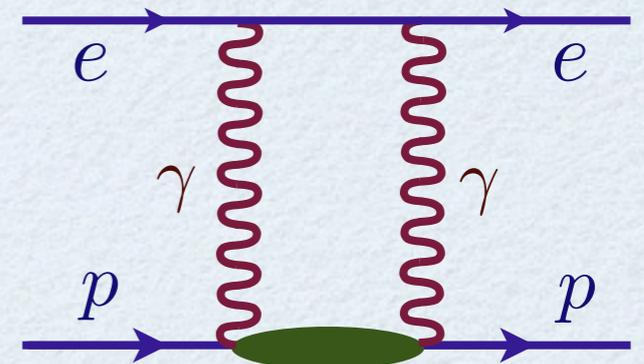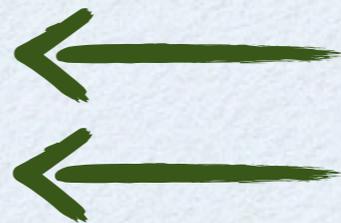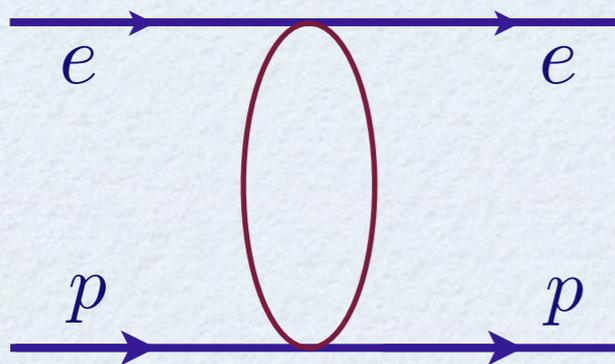# *Two-photon exchange corrections in elastic ep scattering. Dispersive framework*

Oleksandr Tomalak,

Johannes Gutenberg University,

Mainz, Germany

Outline

- Introduction and motivation

- Dispersion relation framework for 2γ corrections
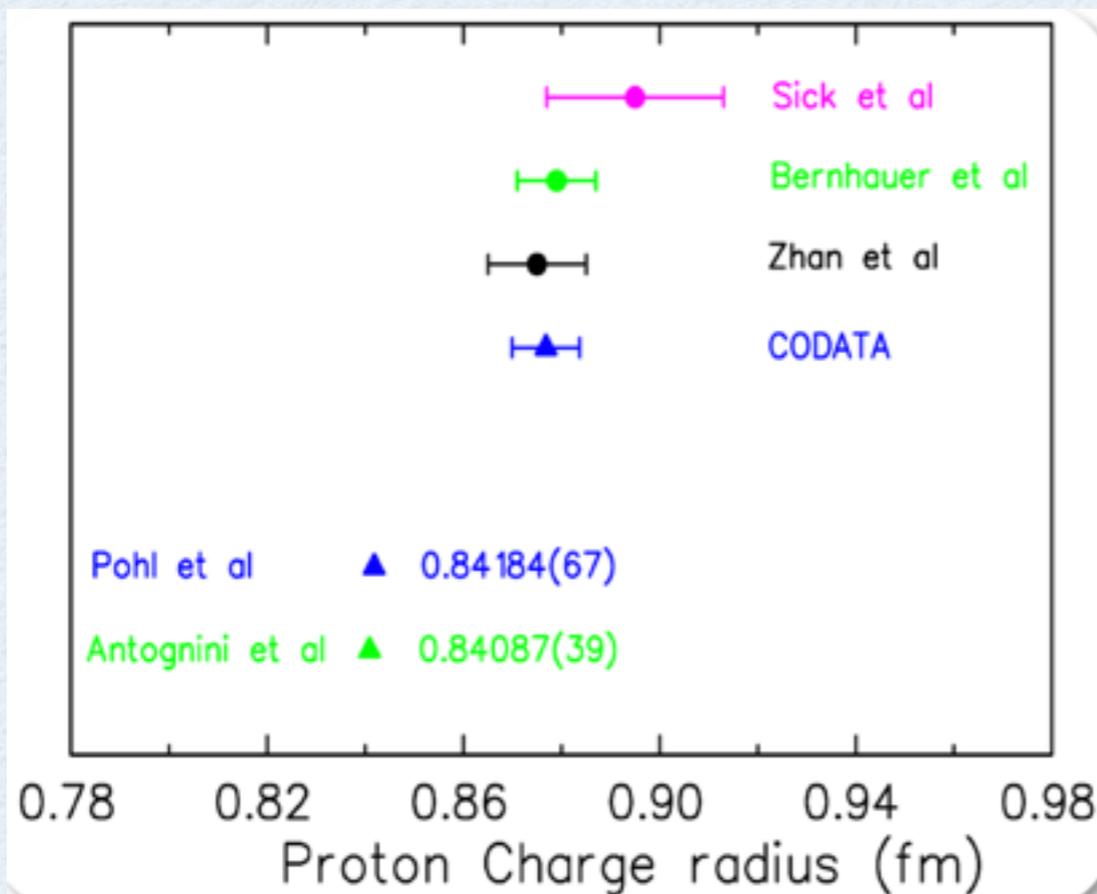
- Results for elastic ep-scattering

- Plans

Proton radius puzzle

$$\mu\mathbf{H}\ \mathbf{data} - R_E = 0.8409 \pm 0.0004\, fm$$

$$\mathbf{ep}\ \mathbf{data} - R_E = 0.8772 \pm 0.0046\, fm$$

$$\mathbf{7.7}\sigma\ \mathbf{difference}\ !$$

Beyond Standard Model physics ?

New aspects of nucleon structure ?

Underestimated uncertainties of ep data ?
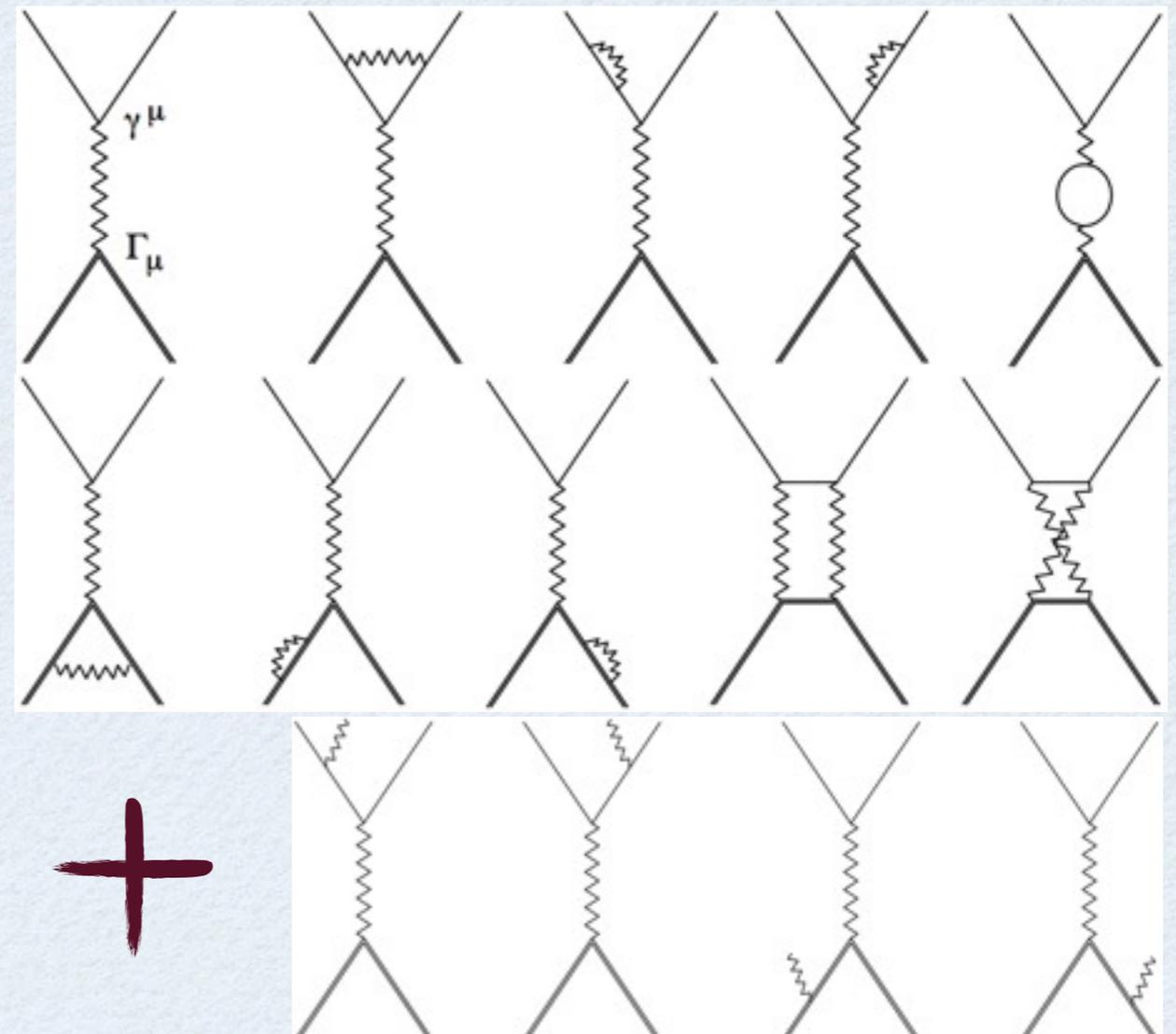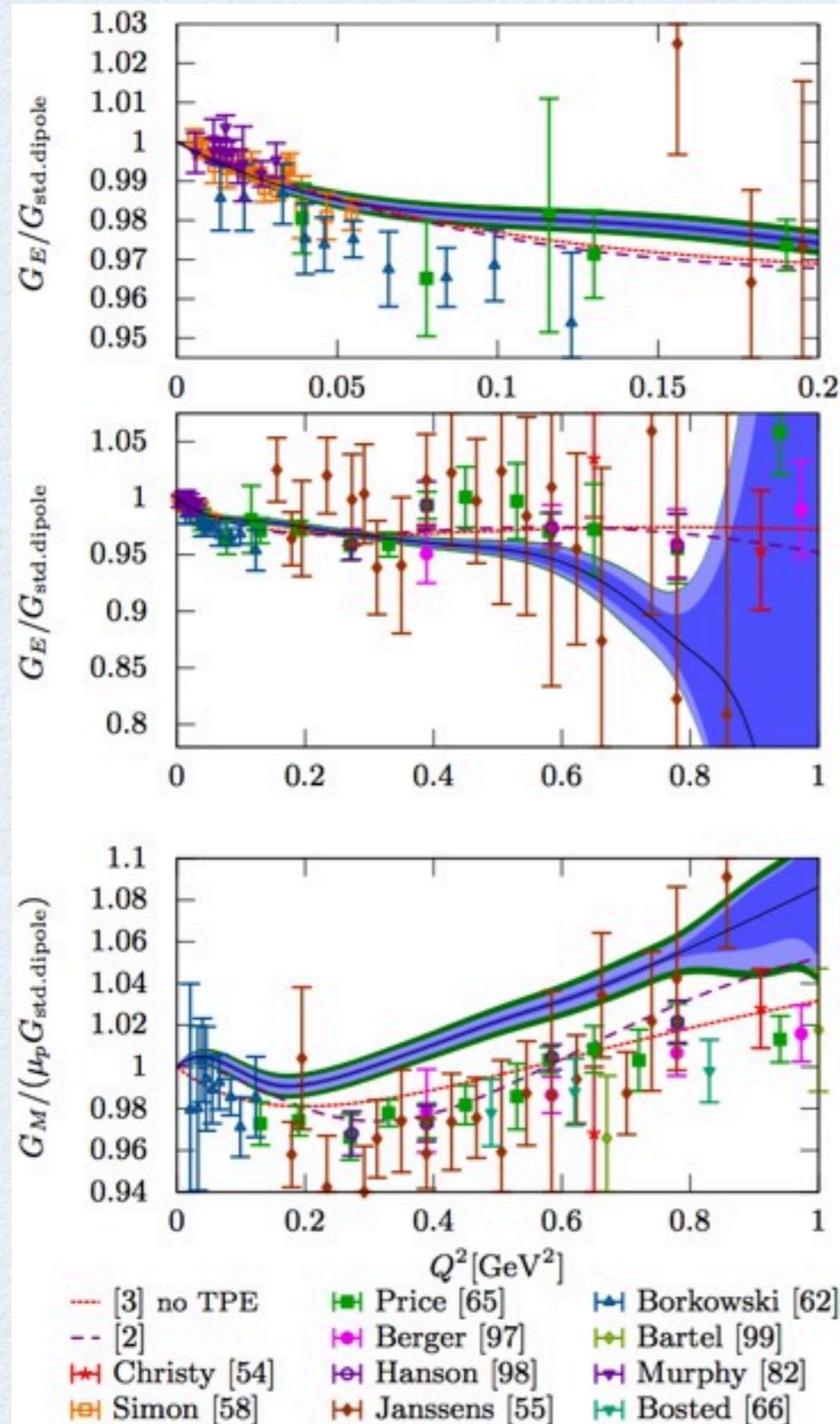
Experimental status

