Net-charge and Net-Proton Higher moments analysis in PbPb@2.76TeV

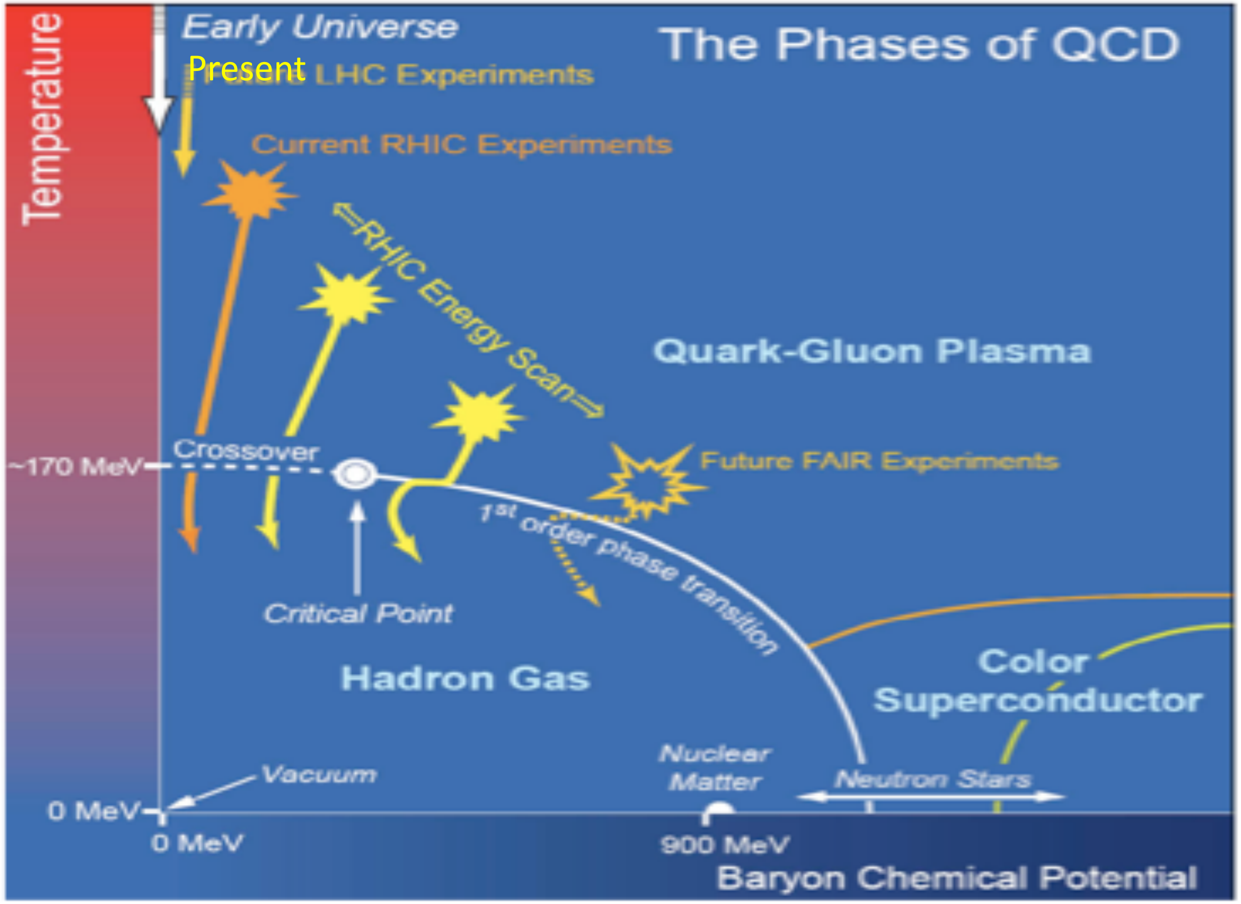
Nirbhay Kumar Behera

IIT Bombay

Outline

- Motivation
- Brief introduction on higher moments
- Results
- Analysis Method
- Baseline study
- Toy model study for efficiency correction
- Summary

Motivation



Goal :
To map the QCD phase Diagram

A parameterized energy dependence baryo-chemical potential: $\mu_B(\sqrt{s}) = \frac{d}{1+e\sqrt{s}}$

J. Cleymans et al. PHYSICAL REVIEW C **73, 034905 (2006)**

=>At LHC, $\mu_B \approx$ 2MeV

Motivation contd.

Higher moments as sensitive probe for fluctuation study

- Event by event analysis of fluctuation of conserved quantities are sensitive to critical phenomena. Signaled by increased and divergence of correlation length

- Higher moments (cumulants) are sensitive to correlation length (ξ)

$$C_n \propto \xi^{2.5n-3}$$

M.A. Stphanov, PRL 102, 032301 (2009)

- LQCD: At LHC, there will be a cross-over and the phase transition is of 2nd order, possible existence of chiral critical point. Only physical observables quark number susceptibilities(χ_q) are accessible in LQCD.

- Higher moments of conserved quantities like net-charge, net-proton can directly compare with various quark number susceptibilities to constraint LQCD predictions.

$$C_n = (VT^3)T^{n-4}\,\chi_n(t,z)$$ where $t = T/T_c$ and $z = \mu_B/T$; μ_B = chemical potential

K. Redlich,Cent. Eur. J. Phys. • 10(6) • 2012 • 1254-1257, **Eur.Phys.J.C71:1694,2011**

Motivation contd.

Ratio of cumulants as a probe to quantify the freezout parameters

(F. Karsch. arXiv:1202.4173v1 and **Eur.Phys.J.C71:1694,2011)**

a) Event to Event ratio of cumulants : Determines freezout temperature(T_f)

b) Even to Odd ratio of cumulants : Determines the brayo-chemical potentional

Motivation contd.

Predictions from Hadron Resonance Gas (HRG) model

(F. Karsch and K. Redlich, Physics Letters B 695 (2011) 136–142)

$\sqrt{s_{NN}}$	$\chi_B^{(2)}/\chi_B^{(1)}$	$\chi_B^{(3)}/\chi_B^{(2)}$	$\chi_Q^{(2)}/\chi_Q^{(1)}$	$\chi_Q^{(3)}/\chi_Q^{(2)}$
7.7	1.01	0.99	4.18	0.49
11.5	1.05	0.95	5.39	0.39
19.6	1.23	0.81	7.95	0.27
39.0	1.87	0.53	14.25	0.15
62.4	2.75	0.36	21.97	0.09
200.0	8.20	0.12	67.80	0.03
2760	111.1	0.09	922.4	0.02

LHC

Freezout condition(B. Friman et al. Eur. Phys. J. C (2011) 71:1694)

Freeze-out conditions	χ_4^B/χ_2^B	χ_6^B/χ_2^B	χ_4^Q/χ_2^Q	χ_6^Q/χ_2^Q
HRG	1	1	~2	~10
QCD: $T^{freeze}/T_{pc} \lesssim 0.9$	$\gtrsim 1$	$\gtrsim 1$	~2	~10
QCD: $T^{freeze}/T_{pc} \simeq 1$	~0.5	<0	~1	<0

If freeze-out appears well in the hadronic phase

If freeze-out appears in the vicinity of the chiral crossover temperature

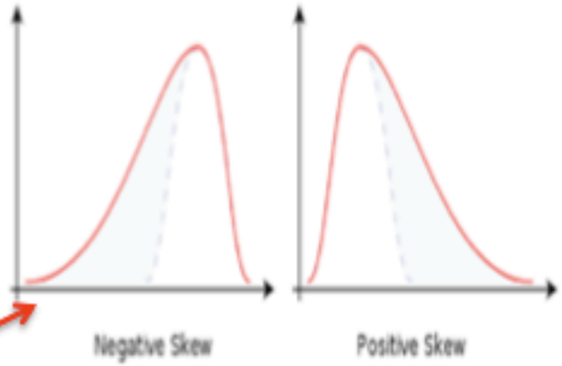
Brief Introduction to Higher Moments

$\Delta N = \Sigma N_{+} - \Sigma N_{-}$

Mean = $[C^1] = < \Delta N >$

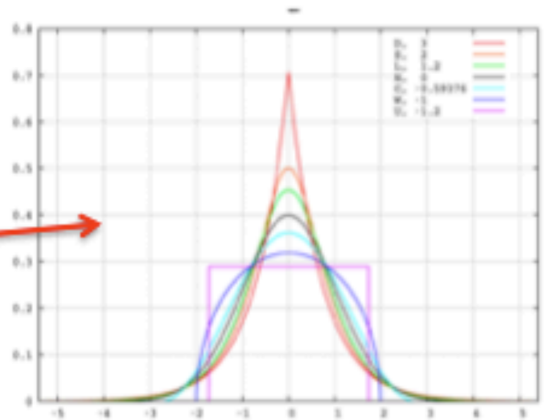
Variance(σ^2) = $[C^2]$ => $sigma(\sigma) = \sqrt{<(\Delta N - <\Delta N>)^2>}$

$$Skewness(S) = \frac{[C^{(3)}]}{([C^{(2)}])^{3/2}} = \frac{<(\Delta N - <\Delta N>)^3>}{\sigma^3}$$



Represents the asymmetry of a distribution

$$Kurtosis(K) = \frac{[C^{(4)}]}{([C^{(2)}])^2} - 3 = \frac{<(\Delta N - <\Delta N>)^4>}{\sigma^4} - 3$$



Represents the peakness of a distribution

$$C_q^{(n)} = VT^3 \chi_q^{(n)}(T, \mu_B)$$

Now, the ratio of cumulants will be volume independent

$$S\sigma = \frac{[C^{(3)}]}{[C^{(2)}]} = \frac{\chi_q^{(3)}}{\chi_q^{(2)}} \qquad K\sigma^2 = \frac{[C^{(4)}]}{[C^{(2)}]} = \frac{\chi_q^{(4)}}{\chi_q^{(2)}}$$

Central Limit Theorem

Assumption : The final multiplicity of particles depends on the sum of the multiplicities of the *identical independent emission source* (IIES).

X.F. Luo etal J. Phys. G: Nucl. Part. Phys. 37 094061

Explains about the evolution of higher moments as a function of centrality

Mean(μ) α N_{part}

Sigma(σ) α $\sqrt{N_{part}}$

Skewness α $1/\sqrt{N_{part}}$

Kurtosis(κ) α $1/N_{part}$

Data set and cuts

DataSet and production Name	Data - LHC10h_AOD086 MC – HIJING_PbPb_LHC10hAOD90
Trigger	kMB
|Vertex Z |	< 10.cm
|Vertex X,Y |	< 3.0cm
Number of Events	~14 M
Centrality Estimator	V0M

For Net-Charge

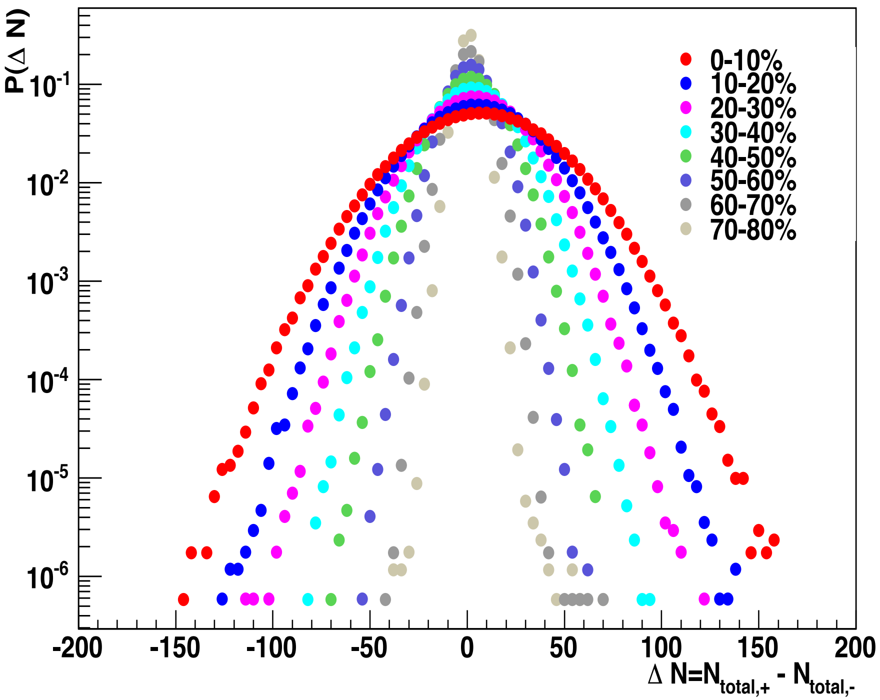
Track Cut Name	Cut
Track(AOD)	TPC only(filterbit 128)
TPC ClusterInfo	**> 80**
TPC chi2/ndf	**< 4.**
DCA_Z	< 3.
DCA_{xy}	< 2.8
p_T	$0.3 < p_T < 1.5$ GeV
|η|	< 0.8

For Net-proton

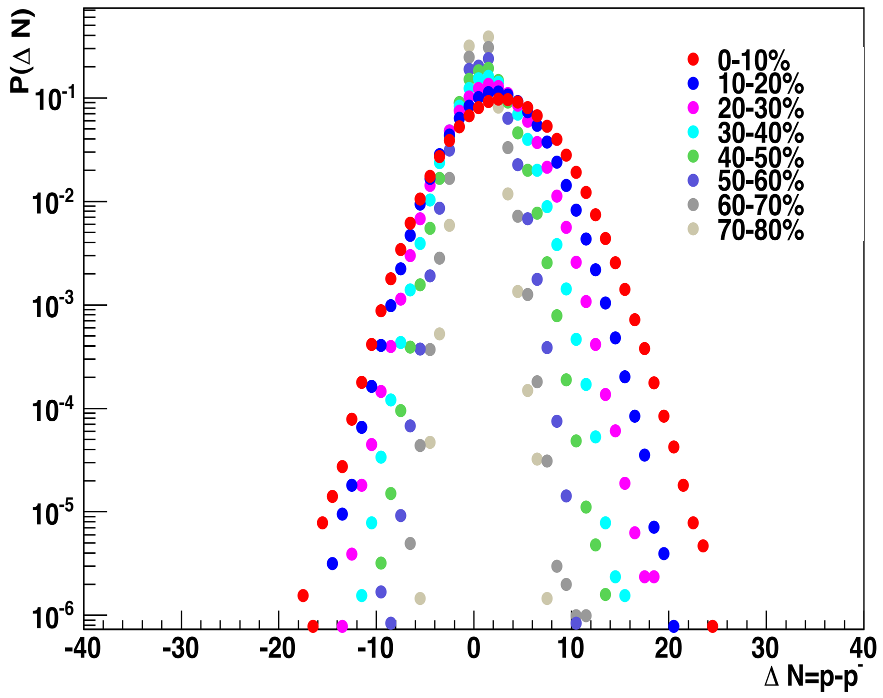
TPC nSigma	**< 3**
DCA_Z	< 2.
DCA_{xy}	< 2.
p_T	$0.4 < p_T < 0.8$ GeV

