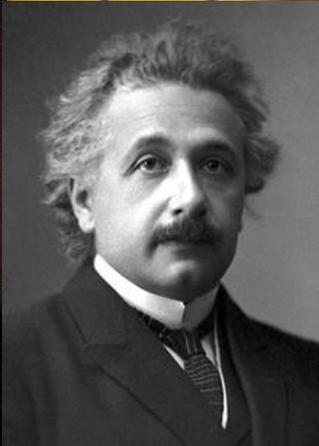
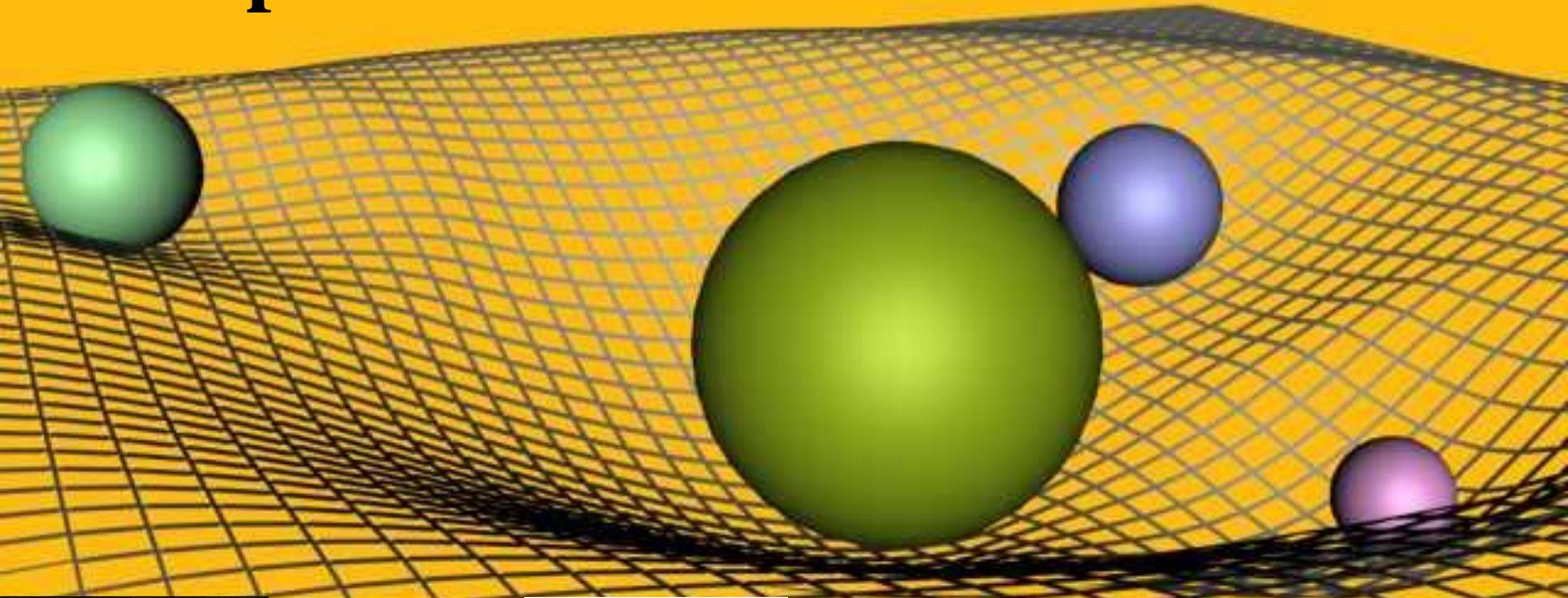


Universal Gravity: A History of Surprises



Steven Balbus
Savilian Professor of Astronomy
The University of Oxford
Inaugural Lecture
15 June 2013

The Languages of Gravity

Gravity, today is spoken in 3 languages:

1) Laplacian (Potential theory), $\mathbf{g} = -\nabla\Phi$

2) Riemannian (space-time manifolds), and

3) Feynmanian (spin 2 gravitons, propagators).

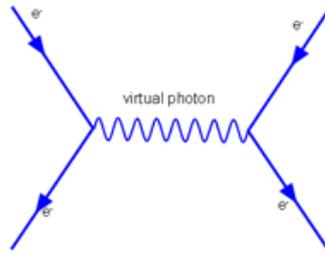
We understand how 1 and 2 combine, but not yet 3.

Three languages for gravity:

$$\Phi = -G \int \frac{\rho(\mathbf{r}') d^3 r'}{|\mathbf{r} - \mathbf{r}'|} \quad (\text{Isaac Newton})$$

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R = -8\pi G T_{\mu\nu} \quad (\text{Albert Einstein})$$

$$W = -\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} T^{*\mu\nu} \frac{(g_{\mu\lambda} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\lambda}) - g_{\mu\nu} g_{\lambda\sigma}}{k^2 + i\epsilon} T^{\lambda\sigma}$$

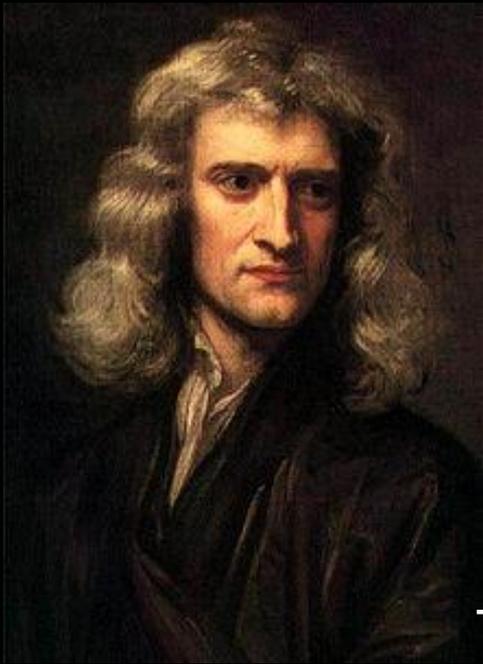


(Richard Feynman)

Electromagnetism also comes in
3 analogous languages:

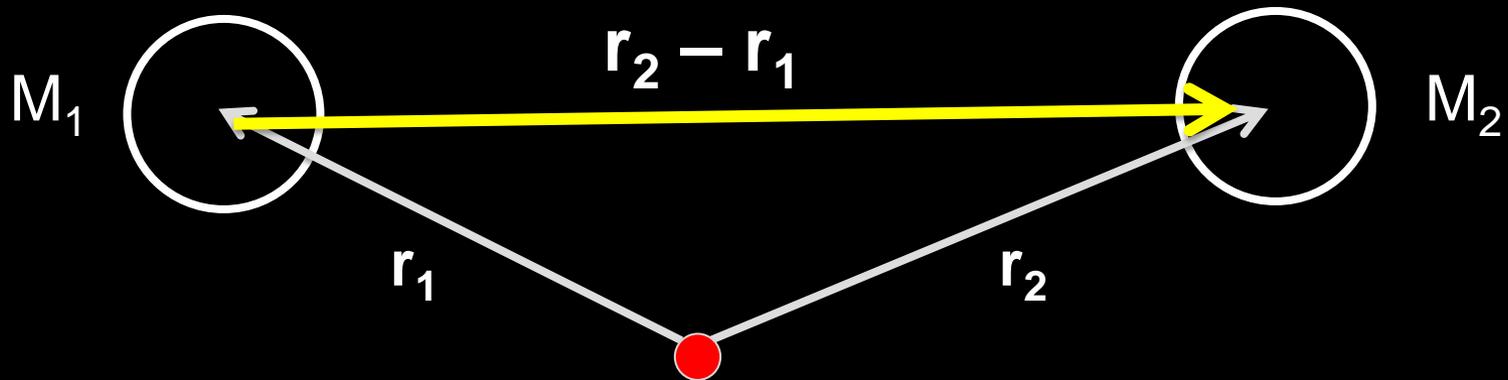
- 1) Coulombian (Electrostatic Potential Theory),
- 2) Maxwellian (Classical Field Theory), and
- 3) Feynmanian (photons and propagators).

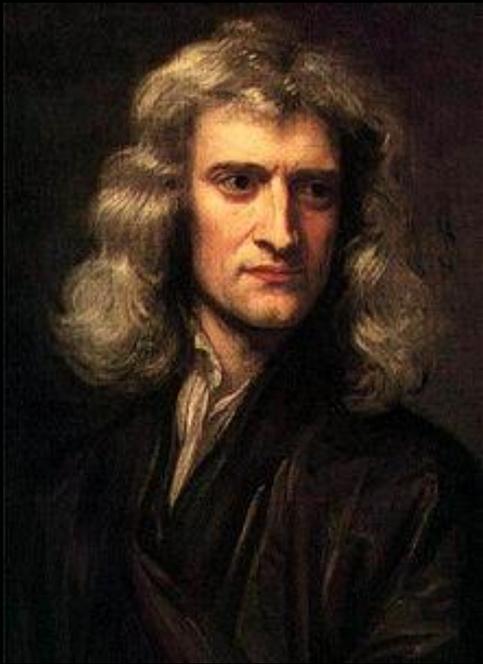
Here, we do understand how 1, 2, and 3
combine fairly seamlessly.



