AN APPROXIMATE NNNLO HIGGS CROSS SECTION FROM ANALYTICITY

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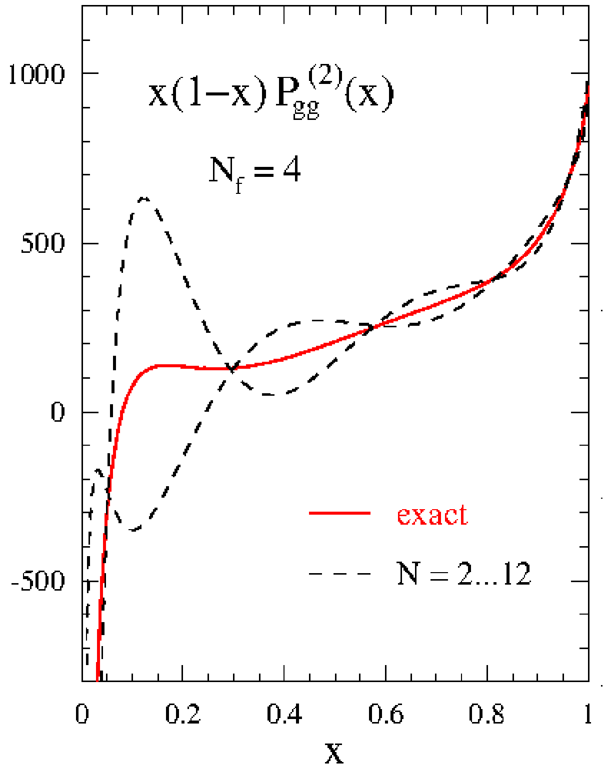
APPROXIMATE PARTONIC CROSS SECTIONS?

- VARIOUS PREVIOUS ATTEMPTS TO ESTIMATE UNKNOWN HIGHER ORDERS FROM PARTIAL INFORMATION
- POSSIBLE INGREDIENTS: LARGE x RESUMMATION, SMALL x RESUMMATION, KNOWN MELLIN MOMENTS

SOME SUCCESSFUL...

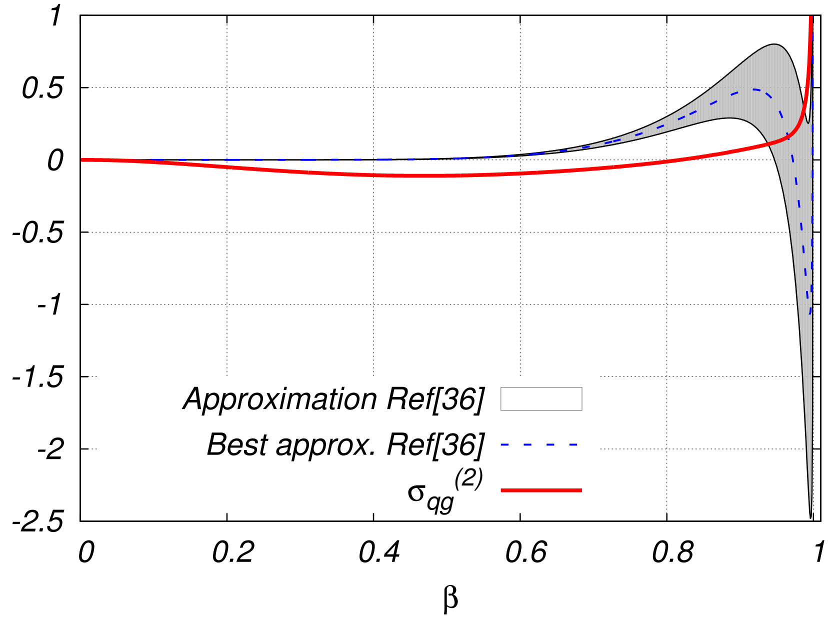
THE NNLO P_{GG} SPLITTING FUNCTION



(Moch, Vermaseren, Vogt 2005)

SOME LESS SO...

THE NNLO QG TOP CROSS SECTION



(Czakon, Mitov, 2013)

- IS IT JUST GUESSWORK?
- CAN ONE CONSTRUCT A CONTROLLED APPROXIMATION?

SUMMARY

- THE BASIC IDEA
  - ANALYTICITY AND SADDLE POINTS
  - APPROXIMATION FROM RESUMMATION
- "SUDAKOV" TERMS
  - THE STRUCTURE OF RESUMMED RESULTS
  - SUBLEADING IMPROVEMENTS AND SMALL N SINGULARITIES
- "BFKL" TERMS
  - SINGLE LOGS AND DOUBLE LOGS
  - MOMENTUM CONSERVATION AND SUBLEADING TERMS
- RESULTS
  - MATCHED PARTONIC CROSS SECTIONS
  - HADRONIC K-FACTORS
  - SCALE DEPENDENCE

APPROXIMATION FROM ANALYTICITY

SOME DEFINITIONS

THE FACTORIZED CROSS SECTION

$$\sigma(\tau, m_H^2) = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathscr{L}_{ij} \left(\frac{\tau}{z}, \mu_{\mathrm{F}}^2\right) \frac{1}{z} \hat{\sigma}_{ij} \left(z, m_H^2, \alpha_s(\mu_{\mathrm{R}}^2), \frac{m_H^2}{\mu_{\mathrm{F}}^2}, \frac{m_H^2}{\mu_{\mathrm{R}}^2}\right) \qquad \tau = \frac{m_H^2}{s}$$

PARTON LUMINOSITIES

$$\mathscr{L}_{ij}(z, \mu^2) = \int_z^1 \frac{dx}{x} f_i\left(\frac{z}{x}, \mu^2\right) f_j(x, \mu^2)$$

COEFFICIENT FUNCTIONS

$$\hat{\sigma}_{ij} \left(z, m_H^2, \alpha_s(\mu_{\mathrm{R}}^2), \frac{m_H^2}{\mu_{\mathrm{F}}^2}, \frac{m_H^2}{\mu_{\mathrm{R}}^2}\right) = z\,\sigma_0 \left(m_H^2, \alpha_s(\mu_{\mathrm{R}}^2)\right) C_{ij} \left(z, \alpha_s(\mu_{\mathrm{R}}^2), \frac{m_H^2}{\mu_{\mathrm{F}}^2}, \frac{m_H^2}{\mu_{\mathrm{R}}^2}\right)$$

$$C_{ij}(z, \alpha_s) = \delta(1-z)\delta_{ig}\delta_{jg} + \alpha_s C_{ij}^{(1)}(z) + \alpha_s^2 C_{ij}^{(2)}(z) + \alpha_s^3 C_{ij}^{(3)}(z) + \mathcal{O}(\alpha_s^4)$$

MELLIN-SPACE FACTORIZATION

$$\sigma(N, m_H^2) = \sigma_0 \left(m_H^2, \alpha_s\right) \mathscr{L}(N) C(N, \alpha_s),$$

$$\sigma(N, m_H^2) \equiv \int_0^1 d\tau\, \tau^{N-2} \sigma(\tau, m_H^2);\ \mathscr{L}(N) \equiv \int_0^1 dz\, z^{N-1} \mathscr{L}(z)\ C(N, \alpha_s) \equiv \int_0^1 dz\, z^{N-1} C(z, \alpha_s)$$

WHAT'S THE PROBLEM

- THE COEFFICIENT FUNCTION $C(z, \alpha_s)$ IS A DISTRIBUTION, WITH SUPPORT IN $0 \leq z \leq 1$ &, AN ORDINARY FUNCTION FOR $z < 1$
- SUDAKOV RESUMMATION DETERMINES THE BEHAVIOUR OF TERMS WHICH AS $z \to 1$ ARE EITHER SINGULAR, OR DISTRIBUTIONS
- BFKL RESUMMATION DETERMINES THE BEHAVIOUR OF TERMS WHICH AS $z \to 0$ HAVE THE STRONGEST SINGULARITY (UP TO POWERS)

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THE SOLUTION

- THE MELLIN-SPACE COEFFICIENT FUNCTION IS AN ANALYTIC (MEROMORPHIC) FUNCTION OF COMPLEX N, WITH A CONVERGENCE ABSCISSA
- SUDAKOV RESUMMATION DETERMINES ITS BEHAVIOUR AT INFINITY, I.E. ALL TERMS WHICH DO NOT VANISH AS $N \to \infty$
- BFKL RESUMMATION DETERMINES ITS RIGHTMOST SINGULARITIES FOR FINITE N

BASIC IDEA

AN ANALYTIC FUNCTION CAN BE RECONSTRUCTED FROM KNOWLEDGE OF ITS SINGULARITIES (Cauchy-Liouville)

A (USEFUL) SIDE REMARK: SADDLE POINT

- THE CROSS SECTION IS DETERMINED FROM THE N-SPACE RESULT BY MELLIN INVERSION:
$$\frac{\sigma(\tau,m_H^2)}{\tau} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, \tau^{-N}\, \sigma(N,m_H^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, e^{E(N,\tau,m_H^2)}$$
$$E(N,\tau,m_H^2) \equiv N \ln\frac{1}{\tau} + \ln\sigma(N,m_H^2)$$

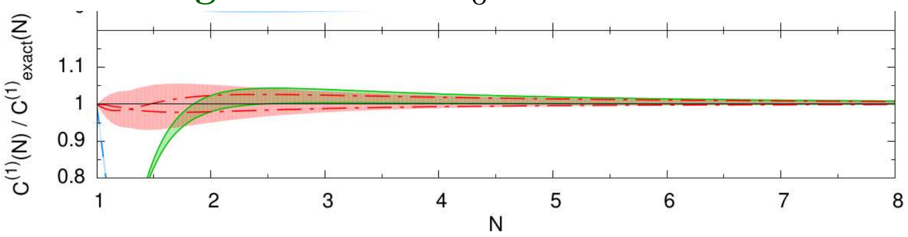
- DOMINATED BY SADDLE POINT $\left.\frac{\partial E(N,\tau,m_H^2)}{\partial N}\right|_{N=N_0} = 0$

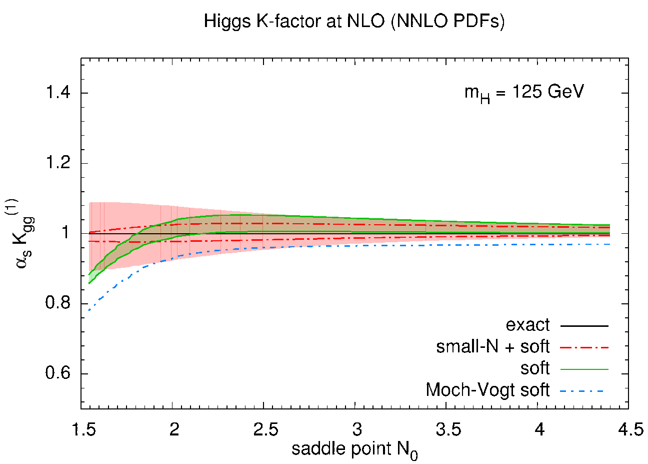
- SADDLE N_0 ON THE REAL AXIS: $\sigma(N)$ MUST BE A DECREASING FUNCTION OF N; AS $\tau \to 0$, N_0 MOVES LEFTWARDS TOWARDS THE RIGHTMOST SING.
- MOST OF THE CONTRIBUTION COMES FROM A SMALL NEIGHBORHOOD OF N_0
- FOR $m_h = 125$ GEV, $N_0 \approx 2$ FOR $s = 7-8$ TEV, $N_0 \approx 1.8$ FOR $s = 14$ TEV

A TEST

SOME RATIO OF COEFFICIENT FUNCTIONS IN MELLIN SPACE
COMPARED TO THE RATIO OF HADRONIC K-FACTORS COMPUTED AS A FUNCTION OF $s(N_0)$

COMPARE $C'(N)/C(N)$ TO $\sigma'(s(N_0))/\sigma(s(N_0))$,
EQUAL IF ONLY N_0 CONTRIBUTES!