Results

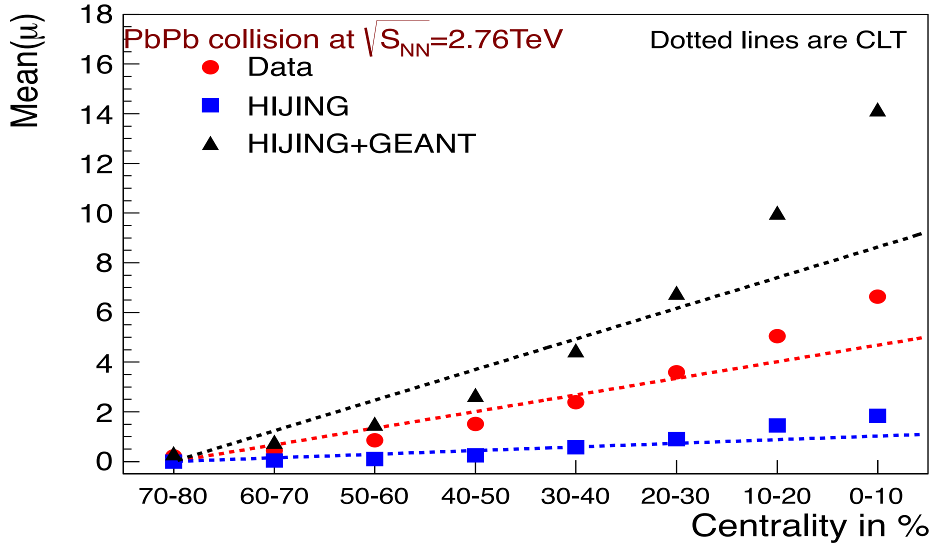
Net-charge distribution

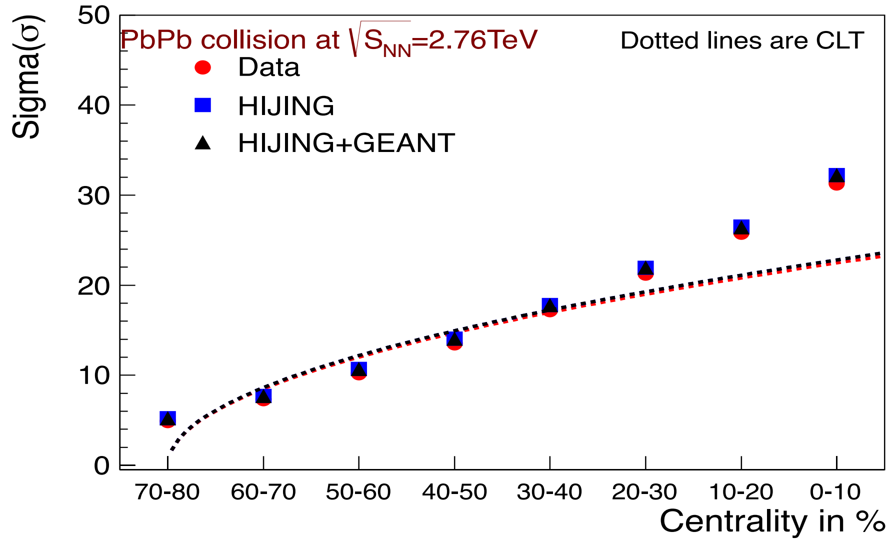


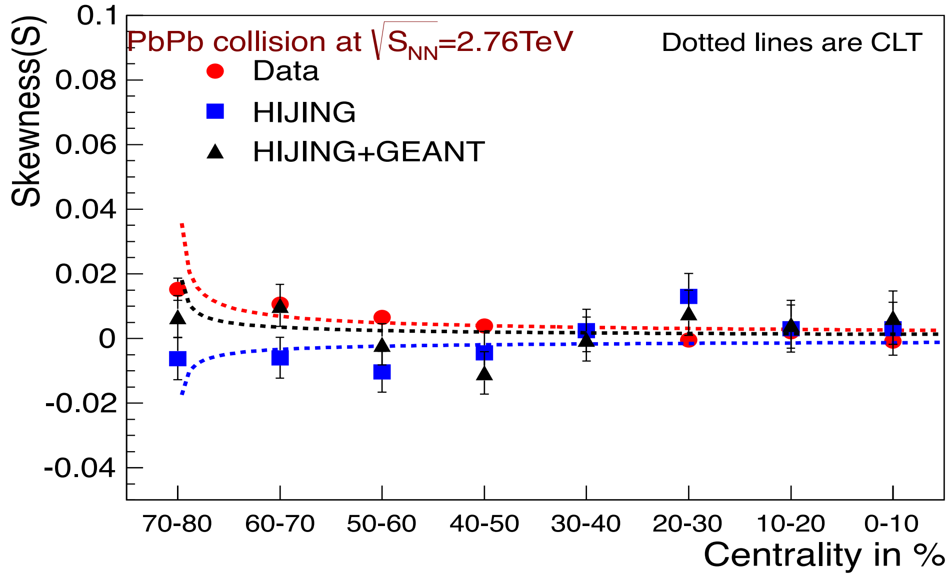
Net-proton distribution

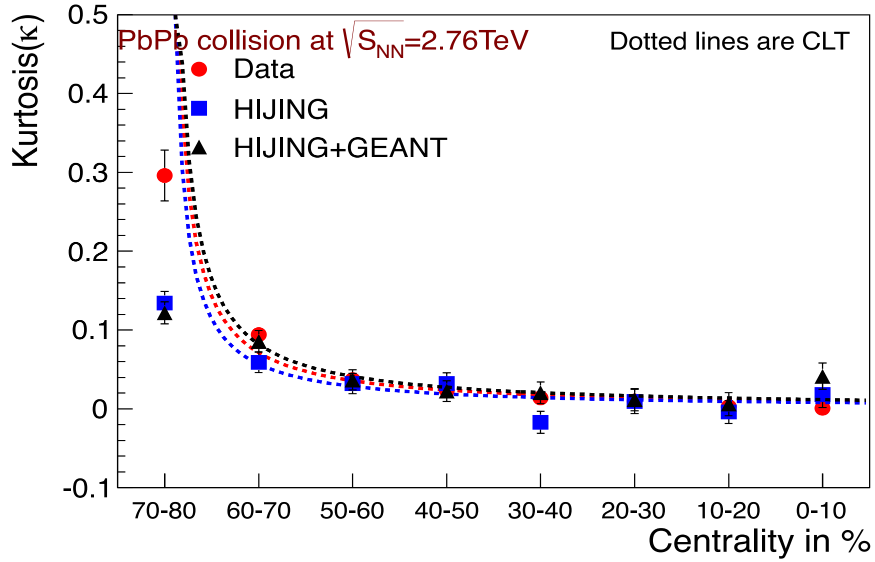


Net-charge higher moments

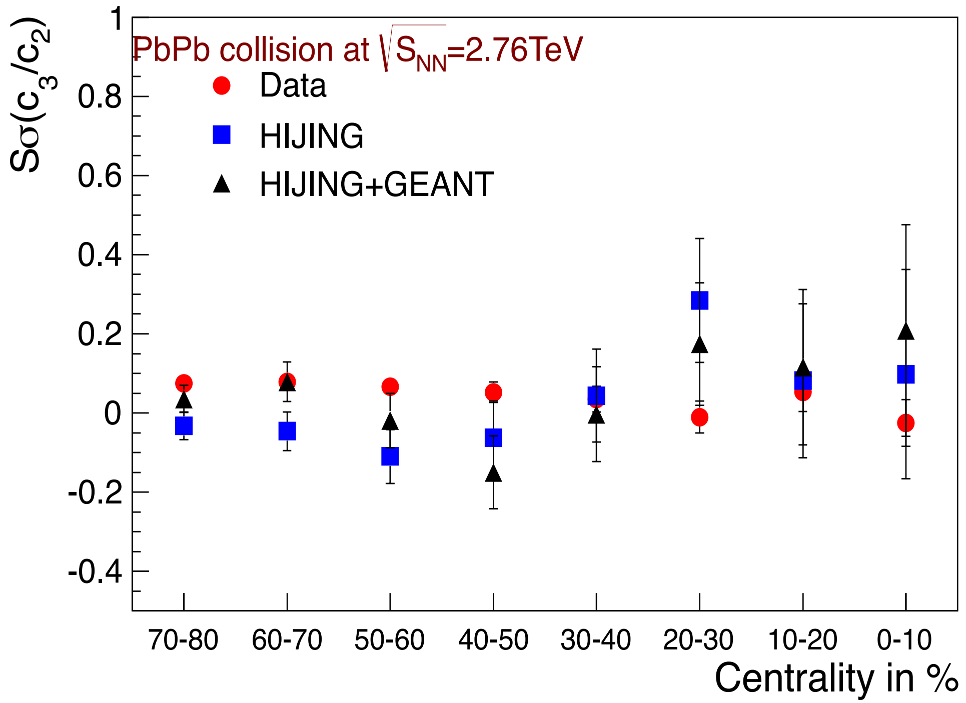


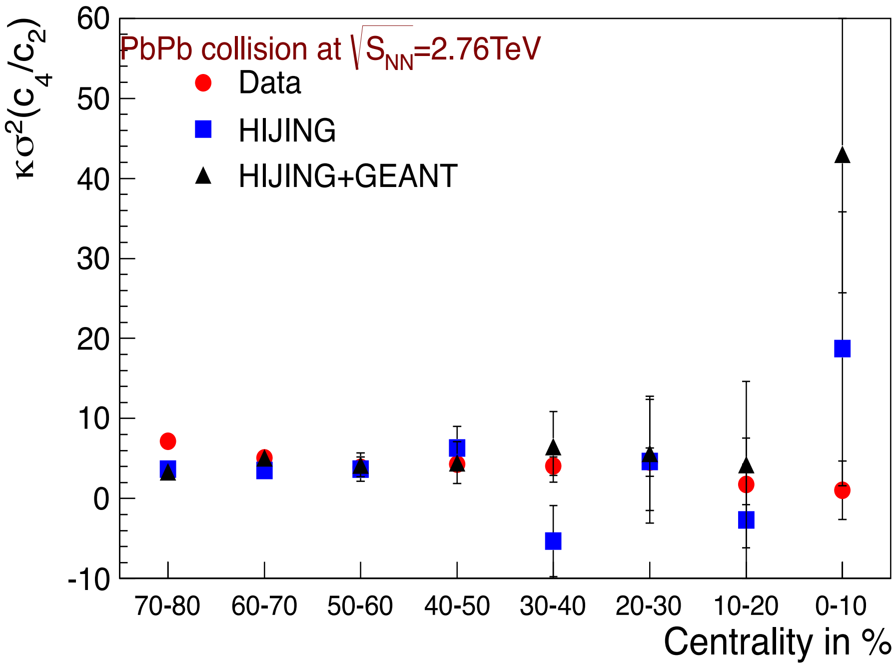






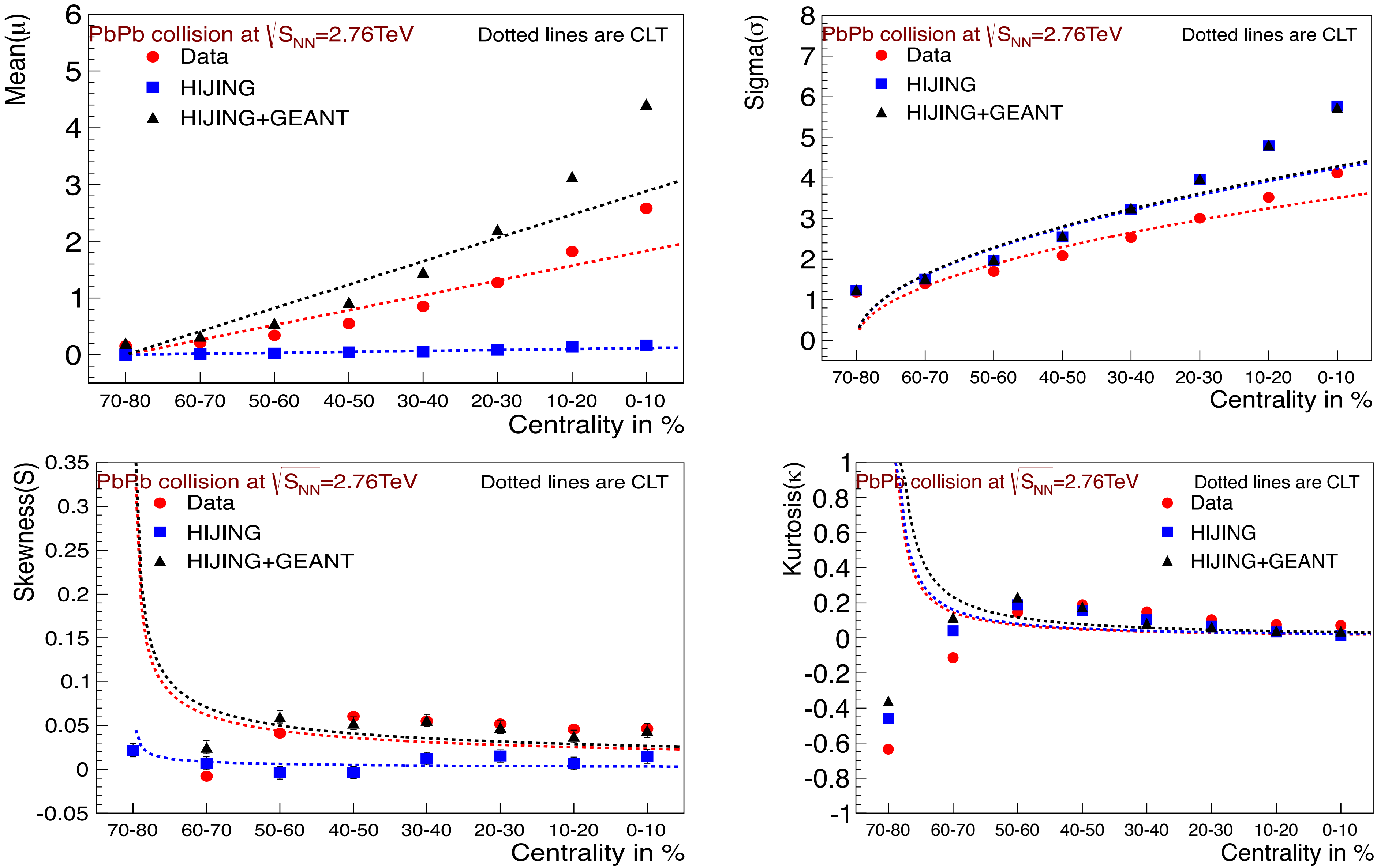
Net-charge higher moments



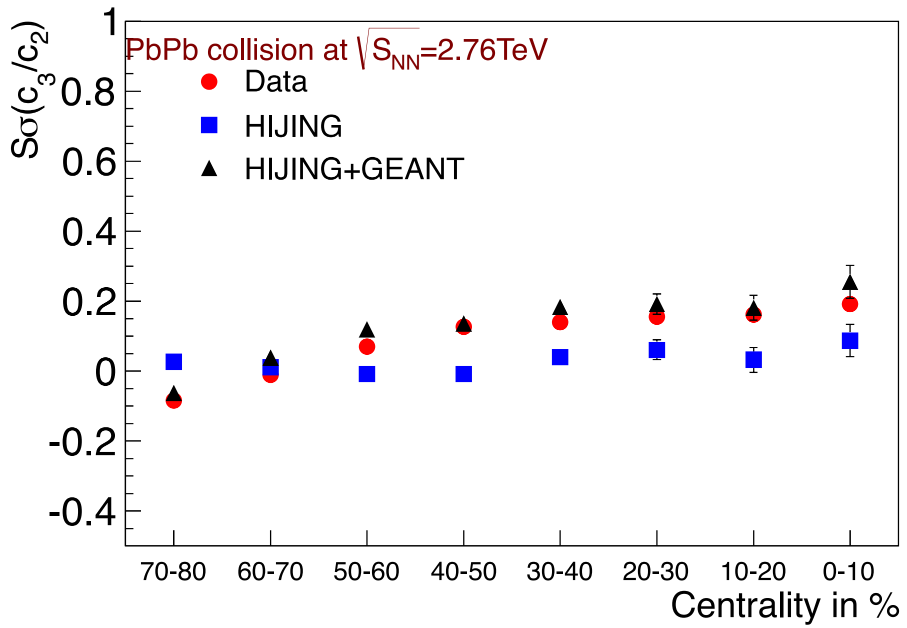


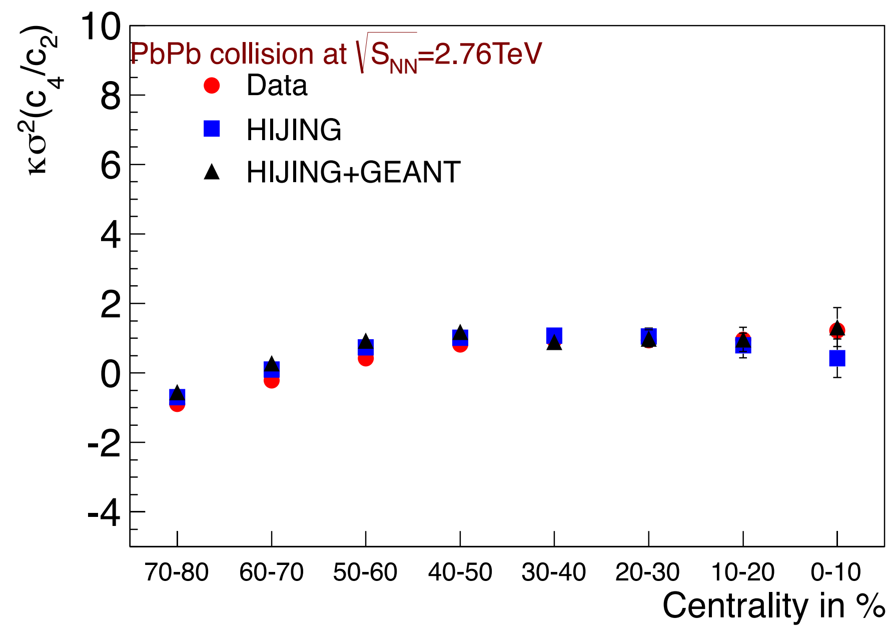
Net-Proton Higher moments results

Net-proton higher moments



Net-proton higher moments





Analysis Methods

- **Bin-width Correction**
- **Error estimation**

Centrality bin-width correction (CBWC)