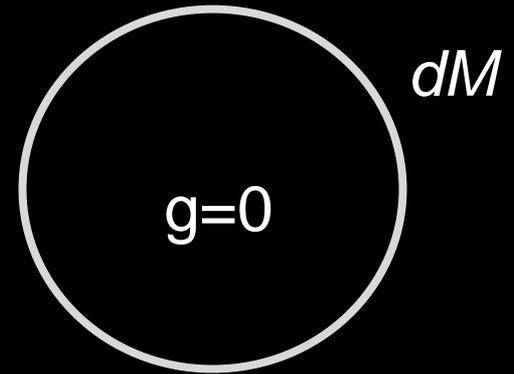
Newtonian Theory

$$\mathbf{F}_{21} = -\frac{GM_1M_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1)$$



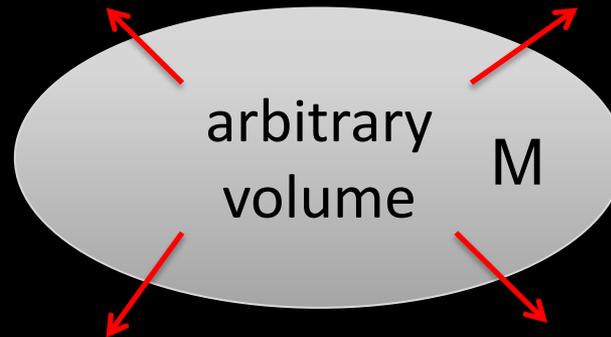


Gauss's Law:



$$\int \mathbf{g} \cdot d\mathbf{S} = -4\pi GM \longrightarrow \text{g inside hollow sphere} = 0$$

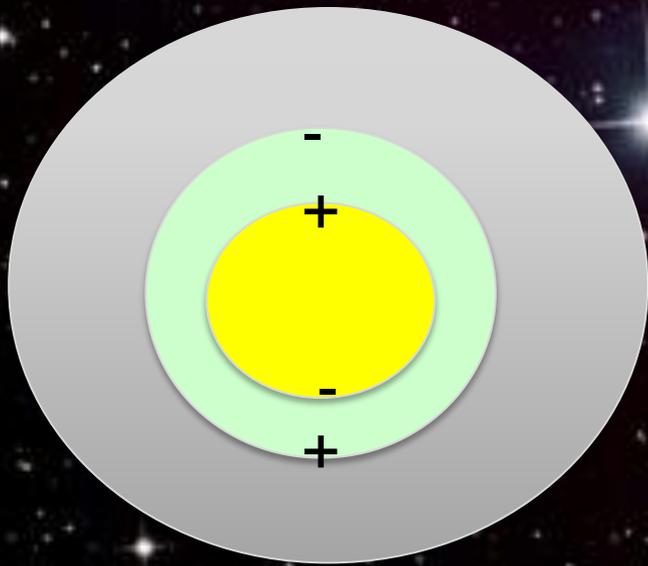
$$\nabla^2 \Phi = 4\pi G \rho$$



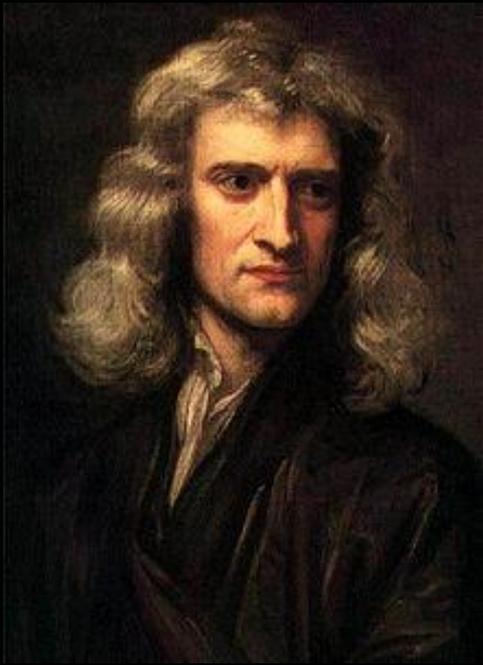
Perfect sphere. Mass M . External gravitational field is exactly GM/r^2 and exactly radial. Must we conclude that the density inside is a function of r only, $\rho = \rho(r)$?



NO. A superposition of two shells, one denser on the north and less dense on the south; the other shell farther out with reversed polarity could superpose to give an exactly spherical exterior field.



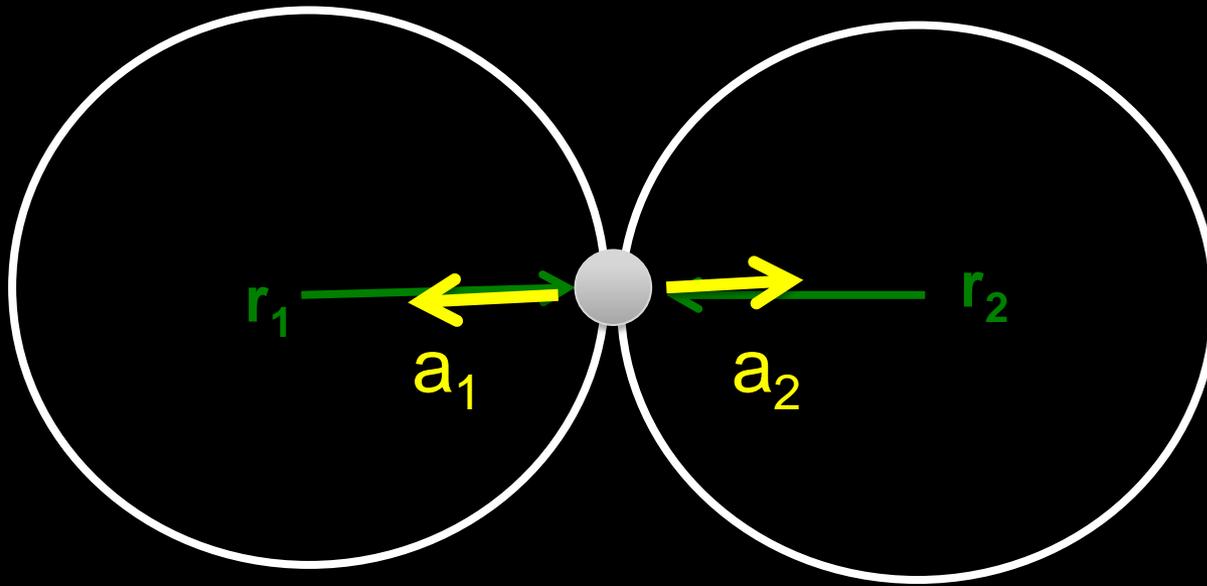
Gravity on Large Scales



“[If] matter were evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered great distances from one to another throughout all that infinite space.”

--- Isaac Newton, 1692

The Problem in a homogeneous, Euclidian static Universe. $\nabla^2\Phi = 4\pi G\rho$ $\Phi = \frac{2\pi}{3}G\rho r^2$

