Density correlations in the Glauber model

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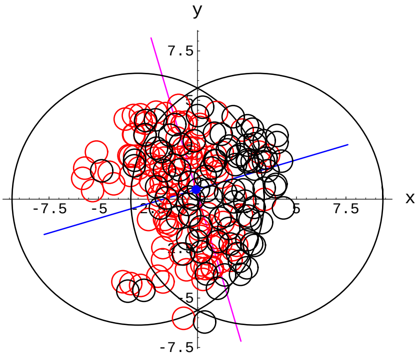
[based on research with J.-Y. Ollitrault and J.-P. Blaizot]

Fluctuations in the initial state



Density-density correlator:

$$S(x,y) = \langle \rho(x)\rho(y) \rangle - \langle \rho(x) \rangle \langle \rho(y) \rangle$$

$< . >$ - e-by-e average

Goal: understand $S(x,y)$ microscopically

[different approach from Coleman-Smith, Petersen, Wolpert 2012 or Floerchinger-Wiedemann 2013]

Density-density correlator

$S(x, y)$ carries information on e-by-e fluctuations of observables (2-body effects): $\mathrm{var}(\mathcal{O}) = \int dx dy \mathcal{O}(x) S(x, y) \mathcal{O}(y)$

Embodies short-range correlations

- autocorrelations
- NN repulsion in colliding nuclei
- correlation formed in the production mechanism

and long-range correlations

- conservation laws
- constraints (e.g., choice of centrality class)
- technical issues (recentering)

(for simplicity all for head-on collisions, $b = 0$)

Glauber sources

In each collision n (point-like) sources (wounded nucleon, binary collisions) are created in the transverse plane with a distribution $f_n(x_1, x_2, \ldots, x_n)$. Marginal distributions are

$$f_n^{(2)}(x_1,x_2) = \int dx_3 ... dx_n f_n(x_1,...,x_n), \quad f_n^{(1)}(x_1) = \int dx_2 f_n^{(2)}(x_1,x_2)$$

The density is $\rho(x) = \sum_{i=1}^{n} \delta(x - x_i)$. Then

$$\langle \rho(x) \rangle = \langle \int dx_1 ... dx_n f_n(x_1,...,x_n) \sum_i \delta(x - x_i) \rangle = \langle n f_n^{(1)}(x) \rangle$$

$$S(x,y) = \langle n f_n^{(1)}(x) \rangle \delta(x-y) + \langle n(n-1) f_n^{(2)}(x,y) \rangle - \langle n f_n^{(1)}(x) \rangle \langle n f_n^{(1)}(y) \rangle$$

Introducing the *pair distribution function*

$$g(x,y) = \frac{\langle n(n-1) f_n^{(2)}(x,y) \rangle}{\langle \rho(x) \rangle \langle \rho(y) \rangle}$$

we may write

$$S(x,y) = \langle \rho(x) \rangle \delta(x-y) + \langle \rho(x) \rangle \langle \rho(y) \rangle [g(x,y) - 1]$$

Some properties

Sum rules:

$$\int dx \rho(x) = n, \quad \int dx dy S(x,y) = \mathrm{var}(n)$$

(sensitivity to constraining n)

The pair distribution function is normalized as

$$\int dx dy \langle \rho(x) \rangle \langle \rho(y) \rangle g(x,y) = \langle n(n-1) \rangle = \mathrm{var}(n) + \langle n \rangle (\langle n \rangle - 1).$$

(increases with the $\mathrm{var}(n)$ as expected)
No correlations, fixed n:

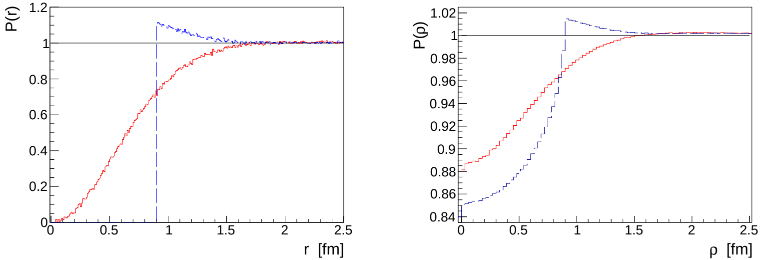
$$g(x,y) = 1 - \frac{1}{n}$$

GLISSANDO 2 is out!