Central Bin-width effect (CBWF) arises due to the impact parameter (or volume) variations due to the finite centrality bin. It can be eliminated/reduced by (CBWC)

Xiaofeng Luo arXiv:1106.2926
arXiv:1302.2332v1

Generalized bin-width correction formula : $x = \frac{\sum_{i=1}^{n} n_i x_i}{\sum_{i=1}^{n} n_i}$

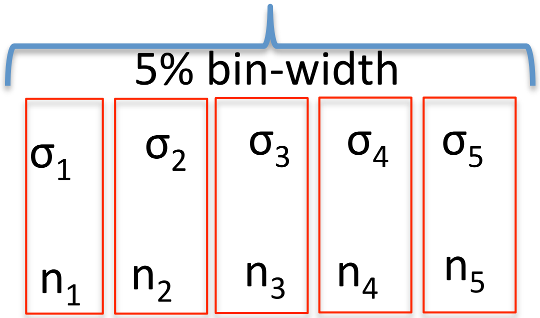
Where n_i is the number of events in i-th centrality.

For example

$$\sigma = \frac{\sum_r n_r \sigma_r}{\sum_r n_r} = \sum_r \omega_r \sigma_r$$

Where n_r is the number of events in the r^{th} centrality bin and corresponding weight ω_r

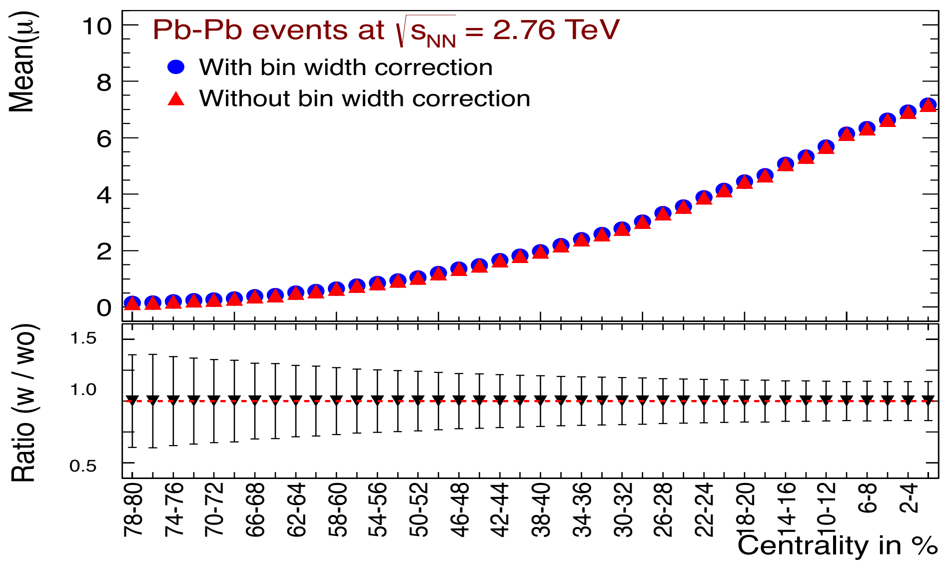
$$\omega_r = \frac{n_r}{\sum_r n_r}$$

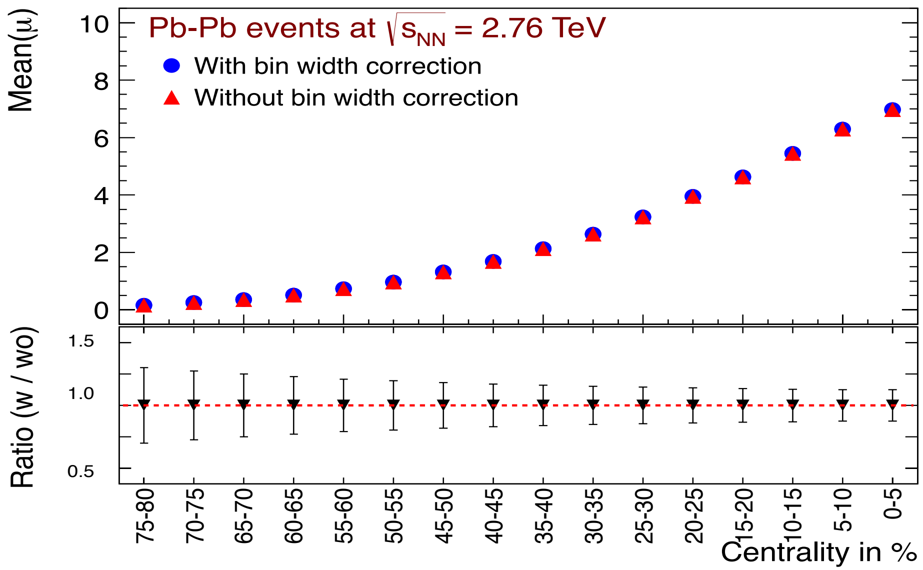


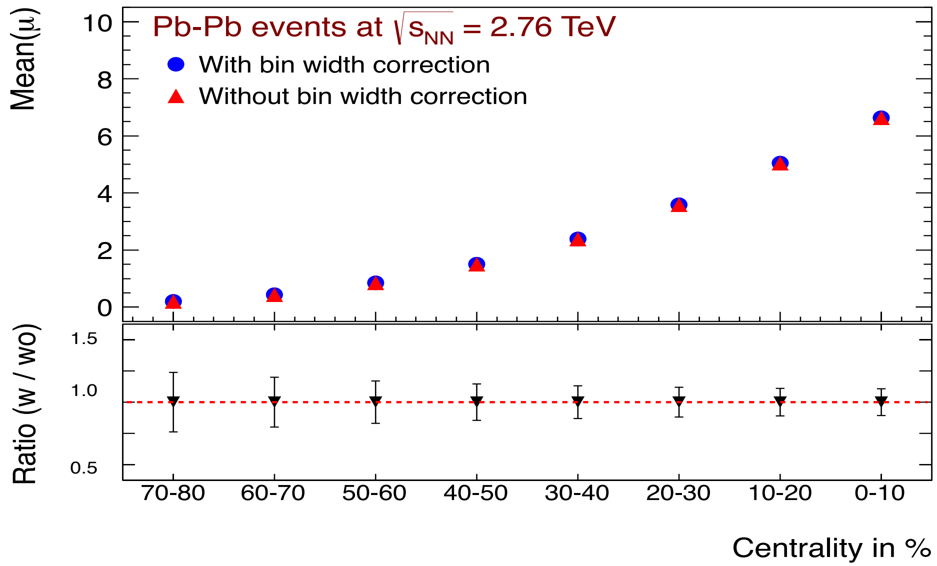
$\sigma = \omega_1\sigma_1 + \omega_2\sigma_2 + \ldots + \omega_5\sigma_5$

