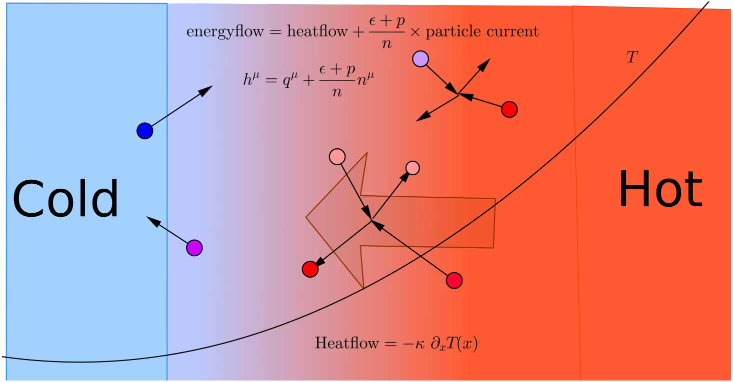
Heat flow in relativistic Navier-Stokes theory

- Heatflow: Energy transfer through *collisions* of particles
- Non-relativistic definition: $Q = -\kappa \bigtriangledown T$
- Navier-Stokes **heat conductivity** κ



Numerical results for elastic cross-sections

This work: $\kappa\sigma_{22} = 2.59 \pm 0.07$, Denicol et al, RTRFD: $\kappa\sigma_{22} = 2.5536$ ($\sigma_{22} = 0.043\ mb - 430\ mb$, elastic, ultrarelativistic Boltzmann particles)

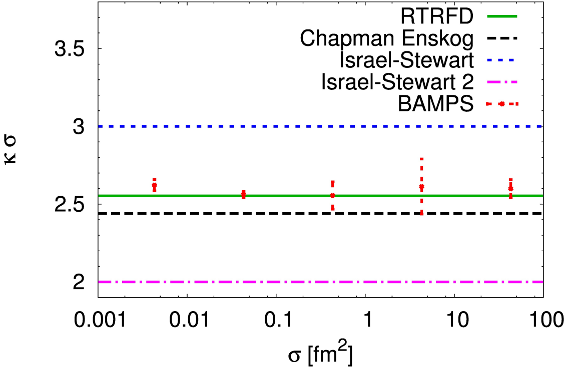


Figure: MG et al., Phys. Rev. E 87, 033019 (2013)

Wanted: value for κ

Useful form for q^μ:

$$q^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \left(\frac{\mu}{T}\right) = \kappa \left(\nabla^\mu T - \frac{T}{\epsilon + p} \nabla^\mu p \right)$$

(Valid for a small Knudsen number $\lambda_{\mathbf{mfp}}/L_{mac}$ and first order in deviation from equilibrium)

AND

$$q^\mu \equiv \Delta^{\mu\alpha} u^\beta T_{\alpha\beta}$$

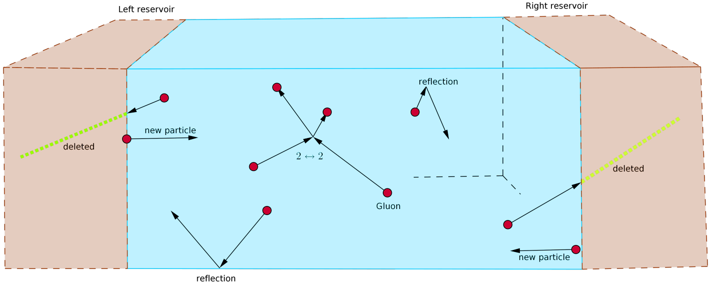
with the dissipative energy-momentum tensor $T_{\alpha\beta}$.

$\nabla^\mu = \partial^\mu - u^\mu D$: space-like Gradient, $D = u^\mu \partial_\mu$: comoving time derivative, $\Delta^{\mu\alpha} = u^\mu u^\alpha - g^{\mu\alpha}$

Const. pressure, static $0 + 1$-dim. system:

$$\kappa = \frac{q^x}{\gamma^2 \partial_x T(x)}$$

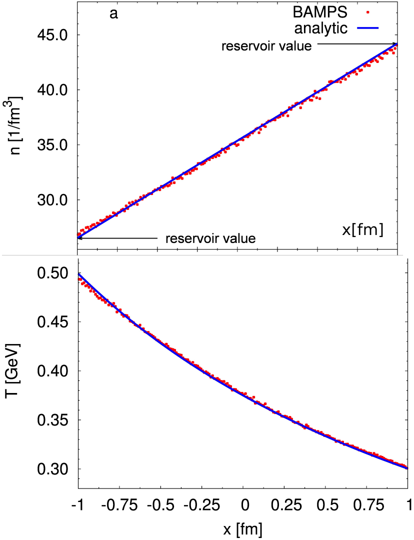
BAMPS - partonic cascade



- Different temperatures in reservoirs ($T_l = 0.5\ GeV,\ T_r = 0.3\ GeV$)
- Fugacity in left reservoir was (arbitrarily) set to 1
- We required $p =$ const. everywhere
- ...$\epsilon_l, \epsilon_r, n_l, n_r$ follow via $\epsilon = 3p = 3nT$