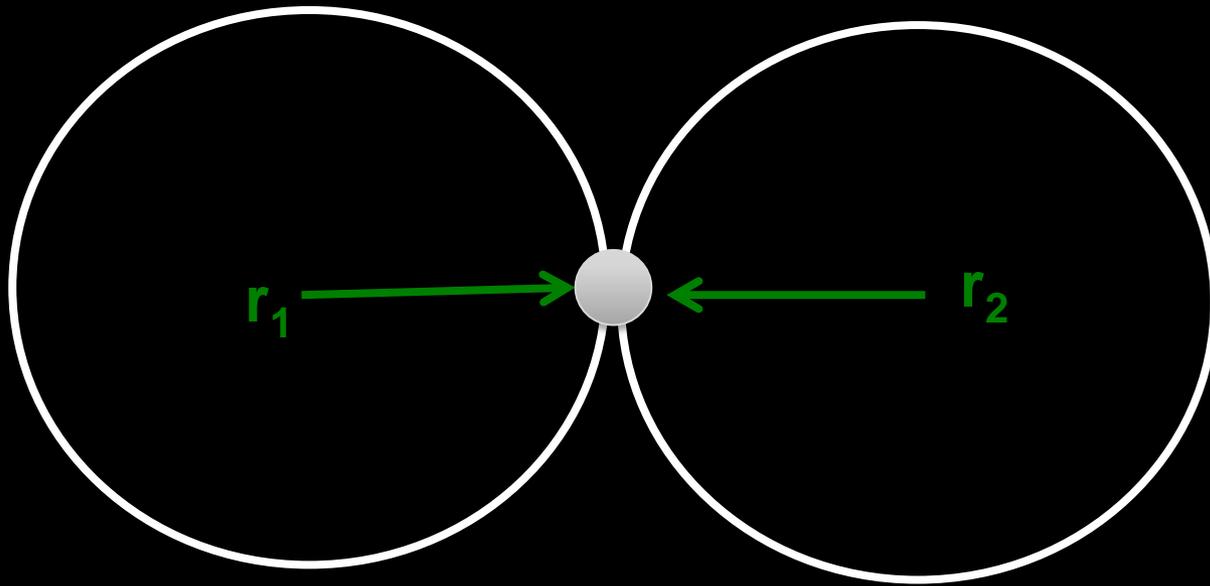


$$\mathbf{a}_1 = -\frac{4\pi G\rho}{3}\mathbf{r}_1$$

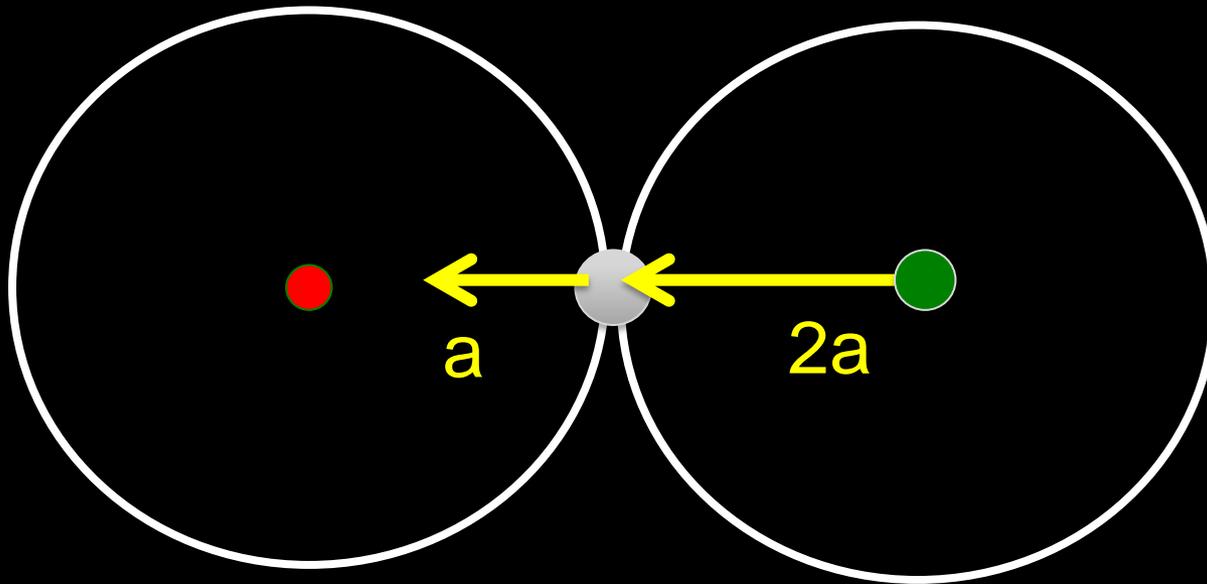
$$\mathbf{a}_2 = -\frac{4\pi G\rho}{3}\mathbf{r}_2$$

$$\mathbf{a}_1 \neq \mathbf{a}_2$$

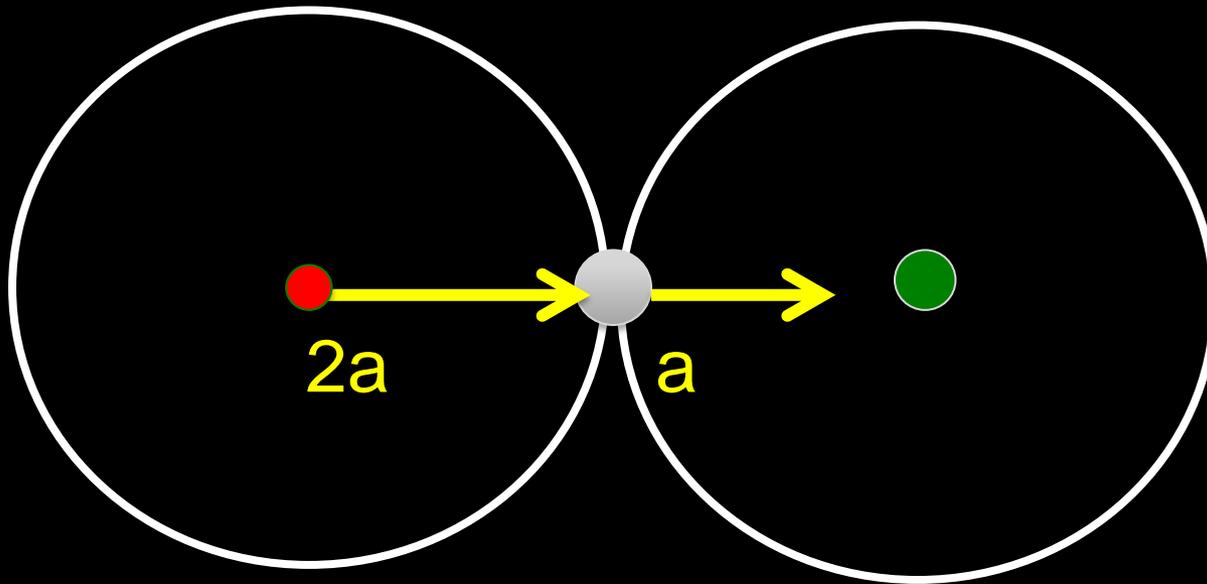
... is resolved in a Euclidian expanding
Universe:



Point of view of the red observer:



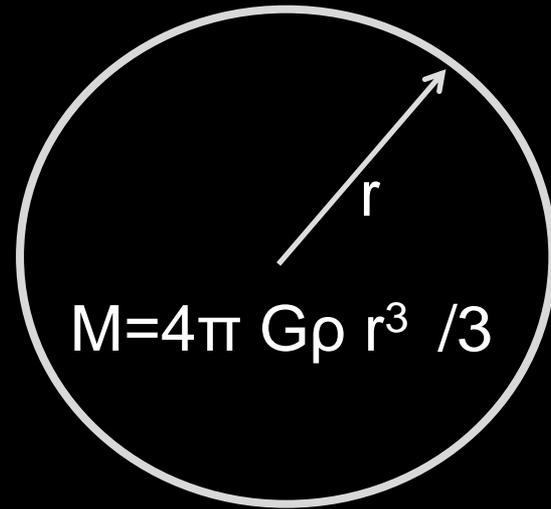
Point of view of the green observer:



The flip in direction of the white force is now a consequence of going from one accelerating frame to another!

