Workshop on Low-Energy Precision Physics

September 27, 2013

Nuclear structure uncertainties in parity-violating electron scattering from carbon 12

A low energy, high precision experimental approach

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Massachusetts Institute of Technology

SUMMARY

- Introduction

  Parity-violating asymmetry / Theoretical uncertainty / Statistical uncertainty

- Goals of this work

- Analysis of contributions

  Coulomb distortion / Nuclear isospin mixing / Nucleon strangeness content / Meson exchange currents / Inelastic transitions

- Strategy outlook

- References

INTRODUCTION: PARITY-VIOLATING ASYMMETRY

The parity-violating asymmetry in polarized electron scattering by nuclei is defined as:

$$A = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

When some conditions are met this asymmetry takes a simple dependence on momentum transfer ('reference asymmetry'):

$$A = A^0 \equiv - \left[\frac{G_F Q^2}{\pi \alpha \sqrt{2}} \right] 2c_A^e \sin^2 \theta_W$$

A precise measurement of the PV asymmetry allows for the extraction of accurate values of Standard Model constants.

INTRODUCTION:
PARITY-VIOLATING ASYMMETRY

- Projectile wave functions are not distorted by the target.
- Only one boson per interaction is exchanged.
- Target state has 0 spin and 0 isospin, exactly.
- Target has no strangeness content.

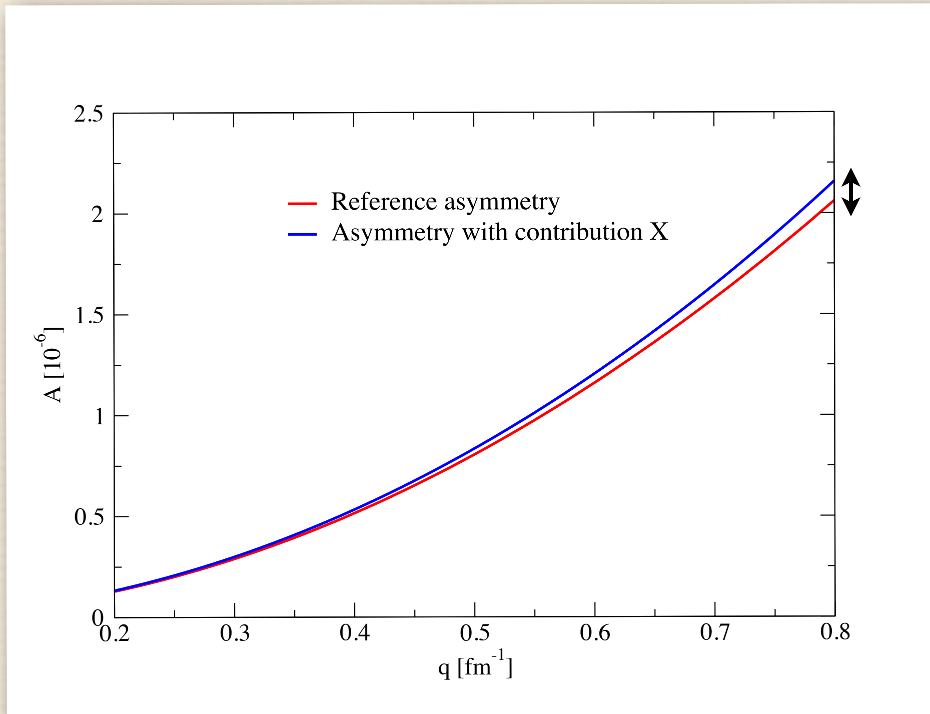
When some conditions are met this asymmetry takes a simple dependence on momentum transfer ('reference asymmetry'):

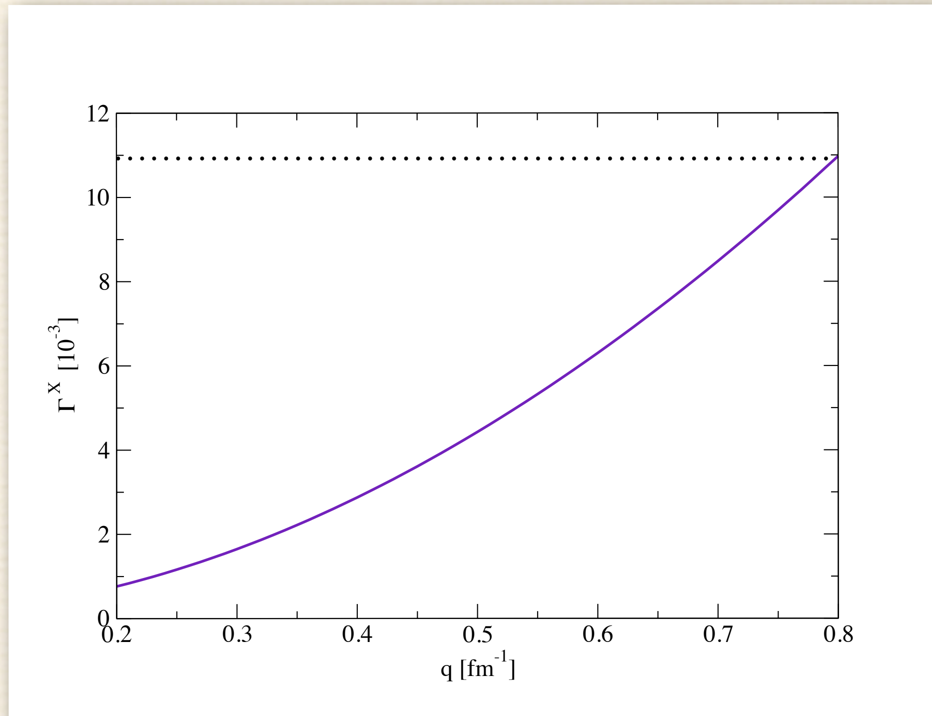
$$A = A^0 \equiv - \left[\frac{G_F\, Q^2}{\pi \alpha \sqrt{2}} \right] 2 c_A^e \sin^2 \theta_W$$

In this situation a precise measurement of the PV asymmetry allows for the extraction of accurate values of Standard Model constants.

INTRODUCTION: THEORETICAL UNCERTAINTY

Theoretical size and uncertainty of deviations from reference PV asymmetry





Absolute effect of contribution X:
Reference asymmetry (A^0) vs. asymmetry containing contribution X (A^X).

Relative effect of contribution X:
Deviation of asymmetry with contribution X with respect to reference asymmetry:
$\Gamma^X = (A^X - A^0) / A^0$.

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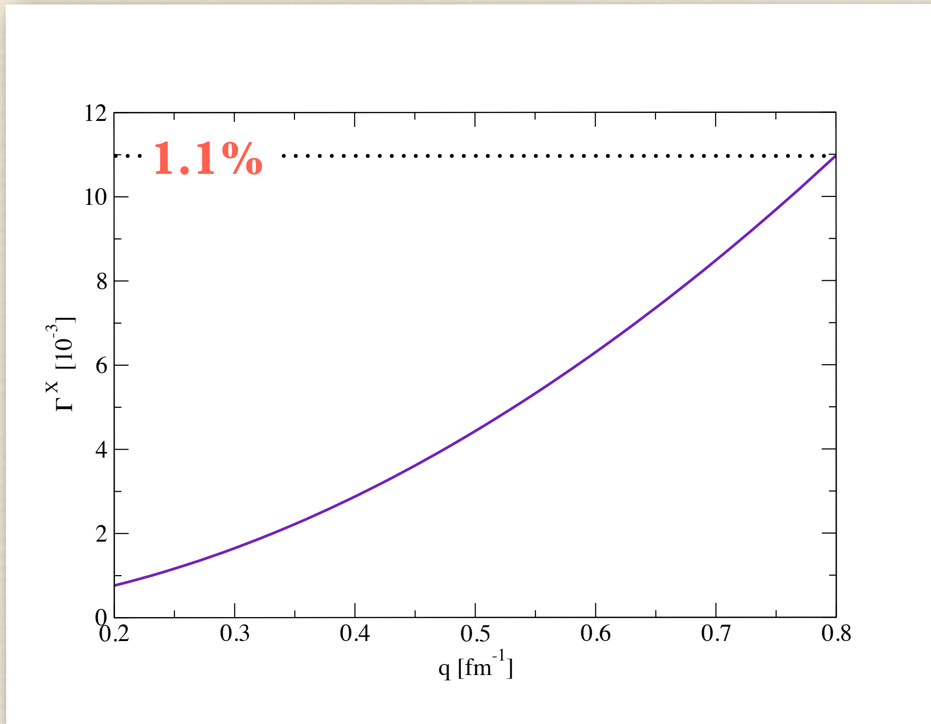

