot cm}$$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30}\text{–}10^{-32}\,\mathrm{e\cdot cm}$$

which is the 6-order magnitude below the experimental upper limit.
(to confirm, non-pertubative estimate is also needed)

Nucleon EDM in QCD

- θ term in QCD Lagrangian

$$\mathcal{L} = \bar{q}_L M q_R + \bar{q}_R M^\dagger q_L + \theta/(64\pi^2) G\widetilde{G}$$

$$\Rightarrow \mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M, \quad \arg\det M \sim \eta\frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}$$

  - Renormalizable and CPV.
  - $d_N/\theta \sim 10^{-16}$ e·cm (quark model, current algebra, etc)

    θ and arg det M is *unnaturally* canceled. Crewther, et al. (1979), Ellis, Gaillard (1979)

- Possible solution

  1. Massless quark (m_u = 0) Blum, et al. (2010)

  from lattice QCD+QED, m_u = 2.24(35) MeV, m_d = 4.65(35) MeV.

  It is hard to explain $\bar{\theta} = 0$.

  2. *Axion model (assumption of PQ symm.),* invisible axion model

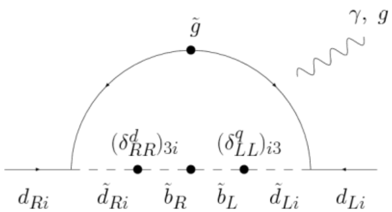
  3. *Spontaneous CP breaking.* θ is calculable in loop order.

Nucleon EDM in BSM

- Higher dimension operators of CPV

$$H_{C\not{P}} = \sum_k C_k(\mu)\,\mathcal{O}_k \qquad \begin{array}{rcll} \mathcal{O}_{\rm qEDM} &=& d_q \bar{q}(\sigma\cdot F)\gamma_5 q & \text{: Quark-photon (5-dim)} \\ \mathcal{O}_{\rm cEDM} &=& d_q^c \bar{q}(\sigma\cdot G)\gamma_5 q & \text{: Quark-gluon (5-dim)} \\ \mathcal{O}_{\rm Weinberg} &=& d^G G G \widetilde{G} & \text{: Pure gluonic (6-dim)} \end{array}$$

- In effective Hamiltonian, the new CPV term appears.
- In BSM, q(c)EDM corresponds to CP phase of heavy particle.
- d_q d_q^c are determined by BSM



Hisano, et al. (2009)

To obtain EDM, we need to estimate QCD effect in nucleon.

1. Introduction

Nucleon EDM in BSM

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: Quark-photon (5-dim)
: Quark-gluon (5-dim)
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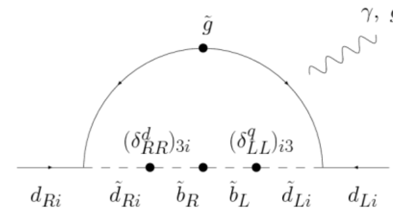
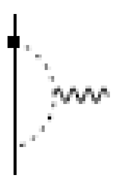


Hisano, et al. (2009)

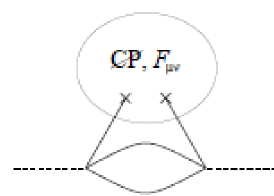
To obtain EDM, we need to estimate QCD effect in nucleon.

Using baryon CHPT or QCD sum rule, there are several evaluations,



…

Mereghetti, Vries, Hockings, Maekawa, Kolck, Timmermans, …



…

Pospelov, Ritz, Hisano, Shimizu, Nagata, Lee, Yang, …

$$\begin{aligned} d_N &= d_N^{\mathrm{QCD}}\bar{\theta} + d_N(d_q, d_q^c) + d_N(d^G) \\ &\sim 10^{-17}[\mathrm{e}\cdot\mathrm{cm}]\bar{\theta} + (1.4\text{–}0.47)d_d - (0.12\text{–}0.35)d_u + O(10^{-2})d_q^c \\ &\sim O(10^{-25}\text{–}10^{-27})\,\mathrm{e}\cdot\mathrm{cm} \end{aligned}$$

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)

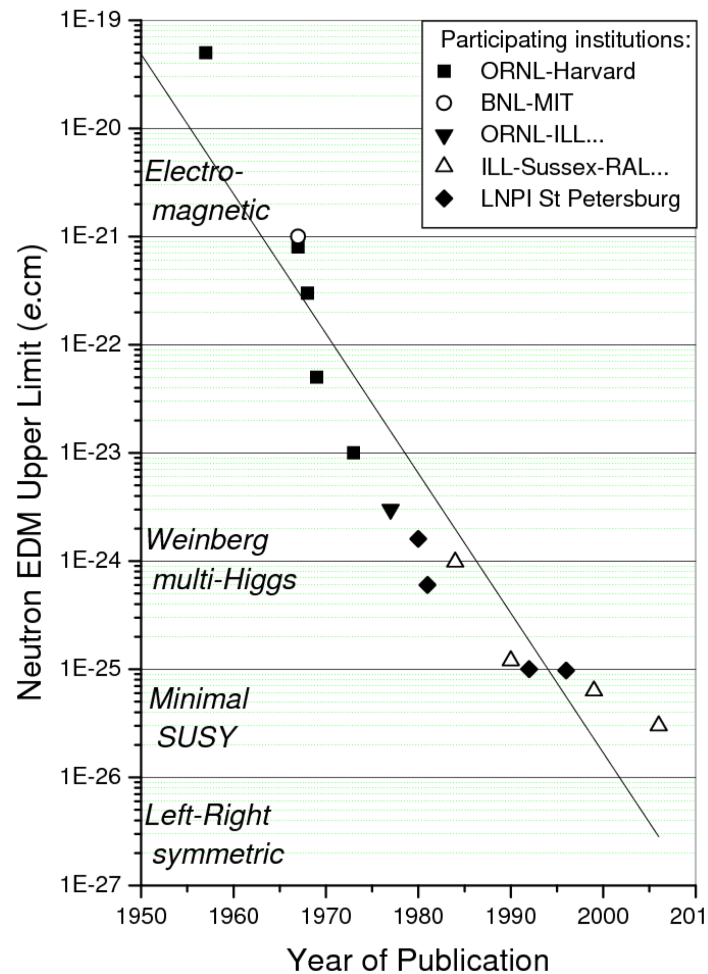
Constraint on nEDM

- ~10 new proposals of EDM experiment

  (p,d)EDM experiment @ BNL,
  nEDM experiment @ ORNL, ILL, FRM-2,
  FNAL, PSI/KEK/TRIUMF, …
  Charged particle (d, p)EDM @ COSY
  Lepton EDM @ J-PARC, FNAL

  aiming for a sensitivity to 10^{-29} e・cm !

- Current estimate of QCD effect is based on quark model, and so that it includes model dependence.
- Computation of non-perturbative contribution in θ term, qEDM, cEDM, etc is important test.



Harris, 0709.3100

What lattice QCD can do for nEDM

- In principle
  - Direct estimate of hadronic contribution to neutron and proton EDM for θ term, higher dim. CPV operators
  - Matrix elements (or condensate) including higher dimension operators

    $\rightarrow$ for QCD sum rule, ChPT, …

Bhattacharya et al, Lattice 2012

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- In practice there are some difficulties
  - Statistical noise

    gauge background (topological charge, sea quark) and *disconnected diagram* (flavor singlet contraction) are intrinsically noisy.

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What lattice QCD can do for nEDM

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- In practice there are some difficulties
  - Statistical noise

    gauge background (topological charge, sea quark) and *disconnected diagram* (flavor singlet contraction) are intrinsically noisy.
  - Systematic study

    Volume effect may be significant. (e.g. BChPT discussion)

    O'Connell, Savage, PLB633, 319(2006), Guo, Meissner, 1210.5887

    Chiral behavior is also important check, $d_N \sim O(m)$.

Strategy and method in lattice QCD

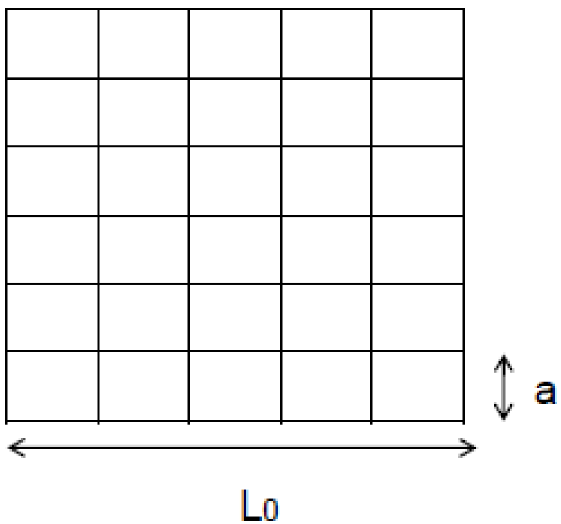
Lattice QCD

In lattice regularization, the path integral is computed by Monte-Carlo integral:

$$\langle O \rangle = Z^{-1} \int D\Psi\, O(\Psi) e^{-S(\Psi)} \simeq \frac{1}{N} \sum_i O(\Psi_i)$$

- Exact QCD calculation (enough large number of sampling N)
- Gauge invariant
- Translational invariant
- Ultraviolet cut-off *a* (lattice spacing)

  Infrared cut-off $V = L_0^D$ (lattice volume)
- Continuum limit, and infinite volume are important.
- The development of machine (BG, GPGPU, …) and algorithm, which make much progress.