All simulations are carried out with

GLISSANDO 2: GLauber Initial-State Simulation AND mOre..., ver. 2
Maciej Rybczynski, Grzegorz Stefanek, Wojciech Broniowski, Piotr Bozek
e-Print: arXiv:1310.5475

NN repulsion



Left: The pair distribution function in the relative distance for the Pb nucleus for the hard-sphere expulsion (dashed line) and Gaussian correlation (solid line)

Right: projection on the transverse plane → "geometric quenching"

These correlations sneak into the fireball! (∼10% effect)

Analytic model for p+Pb

The probability that p incident at an impact parameter b interacts with N participants in a given configuration and does not interact with the remaining $A-N$ nucleons

$$f(s_1,\dots s_A;b) = c_1\theta(b-s_1)\dots\theta(b-s_N)(1-\theta(b-s_{N+1}))\dots(1-\theta(b-s_A))T(s_1,\dots,s_A)$$

c_1 - normalization, T - thickness function, $\theta(u)$ - *wounding profile*
($\int du\theta(u) = \sigma_{\mathrm{inel}}^{NN} \equiv \sigma_w$)
Since d is small, we can include the 2-body correlations perturbatively

$$T(s_1,\dots,s_A) \simeq c_2 T_0(s_1)\dots T_0(s_A)\prod_{\substack{i,j=1\\ i\neq j}}^{A}(1-d(s_i,s_j)) \simeq c_2 T_0(s_1)\dots T_0(s_A)\sum_{i\neq j}(1-d(s_i,s_j))$$

Then

$$f^{(2)}(s_1,s_2;b) = \frac{\theta(b-s_1)\theta(b-s_2)T_0(s_1)T_0(s_2)(1-d(s_1,s_2))}{\int ds_1' ds_2' \theta(b-s_1')\theta(b-s_2')T_0(s_1')T_0(s_2')(1-d(s_1',s_2'))}$$

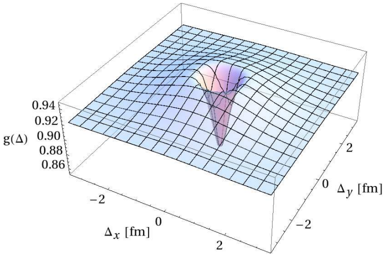
$$f^{(1)}(s_1;b) = \int ds_2 f^{(2)}(s_1,s_2;b)$$

Results of the analytic model for p+Pb

For simplicity, we take $b = 0$ and use the Gaussian parameterizations

$$\theta(u) = A\exp\left(-\frac{u^2}{2\sigma_w^2}\right), \quad A = 0.92, \quad \sigma_w = 1.08 \text{ fm} \quad \text{(LHC)}$$

$$d(s_1, s_2) = B\exp\left(-\frac{(s_1 - s_2)^2}{2\sigma_d^2}\right), \quad B = 0.11, \quad \sigma_d = 0.56 \text{ fm}$$



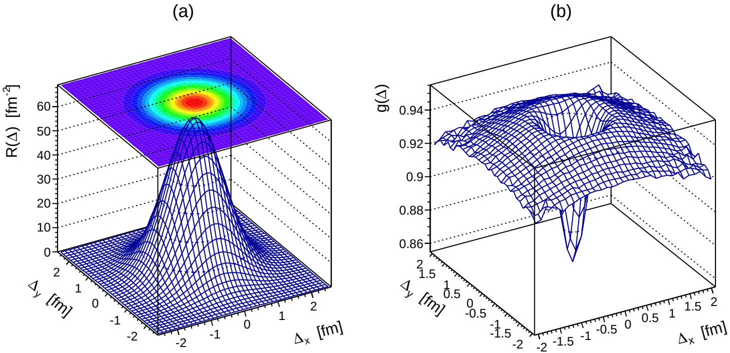
("Aetna plot")

