# Light muonic atoms in search of new interactions at the Compton wavelength scale

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# Proton charge radius puzzle

- global fit to H and D spectrum: $r_p = 0.8758(77)$ fm (CODATA 2010)
- $e - p$ scattering: $r_p = 0.8791(79)$ (Bernauer, 2010)
- from muonic hydrogen: $r_p = 0.84089(39)$ fm (PSI, 2010, 2012)

There is no widely accepted explanation for this discrepancy

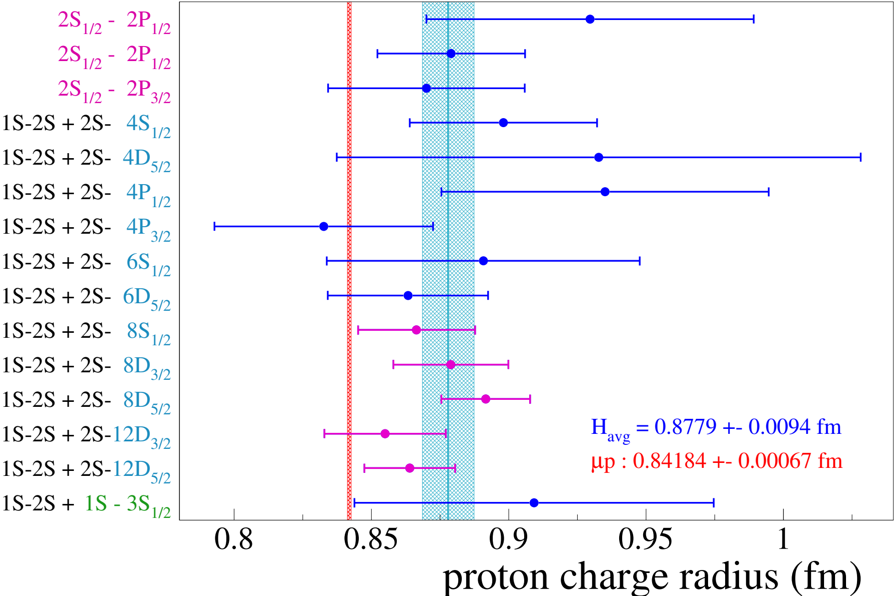
# Proton charge radius and the Rydberg

- Hydrogenic energy levels depend on $R_\infty$, $r_p$ and other constants which uncertainties are irrelevant.

$$E = R_\infty f(\alpha, m_e/m_p) + \frac{2\pi\alpha}{3}\phi^2(0)\langle r_p^2\rangle$$

- Energy shift due to finite nuclear size depends mainly on $r^2$, the mean square nuclear charge radius.
- The remainder $\sim r^3$ is negligible for light (electronic) atoms, but not for muonic atoms !
- One fits two constants $R_\infty$, and $r_p$ to match the well known hydrogen $1S - 2S$ with the other transition
- $He^+$ project: Kield Eikema, Amsterdam

# Hydrogen and $r_p$: Pohl *et al.*, 2013



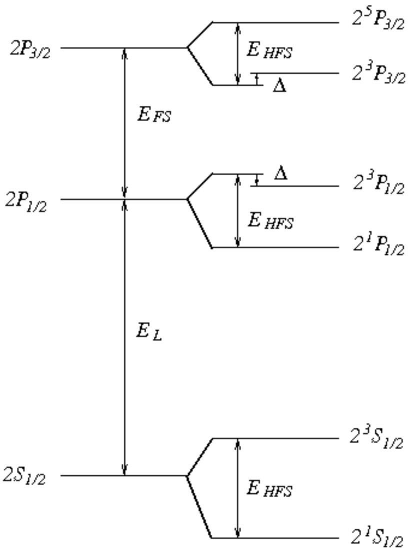
# Proton charge radius puzzle

- Is it obvious that the Standard Model predicts the same $e-p$ and $\mu-p$ interaction at the 1fm scale ?
- If $e-p$ experiments and $\mu H$ theory are correct the plausible solution of this puzzle is the additional interaction at the 1 fm or the electron Compton wavelength scales

How it can be verified ?

Let us say few words about $\mu H$ theory, why is it so reliable.

