Surprisingly Close Tsallis Fits to High Transverse Momentum Hadrons Produced at LHC

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- (*) Introduction (Tsallis fits to hadron p_T spectra ,power laws and power index)
- (*) Relativistic hard-scattering model
- (*) Extraction of the power index *n* from experimental jet and hadron p_T distributions
- (*) Phenomenological input necessary to get the low p_T limit
- (*) Conclusions

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Example of Tsallis distribution: application to PHENIX data



Phenix Coll., PRD 83, 052004 (2011)

Fig. 12 Invariant differential cross sections of different particles measured in p p collisions at $\sqrt{s} = 200$ GeV in various decay modes.

PHENIX ci (π*+π') $(K^{+}K)/2$ p+p√s=200GeV o 🕦 AA $\pi^0\pi^*\pi^-$ yo π^0 ± ω → e⁺e΄. p (p+p)/2 $\circ \eta' \rightarrow \eta \pi' \pi'$ K*K* → e'e' Ed^ªc/dp³ (mb GeV² c³) a-J/w → e⁺e പു **ឃ**'→ e*e 10 10 $\omega \times 10^3$ $\eta \times 10^4$ 10-8 18 20 12 14 16 p_T (GeV/c)

q=1.1

n=10

Tsallis distribution can describe LHC p_T distributions



Wong and Wilk, ActaPhysPol.B43,2047(2012)

Tsallis distribution can describe LHC p_T distributions



Wong and Wilk, ActaPhysPol.B43,2047(2012)

<u>Good Tsallis p_T fits raise questions</u>

- What is the physical meaning of *n*?
- If *n* is the power index of $1/p_T^n$, then why is $n \sim 7$, whereas pQCD predicts $n \sim 4$?
- Why are there only few degrees of freedom over such a large p_T domain ?
- Do multiple parton collisions play any role in modifying the power index *n*?
- Does the hard scattering process contribute significantly to the production of low-p_T hadrons?
- What is the origin of low- p_T part of Tsallis fits ?

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Parton Multiple Scattering

For the collision of a parton a with a target of A partons in sequence without centrality selection, the differential c_{τ} distribution is given by

$$\frac{d\sigma_{H}(aA \to cX)}{d\vec{c}_{T}} = A \frac{\alpha_{s}^{2}}{c_{T}^{4}} \int d\vec{b} T(b) + \frac{A(A-1)}{2} \frac{16\pi\alpha_{s}^{4}}{c_{T}^{6}} \ln\left(\frac{c_{T}}{2p_{0}}\right) \int d\vec{b} [T(b)]^{2} + \frac{A(A-1)(A-2)}{6} \frac{936\pi^{2}\alpha_{s}^{4}}{c_{T}^{8}} \left[\ln\left(\frac{c_{T}}{2p_{0}}\right)\right]^{2} \int d\vec{b} [T(b)]^{3}$$

The contribution from the single collision dominates, but high multiple collisions comes in at lower \mathbf{p}_{T} and for more central collisions

Parton Multiple Scattering

For the collision of a parton a with a target of A partons in sequence without centrality selection, the differential c_{τ} distribution is given by

$$a \xrightarrow{q' \quad q'' \quad q''' \quad q'''}_{b_1 \quad d_1 \quad b_2 \quad d_2 \quad b_3 \quad d_3 \quad b_N \quad d_N} c$$

One expects then that for more and more central collisions, contributions with a greater number of multiple parton collisions gains in importance. As a consequence, the power index n is expected to become greater when we select more central collisions (subject to experimental verification).

The contribution from the single collision dominates, but high multiple collisions comes in at lower p_T and for more central collisions

<u>Good Tsallis p_T fits raise questions</u>

- What is the physical meaning of *n*?
- If *n* is the power index of $1/p_{-n}$, then why is $n \sim 7$.

We seek answers to these questions from the

relativistic hard-scattering (RHS) model

[Blankebecler, Brodsky et al., PRD10(1974)2973; D12(1975)3469; D15(1977)3321]

• What is the origin of low- p_T part of Tsallis fits ?

