Overview of γZ Box Calculations

Ben Rislow

25 September, 2013

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

COLLEGE OF WILLIAM & MARY 1693

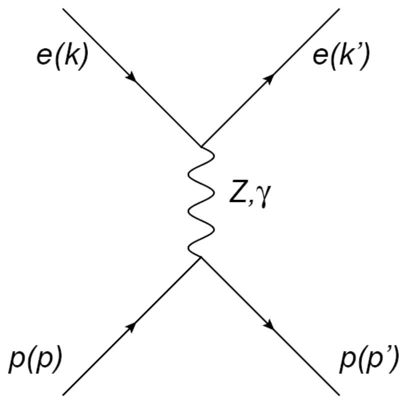
Context

Weak Charge of the Proton

- Q_w^P extracted from parity-violating, *ep* scattering:

$$A_{PV} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

- Lowest order diagrams and result:



$$A_{PV}\big|_{LO} = \frac{\mathcal{M}_Z|_{\lambda=-1/2} - \mathcal{M}_Z|_{\lambda=1/2}}{\mathcal{M}_\gamma} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} Q_W^{p,LO}.$$

- Lowest order definition of Q_w^P :

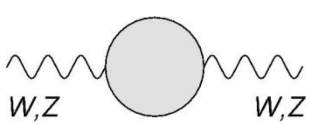
$$Q_W^{p,LO} = -4 g_A^e g_V^p = 1 - 4\sin^2\theta_W$$

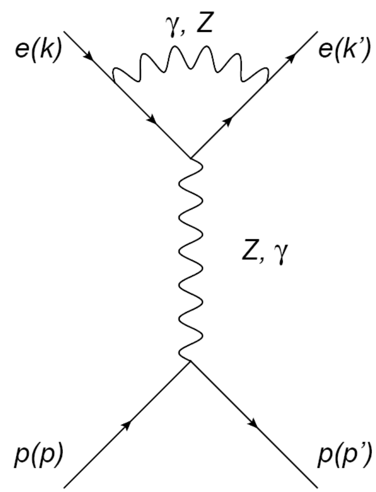
- One loop result:

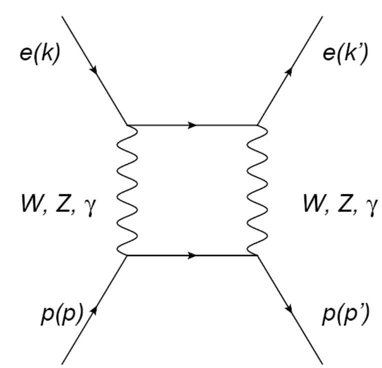
$$A_{PV}\Big|_{1\text{ Loop}} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W^p + B_4 Q^2 + \ldots\right)$$

- One loop weak charge (defined at Q^2=0, E=0):

$$Q_W^p = [\rho_{NC} + \Delta_e][1 - 4\sin^2\theta_W(0) + \Delta_e'] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

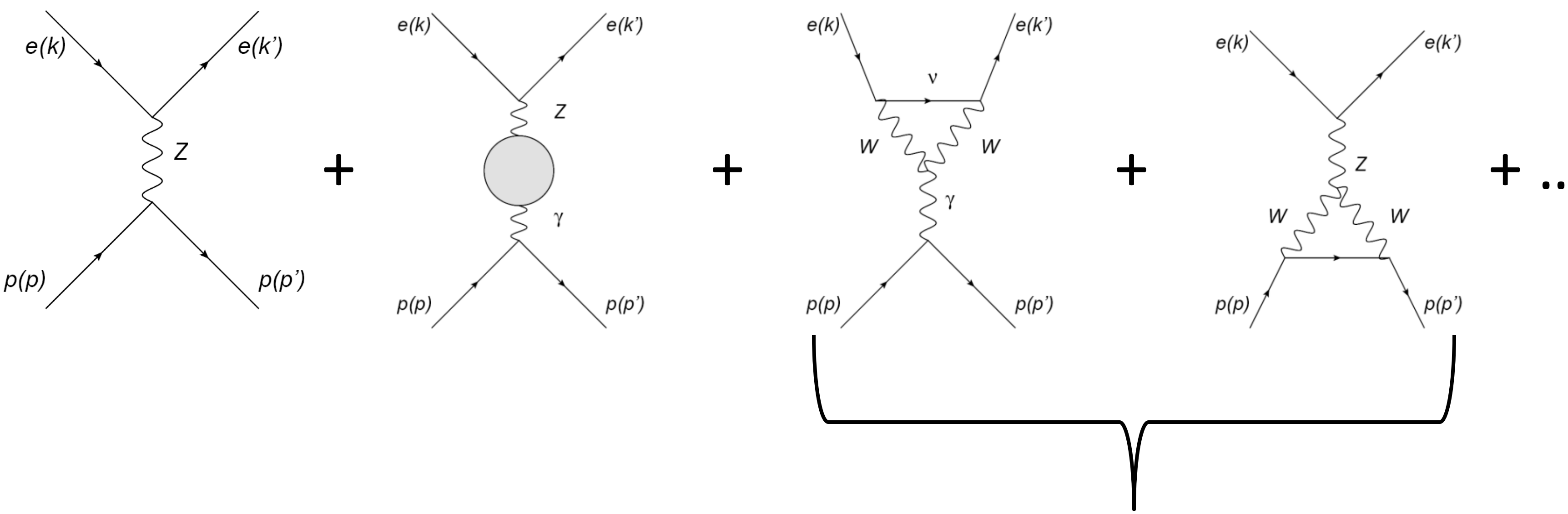






Running Weinberg Angle (Czarnecki and Marciano)

$$\sin^2\theta_W \to \sin^2\theta_W(Q^2) = \kappa(Q^2)\sin^2\theta_W(M_Z^2)$$



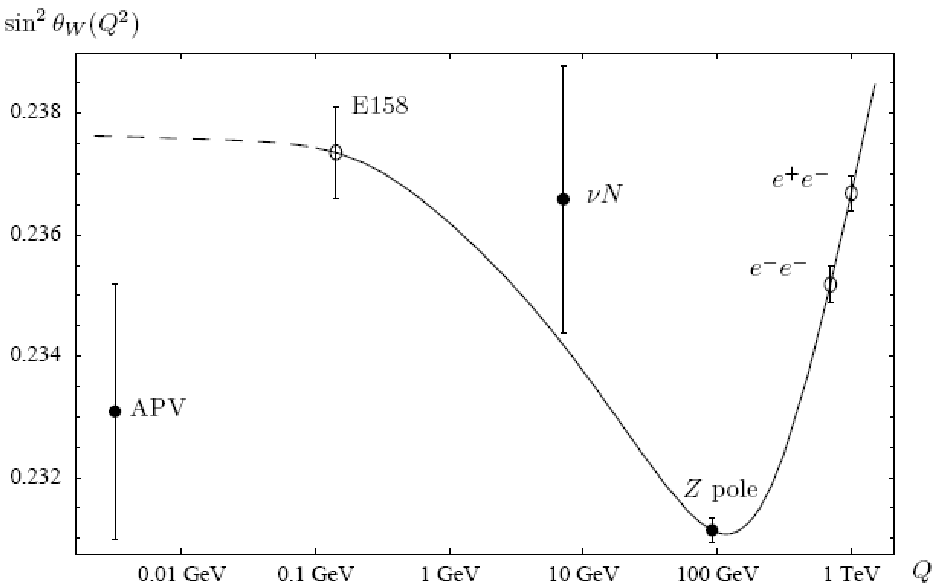
Only "Pinched" Part

Degrassi and Sirlin, PRD 46, 3104 (1992)

Running Weinberg Angle

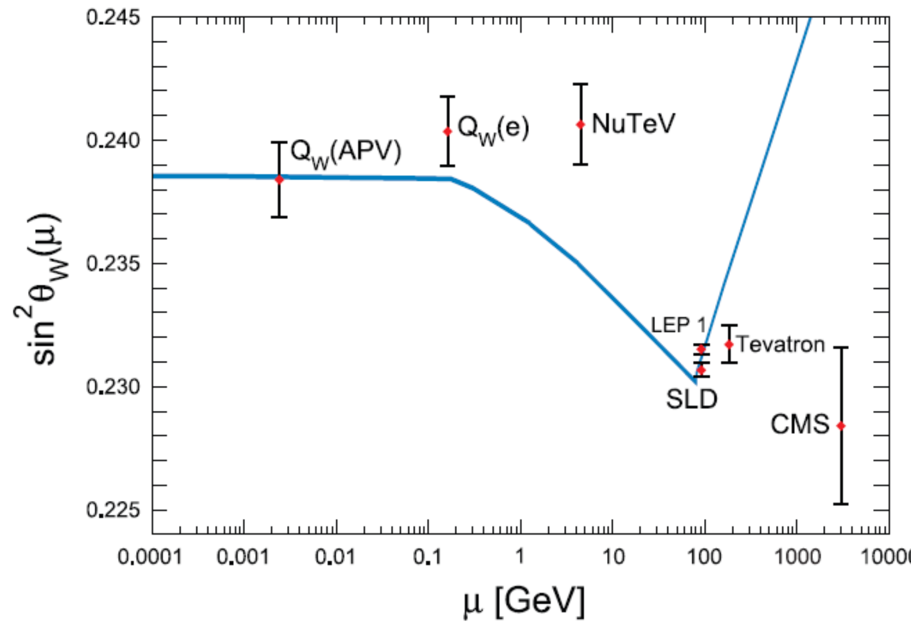
Czarnecki and Marciano Running

Int. J. Mod. Phys. A15, 2365 (2000)



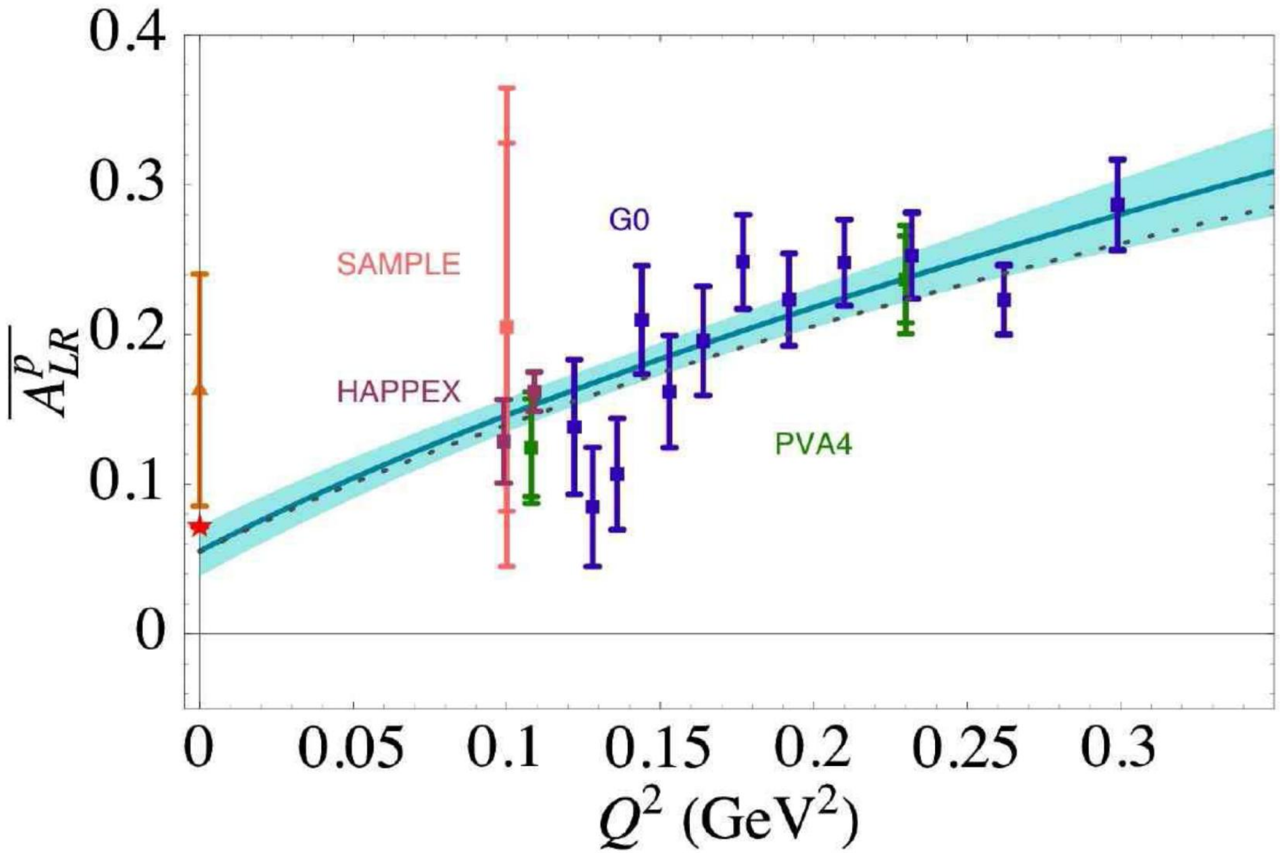
Erler and Langacker

Particle Data Group (2012)