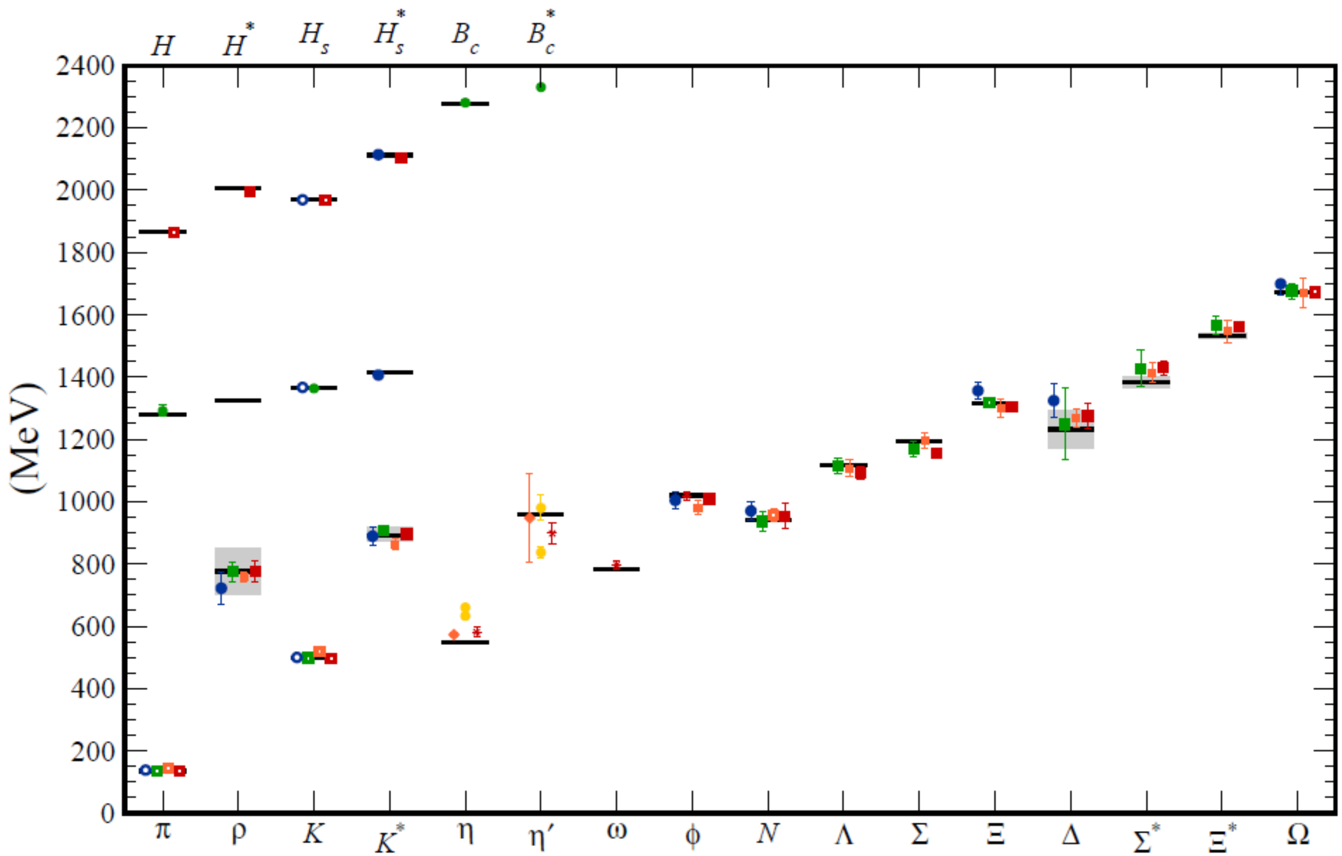


2. Strategy and method in lattice QCD

Hadron spectrum in lattice QCD

- Good agreement with <u>various lattice action and fermion</u> with experimental results !

Kronfeld, 1209.3468



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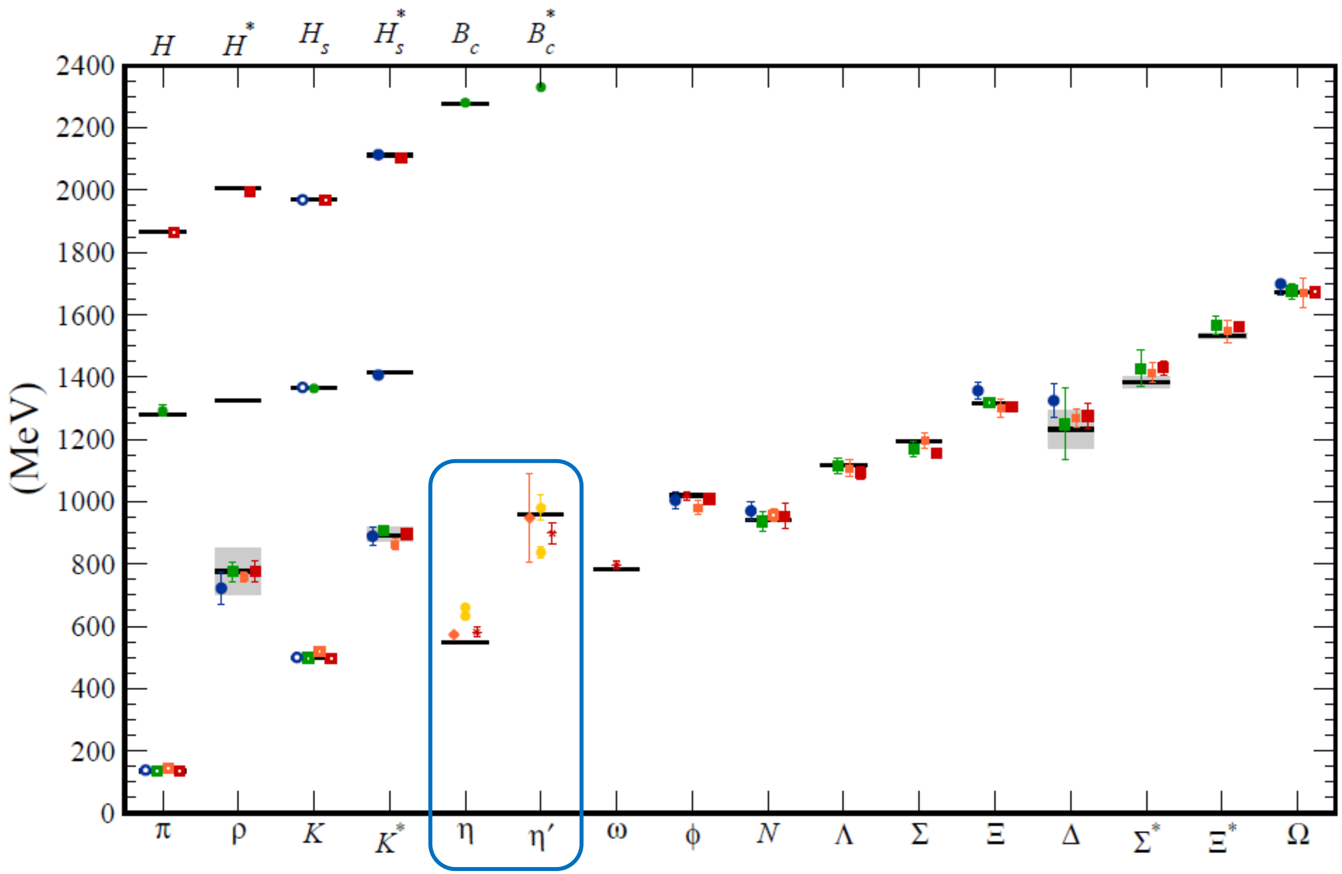
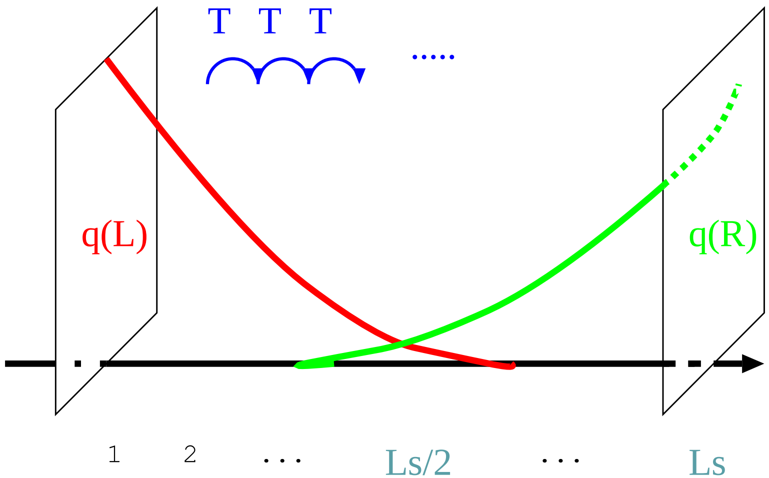


Choice of lattice fermion

- There are several kinds of fermion definition on the lattice.
- Require "realistic" fermion for the precise calculation
  - which has good approximated chiral symmetry on the lattice.
  - Good suppression of O(a) effect
  - Domain-wall fermion is appropriate selection.
- Domain-Wall fermion (DWF)
  - L, R fermion are localized on boundaries
    Exact chiral symmetry is realized if $L_s \to \infty$.
  - In finite L_s
    Violation of chiral symm. is suppressed as
    $am_{res} \sim \exp(-L_s) \ll 1$.

[Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05 --)]



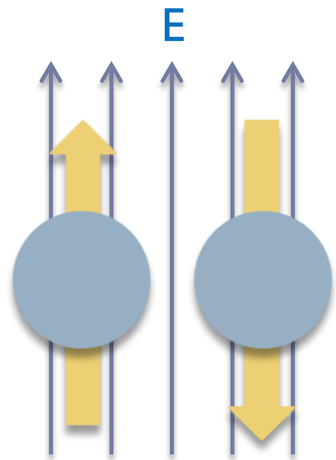
Lattice methods of nEDM

- Spectrum method
  1. S. Aoki and A. Gocksch, Phys. Rev. Lett. 63, 1125 (1989).
  2. S. Aoki, A. Gocksch, A. V. Manohar, S. R. Sharpe, Phys. Rev. Lett. 65, 1092 (1990), in which they discussed about the possible lattice artifact in ref.1 results
  3. ES, et al., for CP-PACS collaboration, Phys. Rev. D75, 034507 (2007)
  4. ES, S. Aoki, Y. Kuramashi, Phys. Rev. D78, 014503 (2008)
- Form factor
  1. ES, et al., for CP-PACS collaboration, Phys. Rev. D72, 014504 (2005).
  2. Berruto, et al. for RBC collaboration, Phys. Rev. D73, 05409 (2006).
  3. ES et al., Lattice 2008.
- Imaginary θ
  1. T. Izubuchi, Lattice 2007.
  2. Horsley et al., arXiv:0808.1428 [hep-lat]

Spectrum method

- given by 2-pt function: $m_{\uparrow\,\text{spin}} - m_{\downarrow\,\text{spin}} = 2d_N\theta E$
- Direct measurement of EDM

  It is simple extraction method



Spectrum method

- given by 2-pt function: $m_{\uparrow \text{spin}} - m_{\downarrow \text{spin}} = 2d_N\theta E$
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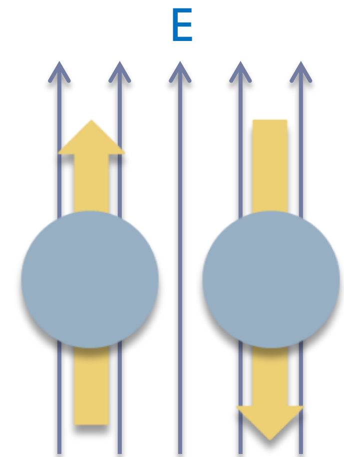
- Ratio of spin up and down

$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\text{up}}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\text{down}}_{\theta,E}} \simeq 1 + d_N E\theta t$$

→ Linear response, gradient is a signal of EDM.