Bernauer et al. (2013)

A1@MAMI fit of measured EM form factors normalized to dipole form

$$G_E(Q^2) = \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2}, \quad G_M(Q^2) = \frac{\mu_p}{(1 + \frac{Q^2}{\Lambda^2})^2} \quad \begin{array}{l} \Lambda^2 = 0.71 GeV^2 \\ \mu_p = 2.793 \end{array}$$
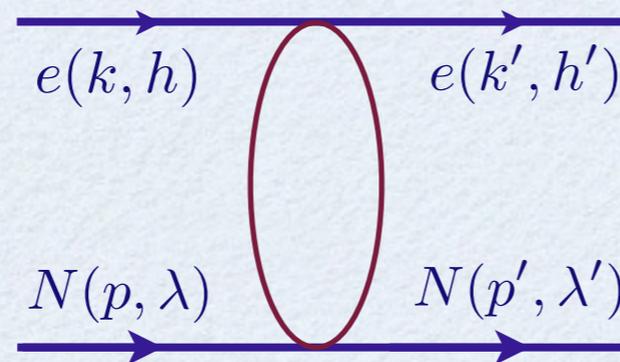
TPE corrections

$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

Structure amplitudes. TPE correction

$$P = \frac{p + p'}{2}$$

$$K = \frac{k + k'}{2}$$

$e(k, h)$ $e(k', h')$

$N(p, \lambda)$ $N(p', \lambda')$

$$t = (k - k')^2 \qquad u = (k - p')^2$$

$$s = (p + k)^2 \qquad \nu = \frac{s - u}{4}$$
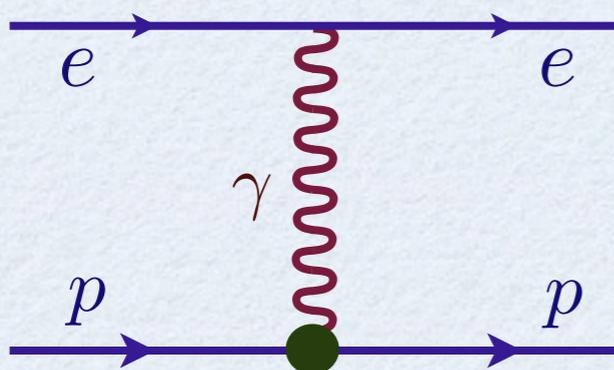
Discrete symmetries $+$ $m_e = 0$ $\longrightarrow$ 3 structure amplitudes

$$T = \frac{e^2}{Q^2} \bar{e}(k', h')\gamma_\mu e(k, h) . \bar{N}(p', \lambda')[\mathcal{G}_M(\nu, t)\gamma^\mu - \mathcal{F}_2(\nu, t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu, t)\frac{\hat{K}P^\mu}{M^2}]N(p, \lambda)$$
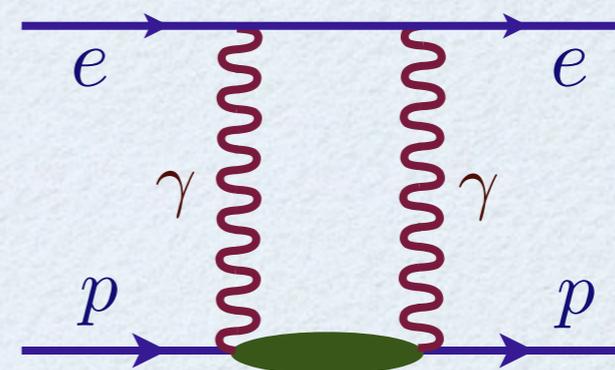
P.A.M. Guichon and M. Vanderhaeghen
Phys. Rev. Lett. 91, 142303 (2003)

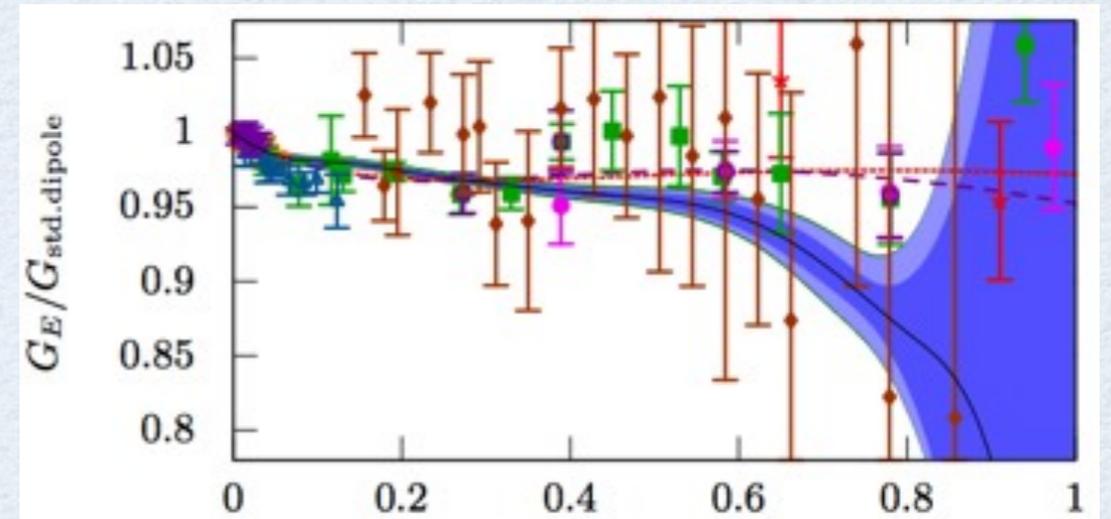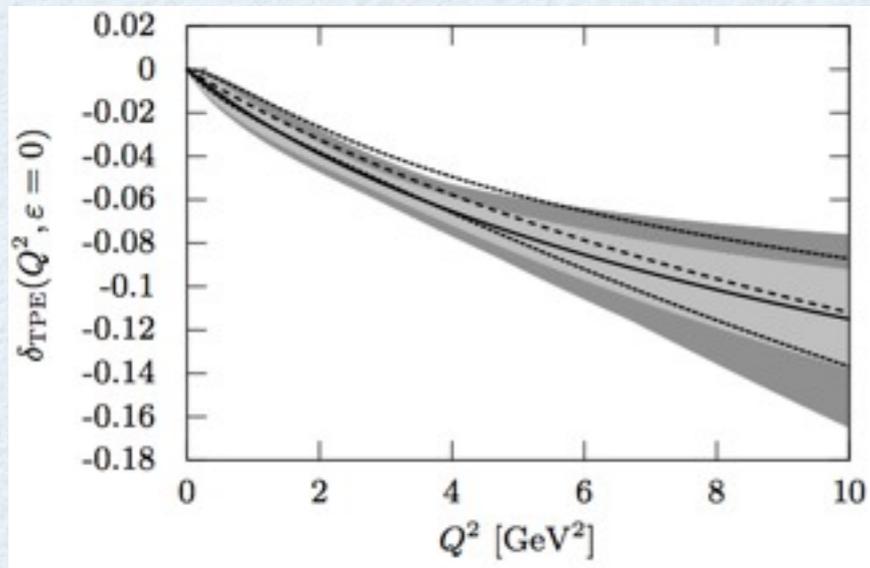## Leading contribution to cross section - interference term