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Relativistic Hard-Scattering (RHS) Model

- Approximate Hard-Scattering Integral -

$$\begin{split} E_p \frac{d\sigma(AB \to cX)}{d^3 p} &= \sum_{ab} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) E_c \frac{d\sigma(ab \to cd)}{d^3 p} \\ \text{The basic differential cross section is} \\ E_c \frac{d\sigma(ab \to cd)}{d^3 p} &= \frac{\hat{s}}{\pi} \frac{d\sigma(ab \to cd)}{dt} \delta(\hat{s} + \hat{t} + \hat{u}). \end{split}$$

$$\begin{aligned} \text{We assume :} \qquad x_a G_{a/A}(x_a) &= A_a (1 - x_a)^g, \\ \text{For central rapidity, } |\eta| \approx 0, \quad \text{we obtain} \\ E_p \frac{d^3 \sigma(AB \to cX)}{d^3 c} &= \sum_{ab} \frac{A_a A_b}{\sqrt{\pi g}} (1 - x_{a0})^g (1 - x_{b0})^g \times \\ &\qquad \times \frac{1}{\sqrt{\tau_c}} \left\{ \frac{1 - x_c}{1 - \tau_c^2 / x_c} \right\}^{1/4} \sqrt{\frac{(1 - x_{b0})}{1 - (x_{b0} + \tau_c^2 / x_c) / 2}} \frac{d\sigma(ab \to cd)}{dt} \\ \\ x_c &= \frac{c_0 + c_z}{\sqrt{s}}, \quad \tau_c = \frac{c_T}{\sqrt{s}}, \quad x_{a0} = x_c + \tau_c \sqrt{\frac{1 - \tau_c^2 / x_c}{1 - x_c}}, \quad x_{b0} = \frac{\tau_c^2}{x_c} + \tau_c \sqrt{\frac{1 - \tau_c^2 / x_c}{1 - x_c}} 10 \end{aligned}$$

RHS Model - The Power Index in Jet Production

For
$$gg \to gg, qq' \to qq'$$
, and $qg \to qg$, $\frac{d\sigma(ab \to cd)}{dt} \propto \frac{\alpha_s^2(c_T)}{c_T^4}$

The analytical formula is

$$E_{c} \frac{d\sigma(AB \to cX)}{d^{3}c} \propto \frac{1}{(c_{T} / \sqrt{s})^{1/2}} \frac{\alpha_{s}^{2}(c_{T})}{c_{T}^{4}} (1 - x_{a0}(c_{T}))^{g} (1 - x_{b0}(c_{T}))^{g}$$

For $\eta \sim 0$, $x_{a0}(c_T) = x_{b0}(c_T) = 2x_c(c_T) = 2c_T / \sqrt{s}$, the analytical formula is

$$E_{c} \frac{d\sigma(AB \to cX)}{d^{3}c} \propto \frac{\alpha_{s}^{2}(c_{T}) (1 - 2x_{c}(c_{T}))^{g} (1 - 2x_{c}(c_{T}))^{g}}{c_{T}^{4 + 1/2} / (\sqrt{s})^{1/2}}$$

We change notations $c \rightarrow p$ and introduce power index *n*

$$E_{p} \frac{d\sigma(AB \to pX)}{d^{3}p} \propto \frac{\alpha_{s}^{2}(p_{T}) (1 - 2x_{c}(p_{T}))^{g} (1 - 2x_{c}(p_{T}))^{g}}{p_{T}^{n} / (\sqrt{s})^{1/2}}$$

where n = 4 + 1/2 for LO pQCD. $g_{a,b} = g = 6 - 10$ (we take g = 6 [18])

[18] D. W. Duke and J. F. Owens, Phy. Rev D30, 49 (1984).

How to get n from data ?

Method (I),

Work at a fixed \sqrt{s} , and make a log-log plot of σ_{inv} and p_T



How to get n from data ?

Method (II) [10]

Write the analytical formula

$$E_{p} \frac{d\sigma(AB \to pX)}{d^{3}p} \propto \frac{\alpha_{s}^{2}(p_{T}) (1 - 2x_{c}(p_{T}))^{g} (1 - 2x_{c}(p_{T}))^{g}}{p_{T}^{n}/(\sqrt{s})^{1/2}}$$

with $x_{c} = p_{T}/\sqrt{s}$ as
$$\sigma_{inv}(\sqrt{s}, x_{c}) = E_{p} \frac{d\sigma(AB \to pX)}{d^{3}p} \propto \frac{\alpha_{s}^{2}(p_{T}) (1 - 2x_{c}(p_{T}))^{g} (1 - 2x_{c}(p_{T}))^{g}}{(x_{c}\sqrt{s})^{n}/(\sqrt{s})^{1/2}}$$

At a fixed x_{c} , we look at two different energies, $\sqrt{s_{1}}$ and $\sqrt{s_{2}}$,
$$\underbrace{\frac{\ln[\sigma_{inv}(\sqrt{s_{1}}, x_{c})/\sigma_{inv}(\sqrt{s_{2}}, x_{c})]}{\ln[\sqrt{s_{2}}/\sqrt{s_{1}}]} \approx n(x_{c}) - \frac{1}{2}}$$

[10] F. Arleo, S. Brodsky, D. S. Hwang, and A. M. Sickles, Phys. Rev. Lett. 105, 062002 (2010).