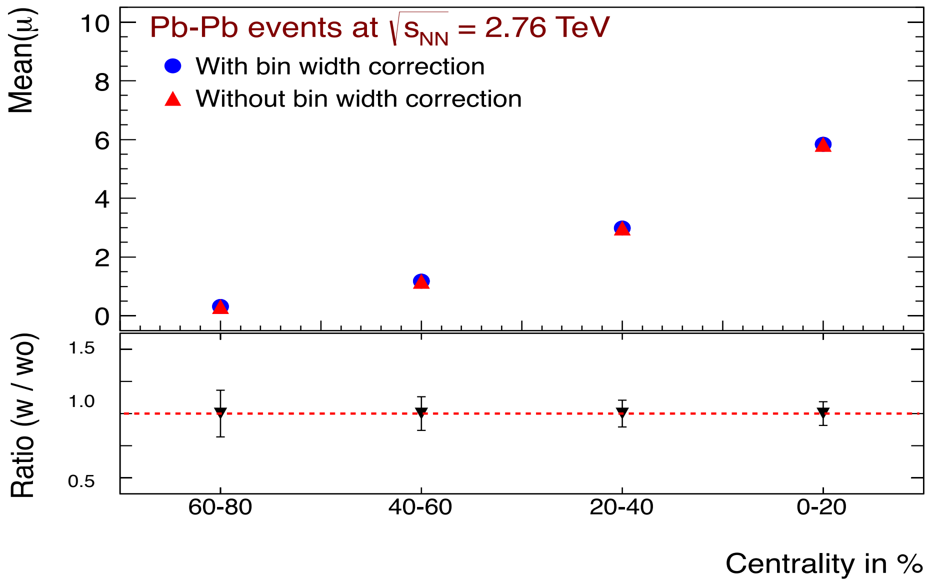
$\omega_1 = n_1/(n_1+n_2+\ldots+n_5)$

Bin-width correction for mean



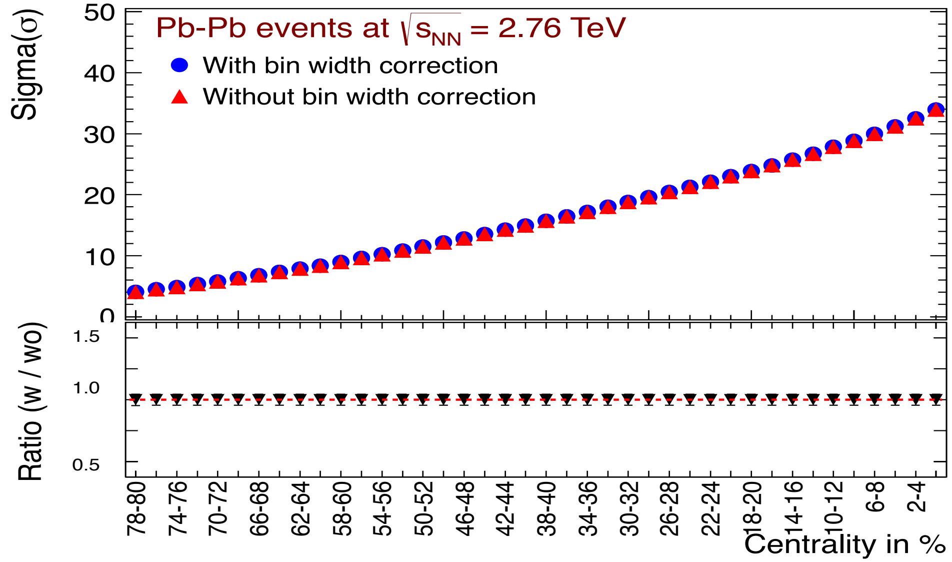


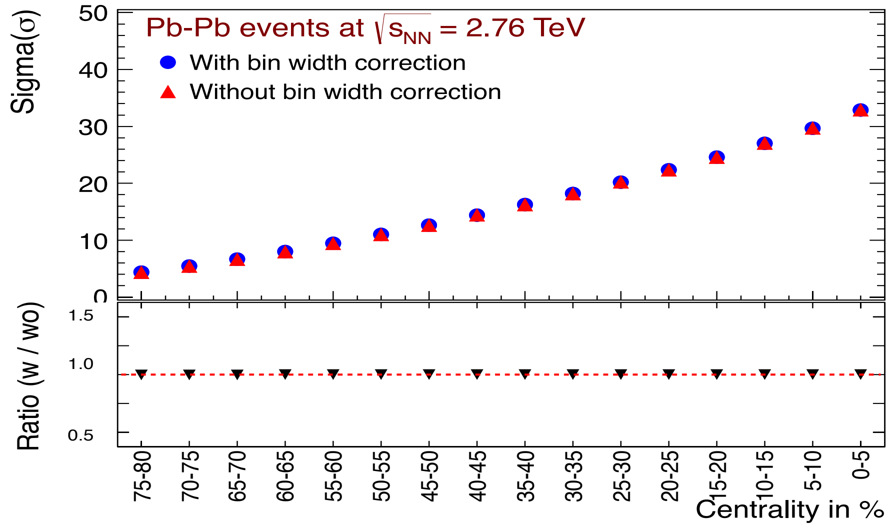


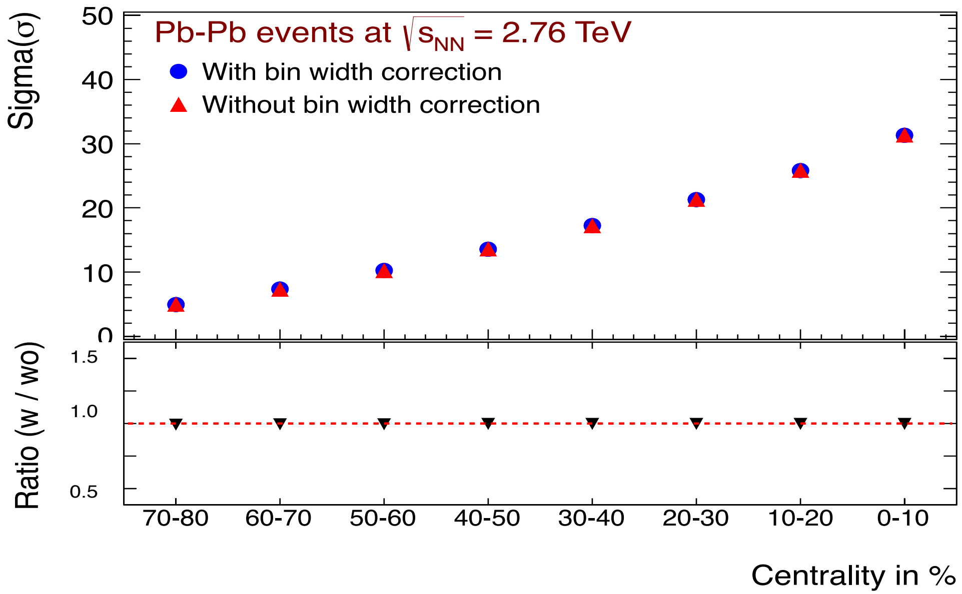


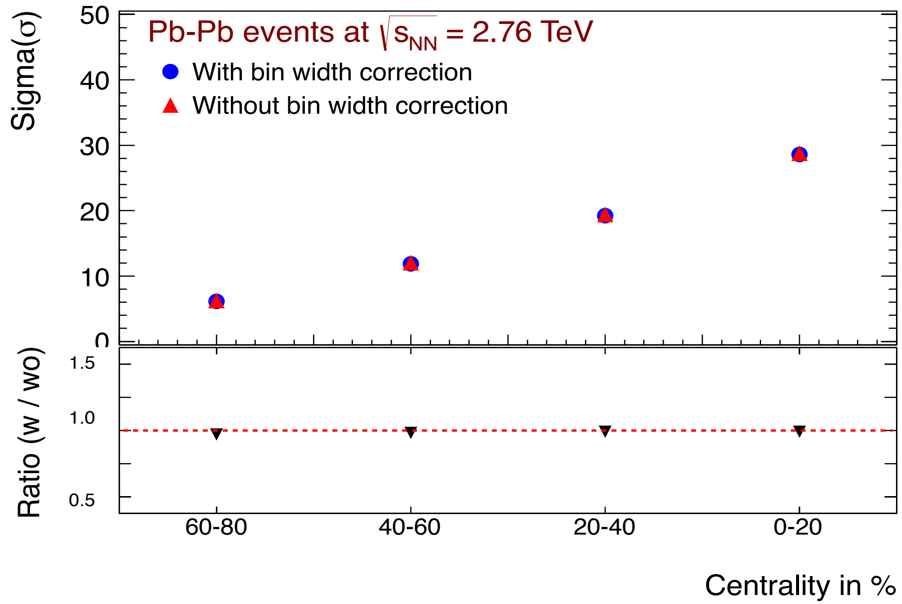
With and without BWC are same for mean

Bin-width correction for sigma



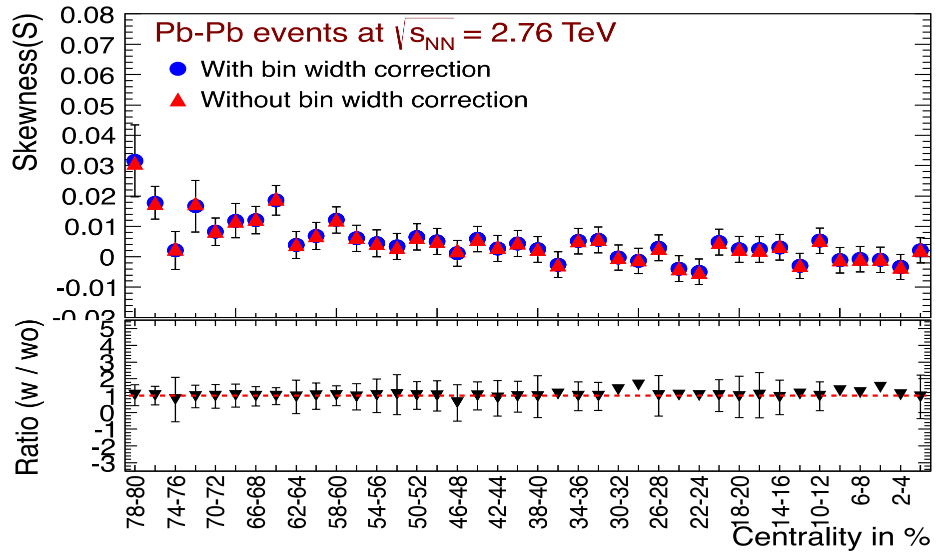


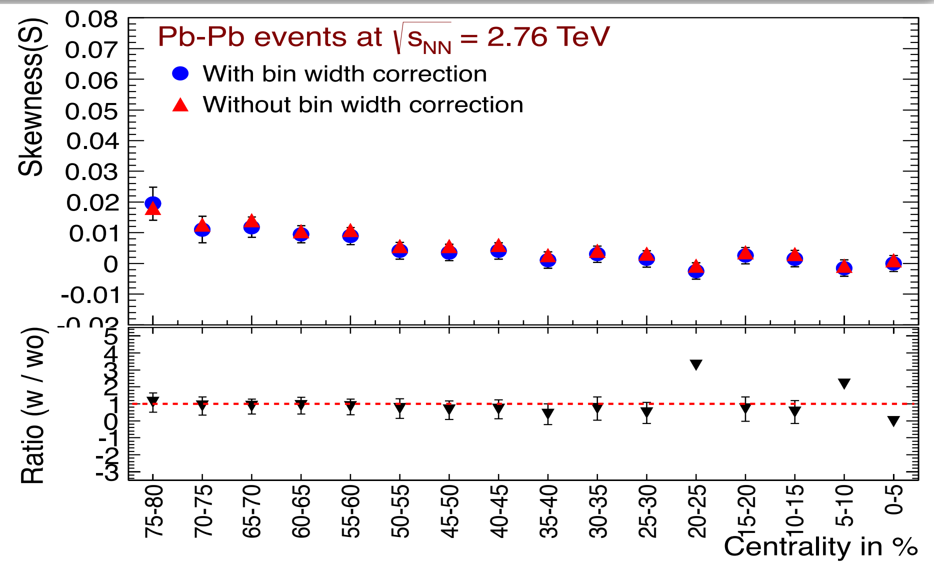


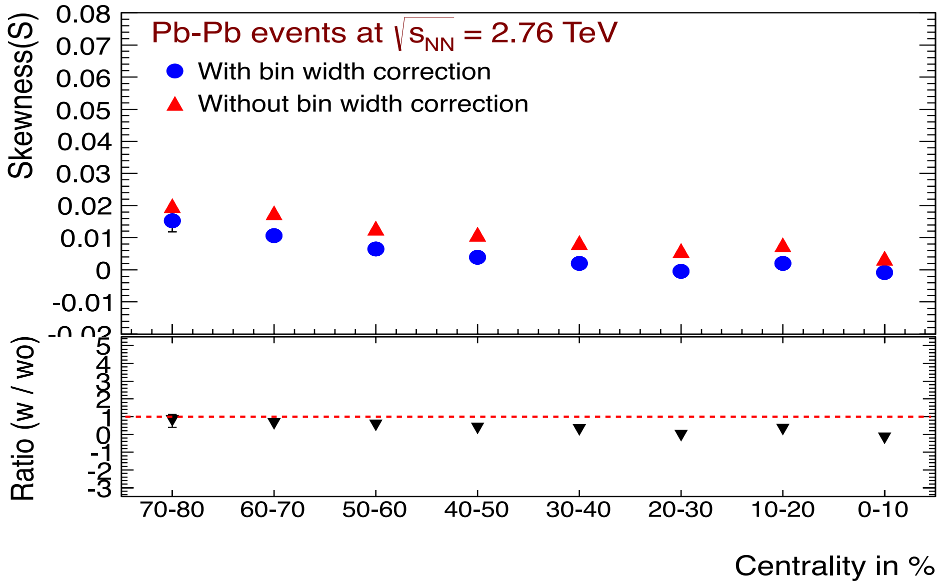


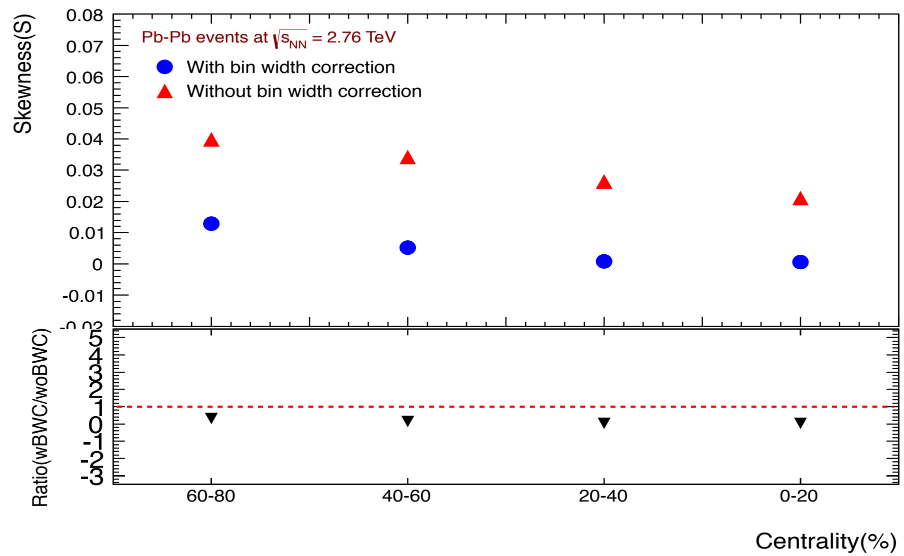
with and w/o BWC, sigma doesn't show any difference

Bin-width correction for skewness



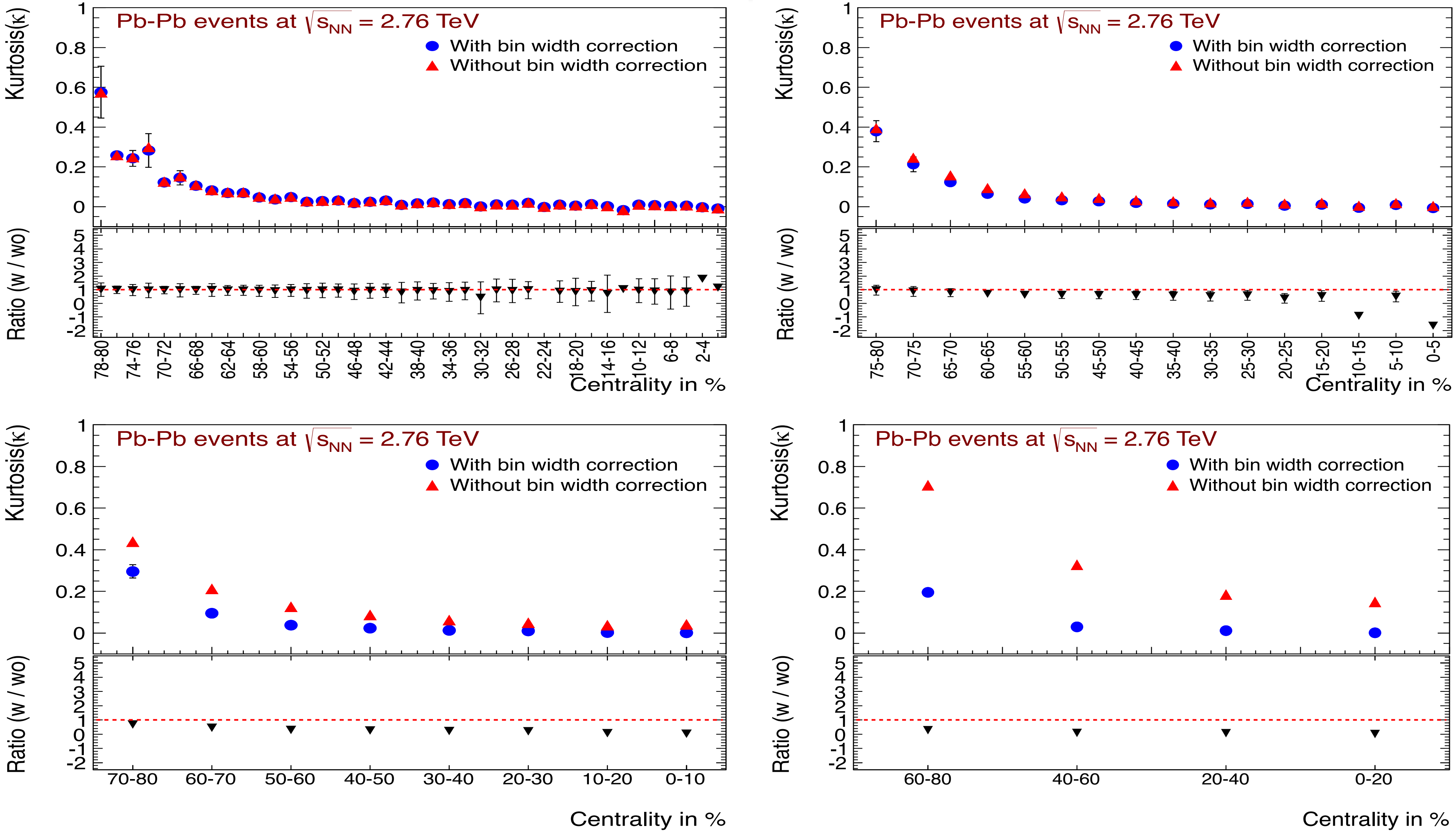






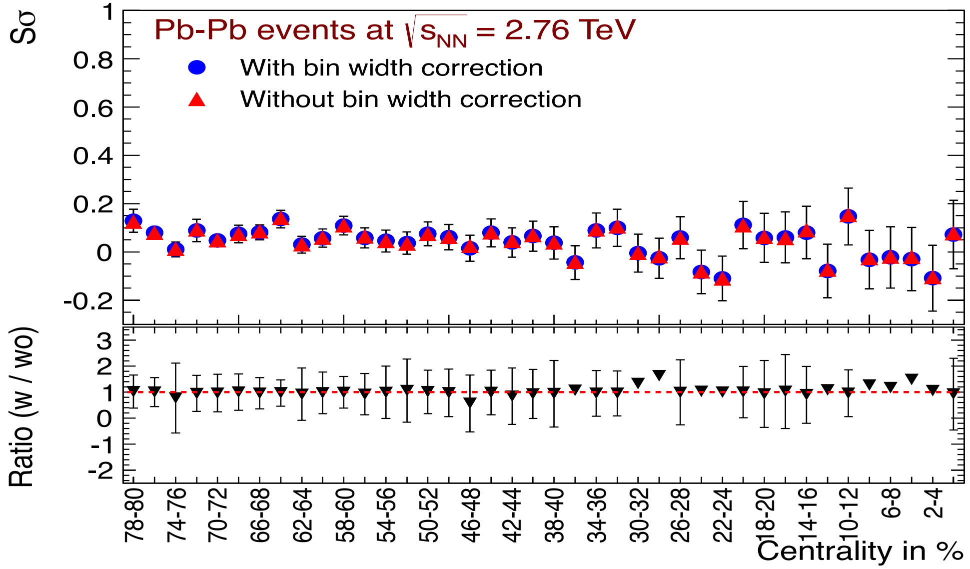
There is a a difference between w/ and w/o BWC