1 photon diagram

e e

γ

p p

2 photon exchange diagram

e e

γ γ

p p

$$\delta_{TPE} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3$$

5

Dispersion relation framework

2γ corrections

$f(z)$

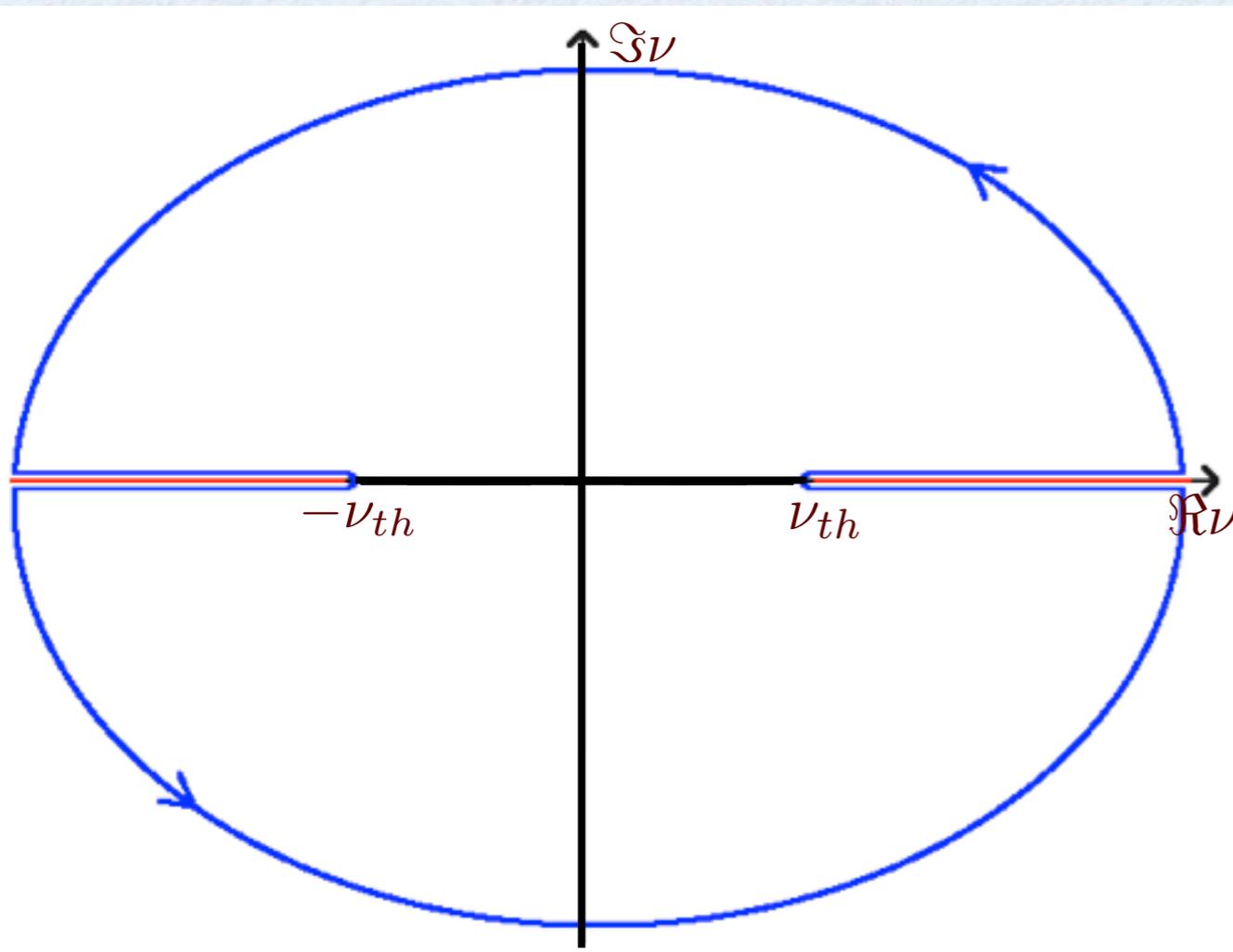analiticity

cross section correction

exp. data/phenomenology

unitarity

DR

amplitudes: real parts

amplitudes: imaginary parts

Fixed-t dispersion relations

Under crossing $\quad \nu \to -\nu$

two-photon exchange amplitudes

$\mathcal{G}_\mathcal{M}(\nu,t), \mathcal{F}_2(\nu,t) \Leftrightarrow \mathcal{G}_1(\nu,t), \mathcal{G}_2(\nu,t)$ are odd

$\mathcal{F}_3(\nu,t) \qquad\qquad$ is even

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3$$

good

$$\mathcal{G}_2 = \mathcal{G}_E + \frac{\nu}{M^2}\mathcal{F}_3$$

HE behavior

Real part can be reconstructed with DRs

$$\Re G^{odd}(\nu,t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{th}}^{\infty} \nu \frac{\Im G^{odd}(\nu'+i0,t)}{\nu'^2 - \nu^2}d\nu'$$

$$\Re G^{even}(\nu,t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{th}}^{\infty} \nu' \frac{\Im G^{even}(\nu'+i0,t)}{\nu'^2 - \nu^2}d\nu'$$

Regge theory gives high energy behavior:

$$\mathcal{G}_M < \nu^0 \qquad\qquad \mathcal{F}_2 < \nu^{-0.4} \qquad\qquad \mathcal{F}_3 < \nu^{-0.92}$$

axial meson $\qquad\qquad\qquad$ ρ-meson $\qquad\qquad\qquad$ pomeron
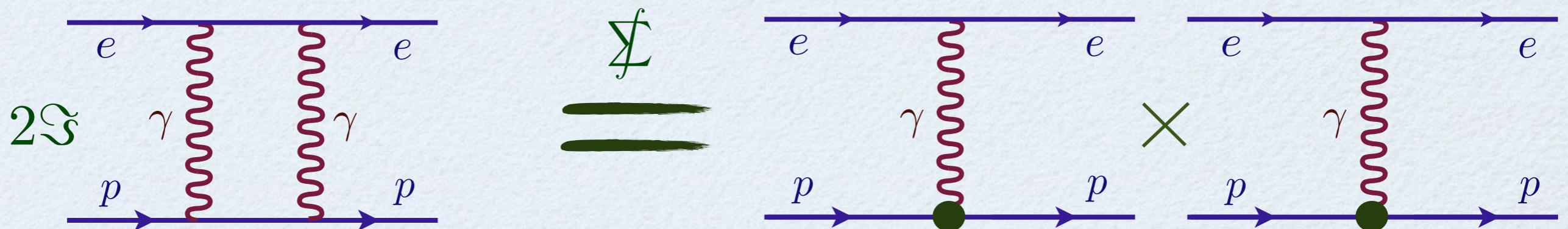
Unitarity relation gives imaginary part

$$S = 1 + iT \qquad S^+ S = 1 \qquad \longrightarrow \qquad \Im T_{h'\lambda', h\lambda}$$

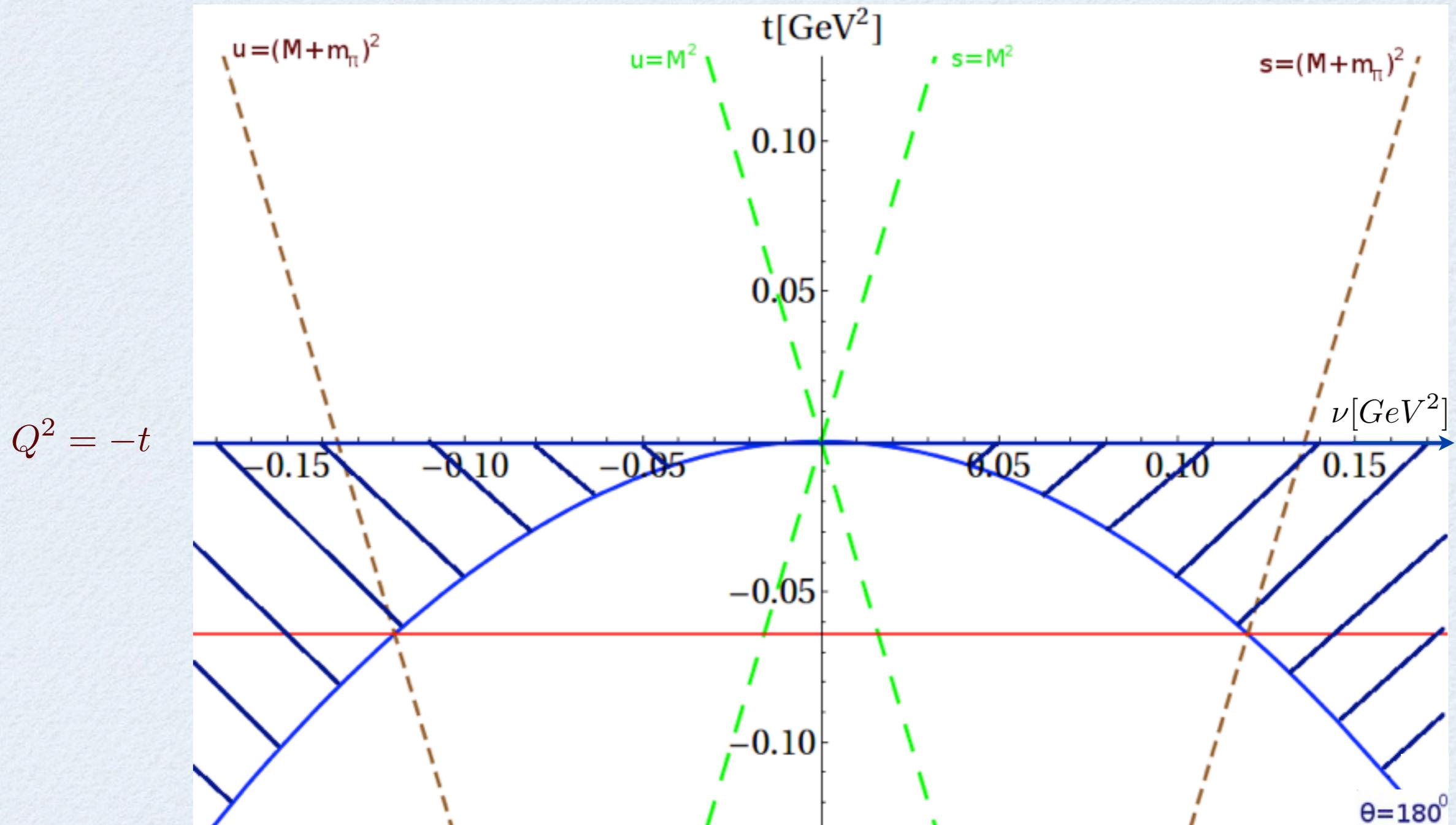### only on-shell information is required

$$2\Im T_{h'\lambda', h\lambda} = \sum\!\!\!\!\!\!\int d\Pi'' T^+_{h'\lambda', \mu} T_{\mu, h\lambda} (2\pi)^4 \delta^4(k + p - \sum_i q_i)$$

e and N intermediate state

on-shell one-photon amplitudes

Kinematic regions

$$Q^2 = m_\pi^2 \left(\frac{2M + m_\pi}{M + m_\pi}\right)^2 \sim 3.5 m_\pi^2$$

- intersection of phys. region and inelastic threshold

Proton intermediate state is outside physical region

Analytical continuation

$$\int d\Omega$$

symmetric coordinates wrt electron momentum transfer

$$\cos\theta_1 = \sqrt{1-\alpha^2}(b\cos\phi + c\sin\phi) \qquad \cos\theta_2 = \sqrt{1-\alpha^2}(b\cos\phi - c\sin\phi)$$