THE SINGULARITY STRUCTURE OF (INCLUSIVE) COEFFICIENT FUNCTIONS

- AT LARGE N
  - THE COEFFICIENT FUNCTION BEHAVES AS A POWER OF $\ln N$
    THE ORDER OF THE POWER INCREASES BY TWO UNITS AT EACH SUBSEQUENT PERTURBATIVE ORDER (DOUBLE LOGS)
  - NOTE THAT THERE ALSO IS A CONSTANT, MELLIN OF $\delta(1-z)$.
  - SUBDOMINANT TERMS BEHAVE AS $\frac{1}{N}$ (OR HIGHER INVERSE POWERS OF N), TIMES POWERS OF $\ln N$
- AT SMALL N
  - THE COEFFICIENT FUNCTION HAS ISOLATED POLES ON THE REAL AXIS
    CANNOT HAVE SINGULARITIES IN THE COMPLEX PLANE
    (WOULD BE IN PAIRS BY SCHWARTZ REFLECTION) $\Rightarrow$
    BY SADDLE POINT, WOULD LEAD TO OSCILLATING XSECT IN THE $s \to \infty$ LIMIT
  - RIGHTMOST (LEADING) POLE IS AT $N = 1$, IN GLUON CHANNELS
  - ORDER OF LEADING POLE INCREASES BY ONE UNIT AT EACH SUBSEQUENT PERTURBATIVE ORDER (SIMPLE LOGS)
    (RECALL $\frac{1}{N^k}$ IS THE MELLIN OF $\ln^{k-1} x$)

THE LARGE N TERMS

THE STRUCTURE OF RESUMMED RESULTS

THE RESUMMED COEFFICIENT FUNCTION

$$C_{\rm res}(N,\alpha_s) = g_0(\alpha_s)\exp\left[\frac{1}{\alpha_s}g_1(\alpha_s\ln N) + g_2(\alpha_s\ln N) + \alpha_s g_3(\alpha_s\ln N) + \dots\right];$$

$$g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3);\quad g_1(\lambda) = \sum_{k=2}^{\infty} g_{1,k}\lambda^k,\ g_i(\lambda) = \sum_{k=1}^{\infty} g_{i,k}\lambda^k \text{ FOR } i\geq 2$$

LOG APPROX.	XSECT ACCURACY	EXP. ACCURACY: $\alpha_s^n L^k$	g_0 ACCURACY: α_s^i
LL	$k=2n$	$k=n+1$	0
NLL	$2n-2\leq k\leq 2n$	$k=n$	1
NNLL	$2n-4\leq k\leq 2n$	$k=n-1$	2

FOR HIGGS INCLUSIVE:

- g_1, g_2, g_3 KNOWN; g_0 KNOWN UP TO $O(\alpha_s^2)$
- N^3LL KNOWN FOR EXPONENT BUT FULLY NOT CROSS-SECTION
- TO $O(\alpha_s^3)$ (N^3LO) ALL LOGS KNOWN, BUT NOT CONSTANT
- THE m_t DEPENDENCE IS ENTIRELY CONTAINED IN THE $g_{0,i}$ COEFFICIENTS; $g_{0,1}(m_t)$ KNOWN EXACTLY, $g_{0,2}(m_t)$ KNOWN AS A POWER SERIES IN $\left(\frac{m_h}{m_t}\right)^2$.

CONSTRUCTING THE LARGE N TERMS

A SIMPLE EXAMPLE

THE NLO CROSS SECTION IN THE POINTLIKE LIMIT

THE z-SPACE RESULT

$$C^{(1)}(z) = 4A_g(z)\left[\frac{\log\frac{1-z}{\sqrt{z}}}{1-z}\right]_+ + \left(\frac{4C_A}{\pi}\zeta_2 + \frac{11}{2\pi}\right)\delta(1-z) - \frac{11}{2\pi}\frac{(1-z)^3}{z};$$

$$P_{gg}(z) = \frac{A_g(z)}{[1-z]_+} + \beta_0\delta(1-z); \quad \beta_0 = \frac{11C_A - 2n_f}{12\pi}; \quad A_g(z) = \frac{C_A}{\pi}\frac{1-2z+3z^2-2z^3+z^4}{z}$$

- THE LOGS ARE ENTIRELY CONTAINED IN THE FIRST TERM: $\tilde{\mathcal{D}}_1(z) \equiv \left[\frac{\log\frac{1-z}{\sqrt{z}}}{1-z}\right]_+$
- THE SECOND TERM CONTRIBUTES TO THE CONSTANT
- THE THIRD TERM IS SUPPRESSED BY A (HIGH) NEGATIVE POWER OF N AS $N \to \infty$

N-SPACE LOGS

MELLIN TRANSF.: $\tilde{\mathcal{D}}_1(N) \equiv \mathcal{M}\left[\tilde{\mathcal{D}}_1(z)\right] = \frac{1}{2}\left[\psi_0^2(N) + 2\gamma_E\psi_0(N) + \gamma_E^2\right]$

THE LARGE N LOGS: $\psi_0(N) = \log\left(N - \frac{1}{2}\right) + \mathcal{O}\left(\frac{1}{N^2}\right) = \log(N) + \mathcal{O}\left(\frac{1}{N}\right)$

THE NAIVE RESULT...

- EXPAND THE RESUMMED RESULT: $C_{\rm res}(N,\alpha_s) = 1 + \alpha_s C^{(1)}_{\rm res}(N) + \mathcal{O}(\alpha_s^2)$
- USE RESULT AS NLO ESTIMATE: $C^{(1)}_{\rm res}(N) = g_{1,2}\ln^2 N + g_{2,1}\ln N + g_{0,1}$; $g_{1,2} = \frac{2C_A}{\pi}$, $g_{2,1} = \frac{4C_A}{\pi}\gamma_E$
- HOWEVER: $\ln N$ HAS A CUT AT $N = 0$! SUBLEADING,M BUT SINGULARITY STRUCTURE RADICALLY CHANGED

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...AND ITS IMPROVEMENTS

FIRST IMPROVEMENT: SMALL N POLES VS CUTS

- THE INVERSE MELLIN OF $C^{(1)}_{\rm res}(N)$ IS $C^{(1)}_{\rm res}(z,\alpha_s)=\frac{4C_A}{\pi}\mathcal{D}^{\log}_1(z)$ (UP TO DELTA);
  $\mathcal{D}^{\log}_1(z)\equiv\left[\frac{\ln\ln\frac{1}{z}}{\ln\frac{1}{z}}\right]_+$
- BUT THE EXACT RESULT CONTAINS $\mathcal{D}_1(z)\equiv\left[\frac{\log(1-z)}{1-z}\right]_+$, NOT $\mathcal{D}^{\log}_1(z)$
- IMPROVEMENT I: EXPRESS THE RESUMMED RESULT IN TERMS OF
  $\mathcal{D}_1(N)\equiv\mathcal{M}\left[\mathcal{D}_1(z)\right]=\frac{1}{2}\left[\psi_0^2(N)-\psi_1(N)+2\gamma_E\psi_0(N)+\zeta_2+\gamma_E^2\right]$
- THIS HAS SIMPLE POLES, NOT CUTS

MORE IMPROVEMENTS

SECOND IMPROVEMENT: PHASE SPACE

- ACTUALLY, THE EXACT RESULT HAS $\tilde{\mathcal{D}}_1(z) = \left[\frac{\log\frac{1-z}{\sqrt{z}}}{1-z}\right]_+$, NOT $\mathcal{D}_1(z) = \left[\frac{\log(1-z)}{1-z}\right]_+$
- DUE TO UNIVERSAL STRUCTURE OF PHASE SPACE FOR REAL SOFT GLUON EMISSION (S.F., RIDOLFI, 2003): $p_{gg}(z)\int_\Lambda^{\frac{M(1-z)}{\sqrt{z}}} \frac{dk_T}{k_T} = \frac{A_g(z)}{1-z}\left(\ln\frac{1-z}{\sqrt{z}} + \ln\frac{M}{\Lambda}\right)$
- IMPROVEMENT II: EXPRESS THE RESUMMED RESULT IN TERMS OF $\tilde{\mathcal{D}}_1(N) \equiv \mathcal{M}\left[\tilde{\mathcal{D}}_1(z)\right] = \frac{1}{2}\left[\psi_0^2(N) + 2\gamma_E\psi_0(N) + \gamma_E^2\right]$
- NOTE: SIMPLER THAN PREVIOUS N-SPACE RESULTS (SOME SINGULARITIES CANCEL)

MORE IMPROVEMENTS

THIRD IMPROVEMENT: SOFT-COLLINEAR

- AGAIN: SOFT EMISSION LEADS TO $p_{gg}(z)\int_{\Lambda}^{\frac{M(1-z)}{\sqrt{z}}}\frac{dk_T}{k_T}=\frac{A_g(z)}{1-z}\left(\ln\frac{1-z}{\sqrt{z}}+\ln\frac{M}{\Lambda}\right)$

- PREFACTOR OF $A_g(z)$ IS THE ALTARELLI-PARISI NUMERATOR $P_{gg}(z)=\frac{A_g(z)}{[1-z]_+}$

- CAN EXPAND $A_g(z)=\frac{C_A}{\pi}\left[1-(1-z)+2(1-z)^2+\mathcal{O}\left[(1-z)^3\right]\right]$

- FIRST-ORDER EXPANSION $A_{g,1}(z)=\frac{C_A}{\pi}\left[1-(1-z)\right]=z\,A_g(1)$ SUMS ALL $\frac{\ln^k N}{N}$ TERMS; ALL OF WHICH ARE THUS SUMMED TO ALL ORDERS (Catani, de Florian, Grazzini, 2001)

- SUBSEQUENT ORDERS SUPPRESSED BY POWERS OF $\frac{1}{N}$; UNIVERSAL, BUT NOT ALL SUCH TERMS THUS INCLUDED

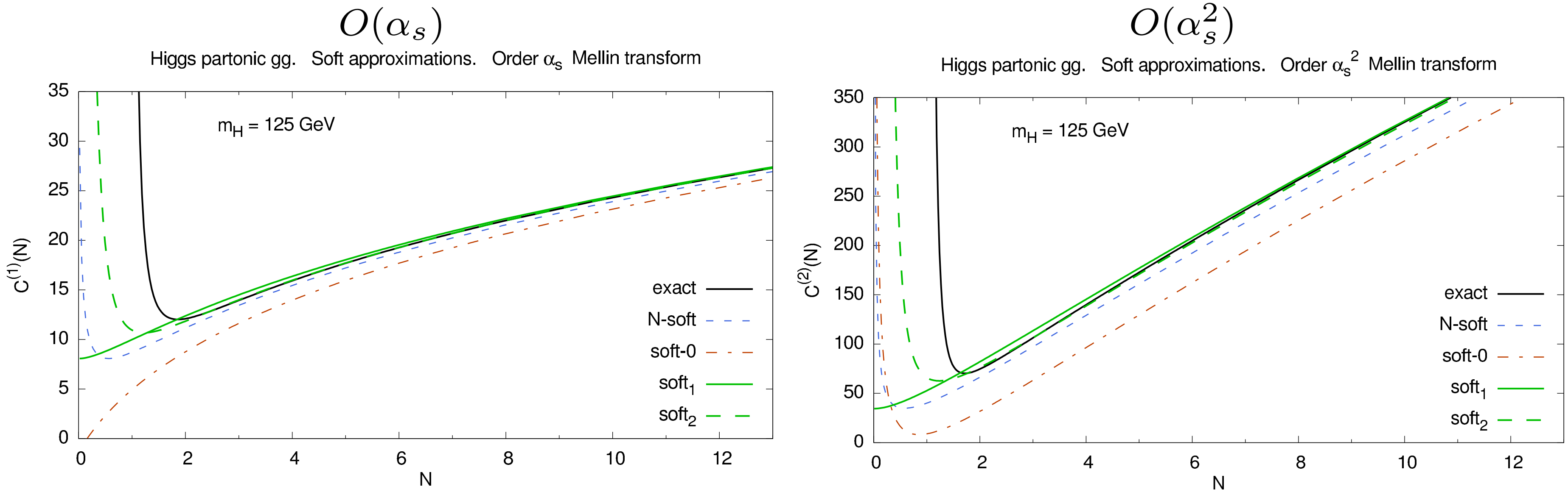
- IN N SPACE, 1ST ORDER EXPANSION AMOUNTS TO $\mathcal{D}_1^{\log}(N)\to\tilde{\mathcal{D}}_1(N+1)$,
  2ND ORDER EXPANSION AMOUNTS TO $\mathcal{D}_k^{\log}(N)\to 2\tilde{\mathcal{D}}_k(N)-3\tilde{\mathcal{D}}_k(N+1)+2\tilde{\mathcal{D}}_k(N+2)$

FINAL PRESCRIPTION

- EXPAND OUT RESUMMED RESULT, WRITE IT IN TERMS OF $\mathcal{D}_k^{\log}(N)$, MELLIN TRANSFORM OF $\mathcal{D}_k^{\log}(z) \equiv \left[\frac{\ln^k \ln \frac{1}{z}}{\ln \frac{1}{z}}\right]_+$
- REPLACE $\mathcal{D}_k^{\log}(N) \to \tilde{\mathcal{D}}_k(N)$, MELLIN TRANSFORM OF $\tilde{\mathcal{D}}_1(z) = \left[\frac{\log \frac{1-z}{\sqrt{z}}}{1-z}\right]_+$

  (PROPER SMALL N SINGULARITY+ PHASE SPACE FACTOR)
- FURTHER REPLACE $\tilde{\mathcal{D}}_k(N) \to \tilde{\mathcal{D}}_k(N+1)$ OR COMBINATION OF SHIFTS FOR 2ND ORDER (SOFT-COLLINEAR IMPROVEMENT)
- ADJUST THE CONSTANT
- INVERT THE MELLIN TRANSFORM

HOW WELL DOES IT WORK?