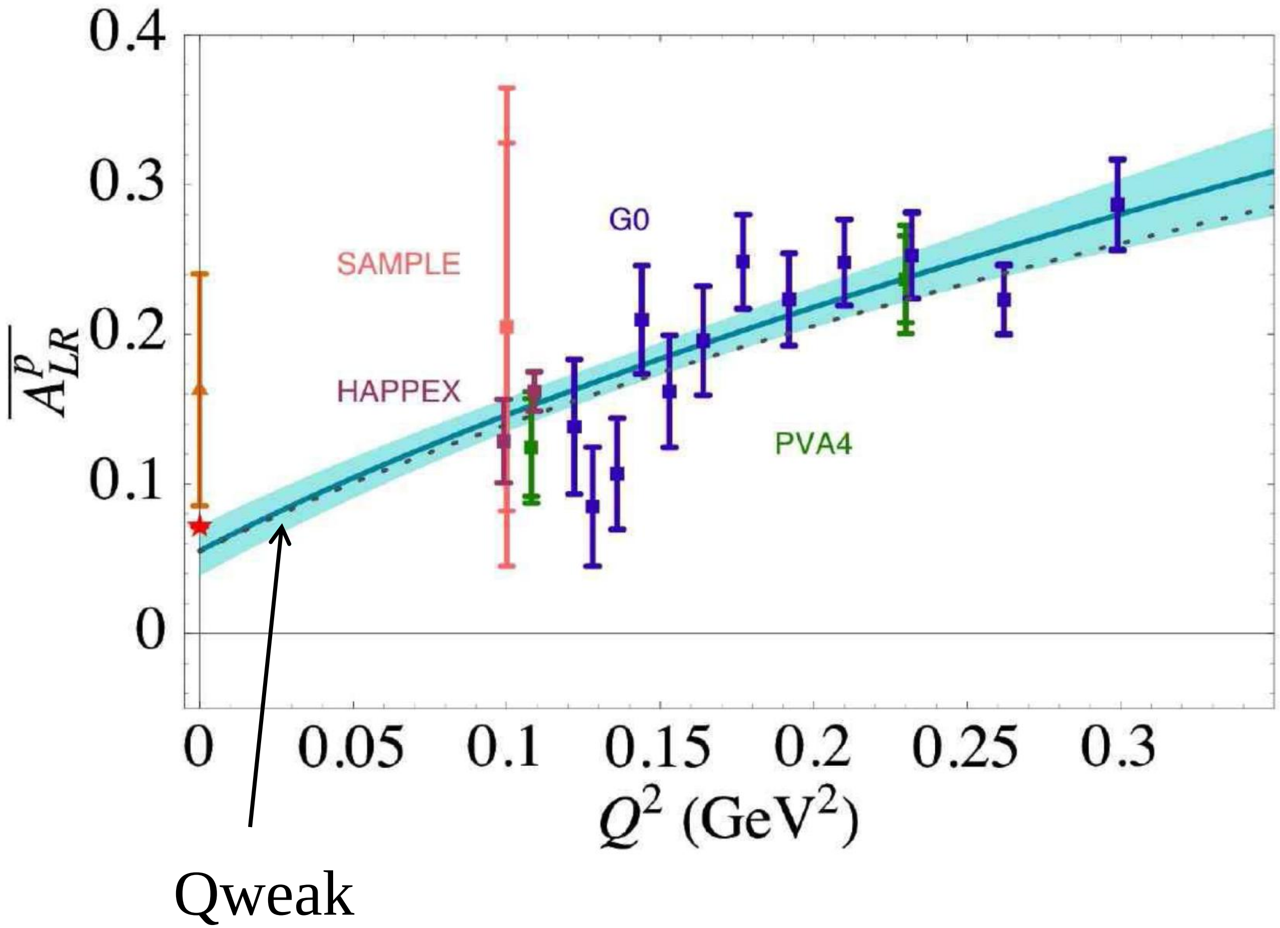
Weak Charge Extrapolation (PRL 99,122003 (2007))

$$A_{PV}\Big|_{1\ \mathrm{Loop}} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}(Q_W^p + B_4 Q^2 + \ldots)$$



Weak Charge Extrapolation (PRL 99,122003 (2007))

$$A_{PV}\Big|_{1\ \mathrm{Loop}} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}(Q_W^p + B_4 Q^2 + \ldots)$$



- All terms in Q_w^P are straightforward except γZ Box

???

$$Q_W^p = [\rho_{NC} + \Delta_e][1 - 4\sin^2\theta_W(0) + \Delta_e'] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

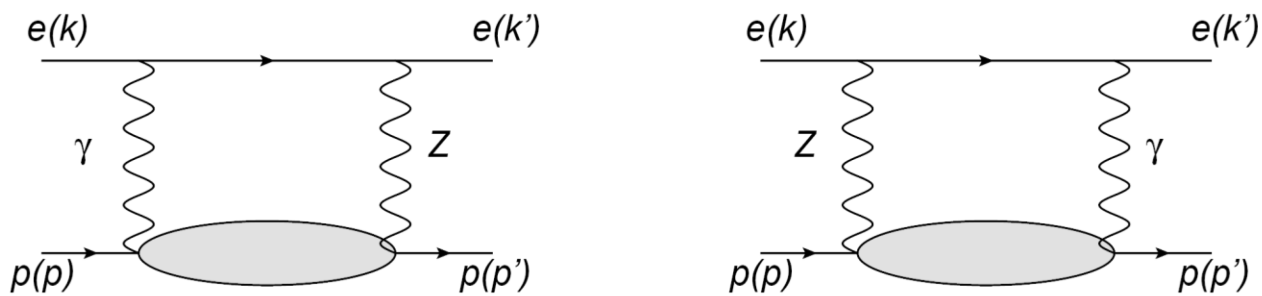
Box Calculation

γZ Box

- Definition:

$$\Box_{\gamma Z} = \frac{\mathcal{M}_{\gamma Z}|_{\lambda=-1/2} - \mathcal{M}_{\gamma Z}|_{\lambda=1/2}}{\mathcal{M}_{Z}|_{\lambda=-1/2} - \mathcal{M}_{Z}|_{\lambda=1/2}} Q_W^{p,LO}$$

- Unlike WW and ZZ boxes, the γZ box cannot be analyzed using pQCD.

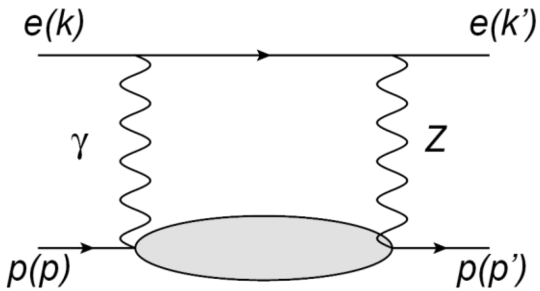


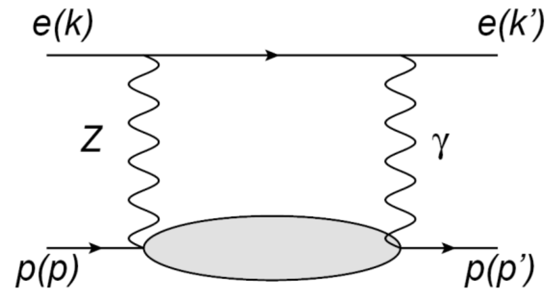
$$< 0|A^\mu(x)A^\nu(y)|0 > \propto \frac{1}{q^2}$$

$$< 0|Z^\mu(x)Z^\nu(y)|0 > \propto \frac{1}{q^2 - M_Z^2}$$

γZ Box

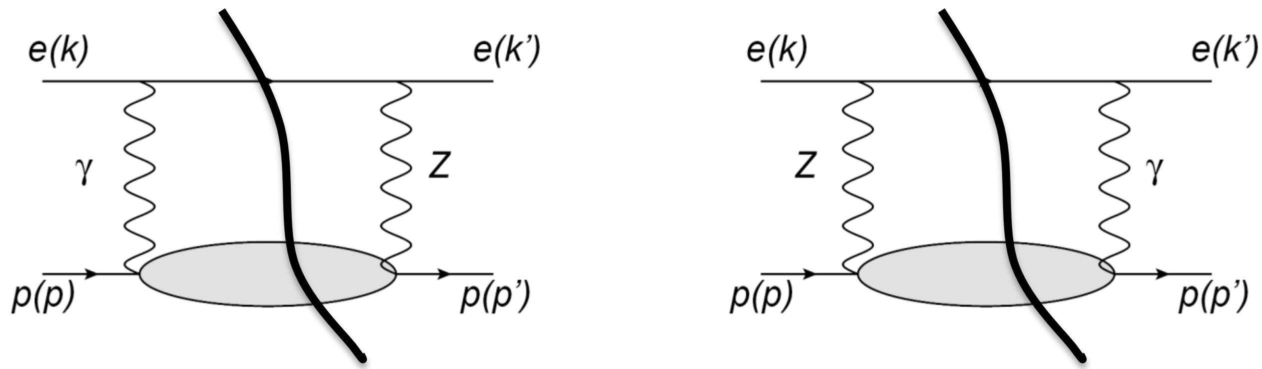
- Gorchtein and Horowitz (PRL 102, 091806 (2009)) had insight to calculate the amplitude dispersively





γZ Box

- Gorchtein and Horowitz (PRL 102, 091806 (2009)) had insight to calculate the amplitude dispersively



- Optical Theorem

$$\mathrm{Im}\,\mathcal{M}_{aa} = \frac{1}{2}\sum_b (2\pi)^4 \delta^{(4)}(p_a - p_b)\mathcal{M}_{ab}\mathcal{M}_{ba}$$

γZ Box Equations

- Imaginary Part:

$$
\begin{aligned}
\operatorname{Im}\Box_{\gamma Z}(E_{Lab}) = {} & \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \left\{ \frac{F_1^{\gamma Z}(x,Q^2) + AF_2^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right. \\
& \left. + \frac{g_V^e}{g_A^e}\frac{BF_3^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right\} \\
= {} & \operatorname{Im}\Box_{\gamma Z}^{V}(E_{Lab}) + \operatorname{Im}\Box_{\gamma Z}^{A}(E_{Lab}).
\end{aligned}
$$

- Dispersion Relations:

$$
\operatorname{Re}\Box_{\gamma Z}^{V}(E_{Lab}) = \frac{2E_{Lab}}{\pi}\int_{\nu_\pi}^{\infty} \frac{dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \operatorname{Im}\Box_{\gamma Z}^{V}(E'_{Lab})
$$

$$
\operatorname{Re}\Box_{\gamma Z}^{A}(E_{Lab}) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} \frac{E'_{Lab}dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \operatorname{Im}\Box_{\gamma Z}^{A}(E'_{Lab})
$$

γZ Box Equations

- Imaginary Part:

$$\mathrm{Im}\,\Box_{\gamma Z}(E_{Lab}) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \left\{ \frac{F_1^{\gamma Z}(x,Q^2) + AF_2^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} + \frac{g_V^e}{g_A^e}\frac{BF_3^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right\}$$

$$= \mathrm{Im}\,\Box^V_{\gamma Z}(E_{Lab}) + \mathrm{Im}\,\Box^A_{\gamma Z}(E_{Lab}).$$

- Dispersion Relations:

Labels proton current.

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E_{Lab}) = \frac{2E_{Lab}}{\pi}\int_{\nu_\pi}^{\infty} \frac{dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^V_{\gamma Z}(E'_{Lab})$$

$$\mathrm{Re}\,\Box^A_{\gamma Z}(E_{Lab}) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} \frac{E'_{Lab}dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^A_{\gamma Z}(E'_{Lab})$$

γZ Box Equations

- Imaginary Part:

$$\mathrm{Im}\,\Box_{\gamma Z}(E_{Lab}) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \left\{ \frac{F_1^{\gamma Z}(x,Q^2) + AF_2^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right.$$

$$\left. + \frac{g_V^e}{g_A^e}\frac{BF_3^{\gamma Z}(x,Q^2)}{\frac{Q^2}{M_Z^2}+1} \right\}$$

$$= \mathrm{Im}\,\Box^V_{\gamma Z}(E_{Lab}) + \mathrm{Im}\,\Box^A_{\gamma Z}(E_{Lab}).$$

Structure Functions must be modeled.

- Dispersion Relations:

Labels proton current.