$$2\int_0^1 d\alpha \int_0^{2\pi} d\phi$$

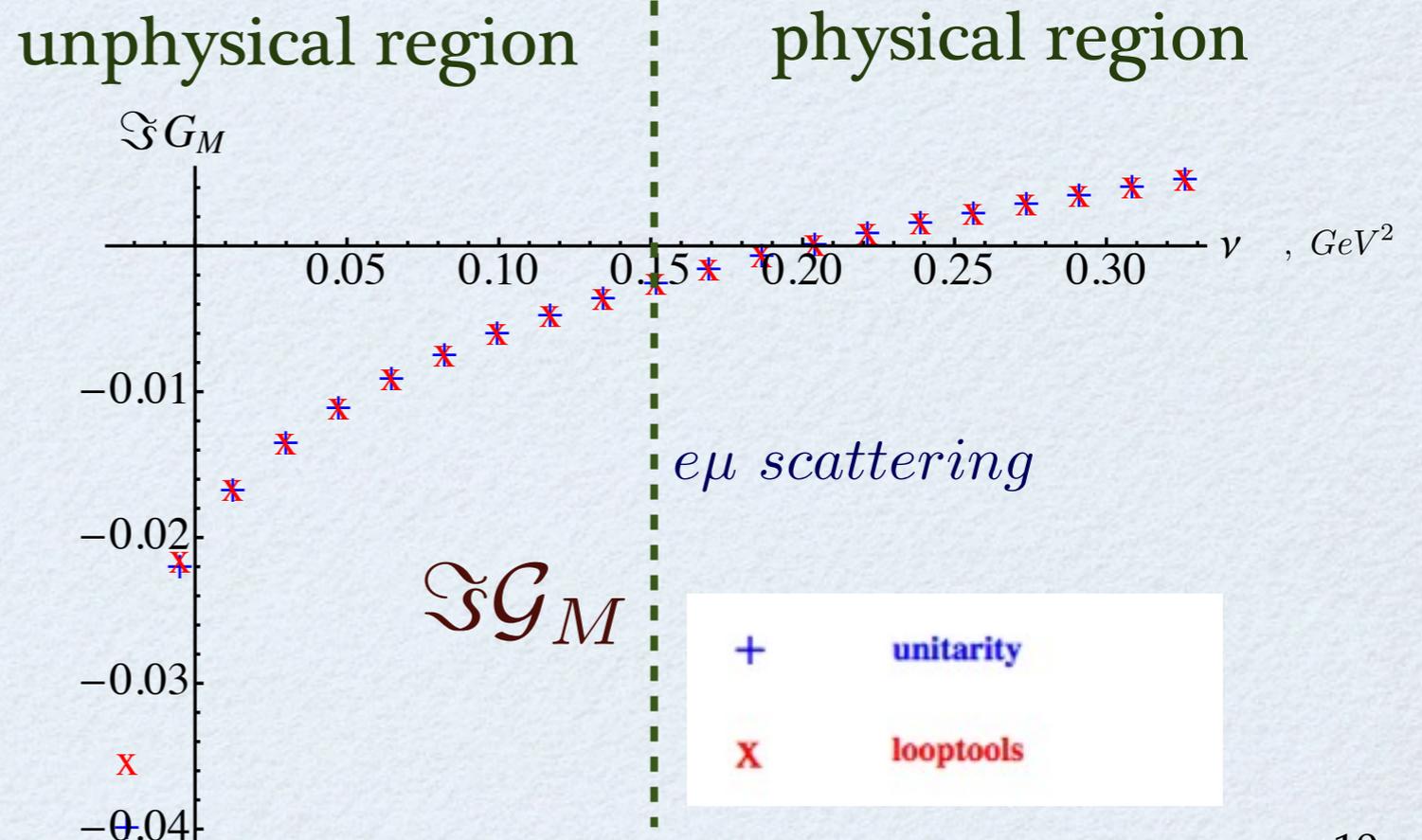→ angular integration to integration on curve in complex plane

→ choose contour keeping poles after transition to unph. region

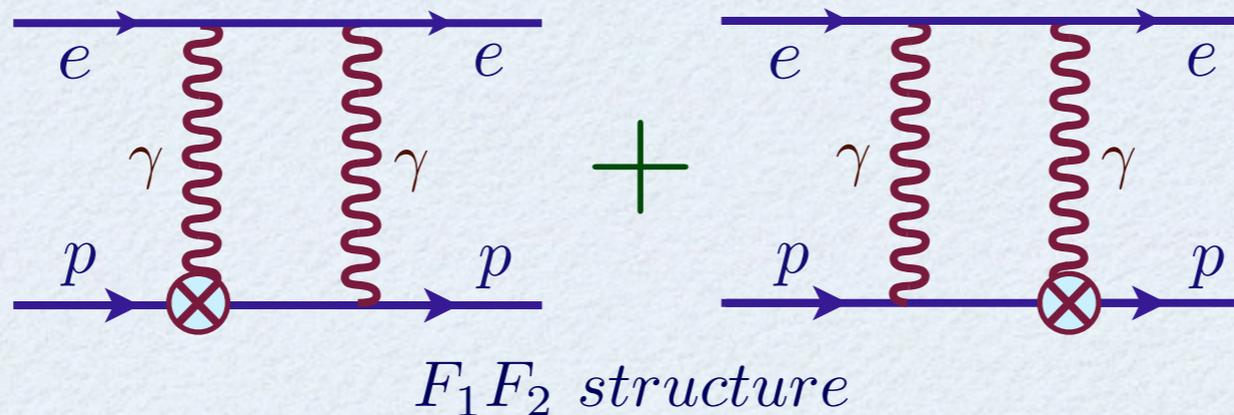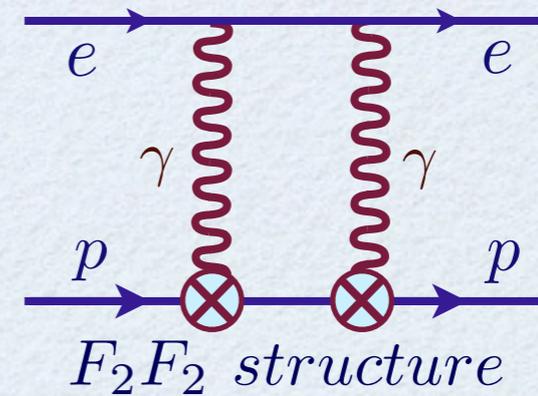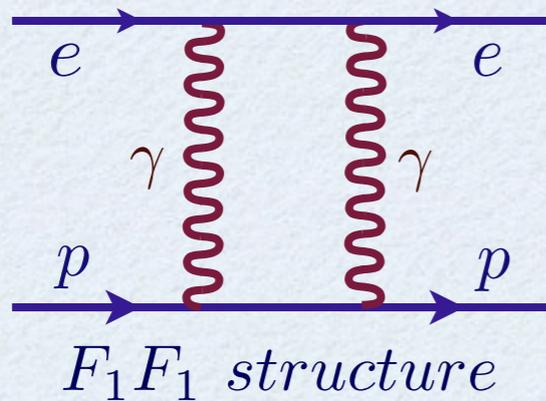Analytical continuation reproduces results in unphysical region

$$Q^2 = 0.1 \; GeV^2$$

$$\nu_{ph} = 0.15 \; GeV^2$$

unphysical region | physical region

$\Im G_M$

$\nu \quad , GeV^2$

$e\mu \; scattering$

$\Im \mathcal{G}_M$

+ unitarity

X looptools

Models with exact results

The one-photon exchange on-shell vertex is described by

$$\Gamma^\mu(Q^2) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2)$$

$F_1 F_1$ *structure*

$F_2 F_2$ *structure*

$F_1 F_2$ *structure*

Point-like couplings $\qquad\longrightarrow\qquad F_1 = 1 \qquad\qquad F_2 = \mu_p - 1$
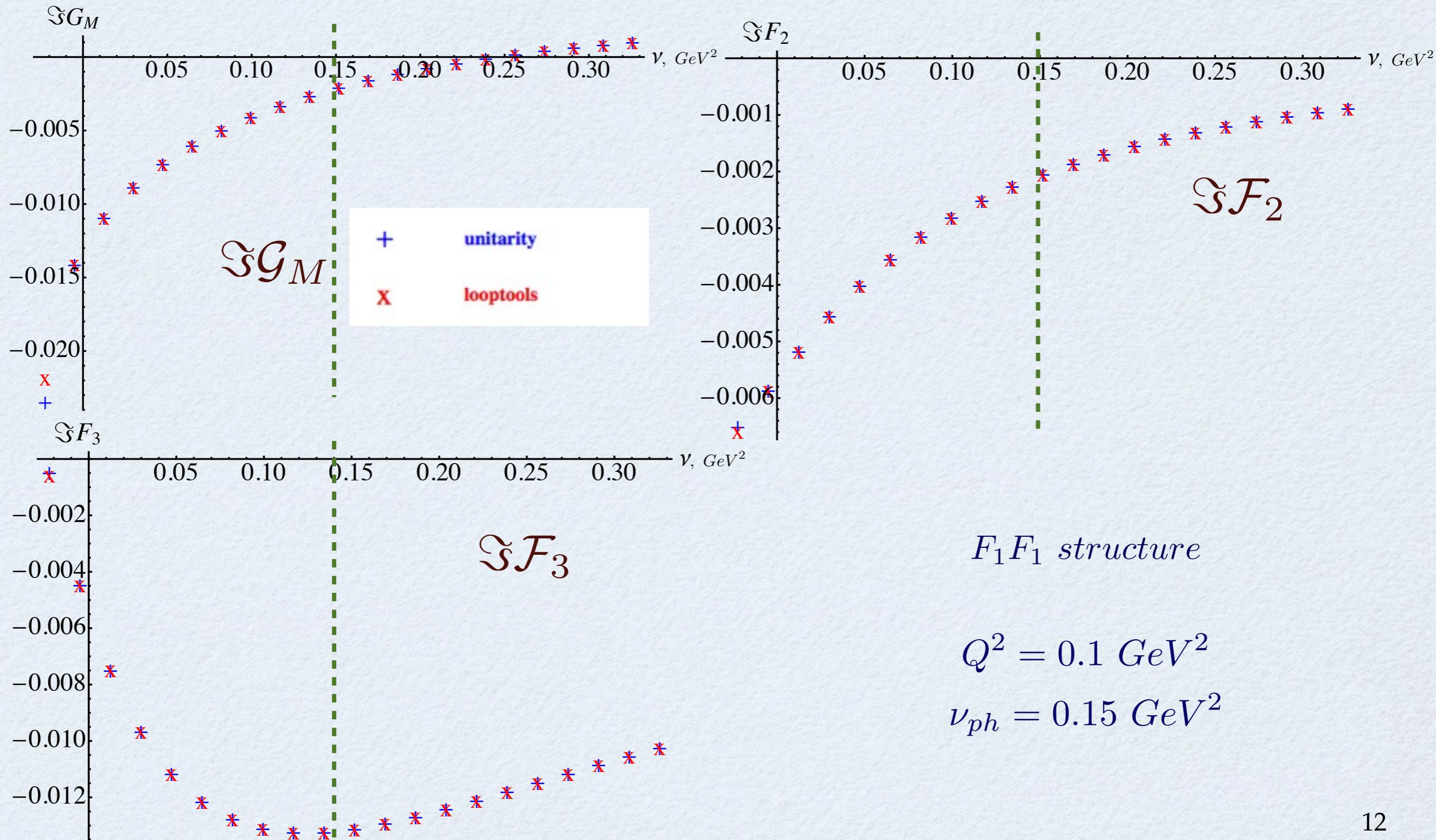
Dipole FFs for $G_M, G_E \qquad\longrightarrow\qquad G_E = F_1 - \tau F_2 \qquad G_M = F_1 + F_2$