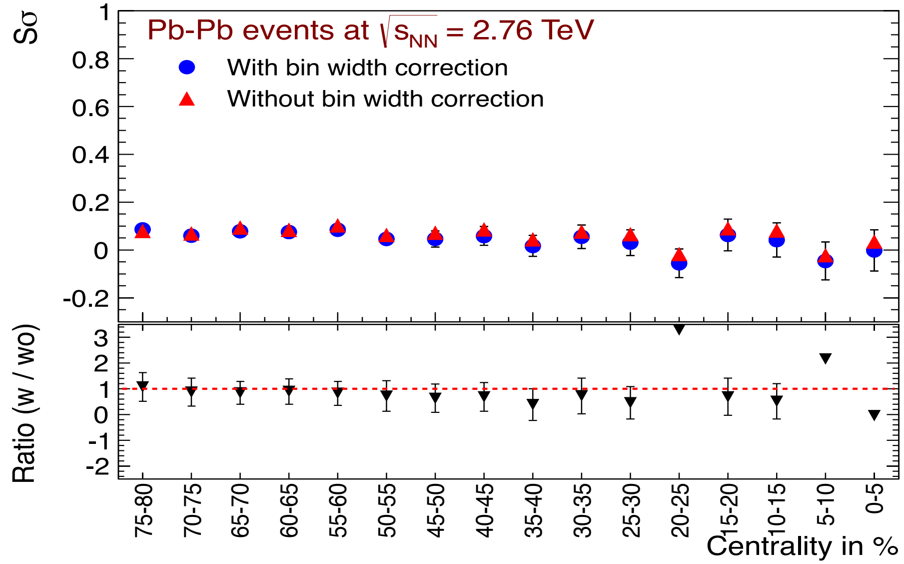
Bin-width correction for kurtosis

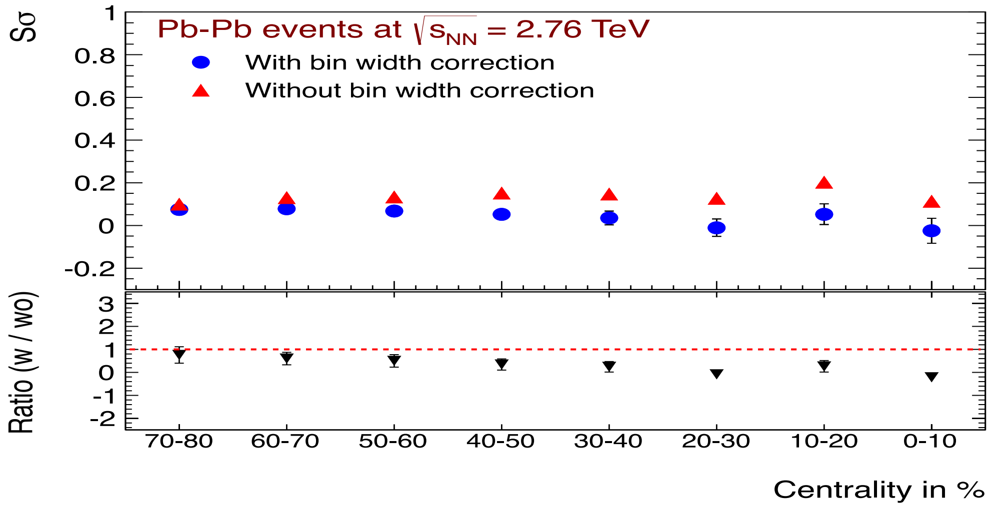


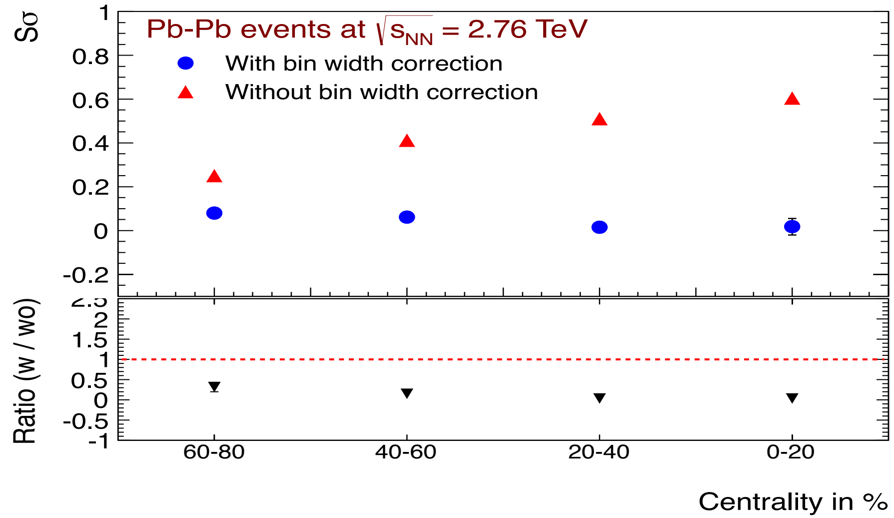
There is a a difference between w/ and w/o BWC

Bin-width correction for Sσ



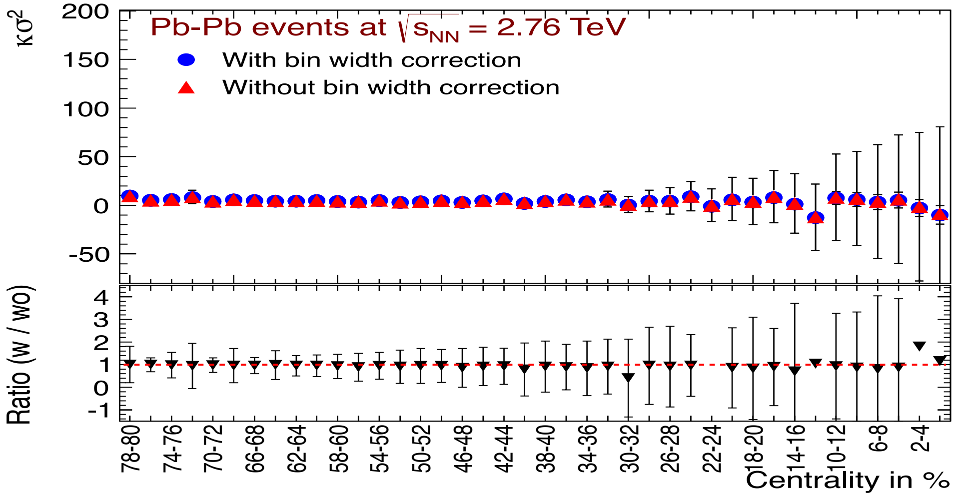


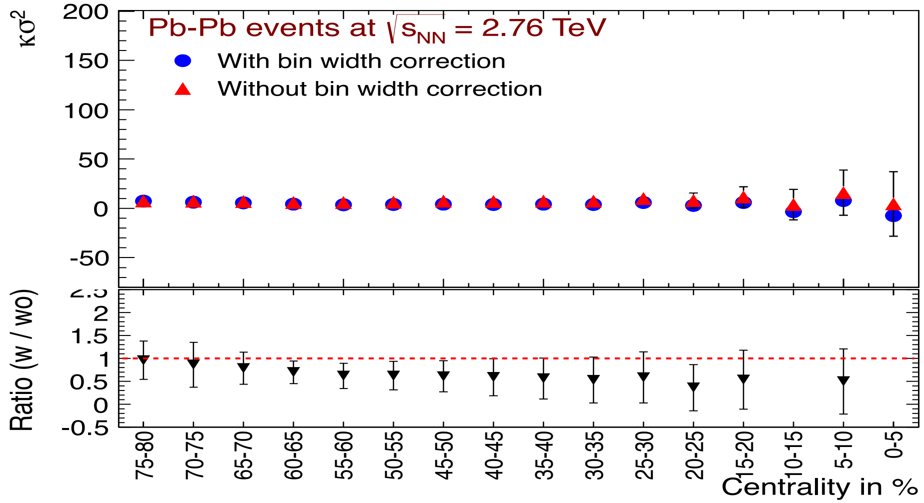


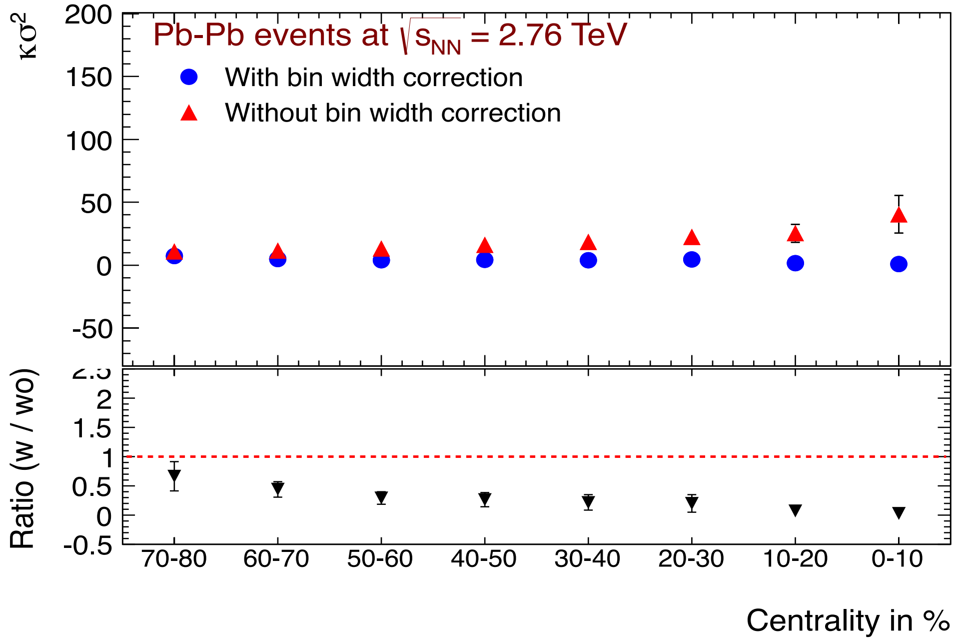


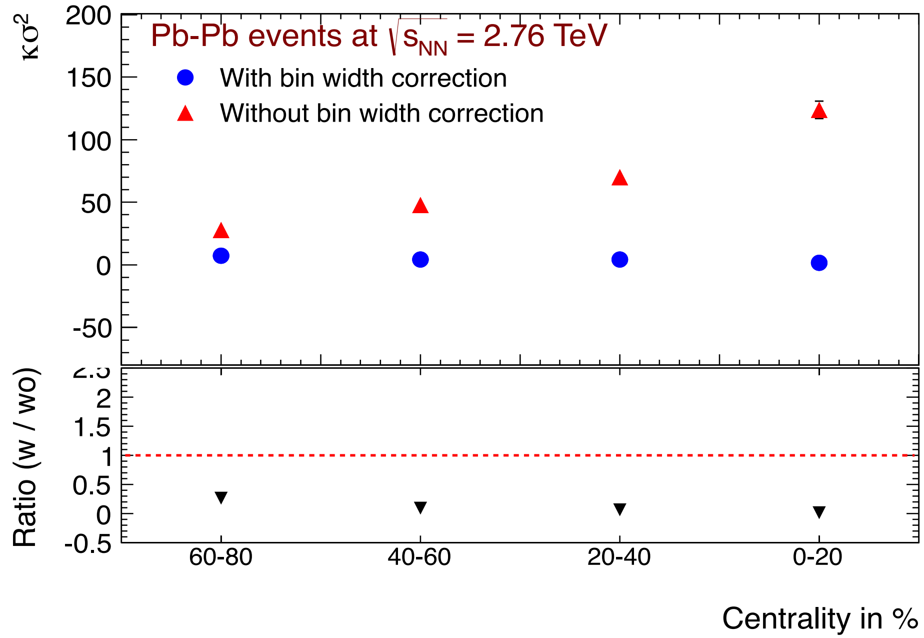
There is a a difference between w/ and w/o BWC

Bin-width correction for $\kappa\sigma^2$







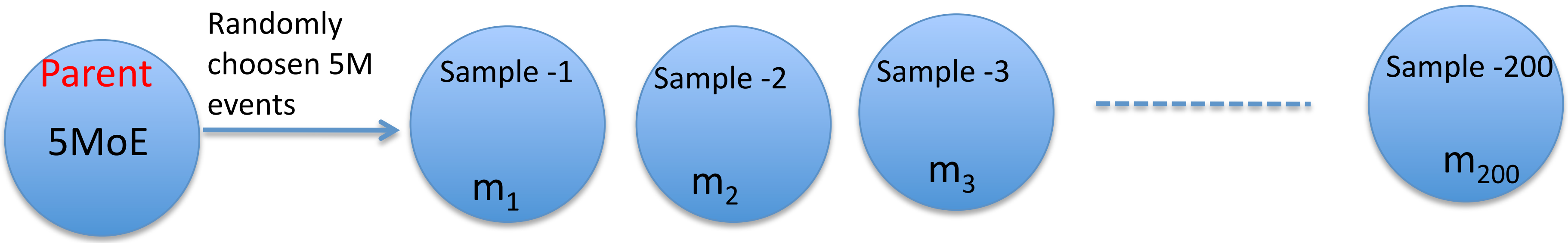


There is a a difference between w/ and w/o BWC

Method – 1 :

Bootstrap Method: Make a sample equal to the numbers of events in the data by choosing randomly from the data. Produce such 200 sample and calculate the higher moments of each sample. The sigma(σ) of the distribution of individual moments distribution is the error of that moments

B. Efron,R. Tibshirani, An introduction to the bootstrap,Chapman & Hall (1993).



TH1D *h1; h1->Fill(m_i); i = 1 to 200

Error of the moment = h1->GetRMS()

To be done for each centrality

Method – 2 : **Delta Theorem** (X. Luo, arxiv-1109.0593v1)

Variance(σ) = $(m_4 - 1)\sigma^2/(4n)$

Variance(S) = $[9 - 6m_4 + m_3^2(35 + 9m_4)/4 - 3m_3m_5 + m_6]/n$

Variance(k) = $[-m_4^2 + 4m_4^3 + 16m_3^2(1 + m_4) - 8m_3m_5 - 4m_4m_6 + m_8]/n$

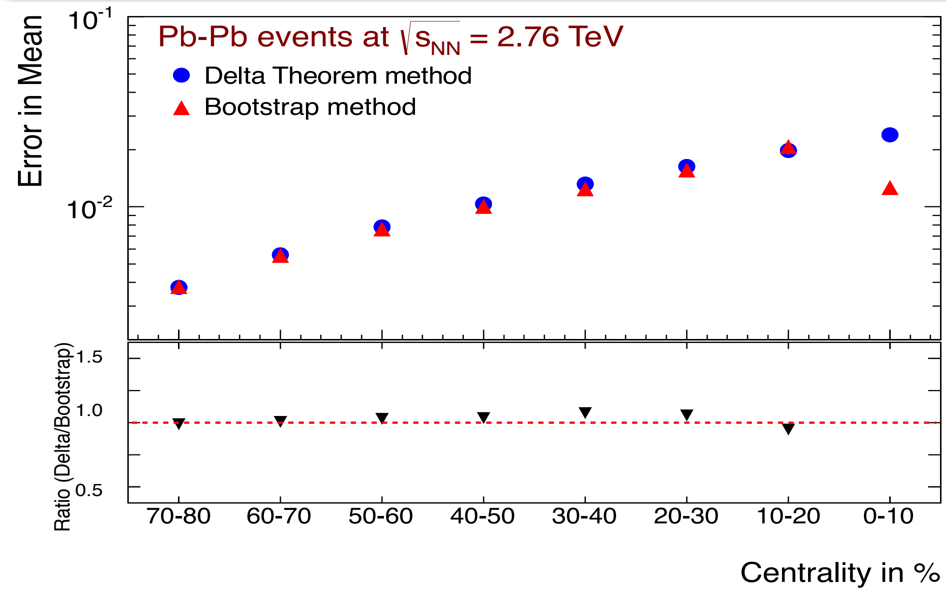
Variance(Sσ) = $[9 - 6m_4 + m_3^2(6 + m_4) - 2m_3m_5 + m_6]\sigma^2/n$

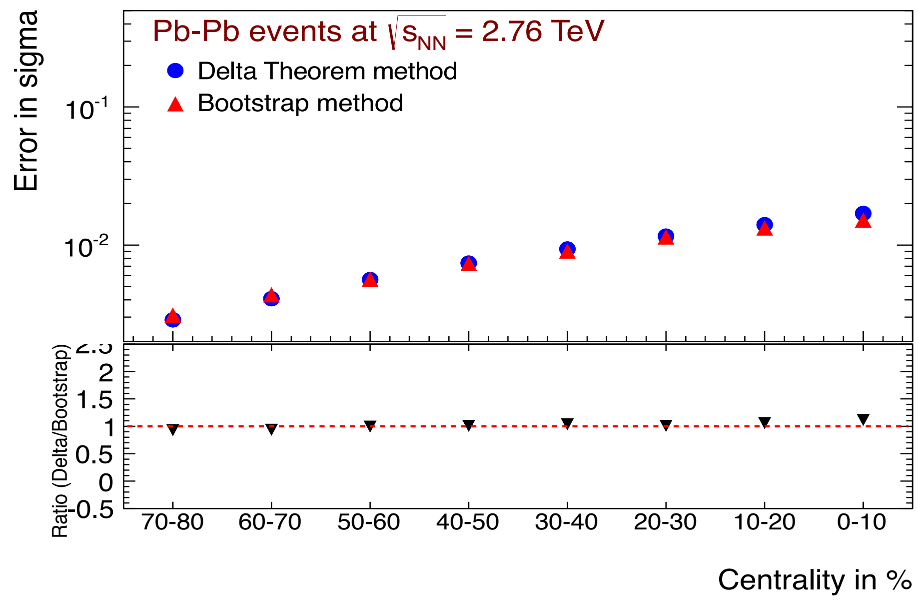
Variance(Kσ²) = $[-9 + 6m_4^2 + m_4^3 + 8m_3^2(5 + m_4) - 8m_3m_5 + m_4(9 - 2m_6) - 6m_6 + m_8]\sigma^4/n$

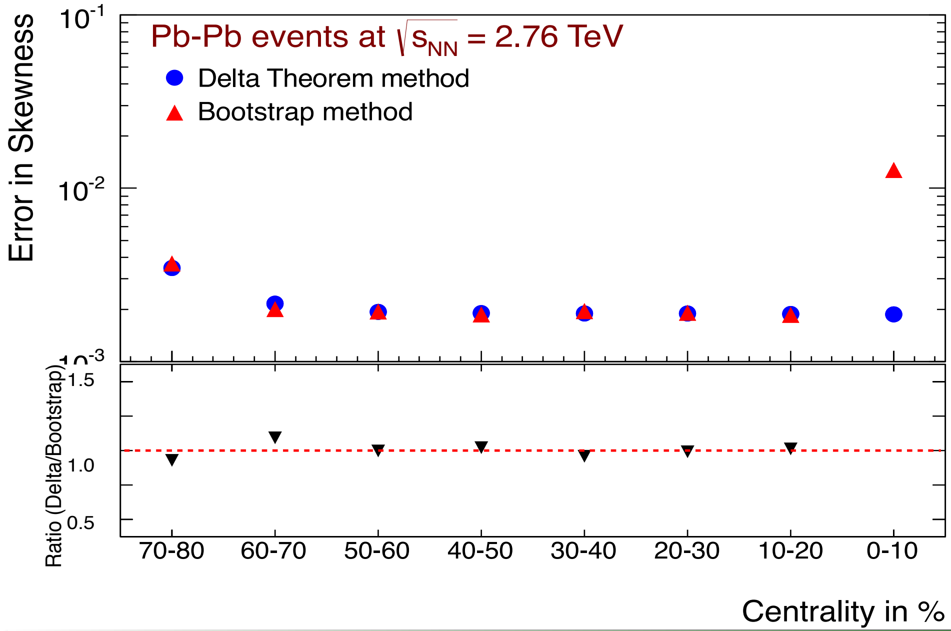
Where $m_r = \mu_r/\sigma^r$ and μ_r is the r-th moment. n = Number of events in that i-th centrality bin.

