Revealing the Quark and Gluon Structure of Hadrons and Nuclei (including charge symmetry breaking effects)

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The Challenge of QCD



- QCD is the only known example in nature of a fundamental quantum field theory that is innately non-perturbative
 - *a priori* no idea what such a theory can produce
 - Solving QCD will have profound implications for our understanding of the natural world
 - e.g. it will explain how massless gluons and light quarks bind together to form hadrons, and thereby explain the origin of ~98% of the mass in the visible universe
 - given QCDs complexity, the best promise for progress is a strong interplay between experiment and theory
- QCD is characterized by two emergent phenomena:
 - confinement & dynamical chiral symmetry breaking (DCSB)
 - a world without DCSB would be profoundly different, e.g. $m_{\pi} \sim m_{\rho}$
- Must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic obserables

Meeting this Challenge – QCDs Dyson–Schwinger Eqns



- The equations of motion of QCD \iff QCDs Dyson–Schwinger equations
 - an infinite tower of coupled integral equations
 - must implement a symmetry preserving truncation
- The most important DSE is QCDs gap equation \implies quark propagator



• ingredients - dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i \not p + M(p^2)}$$

• S(p) has correct perturbative limit

- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
 - no real mass shell \Longrightarrow confinement



- In equal-time quantization the wave function for a hadron is a frame dependent concept
 - as it is defined by observations of different space points at a fixed time
 - boost operators are interaction dependent, i.e. are dynamical
- In high energy scattering experiments particles move at near speed of light
 - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$
- Light-front wave functions, $\psi(x_i, \vec{k}_{\perp i})$, have many remarkable properties
 - provide a frame-independent representation of hadrons
 - have a probability interpretation as close as QFT gets to QM
 - do not depend on the hadrons 4-momentum; only internal variables: $x_i \And ec{k}_{\perp i}$
 - boosts are kinematical not dynamical!!
- Parton distribution amplitudes (PDAs) are (almost) obserables & are related to light-front wave functions

$$arphi(x_i) = \int d^2 ec{k}_{\perp i} \; \psi(x_i, ec{k}_{\perp i})$$

Pion's Parton Distribution Amplitude



- pion's PDA $\varphi_{\pi}(x)$: is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state
 - it's a function of the Bjorken scaling variable $x = \frac{k^+}{p^+}$ and the scale Q^2
 - The pion's PDA is defined by

$$f_{\pi} \varphi_{\pi}(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \,\delta\left(k^+ - x \,p^+\right) \operatorname{Tr}\left[\gamma^+ \gamma_5 \,S(k) \,\Gamma_{\pi}(k,p) \,S(k-p)\right]$$

• $S(k) \Gamma_{\pi}(k,p) S(k-p)$ is the pion's Bethe-Salpeter wave function

- in the non-relativistic limit it corresponds to the Schrodinger wave function
- φ_π(x): is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light front [pseudo-scalar projection also non-zero]
- Pion PDA is interesting because it is calculable in perturbative QCD and, e.g., in this regime governs the Q² dependence of the pion form factor

$$Q^2 F_{\pi}(Q^2) \stackrel{Q^2 \to \infty}{\longrightarrow} 16 \pi f_{\pi}^2 \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_{\pi}^{\text{asy}}(x) = 6 x (1-x)$$

QCD Evolution & asymptotic PDA



Scale (Q²) dependence of pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \, \varphi(x,\mu) = \int_0^1 dy \, V(x,y) \, \varphi(y,\mu)$$

This evolution equation has a solution of the form

$$\varphi_{\pi}(x,Q^2) = 6 x (1-x) \left[1 + \sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1) \right]$$

- α = 3/2 because in Q² → ∞ limit QCD is invariant under the collinear conformal group SL(2; ℝ)
- Gegenbauer- $\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$
- The coefficients of the Gegenbauer polynomials, $a_{\pi}^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_{\pi}(x) \to \varphi_{\pi}^{asy}(x) = 6 x (1-x)$
- At what scales is this a good approximation to the pion PDA

