Locally anisotropic momentum distributions of hadrons at freeze-out in relativistic heavy-ion collisions

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Outline

- Thermal analysis of hadronic yields has become a popular tool for the interpretation of the data collected at the AGS, SPS, and RHIC energies. The successful results of such analyses are understood as the evidence for high-level thermalization of the produced systems (Becattini, Braun-Munzinger, Broniowski, Cleymans, Gaździcki, Gorenstein, Stachel, Rafelski, Redlich, ...)
- In this talk I will argue that the agreement of the thermal models with the data does not necessarily imply the fact that the system is locally equilibrated. I will take into account possible differences in the local distribution of *transverse* and *longitudinal* momenta (using Romatschke-Strickland form).
- I will show that anisotropic distributions lead to exactly the same ratios of hadronic abundances as the equilibrium distributions. Moreover, to a very good approximation the transverse-momentum spectra of hadrons are the same for isotropic and anisotropic systems, provided the size of the system at freeze-out is appropriately adjusted.
- I will further show that the inclusion of anisotropy may be used to improve the agreement between the model and experimental HBT results.

Cooper-Frye formalism
Equilibrium

- Cooper-Frye formula

$$E_p \frac{dN}{d^3p} = \int d\Sigma_\mu(x) p^\mu f(x,p)$$

equilibrium distribution

$$f_{\rm eq}(x,p) = g\left\{\exp\left[\frac{p \cdot U - \mu}{T}\right] - \epsilon\right\}^{-1}$$

- Standard parametrization of the four-momentum

$$p^\mu = \left(E_p, \vec{p}_\perp, p_\parallel\right) = \left(m_\perp \cosh y, \vec{p}_\perp, m_\perp \sinh y\right)$$

- Boost-invariant form of flow

$$U^\mu = \gamma(1, \vec{v}_\perp, v_\parallel)$$

with the Lorentz γ factor

$$\gamma = (1 - v^2)^{-1/2}$$

Cooper-Frye formalism
Off equilibrium

- Romatschke-Strickland form

$$f_{\mathrm{RSF}}(x,p) = g\left\{\exp\left[\frac{\sqrt{(p\cdot U)^2+\xi(p\cdot V)^2}}{\Lambda}\right]-\epsilon\right\}^{-1}$$

  ξ is the anisotropy parameter and Λ is a typical transverse-momentum scale characterizing the particles in the system

- The four-vector V^μ defines the direction of the beam

$$V^\mu = \gamma_z(v_z,0,0,1), \quad \gamma_z = (1-v_z^2)^{-1/2}$$

- Orthogonality relations

$$U^2 = 1, \quad V^2 = -1, \quad U\cdot V = 0$$

- Local rest frame

$$f_{\mathrm{RSF}}(x,p) = g\left\{\exp\left[\frac{\sqrt{m^2+p_\perp^2+(1+\xi)p_\parallel^2}}{\Lambda}\right]-\epsilon\right\}^{-1}$$

Ratios of hadronic yields

- Hadronic yields

$$N_i = \int d\Sigma_\mu(x) \int \frac{d^3p}{E_p} p^\mu f_i(x,p)$$

We assume that freeze-out is defined by the fixed value of the hard scale Λ, but the anisotropy parameter ξ may depend on the space-time position x

$$\int \frac{d^3p}{E_p} p^\mu f^i_{\rm RSF}(p \cdot U, p \cdot V) = n_i\left(\Lambda, \xi(x)\right) U^\mu$$

- The calculation of the density $n_i\left(\Lambda, \xi(x)\right)$ may be done in the LRF

$$n_i\left(\Lambda, \xi(x)\right) = \frac{n_{i,\rm eq}\left(\Lambda\right)}{\sqrt{1+\xi(x)}}$$

- $$N_i = n_{i,\rm eq}\left(\Lambda\right) \int \frac{d\Sigma_\mu(x) U^\mu(x)}{\sqrt{1+\xi(x)}} \quad \rightarrow \quad \frac{N_i}{N_j} = \frac{n_{i,\rm eq}\left(\Lambda\right)}{n_{j,\rm eq}\left(\Lambda\right)}$$

Transverse-momentum spectra

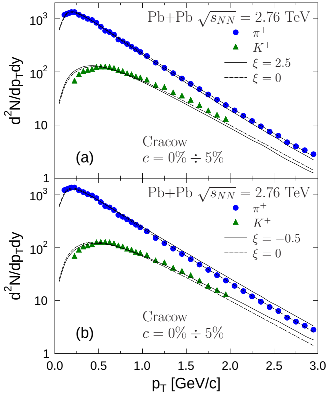
Cracow model (Broniowski & Florkowski)