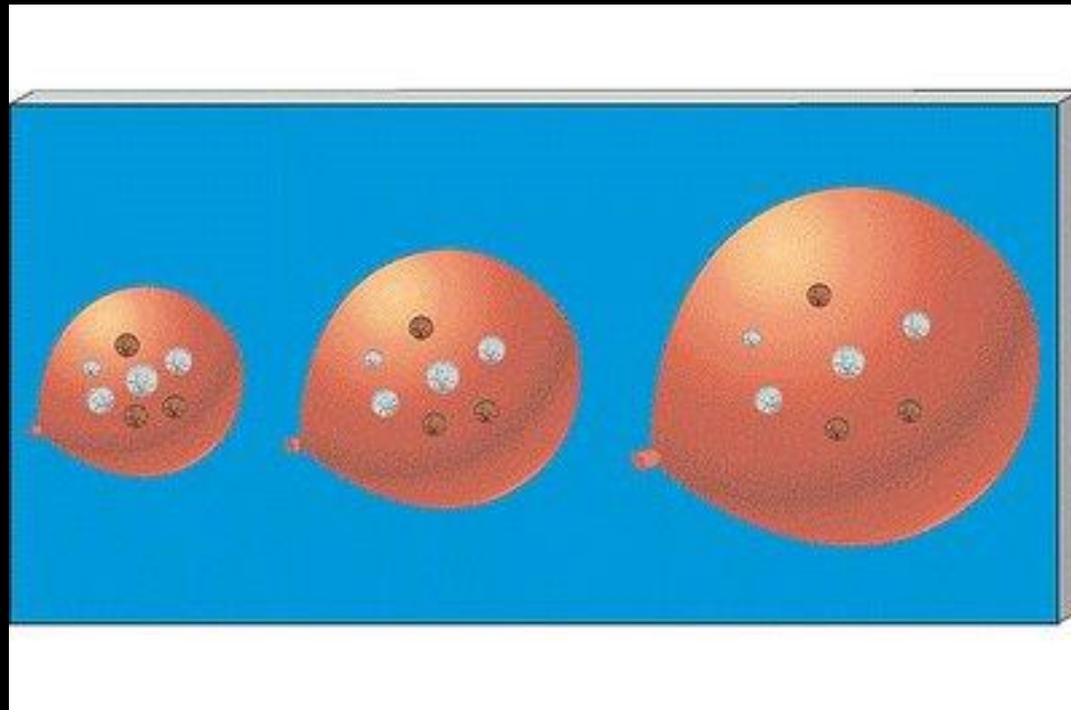
$$r = R(t) s$$

$$\ddot{r} = -\frac{4\pi G \rho r}{3}$$



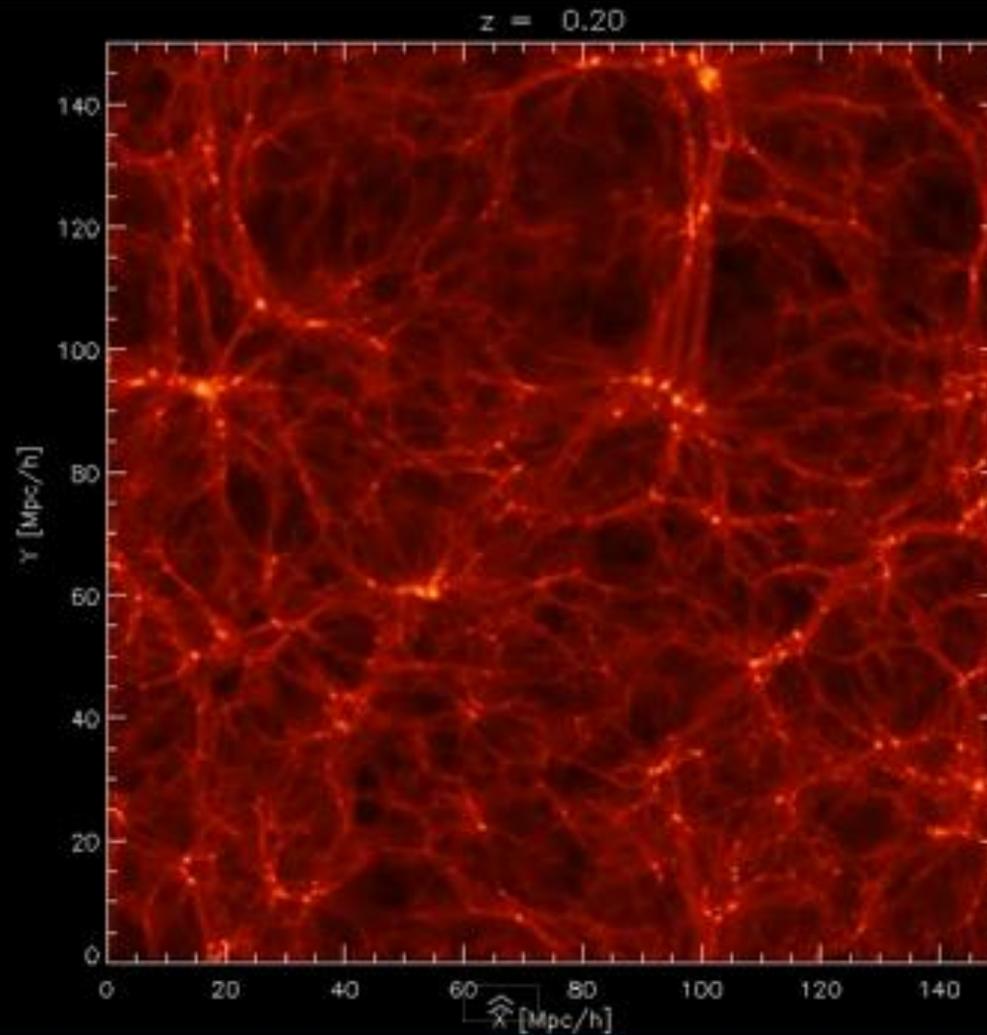
$$\dot{R}^2 - \frac{8\pi G \rho R^2}{3} = 2E$$

This cosmological equation is independent of comoving coordinate s , and much more general than Newtonian, as it happens.



When the Universe grows by a factor R , the density goes down by a factor of $1/R^3$. An embedded “growing” disturbance does not, in fact, initially grow. Density goes down, but only by a factor of $1/R^2$. Then, it collapses.

Grecco Simulation:
1.34 E+08 particles



Gravitational Timescales

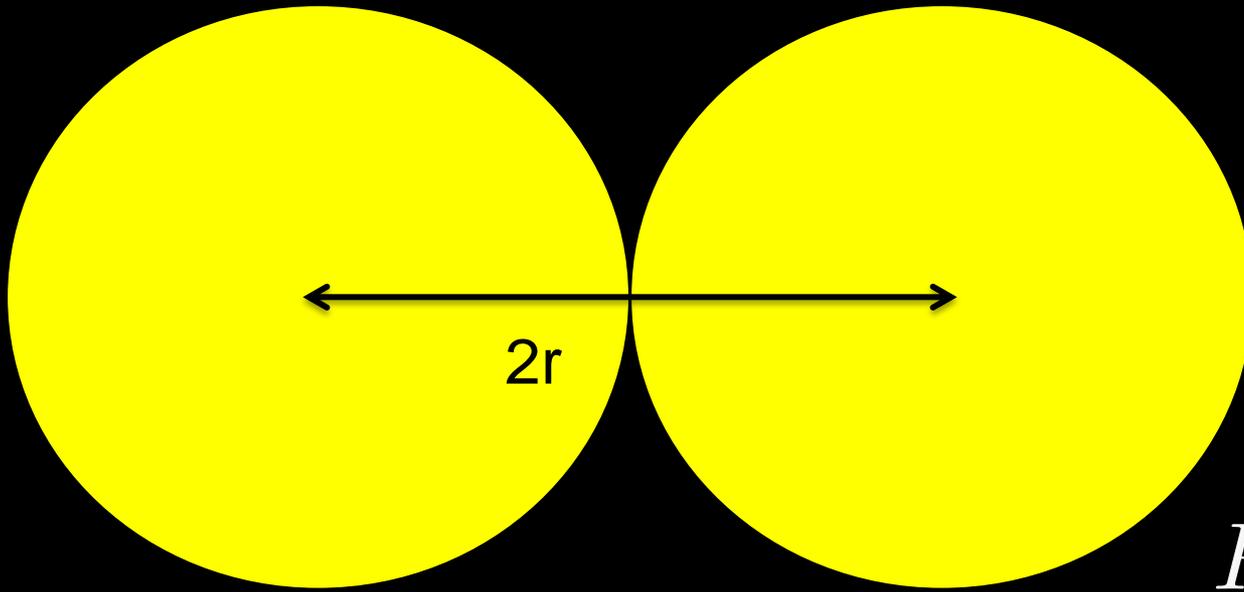
The time for gravitational processes is always $1/\sqrt{G\rho}$, where ρ is the average density spread out through the system. This is interesting, because it does not depend explicitly on the *size*, and all stars and solids have a density $\sim 1 \text{ g cm}^{-3}$.



$$P = \sqrt{\frac{3\pi}{G\rho}}$$

$$\rho = 5.52 \text{ g cm}^{-3}$$

$$P = 1 \text{ h } 24 \text{ m}$$



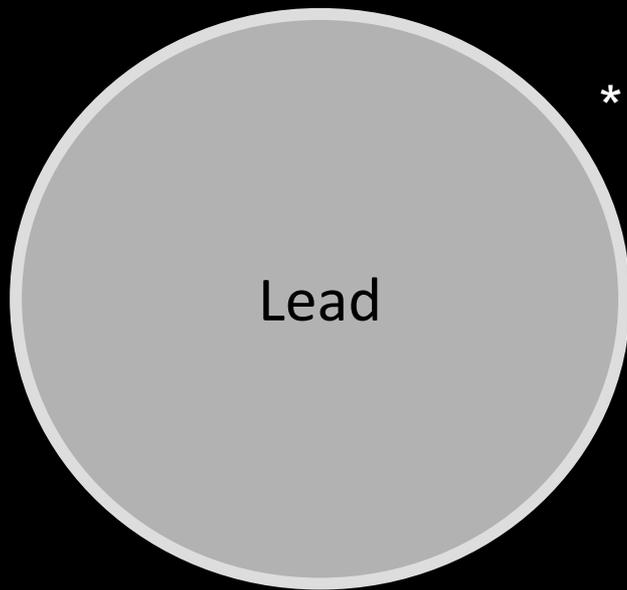
Two stars similar to the sun in a contact orbit (neglects tidal effects!).

$$P = 2\sqrt{\frac{3\pi}{G\rho}}$$

$$\rho = 1.4 \text{ g cm}^{-3}$$

$$P = 5\text{h } 35 \text{ m}$$

A lead sphere with a tiny pebble orbiting at 1% of the radius above the sphere's surface would be an ideal gravitational hour-minder! The orbital period is precisely one hour.



