Transverse Electric Contributions to Atomic EDM

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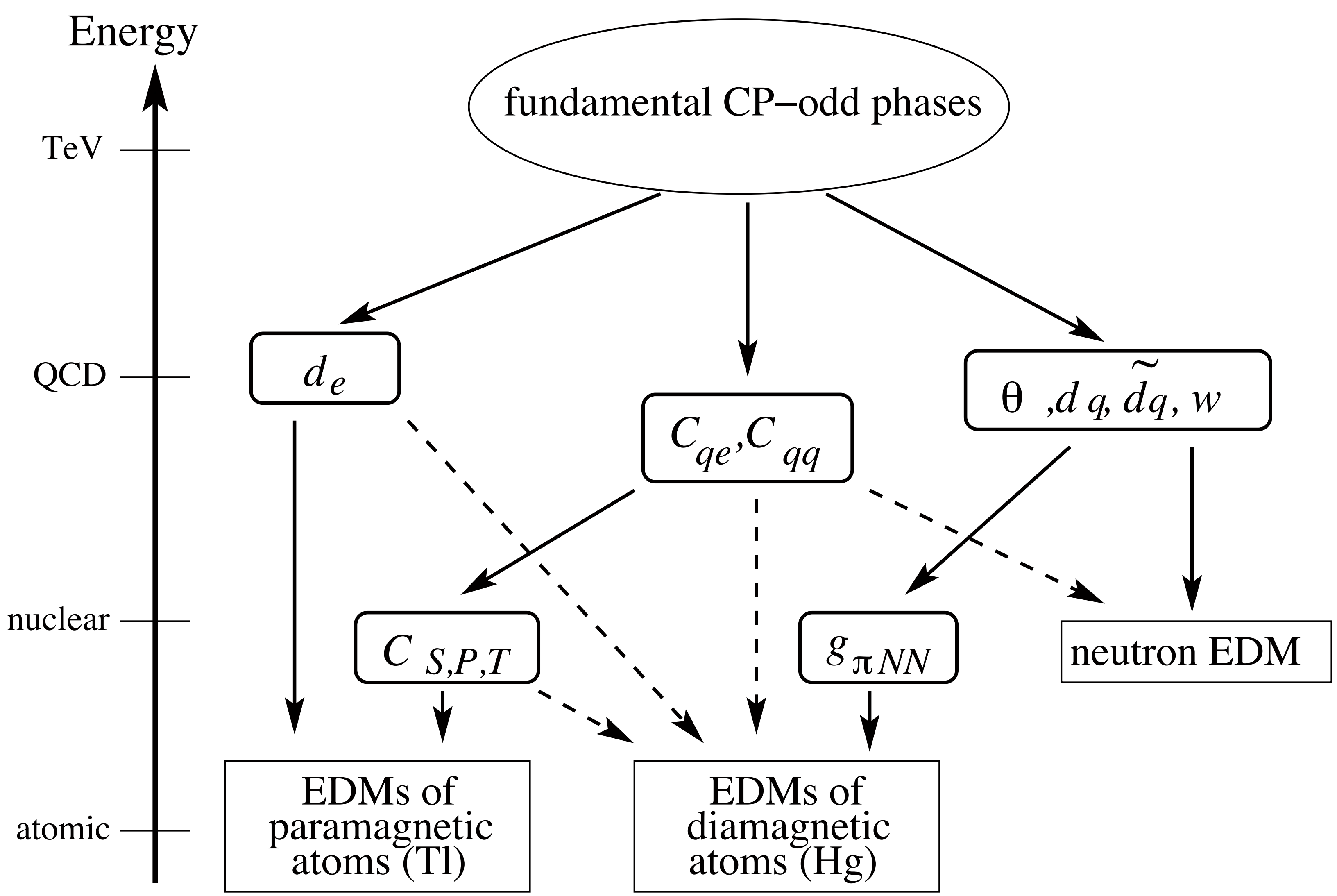
(work w/ M. Ramsey-Musolf & W. Haxton)

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Why diamagnetic atoms?