$$(\tau = \frac{Q^2}{4M^2})$$

Amplitudes imaginary parts

Dipole form of G_M, G_E

$\Im G_M$

$\nu, \ GeV^2$

$\Im \mathcal{G}_M$

| + | unitarity |
| x | looptools |

$\Im F_2$

$\nu, \ GeV^2$

$\Im \mathcal{F}_2$

$\Im F_3$

$\nu, \ GeV^2$

$\Im \mathcal{F}_3$

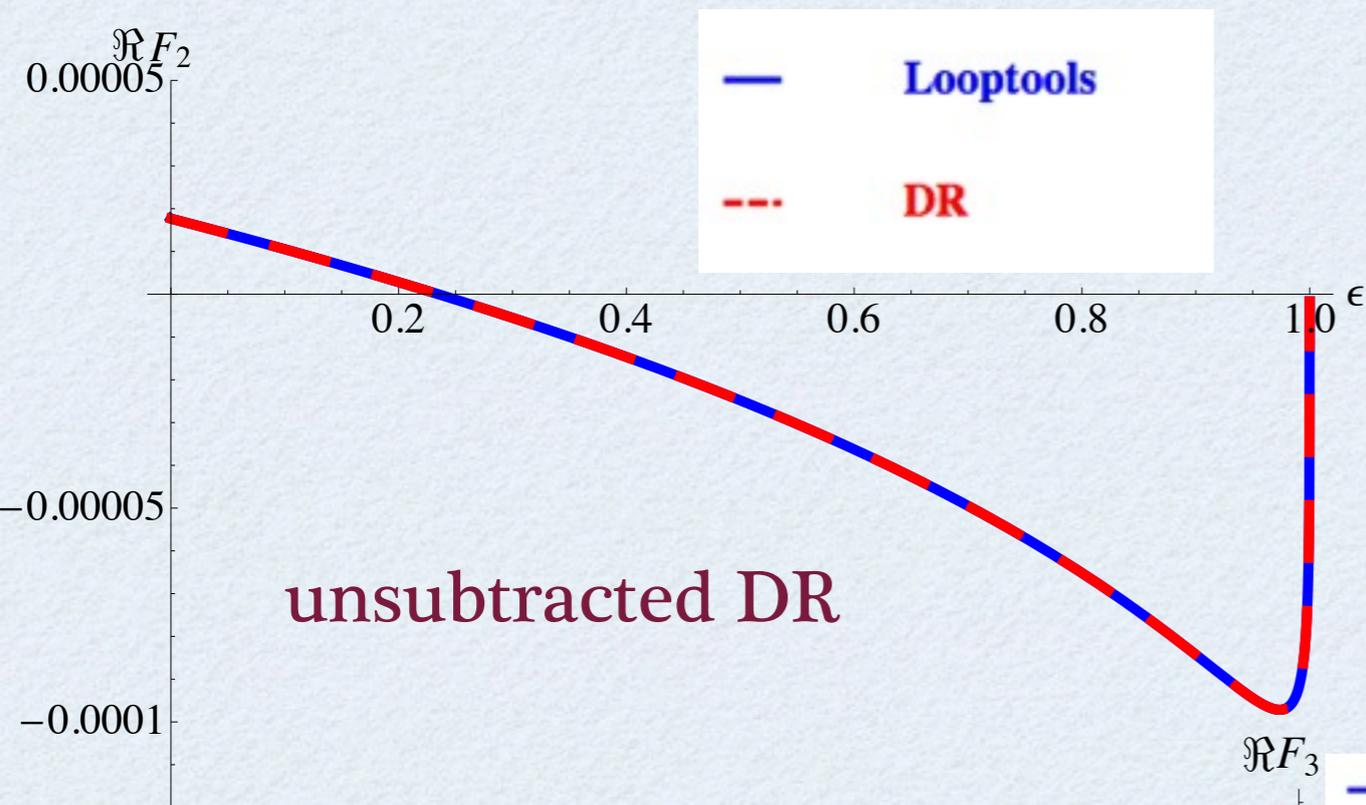$F_1 F_1 \ structure$

$Q^2 = 0.1 \ GeV^2$

$\nu_{ph} = 0.15 \ GeV^2$

Amplitudes real parts

dispersion relation calculation

for point-like couplings

$$\nu = \sqrt{\frac{1+\epsilon}{1-\epsilon}}\,\nu_{ph}$$
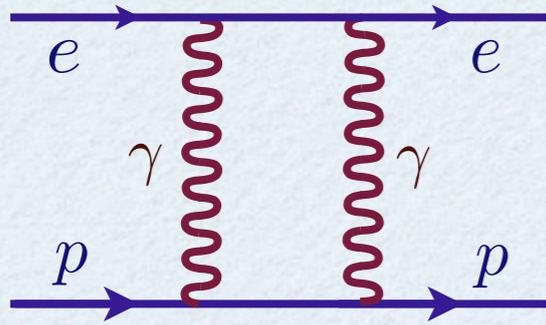
unsubtracted DR

subtracted DR

$Q^2 = 0.1 GeV^2$
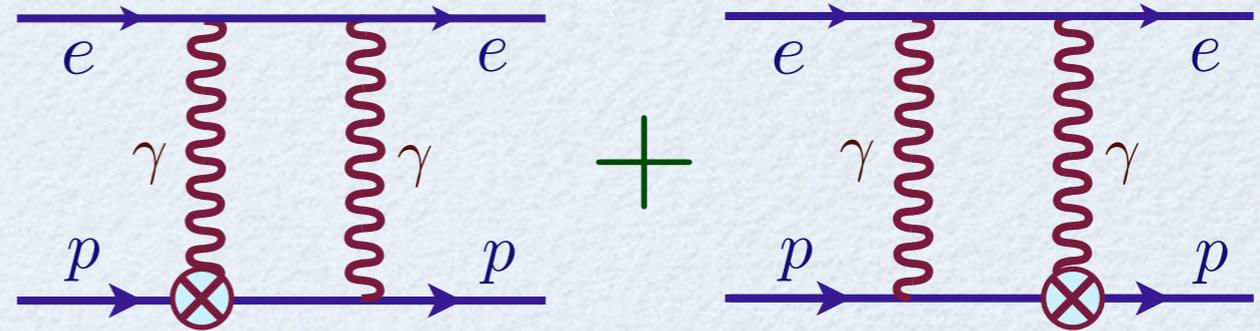
F₂F₂ structure amplitudes
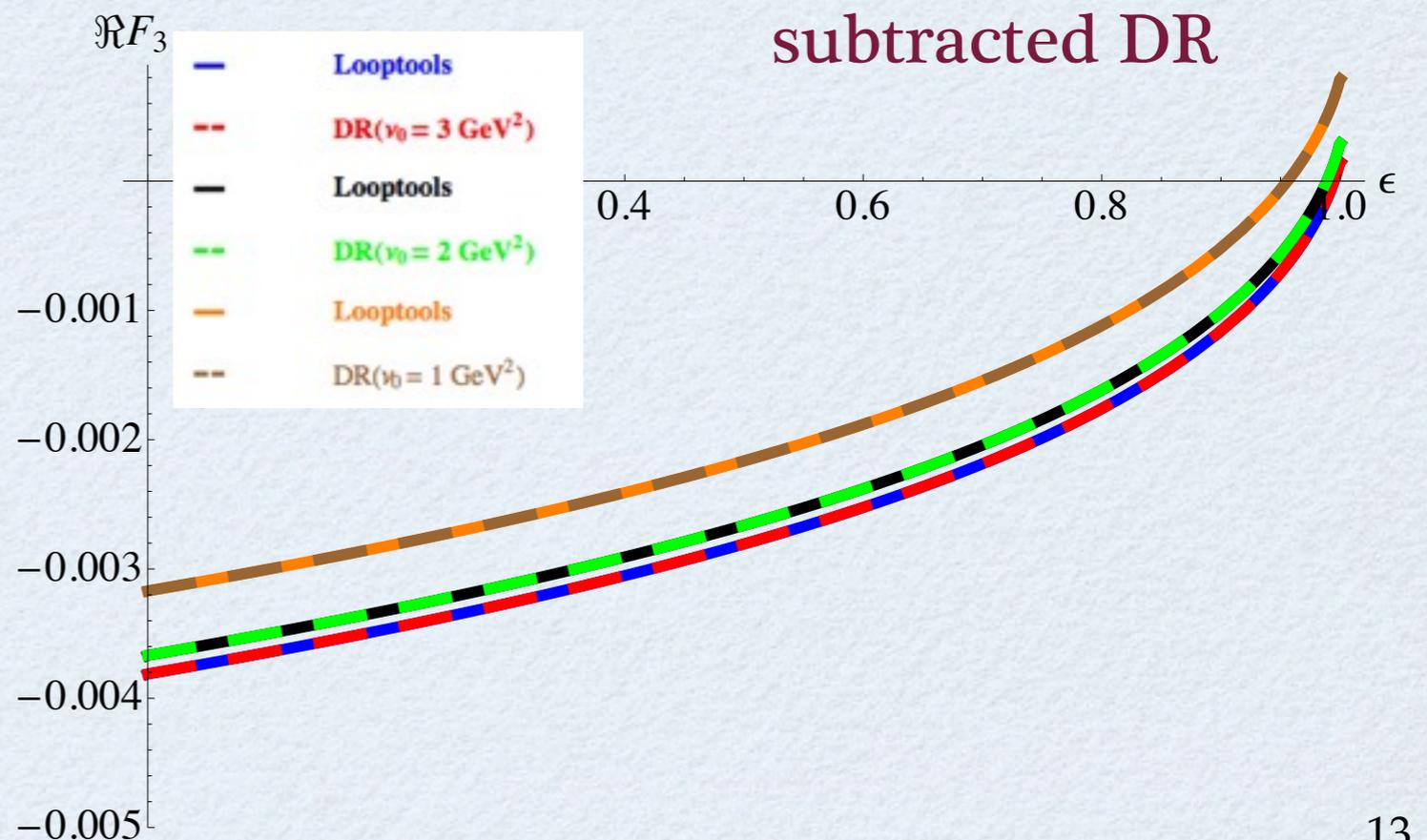
Dispersion relation tests
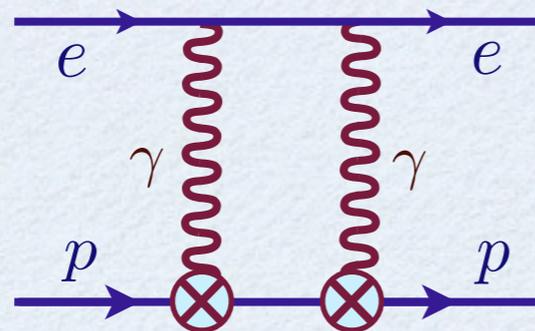
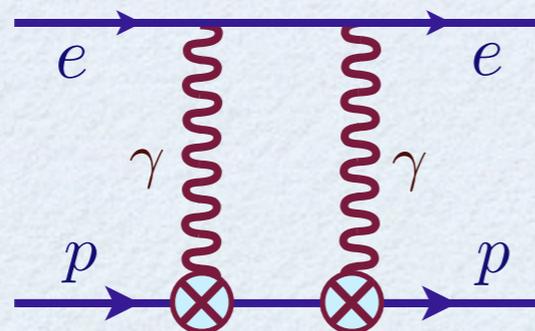## Fixed-t unsubtracted dispersion relation