• E.g., AdS/QCD find $\varphi_{\pi}(x) \sim x^{1/2} (1-x)^{1/2}$ at $Q^2 = 1 \text{ GeV}^2$ expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\%$

Pion PDA in DSEs





Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA

- scale of calculation is given by renormalization point $\zeta = 2 \,\text{GeV}$
- Broading of the pion's PDA is directly linked to DCSB
 - if there is no DCSB, DSEs give $\varphi_{\pi}^{asy}(x) = 6 x (1-x)$
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor, which is measurable at JLab

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Pion PDA from lattice QCD





Standard practice to fit first coefficient of "asymptotic expansion" to moment

$$\varphi_{\pi}(x,Q^2) = 6 x (1-x) \left[1 + \sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1) \right]$$

- however this expansion is guaranteed to converge rapidly only when $Q^2
 ightarrow \infty$
- this procedure results in a *double-humped* pion PDA
- Advocate using a generalized expansion

$$\varphi_{\pi}(x,Q^2) = N_{\alpha} x^{\alpha - 1/2} (1-x)^{\alpha - 1/2} \left[1 + \sum_{n=2,4,\dots} a_n^{\alpha}(Q^2) C_n^{\alpha}(2x-1) \right]$$

• Find
$$\varphi_{\pi} \simeq x^{\alpha} (1-x)^{\alpha}$$
, $\alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \simeq 0.30$

When is the Pion's PDA Asymptotic





• Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100 \text{ GeV}^2$, compared with $\varphi_{\pi}^{\text{asy}}(x) = 6 x (1 - x)$

- Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors
- Importantly, $\varphi_{\pi}^{asy}(x)$ can only be an accurate approximation to $\varphi_{\pi}(x)$ when the pion valence quark PDF is proportional to a delta function: $q_v^{\pi}(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





LO QCD evolution of momentum fraction carried by valence quarks

$$\left\langle x \, q_v(x) \right\rangle(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \left\langle x \, q_v(x) \right\rangle(Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

• therefore, as $Q^2 \to \infty$ we have $\langle x q_v(x) \rangle \to 0$ implies $q_v(x) = \delta(x)$

• At LHC energies valence quarks still carry 20% of pion momentum

• we find that the gluon distribution saturates at $\langle x \, g(x) \rangle \sim 55\%$

Asymptotia is a long way away! Practically QCD is always non-perturbative

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The QCD prediction can be expressed as

Pion Elastic Form Factor

$$Q^2 F_{\pi}(Q^2) \overset{Q^2 \gg \Lambda^2_{\text{QCD}}}{\sim} 16 \pi f_{\pi}^2 \alpha_s(Q^2) w_{\pi}^2; \qquad w_{\pi} = rac{1}{3} \int_0^1 dx \, rac{1}{x} \, arphi_{\pi}(x)$$

- Using $\varphi_{\pi}^{asy}(x)$ significantly underestimates experiment
- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
 - 15% disagreement explained by higher order/higher-twist corrections

• We prediction that QCD power law behaviour sets in at $Q^2 \sim 8 \, GeV^2$

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 Q^2



Charge Symmetry and Charge Independence



- Charge symmetry is a particular type of isospin invariance
 - namely the invariance under an isospin rotation of 180° about the *y*-axis (with charge associated with the *z*-axis)
 - corresponds to the interchange of u and d quarks: $u \leftrightarrow d, \bar{u} \leftrightarrow \bar{d}$

$$P_{cs} \ket{u} = -\ket{d}, \qquad P_{cs} \ket{d} = \ket{u}, \qquad P_{cs} = e^{i \pi T_2}$$

- Charge symmetry does not imply *charge independence*, which is associated with invariance under arbitrary rotations in isospin space
- E.g.: pion masses break charge independence but respect charge symmetry

 $\begin{aligned} |\pi^{+}\rangle &= |u\bar{d}\rangle & m_{\pi^{+}} = 139.57 \,\mathrm{MeV} \\ |\pi^{0}\rangle &= \frac{1}{\sqrt{2}} \left[|u\bar{u}\rangle - |d\bar{d}\rangle \right] & m_{\pi^{0}} = 134.98 \,\mathrm{MeV} \\ |\pi^{-}\rangle &= |d\bar{u}\rangle & m_{\pi^{-}} = 139.57 \,\mathrm{MeV} \end{aligned}$