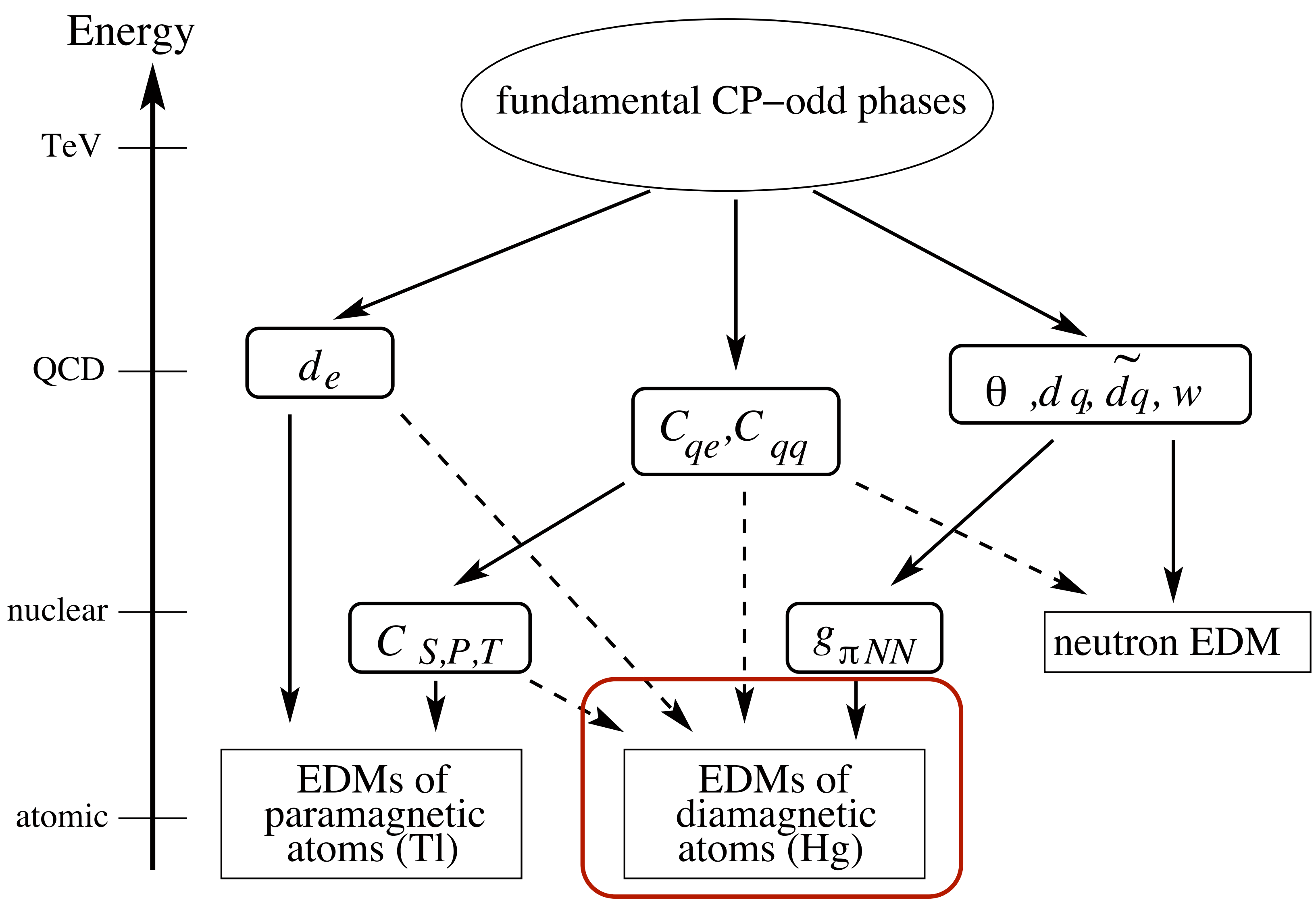
We want more than 1 number for hadronic sector