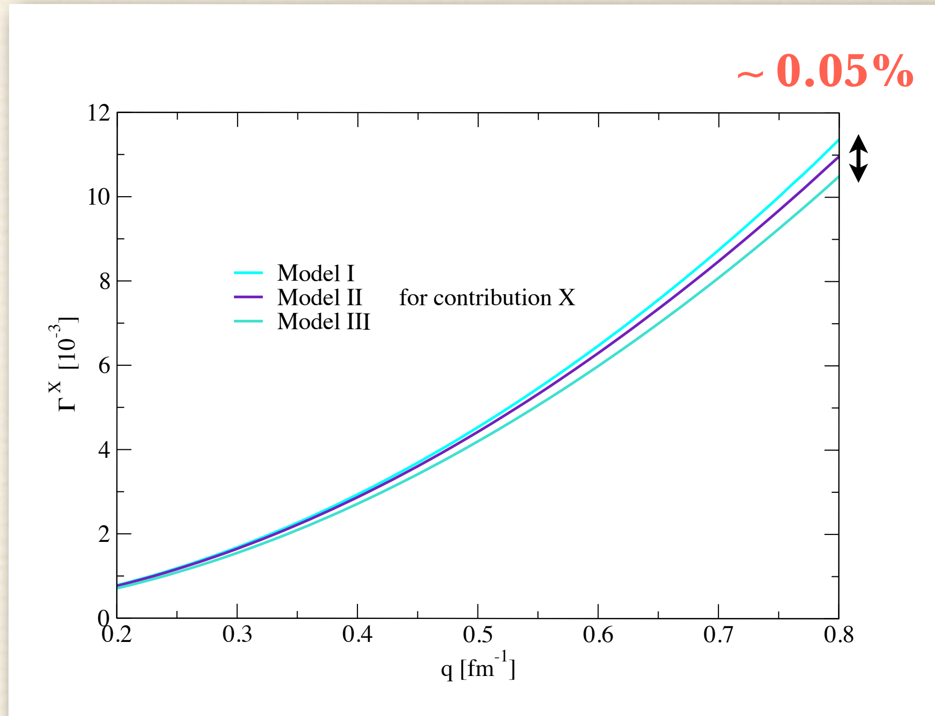

Relative effect of contribution X:
Deviation of asymmetry with contribution X with respect to reference asymmetry:
$\Gamma^X = (A^X - A^0) / A^0$.

Relative uncertainty from contribution X:
Different (reasonable) models to account for contribution X.

INTRODUCTION: THEORETICAL UNCERTAINTY

Definition of PV asymmetry deviation due to the contribution X:

$$\Gamma^X \equiv A^X / A^0 - 1$$

The total PV asymmetry consists of the reference value plus the deviations induced by every possible effect:

$$A^T \approx A^0 \left(1 + \sum_i \Gamma^{X_i} \right)$$

The variability (uncertainty) in the deviation translates directly in a PV relative uncertainty:

$$\Delta\Gamma^X = \Gamma^{X_a} - \Gamma^{X_b} = \frac{A^{X_a} - A^{X_b}}{A^0} = \frac{\Delta A^X}{A^0}$$

INTRODUCTION: STATISTICAL UNCERTAINTY

$$\frac{\Delta A}{A} = \frac{1}{[\Delta\phi \;\; L \;\; T]^{1/2}} \;\; \frac{\left[\int_{\theta_i}^{\theta_f} d\theta \; \frac{d\sigma}{d\Omega}(\theta) \; \sin\theta\right]^{1/2}}{\int_{\theta_i}^{\theta_f} d\theta \; A(\theta) \; \frac{d\sigma}{d\Omega}(\theta) \; \sin\theta}$$

INTRODUCTION: STATISTICAL UNCERTAINTY

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Azimuthal angular coverage: 2π

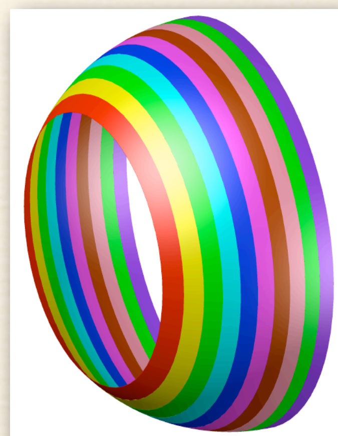
Beam luminosity: $5 \cdot 10^{38}$ cm^{-2}s^{-1}

Data acquisition time: 10^7 s (~ 4 months)

Polar angular coverage

PV asymmetry (100% beam polarization)

Differential cross section



INTRODUCTION: STATISTICAL UNCERTAINTY

$$\frac{\Delta A}{A} = \frac{1}{[\Delta\phi \;\; L \;\; T]^{1/2}} \; \frac{\left[\int_{\theta_i}^{\theta_f} d\theta \; \frac{d\sigma}{d\Omega}(\theta) \; \sin\theta\right]^{1/2}}{\int_{\theta_i}^{\theta_f} d\theta \; A(\theta) \; \frac{d\sigma}{d\Omega}(\theta) \; \sin\theta}$$

$\Delta\phi = 2\pi$

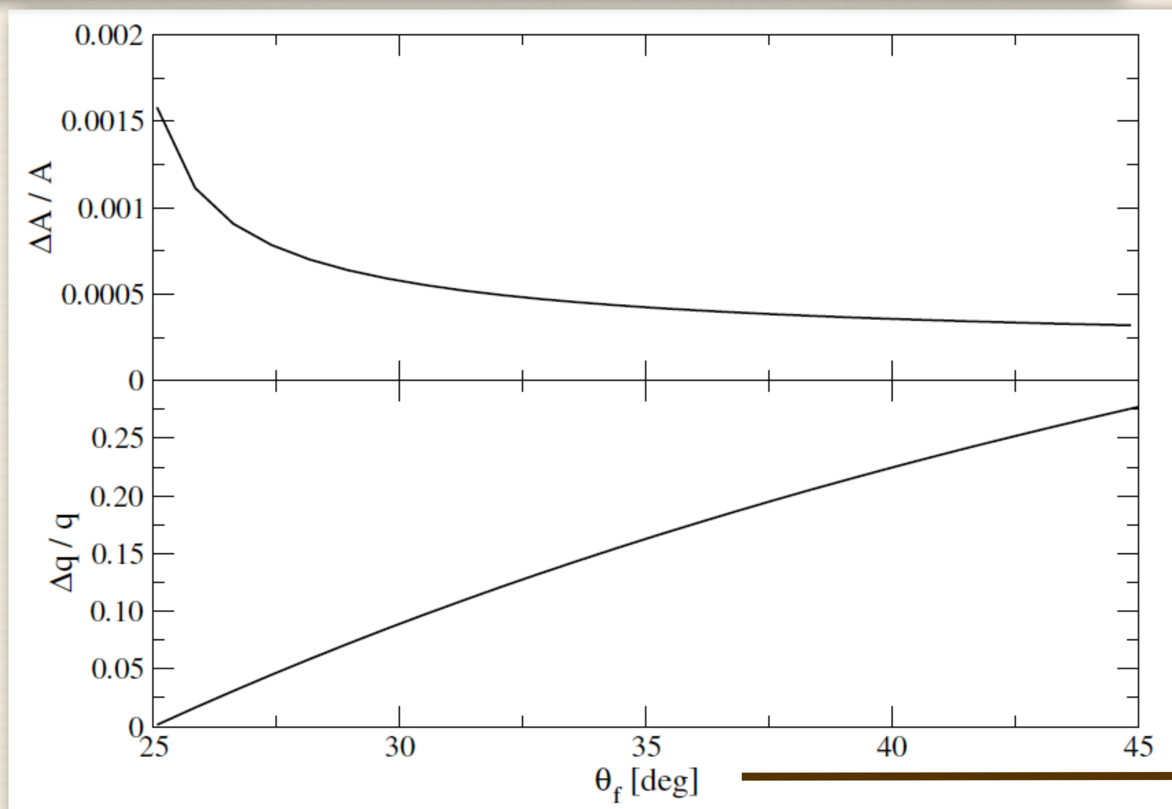
$L = 5 \cdot 10^{38}\ \mathrm{cm^{-2} s^{-1}}$

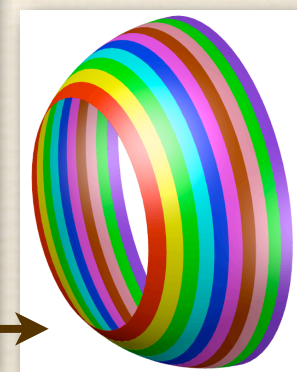
$T = 10^7\ \mathrm{s}$

$P_e = 100\ \%$

$\theta_i = 25^\circ$

$\varepsilon = 150\ \mathrm{MeV}$





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- Estimate the current uncertainties in the theoretical/ empirical descriptions of these effects and translate them into relative uncertainties of the PV asymmetry.
- Estimate the statistical uncertainties of the PV asymmetry and combine them with the above mentioned ‘theoretical’ uncertainties to determine the appropriate conditions and kinematic ranges for the experiments.

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- Strangeness content of the nucleons modifying the isoscalar (but not the isovector) nuclear responses.
- Meson exchange currents among the nucleons affecting differently the isoscalar and the isovector nuclear responses.
- Inelastic transitions where the excited nuclear states differ significantly from the ground state (e.g. different nominal isospin).

Coulomb distortion effect

Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the nuclear charge distributions.

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They generate different Coulomb fields which distort the electron wave functions differently.