• isospin breaking effects of the order: $\frac{m_{\pi^+} - m_{\pi^0}}{m_{\pi^+}} \sim 3 - 4\%$

Charge Symmetry Breaking & $M_n - M_p$



- In Standard Model two main sources of charge symmetry breaking
 - quark mass differences: $\delta m = m_d m_u \sim 4 \,\mathrm{MeV}$
 - quark charges: $e_u = +2/3, \quad e_d = -1/3$
- The proton (*uud*) neutron (*udd*) mass difference, $\delta M_N = M_n - M_p = 1.29$ MeV, indicates charge symmetry breaking
 - charge symmetry breaking effects of the order: $\frac{M_n M_p}{M_p} < 1\%$
- For $m_u = m_d$ electrostatic replusion implies $M_p > M_n$
 - recent analysis finds: $\delta M_N^{\text{QED}} = -1.30(03)(47) \text{ MeV}$ [Walker-Loud et al. PRL 108, 232301 (2012)]
 - Therefore the proton-neutron mass splitting from $m_u \neq m_d$ alone is:

 $\delta M_N^{\rm QCD} = 2.60(03)(47)\,{\rm MeV}$

We will focus on charge symmetry breaking effects in nucleon PDFswhy is this interesting?

Weak mixing angle and the NuTeV anomaly





Fermilab press conference

"The predicted value was 0.2227. The value we found was 0.2277, a difference of 0.0050. It might not sound like much, but the room full of physicists fell silent when we first revealed the result"

"99.75% probability that the neutrinos are not behaving like other particles . . . only 1 in 400 chance that our measurement is consistent with prediction"

• NuTeV: $\sin^2 \theta_W = 0.2277 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$

[G. P. Zeller et al. Phys. Rev. Lett. 88, 091802 (2002)]

- Standard Model: $\sin^2 \theta_W = 0.2227 \pm 0.0004 \Leftrightarrow 3\sigma \implies$ "NuTeV anomaly"
- Huge amount of experimental & theoretical interest [500+ citations]
- Evidence for physics beyond the Standard Model?
- No universally accepted *complete* explanation



Paschos-Wolfenstein ratio motivated the NuTeV study:

$$R_{PW} = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\bar{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\bar{\nu} A}} = \frac{\left(\frac{1}{6} - \frac{4}{9}\sin^2\theta_W\right) \langle x_A \, u_A^- \rangle + \left(\frac{1}{6} - \frac{2}{9}\sin^2\theta_W\right) \langle x_A \, d_A^- + x_A \, s_A^- \rangle}{\langle x_A \, d_A^- + x_A \, s_A^- \rangle - \frac{1}{3} \langle x_A \, u_A^- \rangle}$$

• $\langle x_A q_A^- \rangle$ fraction of target momentum carried by valence quarks of flavor q

• For an isoscalar target $u_A \simeq d_A$ and if $s_A \ll u_A + d_A$

$$R_{PW} = \frac{1}{2} - \sin^2 \theta_W + \Delta R_{PW}; \quad \Delta R_{PW} = \left(1 - \frac{7}{3}\sin^2 \theta_W\right) \frac{\langle x_A \, u_A^- - x_A \, d_A^- - x_A \, s_A^- \rangle}{\langle x_A \, u_A^- + x_A \, d_A^- \rangle}$$

- ΔR_{PW} well constrained \implies excellent way to measure weak mixing angle
- NuTeV "result" for R_{PW} is smaller than Standard Model value
- Studies suggest that largest contributions to ΔR_{PW} maybe:
 - strange quarks
 - charge symmetry breaking (CSB) $\implies u_p \neq d_n, \ d_p \neq u_n$
 - nuclear effects

• NuTeV target was 690 tons of steel $\stackrel{?}{\Longrightarrow}$ non-trivial nuclear corrections