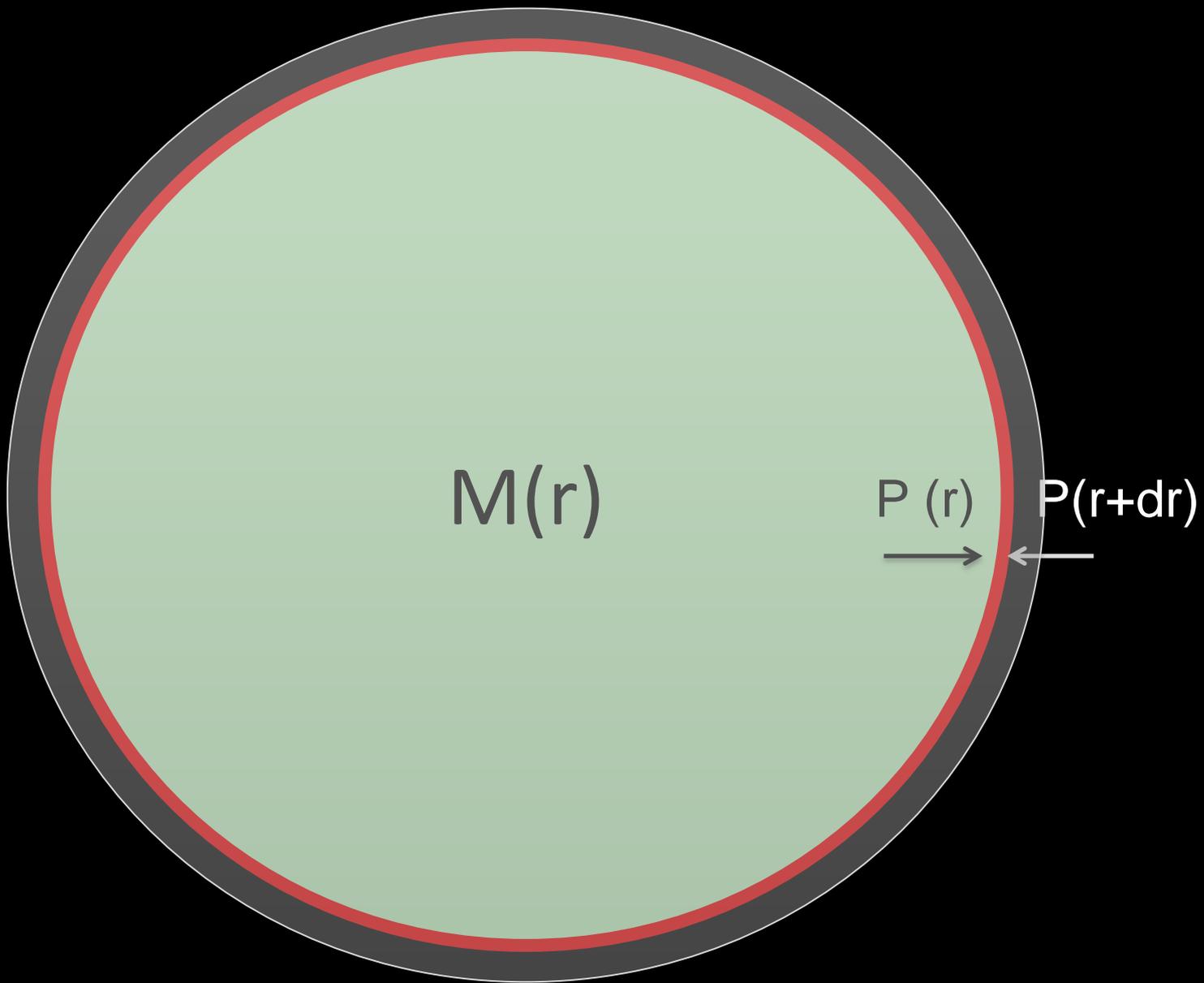
$$\rho_{avg} = \frac{3\pi}{G}$$

$$G = 0.865 \text{ cm}^3 \text{ g}^{-1} \text{ hr}^{-2}$$

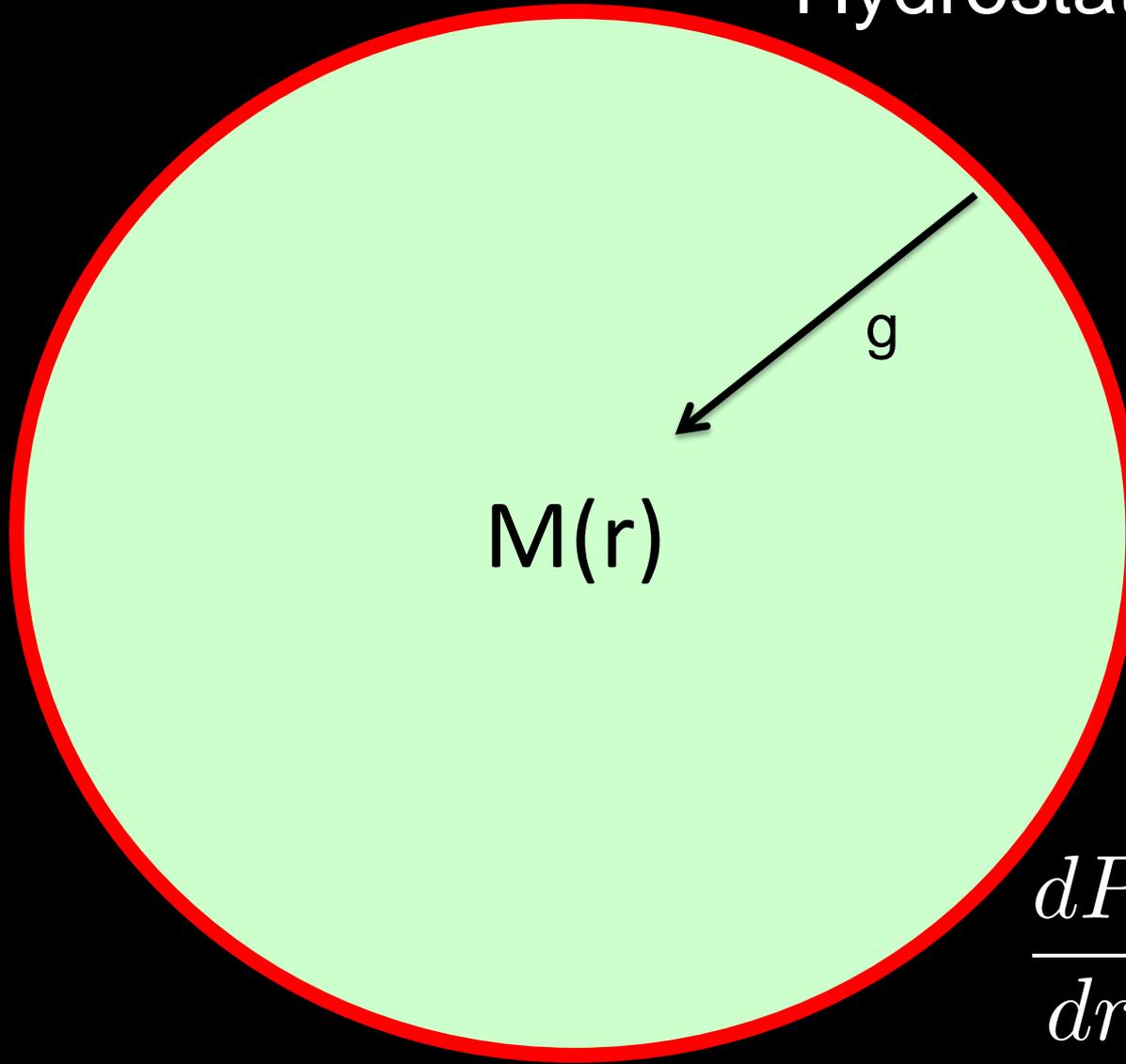
$$\rho_{avg} = 10.9 \text{ g cm}^{-3}$$

$$\approx \rho_{lead} = 11.3$$

Gravitational Equilibrium



Hydrostatic Equilibrium



$$P(r) - P(r+dr) = -dP = g \rho dr$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

$$\nabla P = -\rho \nabla \Phi$$

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P \right) = -\nabla^2 \Phi = -4\pi G \rho$$