- SOFT1 & SOFT2 (ALL THREE IMPROVEMENTS) PROVIDE BEST APPROXIMATION TO EXACT; DIFFERENCE BETWEEN 1ST & 2ND ORDER EXPANSION TAKEN AS ESTIMATE OF UNCERTAINTY
- PURE RESUMMATION N-SOFT (NO IMPROVEMENT) SURPRISINGLY GOOD
- 1ST IMPROVEMENT ONLY (SOFT0) NOT SO GOOD, 2ND IMPROVEMENT (SQRT FACTOR) HAS NEGLIGIBLE IMPACT

TECHNICAL ASIDE: WHY IS N-SOFT SO GOOD

COINCIDENTIALLY, $\frac{\ln^k \ln \frac{1}{z}}{\ln \frac{1}{z}} = \frac{\sqrt{z}}{1-z} \ln^k \frac{1-z}{\sqrt{z}} \times \left[1 + \mathcal{O}\left[(1-z)^2\right]\right]$,

SAME AS IMPROVED, UP TO TERMS SUPPRESSED BY TWO POWERS OF $\frac{1}{N}$.

THE SMALL N TERMS

WHAT DO WE KNOW ABOUT SMALL x?

- AT SMALL N RESUMMATION OF PERTURBATIVE EVOLUTION KNOWN SINCE LONG AT LL LEVEL (Jaroszewicz 1982 using BFKL 1976-78), MORE RECENTLY AT NLL LEVEL (Fadin-Lipatov 1998)
- RESUMMATION OF (INCLUSIVE) COEFFICIENT FUNCTIONS ONLY KNOWN AT LL LEVEL FOR SEVERAL PROCESSS, INCLUDING HIGGS, (Marzani, Ball, del Duca, SF, Vicini, 2008)
- SMALL x BEHAVIOUR FOR HIGGS INCORRECT IN POINTLIKE APPROXIMATION (SPURIOUS DOUBLE LOGS) RESUMMATION OF SPURIOUS DOUBLE LOGS ALSO KNOWN (Hautmann, 2002)
- RESUMMATION OF RAPIDITY DISTRIBUTIONS KNOWN AT LL ONLY FOR HIGGS (Caola, SF, Marzani, 2011)

THE STRUCTURE OF RESUMMED RESULTS

- THE SMALL N COEFFICIENT FUNCTION CAN BE WRITTEN AS A FUNCTION OF THE SMALL N ANOMALOUS DIMENSION $\gamma^+(N)$ ("LARGE" EIGENVECTOR OF SINGLET ANOMALOUS DIMENSION MATRIX)
(Catani, Ciafaloni, Hautmann, 1990), UP TO RUNNING COUPLING CORRECTIONS (REQUIRED FOR CORRECT ALL-ORDER SINGULARITY) (Ball, 2007; Altarelli, Ball, SF, 2008):$\Rightarrow$ $\left[\gamma^{+\,n}\right]$ DENOTES R.C. CORRECTED n-TH POWER OF AN. DIM. γ^+
$C_{\rm ABF}(N,\alpha_s) = \sum_{i_1,i_2\geq 0} c_{i_1,i_2}\left[\gamma^{+\,i_1}\right]\left[\gamma^{+\,i_2}\right] - 1$
$= \alpha_s 2c_{1,0}\gamma^{(0)} + \alpha_s^2\left(\left(2c_{2,0}+c_{1,1}\right){\gamma^{(0)}}^2 - 2c_{2,0}\beta_0\gamma^{(0)} + 2c_{1,0}\gamma^{(1)}\right) + \ldots$

- THE SMALL N ANOMALOUS DIMENSION IS A SERIES OF POLES;
THE HIGHEST TWO (LL & NLL) ARE KNOWN EXACTLY:
$\gamma^+ = \alpha_s\gamma^{(0)} + \alpha_s^2\gamma^{(1)} + \alpha_s^3\gamma^{(2)} + \mathcal{O}(\alpha_s^4)$
$\gamma^{(0)} = \frac{e_{0,-1}}{N-1} + e_{0,0} + \mathcal{O}\left(N-1\right);\quad \gamma^{(1)} = \frac{e_{1,-2}}{(N-1)^2} + \frac{e_{1,-1}}{N-1} + \mathcal{O}\left(1\right)$
$\gamma^{(2)} = \frac{e_{2,-3}}{(N-1)^3} + \frac{e_{2,-2}}{(N-1)^2} + \mathcal{O}\left((N-1)^{-1}\right)$

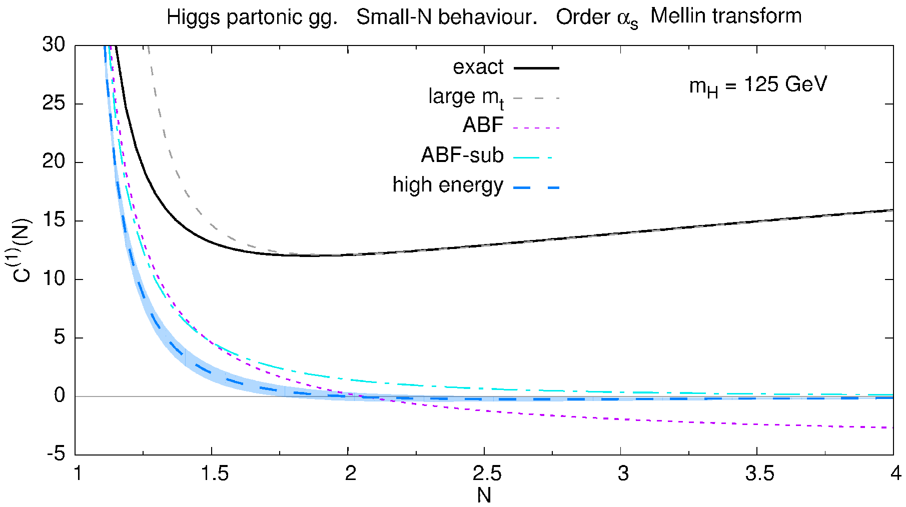
- THE SMALL N CONTRIBUTION MUST VANISH AT LARGE N, & PRESERVE $N=1$ SINGULARITIES (TRADEOFF)
INTRODUCE SUBTRACTION
$C^{(n)}_{\text{ABF-SUB}}(N) = C^{(n)}_{\rm ABF}(N) - 2C^{(n)}_{\rm ABF}(N+1) + C^{(n)}_{\rm ABF}(N+2)$
IN x SPACE CORRESPONDS TO $(1-z)^2$ DAMPING

SUBLEADING SMALL N TERMS

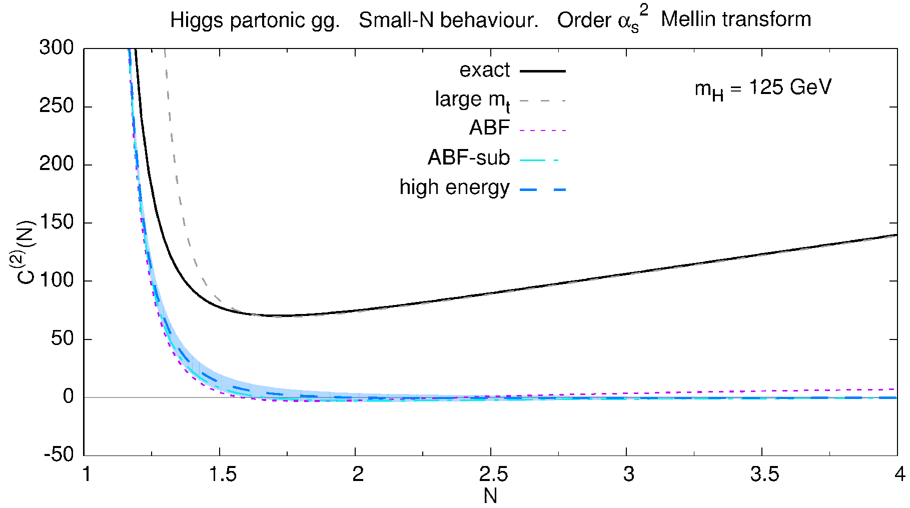
- $\gamma^+(2) = 0$ BY MOMENTUM CONSERVATION, ORDER BY ORDER IN PERTURBATION THEORY
- $N = 2$ IS THE NATURAL TRANSITION BETWEEN SMALL N & LARGE N REGION: ENFORCE VANISHING OF SMALL N TERMS BY SUBTRACTION
  $\gamma^+(N) \to \gamma^+(N) - f_{\text{MOM}}(N)$;
  $f_{\text{MOM}}(N) = \frac{c}{N}$; $f_{\text{MOM}}(2) = \gamma^+(2)$
  FUNCTION $f_{\text{MOM}}(N)$ AS IN ABF (OPTIMIZED MATCHING OF LLx & LLQ^2 RESUMMATION),
  DOES NOT CHANGE LARGE N BEHAVIOUR & SMALL N SINGS: AMOUNTS TO
  $C^{(n)}_{\text{H.E.}}(N) = C^{(n)}_{\text{ABF-SUB}}(N) - \frac{4!\, k_{\text{mom}}}{N(N+1)(N+2)}$
- BUT FULL COEFFICIENT FUNCTION DOES NOT VANISH AT $N = 2$ DUE TO SUBLEADING POLES.
  ESTIMATE THEIR EFFECT BY VARYING $k_{\text{mom}} = C_{\text{ABF-SUB}}(2) \pm 0.05 \times C_{\text{SOFT}}(2)$

THE IMPACT OF SMALL N TERMS

$O(\alpha_s)$



$O(\alpha_s^2)$



- POINTLIKE & FINITE m_t RESULTS START DIFFERING FOR $N \lesssim 1.5$
- EXACT ONLY REDUCES TO LEADING POLE FOR $N \lesssim 1.25$
- IMPACT OF SUBTRACTION SIGNIFICANT, BUT IN REGION WHERE SMALL N POLES ARE SURELY IRRELEVANT
- RESIDUAL UNCERTAINTY NOT NEGLIGIBLE IN $N \sim 1.5$ REGION DUE TO LACK OF KNOWLEDGE OF NLL AND SUBLEADING TERMS

THE HIGGS CROSS SECTION

COMBINING INFORMATION

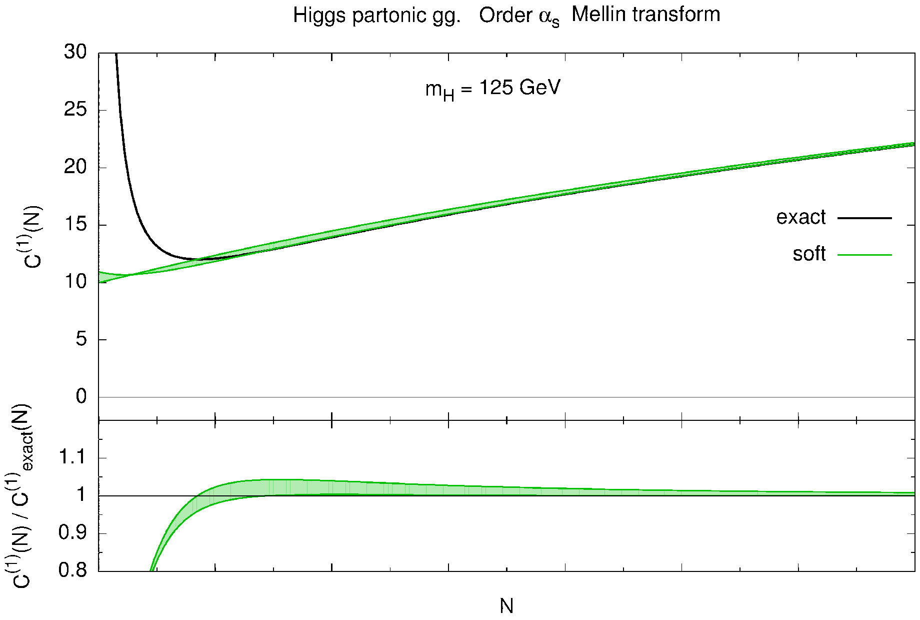
$$C^{(k)}_{\mathrm{APPROX}}(N) = C^{(k)}_{\mathrm{SOFT}}(N) + C^{(k)}_{\mathrm{H.E.}}(N)$$