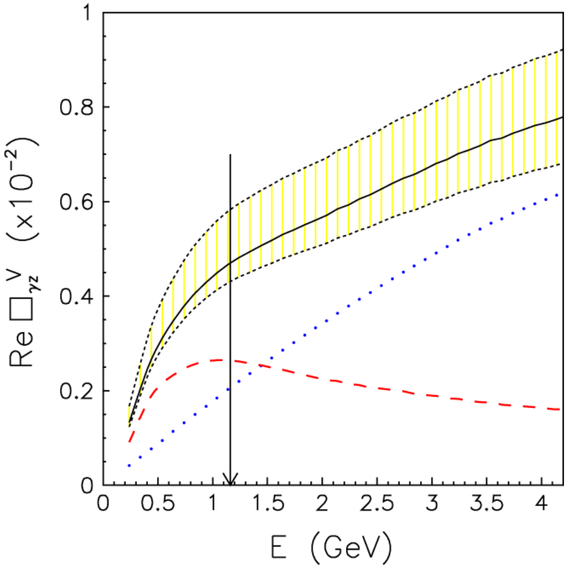
$$\mathrm{Re}\,\Box^V_{\gamma Z}(E_{Lab}) = \frac{2E_{Lab}}{\pi}\int_{\nu_\pi}^{\infty} \frac{dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^V_{\gamma Z}(E'_{Lab})$$

$$\mathrm{Re}\,\Box^A_{\gamma Z}(E_{Lab}) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} \frac{E'_{Lab}dE'_{Lab}}{E'^2_{Lab} - E^2_{Lab}} \mathrm{Im}\,\Box^A_{\gamma Z}(E'_{Lab})$$

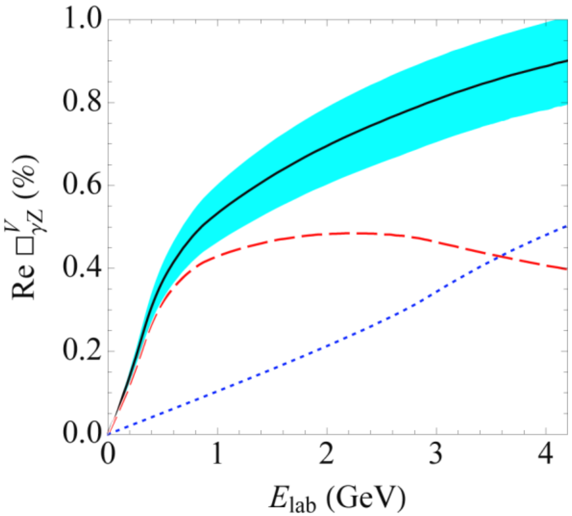
Vector Box Analyses

The Vector Box Plots

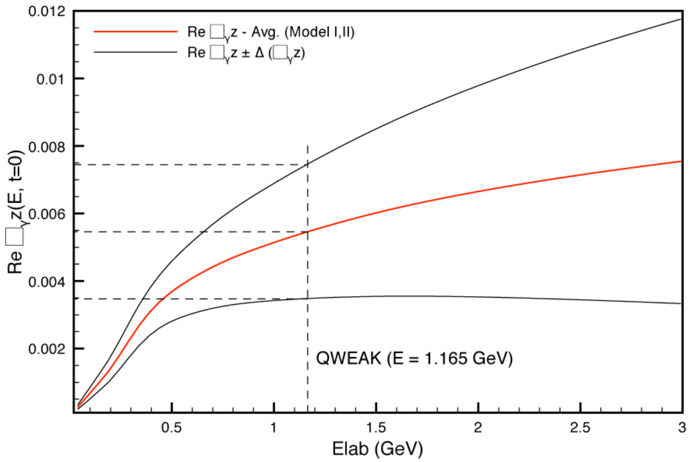
Sibirtsev *et al.*
PRD 82, 013011 (2010)



Carlson and Rislow
PRD 83, 113007 (2011)



Gorchtein *et al.*
PRC 84, 015502 (2011)

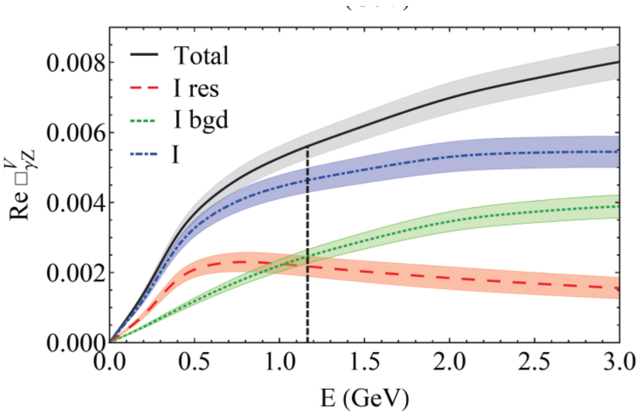


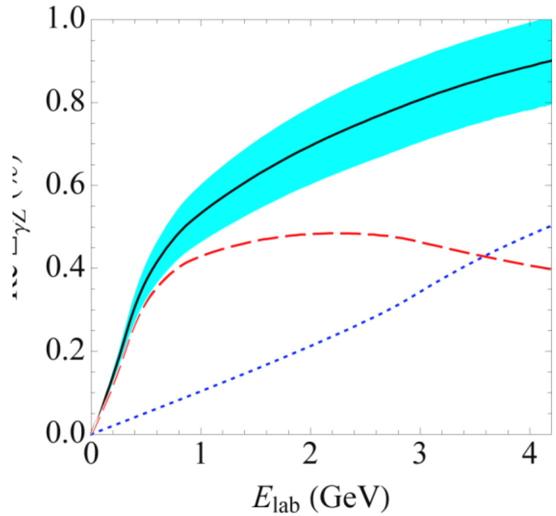
$$\mathrm{Re}\Box^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

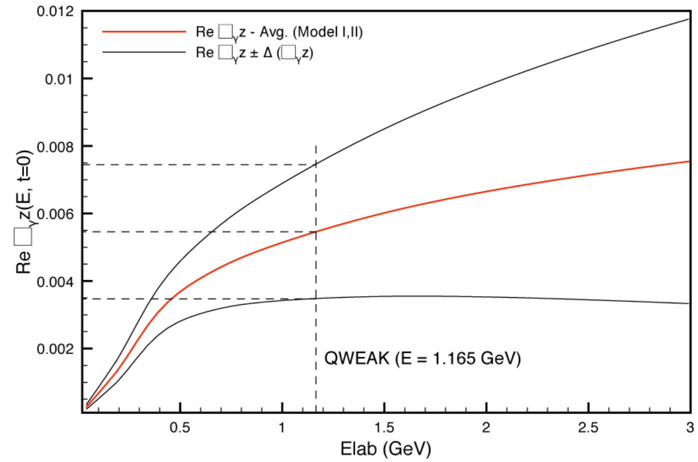
$$(4.7^{+1.1}_{-0.4}) \times 10^{-3} \qquad (5.7 \pm 0.9) \times 10^{-3} \qquad (5.4 \pm 2.0) \times 10^{-3}$$

The Vector Box Plots

Hall *et al.*	Carlson and Rislow	Gorchtein *et al.*
PRD 88, 013011 (2013)	PRD 83, 113007 (2011)	PRC 84, 015502 (2011)







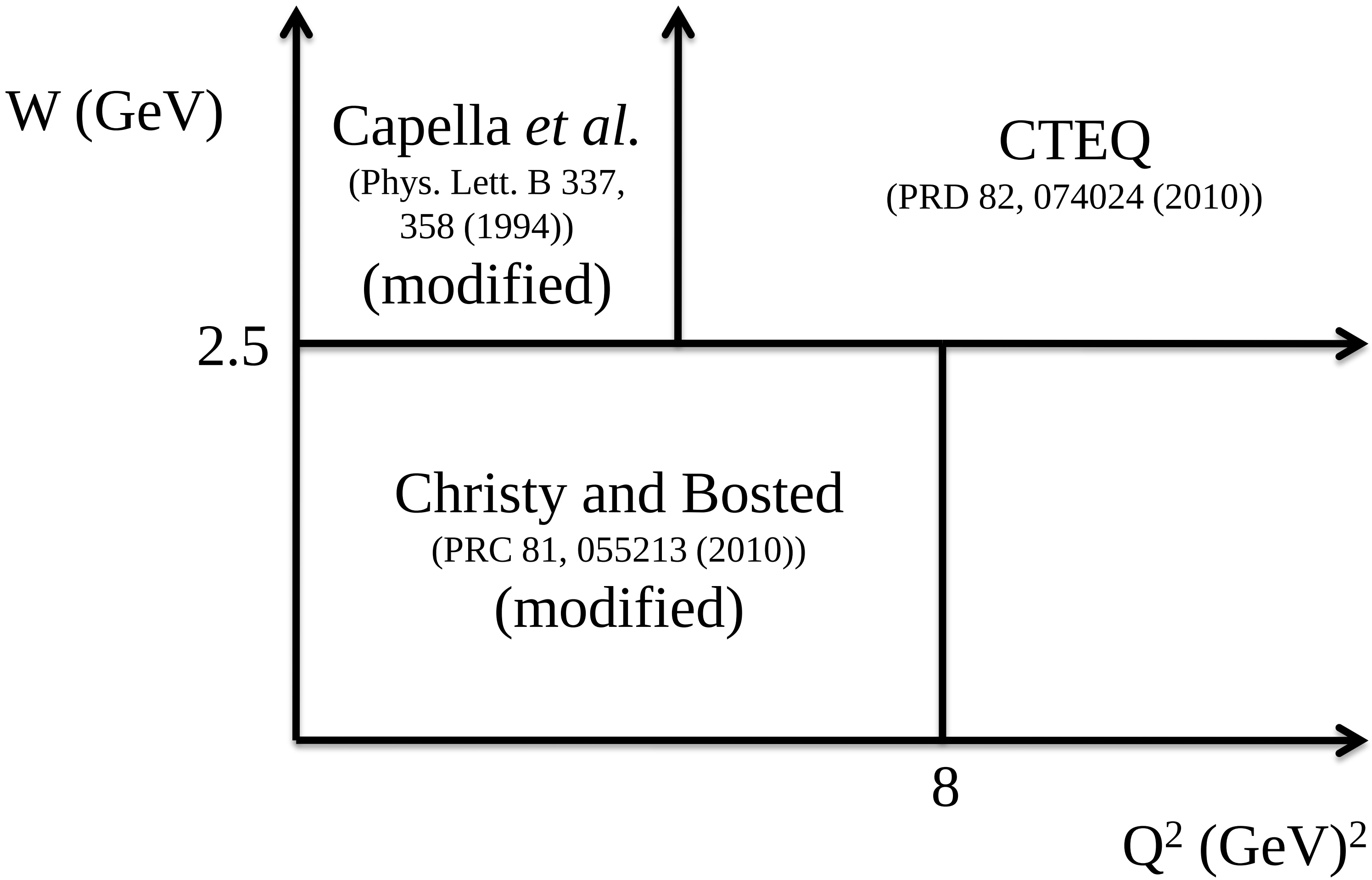
$$\mathrm{Re}\,\Box^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$$(5.6 \pm 0.36) \times 10^{-3} \quad (5.7 \pm 0.9) \times 10^{-3} \quad (5.4 \pm 2.0) \times 10^{-3}$$

- Differences come from the treatment of the structure functions.