all structure amplitudes

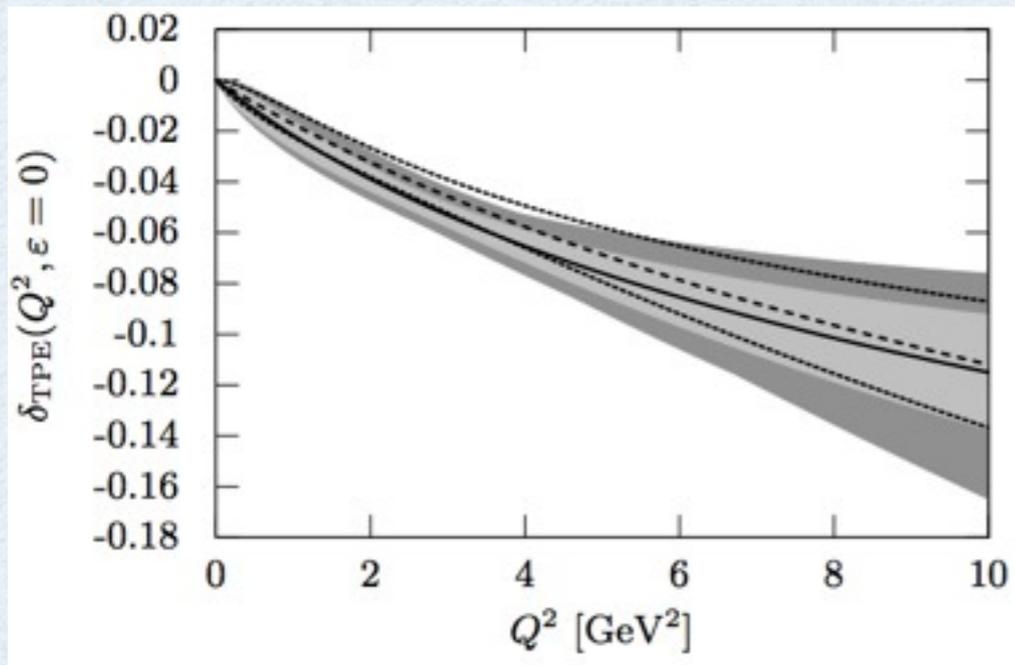all structure amplitudes

UV finite structure amplitudes

$$\mathcal{F}_2 \quad \mathcal{G}_1 \quad \mathcal{G}_2$$

Fixed-t subtracted dispersion relation
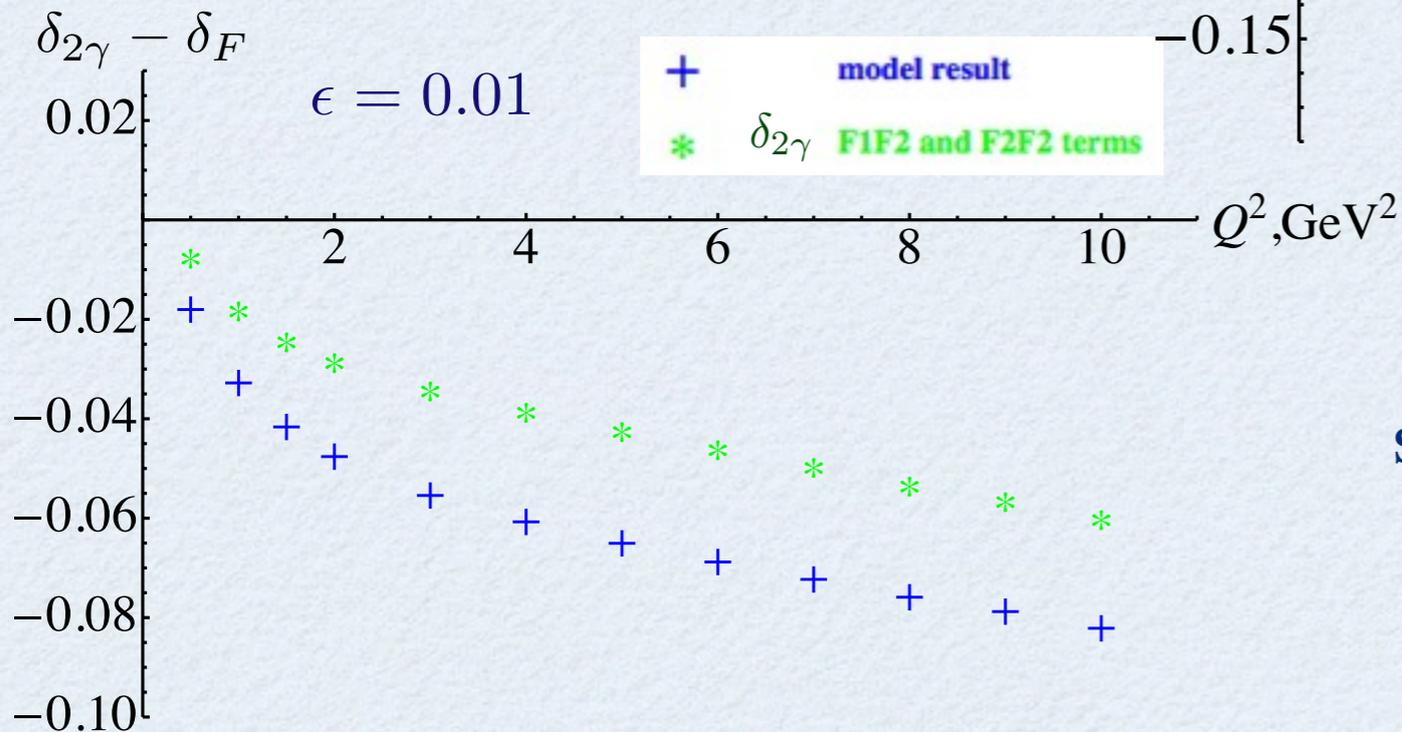

$\mathcal{F}_3$ structure amplitude

Comparison with experiment
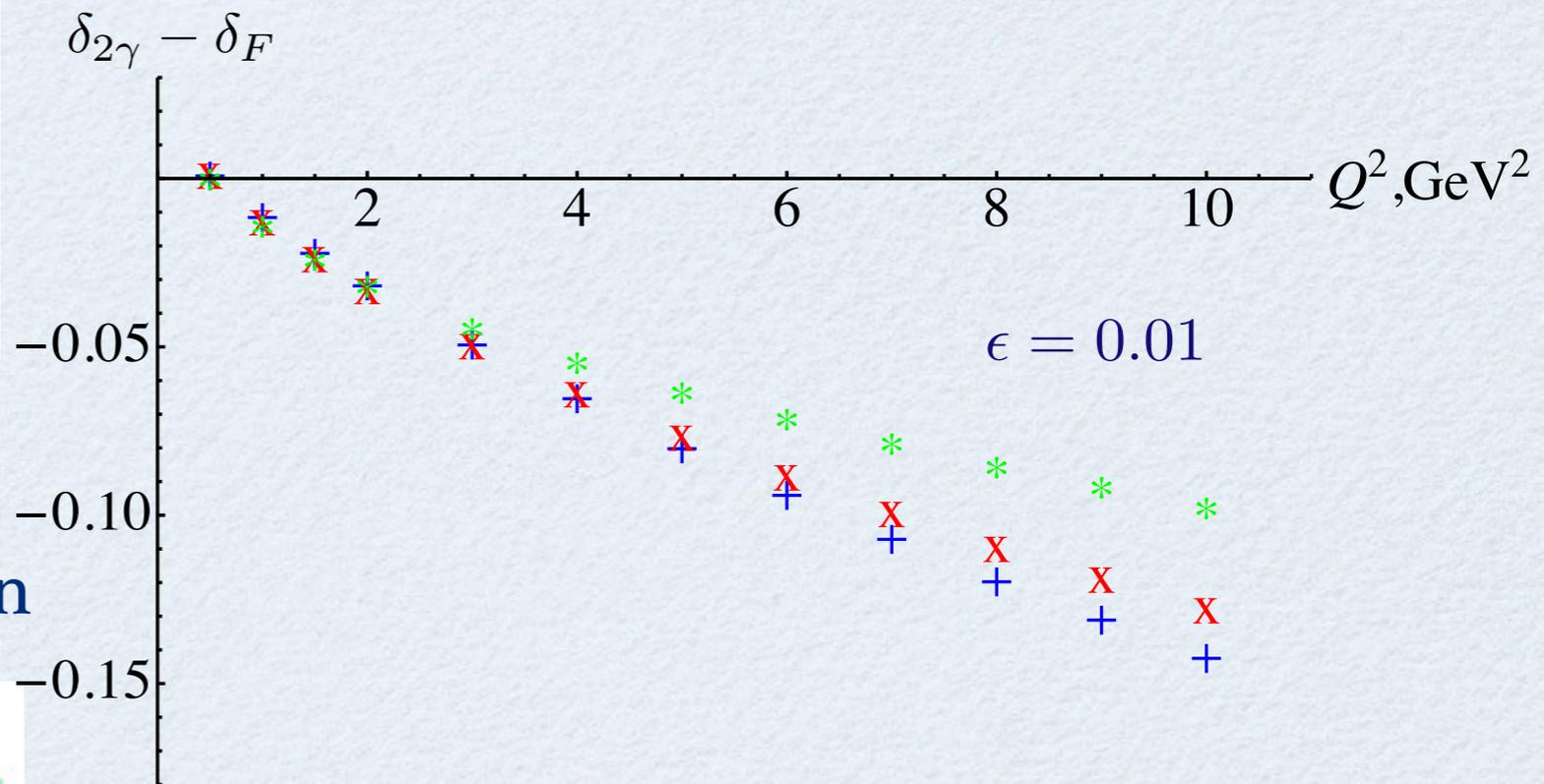

$$\delta_{2\gamma} = \delta_F + \delta_{TPE}$$

$$\delta_{TPE} = -(1-\epsilon)a\,ln(bQ^2+1)$$

box diagram model calculation

$\delta_{2\gamma} - \delta_F$

$\epsilon = 0.01$