(See arXiv:hep-ph/0406278v2, arXiv:1003.4380v1, ...)

Numerical details: How to set up a Temp.-Gradient

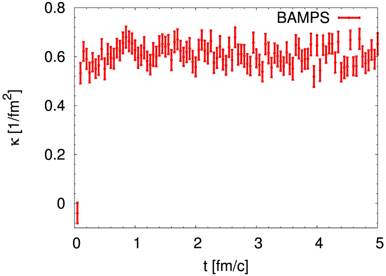


Reservoirs have different density n:

$$n(x) = ax + b$$

$\Rightarrow$ Temperature (const $= p = nT$)

$$T(x) = p/(ax + b)$$



Different analytic formulae for κ

Kinetic theory approaches: Solve BE: $p^\mu \partial_\mu f(x,p) = C[22]$

Israel-Stewart 14-moment approach (ambiguity!)[1]:

$$f(x,p) = f^{eq}(x,p)\,(1+\phi(x,p))$$

$$\kappa\sigma_{22} = 2$$

Denicol et al.[2]: Put $f = f_0\left(1+\tilde{f}_0\phi\right)$, expand ϕ in $k^{\langle\mu\rangle}, k^{\langle\mu}k^{\nu\rangle}, k^{\langle\mu}k^{\nu}k^{\lambda\rangle}, ...$

Resummed Transient Relativistic Fluid Dynamics RTRFD

(does not truncate the moment expansion of the momentum distribution function, uses systematic power-counting scheme using the inverse Reynolds and the Knudsen numbers)

$\kappa\sigma_{22} = 2.5536$ good agreement with BAMPS!

[1] S. Groot, W. van Leeuwen, and van Weert Ch.G., Relativistic Kinetic Theory. Elsevier, 1980/arxiv:1207.5331

[2] Phys.Rev. D85 (2012) p.114047/Eur. Phys. J. A, 4811,170/

Electric conductivity σ_{el} of the QGP

Electric current $J^\mu = \sum_i^{\text{species}} (\text{electric charge})_i N_i^\mu$

Electric conductivity

describes current response to external E-field: $J^\mu = \sigma_{\text{el}}^{\mu\nu} E_\nu$

Methods:

1. Apply external force $q\vec{E}$, observe the current N^μ
2. Keep simulation in equilibrium, observe fluctuations in N^μ

Compare $1) \leftrightarrow 2)$ and with analytic value (elastic $2 \leftrightarrow 2$ cs, massless particles)

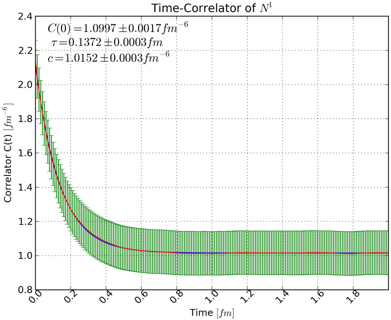
Goal:

Apply 1) & 2) to pQCD-BAMPS, $2, \leftrightarrow 2 + 2 \leftrightarrow 3$ processes

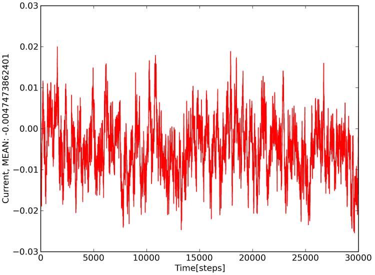
Preliminary: Corellation functions & Green Kubo method

Green-Kubo relation: $\sigma_{\text{el}} \sim \int dt\, C(t)$

$N^x(0)N^x(t)$-Correlator: $C(t) = \frac{1}{s_{\text{max}}} \sum_{s=0}^{s_{\text{max}}} N^x(s)N^x(s+t);\; N^\mu = \frac{1}{V} \sum_{i=1}^{N} \frac{p_i^\mu}{p_i^0}$