# energy levels of $\mu$H




$E_L = 202.1$ meV

$E_{FS} = 8.4$ meV

$E_{HFS}(2S_{1/2}) = 22.7$ meV

$E_{HFS}(2P_{1/2}) = 8.0$ meV

$E_{HFS}(2P_{3/2}) = 3.4$ meV

$\Delta = 0.1$ meV

# $\mu H$ energy levels

- $\mu H$ is essentially a nonrelativistic atomic system
- muon and proton are treated on the same footing
- $m_\mu/m_e = 206.768 \Rightarrow \beta = m_e/(\mu\,\alpha) = 0.737$
  the ratio of the Bohr radius to the electron Compton wavelength
- the electron vacuum polarization dominates the Lamb shift in muonic hydrogen

# Theory of $\mu H$ energy levels

- nonrelativistic Hamiltonian $H_0 = \frac{p^2}{2\,m_\mu} + \frac{p^2}{2\,m_p} - \frac{\alpha}{r}$
- and the nonrelativistic energy $E_0 = -\frac{m_r\,\alpha^2}{2\,n^2}$
- the evp dominates the Lamb shift

$$
\begin{aligned}
E_L = \int d^3r\, V_{vp}(r)\,(\rho_{2P} - \rho_{2S}) &= 205.007\,3\,\mathrm{meV} \\
\text{without finite size} &= 206.033\,6(5)\,\mathrm{meV}
\end{aligned}
$$

- important corrections: second order, two-loop vacuum polarization, and the muon self-energy
- other corrections are much smaller than the discrepancy of 0.3 meV.

# Leading relativistic correction

Breit-Pauli Hamiltonian

$$
\begin{aligned}
H_{BP} &= H_0 + \delta H_{BP} \\
\delta H_{BP} &= -\frac{p^4}{8\,m_\mu^3} - \frac{p^4}{8\,m_p^3} - \frac{\alpha}{2\,m_\mu\,m_p}\,p^i\left(\frac{\delta^{ij}}{r} + \frac{r^i\,r^j}{r^3}\right)p^j \\
&+ \frac{2\,\pi\,\alpha}{3}\left(\langle r_p^2\rangle + \frac{3}{4\,m_\mu^2} + \frac{3}{4\,m_p^2}\right)\delta^3(r) \\
&+ \frac{2\,\pi\,\alpha}{3\,m_\mu\,m_p}\,g_\mu\,g_p\,\vec{s}_\mu\cdot\vec{s}_p\,\delta^3(r) - \frac{\alpha}{4\,m_\mu\,m_p}\,g_\mu\,g_p\,\frac{s_\mu^i\,s_p^j}{r^3}\left(\delta^{ij} - 3\,\frac{r^i\,r^j}{r^2}\right) \\
&+ \frac{\alpha}{2\,r^3}\,\vec{r}\times\vec{p}\left[\vec{s}_\mu\left(\frac{g_\mu}{m_\mu\,m_p} + \frac{(g_\mu - 1)}{m_\mu^2}\right) + \vec{s}_p\left(\frac{g_p}{m_\mu\,m_p} + \frac{(g_p - 1)}{m_p^2}\right)\right]
\end{aligned}
$$

# Leading relativistic correction

$$\delta_{\rm rel} E_L = \langle 2P_{1/2}|\delta H_{BP}|2P_{1/2}\rangle - \langle 2S_{1/2}|\delta H_{BP}|2S_{1/2}\rangle = \frac{\alpha^4\, m_r^3}{48\, m_p^2} = 0.05747\,\mathrm{meV}$$

- valid for an arbitrary mass ratio
- quite small and highr order relativistic corrections are negligible

# Leading vacuum polarization

$$V_{\rm vp}(r) = -\frac{Z\,\alpha}{r}\,\frac{\alpha}{\pi}\int_4^\infty \frac{d(q^2)}{q^2}\,e^{-m_e\,q\,r}\,u(q^2)$$

$$u(q^2) = \frac{1}{3}\sqrt{1-\frac{4}{q^2}}\left(1+\frac{2}{q^2}\right)$$

$$\delta_{\rm vp}E_L = \langle 2P_{1/2}|V_{\rm vp}|2P_{1/2}\rangle - \langle 2S_{1/2}|V_{\rm vp}|2S_{1/2}\rangle = 205.0073\,\mathrm{meV}$$