from Ritz & Pospelov, hep-th/0504231

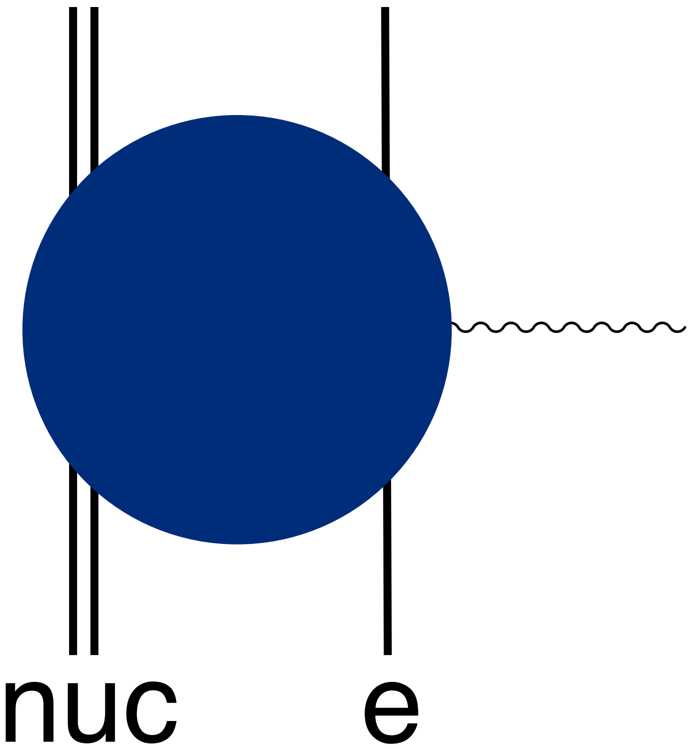
Scope of this talk

Q: What nuclear calculations do we need?



from Ritz & Pospelov, hep-th/0504231

What to calculate



Atomic EDM is defined by $\Delta E = -\vec{d}_A \cdot \vec{E}^{\rm ext}$

Perturbation theory

$$H_0 = \boxed{H_N} + \boxed{H_e - Z\alpha \sum_{i=1}^{Z} \frac{1}{x_i}}$$

Decouple nuclear and atomic problems

Perturbation theory

$$H_0 = \boxed{H_N} + \boxed{H_e - Z\alpha \sum_{i=1}^{Z} \frac{1}{x_i}}$$

Decouple nuclear and atomic problems

$$V = \left(V_{eN} + Z\alpha \sum_{i=1}^{Z} \frac{1}{x_i} \right) + V_e^{\text{ext}} + V_N^{\text{ext}}$$

Calculate energy shift, pick out terms $\propto E^{\text{ext}}$

Breit interaction

Order α correction to Coulomb

$$V_{eN} = -\alpha \iint d^3x d^3y \left[\frac{\rho_e \rho_N}{|\vec{x}-\vec{y}|} - \frac{1}{2}\left(\frac{\vec{j}_e \cdot \vec{j}_N + \vec{j}_e \cdot \hat{n} \vec{j}_N \cdot \hat{n}}{|\vec{x}-\vec{y}|} \right) \right]$$

$\vec{x}$ - electron coordinate, $\vec{y}$ - nuclear coordinate, $\vec{n} = \vec{x} - \vec{y}$

Breit interaction - multipoles

Order α correction to Coulomb

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$\vec{x}$ - electron coordinate, $\vec{y}$ - nuclear coordinate, $\vec{n} = \vec{x} - \vec{y}$

$$\frac{1}{|\vec{x} - \vec{y}|} = \sum_{lm} \frac{4\pi}{2l+1} \left(\frac{y^l}{x^{l+1}} \theta(x-y) + \frac{x^l}{y^{l+1}} \theta(y-x) \right) Y^*_{lm}(\hat{x}) Y_{lm}(\hat{y})$$

Breit interaction - multipoles

Order α correction to Coulomb

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$$= \sum_{lm} \frac{4\pi}{2l+1} \left[\boxed{\frac{y^l}{x^{l+1}}} + \boxed{\left(\frac{x^l}{y^{l+1}} - \frac{y^l}{x^{l+1}} \right) \theta(y-x)} \right] Y^*_{lm}(\hat{x}) Y_{lm}(\hat{y})$$

Pointlike nucleus   Penetration correction

Multipoles - classification

From charge-charge interaction:

$$V^{Cl}_{\mathrm{pt/pen}} \propto C^A_l \odot C^N_l$$

Pointlike C0 - total nuclear charge (in H_0)

Pointlike C1, C2, ... - EDM, EQM, ...

Penetration terms -> Schiff moment

Multipoles - classification

From current-current interaction:

$$V^{Ml}_{\mathrm{pt/pen}} \propto T^{\mathrm{mag},A}_{l} \odot T^{\mathrm{mag},N}_{l}$$

$$V^{El}_{\mathrm{pt/pen}} \propto T^{\mathrm{el},A}_{l} \odot T^{\mathrm{el},N}_{l}$$

More than 1 way to couple current vector to spherical harmonics

So many terms!