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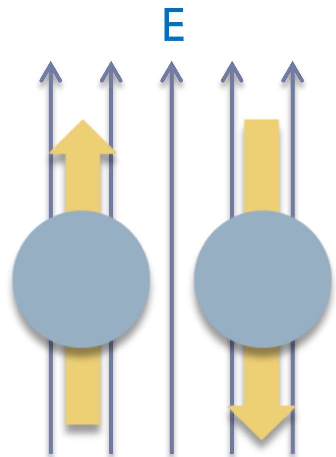
→ Linear response, gradient is a signal of EDM.

  - Reweighting with small θ: $\langle O\rangle_\theta = \langle O e^{i\theta Q}\rangle$

    and introduce external Minkowski E field: $U_t \to U_t e^{qEt}$, $U_t^\dagger \to U_t^\dagger e^{-qEt}$



2. Strategy and method in lattice QCD

Spectrum method

- given by 2-pt function: $m_{\uparrow \text{spin}} - m_{\downarrow \text{spin}} = 2d_N\theta E$
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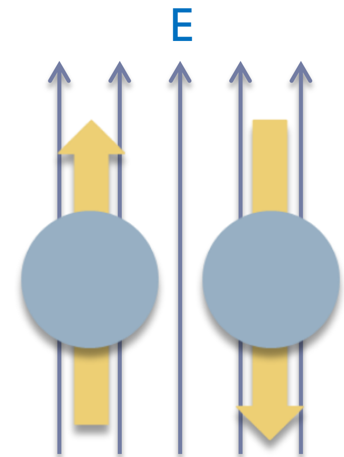

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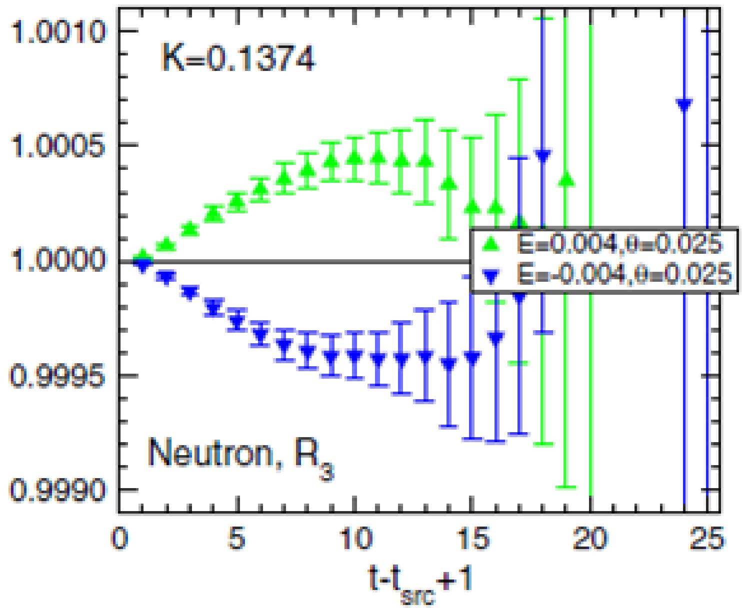
  and introduce external Minkowski E field: $U_t \to U_t e^{qEt}$, $U_t^\dagger \to U_t^\dagger e^{-qEt}$
- Temporal periodicity is broken by electric field.

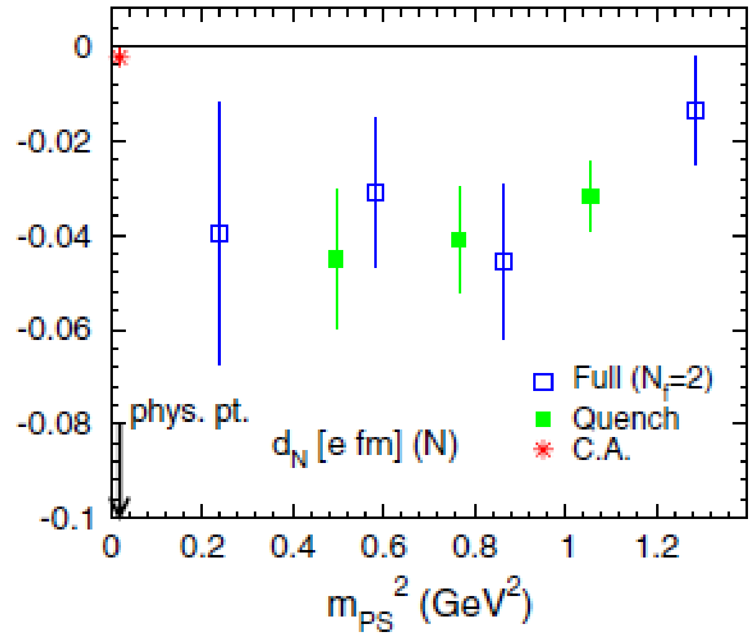
  ⇒ additional systematic error

  In <u>imaginary θ method</u> we can avoid this issue.

Spectrum method

ES et al. (06, 07)





N_f=2 clover (Wilson-type) fermion

- $24^3 \times 48$ lattice (~2 fm^3), pion mass ~ 500 MeV
- Signal of EDM in full QCD ensembles, O(1000) statistics
- Central value is larger than other phenomenological model.
- Statistical noise (and boundary effect) is still large contribution.

EDM Form factor

ES, et al., Phys. Rev. D72, 014504 (2005),
Berruto, et al., Phys. Rev. D73, 05409 (2006).

$$\begin{aligned}\langle n(P_1)|J_\mu^{\mathrm{EM}}|n(P_2)\rangle_\theta \;&=\; \bar{u}_N^\theta\Big[\underbrace{\frac{F_3^\theta(q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}q_\nu}_{\text{P,T-odd}}+\underbrace{F_A(q^2)(iq^2\gamma_\mu\gamma_5-2m_Nq_\mu\gamma_5)}_{\text{P-odd}}\\ &+\;\underbrace{F_1(q^2)\gamma_\mu+\frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu}_{\text{P,T-even}}\Big]u_N^\theta\end{aligned}$$

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&+ \underbrace{F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu}_{\text{P,T-even}} \Big] u_N^\theta
\end{aligned}
$$

$$
\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N}
$$

CPV phase α_N in nucleon propagator

2. EDM calculation on the lattice

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\end{aligned}
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\left.\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N}\right\}
$$

CPV phase α_N in nucleon propagator

$$
\langle\theta|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|\theta\rangle = \langle 0|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|0\rangle + i\theta\langle 0|\eta_N J_\mu^{\mathrm{EM}} Q\bar{\eta}_N|0\rangle
$$

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$$+ \underbrace{F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu}_{\rm P,T\text{-}even} \Big] u_N^\theta$$

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CPV phase α_N in nucleon propagator

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$$\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)|0\rangle$$

Computation

$$= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5$$

Subtraction

$$+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}$$

Extraction

- Subtraction of CP-odd phase, α_N, in n propagator and CP-even part $F_{1,2}$

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$$\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N}$$

CPV phase α_N in nucleon propagator

$$\langle\theta|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|\theta\rangle = \langle 0|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|0\rangle + i\theta\langle 0|\eta_N J_\mu^{\mathrm{EM}} Q\bar{\eta}_N|0\rangle$$

$$\langle 0|\eta_N(t_1)J_\mu^{\mathrm{EM}}(t)Q\bar{\eta}_N(t_0)|0\rangle$$

Computation

$$= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5$$

Subtraction

$$+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}$$

Extraction

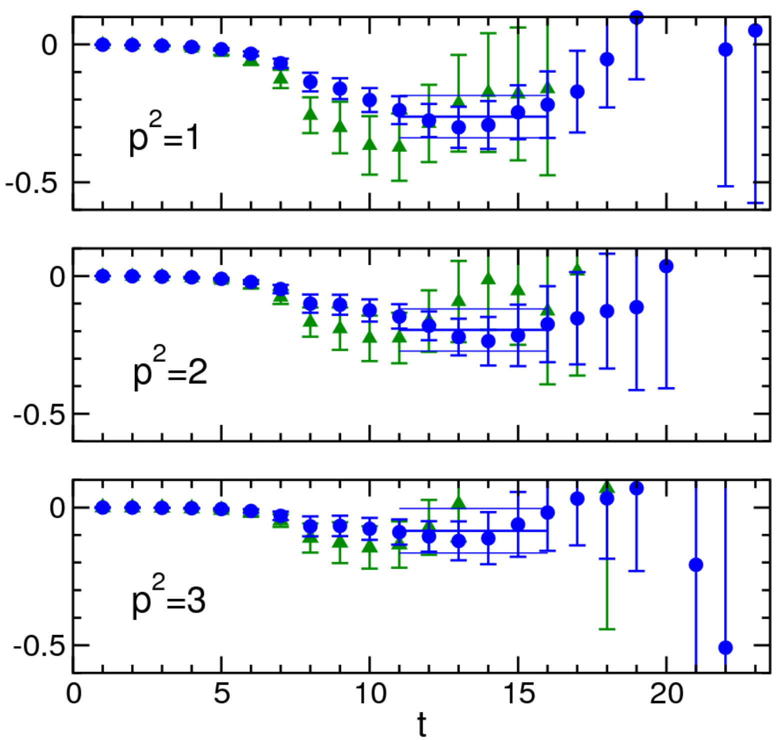
- Subtraction of CP-odd phase, α_N, in n propagator and CP-even part $F_{1,2}$
- EDM is given by zero momentum transfer $d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$

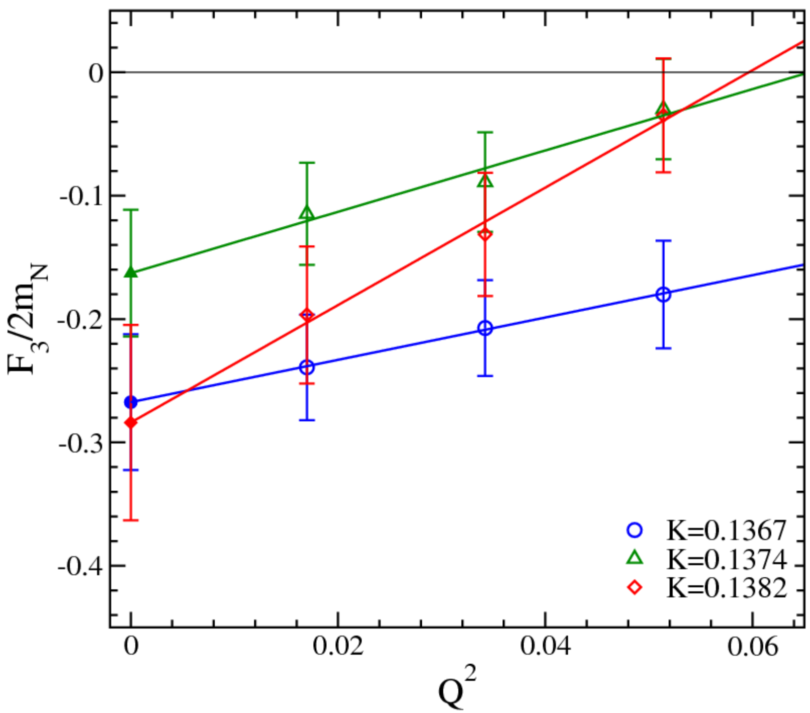
Form factor

ES et al.(05, 08)

Nf=2 clover fermion

- Size is $24^3 \times 48$ lattice (~2 fm^3), pion mass is around 500 MeV
- Ignoring disconnected diagram in 3-pt function
- momentum transfer $Q^2 \rightarrow 0$ limit is with linear func.