How to get n from data ?

Method (III)

At a fixed \sqrt{s} , consider running coupling constant

$$\alpha_{s}(\mathbf{Q}(\mathbf{c}_{T})) = \frac{12\pi}{27\ln(\mathbf{C} + \mathbf{Q}^{2}/\Lambda_{\text{QCD}}^{2})},$$

$$\Lambda_{\text{QCD}} = 0.25 \text{ GeV} \Rightarrow \alpha_{s} \left(M_{Z}^{2}\right) = 0.1184;$$

$$C = 10 \Rightarrow \alpha_{s} \left(Q \propto \Lambda_{QCD}\right) \approx 0.6 \text{ in hadron spectroscopy studies} [17]$$

For
$$Q = c_T$$
 search for *n* by fitting experimental data at $\eta \approx 0$

$$E_c \frac{d\sigma^3(AB \rightarrow cX)}{dc^3} = \frac{A\alpha_s^2(Q^2(c_T)) (1 - x_{a0})^{g_a + \frac{1}{2}} (1 - x_{b0})^{g_b + \frac{1}{2}}}{c_T^n \sqrt{1 - x_c}}$$

Notice: parameter C regularizes the coupling constant for small values of $Q(c_T)$. [17] C. Y. Wong, E. S. Swanson, and T. Barnes, Phy. Rev. C, 65, 014903 (2001)¹⁴

D0 jet data can be described by RHS model



Comparison of the relativistic hard-scattering model results for jet production, Eq. (22) (solid curves), with experimental $d\sigma/d\eta E_T dE_T data$ from the D0 Collaboration, for hadron jet production within $|\eta|<0.5$, in $\bar{p}p$ collision at (a) $\sqrt{s}=1.80$ TeV, and (b) $\sqrt{s}=0.63$ TeV.

ALICE and CMS jet data can be described by RHS model



(without regularization of $\alpha(p_T)$; R is clustering parameter used in jet finding algorithms) ¹⁶

Except for the CMS data at 7 TeV that may need further re-examination, the power indices extracted for hadron jet production are in approximate agreement with the value of n=4.5 in Eq. (19) and with previous analysis in [10], indicating the approximate validity of the hard scattering model for jet production in hadron-hadron collisions, with the predominant α_s^2 / c_T^4 parton-parton differential cross section as predicted by pQCD.

Collaboration	\sqrt{s}	R	η	n
D0	$\bar{p}p$ at 1.80 TeV	0.7	$ \eta < 0.7$	4.39
D0	$\bar{p}p$ at 0.63 TeV	0.7	$ \eta < 0.7$	4.47
ALICE	pp at 2.76 TeV	0.2	$ \eta < 0.5$	4.78
ALICE	pp at 2.76 TeV	0.4	$ \eta < 0.5$	4.98
CMS	pp at 7 TeV	0.5	$ \eta < 0.5$	5.39

[10] F. Arleo, S. Brodsky, D. S. Hwang, and A. M. Sickles, Phys. Rev. Lett. 105, 062002 (2010).

Evolution from jet to hadrons

The evolution from a jet to hadrons passes through the stages of

(i) showering (and/or fragmentation),

(ii) hadronization.

Phenomenological Modifications for Hadron Production



(*) For the case of hadron production, it is necessary to take into account additional effects. Jets undergoes fragmentation and hadronization to produce the observed hadrons.

(*) For example: from the fragmentation function for a parent parton jet to fragment into hadrons, an observed hadron **p** of transverse momentum \mathbf{p}_{T} can be estimated to arise (on the average) from the fragmentation of a parent jet **c** with transverse momentum $\langle \mathbf{c}_{T} \rangle = 2.3 \mathbf{p}_{T}$ [11].

[11] C. Y. Wong and G. Wilk, Phys. Rev. D87, 114007 (2013).