Symmetries to the rescue

	$PCTC$	$PVTC$	$PCTV$	$PVTV$
C	even	–	–	odd
M	odd	–	–	even
E	–	odd	even	–

EDMs are PVTV -> need PVTV coupling to nucleus

Symmetries to the rescue

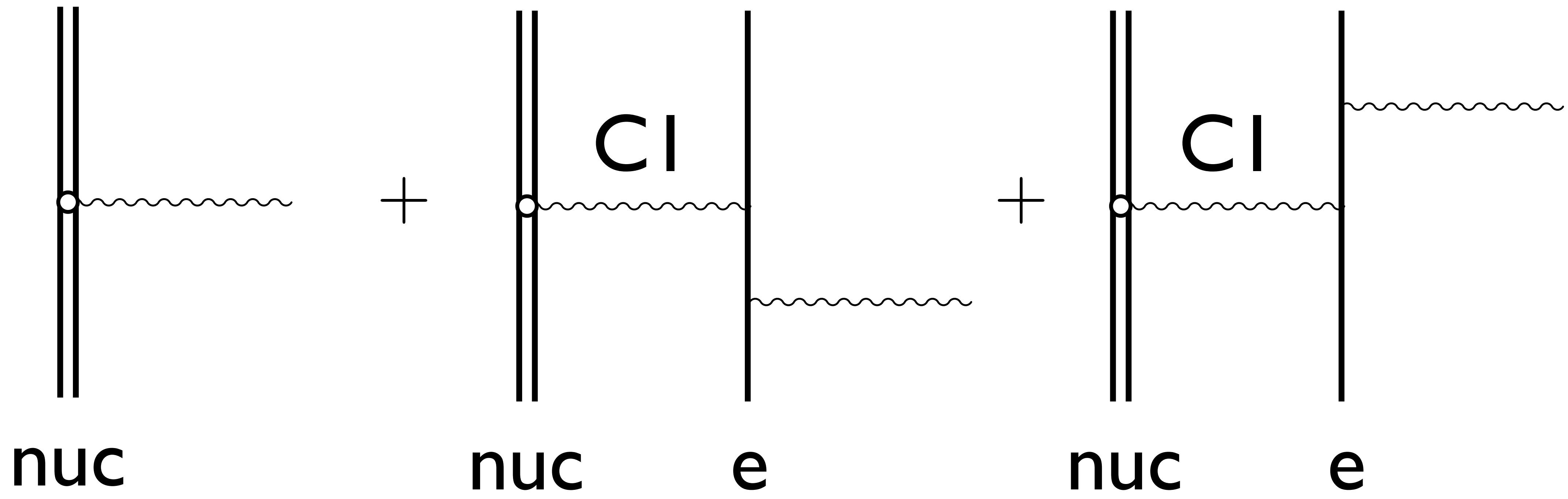
	$PCTC$	$PVTC$	$PCTV$	$PVTV$
C	even	—	—	odd
M	odd	—	—	even
E	—	odd	even	—