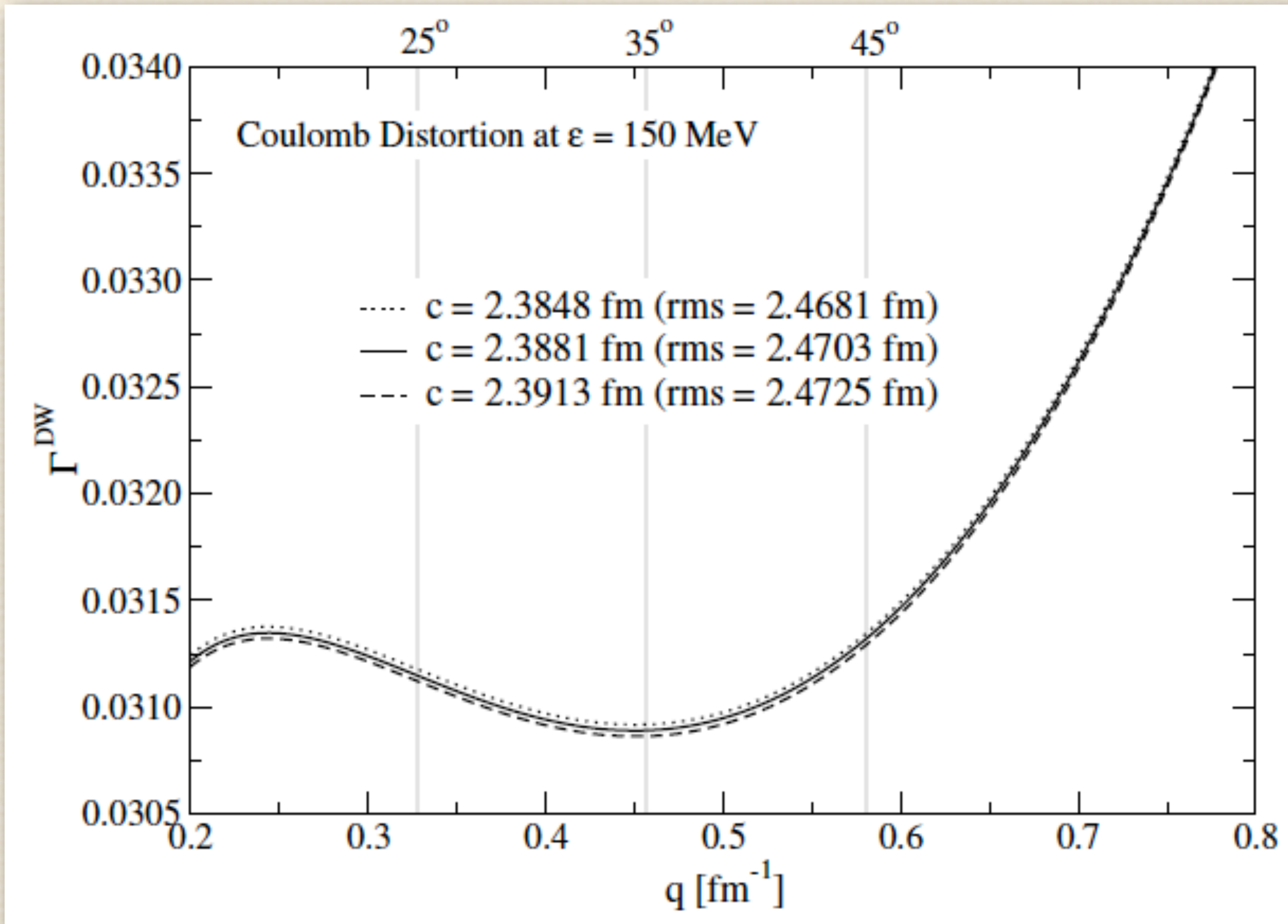
We use a three-parameter Fermi distribution:

$$\rho(r) = \rho_0 \frac{1 + \frac{w\, r^2}{c^2}}{1 + e^{\frac{r-c}{d}}}$$

and keep the rms charge radius within the experimental range.

Coulomb distortion effect

Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the nuclear charge distributions.



Nuclear isospin mixing effect

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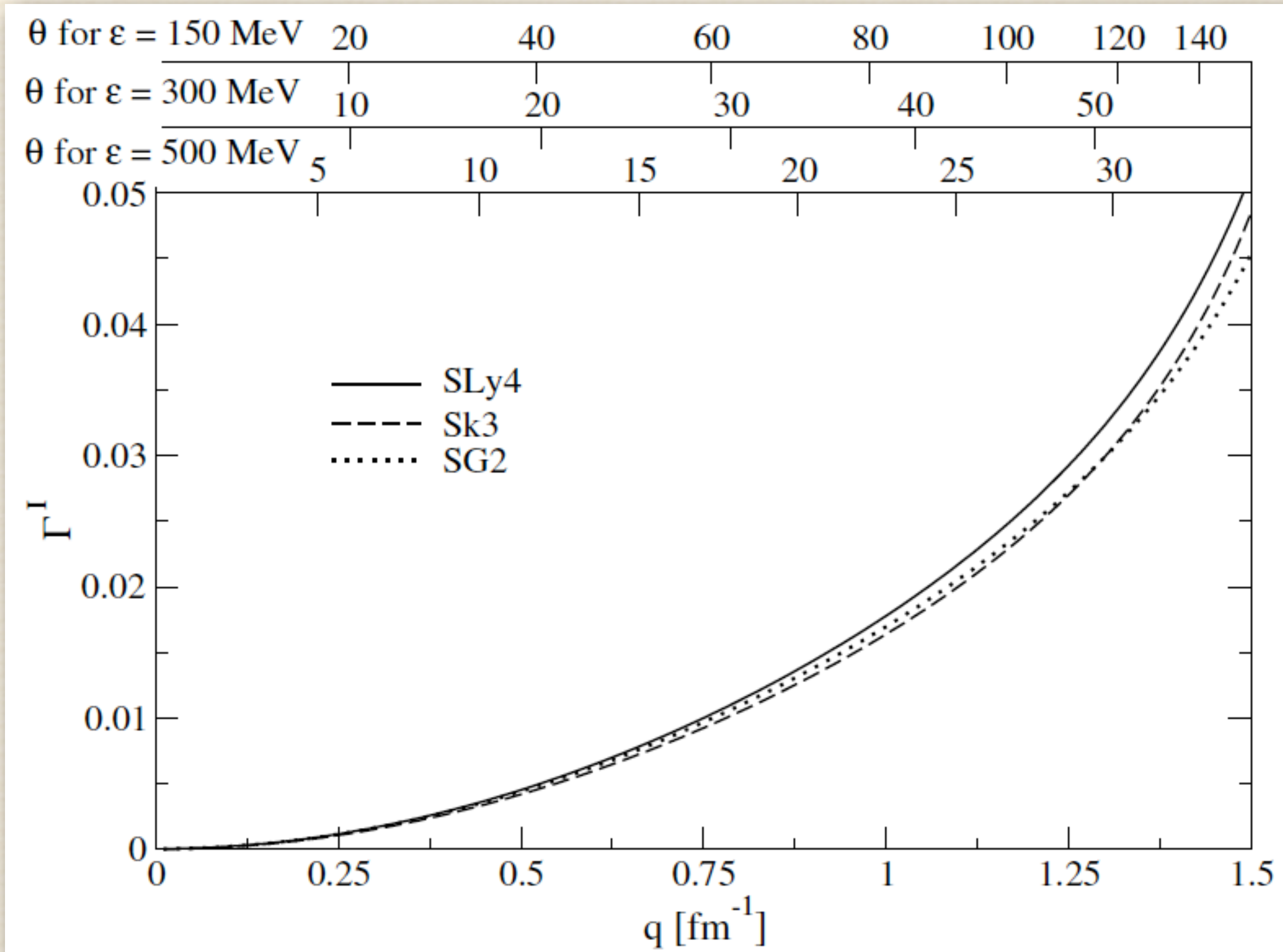
We perform an axially-deformed Hartree-Fock mean field calculation with BCS pairing correlations, using Skyrme nucleon-nucleon interactions:

$$\begin{aligned} V_{12}^{Sk} \quad = \quad & t_0(1+x_0P_\sigma)\delta(\vec{r}_1-\vec{r}_2) + \frac{1}{2}t_1(1+x_1P_\sigma)[\delta(\vec{r}_1-\vec{r}_2)k^2+k'^2\delta(\vec{r}_1-\vec{r}_2)] \\ & +t_2(1-x_2P_\sigma)\vec{k}'\delta(\vec{r}_1-\vec{r}_2)\vec{k} + iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} \\ & +\frac{1}{6}t_3(1+x_3P_\sigma)\delta(\vec{r}_1-\vec{r}_2)\,\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right), \end{aligned}$$

Nuclear isospin mixing effect

Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the neutron vs. the proton distribution within the nucleus.