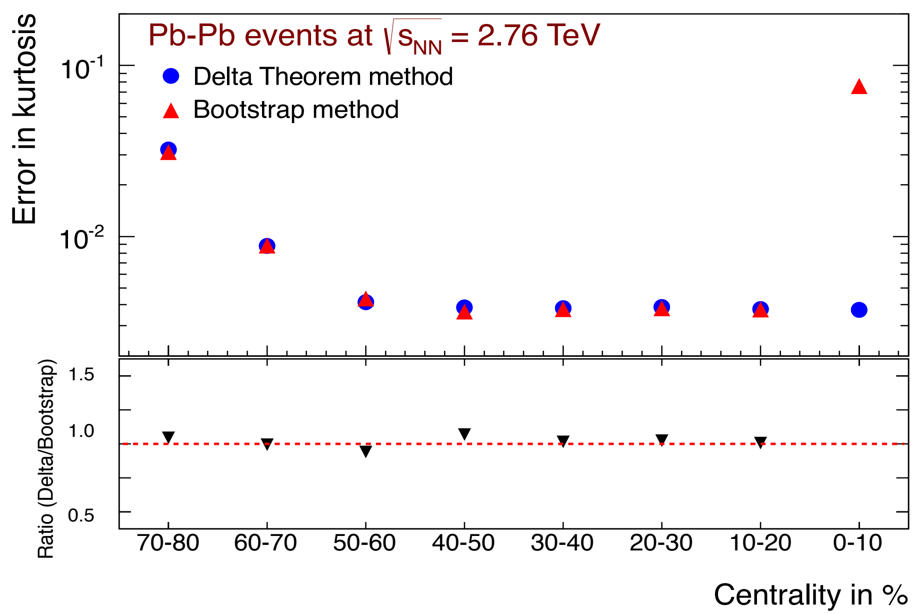
Error for variable $X = \sqrt{\left(\dfrac{\sum_{i=1}^{N} Var(X_i) * n_i^2}{\left(\sum_{i=1}^{n} n_i\right)^2}\right)}$ N = Total centrality Bin

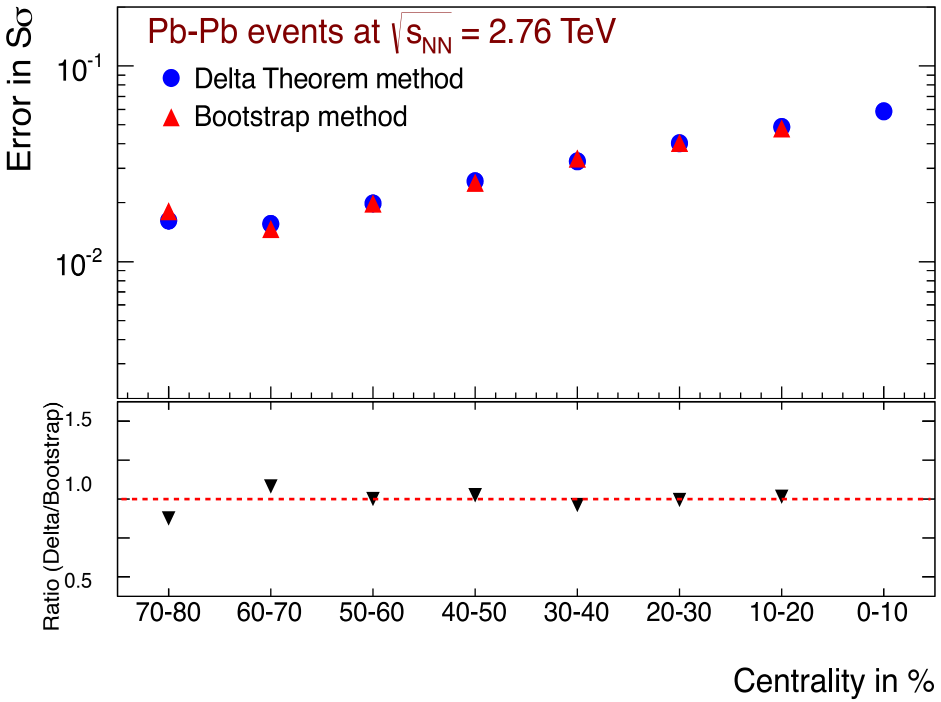
Error estimation

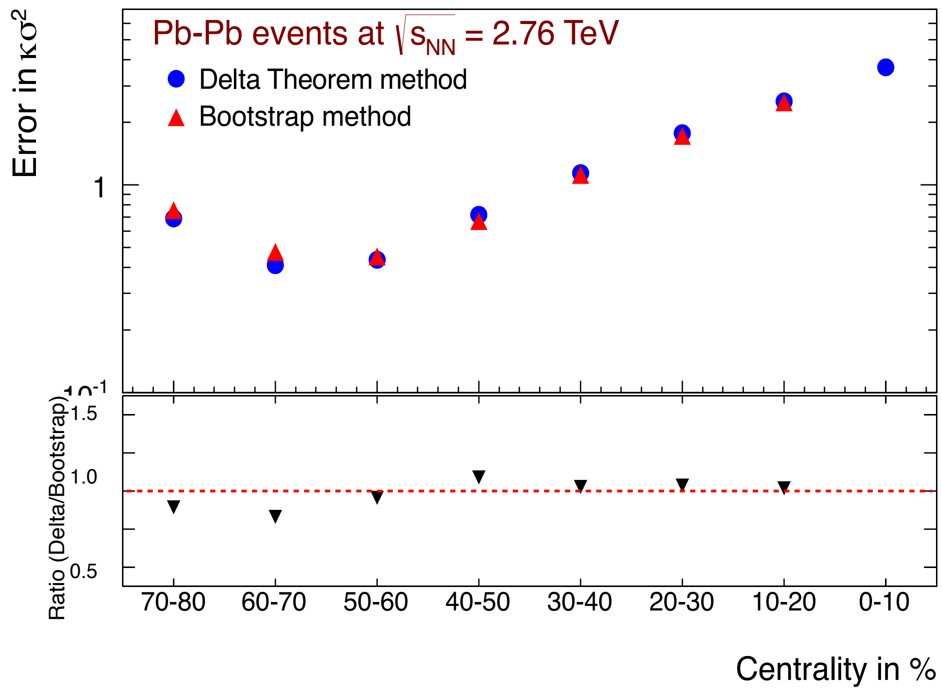












Some problem with most central bin (0-10%) in bootstrap method. Checking!

Poissonian Expectation

Motivation: At Boltzman limit, in HRG model,net-charge, net-baryon distributions are a Skellam distribution.

(P. Braun-Munzinger etal. arXiv:1111.5063v1)

Assumption : Both positive charge and negative charge distribution are of Posissonian kind. Then the difference of two independent poissonian distribution is a **Skellam distribution**.

If m1 and m2 are the means of postive and negative charge respectively,

Then the higher moments of net-charge according to Skellam distribution is

M = m1 – m2
σ = Sqrt(m1+m2)
S = (m1-m2)/(m1+m2)$^{3/2}$
K = 1/(m1+m2)
Sσ =(m1-m2)/(m1+m2)
Kσ^2 = 1

arXiv:1302.2332v1

NBD expectation

(T. J. Tarnowsky and G. D. Westfall arXiv:1210.8102v1)

Motivation : NBD describes the multiplicity distribution better.

In NDB, variance(σ^2) is larger than mean. Poissonian distribution is a limiting case of NBD where mean and variance are same.

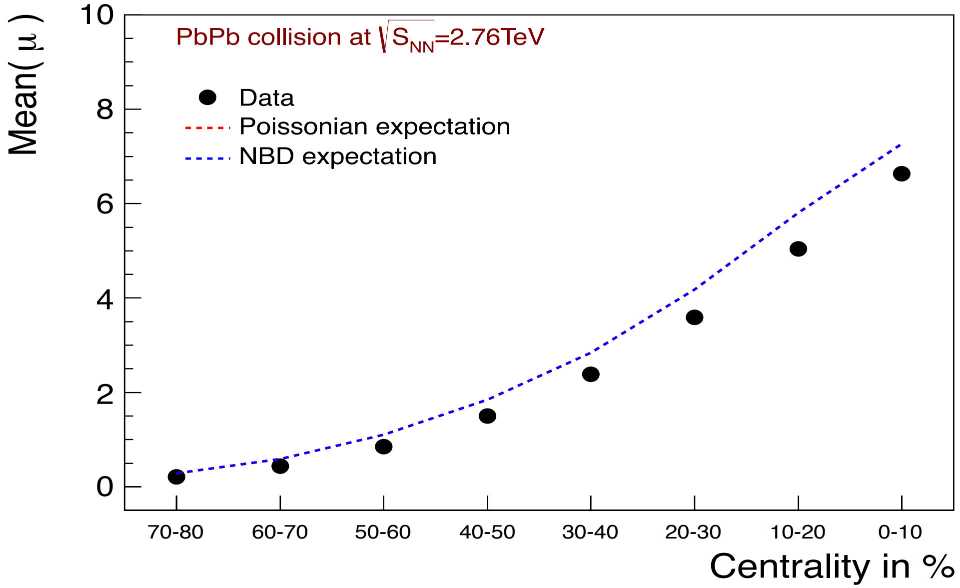
$C_{n,+}$ = n-th order cumulant of +ve charge distribution
$C_{n,-}$ = n-th order cumulant of –ve charge distribution

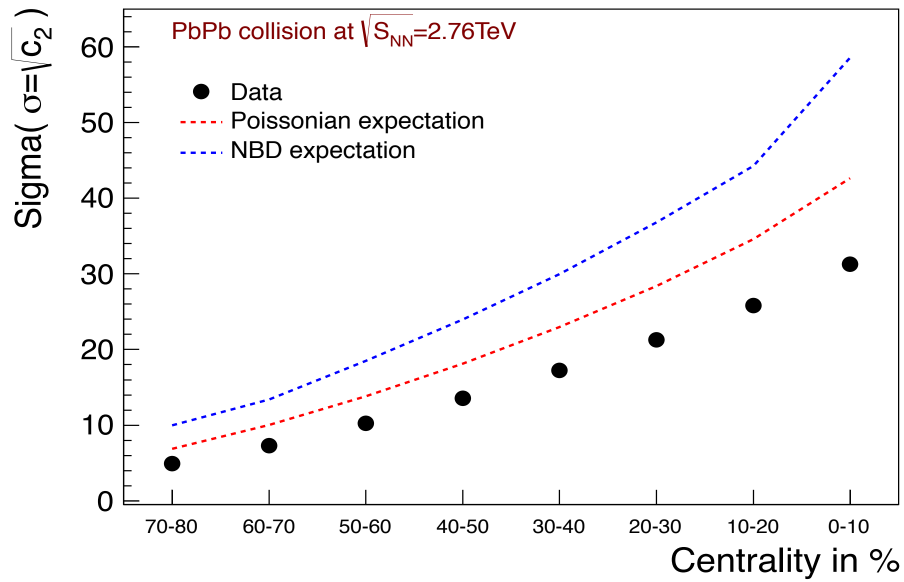
Then nth-order cumulant of net-charge distribution $C_n = C_{n,+} + (-1)^n C_{n,-}$

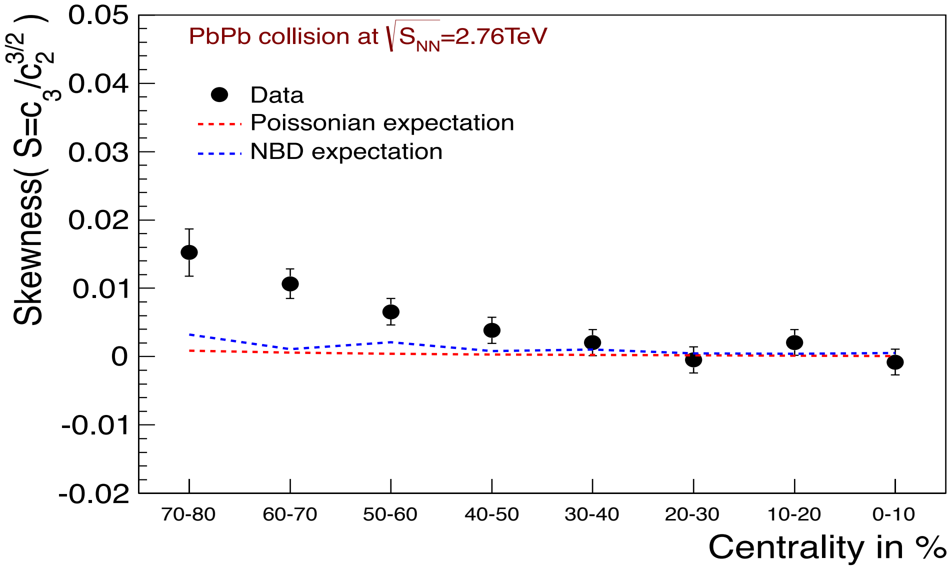
Mean = C_1
Sigma(σ) = Sqrt(C_2)
Skewness(S) = C_3
Kurtosis(K) = C_4/C_2^2