Effects of showering (and/or fragmentation) on power law

If the fragmentation is such that p = zc, $E_{p} \frac{d\sigma(AB \to pX)}{d^{3}p} = \int dz D_{p/c}(z) \int d^{4}c \frac{d\sigma(AB \to cX)}{d^{4}c} \delta^{(4)}(p-zc)$ $= \int \frac{dz}{z^2} D_{p/c}(z) z^{4+1/2} \frac{\alpha_s^2(c_T) (1 - x_c(c_T))^8 (1 - x_c(c_T))^8}{n^{4+1/2}}$ $\approx \frac{\alpha_s^2(\overline{c}_T)(1-x_c(\overline{c}_T))^g(1-x_c(\overline{c}_T))^g}{n^{4+1/2}}$ $\overline{c}_T = p_T \left\langle \frac{1}{\tau} \right\rangle$ where $\left\langle \frac{1}{z} \right\rangle = \int \frac{dz}{z^2} D_{p/c}(z) z^{4+1/2} \left(\frac{1}{z} \right) / \int \frac{dz}{z^2} D_{p/c}(z) z^{4+1/2}$ and

The power law and power index are preserved under p=zc fragmentation

Showering and the power index

As a result of parton showering involving virtuality degradation, the leading hadron momentum *p* and the showering parton momentum **c** may not be linearly related and can expect that

$$\mathbf{p} = \mathbf{z} \, \mathbf{c}^{1-\mu}$$

where parameter μ describes details of virtuality degradation. As a consequence, the power index can be changed under parton showering.

After the fragmentation and showering of the parton c to hadron p, the hard-scattering cross section for the scattering in terms of hadron momentum p_T becomes



Here *a* is a constant relating the scales of virtuality and transverse momentum. Therefore under the fragmentation $c \rightarrow p$, the hard scattering cross section for $AB \rightarrow pX$ becomes:

$$\frac{d^{3}\sigma(AB \rightarrow pX)}{dyd\mathbf{p}_{T}} \propto \frac{\alpha_{S}^{2}(\mathbf{\bar{c}_{T}})(1 - x_{a0}(\mathbf{\bar{c}_{T}}))^{g_{a}}(1 - x_{b0}(\mathbf{\bar{c}_{T}}))^{g_{a}}}{p_{T}^{n'}}$$
$$n' = \left(\frac{n - 2\mu}{1 - \mu}\right) \quad \text{with} \quad n = 4 + 1/2$$

After all this one gets power-law behavior but not Tsallis formula:

$$\frac{d^{3}\sigma(AB \to pX)}{dyd\mathbf{p}_{T}} \propto \frac{\alpha_{s}^{2}(\overline{\mathbf{c}}_{T})(1 - x_{a0}(\overline{\mathbf{c}}_{T}))^{g_{a}}(1 - x_{b0}(\overline{\mathbf{c}}_{T}))^{g_{a}}}{p_{T}^{n'}} \qquad (*)$$

Low p_T is not correct.

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Low p_T is not correct. The proposed possible remedy is

to replace the usual parameter p_0 (~1 ÷ 2 GeV) dividing phase space into part governed by "soft physics " ($p_T < p_0$) from that governed by "hard physics" ($p_T \ge p_0$) by regularizing denominator in (*),

for example by using:

$$\frac{d^{3}\sigma(AB \rightarrow pX)}{dyd\mathbf{p}_{T}} \propto \frac{\alpha_{S}^{2}(\mathbf{\bar{c}_{T}})(1 - x_{a0}(\mathbf{\bar{c}_{T}}))^{g_{a}}(1 - x_{b0}(\mathbf{\bar{c}_{T}}))^{g_{a}}}{\left(1 + \frac{m_{T}}{m_{T0}}\right)^{n}} \qquad (\textcircled{\oplus})$$
$$m_{T} = \sqrt{m^{2} + p_{T}^{2}}$$

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$$m_{T} = \sqrt{m^{2} + p_{T}^{2}}$$

In addition to the replacement

$$(1/p_T)^n \rightarrow 1/(1 + p_T/p_0)^n$$

in actual calculations we also regularize coupling constant for small values of p_T (following method proposed in hadron spectroscopic studies by C. Y. Wong, E. S. Swanson, and T. Barnes, Phy. Rev. C, 65, 014903 (2001)).

$$\alpha_{s}(\mathbf{p}_{T}) = \frac{12\pi}{27\ln(\mathbf{C} + \mathbf{p}_{T}^{2}/\Lambda_{QCD}^{2})},$$

$$\Lambda_{QCD} = 0.25 \text{ GeV} \Rightarrow \alpha_{s} \left(M_{Z}^{2}\right) = 0.1184;$$

$$C = 10 \Rightarrow \alpha_{s} \left(Q \propto \Lambda_{QCD}\right) \approx 0.6 \text{ in hadron spectroscopy studies}$$