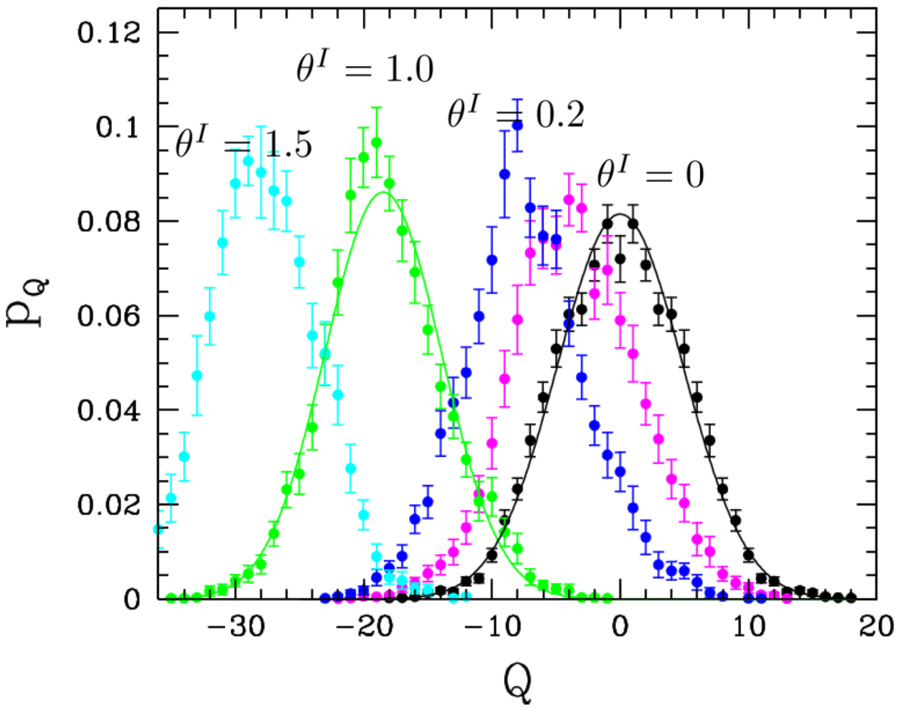
Imaginary θ

Izubuchi(07), Horsley et al. (08)

- Analytically continue to pure imaginary

  $$\langle O e^{i\theta Q} \rangle \rightarrow \langle O e^{-\theta^I Q} \rangle$$

  - There is no sign problem, then expect better signal.



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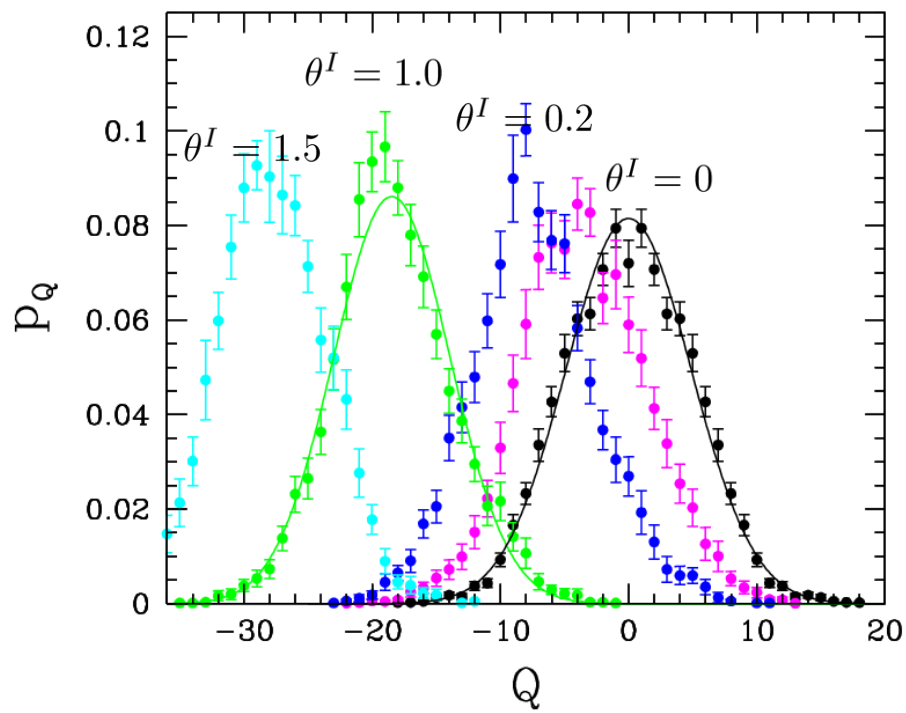
  - There is <u>no sign problem</u>, then expect better signal.

  - <u>Need to generate the new QCD ensemble</u> with θ^I: distribution of Q is shifted by θ^I

    ⇒ it will be challenging work when going to realistic lattice (larger lattice and physical quark mass)



Imaginary θ

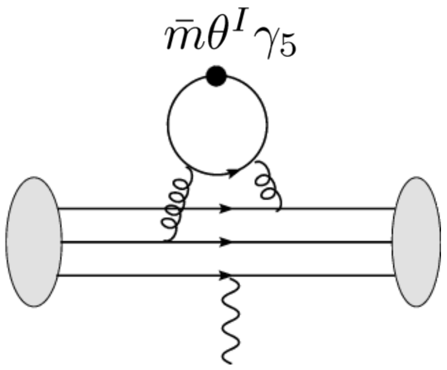
Izubuchi(07), Horsley et al. (08)

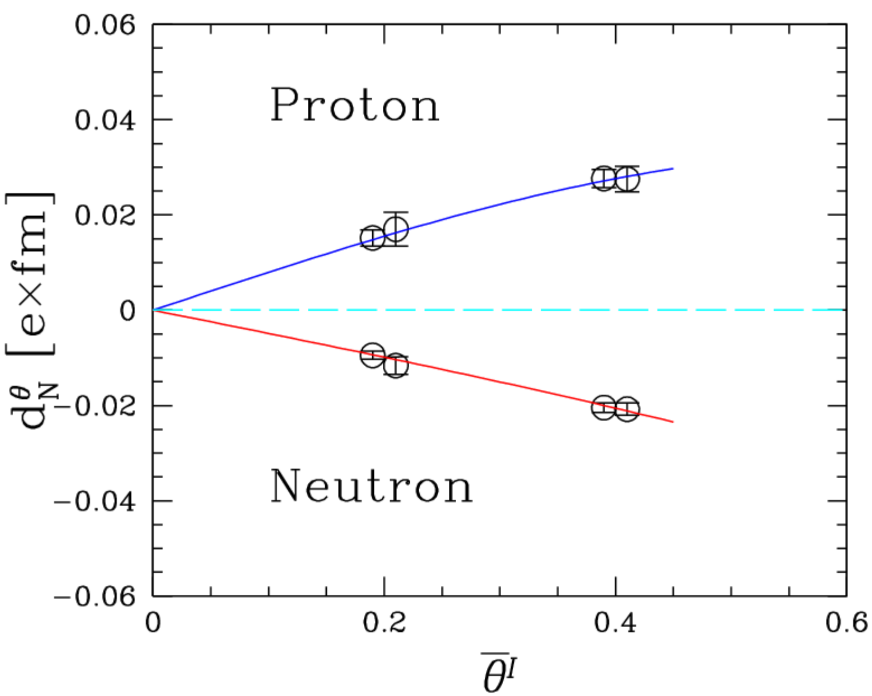
- N_f=2 QCD with Wilson fermion

$16^3 \times 32$ lattice, m_π = 700 MeV

Fermionic insertion of imaginary theta:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2$$





- EDM form factor method
- EDM is given by the slope.
- Clear signal, but systematic error due to chiral symmetry breaking of clover fermion has not been taken into account.

⇒ need careful check

Recent update (preliminary)

Recent work on EDM from lattice

- θ term
  - New developed algorithm, called as AMA method, which is error reduction techniques without additional cost.
  - Extremely high statistics of EDM form factor in DWF

ES, Blum, Izubuchi, Lattice 2013

- Higher dimension term
  - Proposal of calculation in qaurk EDM and chromo EDM term

Bhattacharya, Lattice 2012, Lattice 2013

Error reduction techniques

Blum, Izubuchi, ES, 1208.4349,
ES (lattice 2012),
Chulwoo, plenary on Fri

- Covariant approximation averaging (CAA)
  - For original correlator O, (unbiased) improved estimator is defined as

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

3. Recent update (preliminary)

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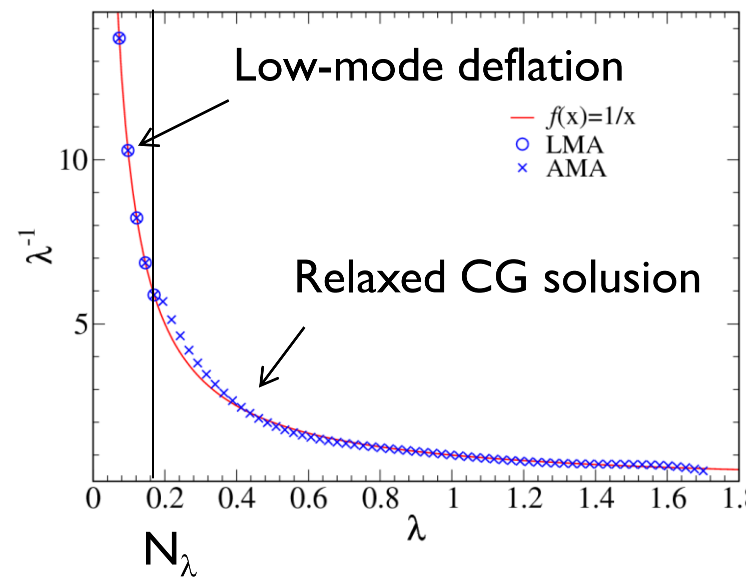
- All-mode-averaging (AMA)
  - Relaxed CG solution for approximation