DIFFERENT NUCLEON-NUCLEON FORCES



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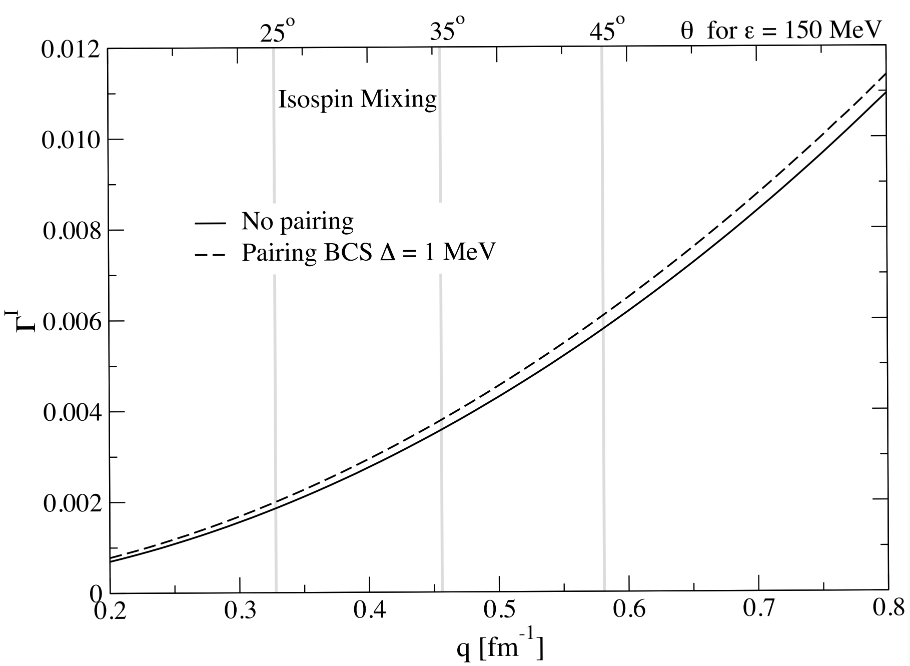
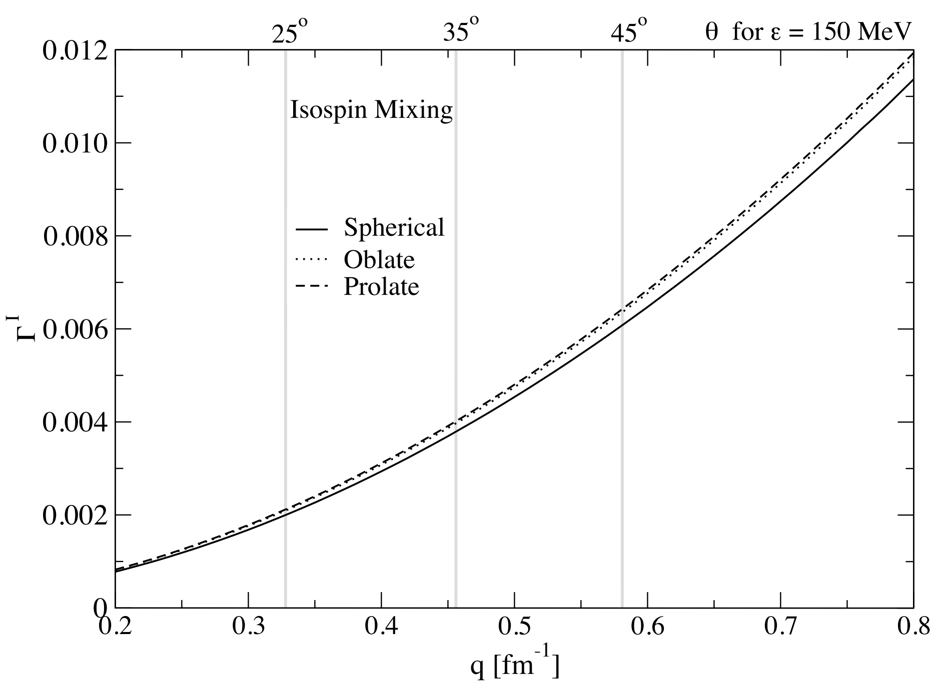
OTHER MODIFICATIONS OF THE MEAN FIELD (PAIRING, SHAPE)





Nuclear strangeness content effect

Theoretical uncertainties in the PV asymmetry due to the experimental uncertainties of the nucleon strangeness content parameters (ρ_s, μ_s).

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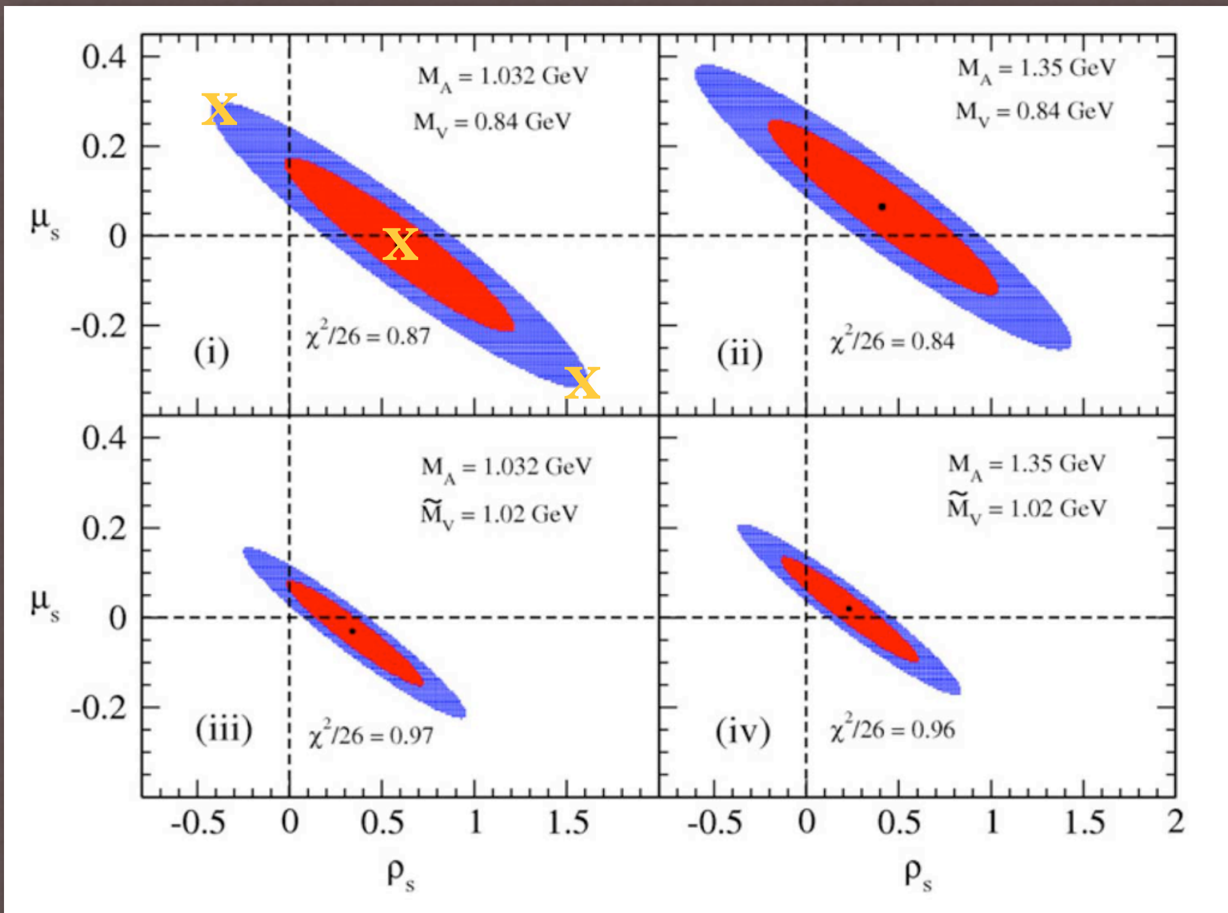
Experimental ranges of strangeness content parameters