NOTHING ELSE TO DO: SINGULARITIES TAKE CARE OF THEMSELVES

NLO AND NNLO

EXACT+SOFT

$O(\alpha_s)$



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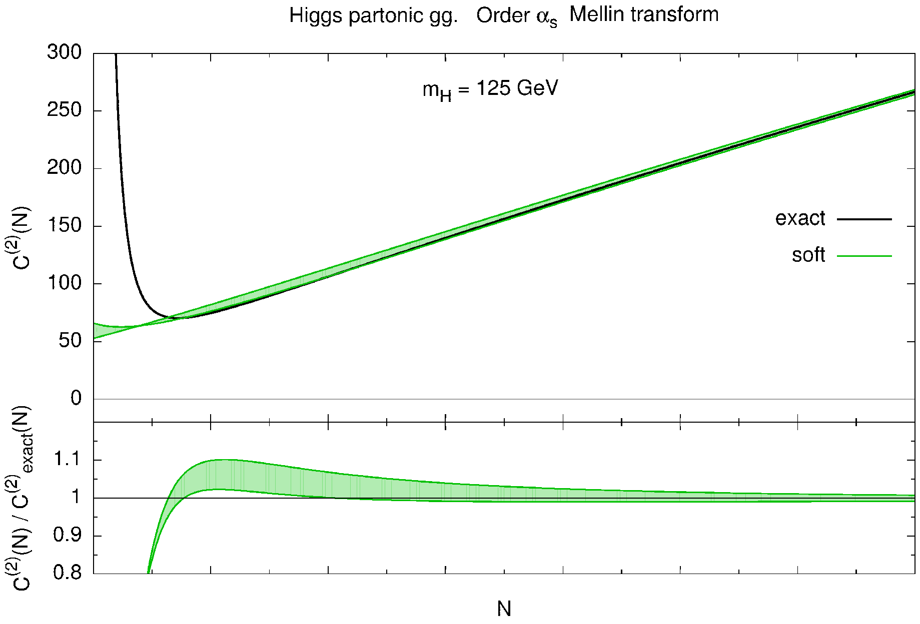

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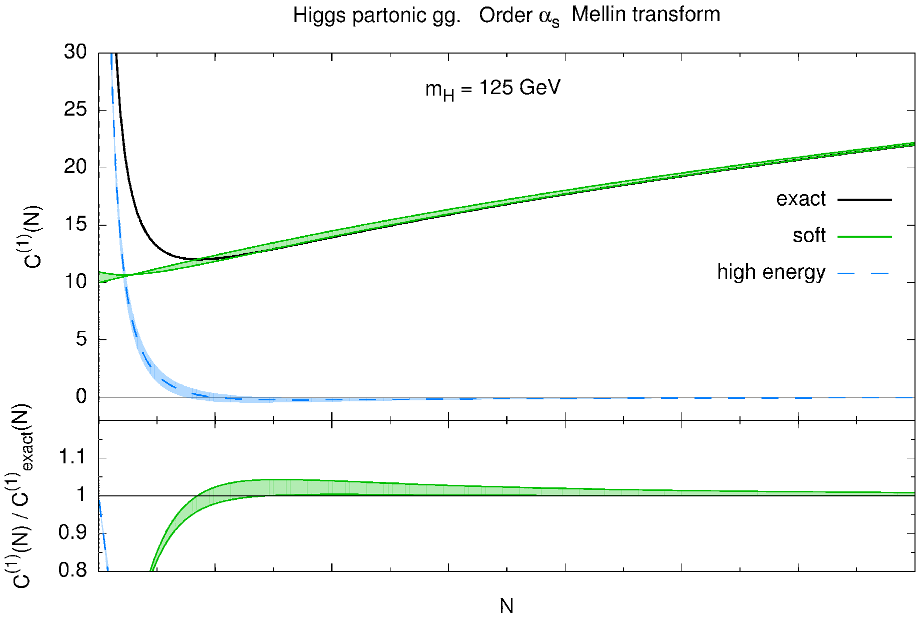
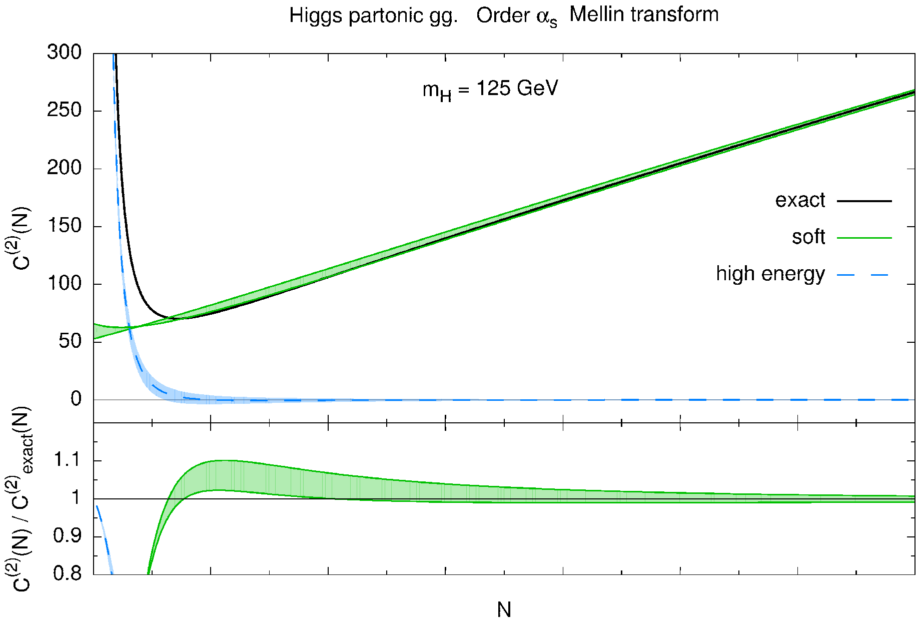
NLO AND NNLO

EXACT+SOFT+HIGH ENERGY

$O(\alpha_s)$



$O(\alpha_s^2)$



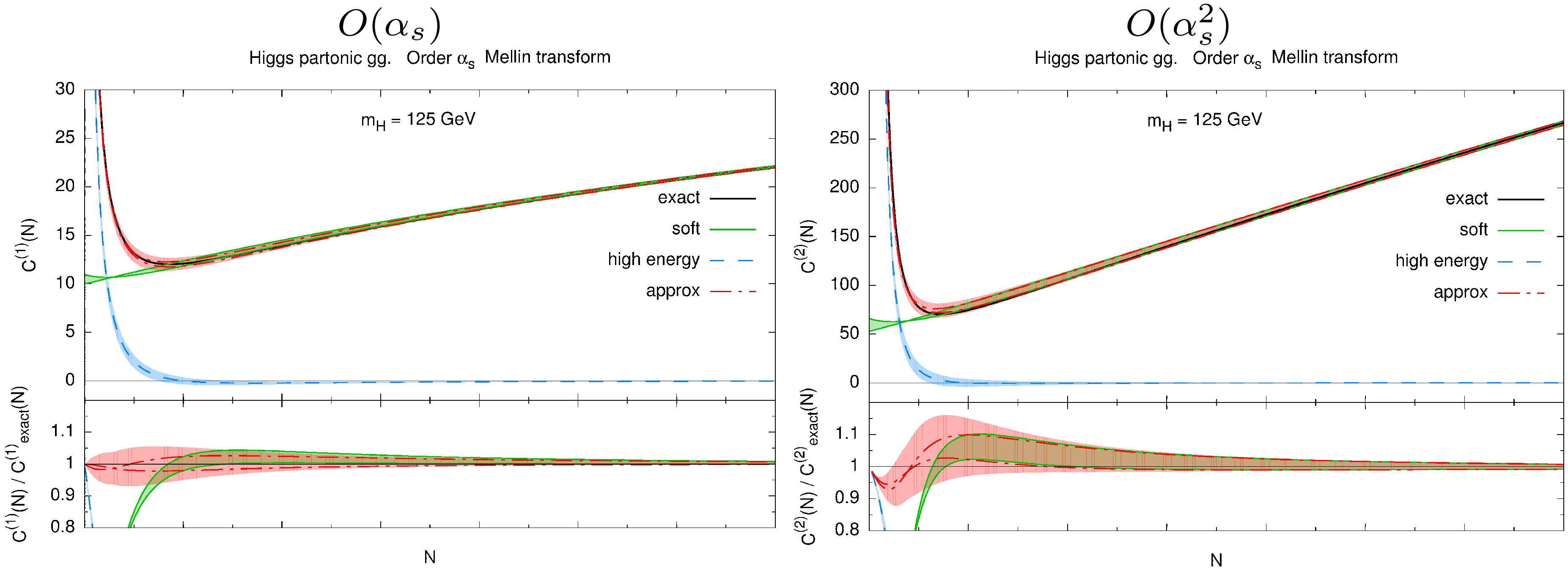
COMBINING INFORMATION

$$C^{(k)}_{\text{APPROX}}(N) = C^{(k)}_{\text{SOFT}}(N) + C^{(k)}_{\text{H.E.}}(N)$$

NOTHING ELSE TO DO: SINGULARITIES TAKE CARE OF THEMSELVES

NLO AND NNLO

EXACT+SOFT+HIGH ENERGY+APPROX.

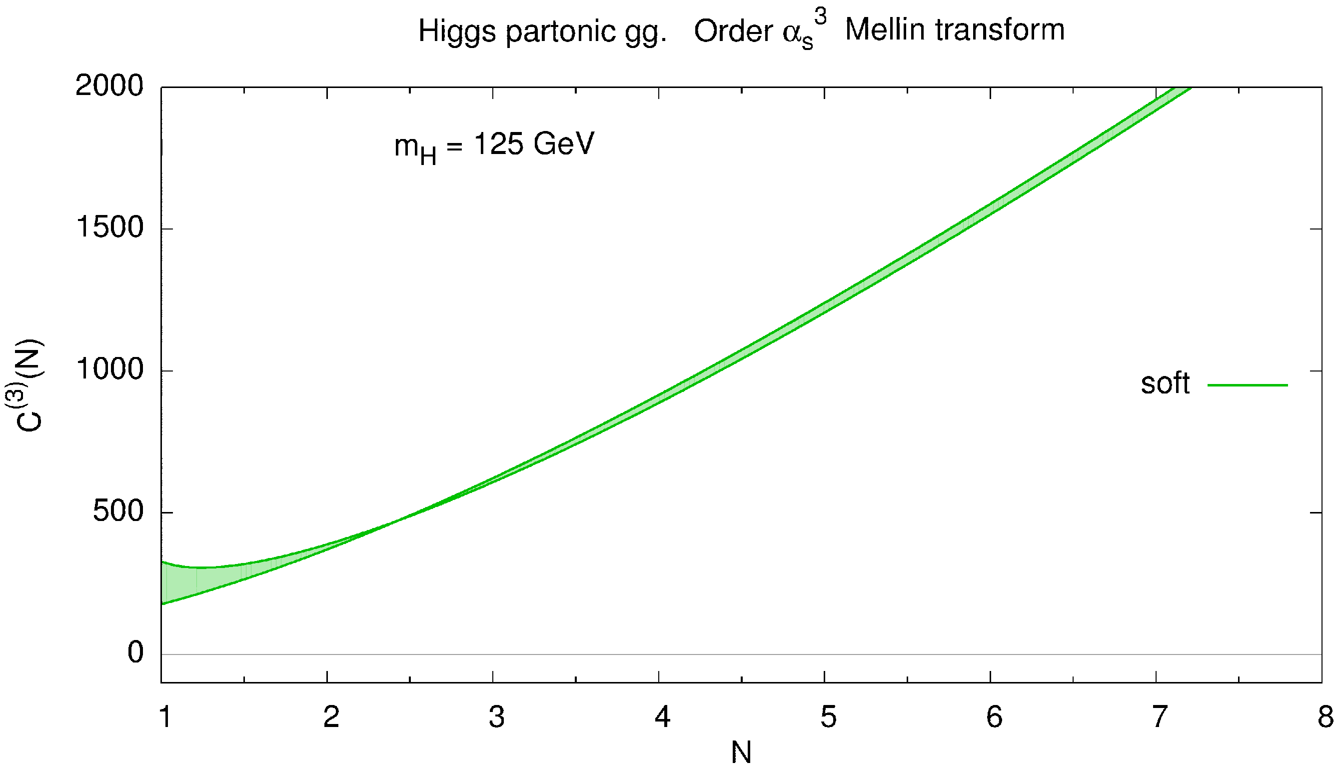


NOTE FOR $N \sim 1.2$ WHERE SMALL DISCRAPANCY WITH NNLO IS OBSERVED, "EXACT" IS BASED ON AN UNRELIABLE APPROX.