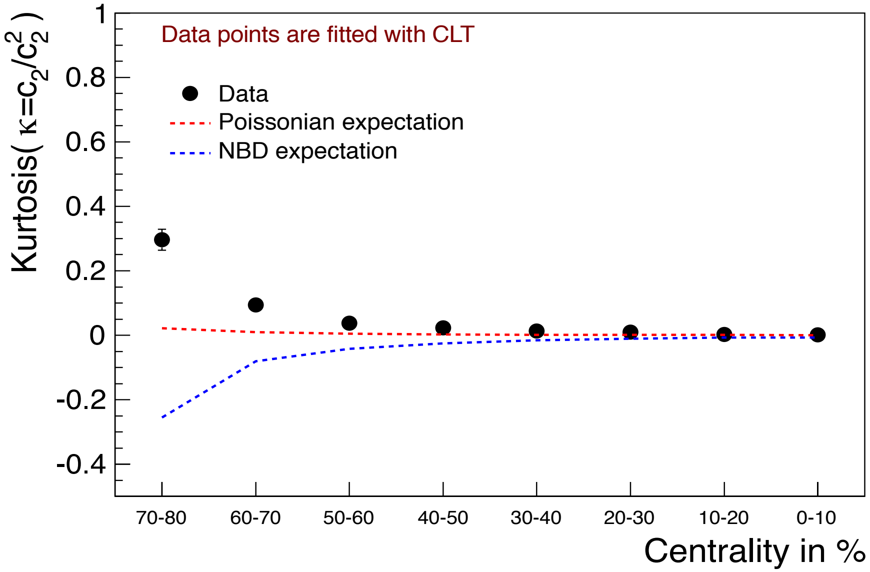
N.B. : The cumulants ($C_{n,+}$, $C_{n,-}$) are caculated according to NBD method.
(http://mathworld.wolfram.com/NegativeBinomialDistribution.html)

Baseline study

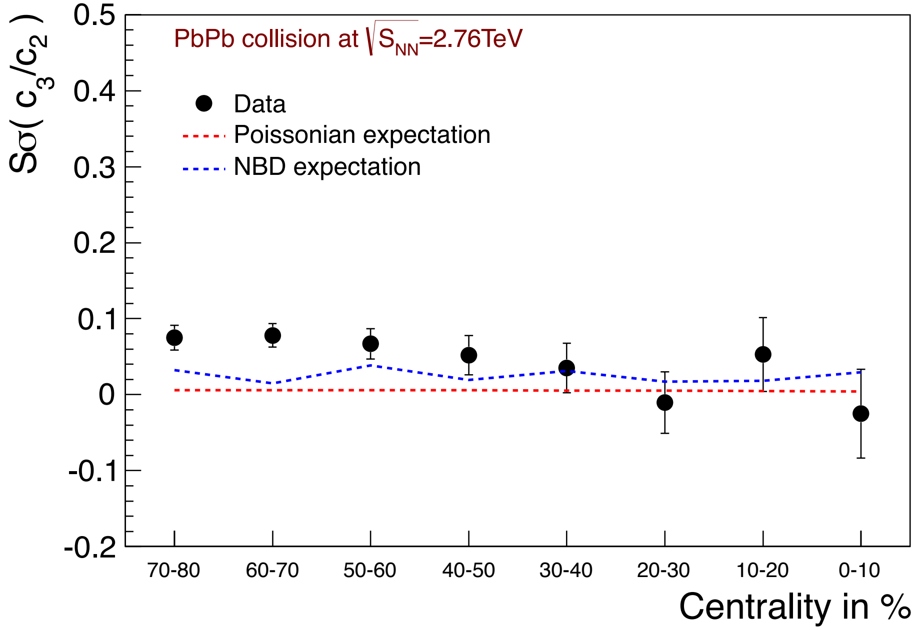


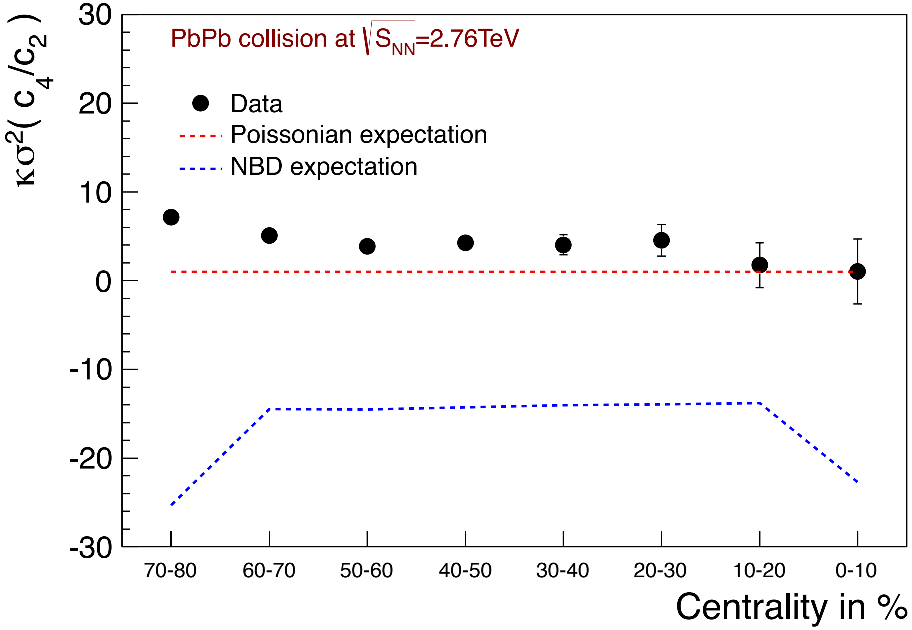






Baseline study





Unfolding net-charge higher moments: a Toymodel study

RooUnfold pakage : http://hepunx.rl.ac.uk/~adye/software/unfold/RooUnfold.html

Bayesian unfolding method is used to remove (correct) the know effects like detection efficiency to get the true net-charge distribution.

P. Garg etal arXiv:1211.2074v2 [nucl-ex] 5 Feb 2013

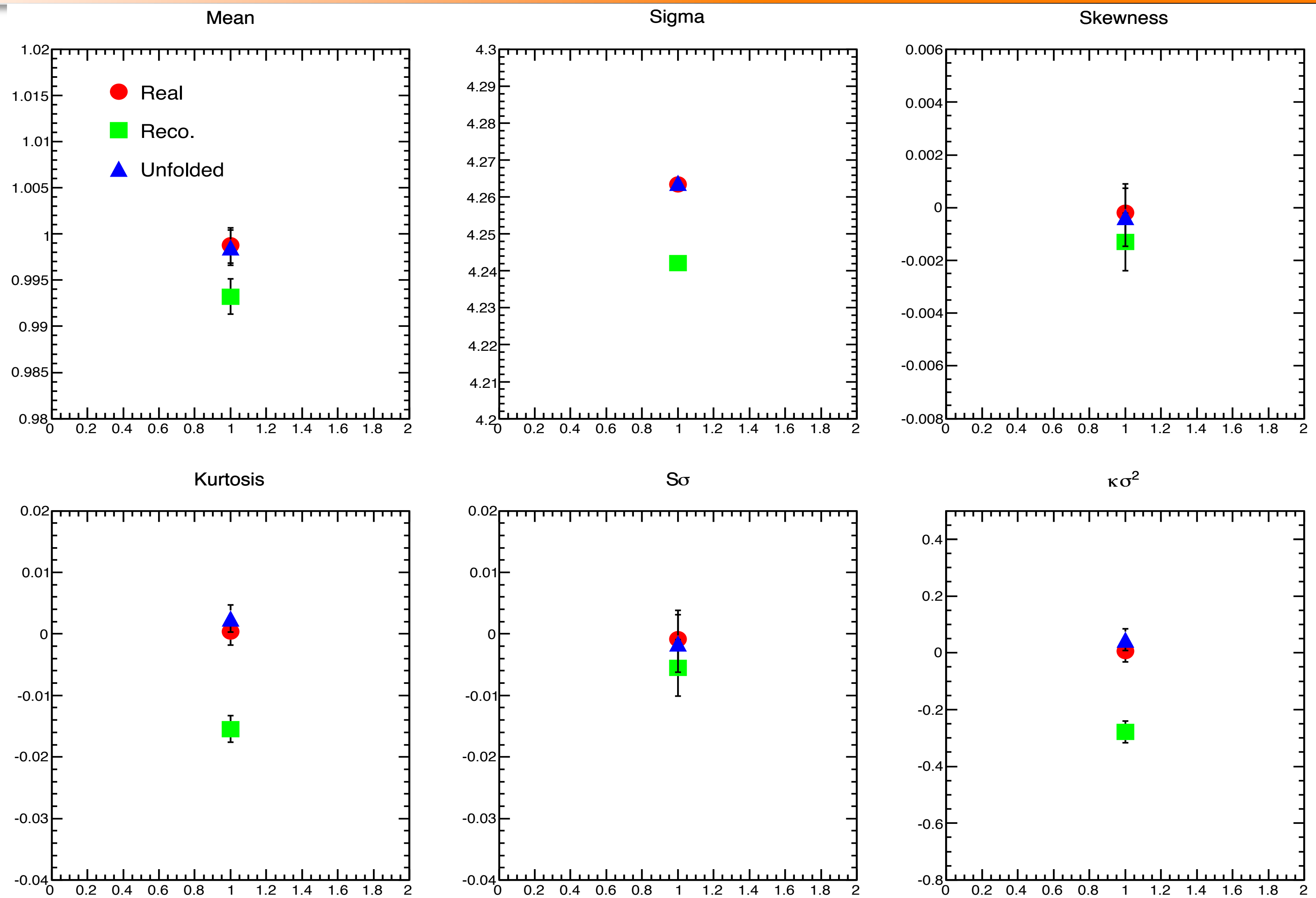
Toy model study

Case -1 : Unfold the same data which is used for training

Case-2 : Unfold another data sample which has mean of net-charge is different from the training sample by keep detection efficiency same.

Test is going on

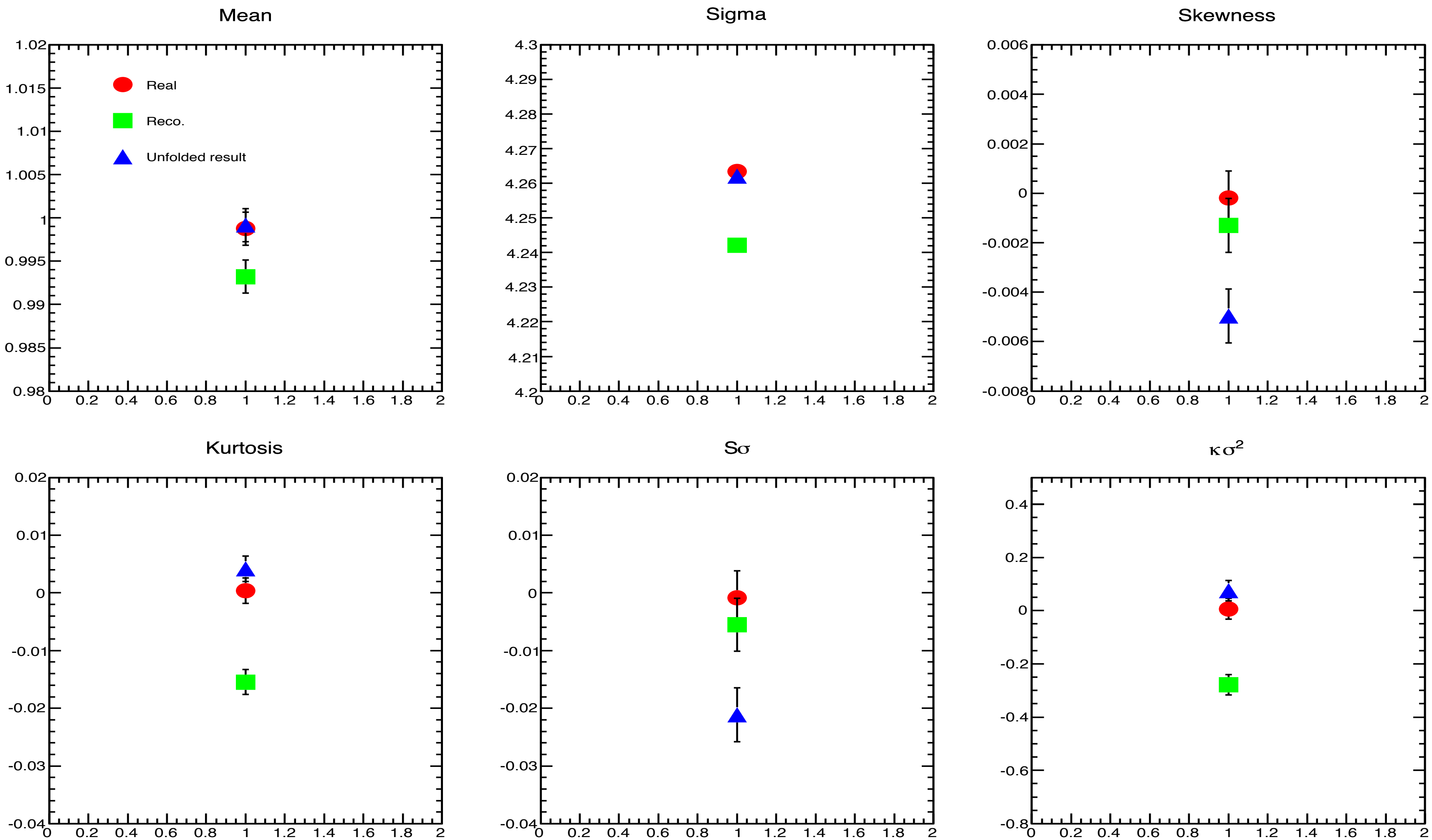
Unfolding net-charge higher moments: a Toy model study



Sample used for training can be unfolded back successfully to get back the true higher moments

Unfolding results

Sample different than training sample



Unfolding method is unable to reproduce the data which mean is other than the training sample.

Summary and future plan

•Higher moments of net-charge and net-proton is studied in Pb-Pb @2.76TeV.

• Effect of without and with bin-width correction are studied. Without bin-width correction shoots the results up for $S\sigma$ and $\kappa\sigma^2$ whereas bin-width correction stabilize the data value. So we need bin-width correction.

• Two different types of methods adopted to estimate the errors for higher moments. Bootstrap and Delta theorem gives similar error.

• Data is compared with Poissonian and NBD expectation. None of them describes the data for $S\sigma$ and $\kappa\sigma^2$

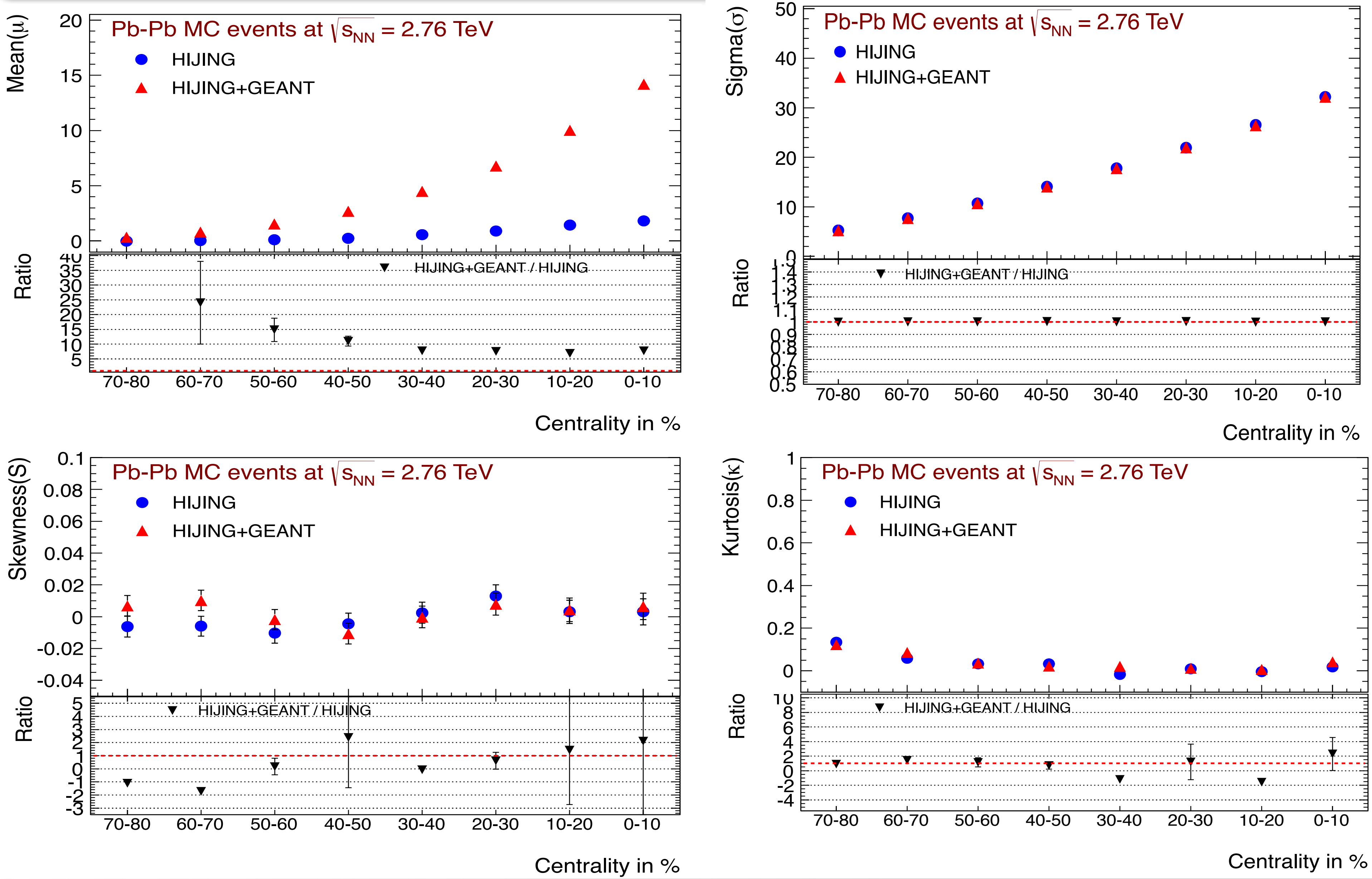
Future Plan:

• To do more extensive study with MC and data and repeating the systematic studies to calculate systematic error. Same analysis will be done for net-proton.