- Cracow model: The freeze-out hypersurface is defined by the condition
  $(t^2 - x^2 - y^2 - z^2)^{1/2} = \tau_{3f}$
  the invariant time τ_{3f} and the transverse size of the system $r_{\rm max}$
  are the two geometric parameters of the model
  the flow of matter at freeze-out has the Hubble form $U^\mu = x^\mu/\tau_{3f}$
- Before we analyzed Pb+Pb collisions at the beam energy $\sqrt{s_{\rm NN}} = 2.76$ TeV for three centrality classes. The freeze-out temperature was always set equal to $T_0 = 165.6$ MeV. The fitted geometric parameters are:
  $\tau^0_{3f} = 9.0$ fm and $r^0_{\rm max} = 11.4$ fm for $c = 0\%–5\%$,
  $\tau^0_{3f} = 7.4$ fm and $r^0_{\rm max} = 9.60$ fm for $c = 10\%–20\%$,
  $\tau^0_{3f} = 5.9$ fm and $r^0_{\rm max} = 7.25$ fm for $c = 30\%–40\%$.
- Change to anisotropic description:

$$\begin{aligned} \Lambda &= T_0 \\ \tau_{3f} &= \tau^0_{3f}(1+\xi)^{1/6}, \\ r_{\rm max} &= r^0_{\rm max}(1+\xi)^{1/6}. \end{aligned}$$

Transverse-momentum spectra

Cracow model (Broniowski & Florkowski)



Transverse-momentum spectra of positive pions and kaons measured by ALICE in Pb+Pb collisions at the beam energy $\sqrt{s_{\rm NN}} = 2.76$ TeV compared to the Cracow model results with $\xi = 2.5$ (a) and $\xi = -0.5$ (b). The experimental and model results are shown for the centrality class $c = 0\% - 5\%$.

Transverse-momentum spectra

Modified blast-wave model (Broniowski & Florkowski & Kisiel)

Modified blast-wave model:

The shape of the freeze-out curve in the Minkowski space is controlled by the parameter A, whereas the magnitude of the flow is controlled by the parameter v_T
In the equilibrium version we used four parameters: A, T_0, $\tau_{\rm 2f}^0 = r_{\rm max}^0$, and v_T.

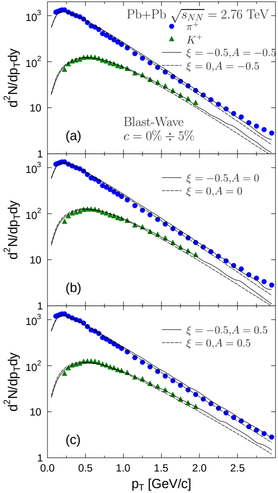
$$\begin{aligned}\Lambda &= T_0\\ \tau_{\rm 2f} &= \tau_{\rm 2f}^0 (1+\xi)^{1/6},\\ r_{\rm max} &= r_{\rm max}^0 (1+\xi)^{1/6}.\end{aligned}$$

Transverse-momentum spectra

Modified blast-wave model (Broniowski & Florkowski & Kisiel)

A	c [%]	$r_{\rm max}$ [fm]	v_T
0.5	0–5	9.9	0.375
0.5	10–20	8.3	0.375
0.5	30–40	6.5	0.425
0.0	0–5	9.9	0.45
0.0	10–20	8.2	0.43
0.0	30–40	6.2	0.46
-0.5	0–5	10.4	0.46
-0.5	10–20	8.9	0.475
-0.5	30–40	6.9	0.58

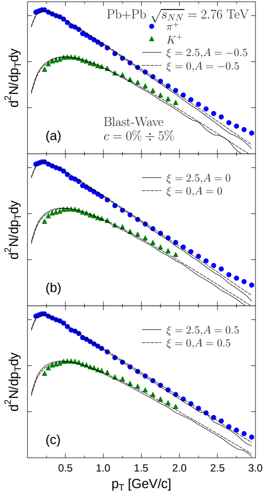
Transverse-momentum spectra
Modified blast-wave model (Broniowski & Florkowski & Kisiel)



The transverse-momentum spectra obtained in the blast-wave model for three different values of the parameter *A*, $\xi = -0.5$ (solid lines) and $\xi = 0$ (dashed lines).

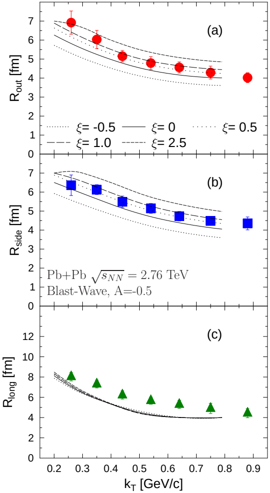
Transverse-momentum spectra

Modified blast-wave model (Broniowski & Florkowski & Kisiel)



The transverse-momentum spectra obtained in the blast-wave model for three different values of the parameter A, $\xi = 2.5$ (solid lines) and $\xi = 0$ (dashed lines).

HBT radii
HBT



The pion HBT radii $R_{\rm out}$ (a), $R_{\rm side}$ (b), and $R_{\rm long}$ obtained in the blast-wave model with the freeze-out slope parameter $A = -0.5$ for five different values of the local momentum anisotropy ξ. The model results are compared to the ALICE data.

Conclusions

- We have considered locally anisotropic momentum distributions of hadrons at freeze-out. We have taken into account the local anisotropy between the longitudinal and transverse momenta.
- Our results show that physical observables, such as the ratios of hadron abundances or the hadronic transverse-momentum spectra, are in practice indistinguishable from those obtained in an analogous equilibrium calculations — the effect of the momentum anisotropy may be compensated by an appropriate change of the geometric models of the system.
- We have also demonstrated that this freedom of the parameters may be used to improve the agreement of the model calculations with the measured HBT radii.
- Our results indicate insensitivity of the thermal approach against specific variations of the model assumptions and show that it may be quite successful even in the situations where matter is out of equilibrium.