+ **model result**

* $\delta_{2\gamma}$ **F1F2 and F2F2 terms**

subtracted DR

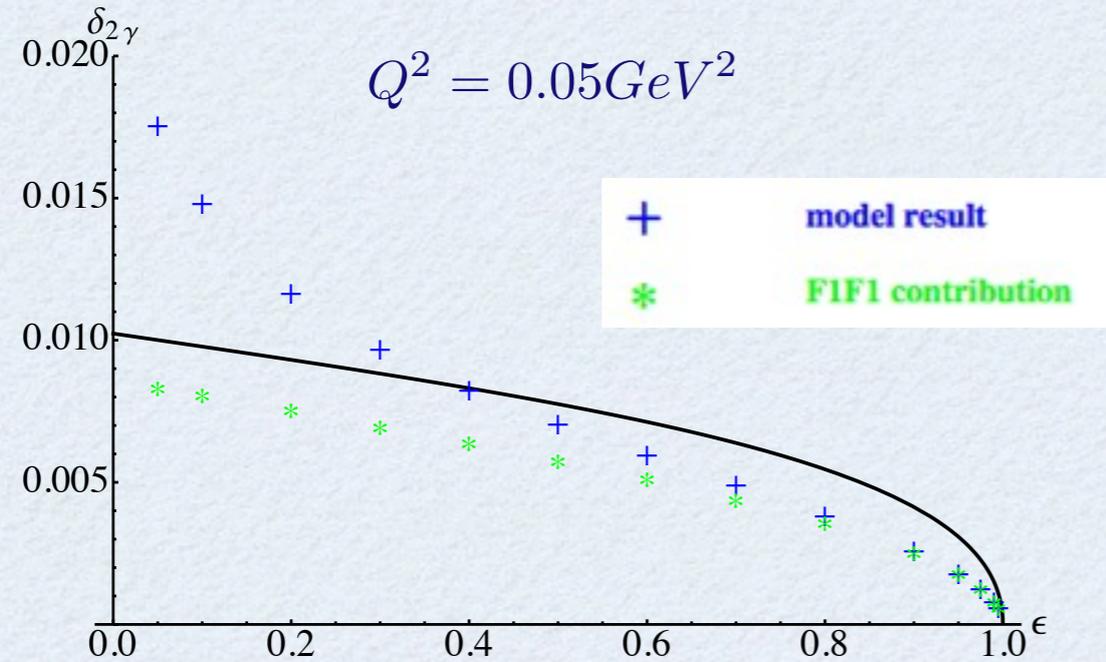$$\delta^{sub}(\nu) = \delta(\nu) - \delta(\nu_0) + \delta^{exp}(\nu_0)$$

$\delta_{2\gamma} - \delta_F$

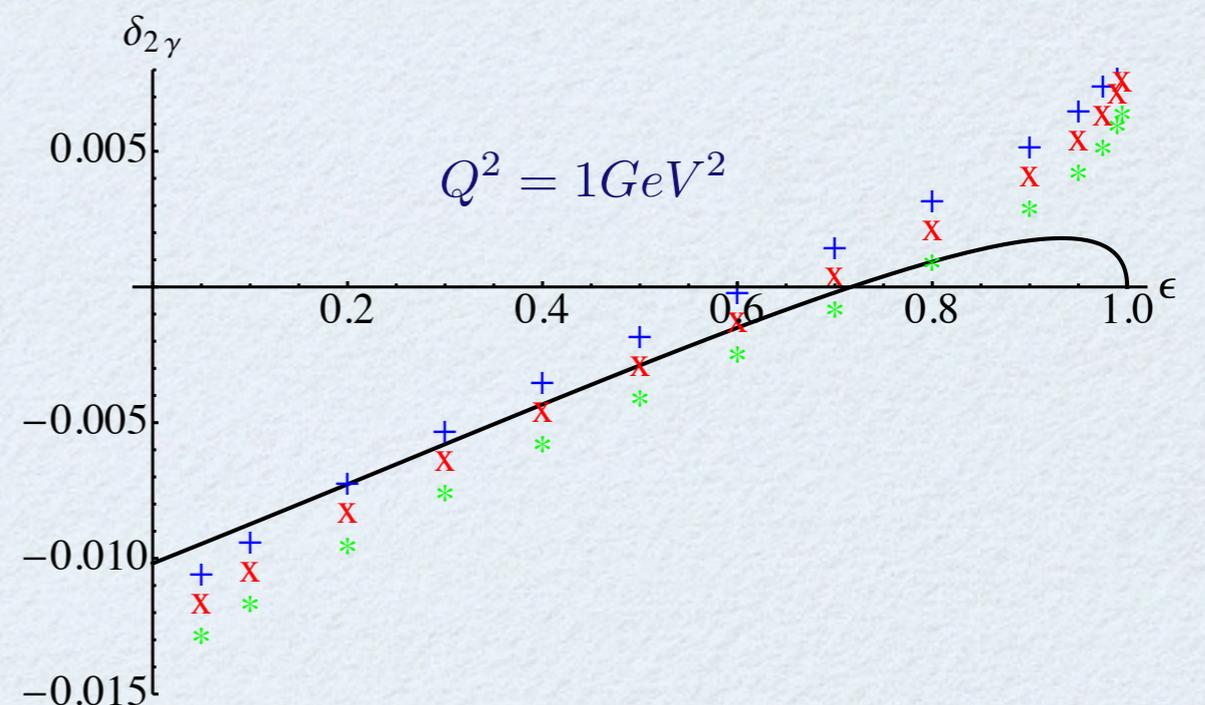$\epsilon = 0.01$

subtraction points

+ $\epsilon_0 = 0.2$

X $\epsilon_0 = 0.5$

* $\epsilon_0 = 0.8$

15

Comparison with experiment

box diagram model calculation

$Q^2 = 0.05 GeV^2$

+ model result

∗ F1F1 contribution

$Q^2 = 1 GeV^2$

subtracted DR (full model)