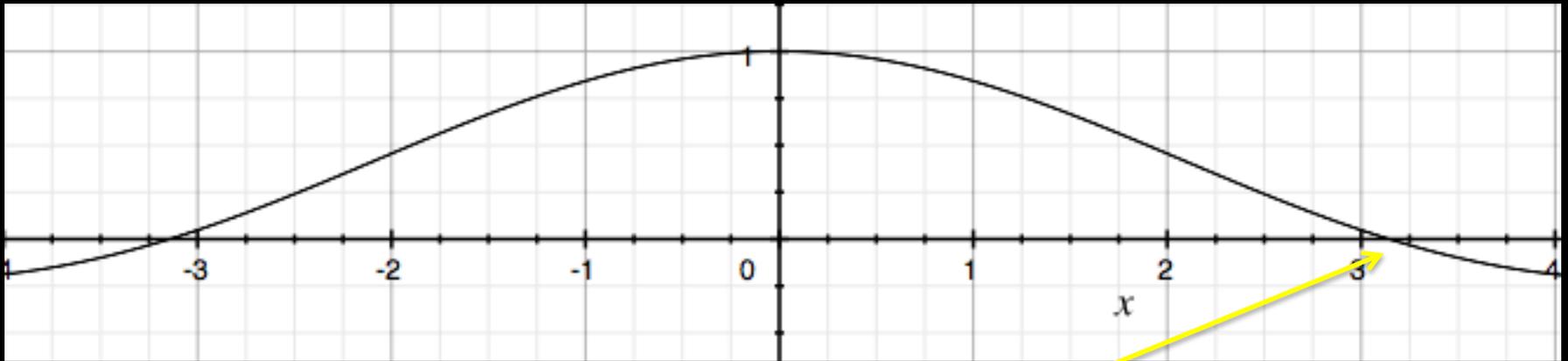
Hence, if $P = P(\rho)$ is given, we have a single (partial) differential equation. It is interesting to consider $P = K\rho^2$:

$$\nabla^2 \rho = -\frac{2\pi G \rho}{K}$$

This, we can solve analytically....

$$\rho = \rho_c \frac{\sin(kr)}{kr}$$

$$k^2 = \frac{2\pi G}{K}$$



radius of star given by $kr = \pi$

But why not solve

$$\nabla^2 \rho = -\frac{2\pi G \rho}{K}$$

in Cartesian coordinates? Then:

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2} = -\frac{2\pi G \rho}{K}$$

$$\rho = \rho_c \cos(k_x x) \cos(k_y y) \cos(k_z z)$$

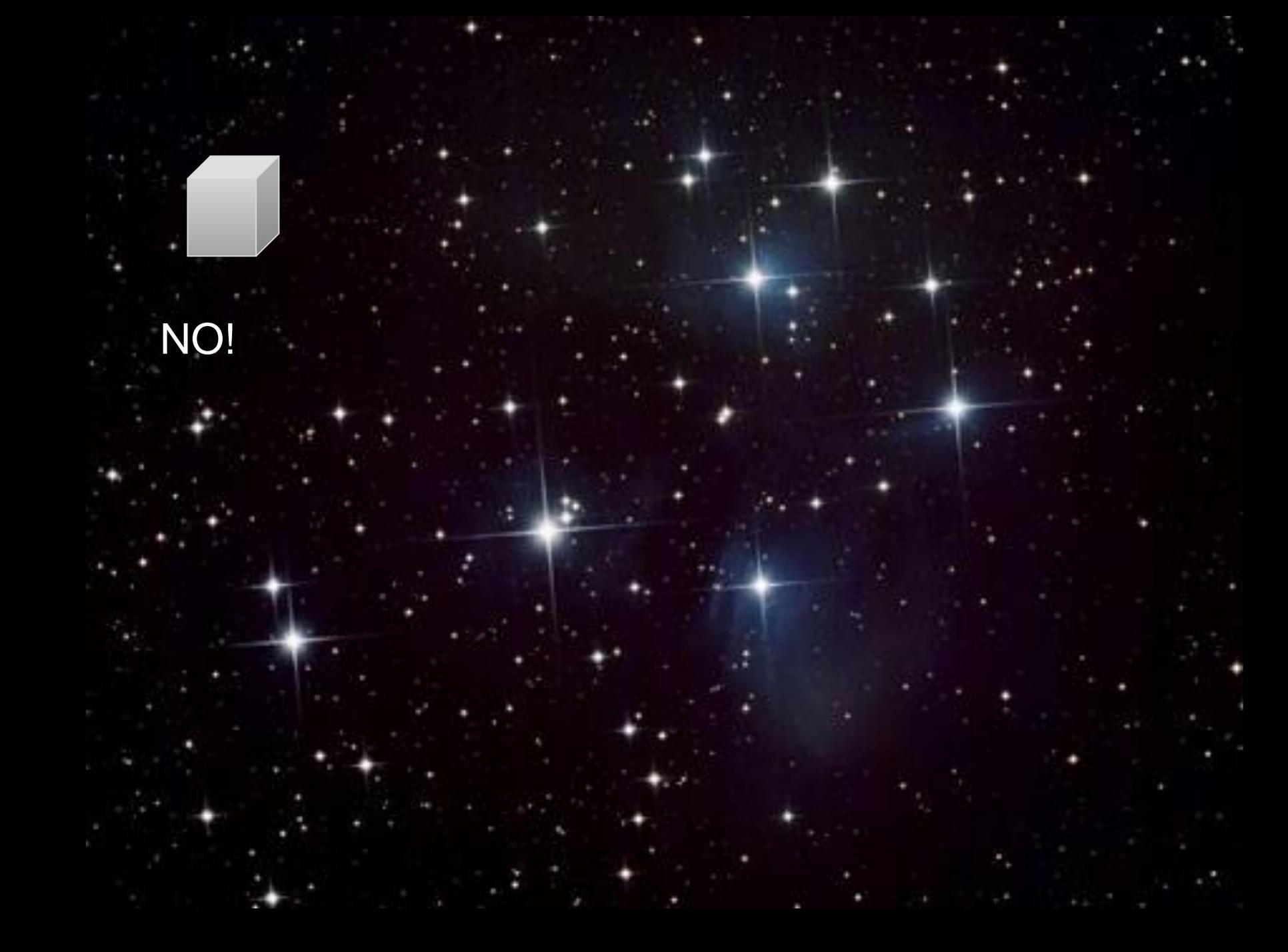
$$k_x^2 + k_y^2 + k_z^2 = \frac{2\pi G}{K}$$