COMBINING INFORMATION
THE N^{3}LO PARTON-LEVEL RESULT

EXACT+SOFT

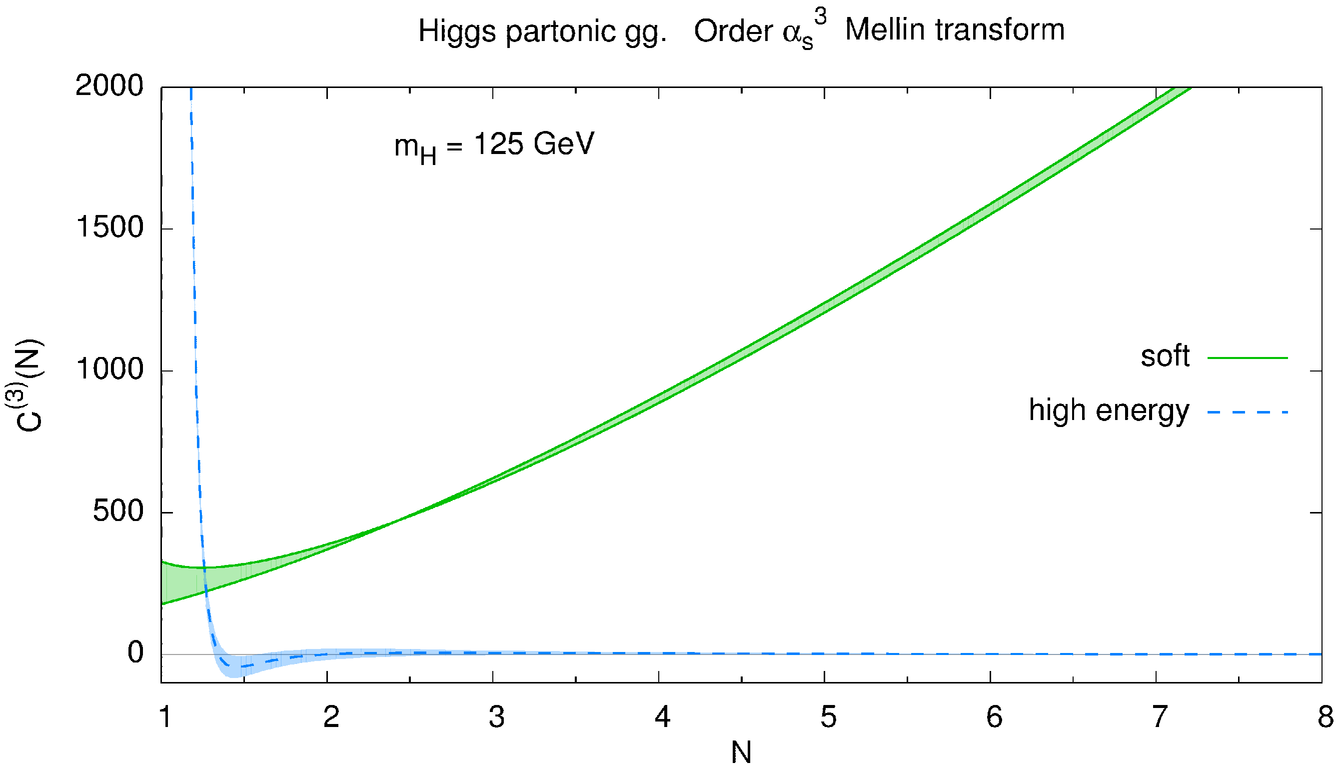
$O(\alpha_s^3)$



COMBINING INFORMATION
THE N^{3}LO PARTON-LEVEL RESULT

EXACT+SOFT+HIGH ENERGY

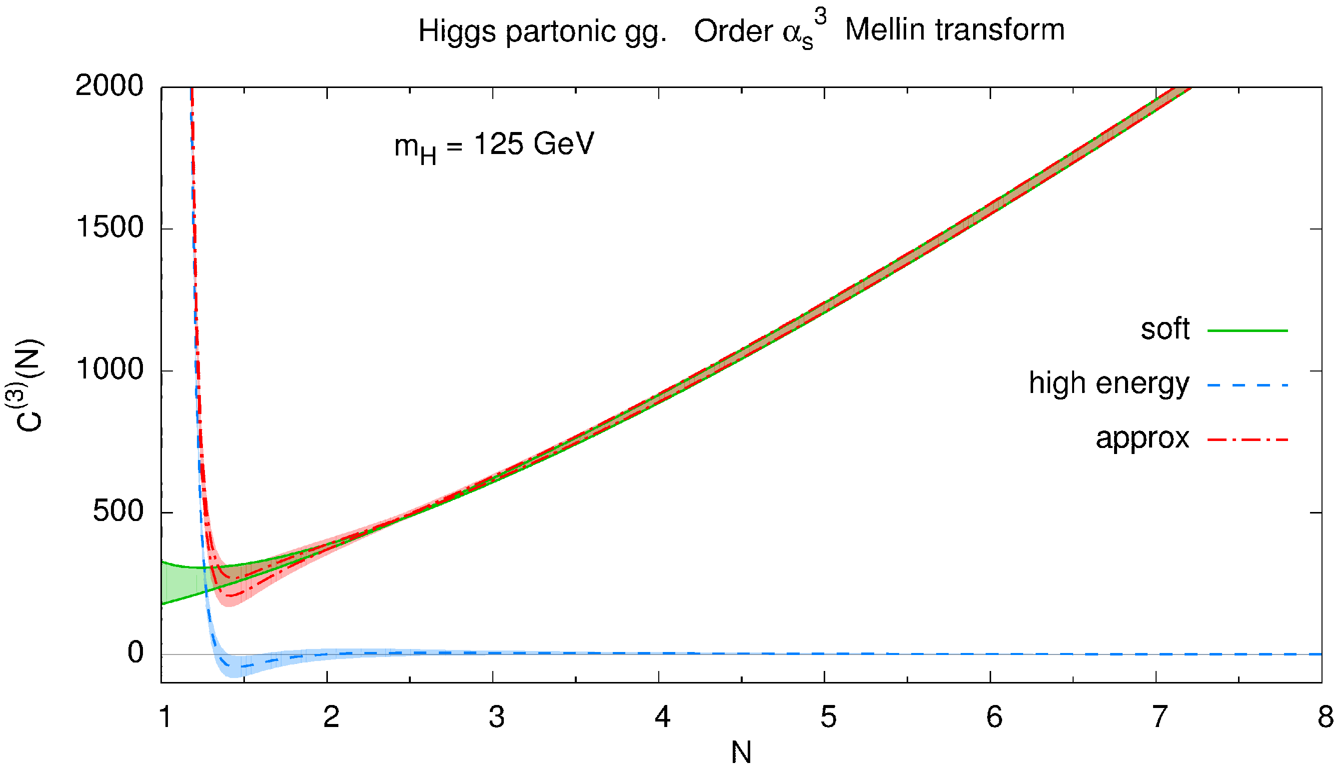
$O(\alpha_s^3)$



COMBINING INFORMATION
THE N^{3}LO PARTON-LEVEL RESULT

EXACT+SOFT+HIGH ENERGY+APPROX.

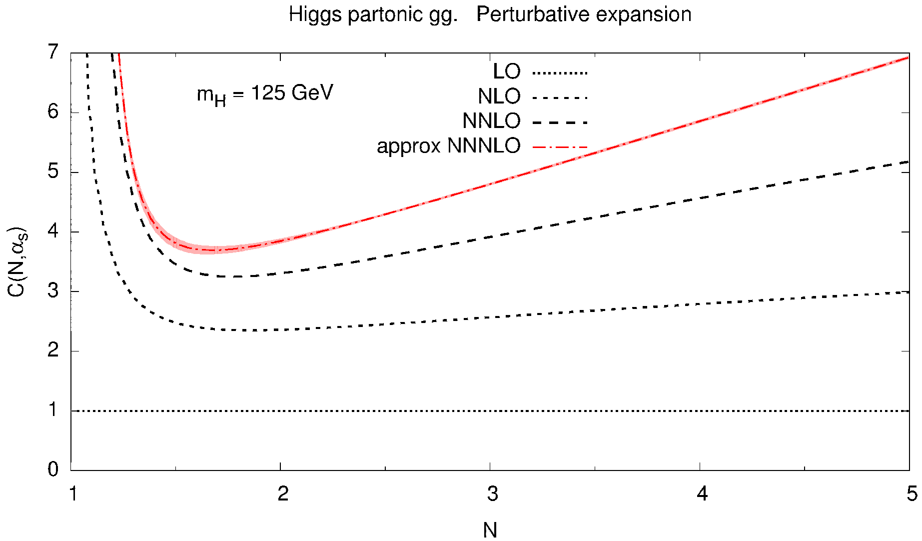
$$O(\alpha_s^3)$$



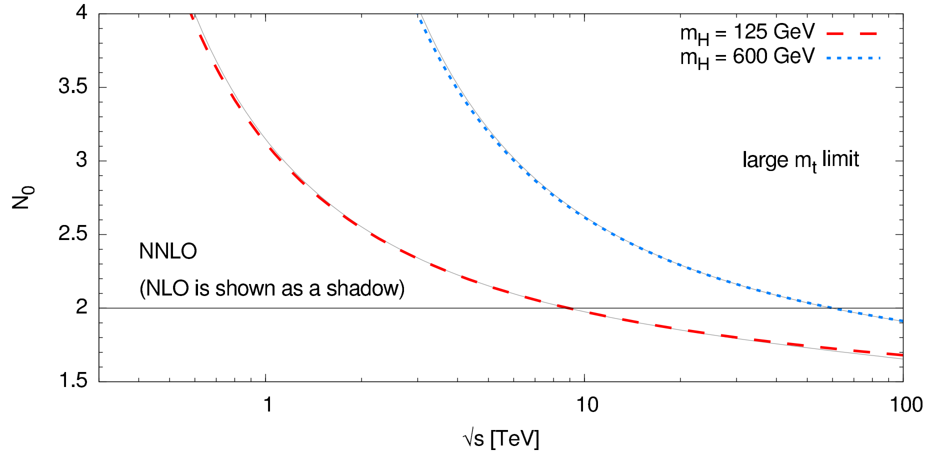
PERTURBATIVE (IN)STABILITY

THE N^{3}LO PARTON-LEVEL RESULT

PERTURBATIVE EXP.