CSB Correction to NuTeV



- Two sources of charge symmetry breaking (CSB) corrections
 - quark mass differences: $\delta m = m_d m_u \sim 4 \,\mathrm{MeV}$
 - quark charge differences: $e_u^2 \neq e_d^2$ [QED splitting/QED evolution of PDFs]
- CSB correction to Paschos-Wolfenstein ratio:

$$\Delta R_{PW}^{CSB} \simeq \left(1 - \frac{7}{3}\sin^2\theta_W\right) \frac{\langle x\,u_A^- x\,d_A^-\rangle}{\langle x\,u_A^- + x\,d_A^-\rangle} \longrightarrow \frac{1}{2} \left(1 - \frac{7}{3}\sin^2\theta_W\right) \frac{\langle x\,\delta u^- - x\,\delta d^-\rangle}{\langle x\,u_p^- + x\,d_p^-\rangle}$$
$$\delta d^-(x) = d_p^-(x) - u_n^-(x) \qquad \delta u^-(x) = u_p^-(x) - d_n^-(x)$$

Mass differences – what do we expect? Consider deuteron:



• $e_u^2 > e_d^2 \implies u$ -quarks lose momentum faster than d-quarks to γ -field

• Expect CSB corrections reduce NuTeV discrepancy with Standard Model

CSB – experiment & theory





MRST has done two studies of CSB in PDFs (includes all CSB sources)

• $\Delta R_{PW}^{CSB} = -0.002$ or 90% confidence of $-0.007 < \Delta R_{PW}^{CSB} < 0.007$

Theory [Rodionov et al.] and MRST in excellent agreement

• Londergan & Thomas [PRD 2003]: $\langle x \, \delta u^- - x \, \delta d^- \rangle \simeq \frac{\delta m}{M_N}$

• The correction to NuTeV is (explains $\sim 30\%$ of anomaly):

$$\Delta R_{PW}^{\delta m} \equiv \Delta^{\delta m} \sin^2 \theta_W = -0.0020$$

Nambu–Jona-Lasinio modelArgonaContinuum QCD"integrate out gluons" $\frac{1}{m_{cr}^2} \Theta(\Lambda^2 - k^2)$

- this is just a modern interpretation of the Nambu-Jona-Lasinio (NJL) model
- model is a Lagrangian based covariant QFT, exhibits dynamical chiral symmetry breaking & quark confinement; elements can be QCD motivated via the DSEs



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Nucleon quark distributions



• Nucleon = quark+diquark • PDFs given by Feynman diagrams: $\langle \gamma^+ \rangle$



Ovariant, correct support; satisfies sum rules, Soffer bound & positivity

 $\langle q(x) - \bar{q}(x) \rangle = N_q, \ \langle x u(x) + x d(x) + \ldots \rangle = 1, \ |\Delta q(x)|, \ |\Delta_T q(x)| \leqslant q(x)$

• q(x): probability strike quark of favor q with momentum fraction x of target



Nucleon transversity quark distributions





$$g_T = \int dx \left[\Delta_T u(x) - \Delta_T d(x) \right]$$

- Non-relativistically: $\Delta_T q(x) = \Delta q(x) a$ measure of relativistic effects
- Helicity conservation: no mixing bet'n $\Delta_T q \& \Delta_T g$: $J \leq \frac{1}{2} \Rightarrow \Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated

• At model scale we find: $g_T = 1.28$ compare $g_A = 1.267$ (input)



Transverse Momentum Dependent PDFs





So far only considered the simplest spin-averaged TMDs $-q(x, k_T^2)$

In phenomenology common to work with parametrization of the form:

$$q(x,k_T^2) = q(x) \frac{e^{-k_T^2/\langle k_T^2 \rangle_0}}{\pi \langle k_T^2 \rangle_0} \qquad \begin{pmatrix} \langle k_T^2 \rangle^{Q^2 = Q_0^2} = 0.36^2 \operatorname{GeV}^2 \sim M^2 \\ \langle k_T^2 \rangle = 0.56^2 \operatorname{GeV}^2 \text{ [hermes]}, \quad 0.64^2 \operatorname{GeV}^2 \text{ [emc]} \end{cases}$$