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],\ S_l = \sum_{\lambda=1}^{N_\lambda} v_\lambda v_\lambda^\dagger \frac{1}{\lambda} + P_n(\lambda)|_{|\lambda|>N_\lambda}$$



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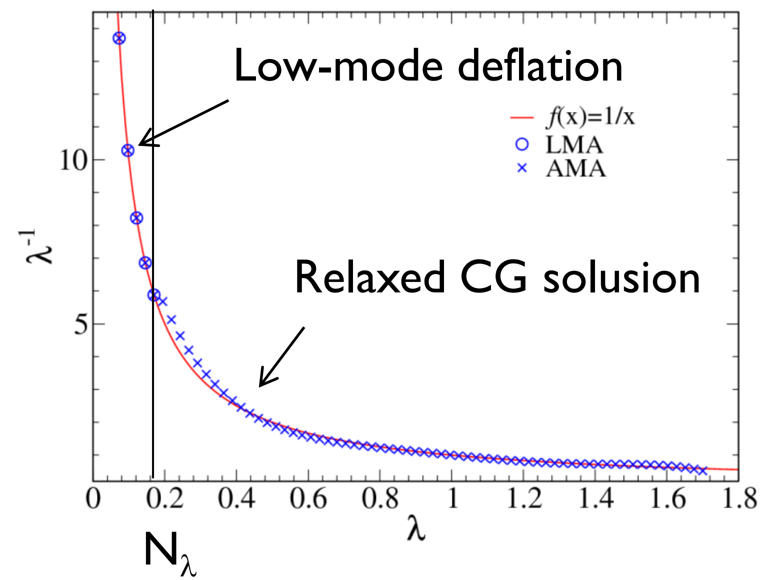
  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.



Parameters

- DWF
  - $24^3 \times 64$ lattice, a^{-1} = 1.73 GeV (~3 fm^3 lattice)
  - L_s = 16 and am_{res} = 0.003
  - m = 0.005, 0.01 corresponding to m_π = 0.33, 0.42 GeV
  - Two temporal separation of N sink and source in 3 pt. function

    t_{sep} = 12 (t_{source} = 0, t_{sink} = 12), t_{sep} = 8 (t_{source} = 0, t_{sink} = 8)
  - # configs = 751 (m=0.005), 700 (m=0.01) [t_{sep} = 12]

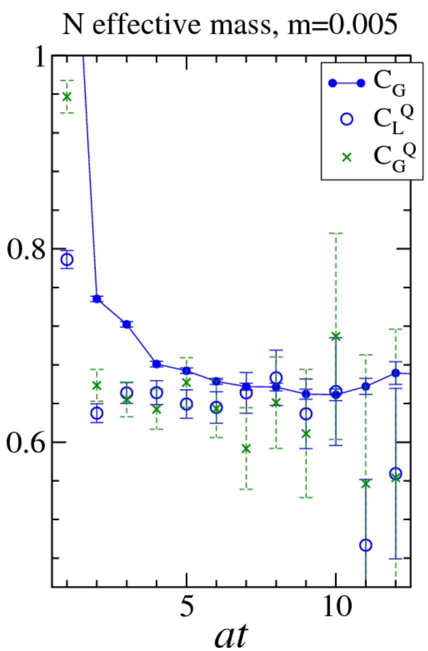
    # configs = 180 (m=0.005) [t_{sep} = 8]
- AMA
  - # of low-mode : N_λ = 400 (m=0.005), 180 (m=0.01)
  - Stopping condition, $|r| < 0.003$
  - N_G = 32 (2 separation for spatial, 4 separation for temporal direction of source localtion) → effectively $O(10^4)$ statistics

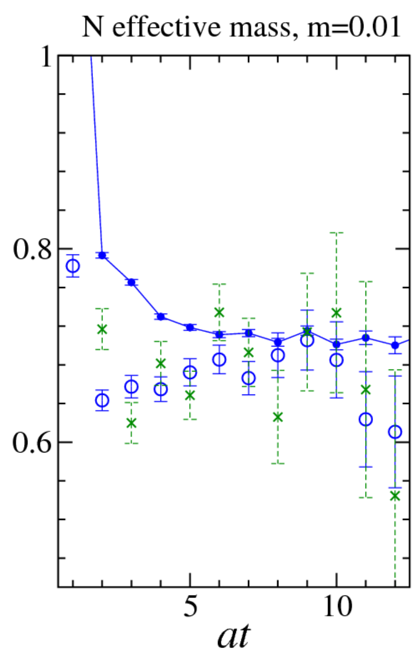
α_N : CP-odd phase of wave function

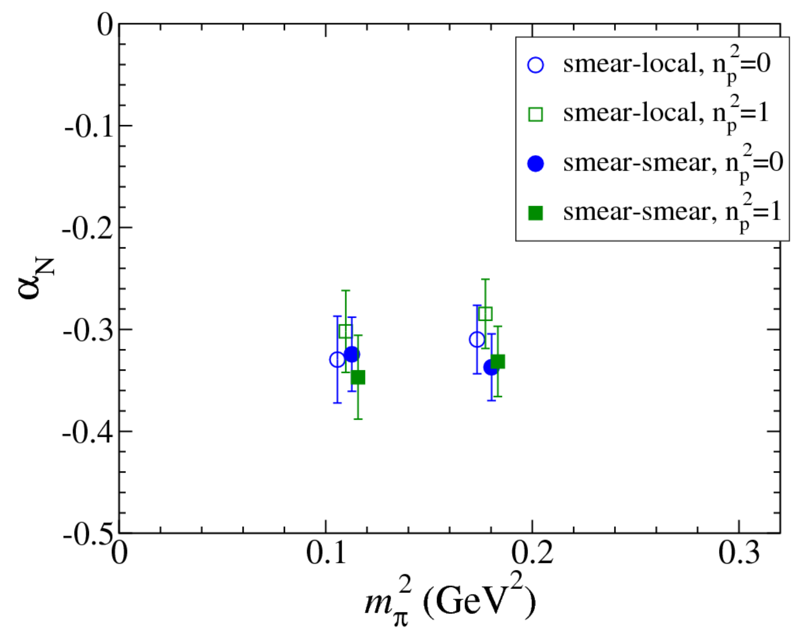
- Projection with γ_5 for 2 pt in θ term, perform global fitting

$$\mathrm{tr}\left[\gamma_5 \langle N(t)\bar{N}(0)Q\rangle\right] = Z_N \frac{2m_N}{E_N}\alpha_N e^{-E_N t} + O(e^{-E_{N^*}})$$

- By using AMA, this factor is determined within15 % error.
- It does not depend on smearing and momentum, but mass dependence is not so clear.





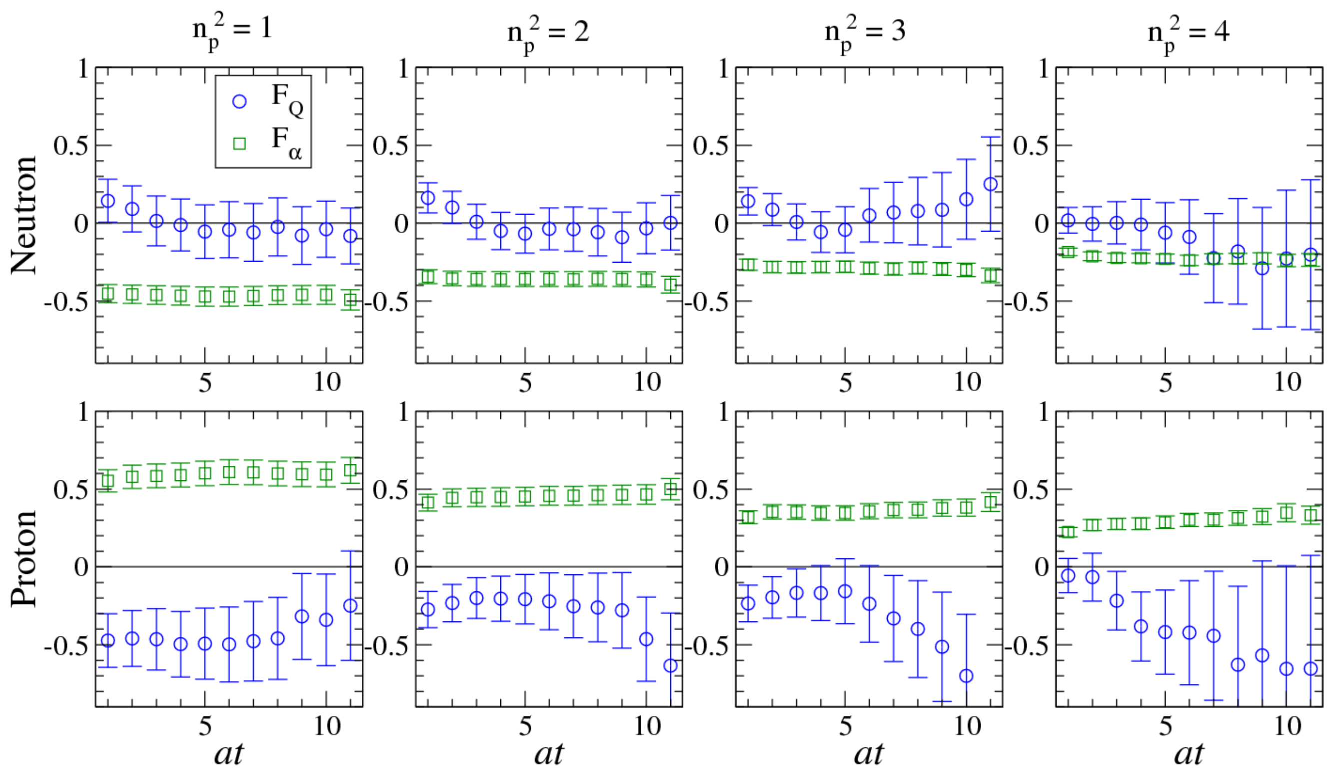


Subtraction term and 3pt function

- Splitting EDM form factor into two parts:

$$F_3 = F_Q + F_\alpha,\; F_Q = C(m_N)\langle N J_t^{\mathrm{EM}} \bar{N} Q\rangle,\; F_\alpha = F(\alpha_N, F_{1,2})$$