POSITION OF SADDLE N_0



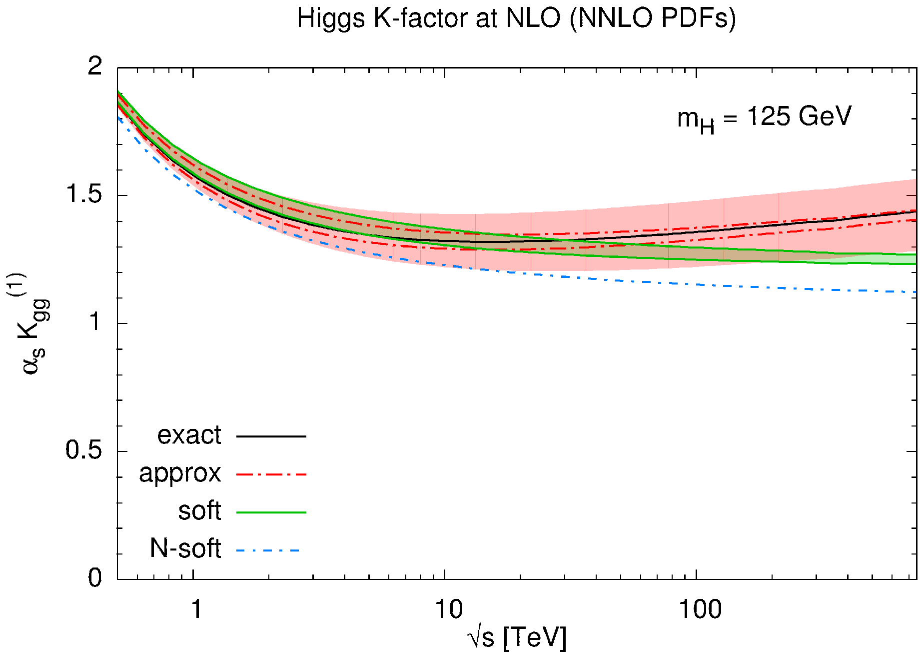
SLOW CONVERGENCE; BEST AT $\sqrt{s} \sim 100$ TeV

THE HADRONIC K-FACTORS: NLO & NNLO

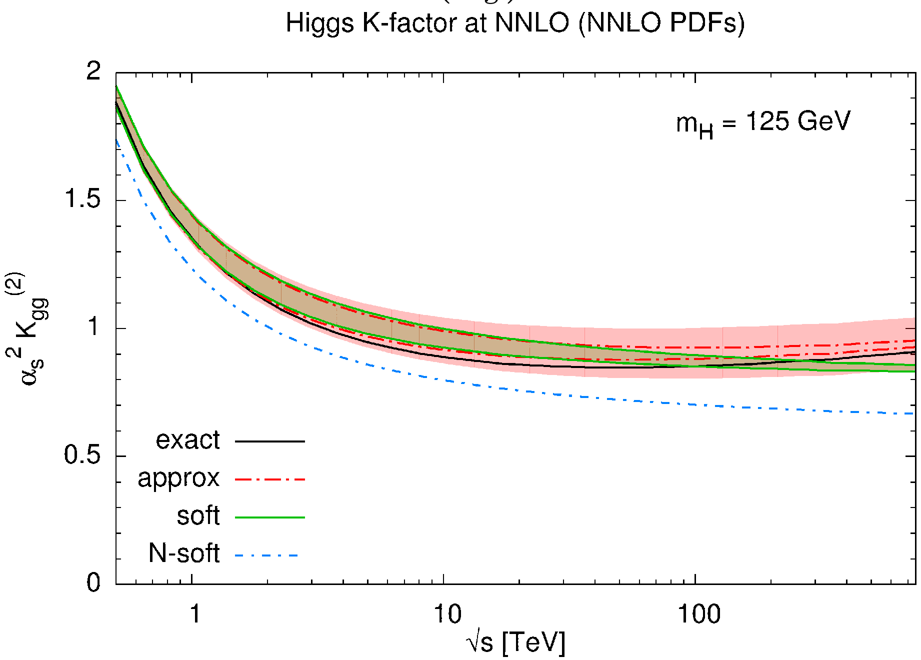
$K_{gg}(\tau, m_H^2) \equiv \sigma_{gg}(\tau, m_H^2)/\sigma^{(0)}(\tau, m_H^2) = 1 + \alpha_s K_{gg}^{(1)} + \alpha_s^2 K_{gg}^{(2)} + \alpha_s^3 K_{gg}^{(3)} + \mathcal{O}(\alpha_s^4)$; FIXED PDF

	$\mu_R = m_H$			$\mu_R = m_H/2$		
	$C^{(n)}_{\text{EXACT}}$	$C^{(n)}_{\text{APPROX}}$	$C^{(n)}_{N\text{-SOFT}}$	$C^{(n)}_{\text{EXACT}}$	$C^{(n)}_{\text{APPROX}}$	$C^{(n)}_{N\text{-SOFT}}$
$\alpha_s K_{gg}^{(1)}$	1.328	1.330 ± 0.099	1.241	1.262	1.265 ± 0.111	1.167
$\alpha_s^2 K_{gg}^{(2)}$	0.903	0.968 ± 0.088	0.815	0.795	0.747 ± 0.109	0.558

$O(\alpha_s)$



$O(\alpha_s^2)$



- APPROX IN PERFECT AGREEMENT WITH EXACT
- SOFT APPROX EXCELLENT FOR $s \lesssim 10$ TEV, THEN FAILS (Bonvini, SF, Ridolfi, 2012)
- N-SOFT (Moch,Vogt 2005) RATHER LESS ACCURATE (EXPANDED RESUMMATION)

THE HADRONIC K-FACTORS
THE N^{3}LO PREDICTION

	$\mu_R = m_H$			$\mu_R = m_H/2$		
	$C^{(n)}_{\text{EXACT}}$	$C^{(n)}_{\text{APPROX}}$	$C^{(n)}_{N\text{-SOFT}}$	$C^{(n)}_{\text{EXACT}}$	$C^{(n)}_{\text{APPROX}}$	$C^{(n)}_{N\text{-SOFT}}$
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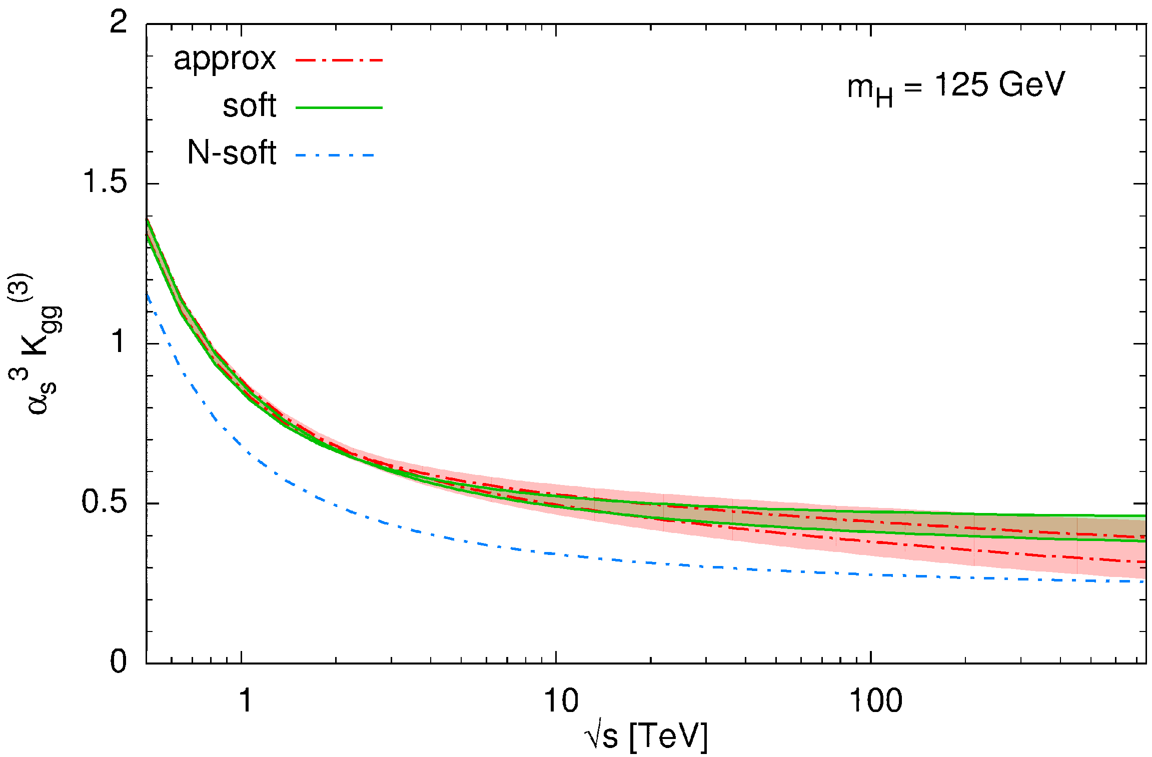
$\alpha_s^3 K^{(3)}_{gg}$	—	0.527 ± 0.043	0.353	—	0.323 ± 0.059	0.085

$O(\alpha_s^3)$

Higgs K-factor at NNNLO (NNLO PDFs)

