Dissipative corrections to anisotropic flow

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The General Idea

- Matter produced in heavy-ion collisions follows ideal fluid dynamics:
  - continuous medium
- Detectors observe:
  - particles
- How does the transition proceed?
  - here: **Sudden Freeze-out Approximation**
- Remark: (initial) fluctuations are ignored

Cooper-Frye Formula

- Cooper-Frye Formula

$$E_{\vec{p}} \frac{d^3 N}{d^3 \vec{p}} = \frac{g}{(2\pi)^3} \int_{\Sigma} f\left(\frac{p \cdot u(x)}{T}\right) p^{\mu} d^3 \sigma_{\mu}(x)$$

- f ist the single particle function
  - ideal fluid: the equilibrium thermal distribution f_0

$$f_0\left(\frac{p \cdot u(x)}{T}\right) \propto \exp\left(-\frac{p \cdot u(x)}{T}\right)$$

Decoupling From An Ideal Fluid

- Compute the Cooper-Frye integral via method of steepest descent
  - Borghini, Ollitrault in 2005
- "Slow particles" ($\frac{p_t}{m} < u_{max},\ p_\mu \propto u_\mu$)
- "Fast particles" ($\frac{p_t}{m} > u_{max}$)
- Stronger constraints on p_t are needed
  - to ensure the validity of the approximation

From An Ideal Fluid To A Dissipative Fluid

- $u(x)$ is changed
- f_0 in the Cooper-Frye formula receives correction terms
  - first-order corrections δf_1: shear and bulk viscousity
  - second-order correction δf_2: ...

- $f_0 \rightarrow f_0 + \delta f_1^{shear} + \delta f_1^{bulk}$
  - $\delta f_1^{shear} = C_{shear}(\frac{p \cdot u(x)}{T}) \pi^{\mu\nu}(x) p_\mu p_\nu f_0(\frac{p \cdot u(x)}{T})$
  - $\delta f_1^{bulk} = C_{bulk}(p \cdot u(x), p^2) \Pi(x) f_0(\frac{p \cdot u(x)}{T})$

- C_{shear} and C_{bulk}: not really known

Decoupling From A Dissipative Fluid

Results for slow particles

- The shear tensor must fullfil: $\pi^{\mu\nu}(x)u_\mu = 0$
  - $\delta f_1^{shear} = 0$ for slow particles because $p_\mu \propto u_\mu$
- The factor C_{bulk} is a function of $p^2 = m^2$
  - contribution of δf_1^{bulk} is identical for particles with the same u_μ
- Conclusion: same as in the ideal case (Borghini, Ollitrault in 2005):

- $E_{\vec{p}} \frac{d^3N}{d^3\vec{p}} = c(m) f(\frac{p_t}{m}, y, \phi)$
- mass ordering of $v_n(p_t, y)$

- **BUT:** few slow particles detected at LHC (until now? ...)

Decoupling From A Dissipative Fluid

Results for fast particles

- Assumption: velocity profile at freeze-out is approximately radial
  - u^{φ} and derivatives vanish
- C_{shear} is given by $\frac{\alpha_{shear}}{2sT^3}$ (Grad prescription)
  - $\delta f_1^{shear} = \frac{\alpha_{shear}}{2T} \frac{\eta}{s} \frac{[p_t - m_t v_{max}]^2}{T^2} f_0(\frac{p \cdot u(x)}{T}) \nabla^{\langle r} u^{r\rangle}$
- δf_1^{bulk} will be neglected
- The particle spectrum:

$$E_{\vec{p}} \frac{d^3 N}{dy d^2 \vec{p}_t} \propto \exp(\frac{p_t u_{max} - m_t \sqrt{1 + u_{max}^2}}{T}) \\ [1 + \frac{\alpha_{shear}}{2T} \frac{\eta}{s} \frac{[p_t - m_t v_{max}]^2}{T^2} \nabla^{\langle r} u^{r\rangle}]$$

Flow Coefficients For Fast Particles

- $u_{max} \;\rightarrow\; \bar{u}_{max}[1+\sum_{n\geq 1} 2V_n \cos(n\varphi)]$

- $\mathbf{v_2(p_t) = V_2[A-B]}$
- $\mathbf{v_3(p_t) = V_3[A-B]} + \mathscr{O}(\mathbf{V_1 V_2})$
- $\mathbf{v_4(p_t) = V_4[A-B] + V_2^2[\frac{A^2}{2} - AB]}$
- $\mathbf{v_5(p_t) = V_2 V_3[A^2 - AB]} + \mathscr{O}(\mathbf{V_5})$