R. González-Jiménez, J. A. Caballero, T. W. Donnelly, Phys. Rep. 524 (2013) 1

Strangeness form factors

$$G_E^{(s)} = \frac{\rho_s \tau}{(1+4.97\tau)^2}$$

$$G_M^{(s)} = \frac{\mu_s}{(1+4.97\tau)^2}$$

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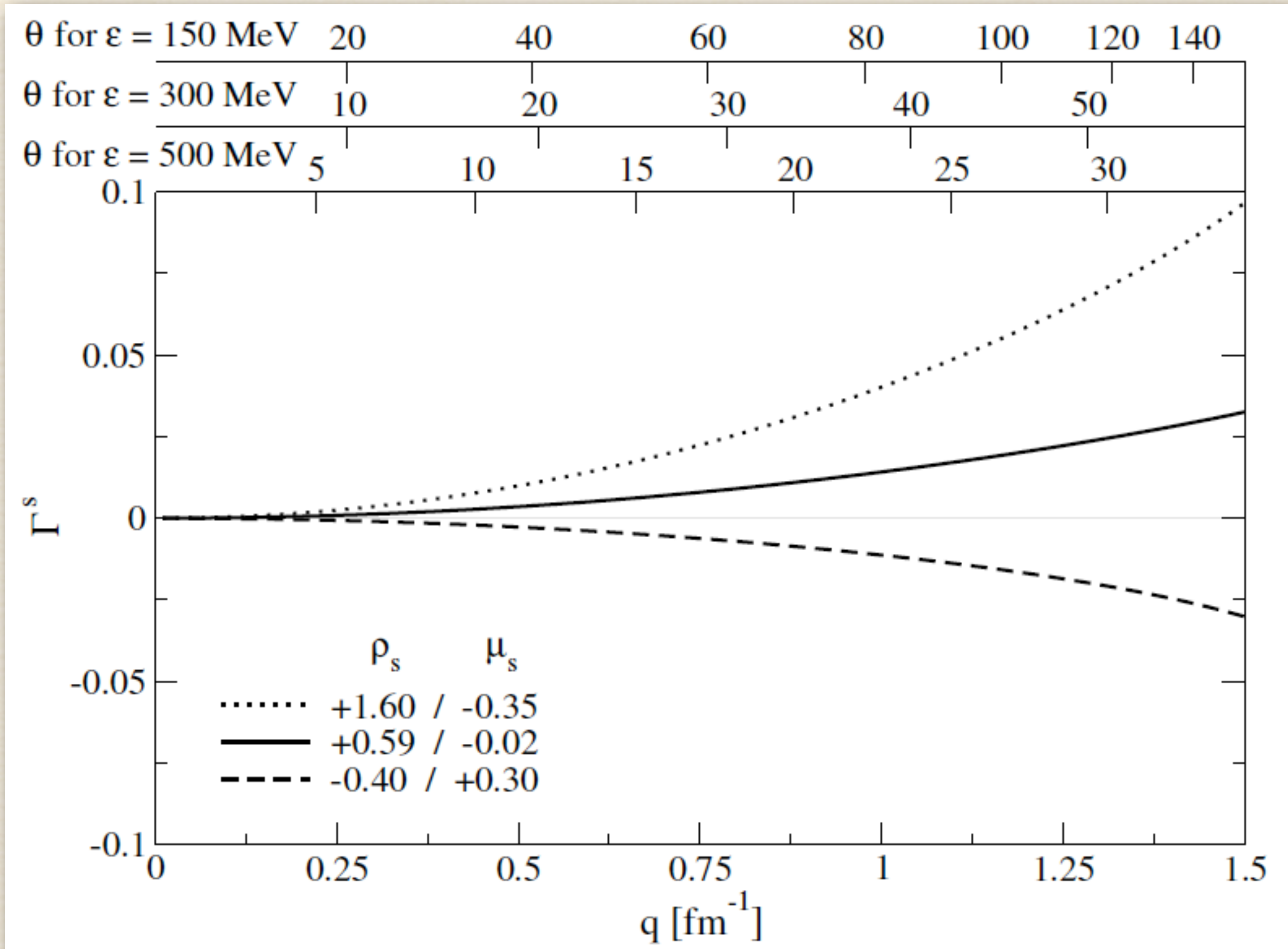

Meson exchange current effect

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- With nuclear isospin mixing and nucleon strangeness content: any MEC uncertainty is clearly exceeded by the strangeness content uncertainties.

- As for the strangeness content of exchanged mesons (two-body strangeness): effects of this exotic MEC expected to be smaller than conventional MEC effects at low momentum transfer.

  [see M. J. Musolf, R. Schiavilla and T. W. Donnelly, Phys. Rev. C50 (1994) 2173]

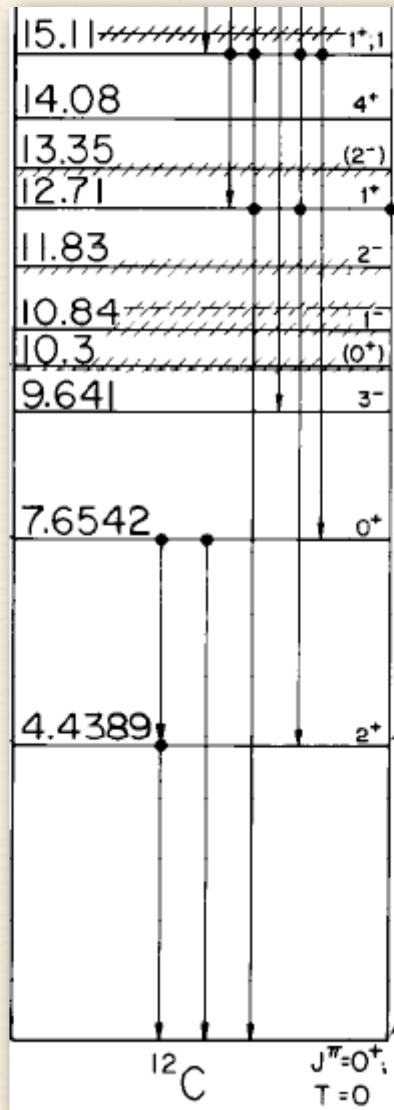
Inelastic transitions

Theoretical uncertainties in the PV asymmetry due to possible excitations of carbon 12 in the scattering process.

Total asymmetry:

$$A = \sum_{i=0}^{n} f_i A_i$$

$$f_i = \frac{\sigma_i^{EM}}{\sum_{k=0}^{n} \sigma_k^{EM}}$$



T = 0 levels:
Same asymmetry as for ground state

(in absence of strangeness and at tree-level)

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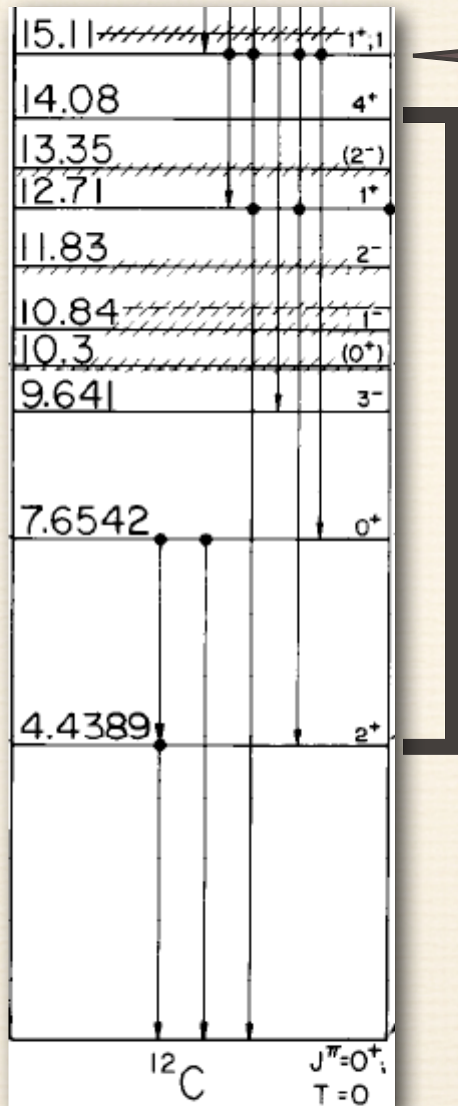


T = 1 levels:
Different asymmetry
(isovector contribution)

T = 0 levels:
Same asymmetry
as for ground state
(in absence of strangeness
and at tree-level)

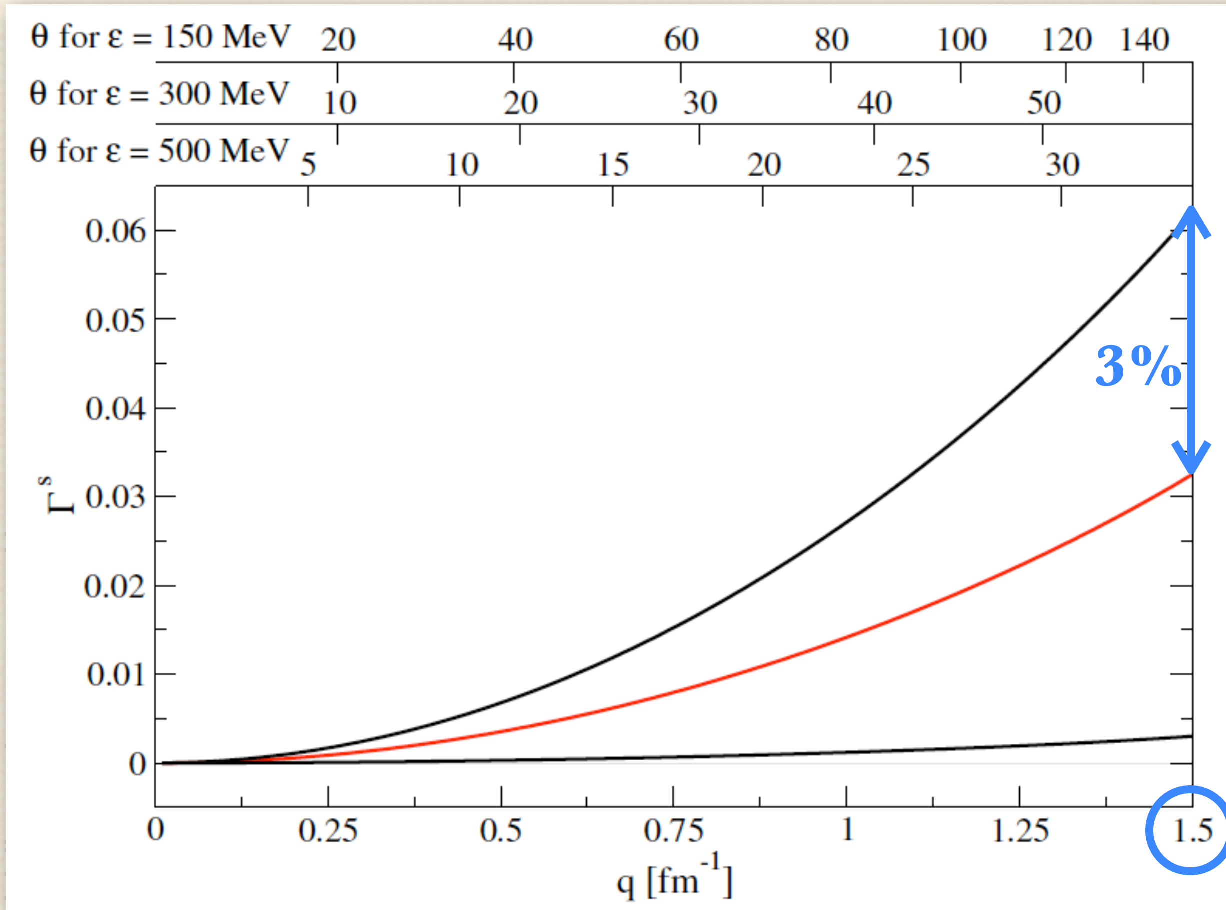
Summary of sizes and uncertainties

(150 MeV incident energy, 25° - 45° scattering angular range)