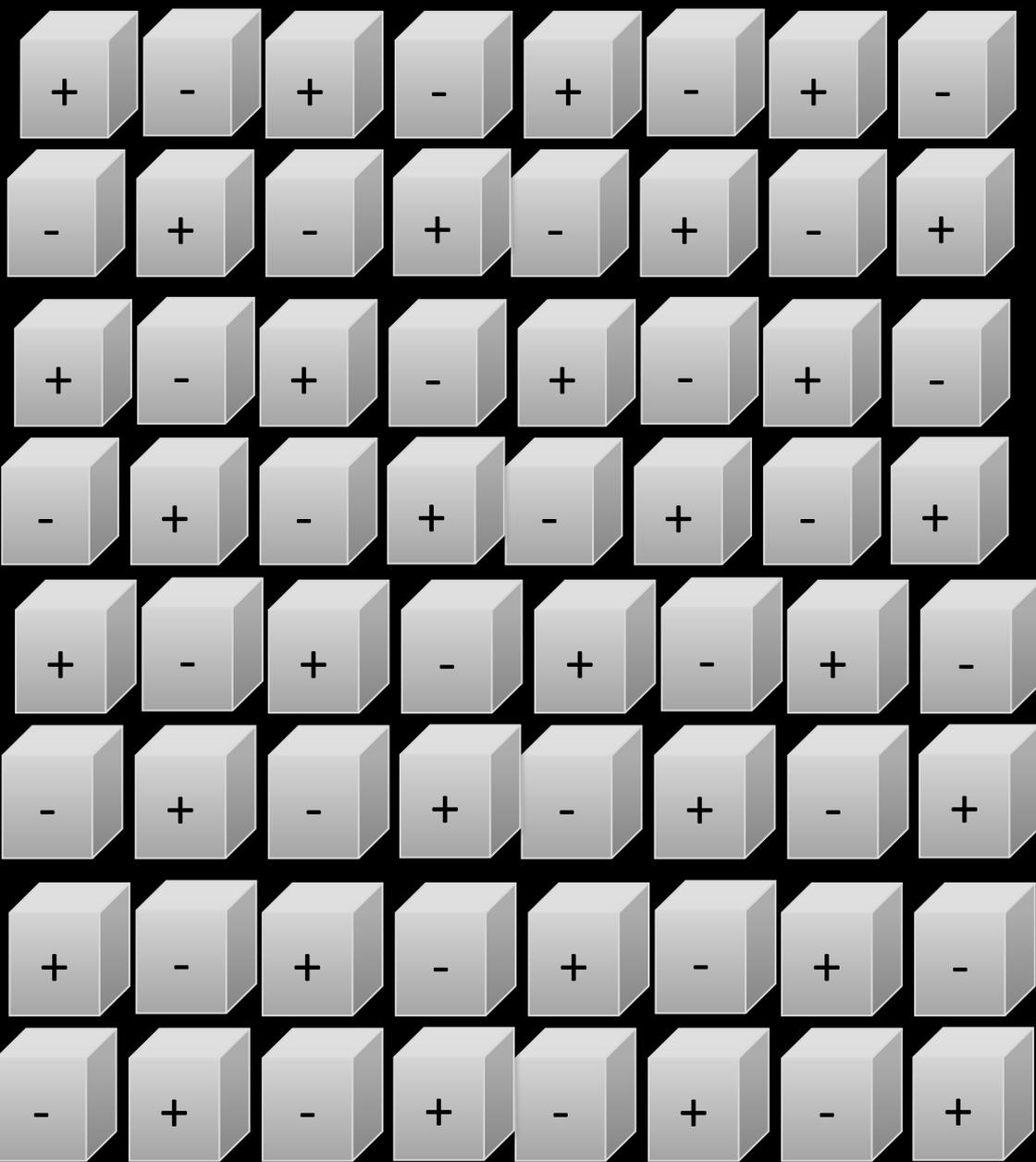


A cubic star???



NO!



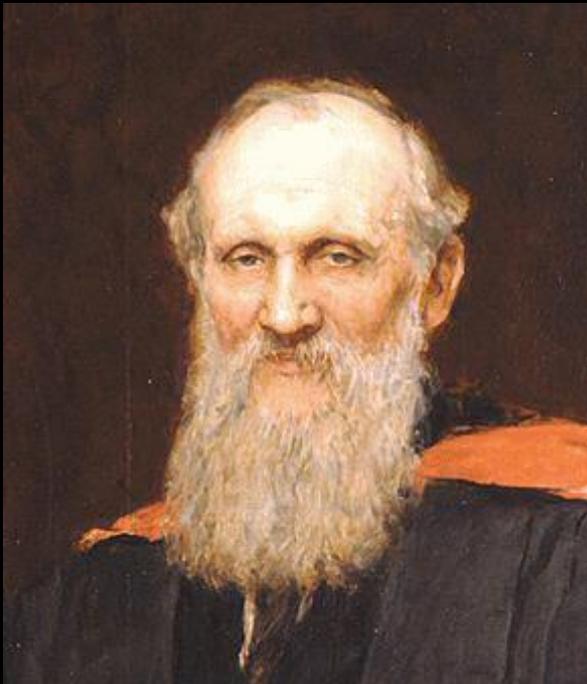


The real solution!

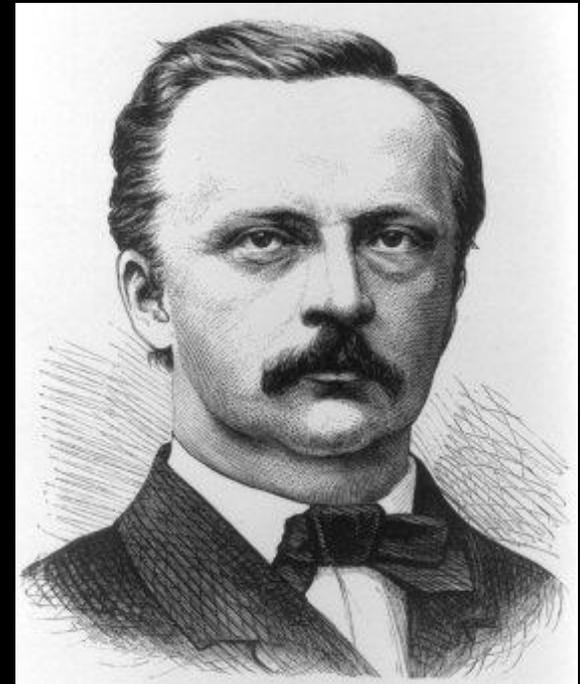
$$\rho = \rho_c \cos(k_x x) \cos(k_y y) \cos(k_z z)$$

Stars.

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$



Kelvin



von Helmholtz

The Virial Theorem

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

$$\int_0^R 4\pi r^3 \times \frac{dP}{dr} dr = - \int_0^R \frac{GM(r)\rho}{r^2} \times 4\pi r^3 dr$$

$$-3 \int_0^R 4\pi r^2 \times P dr = - \int_0^R \frac{GM(r)\rho}{r} \times 4\pi r^2 dr$$

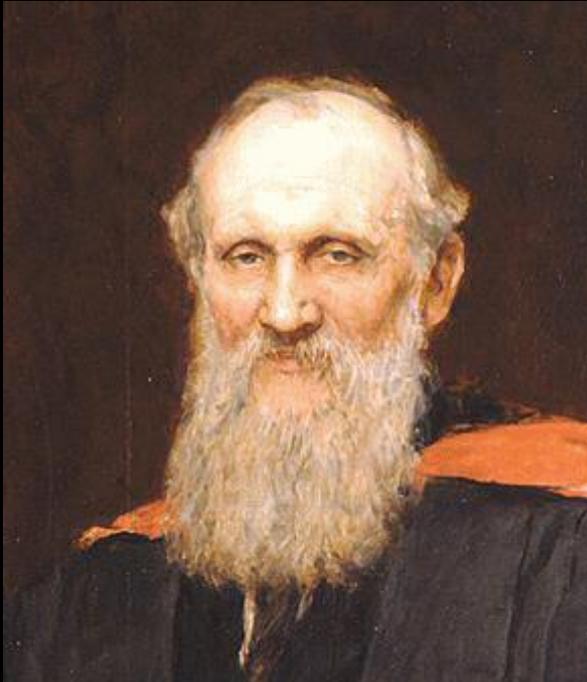
$$-3 \int P dV = -2E_{thermal} = \Phi$$

Classic Kelvin-Helmholtz theory.

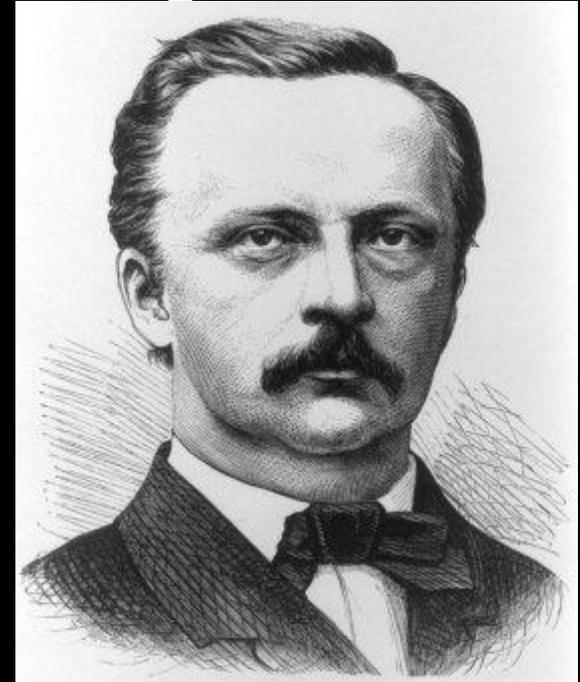
$$E(\text{thermal}) = -\Phi/2 \sim 0.5 GM^2/R$$

$$E(\text{total}) = +\Phi/2 \sim -0.5 GM^2/R$$

As a star loses heat, it contracts & gets hotter!