(a) Typical $C(t)$ @ $T = 0.4\ GeV$



(b) Current (N^x) fluctuations over time in box ($V = 0.8^3 fm,\ \sigma_{22} = 10mb,\ T = 0.2\ GeV$)

Preliminary: external force method

1. Current is $J^\mu = \frac{q}{V} \sum_{i=1}^{N} \frac{p_i^\mu}{p_i^0}$
2. Apply[3] an additional momentum to each particle i with charge q_i at each timestep Δt, using small electric field E^x,

$$p_i^x \longrightarrow p_i^x + (\Delta t E^x q_i)$$

3. Wait until static, non-zero current has established
4. **Read off electric conductivity σ_{el}**

$$\sigma_{\text{el}} = \frac{J^x}{E^x}$$

[3] see e.g. Cassing et al.,arXiv:1302.0906

Preliminary: Chapman-Enskog, rel. time analytic value

Requirement: Massless Boltzmann particles, only elastic $2 \leftrightarrow 2$-cs σ_{22}, charge q
Start from the Boltzmann-Eq:

$$qF^{\alpha\beta}p_\beta \frac{\partial f}{\partial p^\alpha} = -\frac{p^\mu u_\mu}{\tau}\left(f - f_{\text{eq}}\right)$$

Expansion:

$$f = f_{\text{eq}} + f_{\text{eq}}\phi$$

Finally:

$$q\int d^3\vec{p}\,\frac{p^x}{p^0}f \equiv J^x = \sigma_{\text{el}}E^x$$

$$\Rightarrow\ \sigma_{\text{el}} = \frac{2}{3}\frac{q^2}{\beta^2 n}\frac{1}{\sigma_{22}}$$

Thank you for listening!

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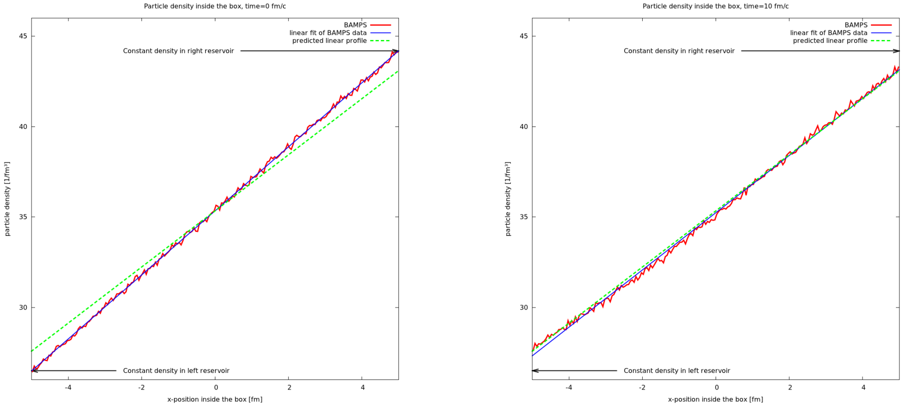
- Andrej El
- Carsten Greiner

Linear density profile in ultrarelativistic cascade

$$n(x) = \frac{n_r - n_l}{L + 2\lambda_{mfp}} x + \frac{n_r - n_l}{2} + n_l, \quad \lambda_{\text{mfp}} = \frac{1}{n\sigma} \sim 0.65 \;\Rightarrow\; Kn = \frac{\lambda_{\text{mfp}}}{L} \sim 0.065 \tag{1}$$



(a) (initialised) density at $t = 0\ fm/c$

(b) density at $t = 10\ fm/c$

Figure: $\sigma = 0.43mb$, density evolution: BAMPS vs. eq.(1)

Coming soon: Heat-Flow Corellation functions & Green Kubo method

Green-Kubo relation for heat conductivity

$$\kappa \sim \frac{1}{T^2} \int_0^\infty \langle q^x(0) q^x(t) \rangle$$

- Calculate volume-averaged heat flow for each timestep
- Calculate heat-flow time-autocorrelator
- Average over many events
- Integrate correlation function
- $q^x(0)q^x(t)$-Correlator:

  $C(t) = \frac{1}{s_{\textbf{max}}} \sum_{s=0}^{s_{\textbf{max}}} q^x(s) q^x(s+t); \; q^\mu = \Delta^{\mu\nu} u^\alpha T_{\nu\alpha}$

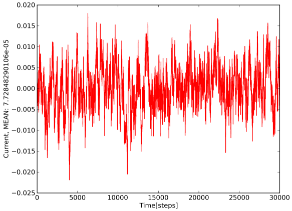


Figure: Typ. q^x fluctuations over time