Contribution to PV asymmetry	**Size (%)**	**Uncertainty (%)**
Coulomb distortion of projectile wave function	3	0.01
Nuclear isospin mixing (electromagnetic origin)	0.4	0.05
Nucleon strangeness content (mainly electric)	0 - 1	1
Meson exchange currents	< 0.1	< 0.1
Dispersion (inelastic contributions)	< 0.1	--

STRATEGY OUTLOOK

Focus on the momentum transfer range where the strangeness content uncertainty is large:



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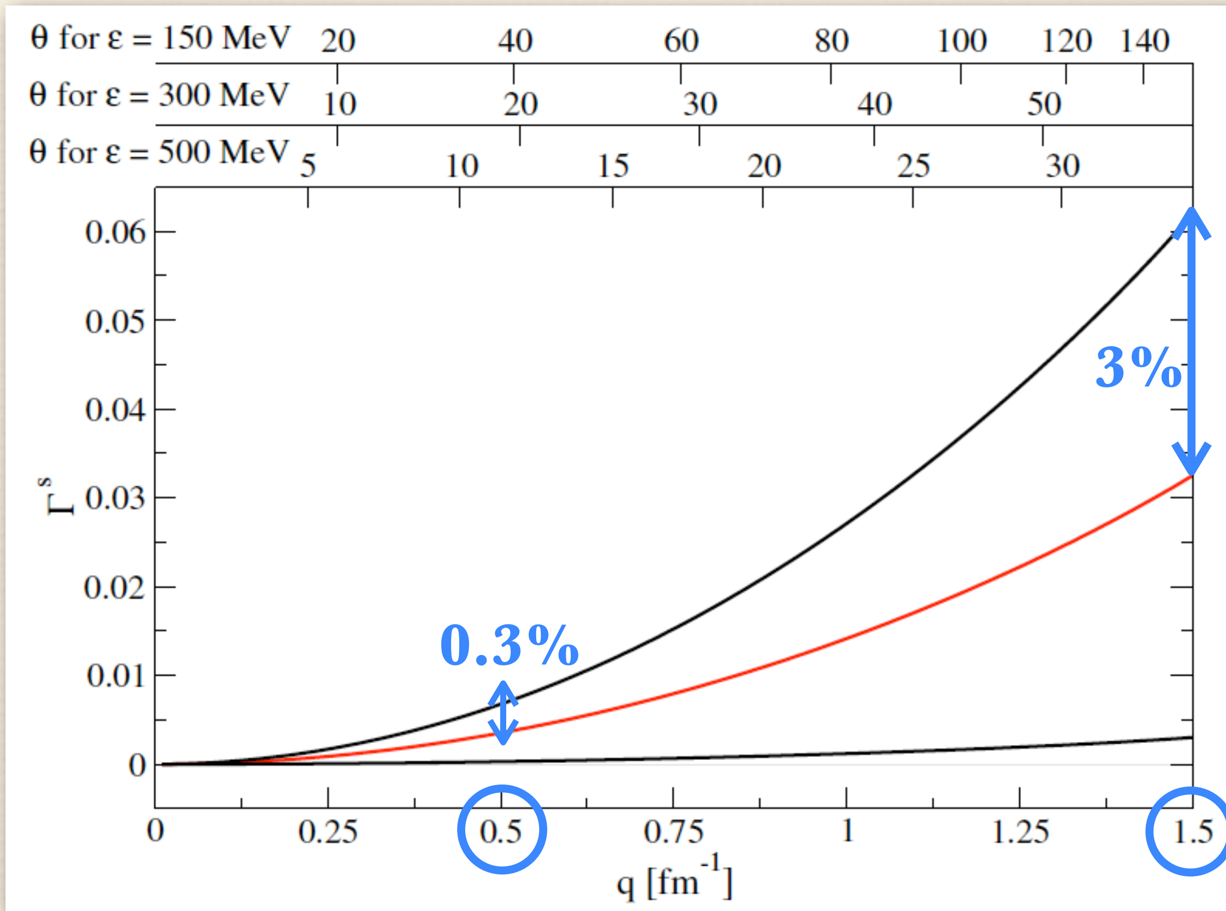
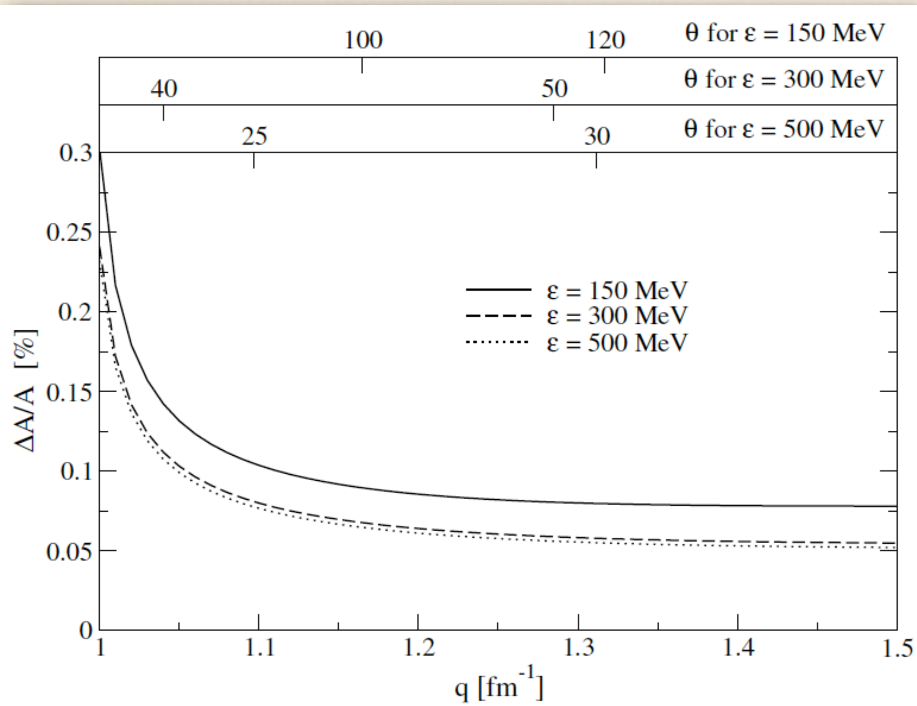
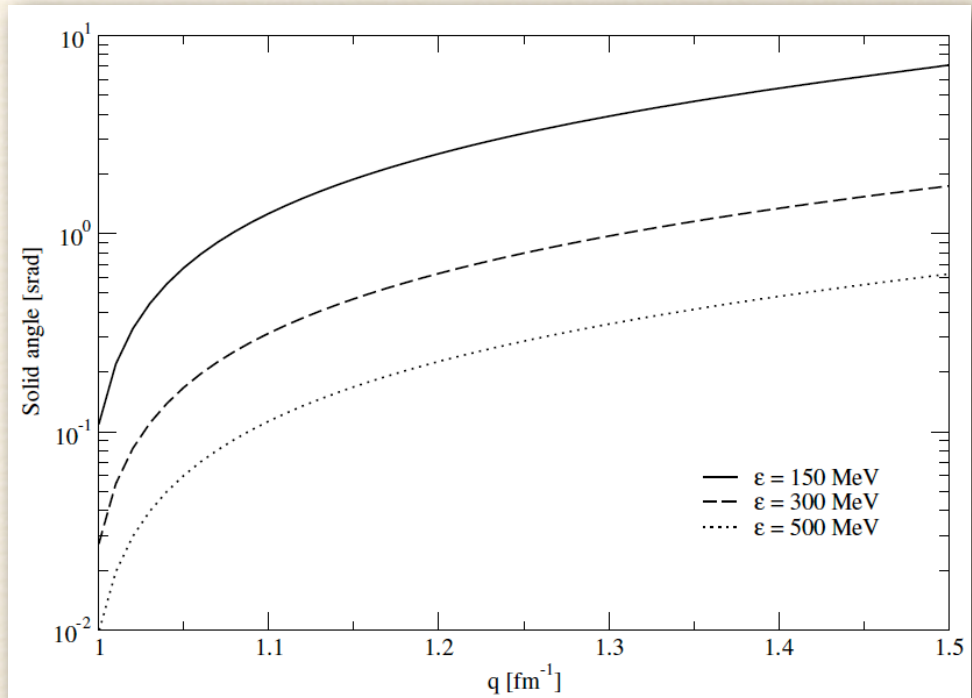


STRATEGY OUTLOOK

Conditions in this momentum transfer region: statistics and solid angle





REFERENCES

- Asymmetry in PVES from nuclei and definition of reference (Standard Model) value:
  G. Feinberg, Phys. Rev. D 12 (1975) 3575 & J. D. Walecka, Nucl. Phys. A 285 (1977) 349.