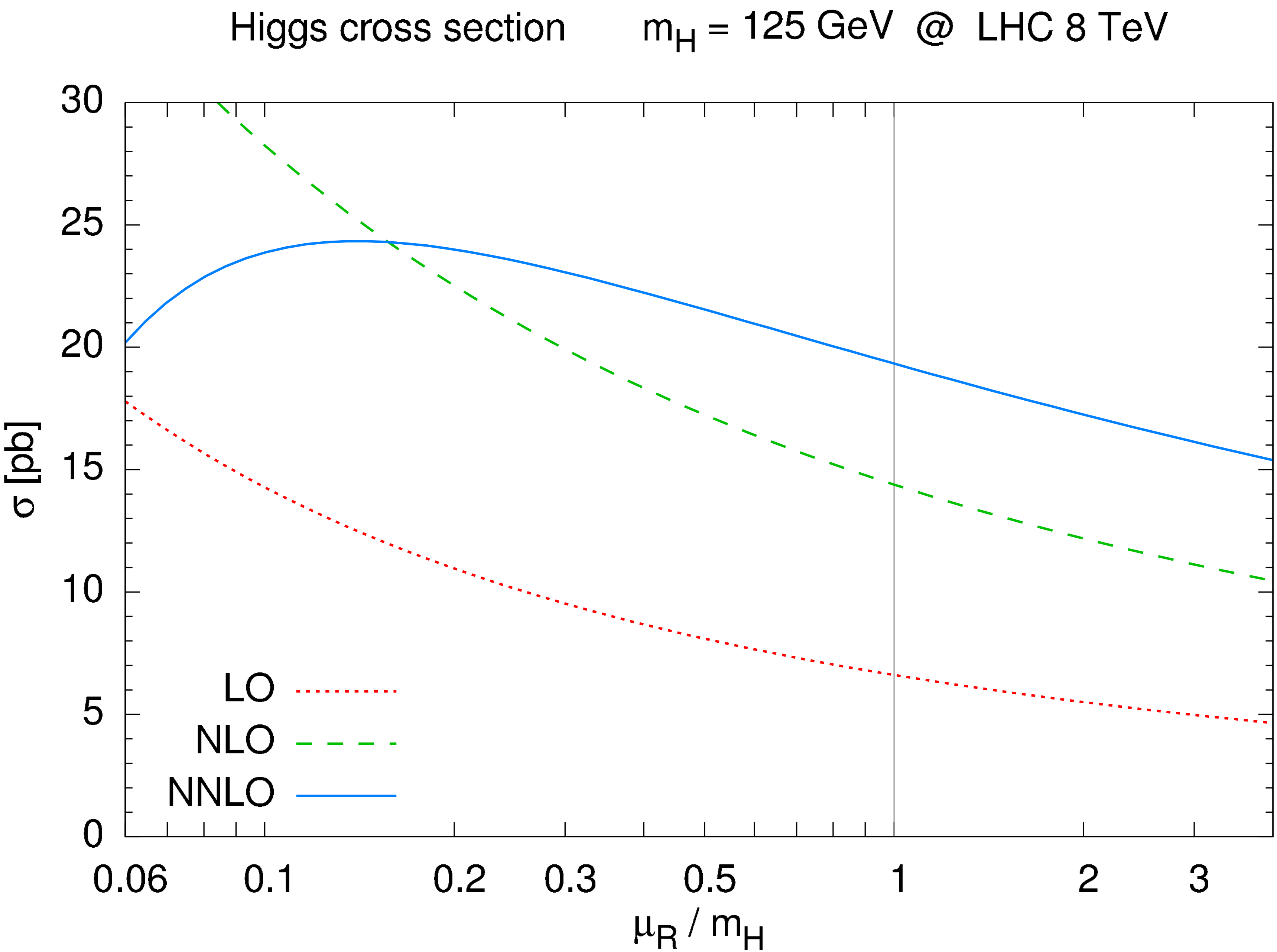


THE N^{3}LO PREDICTION
UNCERTAINTIES, ALTERNATIVE APPROXIMATIONS, COMPARISONS

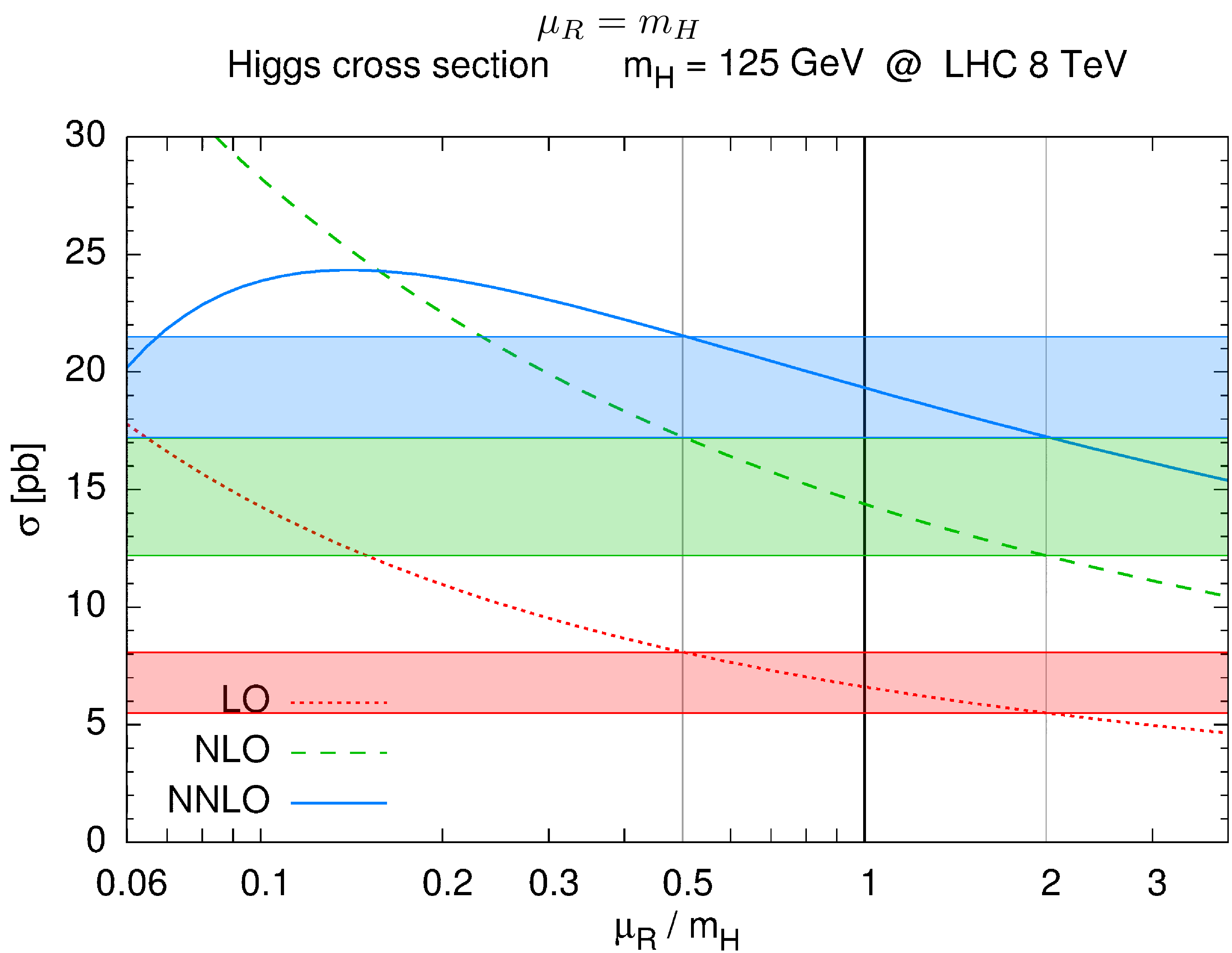
$$\sigma^{\mathrm{N^3LO}}_{\mathrm{APPROX}}(\tau, m_H^2) = \left(22.61 \pm 0.27 + 0.91 \cdot 10^{-2} \bar{g}_{0,3}\right) \text{ PB} \qquad \text{FOR } \mu_{\mathrm{R}} = m_H$$
$$\sigma^{\mathrm{N^3LO}}_{\mathrm{APPROX}}(\tau, m_H^2) = \left(24.03 \pm 0.45 + 1.55 \cdot 10^{-2} \bar{g}_{0,3}\right) \text{ PB} \qquad \text{FOR } \mu_{\mathrm{R}} = m_H/2,$$

- RECALL $O(\alpha_s^3)$ CONSTANT $\bar{g}_{0,3}$; SOME LATITUDE IN DEFINITION OF "CONSTANT", WITH OUR DEF. $\bar{g}_{0,1} \sim 5$, $\bar{g}_{0,2} \sim 10$ (Moch,Vogt 2005) (E.G. WITH DEF. FROM EXPANSION OF RESUMMATION $g_{0,1} \sim 10$, $g_{0,2} \sim 40$)
- N^{3}LO CORRECTS NNLO BY 17% IF $\mu_r = m_H$, BY 11.5% IF $\mu_r = m_H/2$
- NNLL RESUMMATION (de Florian, Grazzini, 2012) CORRECTS NLO BY 8% ($\mu_r = m_H$) OF THIS 6% COMES FROM N^{3}LO (RESUMMATION IS PERTURBATIVE)
- N^{3}LO TERM FROM RESUMMATION IS IDENTICAL TO N-SOFT, WITH $g_{0,3} = 0$ INSTEAD OF $\bar{g}_{0,3} = 0$
- USING $g_{0,3} = 0$ INSTEAD OF $\bar{g}_{0,3} = 0$ REDUCES N^{3}LO BY 5% IF $\mu_r = m_H$, BY 8.5% IF $\mu_r = m_H/2$
- USING THE LESS ACCURATE N-SOFT APPROXIMATION (WHICH CORRESPONDS TO EXPANDING OUT THE RESUMMED RESULT) REDUCES N^{3}LO BY 6% IF $\mu_r = m_H$, BY 8.5% IF $\mu_r = m_H/2$
- OUR APPROX IS THREE TIMES LARGER THAN EXPANSION OF RESUMMATION; HALF DUE TO CONSTANT, HALF DUE TO BETTER APPROX