Half-integrated distributions
(for visualization in the relative coordinate)

$$R(\Delta) = \int dr\, \langle \rho(r + \Delta/2) \rangle \langle \rho(r - \Delta/2) \rangle$$

$$g(\Delta) = \frac{1}{R(\Delta)} \int dr\, \langle n(n-1) f_n^{(2)}(r + \Delta/2, r - \Delta/2) \rangle$$

GLISSANDO for p+Pb



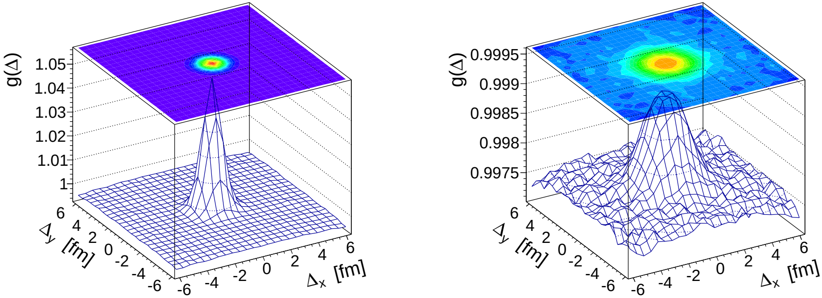
(a) $R(\Delta)$ (b) and $g(\Delta)$ for p+Pb at $b = 0$ and $N = 15$.

(full agreement with the analytic model)

GLISSANDO for Pb+Pb

(for the moment no NN repulsion in the nuclear distributions)

Why do we see the peaks?



Left: $g(\Delta)$ for the Pb+Pb collisions at the impact parameter $b = 0$ with $\sigma_w = 20$ mb and $N_w = 371$

Right: Same for with $\sigma_w = 68$ mb and $N_w = 410$

GLISSANDO for Pb+Pb

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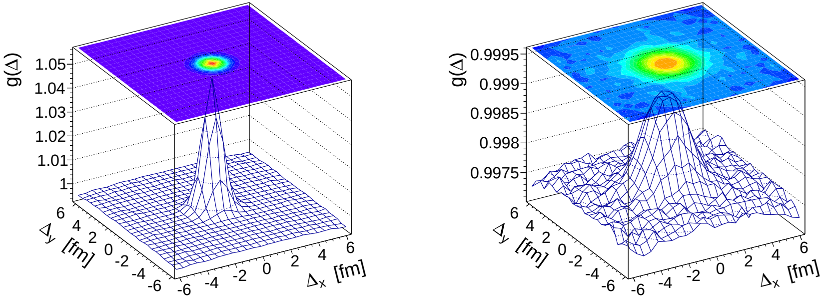

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Peaks from the twin-production!

Twin production

Wounded nucleons are created in partnership: one nucleon from nucleus A and one from nucleus B. The range of the correlation is $\sim \sqrt{\sigma_w/\pi}$. For very small σ_w they come in isolated pairs, hence

$$f_n^{(2)}(x_i, x_j) = \delta(x_i - x_j) f_n^{(1)}(x_i), \quad i \in A, j \in B \quad (\sigma_w \to 0)$$

There are $N_w/2$ such tightly correlated pairs, while the remaining pairs are uncorrelated: $f_n^{(2)}(x_i, x_j) = f_n^{(1)}(x_i) f_n^{(1)}(x_j)$, $i, j \in$ different nuclei. This leads to