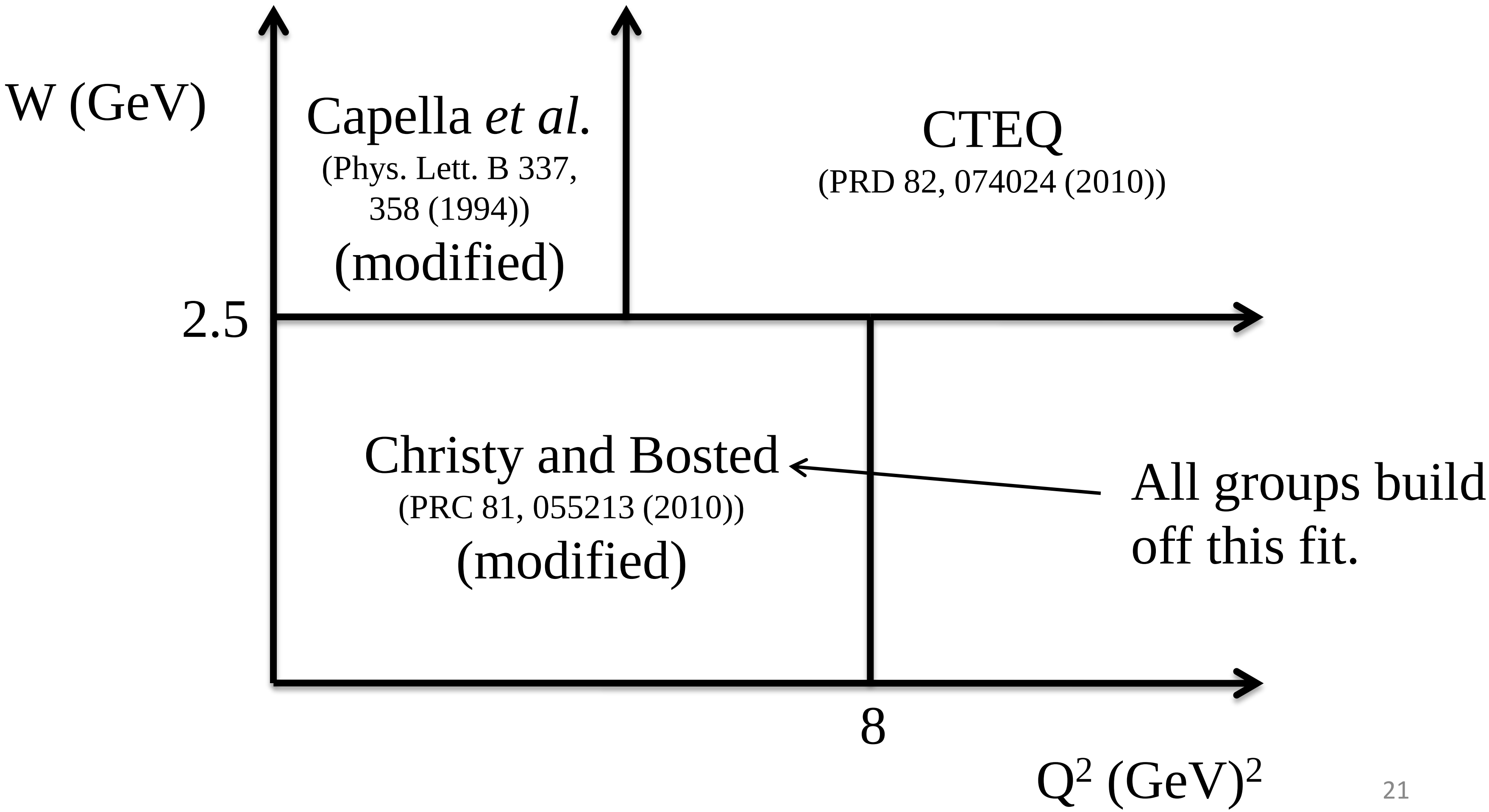
Carlson and Rislow Analysis

- We split the energy regions and modified the structure functions.



Carlson and Rislow Analysis

- We split the energy regions and modified the structure functions.



Evaluation of Scaling Region

- Calculated directly using PDFs

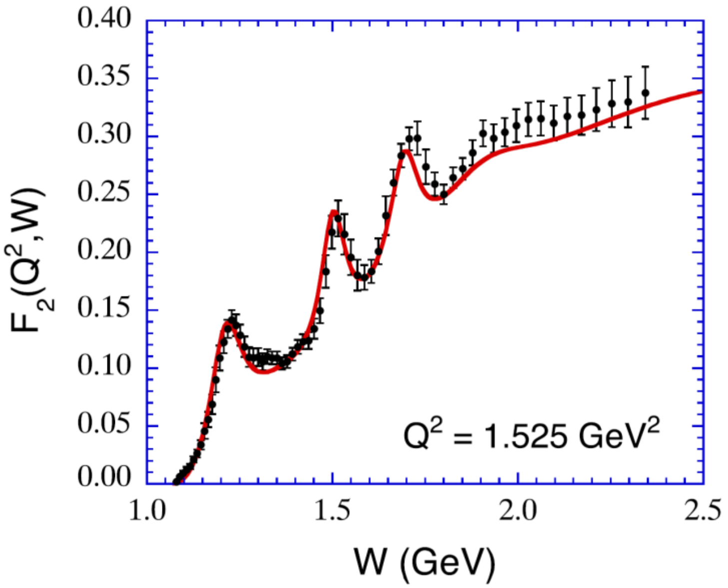
$$F_2^{\gamma Z}(x,Q^2) = 2xF_1^{\gamma Z}(x,Q^2) = x\sum_{q,\bar{q}} 2e_q g_V^q (q(x,Q^2) + \bar{q}(x,Q^2))$$

- We use CTEQ

- Alternative
  - Hall *et al.* use ABM11 (PRD 86, 054009 (2012))

Evaluation of Resonance Region

- All later calculations modify Christy Bosted electromagnetic fits.
- CB fit has 7 resonances and a smooth background



- Resonances modified by corrective ratio:

$$F_1^{\gamma Z} = \sum_{res} C_{res} \times F_1^{\gamma\gamma}\big|_{res} \quad ; \quad C_{res} = \left.\frac{F_1^{\gamma Z}}{F_1^{\gamma\gamma}}\right|_{res}$$

Rislow and Carlson Vector C_{res}

- Definition of structure functions:

$$F_1^{\gamma\gamma(\gamma Z)}\Big|_{N\to res} = \varepsilon_+^{\mu *}\varepsilon_+^{\nu} W_{\mu\nu}^{\gamma\gamma(\gamma Z)}$$
$$= (2)\sum_{\lambda}\int \mathrm{d}^4 \mathrm{z} e^{iqz}\langle N,s\big|\varepsilon_+^{*}\cdot J^{\gamma(Z,V)\dagger}(z)\big|res,\lambda\rangle$$
$$\times\langle res,\lambda\big|\varepsilon_+\cdot J^{\gamma}(0)\big|N,s\rangle$$

- C_{res} in terms of helicity amplitudes:

$$C_{res} = \frac{2\sum_\lambda A_\lambda^\gamma A_\lambda^Z}{\sum_\lambda (A_\lambda^\gamma)^2}$$

Rislow and Carlson Vector C_{res}

- We constructed helicity amplitudes using SU(6) wave functions:

$$\left\langle res, \lambda \middle| \epsilon_+ \cdot J^{\gamma(Z,V)} \middle| N, s \right\rangle = 3 \times e_q^{(3)} \left(g_V^{q(3)} \right)$$

$$\times \left\langle \psi_{res} \phi_{res} \chi_\lambda \middle| \underbrace{\bar{u}(k', \lambda') \epsilon_+ \cdot \gamma u(k, s')}_{= AL_+ + BS_+} \middle| \psi_N \phi_N \chi_s \right\rangle$$

- Phenomenological constraints used to fit A and B:

$$\frac{|A^\gamma_{1/2}|^2 - |A^\gamma_{3/2}|^2}{|A^\gamma_{1/2}|^2 + |A^\gamma_{3/2}|^2} = \begin{cases} -1 & \text{for } Q^2 = 0 \\ +1 & \text{for high } Q^2 \end{cases}$$

Rislow and Carlson Vector C_{res}

- We constructed helicity amplitudes using SU(6) wave functions:

Only operate on 3rd quark

$$\langle res, \lambda | \epsilon_+ \cdot J^{\gamma(Z,V)} | N, s \rangle = 3 \times e_q^{(3)} \left(g_V^{q(3)} \right)$$

$$\times \langle \psi_{res} \phi_{res} \chi_\lambda | \underbrace{\bar{u}(k', \lambda') \epsilon_+ \cdot \gamma u(k, s')} | \psi_N \phi_N \chi_s \rangle$$

$$= AL_+ + BS_+$$

- Phenomenological constraints used to fit A and B:

$$\frac{|A^\gamma_{1/2}|^2 - |A^\gamma_{3/2}|^2}{|A^\gamma_{1/2}|^2 + |A^\gamma_{3/2}|^2} = \begin{cases} -1 \text{ for } Q^2 = 0 \\ +1 \text{ for high } Q^2 \end{cases}$$

C_{res} for $D_{13}(1520)$

- SU(6) wave function for $D_{13}(1520)$:

$$|2^8, 56\rangle = \frac{1}{\sqrt{2}} \underbrace{\psi}_{\text{Spatial}}{}^{S}_{L=0, L_Z=0} \left(\underbrace{\phi}_{\text{Flavor}}{}^{M,S} \underbrace{\chi}_{\text{Spin}}{}^{M,S}_{S_Z=\pm 1/2} + \phi^{M,A} \chi^{M,A}_{S_Z=\pm 1/2} \right)$$

C_{res} for $D_{13}(1520)$

- SU(6) wave function for $D_{13}(1520)$:

$$|2^8, 56\rangle = \frac{1}{\sqrt{2}}\psi^{S}_{L=0,L_Z=0}\left(\phi^{M,S}\chi^{M,S}_{S_Z=\pm 1/2} + \phi^{M,A}\chi^{M,A}_{S_Z=\pm 1/2}\right)$$

Spatial Flavor Spin

- Helicity Amplitudes:

$$\begin{aligned} A^{\gamma(Z)}_{\lambda=1/2} &= 3 \times e_q^{(3)}(g_V^{q(3)})\langle\psi_{\mathrm{res}}\phi_{\mathrm{res}}\chi_{+1/2} \\ &\quad \times |[AL_+ + BS_+]|\psi_N\phi_N\chi_s\rangle \\ &= \frac{1}{\sqrt{6}}\Big(-A_{10}[e_u(g_V^u) - e_d(g_V^d)] \\ &\quad - \sqrt{2}B_{10}\Big[\frac{5}{3}e_u(g_V^u) + \frac{1}{3}e_d g_V^d)\Big]\Big) \end{aligned}$$

$$\begin{aligned} A^{\gamma(Z)}_{\lambda=3/2} &= 3 \times e_q^{(3)}(g_V^{q(3)})\langle\psi_{\mathrm{res}}\phi_{\mathrm{res}}\chi_{+3/2} \\ &\quad \times |[AL_+ + BS_+]|\psi_N\phi_N\chi_s\rangle \\ &= -\frac{1}{\sqrt{2}}A_{10}[e_u(g_V^u) - e_d(g_V^d)]. \end{aligned}$$

C_{res} for $D_{13}(1520)$

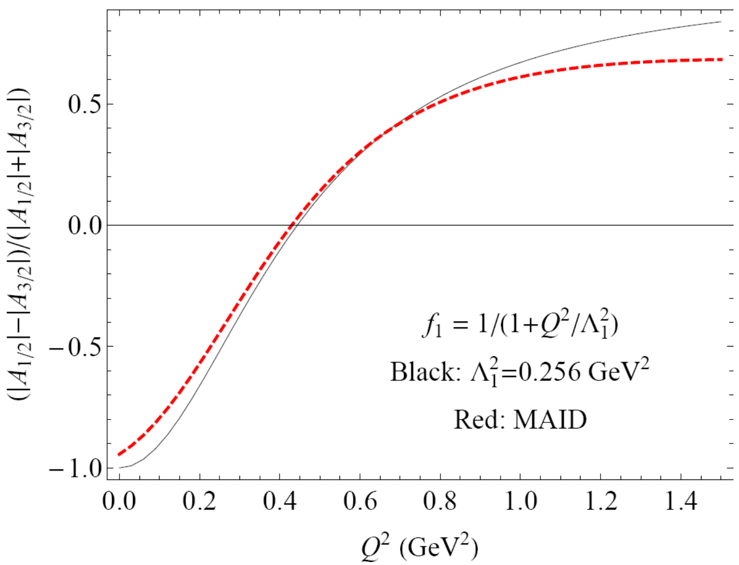
- A and B relations for $D_{13}(1520)$:

$$A_{10}(Q^2=0) = -\sqrt{2}B_{10}(Q^2=0)$$

$$\Downarrow$$

$$\frac{A_{10}(Q^2)}{B_{10}(Q^2)} = -\sqrt{2}f_1(Q^2) = -\sqrt{2}\frac{1}{1+Q^2/\Lambda_1^2}$$

- Λ found by comparing amplitudes to MAID:



Rislow and Carlson Vector C_{res}

resonance	proton electroproduction amplitudes	C^p_{res}	C^d_{res}
$P_{33}(1232)$	$A^\gamma_{1/2} \propto (e_u - e_d)$	$1 + Q_W^{p,LO}$	$1 + Q_W^{p,LO}$
$S_{11}(1535)$	$A^\gamma_{1/2} = \frac{1}{\sqrt{6}}\left(\sqrt{2}A_{10}(e_u - e_d) - B_{10}\left(\frac{5}{3}e_u + \frac{1}{3}e_d\right)\right)$	$\frac{1/3+2f_1}{1+2f_1} + Q_W^{p,LO}$	$2\frac{(1+2f_1)(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2} + Q_W^{p,LO}$
$D_{13}(1520)$	$A^\gamma_{1/2} = \frac{1}{\sqrt{6}}\left(A_{10}(e_u - e_d) + \sqrt{2}B_{10}\left(\frac{5}{3}e_u + \frac{1}{3}e_d\right)\right)$ $A^\gamma_{3/2} = \frac{1}{\sqrt{2}}A_{10}(e_u - e_d)$	$\frac{(1-f_1)(1/3-f_1)+3f_1^2}{(1-f_1)^2+3f_1^2} + Q_W^{p,LO}$	$\frac{2(1-f_1)(1/3-f_1)+6f_1^2}{(1-f_1)^2+(1/3-f_1)^2+6f_1^2} + Q_W^{p,LO}$
$F_{15}(1680)$	$A^\gamma_{1/2} = \sqrt{\frac{2}{5}}A_{20}(2e_u + e_d) + \sqrt{\frac{3}{5}}B_{20}\left(\frac{4}{3}e_u - \frac{1}{3}e_d\right)$ $A^\gamma_{3/2} = \frac{2}{\sqrt{5}}A_{20}(2e_u + e_d)$	$\frac{2/3(1-f_2)}{(1-f_2)^2+2f_2^2} + Q_W^{p,LO}$	$4\frac{1-f_2}{3(1-f_2)^2+6f_2^2+4/3} + Q_W^{p,LO}$
$S_{11}(1650)$	$A^\gamma_{1/2} = -\sqrt{\frac{2}{27}}B_{10}(e_u + 2e_d)$	$\frac{1/3+2f_1}{1+2f_1} + Q_W^{p,LO}$	$2\frac{(1+2f_1)(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2} + Q_W^{p,LO}$
$P_{11}(1440)$	$A^\gamma_{1/2} = B_{00}\left(\frac{4}{3}e_u - \frac{1}{3}e_d\right)$	$2/3 + Q_W^{p,LO}$	$12/13 + Q_W^{p,LO}$
$F_{37}(1950)$	$A^\gamma_{1/2} \propto (e_u - e_d)$	$1 + Q_W^{p,LO}$	$1 + Q_W^{p,LO}$
Background		$\frac{5}{6} + Q_W^{p,LO}$	$\frac{9}{10} + Q_W^{p,LO}$

Rislow and Carlson Background Correction

- In limit where all light quarks (u, d, s) are equally likely:

$$\frac{F_2^{\gamma Z}}{F_2^{\gamma\gamma}} = \frac{\sum_{q=u,d,s} 2e_q g_V^q x f(x)}{\sum_{q=u,d,s} (e_q)^2 x f(x)} = 1 + Q_W^{p,LO}$$

- In valence quark limit (d and 2 u's):

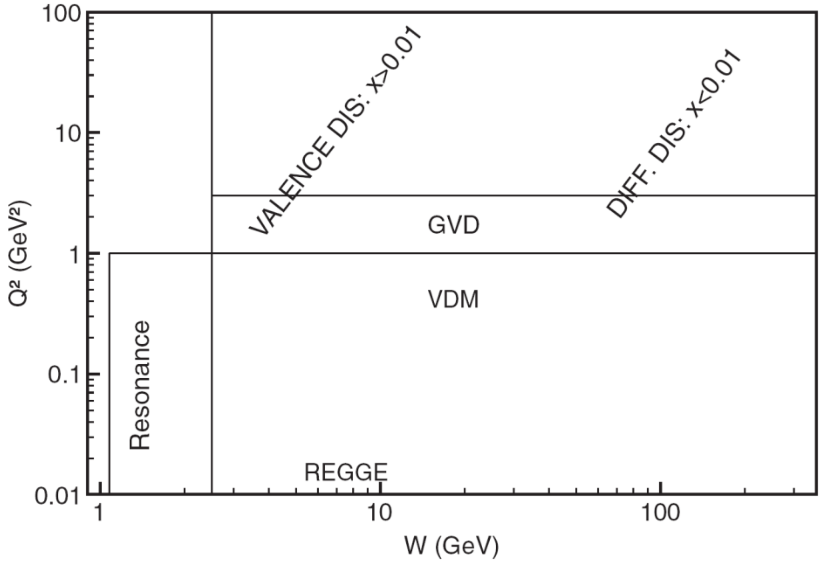
$$\frac{F_2^{\gamma Z}}{F_2^{\gamma\gamma}} = \frac{\sum_{q=d,u,u} 2e_q g_V^q x f(x)}{\sum_{q=d,u,u} (e_q)^2 x f(x)} = \frac{2}{3} + Q_W^{p,LO}$$

- We took their average as the correction:

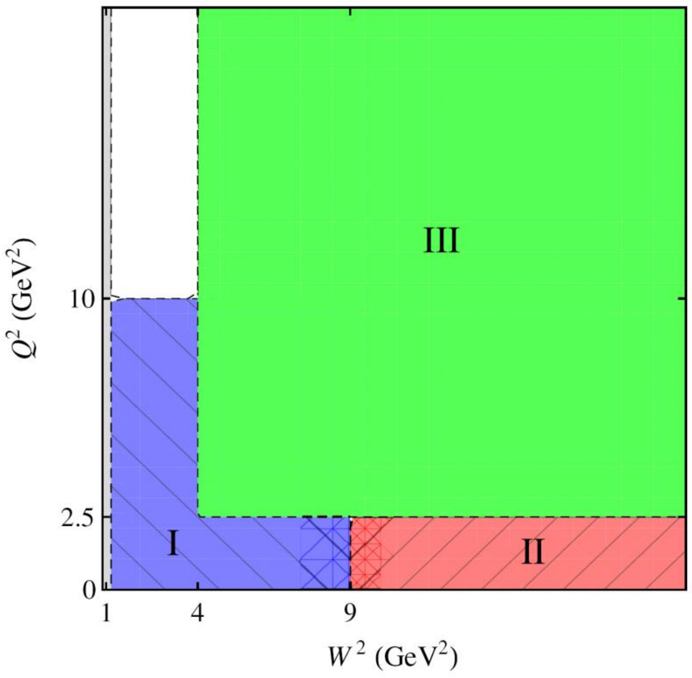
$$Avg = \frac{5}{6} + Q_W^{p,LO}$$

Alternative Analyses

Gorchtein *et al.*
PRC 84, 015502 (2011)

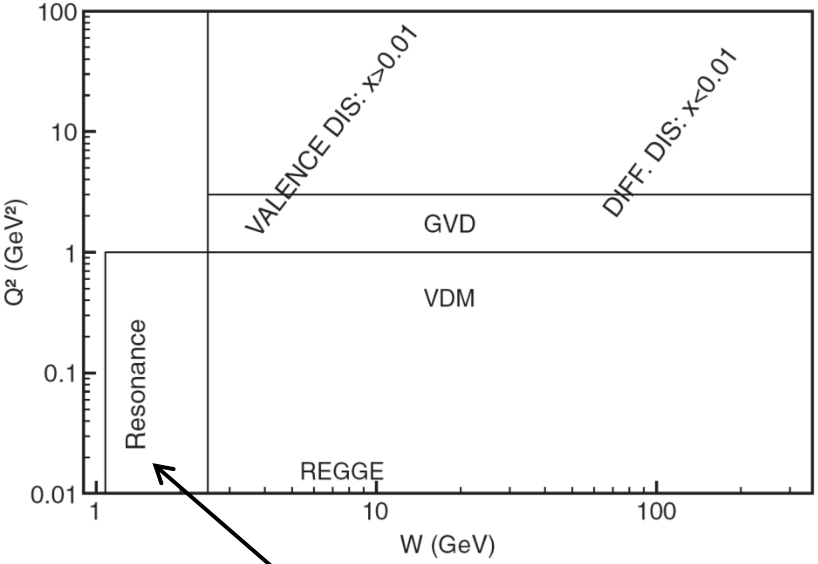


Hall *et al.*
PRD 88, 013011 (2013)

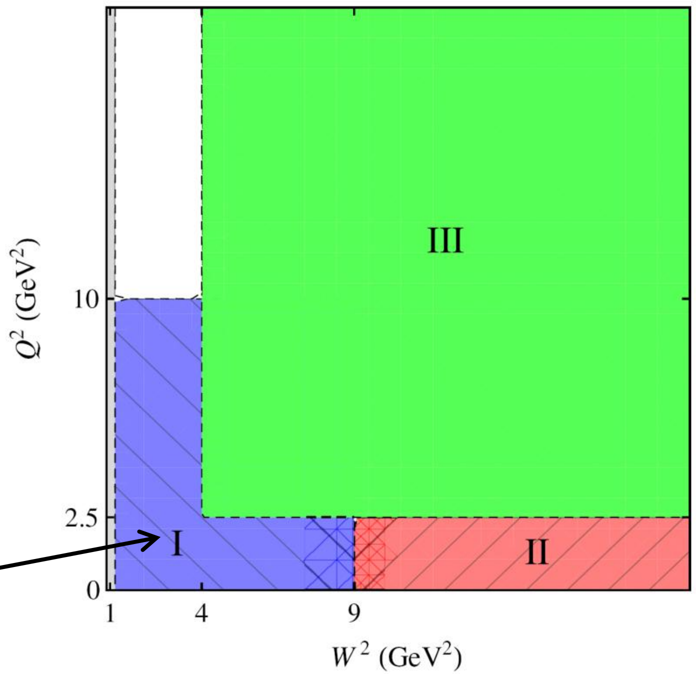


Alternative Analyses

Gorchtein *et al.*
PRC 84, 015502 (2011)



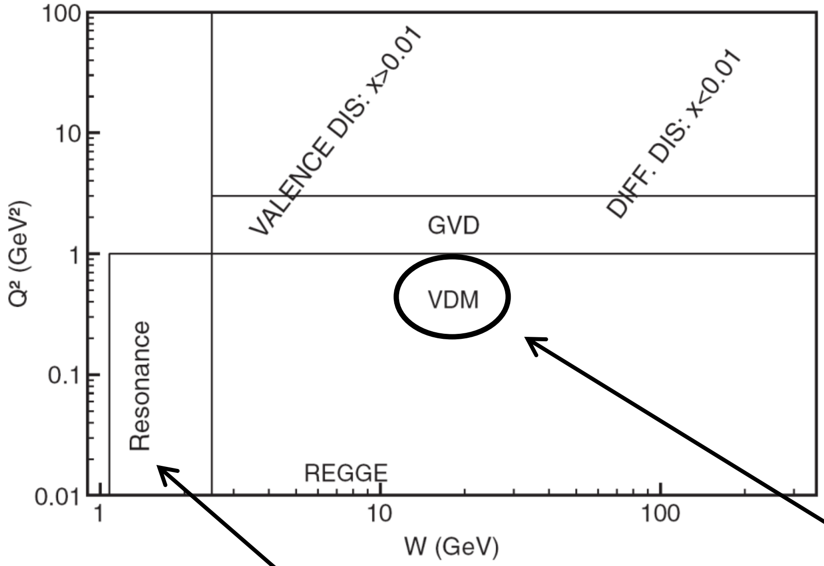
Hall *et al.*
PRD 88, 013011 (2013)



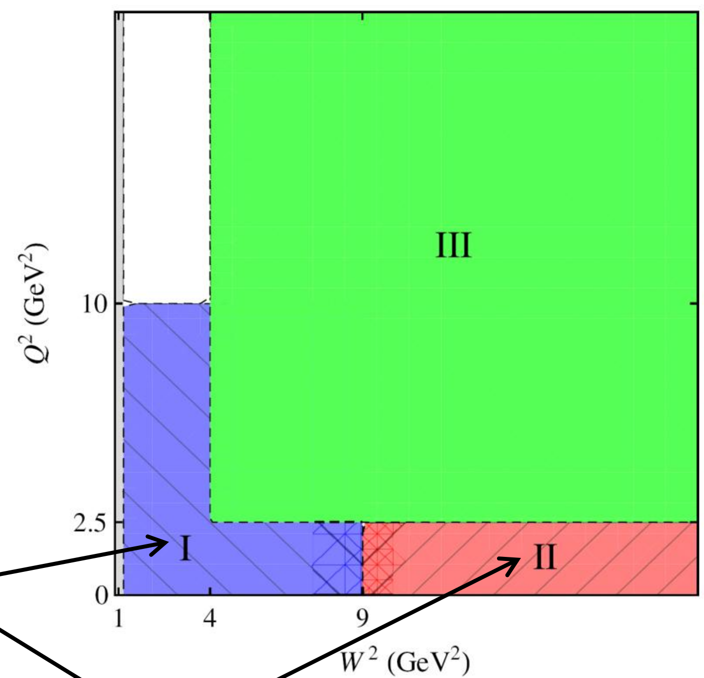
Christy and Bosted

Alternative Analyses

Gorchtein *et al.*
PRC 84, 015502 (2011)



Hall *et al.*
PRD 88, 013011 (2013)