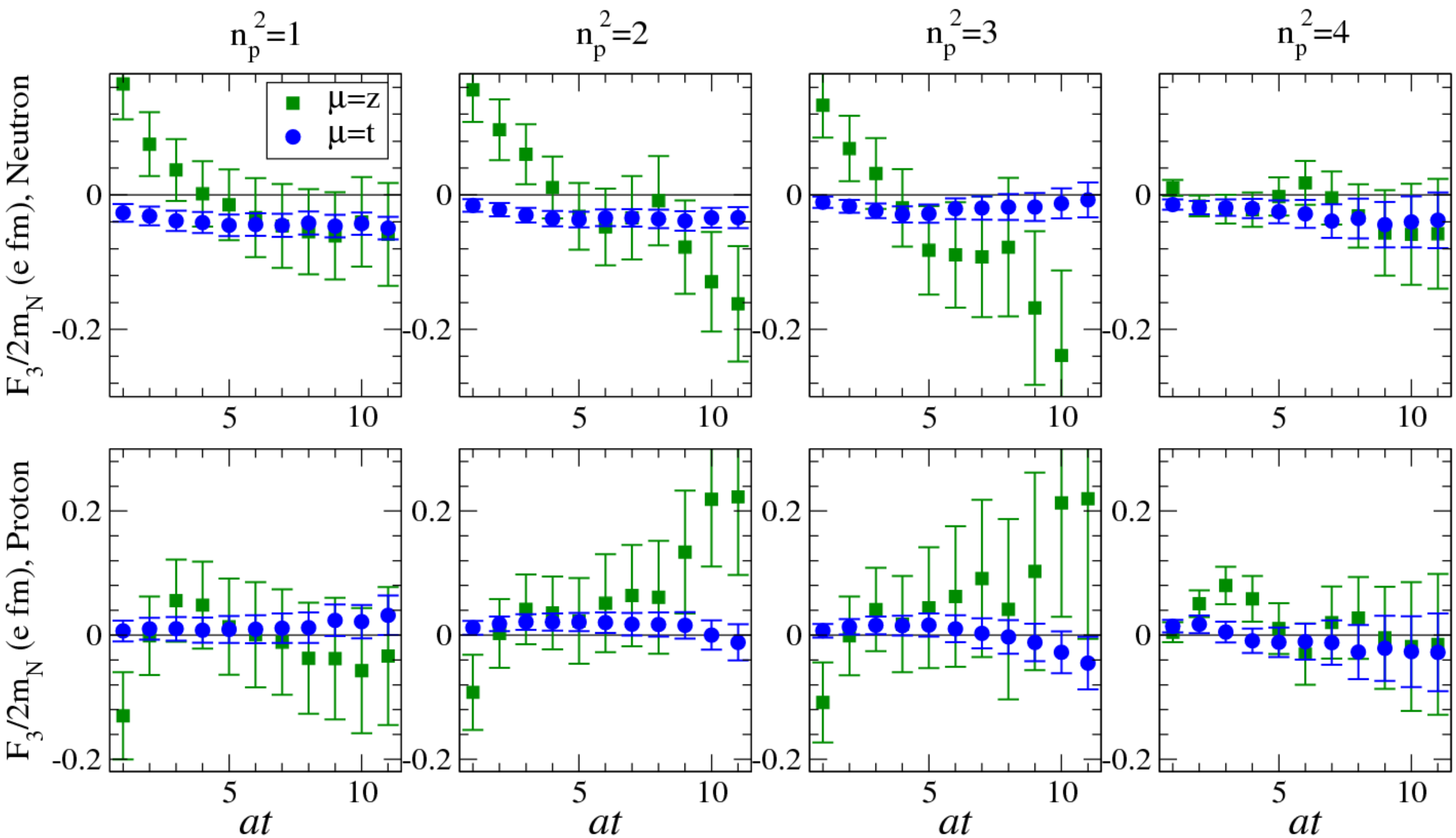
- F_α is good precision, and fluctuation of F_Q is large.



3. Recent update (preliminary)

Comparison with μ = t, z

- EDM form factor is given from two directions of EM current
- Two signals are consistent, and data in t direction is much stable.

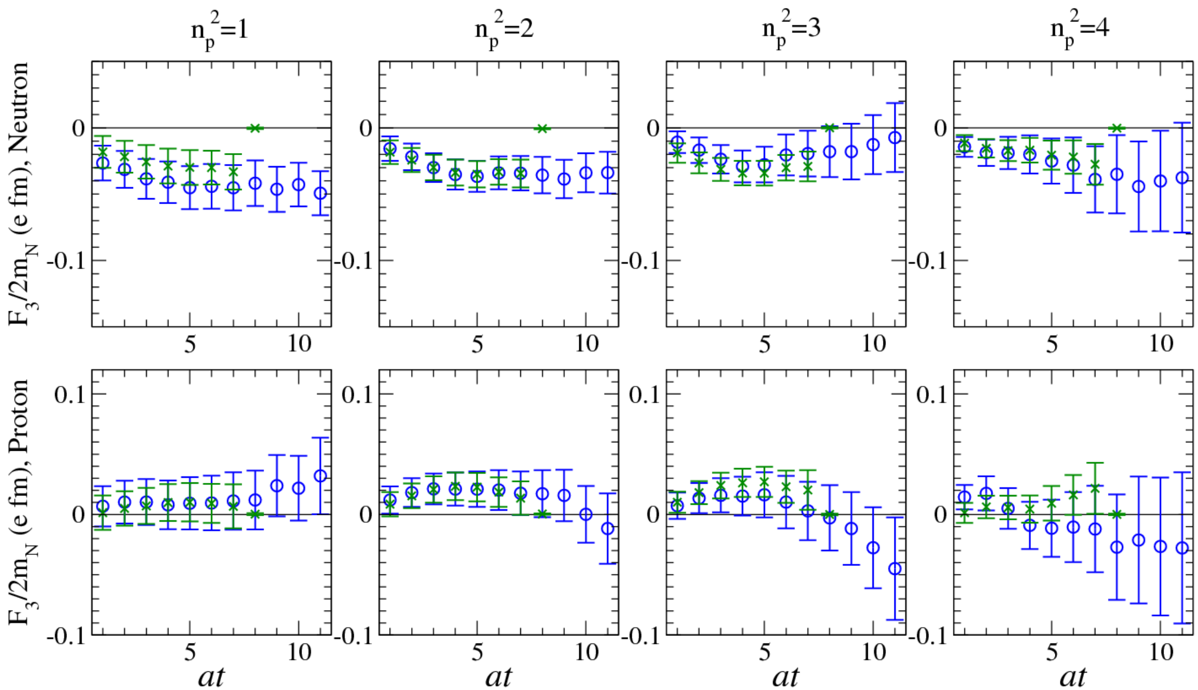


Comparison with different t_{sep}

- The sink and source separation in 3pt function enables us to control the statistical noise and excited state contamination
  - Short: statistical fluctuation < excited state contamination
  - Long: statistical fluctuation > excited state contamination
- Comparison

t_{sep} = 12 (blue), [N_{conf} = 751]
t_{sep} = 8 (green) [N_{conf} = 180]

- Good consistency between them.
- Precision in t_{sep}=8 is much better.



$q^2 = 0$ dependence

- Fitting data of EDM form factor at each momentum
- Open(t_{sep}=8) [N_{conf} = 751],
  filled (t_{sep}=12) [N_{conf} = 180]
- Fitting function

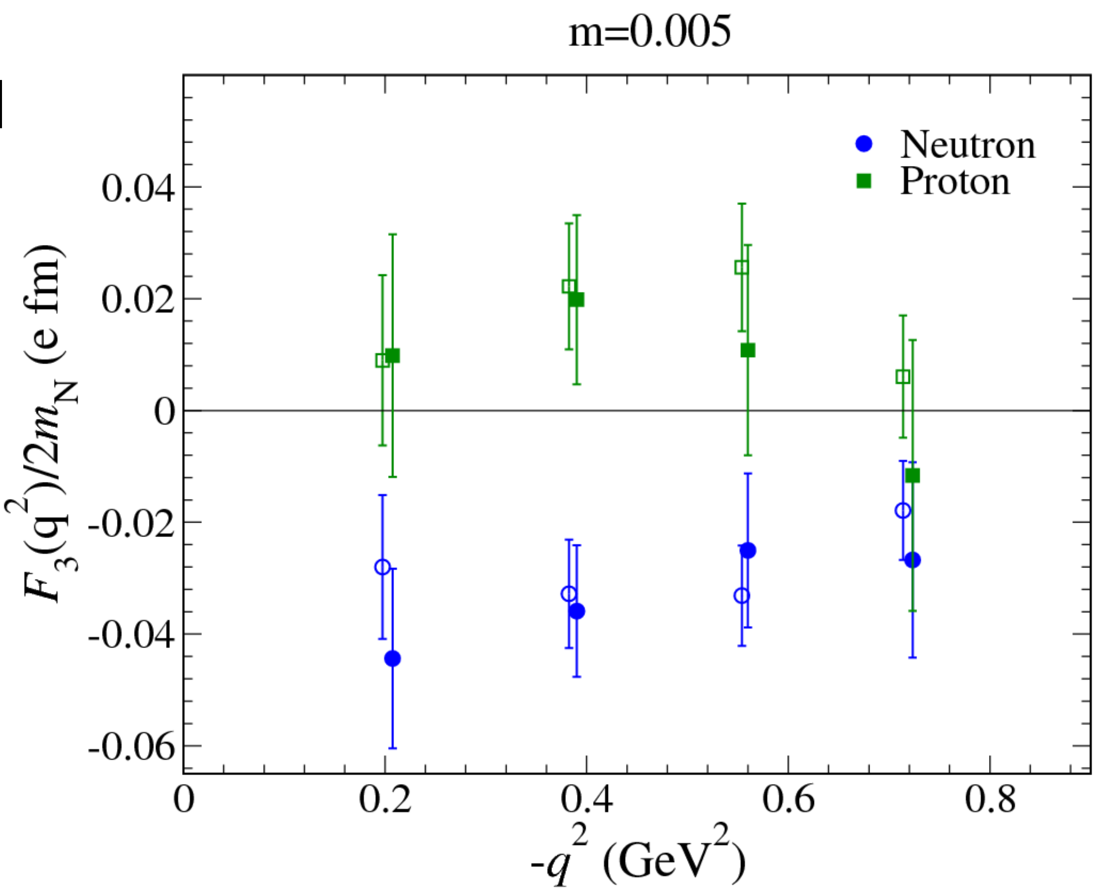
  3 point linear :