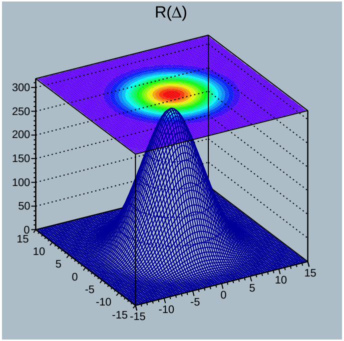
$$S(x,y) = 2\langle n \rangle \left(f^{(1)}(x) \rangle \delta(x-y) - f^{(1)}(x) f^{(1)}(y) \right) + \mathrm{var}(n) f^{(1)}(x) f^{(1)}(y).$$
$$= 4 \left[\langle n_{\mathrm{pairs}} \rangle f^{(1)}(x) \rangle \delta(x-y) + \langle n_{\mathrm{pair}}(n_{\mathrm{pair}} - 1) \rangle f^{(1)}(x) f^{(1)}(y) \right]$$

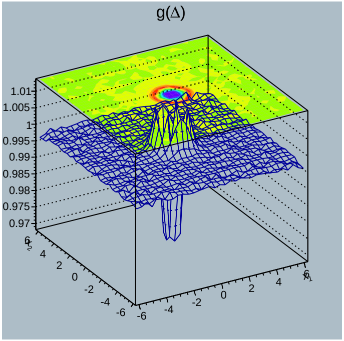
(pairs are the basic objects)

In the limit $\sigma_w \to \infty$ all nucleons are wounded, hence there is no correlation!

For intermediate values of σ_w clusters form and things are complicated

Same with NN repulsion present





(two effects: positive peak from twin production and negative peak from NN repulsion in projectiles)

Observables

Radius^n:

$$\langle r^n\rangle_{\rm incl} = \langle \sum_j r_j^n\rangle = \int dx\, r^n \langle\rho(x)\rangle, \quad {\rm var}(r^n)_{\rm incl} = \int dx\, dy\, r^n r'^n S(x,y)$$

r and r' – transverse radii corresponding x and y

Eccentricities:

$$\langle \epsilon_n e^{i\Psi_n}\rangle_{\rm incl} = \frac{\int d^2x r^n e^{in\phi}\rho(x)}{\langle r^n\rangle_{\rm incl}}, \quad {\rm var}(|\epsilon_n|)_{\rm incl} = \frac{\int dx\, dy\, r^n e^{in\phi} r'^n e^{-in\phi'} S(x,y)}{\langle r^n\rangle^2_{\rm incl}}$$

Ψ_n – event-plane angles, ϕ and ϕ' – azimuths corresponding to x and y

No correlations, $b=0$:

$$\begin{aligned}
\langle r^n\rangle_{\rm incl}^{\rm no\ corr.} &= \langle n\rangle\langle r^n\rangle \\
{\rm var}(r^n)_{\rm incl}^{\rm no\ corr.} &= \langle n\rangle\langle r^{2n}\rangle + ({\rm var}(n) - \langle n\rangle)\langle r^n\rangle^2 \\
\langle \epsilon_n e^{i\Psi_n}\rangle_{\rm incl}^{\rm no\ corr.,central} &= 0 \ \ \text{(symmetry)} \\
{\rm var}(|\epsilon_n|)_{\rm incl}^{\rm no\ corr.,central} &= \frac{\langle r^{2n}\rangle}{\langle n\rangle\langle r^n\rangle^2}
\end{aligned}$$

To focus on the effects of correlations in the fluctuation measures, we introduce

$$R(O) = \frac{{\rm var}(O)_{\rm incl}}{{\rm var}(O)_{\rm incl}^{\rm no\ corr}}$$

Observables in p+Pb

Analytic model:
($B = 0.11$ – depth of the "soft-core", $\sigma_d = 0.56$ fm – its width, $\sigma_w = 1.08$ fm)

$$R(r^2) = 1 - 2B\frac{\omega(n)+\langle n\rangle}{\omega(n)+1}\frac{\sigma_d^2\sigma_w^4}{\left(\sigma_d^2+2\sigma_w^2\right)^3} + \mathcal{O}\left(B^2\right) = 1 - 0.69B + \mathcal{O}\left(B^2\right))$$

$$R(\epsilon_2) = 1 - B(\omega(n)+\langle n\rangle)\frac{\sigma_d^2\sigma_w^4}{\left(\sigma_d^2+2\sigma_w^2\right)^3} + \mathcal{O}\left(B^2\right) = 1 - 0.35B + \mathcal{O}(B^2)$$