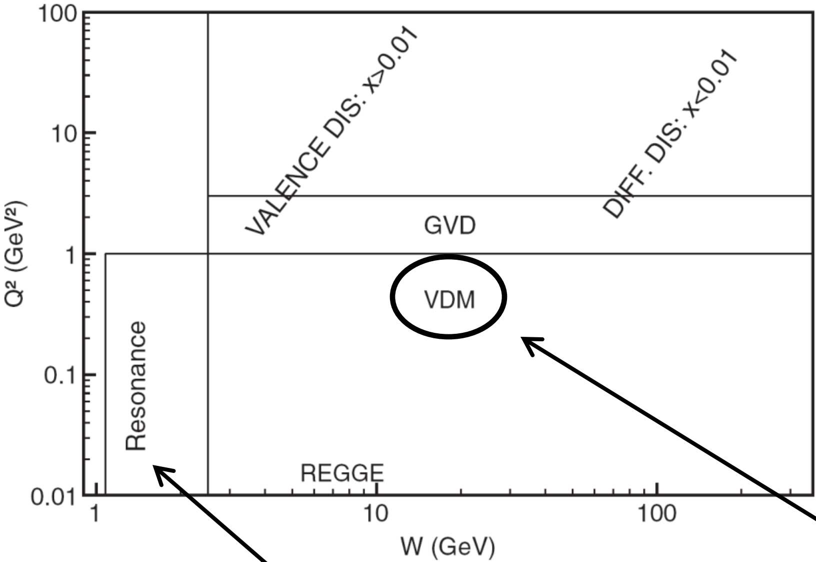
Christy and Bosted

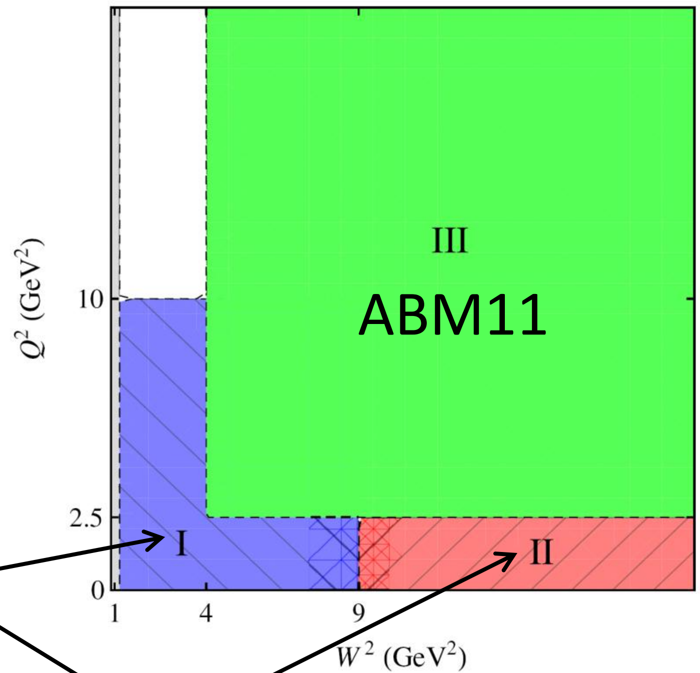
Vector Meson Model
Phys. Lett. B 596, 77 (2004)

Alternative Analyses

Gorchtein *et al.*
PRC 84, 015502 (2011)

Hall *et al.*
PRD 88, 013011 (2013)





Christy and Bosted

Vector Meson Model

Phys. Lett. B 596, 77 (2004)

Christy and Bosted Modification

- Isospin rotate neutron amplitudes:

$$\langle N_n^* | J_\mu^\gamma | n \rangle = e_u \langle N_n^* | \bar{u} \gamma_\mu u | n \rangle + e_d \langle N_n^* | \bar{d} \gamma_\mu d | n \rangle$$

$$\Downarrow$$

$$\langle N_n^* | J_\mu^\gamma | n \rangle = e_u \langle N_p^* | \bar{d} \gamma_\mu d | p \rangle + e_d \langle N_p^* | \bar{u} \gamma_\mu u | p \rangle$$

Christy and Bosted Modification

- Isospin rotate neutron amplitudes:

$$\langle N_n^* | J_\mu^\gamma | n \rangle = e_u \langle N_n^* | \bar{u}\gamma_\mu u | n \rangle + e_d \langle N_n^* | \bar{d}\gamma_\mu d | n \rangle$$

$$\Downarrow$$

$$\langle N_n^* | J_\mu^\gamma | n \rangle = e_u \langle N_p^* | \bar{d}\gamma_\mu d | p \rangle + e_d \langle N_p^* | \bar{u}\gamma_\mu u | p \rangle$$

- Rewrite neutral current:

$$\langle N_p^* | J_\mu^{\gamma(Z,V)} | p \rangle = \frac{2}{3}(g_u^V) \langle N_p^* | \bar{u}\gamma_\mu u | p \rangle - \frac{1}{3}(g_d^V) \langle N_p^* | \bar{d}\gamma_\mu d | p \rangle$$

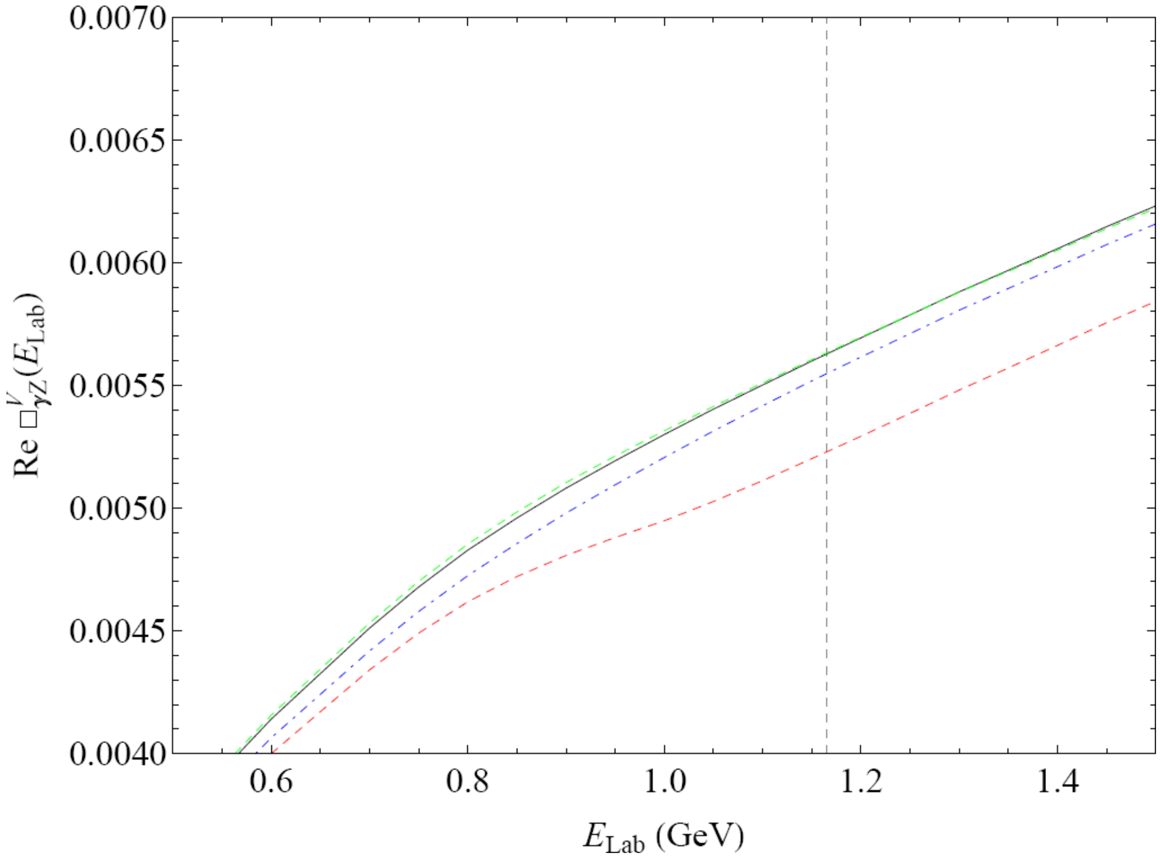
$$\Downarrow$$

$$\langle N_p^* | J_\mu^{Z,V} | p \rangle = \frac{1}{2}(1 - 4\sin^2\theta_W(0)) \langle N_p^* | J_\mu^\gamma | p \rangle - \frac{1}{2} \langle N_n^* | J_\mu^\gamma | n \rangle$$

$$
\begin{aligned}
C_{res} &= 2\frac{\sum_\lambda A_\lambda^{\gamma,p} A_\lambda^{Z,p}}{\sum_\lambda (A_\lambda^{\gamma,p})^2} \\
&= Q_W^{p,LO} - \frac{\sum_\lambda A_\lambda^{\gamma,p} A_\lambda^{\gamma,n}}{\sum_\lambda (A_\lambda^{\gamma,p})^2}
\end{aligned}
$$

- C_{res} calculated using PDG photoproduction data

C_{res} Effect on γZ Box



- Solid: Constituent Quark Model of Carlson and Rislow
- Blue: PDG (used by Gorchtein *et al.* and Hall *et al.*)
- Red: MAID (Eur.Phys.J.ST 198, 141 (2011))
  - Green: MAID without Roper Resonance

Gorchtein *et al.* Background Analysis

- Christy Bosted background thrown away.
- Vector Meson Dominance Model (Alwall and Ingleman Phys.Letts.B 596, 77 (2004)): photon changes to a vector meson before interacting with proton.

$$\sigma_T^{\mathrm{VMD}} = \sigma_{\gamma N}\Big[\sum_V r_V \frac{1}{(1+Q^2/m_V^2)^2} + r_C \frac{1}{1+Q^2/m_0^2}\Big]$$

$$\sigma_L^{\mathrm{VMD}} = \sigma_{\gamma N}\Big[\sum_V r_V \xi_V \frac{Q^2/m_V^2}{(1+Q^2/m_V^2)^2} + r_C \xi_C \Big(\frac{m_0^2}{Q^2}\ln(1+Q^2/m_0^2) - \frac{1}{1+Q^2/m_0^2}\Big)\Big]$$

- V = ρ, ω, φ
- C = Continuum

Gorchtein *et al.* Background Analysis

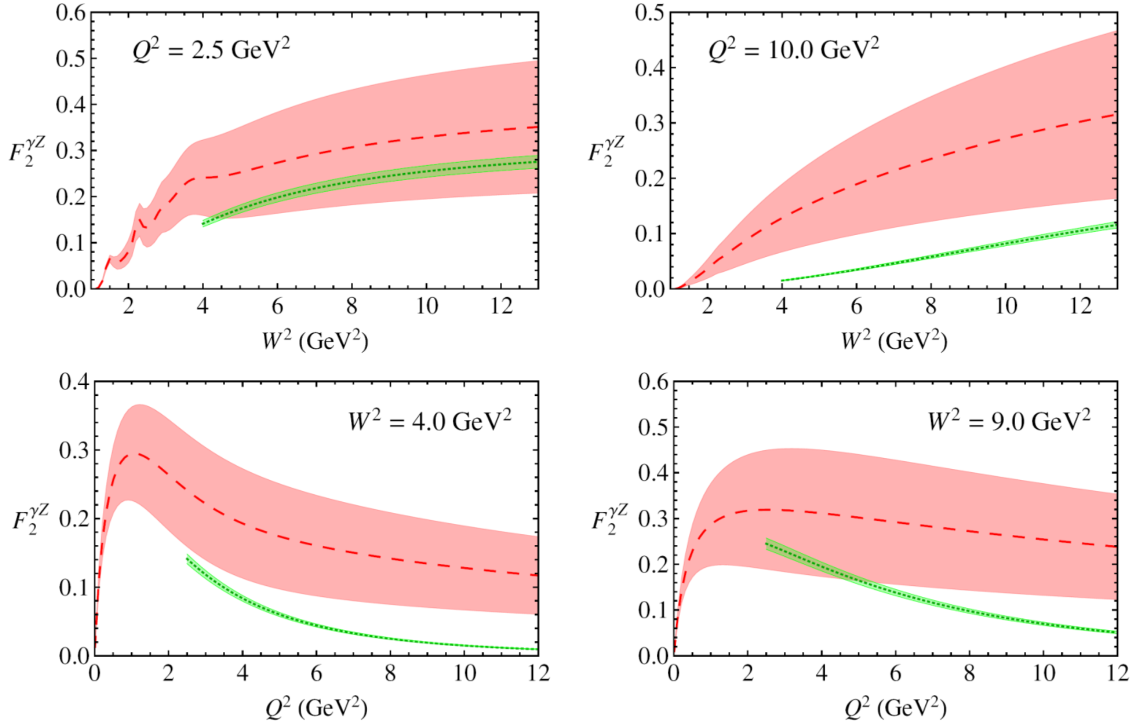
- r_V corrected by:

$$
\begin{aligned}
r_{\gamma^* Z\rho}(W^2,\, Q^2) &= \frac{g_V^{I=1}}{e_q^{I=1}} r_{\gamma^*\rho}(W^2,\, Q^2) \\
&= (2 - 4\sin^2\theta_W) r_{\gamma^*\rho}(W^2,\, Q^2), \\
r_{\gamma^* Z\omega}(W^2,\, Q^2) &= \frac{g_V^{I=0}}{e_q^{I=0}} r_{\gamma^*\omega}(W^2,\, Q^2) \\
&= -4\sin^2\theta_W r_{\gamma^*\omega}(W^2,\, Q^2), \\
r_{\gamma^* Z\phi}(W^2,\, Q^2) &= \frac{g_V^{s}}{e_q^{s}} r_{\gamma^*\phi}(W^2,\, Q^2) \\
&= (3 - 4\sin^2\theta_W) r_{\gamma^*\phi}(W^2,\, Q^2),
\end{aligned}
$$

- Gorchtein *et al.* do not modify r_C and assign it 100% uncertainty.
  - Largest source of their fit's uncertainty.