Gaussian ansatz fits our results well

• argeement with experiment reasonable as $\left\langle k_T^2
ight
angle$ grows with Q^2

Charge Symmetry Breaking Results



- Our dressed quark mass is $M_0 = 400 \text{ MeV}$ in the charge symmetry limit
 - define: $M_u = M_0 \delta_M$ & $M_d = M_0 + \delta_M$; $\delta_M \mid M_n M_p = 3 \text{ MeV}$

$$M_u = 397 \,\mathrm{MeV} \& M_d = 403 \,\mathrm{MeV} \implies \frac{M_d - M_u}{M_d} \simeq 1.5\%; \quad \frac{m_u}{m_d} = 0.58$$

For the CSB PDFs we find, at $Q^2 = 4 \text{ GeV}^2$:

$$\langle x \, \delta u^- \rangle = -0.0019 \qquad \langle x \, \delta d^- \rangle = 0.0019$$

• therefore a small amount of momentum is shifted from the u to the d quarks



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QED splitting & total CSB correction to NuTeV



▶ QED evolution of PDFs – compare DGLAP evolution in QCD

$$\frac{d}{d\ln Q^2} q^-(x,Q^2) = \frac{\alpha_e}{4\pi} e_q^2 P_{q\gamma}(z) \otimes q^-(x,Q^2)$$

- contributes even if $m_u = m_d$
- $e_u^2 > e_d^2 \implies u$ -quarks lose momentum faster than d-quarks to γ -field
- Model independent however where to start &/or stop Q^2 evolution?
- QED splitting: M. Glück et al., PRL 95, 022002 (2005) assign 100% error

 $\Delta R^{QED} = -0.011 \pm 0.011$

- QED evolution started at $Q^2 \sim m_q^2$
- Total CSB correction to NuTeV $\sin^2 \theta_W$ measurement

 $\Delta R^{CSB} = \Delta R^{\delta m} + \Delta R^{QED} \equiv \Delta^{CSB} \sin^2 \theta_W = -0.0024 \pm 0.0012$

Measuring CSB – Parity Violating DIS



0

- Leading contribution to PV-DIS
 - γZ interference

$$\sum_{X} \left| \begin{array}{c} \ell \\ \ell \end{array} \right\rangle \xrightarrow{\gamma} \left| \begin{array}{c} \eta \\ \eta \end{array} \right\rangle$$

Construct asymmetry:

$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \propto a_2(x) = -2g_A^e \frac{F_2^{\gamma Z}}{F_2^{\gamma}} \stackrel{N \sim Z}{=} \frac{9}{5} - 4\sin^2\theta_W - \frac{12}{25} \frac{u^+(x) - d^+(x)}{u^+(x) + d^+(x)}$$



• For CSB, correction term has the form

$$\Delta a_2 = -\frac{6}{25} \frac{\delta u^+(x) - \delta d^+(x)}{\delta u^+(x) + \delta d^+(x)}$$

- Our result too small to be measured in proposed JLab 12 GeV experiment
 - however, still need to include QED splitting CSB increases

Conclusion



- QCD and therefore Hadron Physics is unique:
 - must confront a fundamental theory in which the elementary degrees-of-freedom are intangible (confined) and only composites (hadrons) reach detectors
- QCD will only be solved by deploying a diverse array of experimental and theoretical methods:
 - must define and solve the problems of confinement and its relationship with DCSB
- These are two of the most important challenges in fundamental Science
- At experimental energies pion PDA & PDF are a long way from asymptotia
 - DCSB ⇔ valence quarks remains importance to well beyond LHC energies
- Presented state-of-the-art results for the x dependence of CSB effects in PDFs
 - CSB effects will be measured in PV-DIS experiments at JLab but experimental errors may still be too large see CSB effects
- Highlight the importance of understanding the EMC effect as a critical step towards a QCD based description of nuclei



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