EDMs are PVTV -> need PVTV coupling to nucleus

Also - Atomic J = 0, Nuclear J = 1/2

M2, C3, etc. don't contribute

Schiff screening



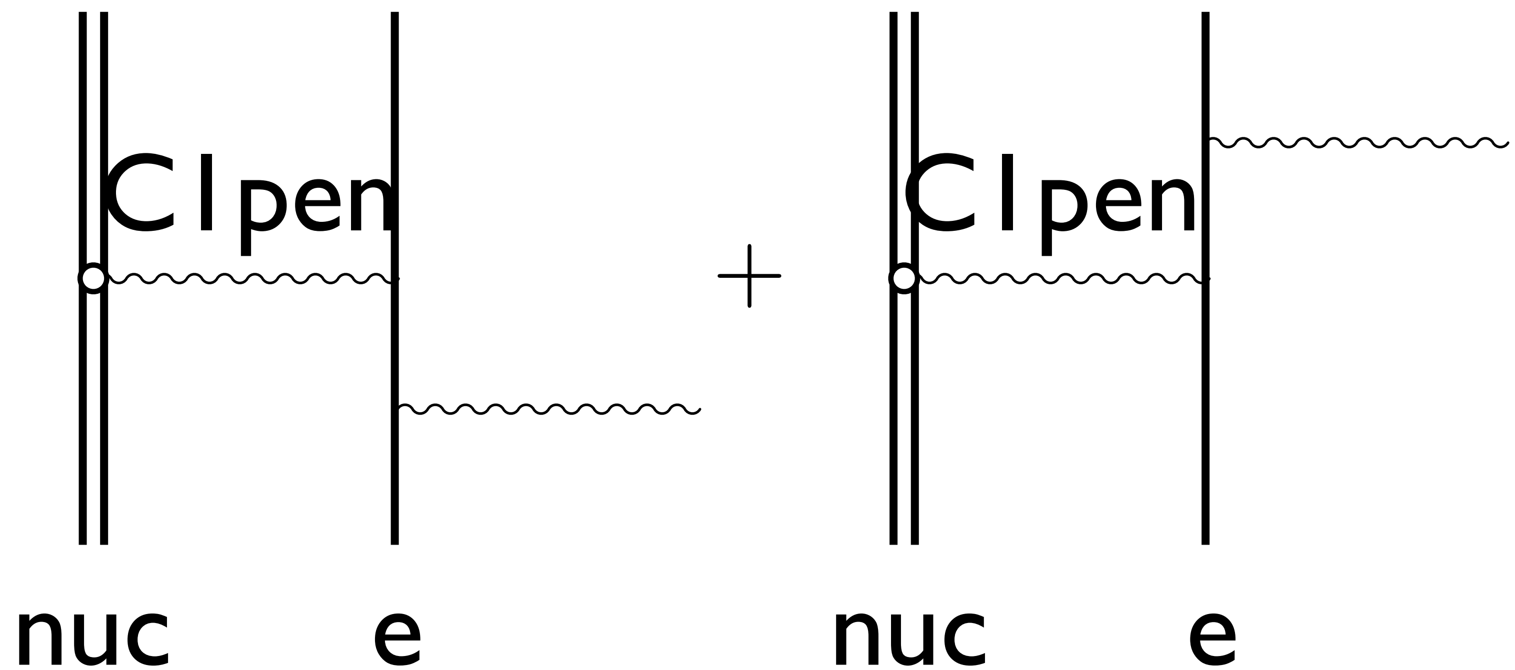
$$\langle 0|V_N^{\rm ext}|0\rangle + \left(\langle 0|V_e^{\rm ext} K V_{\rm pt}^{C1}|0\rangle + {\rm c.c.}\right) = 0$$