  $-q^2 < 0.55$ GeV^2

  2 point linear:

  $-q^2 < 0.4$ GeV^2

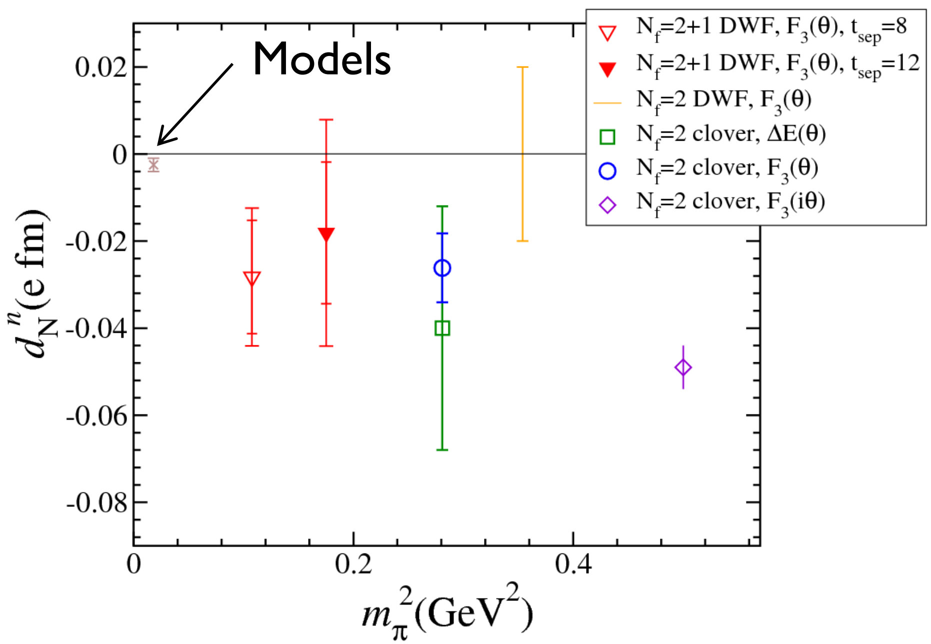
  (ChPT fit is possible ?)
- Estimate of systematic error of extrapolation



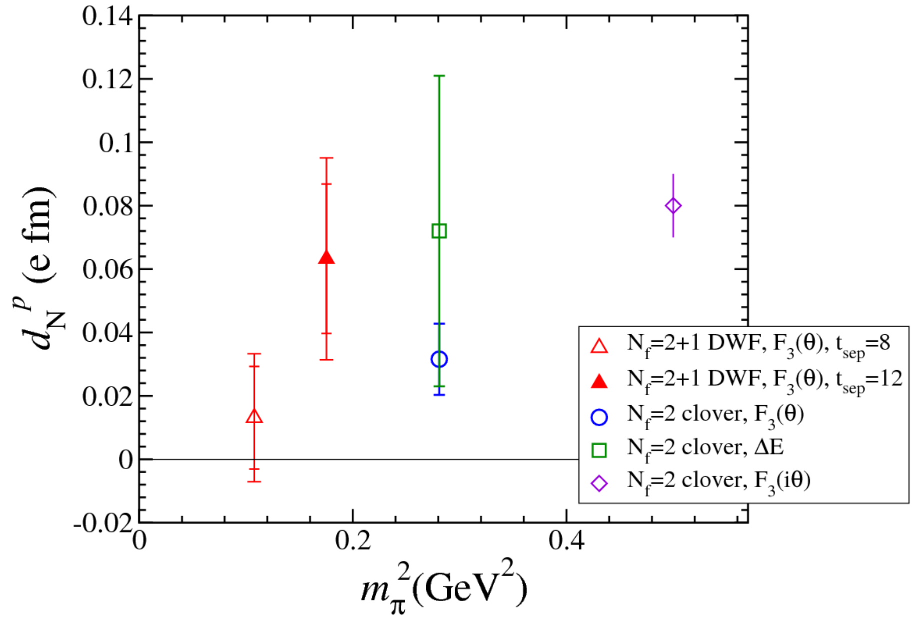
Mass dependence

- Comparison
  - Statistical error is still dominant rather than systematic one.
  - Central value is 10 times larger than models.
  - M_π^2 dependence is not clear, however its sign is consistent with magnetic moment, $d_N \sim \mu_m m_\pi^2 \Delta m$ Abada et al., PLB256(1991), Aoki, Hatsuda PRD45(1992)

Neutron



Proton



Statistical error

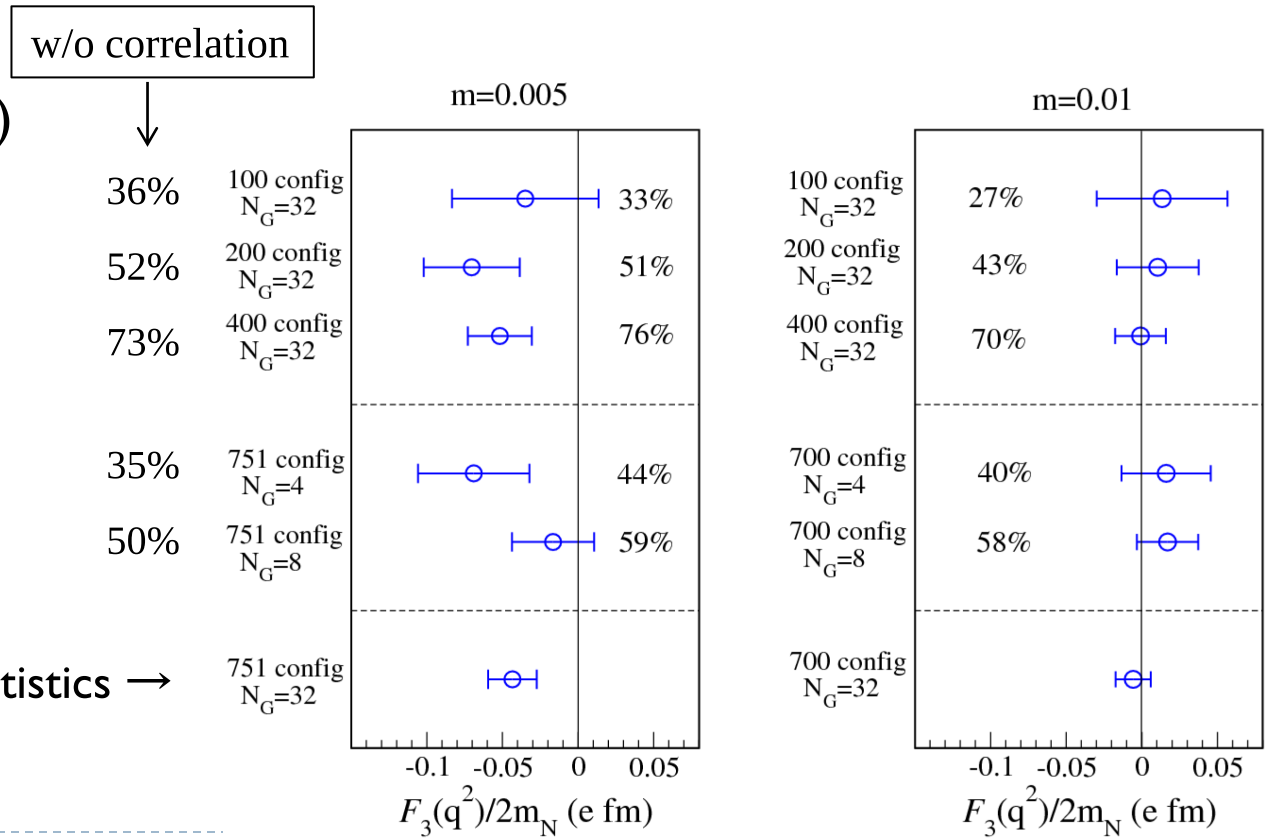
- Comparison between AMA error reduction and number of configurations.
- Number of configurations : reduce stat. error and relating to Q distribution

  AMA error reduction : reduce stat. error
- Error rate

  = Error(full)/Error(N)
- AMA works well
- Reduction rate when

  increase of confgs.

  is slightly better.



Full statistics →

Summary and future plan

- Nucleon EDM in N_f = 2+1 DWF in θ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short t_{sep} allows us to reduce the statistical error without large excited state contamination effect.

Summary and future plan

- Nucleon EDM in N_f = 2+1 DWF in θ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short t_{sep} allows us to reduce the statistical error without large excited state contamination effect.
- (Near) physical point of DWF configurations
  - Ensembles near physical points and large volume are available.
  - AMA with Möbius-DWF approximation is helpful. Hantao, Lattice2013
  - Remove chiral extrapolation → less than 10% precision

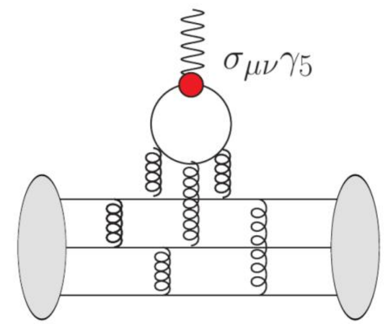
Lattice size	Physical size	a	L_s	Gauge action	Pion mass
$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3 \times 96$	5.5 fm^3	0.115 fm	16	Iwasaki	135 MeV

q(c)EDM term into QCD action

Bhattacharya et al, Lattice 2012

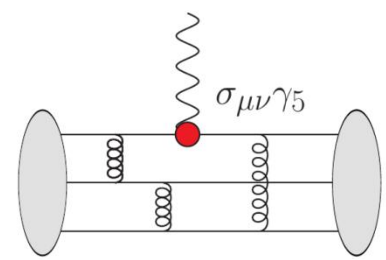
- Plan to do extension toward BSM action
  - Matrix element including BSM operator, quark EDM and chromo EDM (PQ symmetry is assumed)
  - The q(c)EDM term is CP-violating tensor charge of nucleon, connected diagram should be leading contribution→ statistical signal will be clear.
  - External E filed method may be easy way.
  - qEDM
    - Tensor charge matrix element + matrix element with qEDM:

$$\partial_{A_\mu}\langle N|d_q(\bar{q}\gamma_5\sigma\cdot Fq)|N\rangle_E$$
$$= \langle N|d_q(\bar{q}\gamma_5 q^\nu\sigma_{\mu\nu}q)|N\rangle + \langle N|J_\mu d_q(\bar{q}\gamma_5\sigma\cdot Fq)|N\rangle$$



  - Quark chromo EDM
    - Matrix element with chromo EDM term:

$$\partial_{A_\mu}\langle N|d_{cq}(\bar{q}\gamma_5\sigma\cdot Gq)|N\rangle_E = \langle N|J_\mu d_{cq}(\bar{q}\gamma_5\sigma\cdot Gq)|N\rangle$$



Thank you for your attention !