- the dominating part of the muonic hydrogen Lamb shift
- the expectation value is taken with nonrelativistic wave function
- the muon-proton mass ratio $\eta$ is included exactly

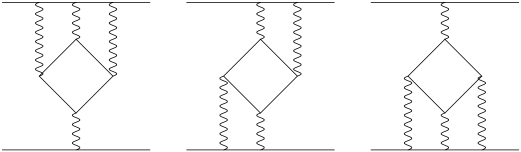
# Higher order vacuum polarization

- second order $V_{\rm vp}$: $\delta E_L = 0.1509$ meV
- two-loop vp: $\delta E_L = 1.5081$ meV
- three-loop vp: $\delta E_L = 0.0053$ meV
- hadronic vp: $\delta E_L = 0.0112(4)$ meV

Muonic vp is included later together with the self-energy

Is there any further corrrection related to vp ?

# Light by light diagrams



- $\delta E_L = -0.0009$ meV
- significant cancellation between diagrams
- S.G. Karshenboim *et al.*, arXiv:1005.4880

# Small corrections

- relativistic correction to vp

$$\delta_{\rm vp,rel} E_L = \langle \delta_{\rm vp} H_{BP} \rangle + 2 \left\langle V_{\rm vp} \frac{1}{(E-H)'} H_{\rm BP} \right\rangle = 0.01876\,{\rm meV}.$$

  If one used the Dirac equation in the infinite nuclear mass limit, the obtained result would be 0.021 meV

- muon self-energy and muon vp: $\delta E_L = -0.6677$ meV
- muon self-energy combined with evp: $\delta E_L = -0.0025$ meV
- pure recoil corrections of order $\alpha^5$: $\delta E_{LS} = -0.0450\,{\rm meV}$

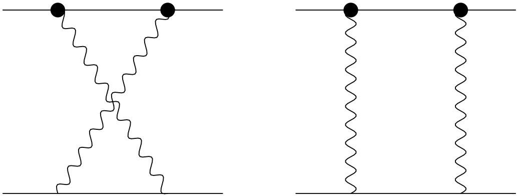
# Summary of theoretical predictions

$$
\begin{aligned}
\Delta E_{\mathrm{LS}} &= 206.0336(15) - 5.2275(10)\, r_p^2 + \Delta E_{\mathrm{TPE}} \\
\Delta E_{\mathrm{FS}} &= 8.3521\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2S_{1/2}} &= 22.8089(51)\,\mathrm{meV}, \quad \text{(exp. value)} \\
\Delta E_{\mathrm{HFS}}^{2P_{1/2}} &= 7.9644\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2P_{3/2}} &= 3.3926\,\mathrm{meV} \\
\Delta &= 0.1446\,\mathrm{meV}
\end{aligned}
$$

where $\Delta E_{\mathrm{TPE}} = 0.0351(20)$ meV is a proton structure dependent two-photon exchange contribution, on the next slide…

# Nuclear structure effects

- if nuclear excitation energy is much larger than the atomic energy, the two-photon exchange scattering amplitude gives the dominating correction
- the total proton structure contribution $\delta E_L = 0.035\,1(20)$ meV is much too small to explain the discrepancy, but its calculation is not very certain [Carlson, Vanderhaeghen, 2011]



# Lepton-proton interaction at the 1 fm scale

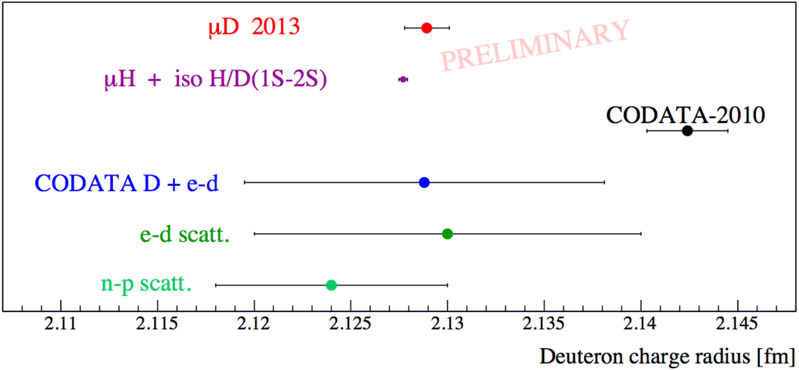
- Question: How to test universality of the lepton-proton interaction ?
- Answer: compare $e-p$ with $\mu-p$ scattering: MUSE project, old $\mu-p$ Brookhaven scatering data (1969) are not conclusive
- Answer: $\mu^4$He and $\mu^3$He measurements, if discrepancy persists, is should be parametrized by

$$\delta E = (Z\,\delta r_p^2 + (A-Z)\,\delta r_n^2)\,\frac{2\,\delta_{l0}}{3\,n^3}\,Z^3\,\alpha^4\,\mu^3$$