Naive LO terms cancel

$$K = \sum_{n\neq 0} \frac{|n\rangle\langle n|}{E_0 - E_n}$$

Schiff moment - term 1

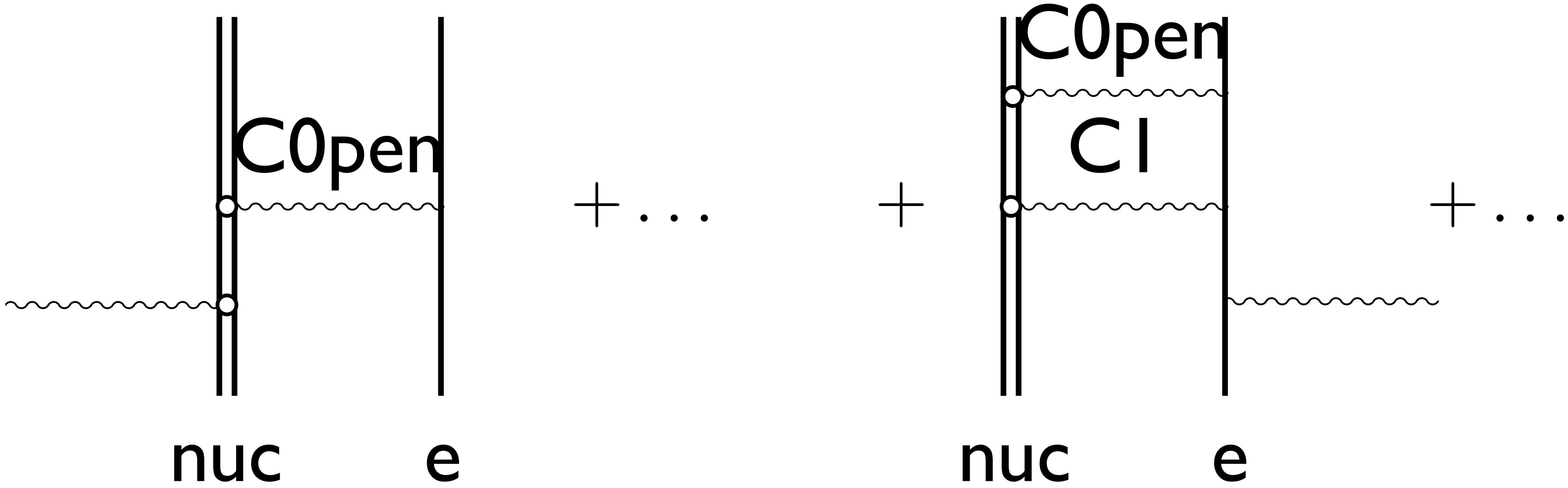
Penetration C1 is not screened



$$\langle 0|V_e^{\rm ext} K V_{\rm pen}^{C1}|0\rangle + {\rm c.c.}$$

Schiff moment - term 2

Higher order terms are only partially screened



Both terms - Penetration effect. Only Coulomb multipoles

Other terms?

	$PCTC$	$PVTC$	$PCTV$	$PVTV$
C	even	–	–	odd
M	odd	–	–	even
E	–	odd	even	–

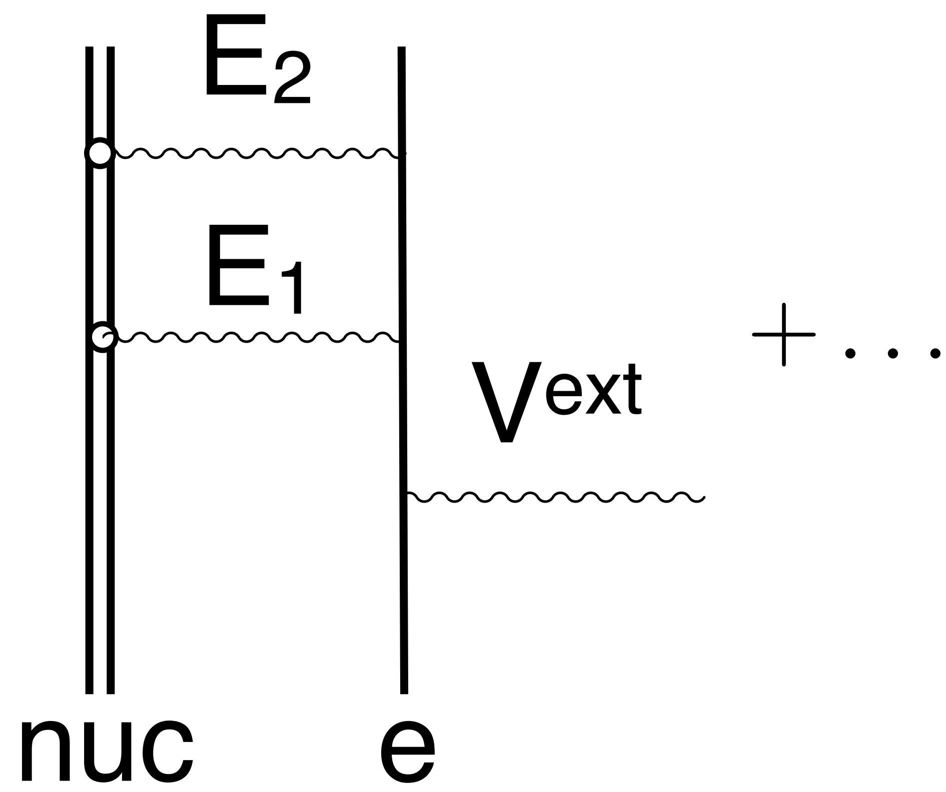
E multipoles don't have PVTV moments, but what if you iterate?

Other terms?

	$PCTC$	$PVTC$	$PCTV$	$PVTV$
C	even	—	—	odd
M	odd	—	—	even
E	—	odd	even	—

E multipoles don't have PVTV moments, but what if you iterate?

Combine E1 & E2 -> PVTV!