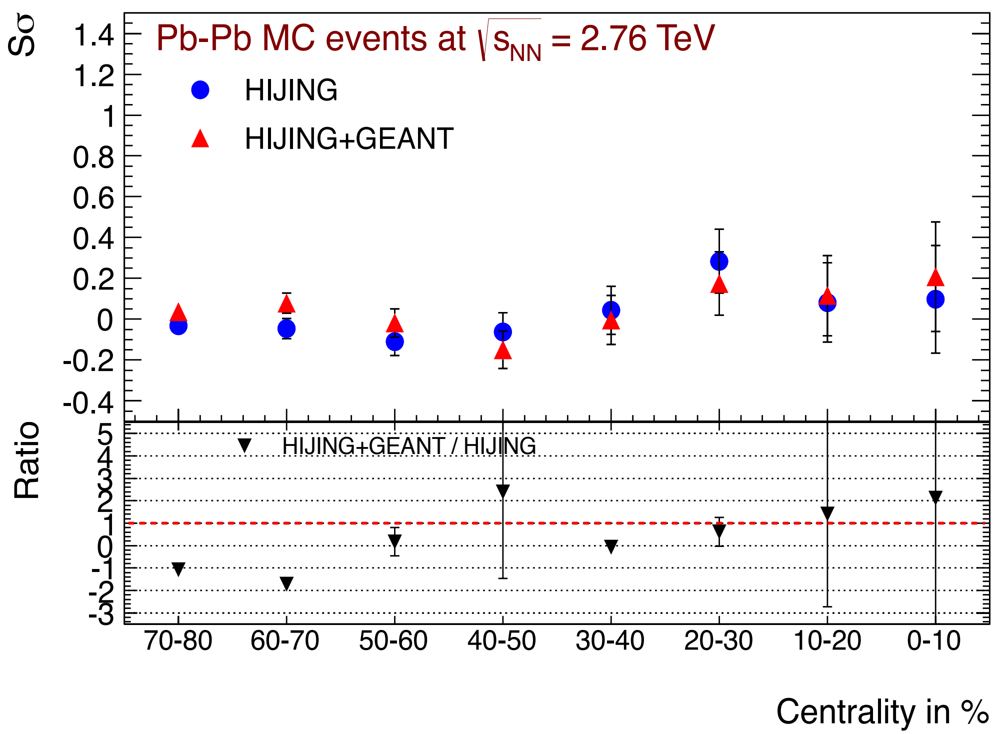
• Work is going on for models to do efficiency acceptance correction proposed by Adam Bzdak and Volker Koch(arXiv:1206.4286v2).

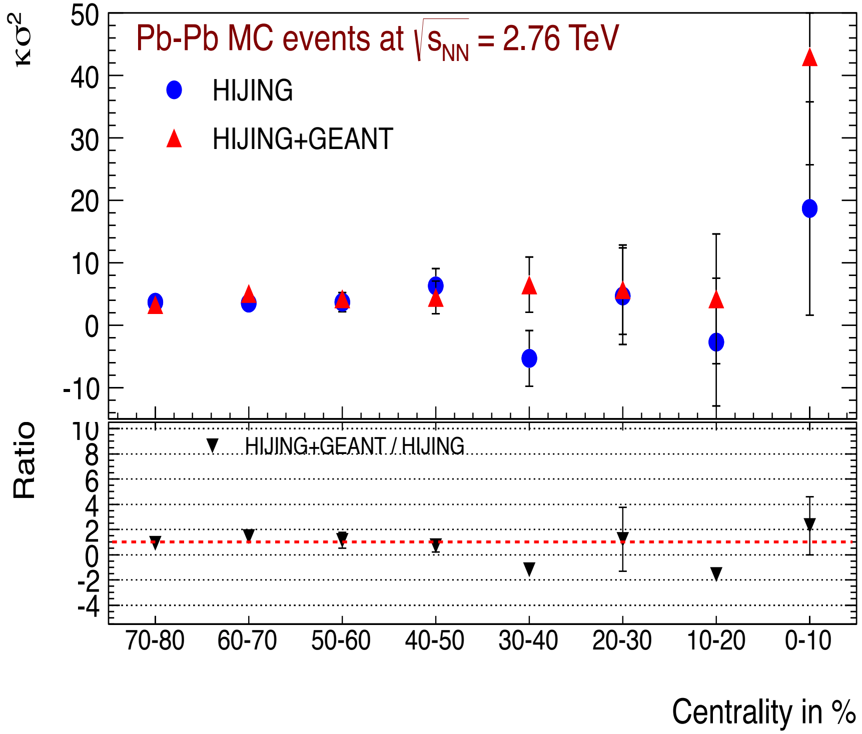
•Work is going on unfolding method. To apply for real data we need a model whose pT spectra, multiplicity and particle ratio matches with experimental data.

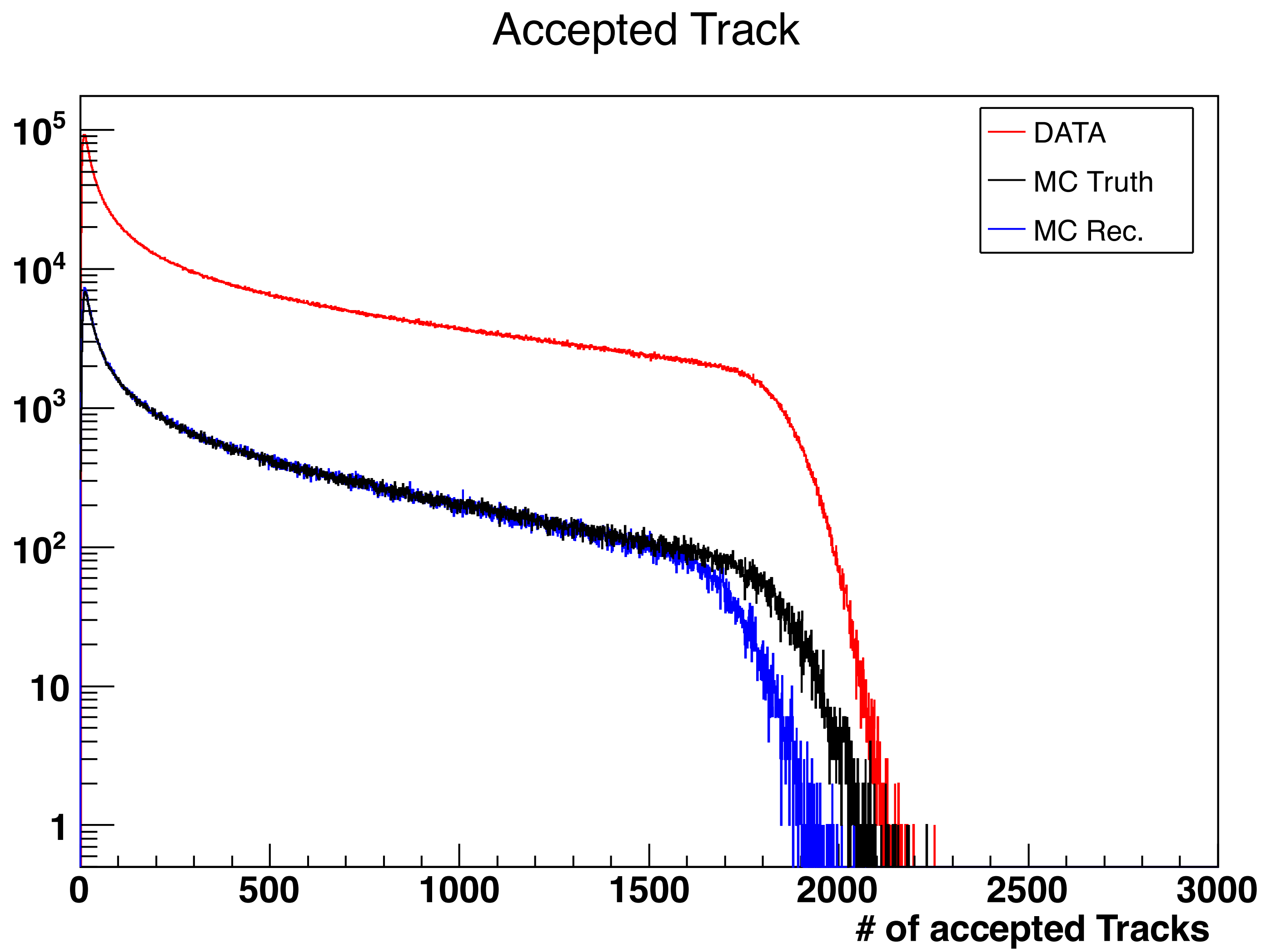
Back Ups



Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
Mean(μ)
Ratio
HIJING+GEANT / HIJING
70-80 60-70 50-60 40-50 30-40 20-30 10-20 0-10
Centrality in %
Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
Sigma(σ)
Ratio
HIJING+GEANT / HIJING
70-80 60-70 50-60 40-50 30-40 20-30 10-20 0-10
Centrality in %
Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
Skewness(S)
Ratio
HIJING+GEANT / HIJING
70-80 60-70 50-60 40-50 30-40 20-30 10-20 0-10
Centrality in %
Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
Kurtosis(κ)
Ratio
HIJING+GEANT / HIJING
70-80 60-70 50-60 40-50 30-40 20-30 10-20 0-10
Centrality in %

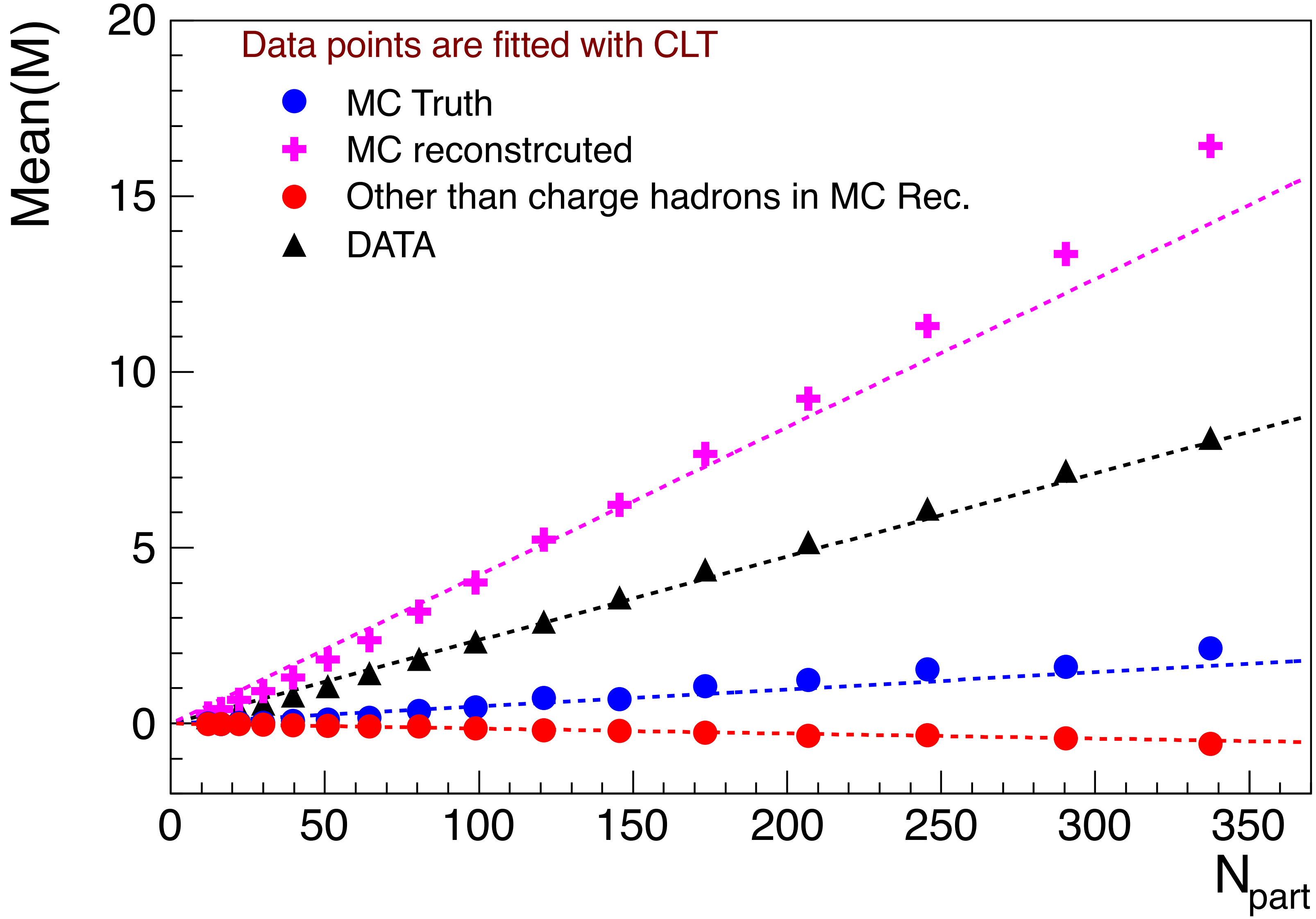

Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
HIJING+GEANT / HIJING
$S\sigma$
Ratio
70-80
60-70
50-60
40-50
30-40
20-30
10-20
0-10
Centrality in %


Pb-Pb MC events at $\sqrt{s_{NN}}$ = 2.76 TeV
HIJING
HIJING+GEANT
HIJING+GEANT / HIJING
$\kappa\sigma^2$
Ratio
70-80
60-70
50-60
40-50
30-40
20-30
10-20
0-10
Centrality in %



Accepted Track
DATA
MC Truth
MC Rec.
10^5
10^4
10^3
10^2
10
1
0
500
1000
1500
2000
2500
3000
# of accepted Tracks

Mean



Mean(M)
20
15
10
5
0
Data points are fitted with CLT
MC Truth
MC reconstrcuted
Other than charge hadrons in MC Rec.
DATA
0
50
100
150
200
250
300
350
N part