Hall *et al.* Background Analysis

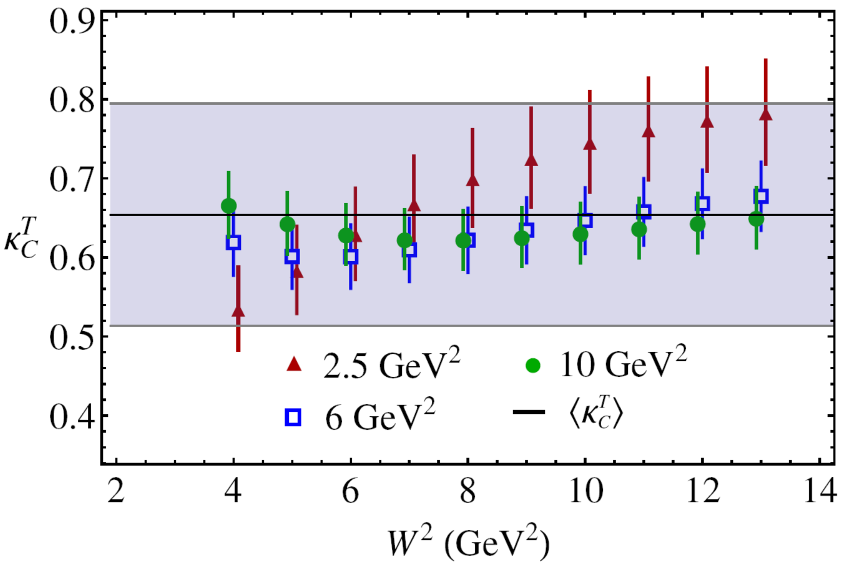
- Argue uncertainty from continuum can be reduced by matching boundary conditions with PDF fits.

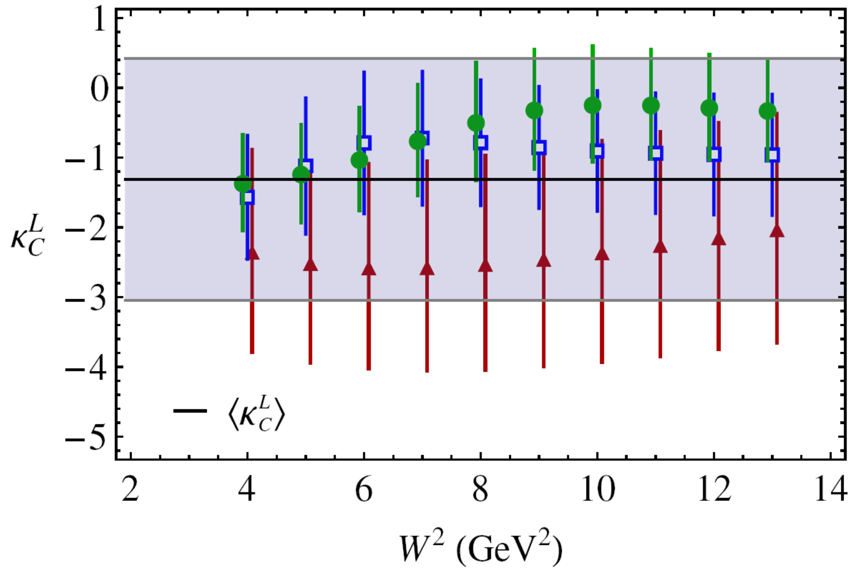


Pink: Gorchtein *et al.* VMD background.
Green: PDFs from ABM11.

Hall *et al.* Background Analysis

- Continuum Fits:

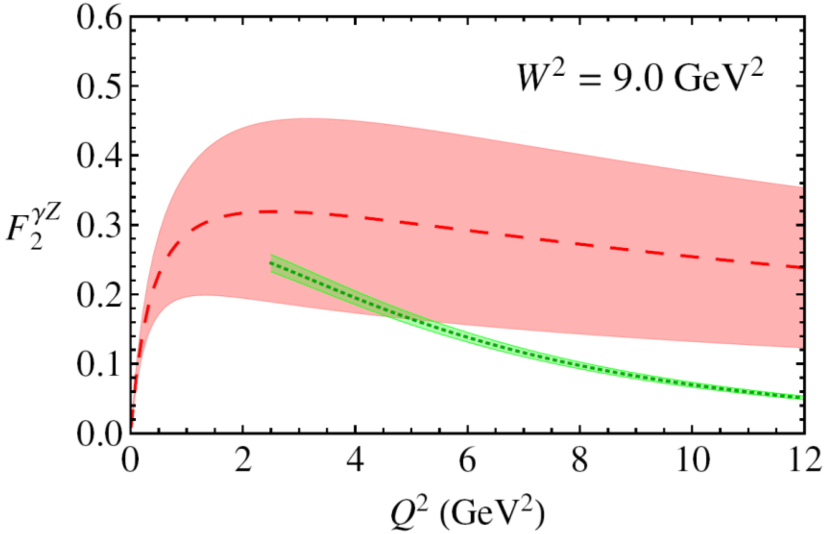




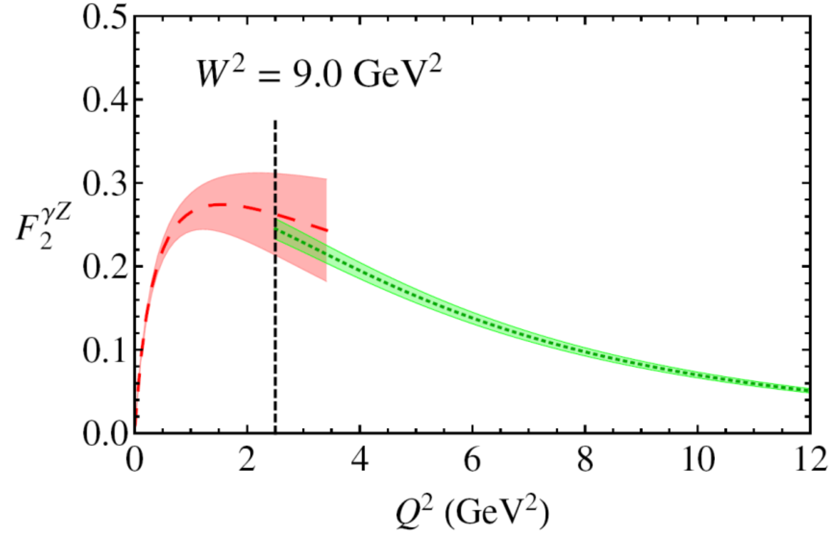
$$\kappa_C^T = 0.65 \pm 0.14, \qquad \kappa_C^L = -1.3 \pm 1.7.$$

Comparison

Gorchtein *et al.*



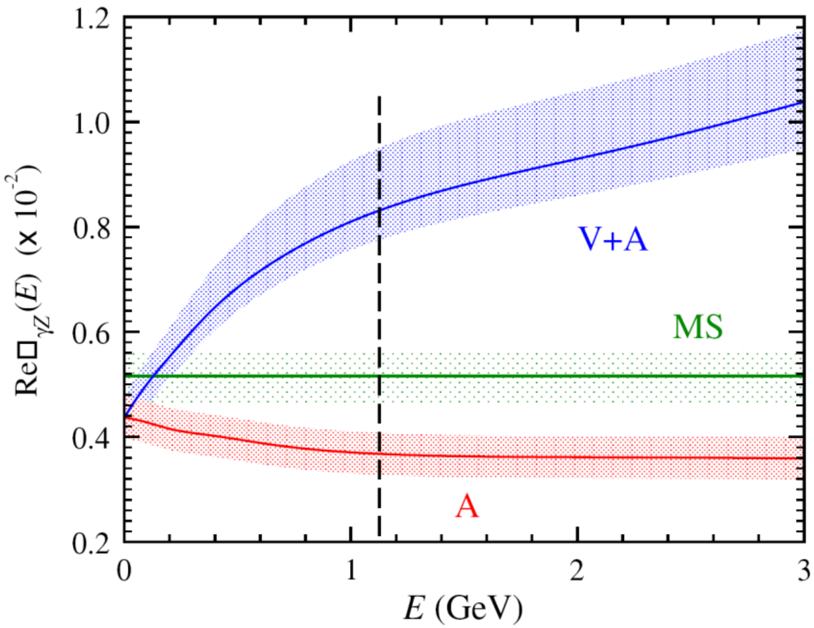
Hall *et al.*



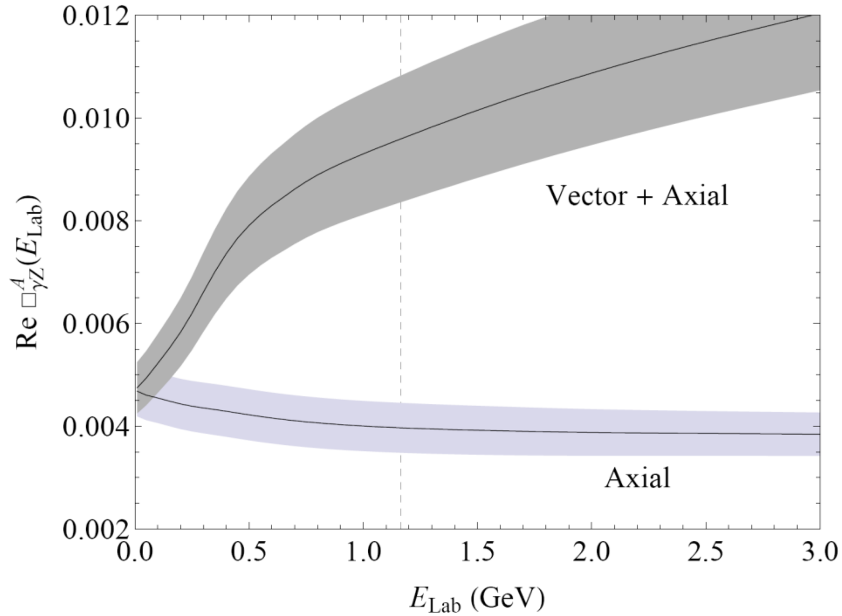
Axial Box Analyses

Axial Box Calculations

Blunden *et al.*
PRL 107, 081801 (2011)



Carlson and Rislow
PRD 88, 013018 (2013)

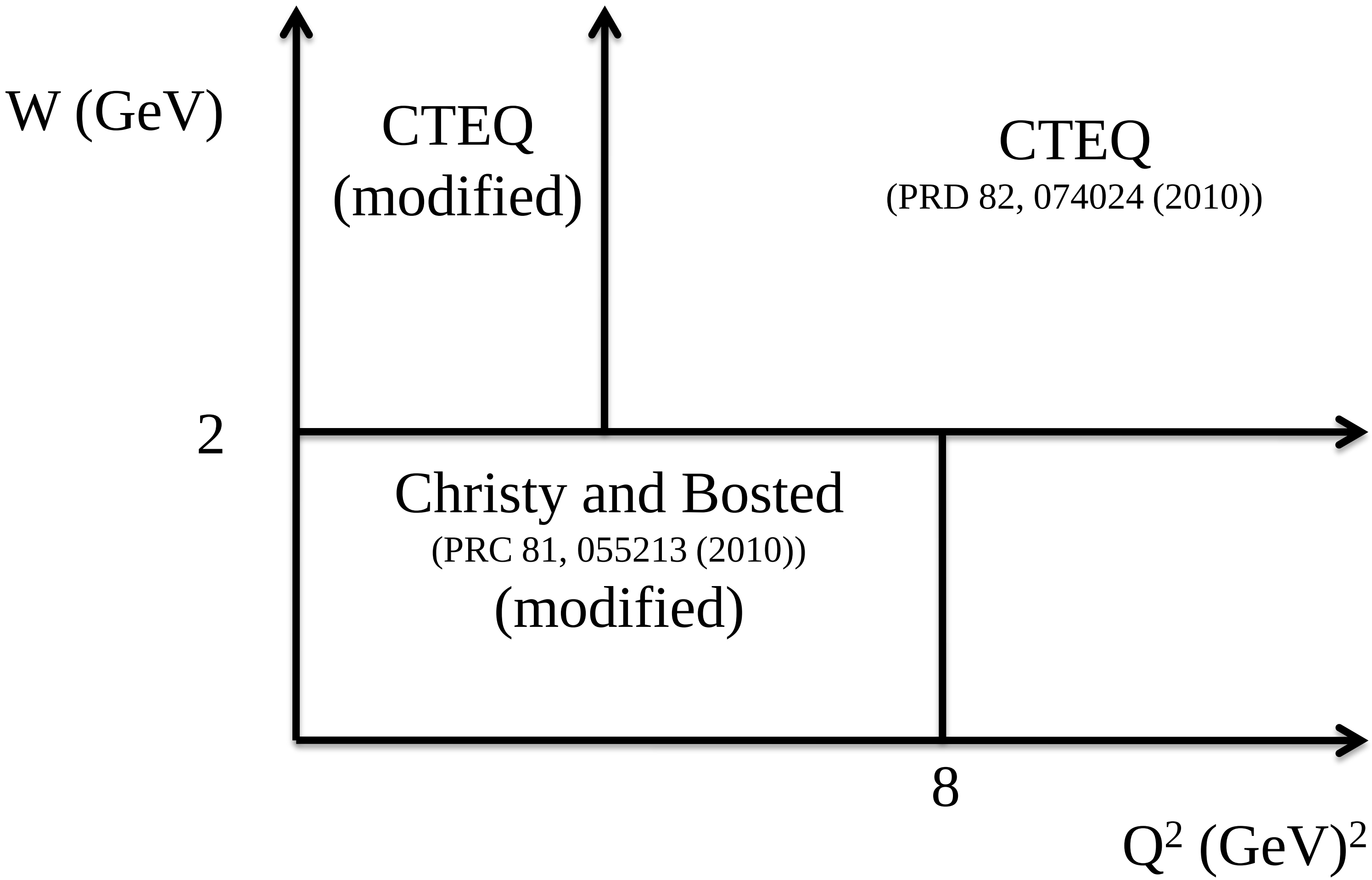


$$\mathrm{Re}\Box^{A}_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$$(3.7 \pm 0.4) \times 10^{-3} \qquad (4.0 \pm 0.5) \times 10^{-3}$$

Carlson and Rislow Analysis

- We split the energy regions and modified the structure functions.