- Exploration of isospin mixing in PVES from nuclei and proposal to use PV electron scattering to measure the neutron distribution in the nucleus (PREX, CREX, etc.):
  T. W. Donnelly, J. Dubach and I. Sick, Nucl. Phys. A503 (1989) 589.

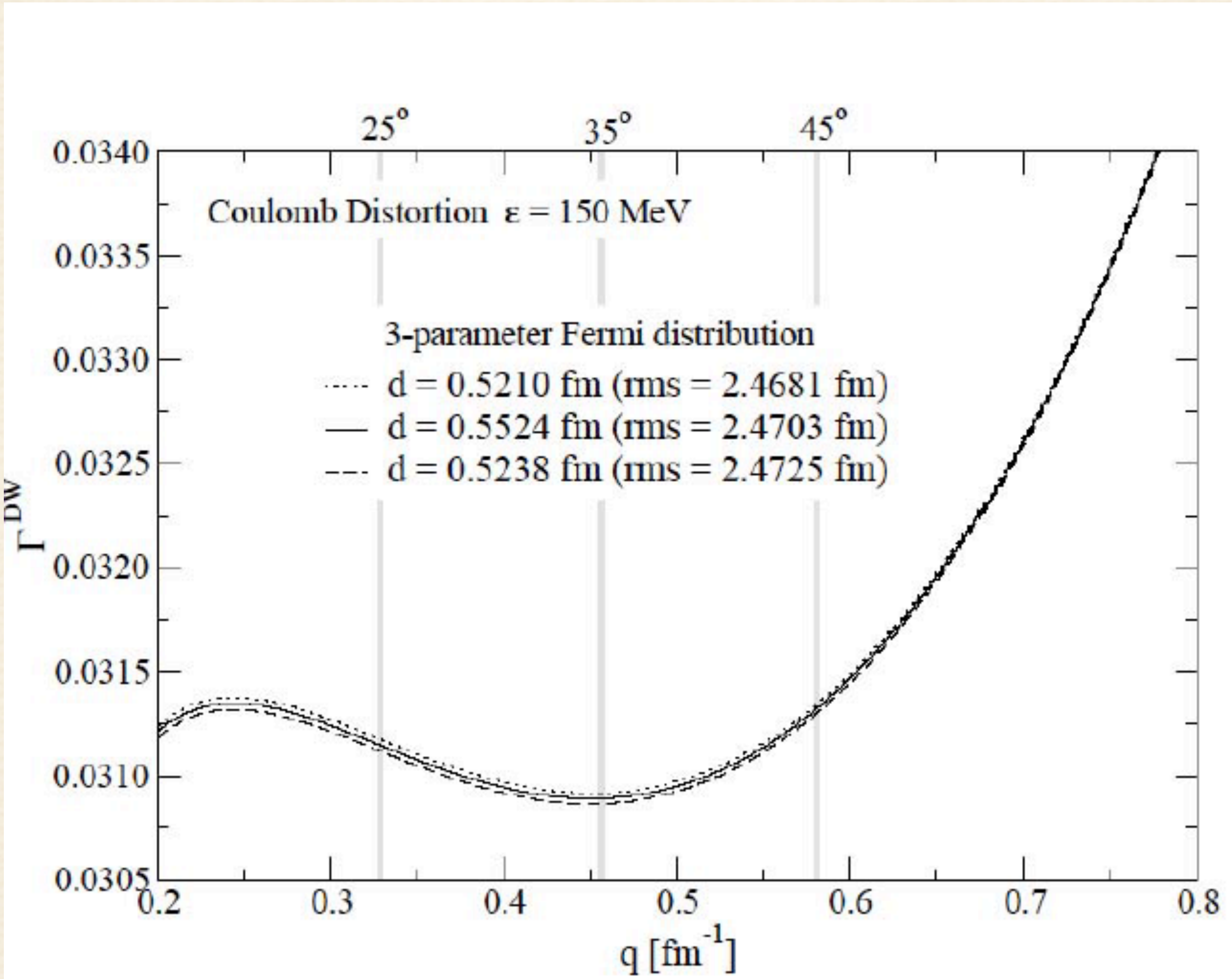
- Extension of the previous study using improved nuclear models of light N=Z nuclei, including carbon 12:
  O. Moreno, P. Sarriguren, E. Moya de Guerra, J. M. Udías, T. W. Donnelly and I. Sick, Nucl. Phys. A828 (2009) 306.

- A first analysis of nuclear uncertainties in PVES from carbon 12:
  T. W. Donnelly and O. Moreno, AIP Conference Proceedings Series, PEB Workshop at MIT, March 2013 (to be published). [A more detailed version of it in preparation.]

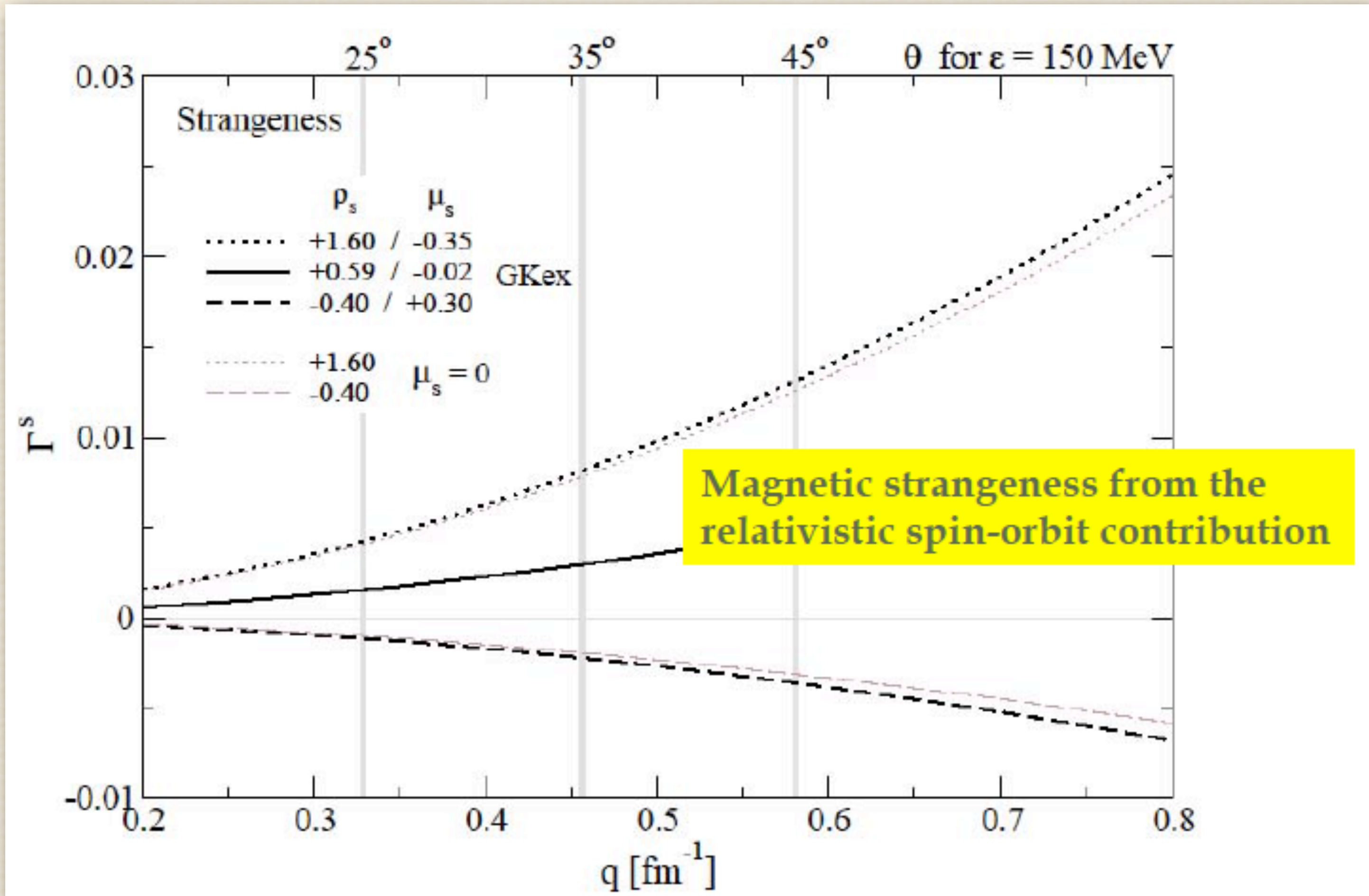
- World data analysis of strangeness content parameters extracted from PVES experiments:
  R. González-Jiménez, J. A. Caballero, T. W. Donnelly, Phys. Rep. 524 (2013) 1.