- $\mu$D $\Rightarrow \delta r_n^2 = 0$ !!!

$$\delta E = \delta r_p^2\,\frac{2\,\delta_{l0}}{3\,n^3}\,Z^4\,\alpha^4\,\mu^3$$

# Muonic deuterium: preliminary results

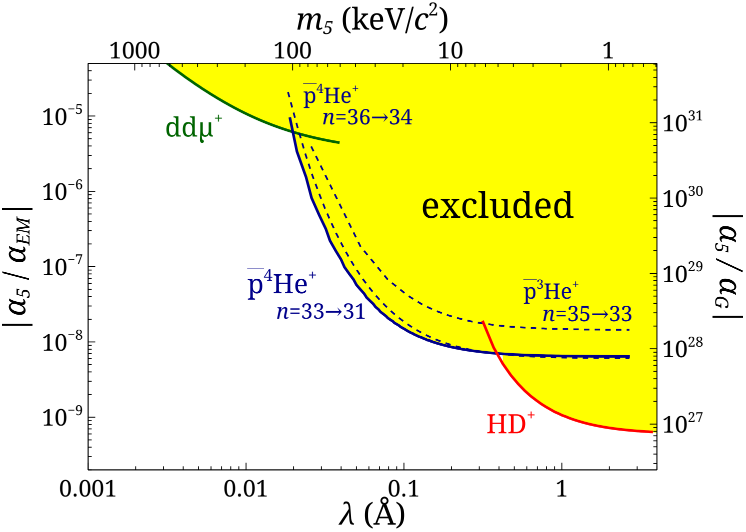



Randolf Pohl, PSI workshop, September 10, 2013.

# 5th force at the Compton wavelength scale

- $\mu$H is very sensitive to the electron vacuum polarisation:
  for H
  $$\frac{E_{\text{discrepancy}}}{E_{\text{Uehling}}} = \frac{94\,\text{kHz}}{216\,676\,\text{kHz}} = 0.00043$$
  for $\mu$H
  $$\frac{E_{\text{discrepancy}}}{E_{\text{Uehling}}} = \frac{0.31\,\text{meV}}{205.0073\,\text{meV}} = 0.0015$$
- This means that a small modification of $V_{\text{vp}}$ may explain discrepancy as the change in $\mu$H is 4 times larger than in H.
- Are there any measurements sensitive to 5th force at the electron Compton wavelength ?

# fifth force: Salumbides, Ubachs, Korobov (2013)



# $\mu$He proposal

- Measurement of 2S-2P transition in $\mu$He (F. Kottman)
- Calculations easier than in $\mu$H: no hfs
- $r_{\rm He} = 1.681(4)$ fm from the electron scattering
- $\mu^4$He nuclear polarizability correction: S. Schlesser at this Workshop and N. Barnea *et al*, soon in PRL

# Proposal to determine $\alpha$ charge radius from the atomic spectroscopy

- $E(2^3S - 2^3P, {}^4\mathrm{He})_{\mathrm{centroid}} = 276\,736\,495\,649.5(2.1)$ kHz, Florence, 2004
- finite size effect: $E_{\mathrm{fs}} = 3\,387$ kHz
- since $E_{\mathrm{fs}}$ is proportional to $r^2$

$$\frac{\Delta r}{r} = \frac{1}{2}\frac{\delta E_{\mathrm{fs}}}{E_{\mathrm{fs}}} \approx \frac{1}{2}\frac{10}{3\,387} = 1.5 \cdot 10^{-3}$$

- electron scattering gives $r_{\mathrm{He}} = 1.681(4)$ fm, what corresponds to about $2.5 \cdot 10^{-3}$ relative accuracy
- can theoretical predictions be accurate enough $\sim 10$ kHz ?