Evaluation of Axial Structure Function

- Scaling Region:

$$F_3^{\gamma Z}(x,Q^2) = \sum_q 2e_q g_q^A \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

- High W, Low Q^2:

$$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)\Big|_{\mathrm{CTEQ}}$$

$$Q_0^2 = 1 \ \mathrm{GeV}^2$$

$$\Lambda^2 = 0.7 \ \mathrm{GeV}^2$$

- Resonance Region:
  - Blunden *et al.* use Lalakulich *et al.* (PRD 74, 014009)
  - We continue modifying the Christy Bosted fit.

$$C_{res} = \frac{F_3^{\gamma Z}}{F_1^{\gamma\gamma}}$$

$$F_3^{\gamma Z}\Big|_{N\to res} =$$

$$3\left(2g_A^{q(3)}\right)\frac{2\nu}{q_z}\langle\psi_N\phi_N\chi_s\Big|\left[\frac{2m_q}{q_z}BS_+\right]^\dagger\Big|\psi_{res}\phi_{res}\chi_\lambda\rangle$$

$$\times\, 3e_q^{(3)}\langle\psi_{res}\phi_{res}\chi_\lambda\Big|[AL_+ + BS_+]\Big|\psi_N\phi_N\chi_s\rangle,$$

Rislow and Carlson Axial C_{res}

resonance	proton axial current amplitudes	C^p_{res}	C^d_{res}
$P_{33}(1232)$	$A^{Z,A}_{1/2} \propto (g^u_A - g^d_A)\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$
$S_{11}(1535)$	$A^{Z,A}_{1/2} = -\frac{1}{\sqrt{6}}B_{10}\left(\frac{5}{3}g^u_A + \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{1}{3(2f_1+1)}\,\frac{16m_q\nu}{3q_z^2}$	$\frac{(1+2f_1)+(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2}\,\frac{16m_q\nu}{3q_z^2}$
$D_{13}(1520)$	$A^{Z,A}_{1/2} = \sqrt{\frac{2}{6}}B_{10}\left(\frac{5}{3}g^u_A + \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$ $A^{Z,A}_{3/2} = 0$	$\frac{1-f_1}{(f_1-1)^2+3f_1^2}\,\frac{16m_q\nu}{3q_z^2}$	$\frac{(1-f_1)-(f_1-1/3)}{(1-f_1)^2+(f_1-1/3)^2+6f_1^1}\,\frac{16m_q\nu}{3q_z^2}$
$F_{15}(1680)$	$A^{Z,A}_{1/2} = \sqrt{\frac{3}{5}}B_{20}\left(\frac{4}{3}g^u_A - \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$ $A^{Z,A}_{3/2} = 0$	$\frac{(1-f_2)}{(1-f_2)^2+2f_2^2}\,\frac{20m_q\nu}{3q_z^2}$	$\frac{(1-f_2)+2/3}{(1-f_2)^2+2f_2^2+4/9}\,\frac{20m_q\nu}{3q_z^2}$
$S_{11}(1650)$	$A^{\gamma}_{1/2} = -\sqrt{\frac{2}{27}}B_{10}\left(g^u_A + 2g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{1}{3(2f_1+1)}\,\frac{16m_q\nu}{3q_z^2}$	$\frac{(1+2f_1)+(1/3+2f_1)}{(1+2f_1)^2+(1/3+2f_1)^2}\,\frac{16m_q\nu}{3q_z^2}$
$P_{11}(1440)$	$A^{Z,A}_{1/2} = B_{00}\left(\frac{4}{3}g^u_A - \frac{1}{3}g^d_A\right)\frac{4m_q\nu}{q_z^2}$	$\frac{20m_q\nu}{3q_z^2}$	$\frac{100m_q\nu}{13q_z^2}$
$F_{37}(1950)$	$A^{Z,A}_{1/2} \propto (g^u_A - g^d_A\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$	$2\frac{4m_q\nu}{q_z^2}$
Background		$\frac{5}{3}$	$\frac{9}{5}$

Rislow and Carlson Background Correction

- Smooth Background.

$$C_{bkgd}\big|_{x\to 0} = \frac{\sum_{q=u,d,s} 2e_q g_q^A f_q(x)}{\frac{1}{2}\sum_{q=u,d,s}(e_q)^2 f_q(x)} = 0$$

$$C_{bkgd}\big|_{\text{valence quarks}} = \frac{\sum_{q=u,u,d} 2e_q g_q^A f_q(x)}{\frac{1}{2}\sum_{q=u,u,d}(e_q)^2 f_q(x)} = \frac{10}{3}$$

- Average 5/3.

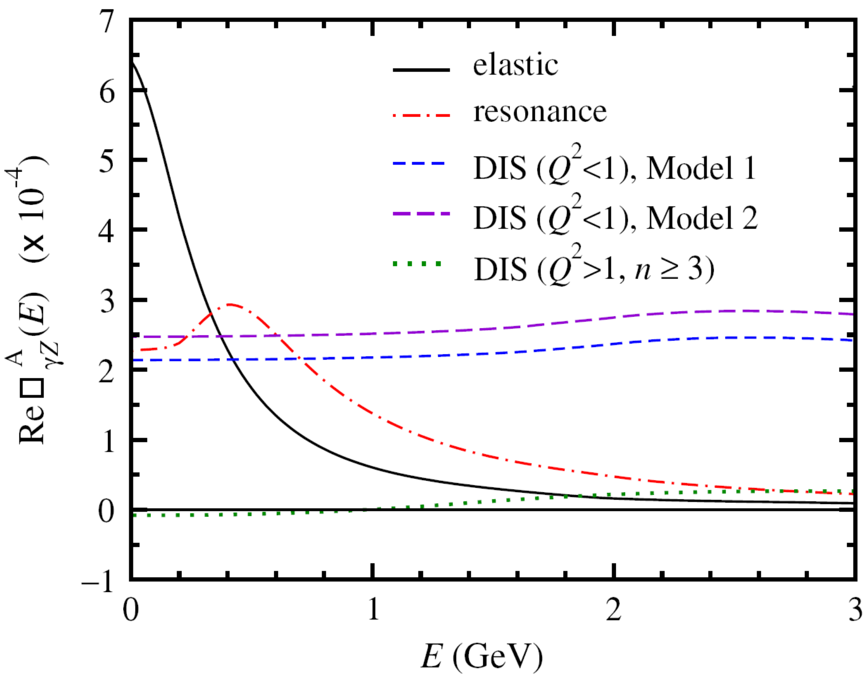
Blunden *et al.* Analysis

- F_3 extracted from Lalakulich *et al.* (PRD 74, 014009)
  - fits for D_{13}(1520), P_{11}(1440), P_{33}(1232), and S_{11}(1535).
- Fewer resonances than the Christy Bosted fit

Axial Box Contributions

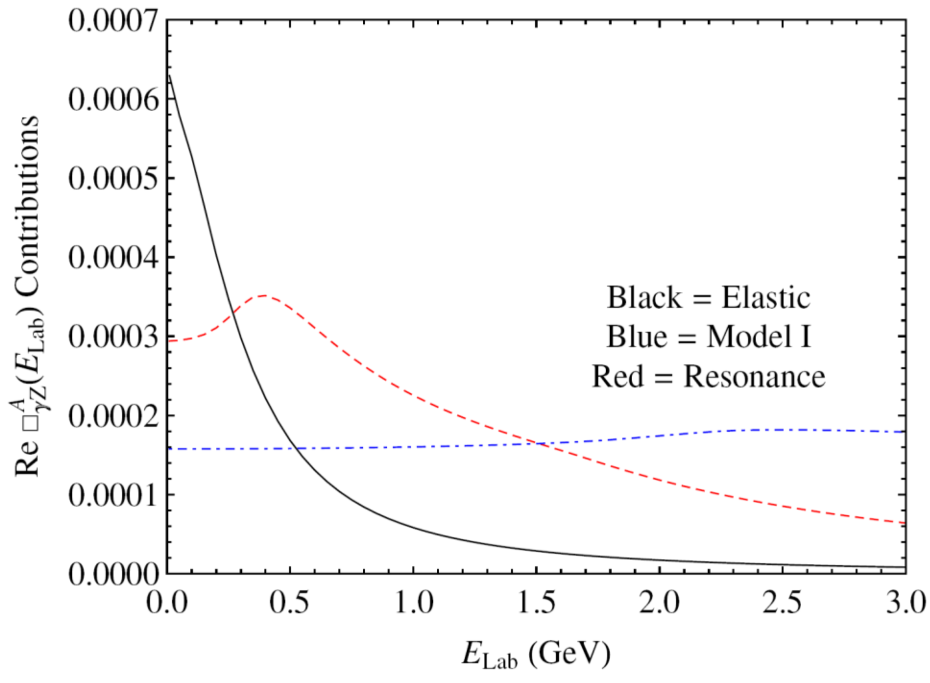
Blunden *et al.*

PRL 107, 081801 (2011)



Carlson and Rislow

PRD 88, 013018 (2013)



Closing Remarks

Experimental Tests of γZ structure functions

- PVDIS Experiment at JLab a first test of γZ structure functions

$$A_{PVDIS} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\frac{Q^2}{Q^2+M_Z^2}$$
$$\times \frac{g_A^e\left\{xy^2 F_1^{\gamma Z}(x,Q^2) + \left(1-y-\frac{M^2x^2y^2}{Q^2}\right)F_2^{\gamma Z}(x,Q^2) + \frac{g_V^e}{g_A^e}\left(y-\frac{y^2}{2}\right)xF_3^{\gamma Z}(x,Q^2)\right\}}{xy^2F_1^{\gamma\gamma}(x,Q^2) + \left(1-y-\frac{M^2x^2y^2}{Q^2}\right)F_2^{\gamma\gamma}(x,Q^2)}$$

- Parity-violating, electron deuteron scattering (arXiv: 1304.7741).

Actual Values

PVDIS kinematics	I	II	III	IV
E (GeV)	4.867	4.867	4.867	6.067
Q^2 (GeV^2)	0.950	0.831	0.757	1.472
W_{eff} (GeV)	1.263	1.591	1.857	1.981

A_{PVDIS} (ppm)				
Measured	-68.97 ± 9.09	-74.12 ± 7.43	-61.80 ± 5.50	-119.56 ± 18.42
CQM	-87.4 ± 8.9	-67.1 ± 7.0	-64.9 ± 6.8	-128.1 ± 13.3
MAID	-75.2 ± 7.7	-56.9 ± 6.1	-63.2 ± 6.5	-125.5 ± 13.0
GHRM	-87.3	-67.6	-64.6	-127.6

Rislow and Carlson Total Result

- For Qweak:

$$\mathrm{Re}\Box_{\gamma Z}(E = 1.165\ \mathrm{GeV})|_{\mathrm{total}} = (9.7 \pm 1.4) \times 10^{-3}$$

- For P2:

$$\mathrm{Re}\,\Box_{\gamma Z}(E = 150\ \mathrm{MeV})|_{\mathrm{total}} = (5.5 \pm 0.6) \times 10^{-3}$$