Size of Schiff moment

Nuclear Schiff moment operator

$$\vec{S} \equiv \frac{1}{10}\left[\int d^3y\left(y^2\vec{y} - \frac{5}{3Z}y^2\langle\vec{d}_N\rangle\right)\rho_N(\vec{y})\right]$$

goes as $\sim R_N^3$

Energy shift due to Schiff moment is $\sim E^{\text{ext}} R_A \left(\frac{R_N}{R_A}\right)^3$

Suppression of $\left(\frac{R_N}{R_A}\right)^2$ compared to nuclear EDM

Power counting

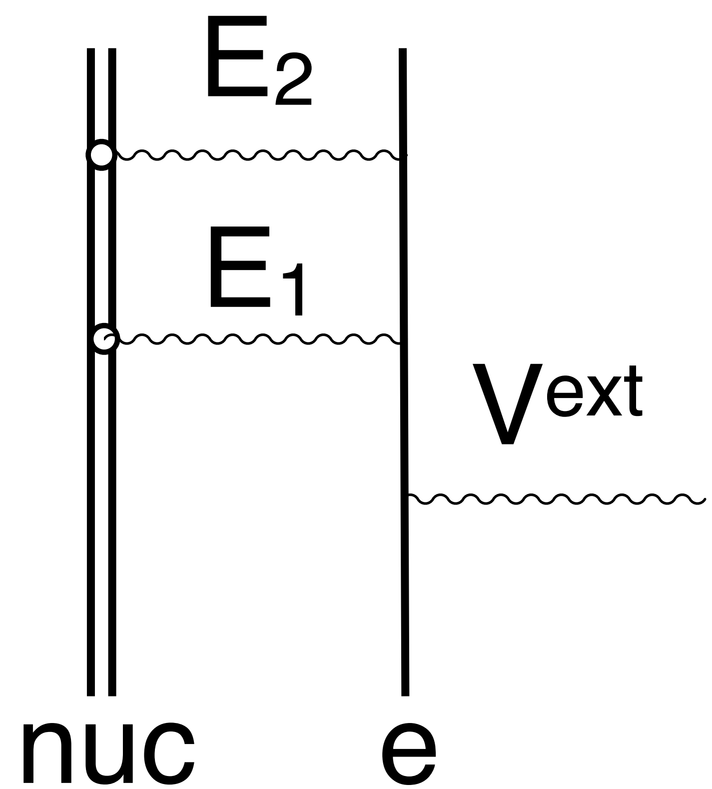
This can be applied to other multipoles

$$V_{\rm pt}^{Cl,Ml} \sim \frac{1}{R_A}\left(\frac{R_N}{R_A}\right)^l$$

$$V_{\rm pt}^{El} \sim \frac{1}{R_A}\left(\frac{R_N}{R_A}\right)^{l-1}$$

Transverse electrics are less suppressed for given L

E1-E2 term



$$\langle 0|V_e^{\text{ext}} K V_{\text{pt}}^{E1} K V_{\text{pt}}^{E2}|0\rangle$$

$$\sim E^{\text{ext}} R_A \cdot R_A \cdot \frac{1}{R_A} \cdot R_N \cdot \left(\frac{1}{R_A}\frac{R_N}{R_A}\right) = E^{\text{ext}} R_A \left(\frac{R_N}{R_A}\right)^2$$

Schiff moment term had exponent of 3

Rough evaluation

Nuclear part of calculation

$$\sum_{n\neq 0} \frac{1}{E_0 - E_N} \langle 0_N | T_1^{\mathrm{el},N} | n_N \rangle \langle n_N | T_2^{\mathrm{el},N} | 0_N \rangle$$

Rough evaluation

Use Siegert's theorem:

$$T_l^{\mathrm{el},N} \propto [C_l^N, H_N]$$

Nuclear part of the calculation becomes

$$\sum_n \frac{1}{-\Delta E} \langle 0_N | T_1^{\mathrm{el},N} | n_N \rangle \langle n_N | T_2^{\mathrm{el},N} | 0_N \rangle$$

$$\propto \sum_n \Delta E \langle 0_N | C_1^N | n_N \rangle \langle n_N | C_2^N | 0_N \rangle$$

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$$\propto \sum_n \Delta E \langle 0_N | C_1^N | n_N \rangle \langle n_N | C_2^N | 0_N \rangle$$