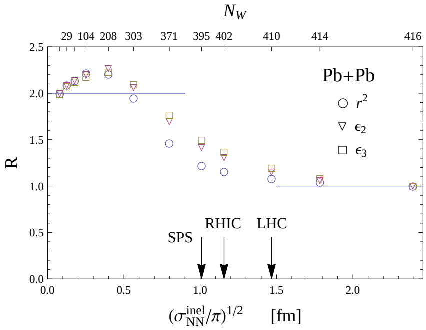
$$R(\epsilon_3) = 1 - 0.09B + \mathcal{O}(B^2)$$

Small but non-negligible effect (a few %)
Maximum when $\sigma_w = \sigma_d$ – matching of the two scales: probing and internal

Observables in Pb+Pb

(R = "correlated/uncorrelated")

GLISSANDO:

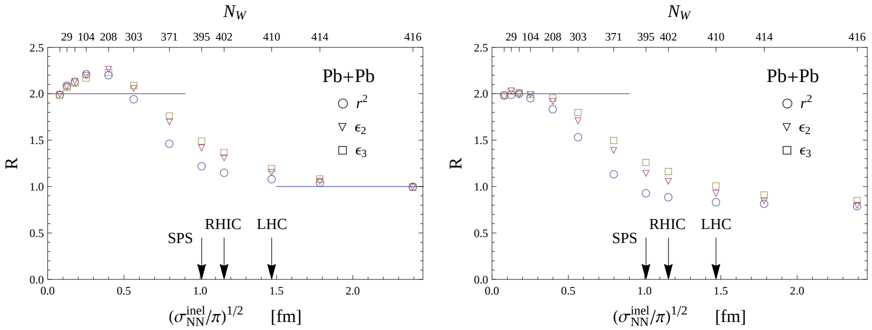


The ratios R at various values of σ_w. The corresponding fixed values of the number of the wounded nucleons is shown on the upper axis

Observables in Pb+Pb

(R = "correlated/uncorrelated")

GLISSANDO:



Left: The ratios R at various values of σ_w. The corresponding fixed values of the number of the wounded nucleons is shown on the upper axis

Right: Same with the NN repulsion included

Superposition model

$$\rho(x) = \sum_{i=1}^{n} w_i \delta(x - x_i) \quad - \text{ varying strength}$$

w_i – weights generated independently according to some suitable distribution. Then $(\omega(w) = \mathrm{var}(w)/\overline{w})$

$$\begin{aligned} \langle \rho(x) \rangle &= \overline{w} \langle n f_n^{(1)}(x) \rangle \\ S(x,y) &= (\omega(w) + \overline{w}) \langle \rho(x) \rangle \delta(x-y) + \langle \rho(x) \rangle \langle \rho(y) \rangle [g(x,y) - 1] \end{aligned}$$

$$g(x,y) = \frac{\overline{w}^2 \langle n(n-1) f_n^{(2)}(x,y) \rangle}{\langle \rho(x) \rangle \langle \rho(y) \rangle}$$

Upon integration over x and y we find the superposition-model formula

$$\mathrm{var}(\rho) = \mathrm{var}(w) \langle n \rangle + \overline{w}^2 \mathrm{var}(n)$$

$\omega(w)/\overline{w}$ enhances the relative contribution of the autocorrelation term

Conclusions

- Microscopic understanding achieved
- Smearing: $\delta(x-y) \to d(x-y)$
- Short-range correlations assume the form $S_{\rm short}(x,y) = A(x)d(x-y)$ (short range dominance)
- Autocorrelations + NN repulsion in projectiles + twin production
- Long range term $\sim f^{(1)}(x)f^{(1)}(y)$ from constraining n (in general, global constraints lead to long-range correlations)
- Correlations affect fluctuation of observables (NN repulsion already studied in [WB & M. Rybczyński, PRC 81 (2010) 064909])
- Fluctuations of eccentricities dominated by $S_{\rm short}(x,y)$