PERTURBATIVE (IN)STABILITY



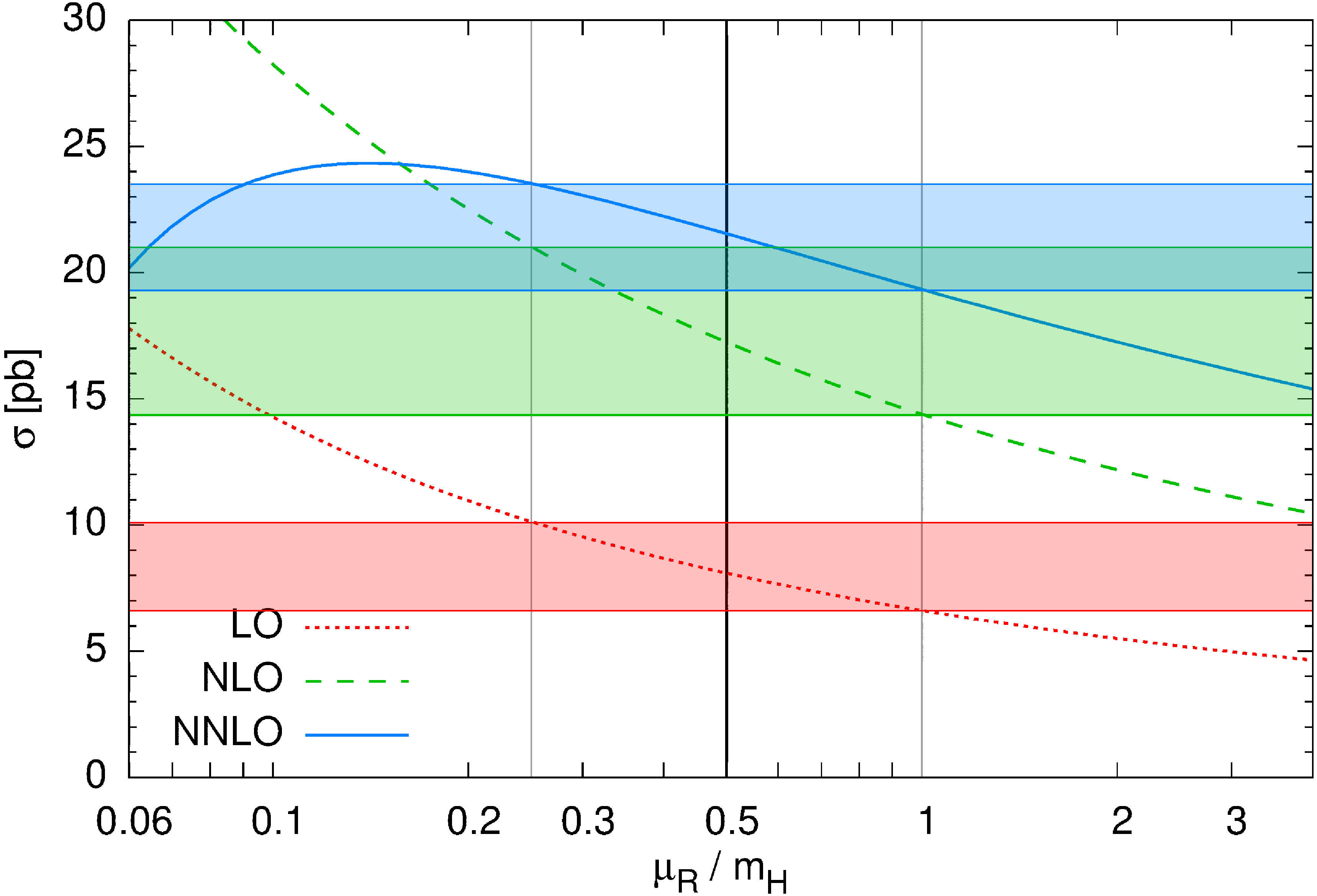
PERTURBATIVE (IN)STABILITY



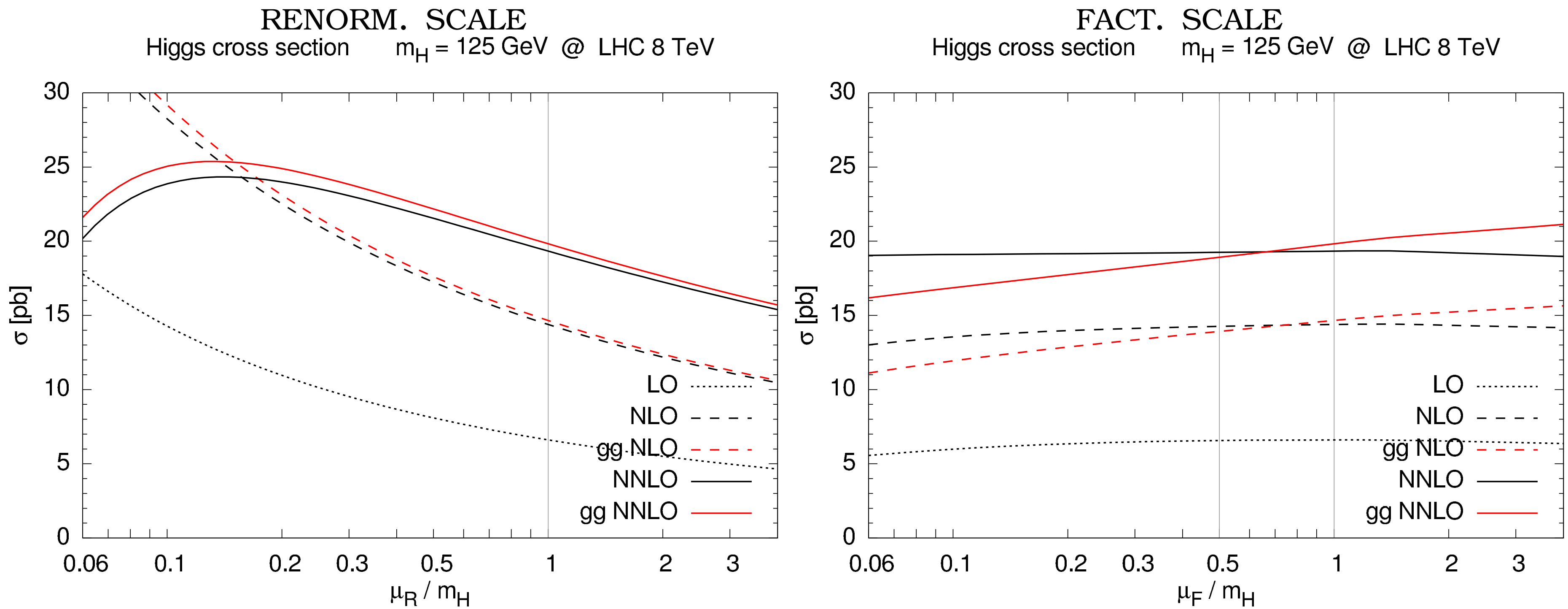
PERTURBATIVE (IN)STABILITY

$\mu_R = m_H/2$

Higgs cross section m_H = 125 GeV @ LHC 8 TeV

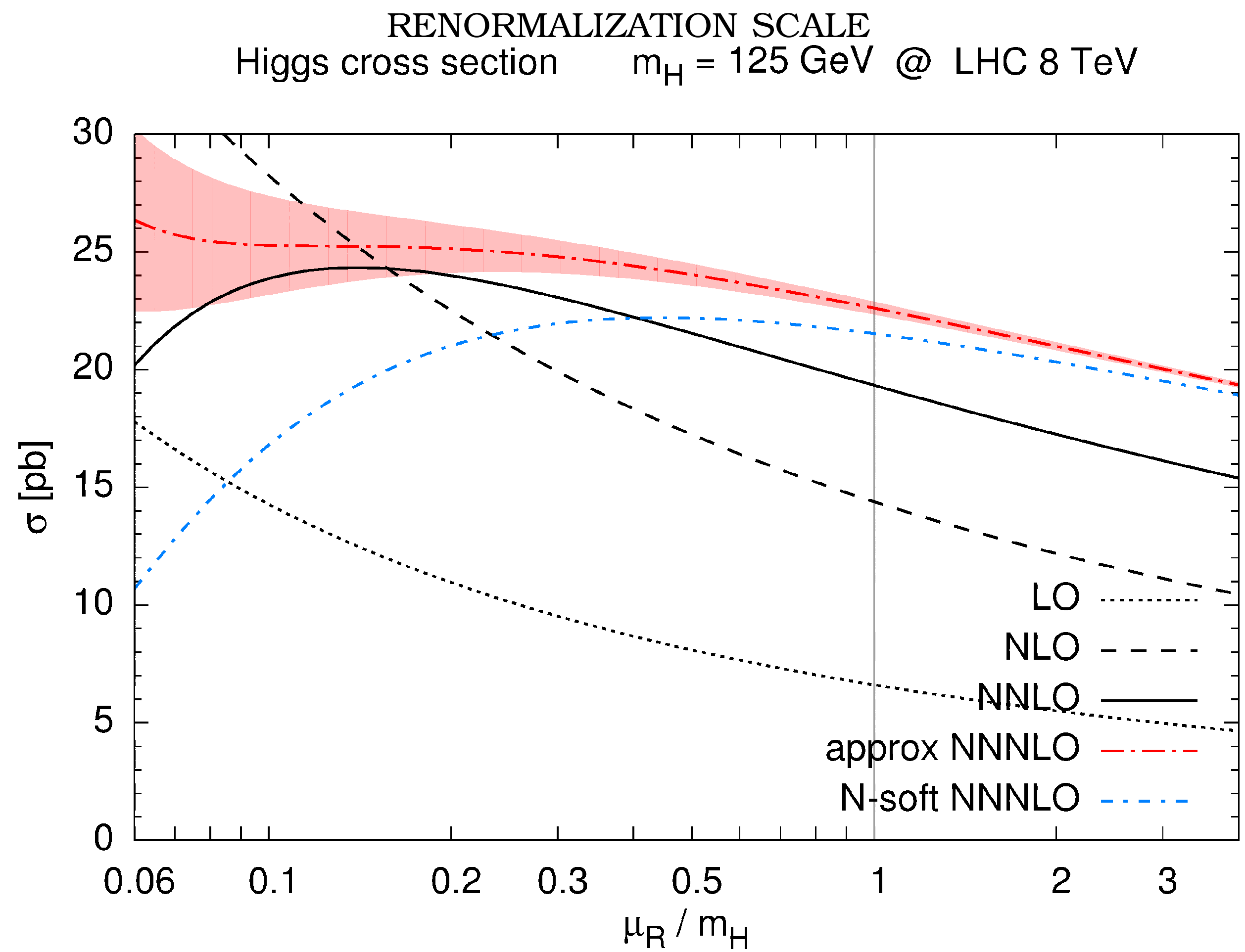


SCALE DEPENDENCE
LO, NLO AND NNLO



- RENORMALIZATION SCALE STRONG EVEN AT NNLO
- FACTORIZATION SCALE WEAK EVEN AT LO
  SIZABLE CANCELLATION BETWEEN PARTONIC CHANNELS

THE N^3LO PREDICTION
SCALE DEPENDENCE



- SCALE DEPENDENCE SIGNIFICANTLY FLATTENED BY N^3LO CORRECTION
- N-SOFT (RESUMMED RESULT) HAS SIMILARLY WEAK SCALE DEP. FOR HIGH SCALES, BUT STRONG SCALE DEP. FOR LOW SCALES
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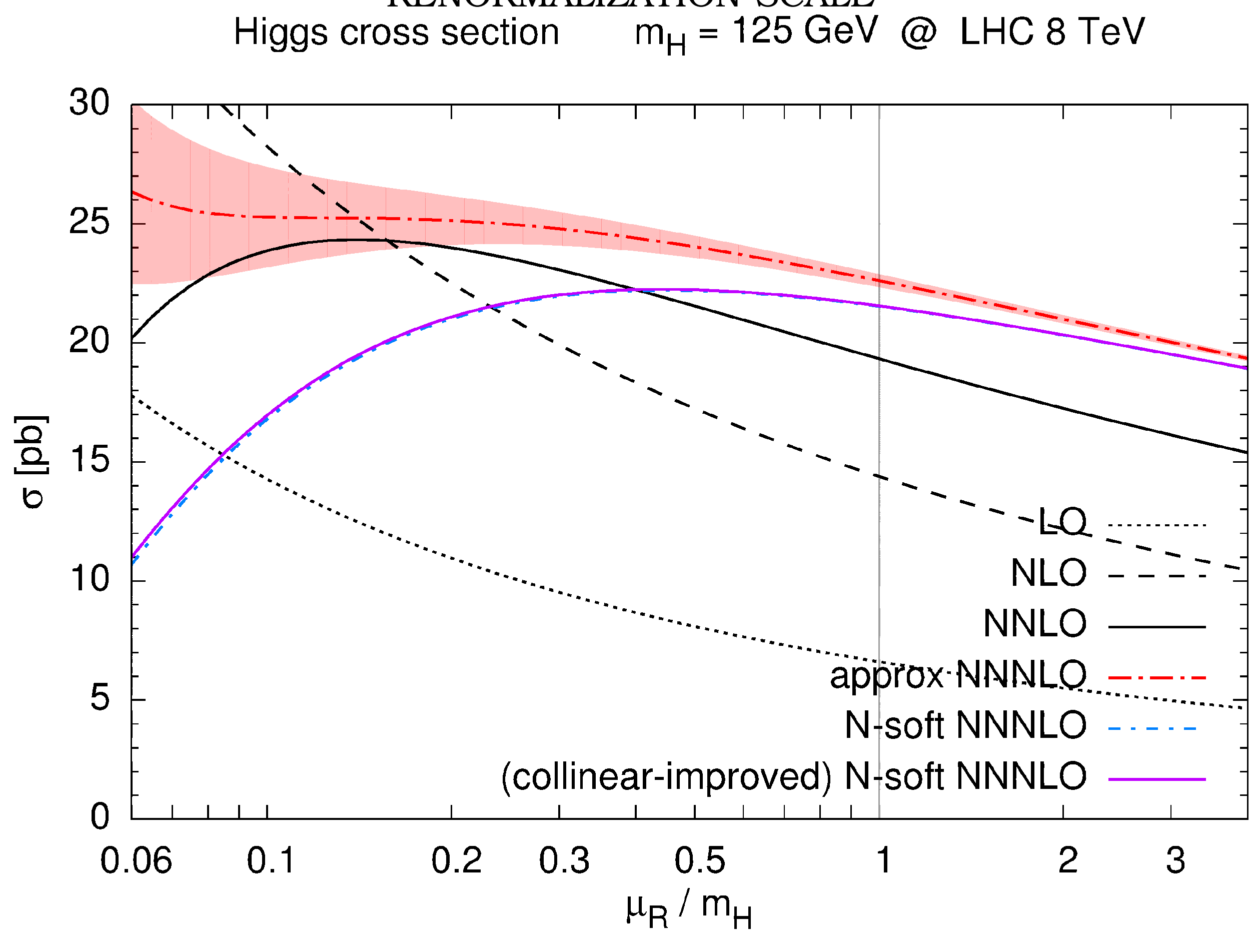
- .

THE N^{3}LO PREDICTION
SCALE DEPENDENCE

RENORMALIZATION SCALE



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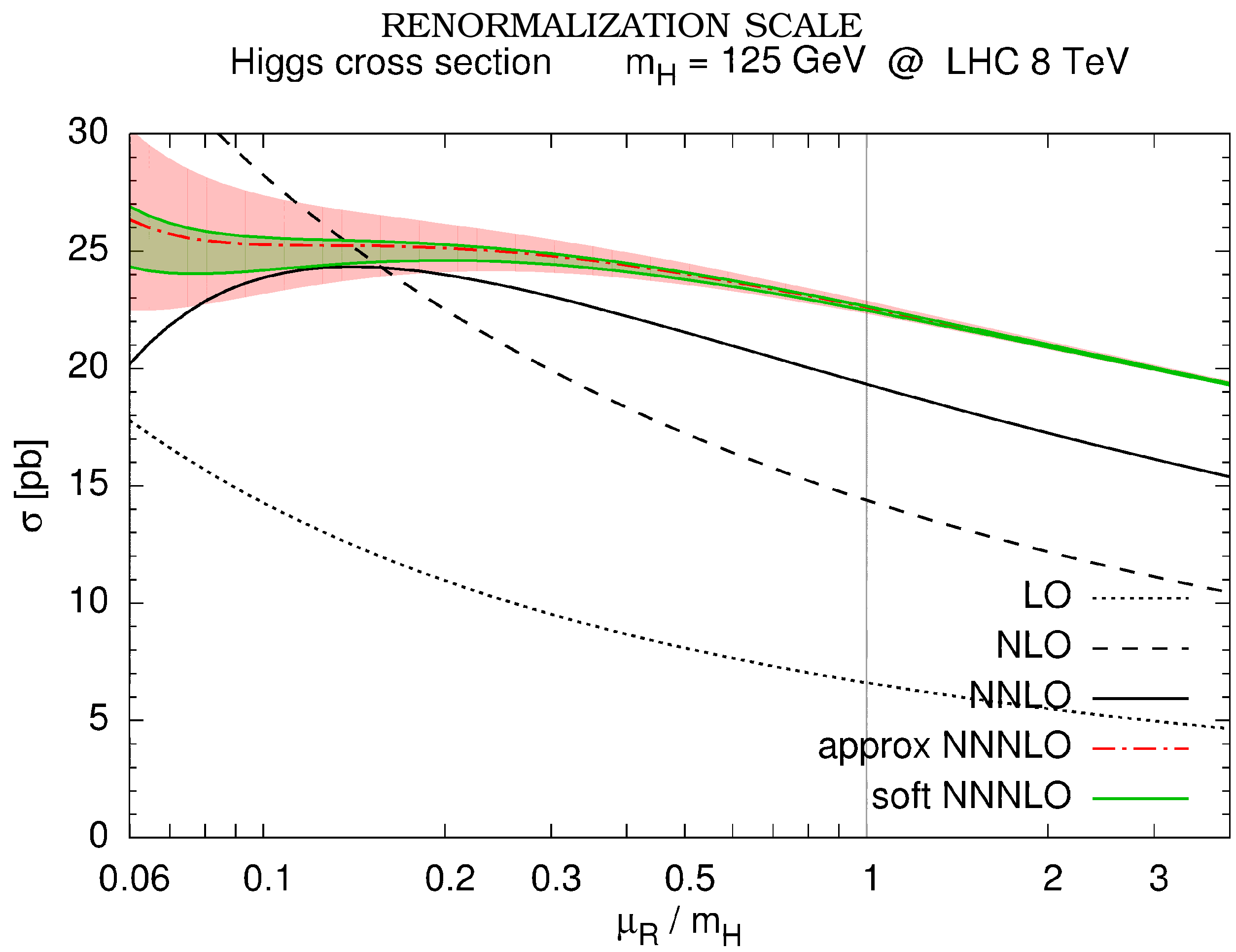
THE N^{3}LO PREDICTION
SCALE DEPENDENCE



- SCALE DEPENDENCE SIGNIFICANTLY FLATTENED BY N^{3}LO CORRECTION
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- PREVIOUS COLLINEAR IMPROVEMENTS (Catani, de Florian, Grazzini, Nason, 2003) INEFFECTIVE
- SMALL x TERMS STABILIZE SCALE DEP. @ VERY LOW $\mu_R \lesssim 0.2$

OUTLOOK

- CAN WE DO RAPIDITY DISTRIBUTIONS?
- MIGHT OTHER PROCESSES BE OF SOME INTEREST (DRELL-YAN)?
- HOW WELL DOES IT WORK?

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WE'LL SOON KNOW!