Looks like a sum rule

Assume the sum is saturated by giant dipole resonance

Compare with Schiff moment term

E1-E2 energy shift (approximate):

$$\frac{(4\pi\alpha)^2 E_{\mathrm{GDR}}}{30\sqrt{3}} \left(\frac{R_N}{R_A}\right)^3 \sum_{n'\neq 0} \frac{1}{E_0 - E_{n'}} \langle j_N \| \left\{C_1^N, C_2^N\right\}_1 \| j_N \rangle$$

$$\times \left(\langle j_e \| \sum_i \vec{x}_i \cdot \vec{E}^{\mathrm{ext}} \| j_e' \rangle \langle j_e' \| \left\{ T_1^{\mathrm{el},A}, T_2^{\mathrm{el},A} \right\}_1 \| j_e \rangle + \mathrm{c.c.} \right)$$

Schiff moment energy shift:

$$-\frac{4\pi\alpha}{3R_A} \left(\frac{R_N}{R_A}\right)^3 \sum_{n'\neq 0} \frac{1}{E_0 - E_{n'}} \langle j_N \| \vec{S} \| j_N \rangle$$

$$\times \left(\langle j_e \| \sum_i \vec{x}_i \cdot \vec{E}^{\mathrm{ext}} \| j_e' \rangle \langle j_e' \| R_A^4 \sum_j \left[\overleftarrow{\nabla}_j \delta^3(\vec{x}_j) + \delta^3(\vec{x}_j) \overrightarrow{\nabla}_j \right] \| j_e \rangle + \mathrm{c.c.} \right)$$

Compare with Schiff moment term

Focus on the numerical factors (S contains factor 1/10)

$$\frac{(4\pi\alpha)^2 E_{\rm GDR}}{30\sqrt{3}} \Bigg/ \left(\frac{4\pi\alpha}{3R_A}\cdot\frac{1}{10}\right) = \frac{4\pi\alpha R_A E_{\rm GDR}}{\sqrt{3}}$$

$$\sim 500$$

E1-E2 term is *larger* assuming comparable matrix elements

Compare with Schiff moment term

Focus on the numerical factors

$$\frac{(4\pi\alpha)^2 E_{\mathrm{GDR}}}{30\sqrt{3}} \Bigg/ \left(\frac{4\pi\alpha}{3R_A}\cdot\frac{1}{10}\right) = \frac{4\pi\alpha R_A E_{\mathrm{GDR}}}{\sqrt{3}}$$

$$\sim 500$$

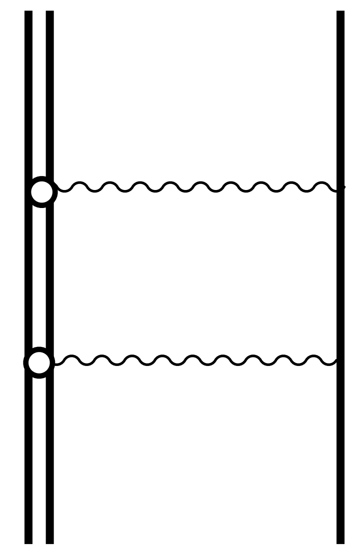
E1-E2 term is *larger* assuming comparable matrix elements

Nuclear matrix elements were of same order in

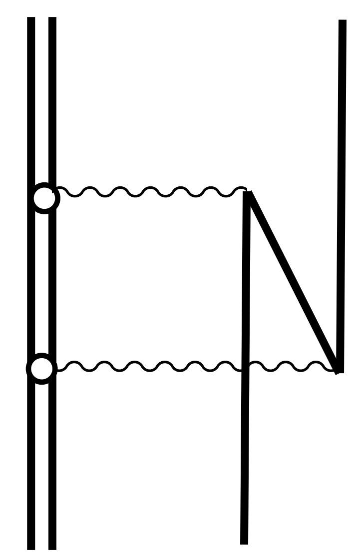
a very simple model of nucleus (nucleons in pure HO potential)

What could go wrong?

1. Iterating Breit interaction



How large is



?

What could go wrong?

1. Iterating Breit interaction

2. R_N/R_A power counting may fail

   due to nuclear? atomic? physics we haven't considered

Conclusions

1. Atomic EDM can come from E multipoles

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2. Naive power counting suggests contributions are large

   -Better constraints on new physics than Schiff, if true

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1. Atomic EDM can come from E multipoles

2. Naive power counting suggests contributions are large

   -Better constraints on new physics than Schiff, if true

3. Nuclear calculation would be harder than Schiff moment

$$\sum_{n\neq 0} \frac{1}{E_0 - E_N} \langle 0_N | T_1^{\mathrm{el},N} | n_N \rangle \langle n_N | T_2^{\mathrm{el},N} | 0_N \rangle$$