Extra material

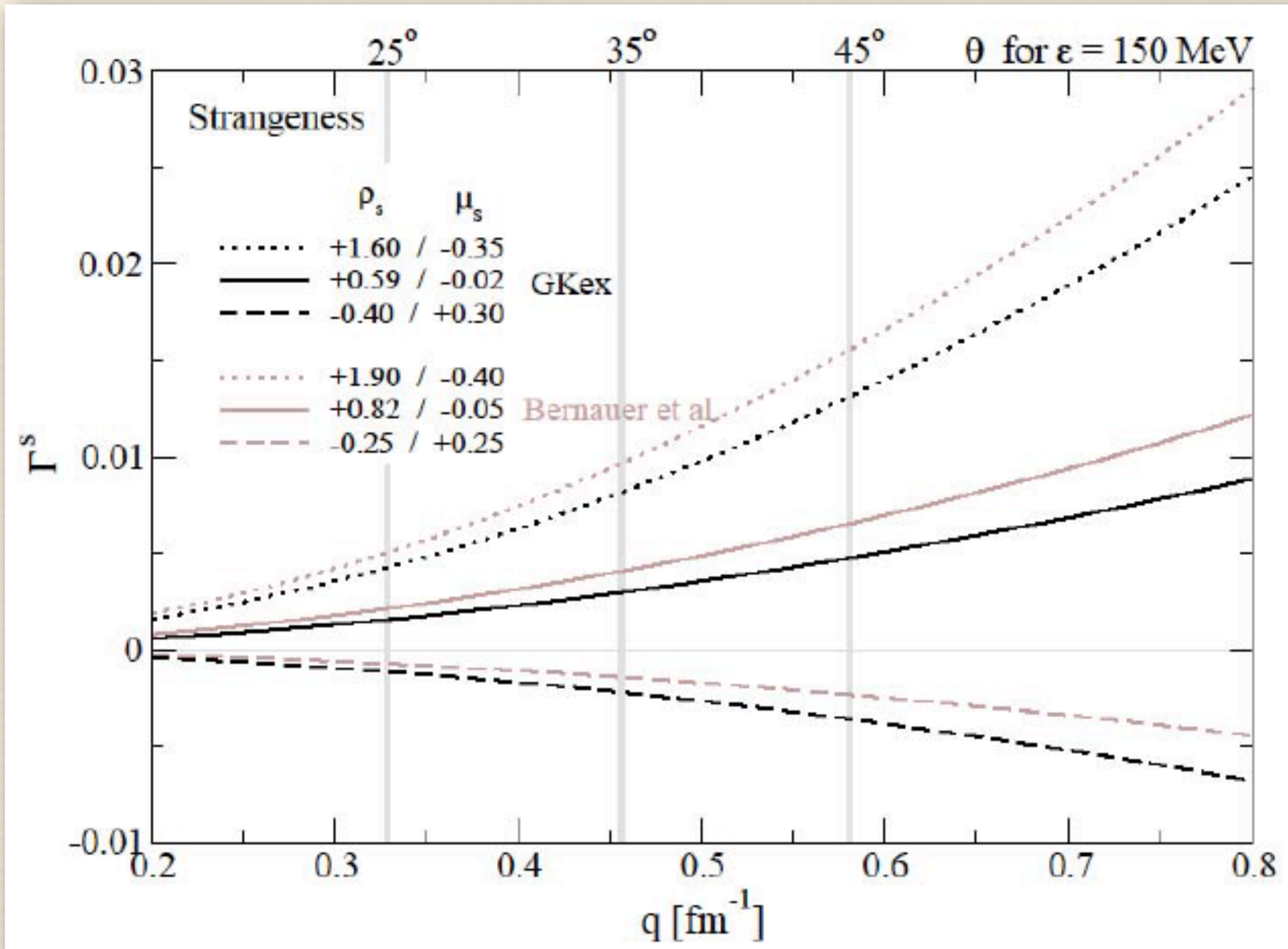
Deviation due to Coulomb distortion for nuclear charge distributions differing in the diffuseness parameter of the Fermi distribution



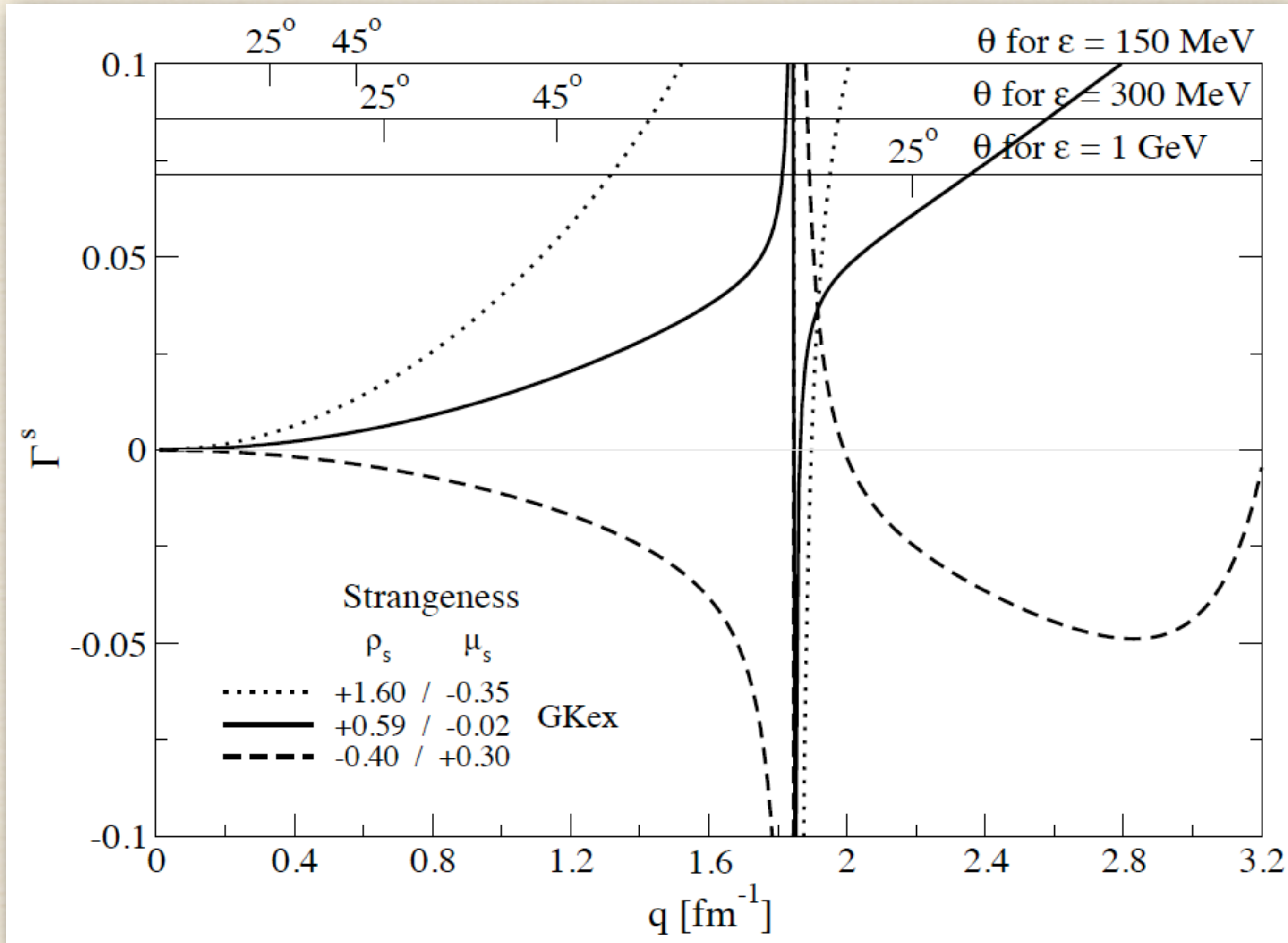
Contribution of magnetic strangeness content (through spin-orbit correction in operator)



Strangeness contribution for two different extractions of experimental ranges of strangeness content parameters



Strangeness contribution within a large range of momentum transfer



Inelastic transitions

Theoretical uncertainties in the PV asymmetry due to possible excitations of carbon 12 in the scattering process.

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T = 0 levels:
Same PV asymmetry as for ground state

(in absence of strangeness and at tree-level)

$$X^{PV} = a_A \left(V_L^{VV} W_L^{VV} + V_T^{VV} W_T^{VV} \right) + a_V V_{T'}^{AV} W_{T'}^{AV}$$

$$X^{PC} = V_L^{VV} W_L^{EM\,VV} + V_T^{VV} W_T^{EM\,VV}$$

Inelastic transitions

Theoretical uncertainties in the PV asymmetry due to possible excitations of carbon 12 in the scattering process.

Consider a situation where all states below the T=1 15.11 MeV state must be summed

Starting point: Given

(1) no isospin mixing (no isoscalar/isovector differences)
(2) no strangeness (implying only non-strange isoscalar currents enter)
(3) tree level WNC couplings (no isoscalar axial-vector coupling, i.e. in T')

… then all of the states (being T=0) have the same asymmetry as the ground state and so the answer for the total is the same

However, with strangeness (and isospin mixing) the dependences in the L and T contributions (still assuming no T' contributions) are different.

For example, in the T responses there are both convection and magnetization currents and so both **electric** and **magnetic** strangeness effects occur at leading order