- $A = \frac{\bar{u}_{max}}{T}[p_t - m_t \bar{v}_{max}]$ and $B = \frac{m_t \bar{v}_{max}}{p_t - m_t \bar{v}_{max}} \frac{2}{1+\bar{u}_{max}^2} h(\frac{p_t - m_t \bar{v}_{max}}{T})$
- $h(\xi) = \frac{g(\xi)}{1+g(\xi)}$ and $g(\xi) = \alpha_{shear} \frac{\eta}{s} \frac{\nabla^{\langle r} u^{r\rangle}}{2T} \xi^2$

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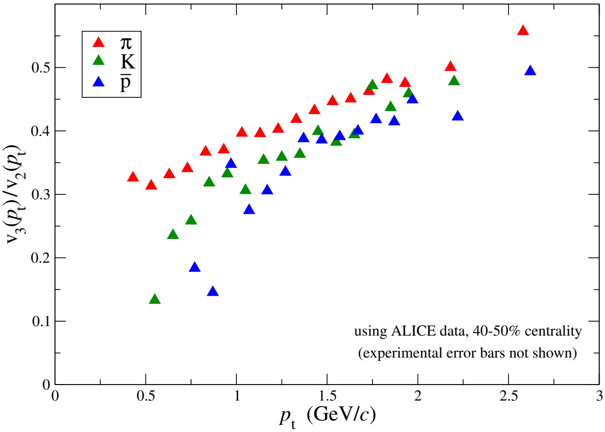

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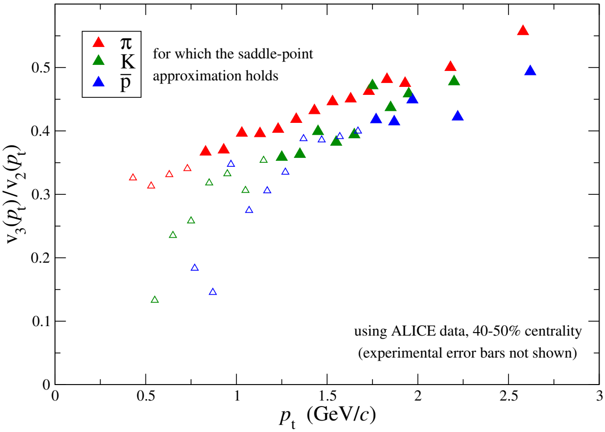

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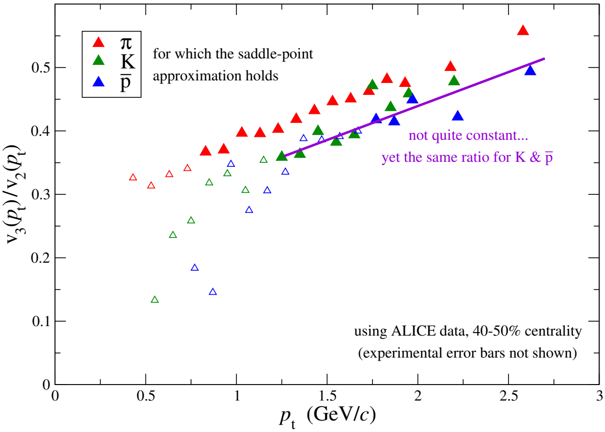

Flow Coefficients For Fast Particles

- For $V_4 \ll V_2$
  - ideal case: $v_4(p_t) = \frac{v_2^2(p_t)}{2}$ (Borghini, Ollitrault in 2005)
  - viscous case: $v_4(p_t) < \frac{v_2^2(p_t)}{2}$
- For $V_5 \ll V_2, V_3$
  - ideal case: $v_5(p_t) = v_2(p_t)v_3(p_t)$ (Teaney, Yan in 2012)
  - viscous case: $v_5(p_t) > v_2(p_t)v_3(p_t)$
    - Better: $\frac{v_5 - v_2 v_3}{v_3} \propto$ viscous term in v_2
- Viscous results are seen in real hydrodynamical simulations

Summary

- For slow particles viscous corrections do not change anything
- Flow coefficients for fast particles gain a viscous term
- Known relations between flow coefficients are modified
  - possibility to access the shape of the viscous correction?

- Outlook
  - other relations (multi-particle correlations, ...)**?**
  - other transport coefficients
  - investigate other freeze-out ansätze (non-polynomial)
  - freeze-out at different temperatures $\rightarrow \eta(T)$**?**

Lang, Borghini, arXiv:1311:????