$Q^2 = 0.05 GeV^2$

+ $\epsilon_0 = 0.2$

✗ $\epsilon_0 = 0.5$

∗ $\epsilon_0 = 0.8$

$Q^2 = 1 GeV^2$
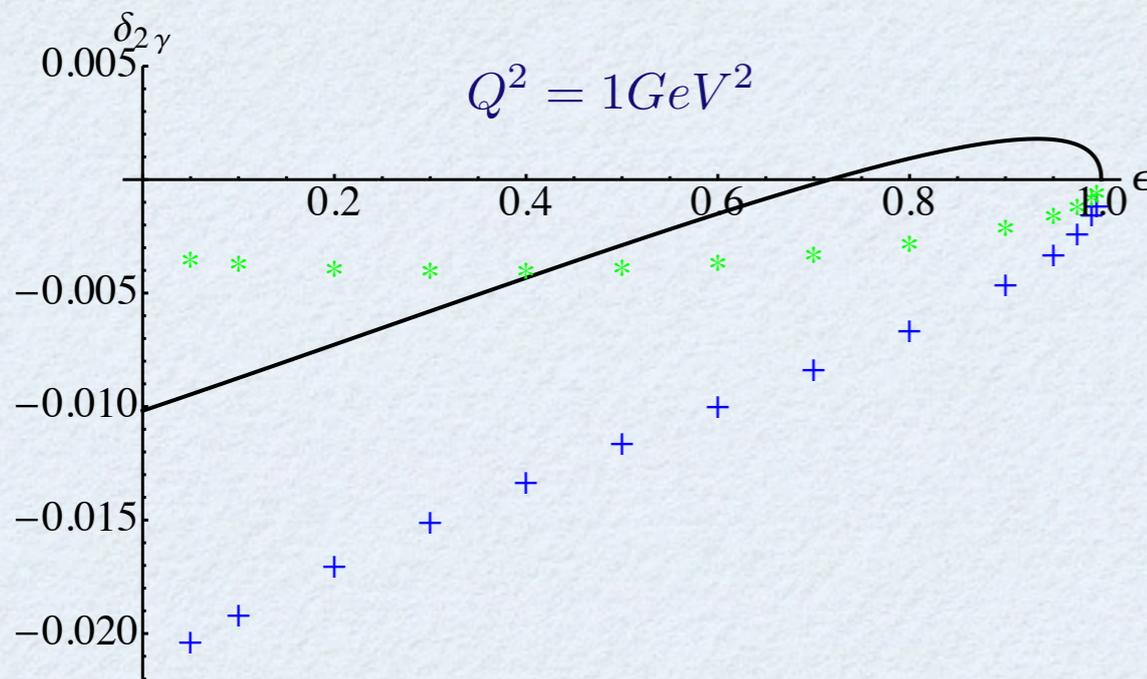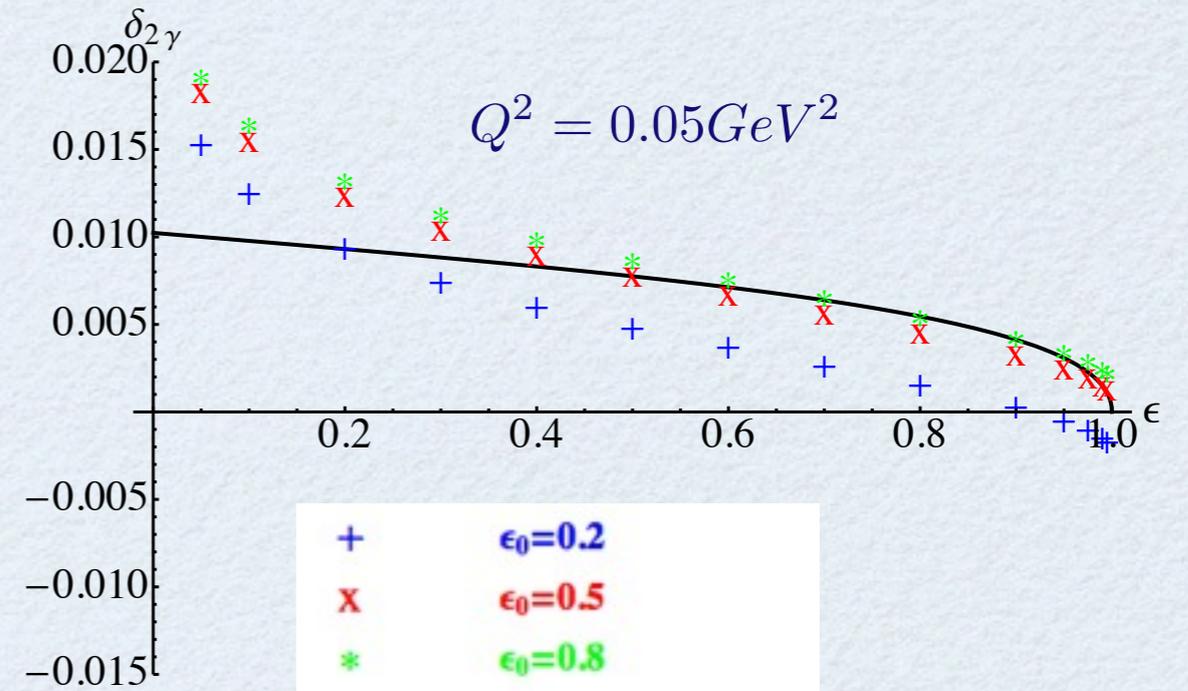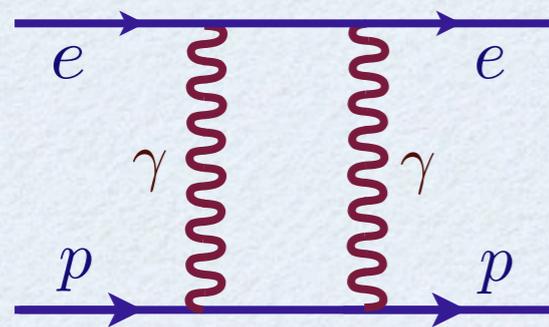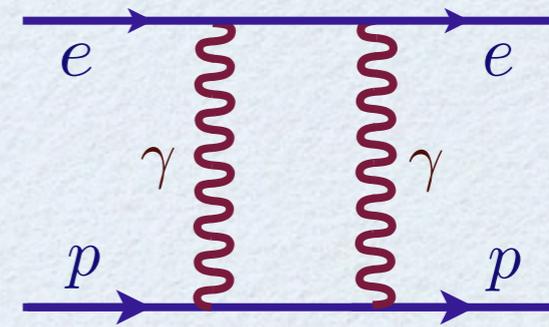
Small momentum transfer limit

Feshbach correction - scattering correction in Dirac theory (HE)
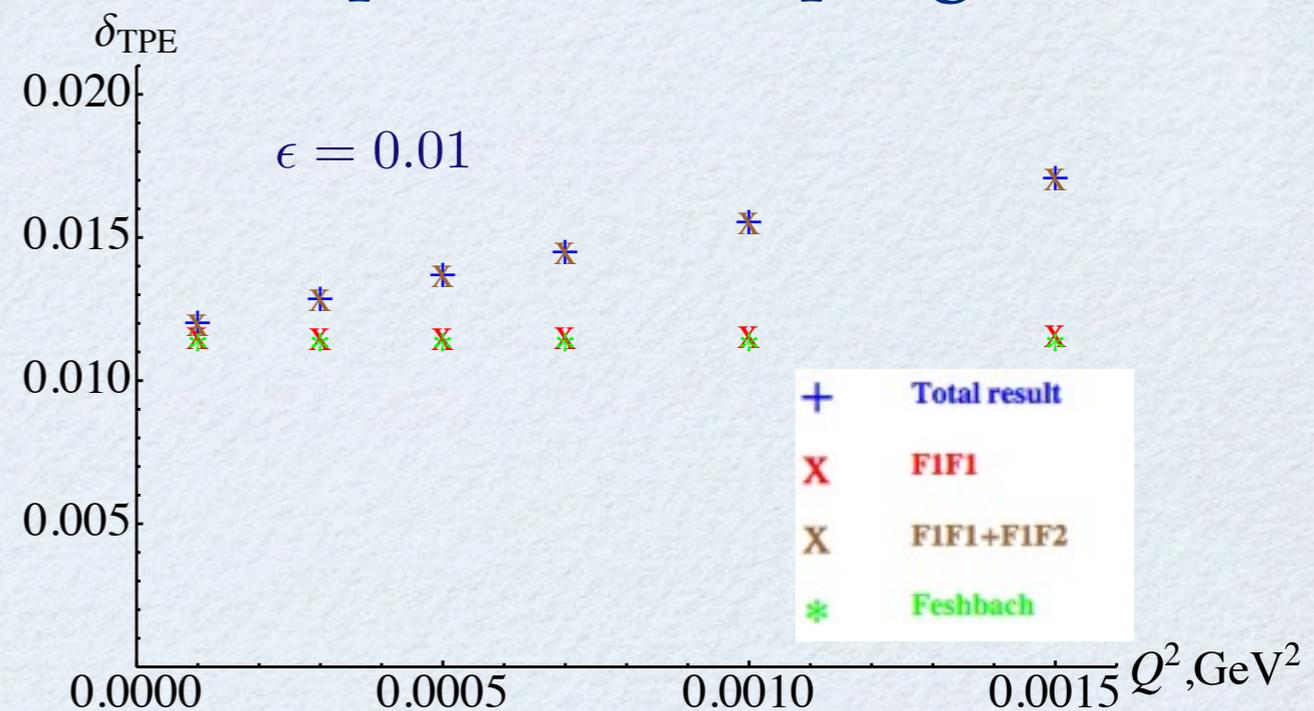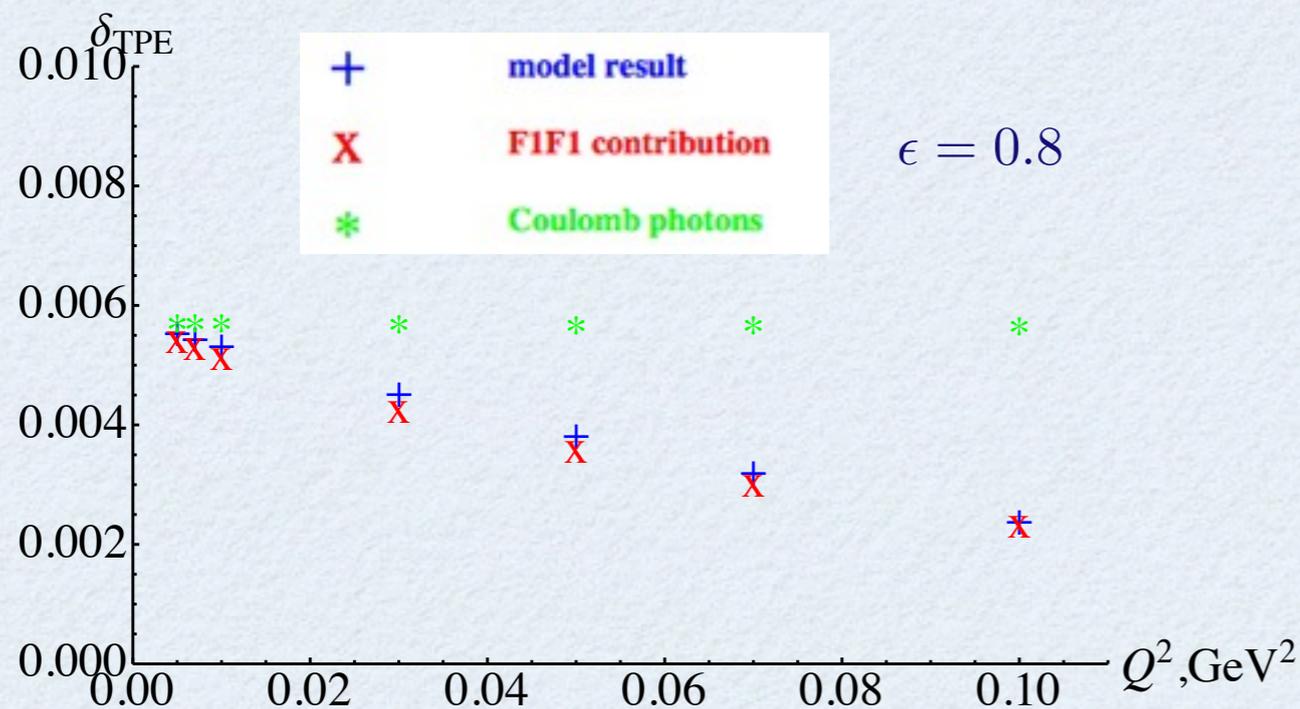

$F_1 F_1$ *structure*

$\Longrightarrow$

Coulomb photons

$$\delta_F = \pi\alpha \frac{\sqrt{1-\epsilon}}{\sqrt{1-\epsilon}+\sqrt{1+\epsilon}} = \pi\alpha \frac{sin(\frac{\theta}{2}) - sin(\frac{\theta}{2})^2}{cos(\frac{\theta}{2})^2}$$

dipole FFs

point-like coupling

$\epsilon = 0.8$

+	model result
X	F1F1 contribution
*	Coulomb photons

$\epsilon = 0.01$

+	Total result
X	F1F1
X	F1F1+F1F2
*	Feshbach

Plans

- Apply dispersion formalism with proton intermediate state

to unpolarized cross section data

- Extend dispersion formalism to inelastic intermediate states

- Compare with EFT expansion at small non-zero momentum transfer of

the box diagram (relevant for proton radius extraction)

Thanks for your attention !!!