Experiments measure the differential yield in nonsingle-diffractive events, which in our case is

$$\frac{d^{3}N(AB \rightarrow pX)}{dyd\mathbf{p}_{T}} = \sqrt{1 - \frac{m^{2}}{m_{T}^{2}\cosh^{2}y}} A \frac{\alpha_{s}^{2}(\mathbf{\bar{c}_{T}})(1 - x_{a0}(\mathbf{\bar{c}_{T}}))^{g_{a}}(1 - x_{b0}(\mathbf{\bar{c}_{T}}))^{g_{a}}}{\left(1 + \frac{m_{T}}{m_{T0}}\right)^{n}} = \sqrt{1 - \frac{m^{2}}{m_{T}^{2}\cosh^{2}y}} A \alpha_{s}^{2}(\mathbf{\bar{c}_{T}})(1 - x_{a0}(\mathbf{\bar{c}_{T}}))^{g_{a}}(1 - x_{b0}(\mathbf{\bar{c}_{T}}))^{g_{a}} \cdot \frac{1 - (1 - q)\frac{m_{T}}{m_{T}}}{1 - q}}{\left(1 - (1 - q)\frac{m_{T}}{T}\right)^{1 - q}}$$
where $q = 1 + \frac{1}{n}$ and $T = \frac{m_{T0}}{q - 1}$

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Analysis of hadron p_T distributions

Two ways to regulate the cross sections at low p_T were used :

I. Linear
$$m_T: p_T \to (m_{T0} + m_T)$$

 $E_p \frac{d\sigma(AB \to pX)}{d^3 p} = \frac{A\alpha_s^2(\bar{c}_T) (1 - 2x_c(\bar{c}_T))^g (1 - 2x_c(\bar{c}_T))^g}{(m_{T0} + m_T)^n}.$ (24)
II. Quadratic $m_T: p_T^2 \to (m_{T0}^2 + m_T^2)$
 $E_p \frac{d\sigma(AB \to pX)}{d^3 p} = \frac{A\alpha_s^2(\bar{c}_T) (1 - 2x_c(\bar{c}_T))^g (1 - 2x_c(\bar{c}_T))^g}{(m_{T0}^2 + m_T^2)^{n/2}}.$ (25)

where
$$\overline{c}_T = p_T \left\langle \frac{1}{z} \right\rangle; \quad \left\langle \frac{1}{z} \right\rangle = \int \frac{dz}{z^2} D_{p/c}(z) z^{4+1/2} \left(\frac{1}{z} \right) / \int \frac{dz}{z^2} D_{p/c}(z) z^{4+1/2} = 2.33$$

We search for *n* that fits the hadron p_T spectra.



Comparison of the experimental data for hadron production in pp collisions at the LHC with the relativistic hard scattering model results (solid and dashed curves) (a) using Eq. (25), with a quadratic m_T dependence of the regulating function, and (b) using Eq. (24), with a linear m_T dependence of the regulating function. In both cases regularized coupling constant α_S was used.

	Line	ar m_T	Quadratic m_T^2		
	Eq. (24)		Eq. (25)		
	$\sqrt{s}=7\text{TeV}$	$\sqrt{s}=0.9$ TeV	$\sqrt{s}=7$ TeV	$\sqrt{s}=0.9$ TeV	
n	5.69	5.86	5.45	5.49	
$m_{T0} (\text{GeV})$	0.804	0.634	1.09	0.837	



(*) For pp collisions at the LHC the power index extracted from hadron spectra has the value of $n\sim6$ and is slightly greater than the power indices of $n\sim4-5$ extracted from jet transverse differential cross sections.

(*) Fragmentation and showering processes increase therefore slightly the value of the power index n of the transverse spectra.

Relevance to Tsallis fits to p_T spectra

- The successes of the Tsallis fits to LHC high p_T spectra arises from the power law of α_s^2/p_T^4 for the production of jets, leading to n = 4.5.
- Showering and hadronization changes n from 4.5 to ~6.
- The additional $\alpha_s^2(\mathbf{p}_T)$ and $(1-2\mathbf{x}_c)^{2\mathbf{g}}$ bring in additional \mathbf{p}_T dependencies that lead to an increase of \mathbf{n} from ~ 6 to ~ 7 in the Tsallis distribution for hadron production..

Conclusions

- A simple Tsallis formula can describe data with a power index of n ~ 6.6 - 7.6
- A power law with a power index of n ~ 4 5 can describe the p_T spectra of jets.
- A regularized power law with a power index of $n \sim 5.5 6$ can describe (together with regularized coupling constant) the p_T spectra of hadrons for all p_T .
- The power index *n* appears to become larger as a jet evolves into hadrons.