Backup

Examples of CAA

- Lowmode averaging (LMA)

Guisti et al.(04), Neff et al.(01), DeGrand et al. (04)

  - Using lowlying eigenmode of Dirac operator to approximate propagator:

$$\mathcal{O}^{(\text{appx})} = \sum_{\lambda}^{N_\lambda} \mathcal{O}_\lambda^{\text{low}}$$

where N_λ is number of lowmode computed by Lanczos.

Except for computational cost of eigenmode, Cost(LMA) $\simeq$ 0, but approximation is only lowmode part (long distance contribution).

- All-mode averaging (AMA)
  - Using sloppy CG (loose stopping condition),

$$\mathcal{O}^{(\text{appx})} = O^{\text{sloppy}}$$

If stopping cond. is 0.003, Cost(AMA) $\simeq$ Cost(CG)/50(without deflation).

Approximation becomes better than LMA for other than lowmode dominanted observables (nucleon, finite momentum hadron, …).

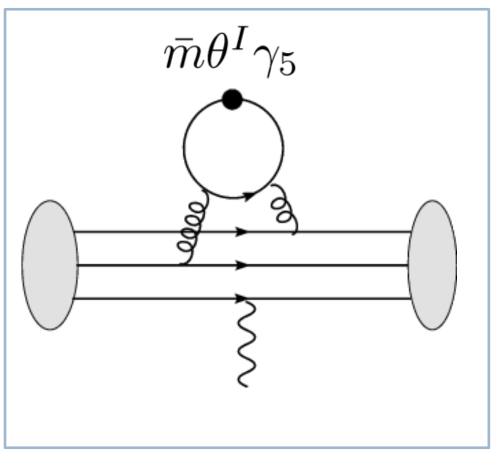
Imaginary θ

Izubuchi(07), Horsley et al. (08)

- Full QCD with Wilson fermion

Fermionic insertion of imaginary theta should be changed by Wilson term:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2 \to \mathcal{L}_\theta^W = \bar{m}(1+\kappa_P)\theta^I \bar{q}\gamma_5 q,\ \kappa_P \sim \mathcal{O}(a) : \mathrm{renom.\ const.}$$



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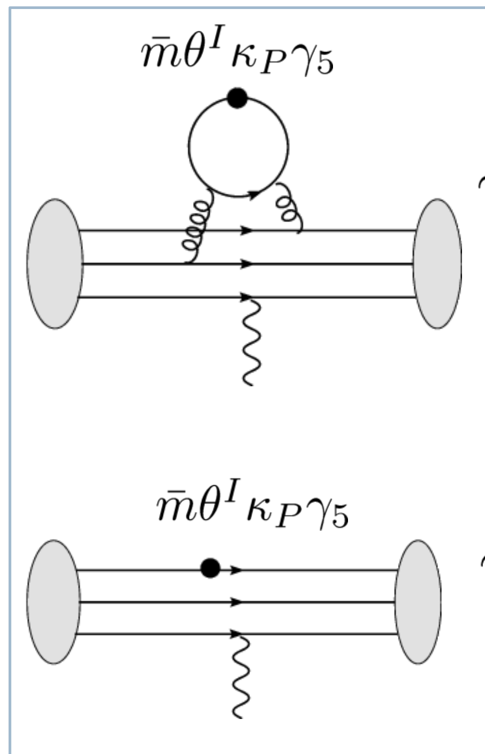


$\sim \mathcal{O}(am_{\mathrm{sea}}\alpha^2\theta^I)$

→ negligible

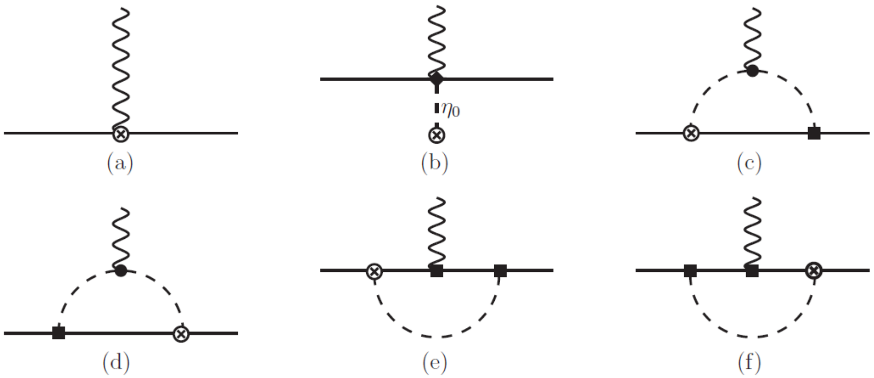
$\sim \mathcal{O}(am_{\mathrm{val}}\theta^I)$

→ significant

Cf. discussion in Aoki, Gocksh, Manohar,Sharpe (1990)

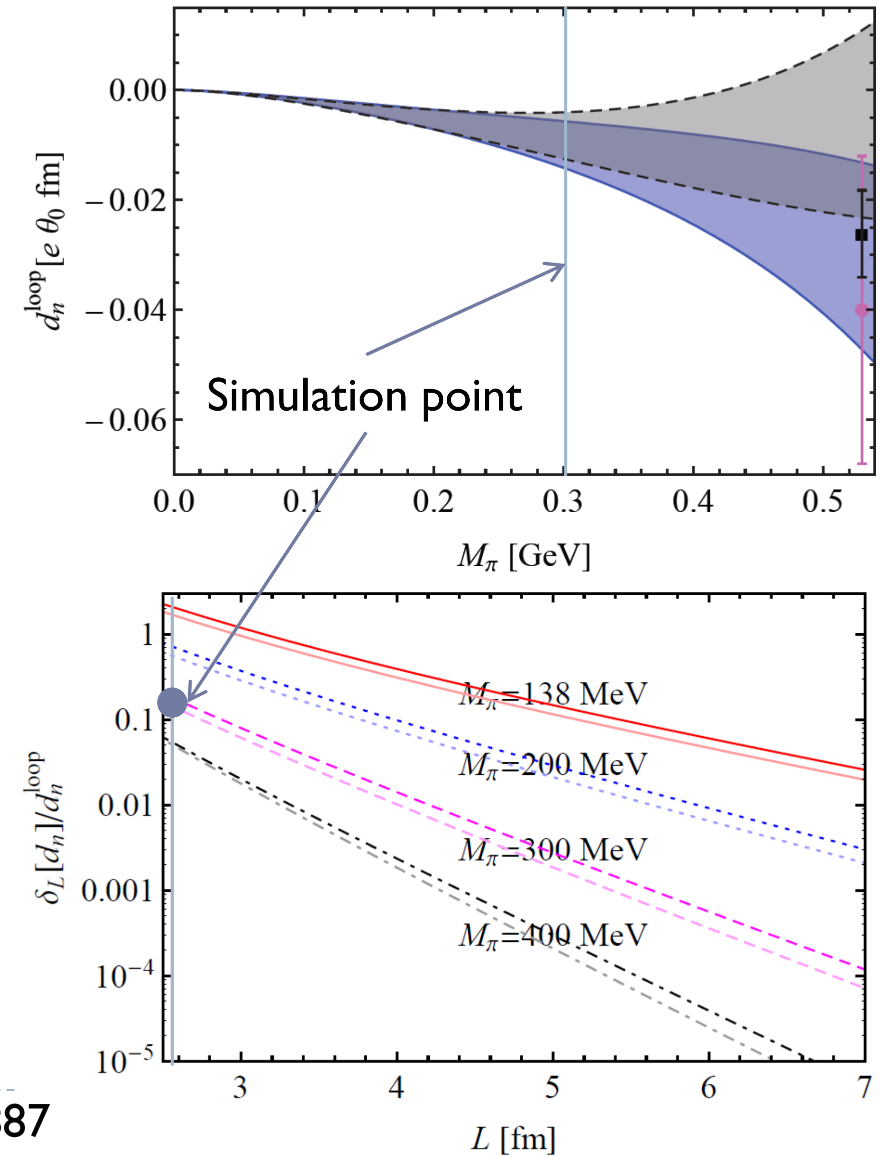
Volume effect ?

- BChPT analysis



$\otimes$ CP violating coupling

In LO, NLO BChPT analysis, there may be more than 20% finite size effect.



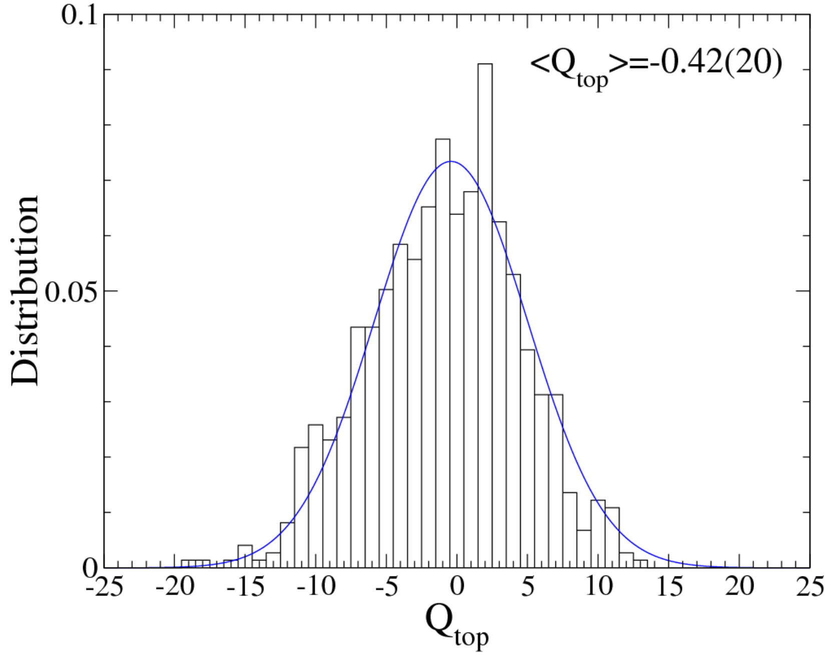
Guo, Meissner, 1210.5887

Topological charge distribution

- Topological susceptibility

$$\begin{aligned}\langle Q^2\rangle/V &= 3.0(1)\times 10^{-4}\,\mathrm{GeV}^4\;(m=0.005)\\ &= 4.6(2)\times 10^{-4}\,\mathrm{GeV}^4\;(m=0.01)\end{aligned}$$

m=0.005



m=0.01