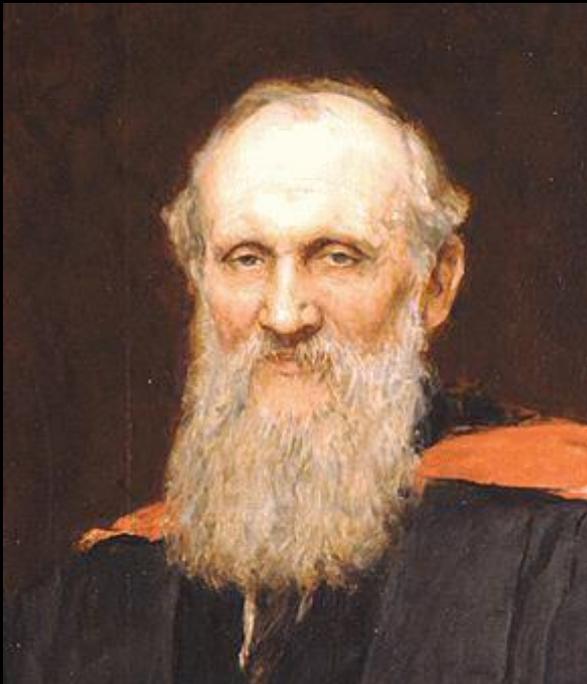


Kelvin

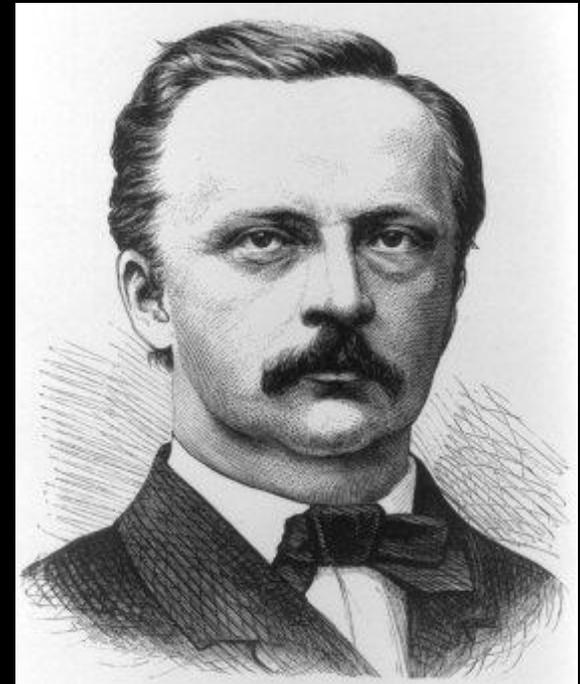


von Helmholtz

In the models of Kelvin and Helmholtz, gravity was everything: it squeezed the gas, heated it, and *let half the energy leak out as light.*

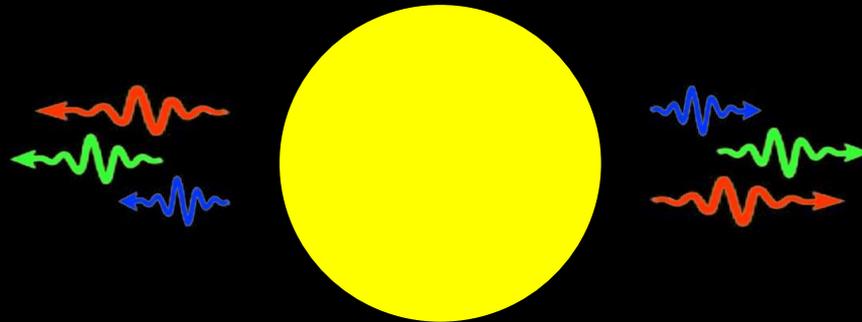


Kelvin



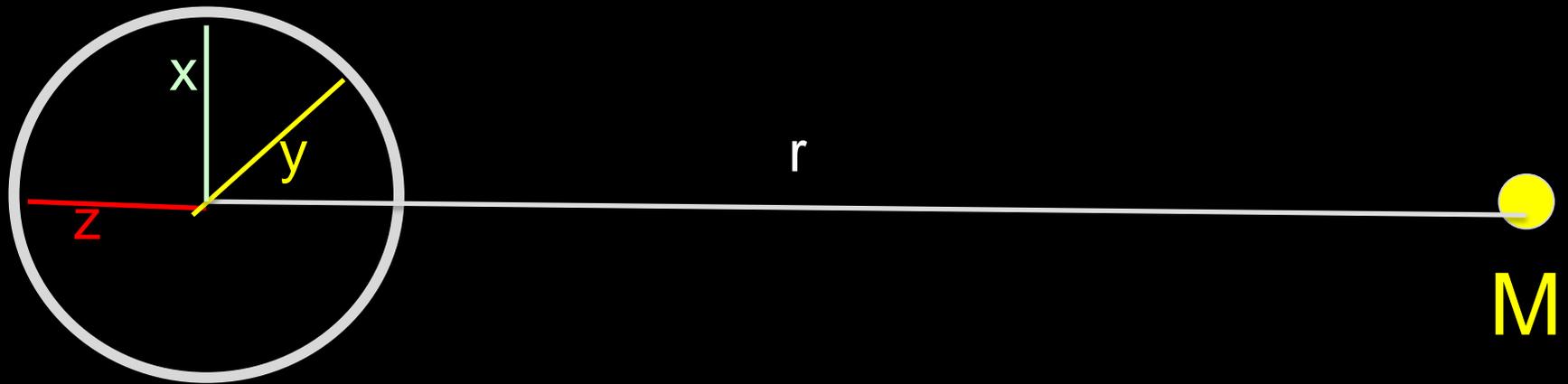
von Helmholtz

The source of a star's energy is the contraction itself (work and release of PE), and with half of the liberated energy released as heat, the star both shines and stays at the “just right” point!



One small problem: the lifetime of a star like the sun would be only $\sim 10^7$ years. Indeed, Kelvin used this together with cooling of the Earth to argue explicitly against the geological time scales and thus indirectly against Darwinian evolution: the work was sharply criticised by TH Huxley.

Gravitational Tidal Forces

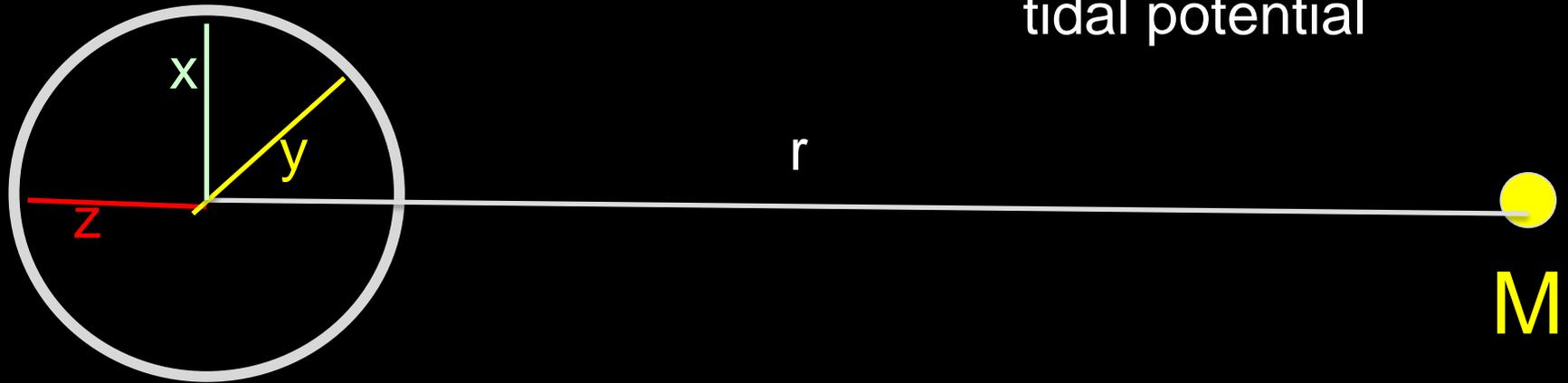


$$\Phi = -\frac{GM}{[(r+z)^2 + x^2 + y^2]^{1/2}}$$

x, y, z are all $\ll r$

$$\Phi = -\frac{GM}{r} + \frac{GMz}{r^2} - \underbrace{\frac{GM}{2r^3}(2z^2 - x^2 - y^2)}_{\text{tidal potential}}$$

tidal potential



$$F_x = -\frac{\partial\Phi}{\partial x} = -\frac{GMx}{r^3} \quad F_y = -\frac{\partial\Phi}{\partial y} = -\frac{GM y}{r^3}$$

$$F_z = -\frac{\partial\Phi}{\partial z} = -\frac{GM}{r^2} + \frac{2GMz}{r^3}$$

$$\Phi = -\frac{GM}{r} + \frac{GMz}{r^2} - \underbrace{\frac{GM}{2r^3}(2z^2 - x^2 - y^2)}_{\text{tidal potential}}$$

The tidal force is proportional to M/r^3 .
For the sun and moon, these quantities are comparable: the sun is very far away but much, much more massive! The sun's tidal amplitude is about 46% that of the moon's.

$$\Phi_{tidal} = -\frac{GM_p}{2r_p^3} (2z^2 - x^2 - y^2)$$

For the moon, say, at latitude l on the earth, x , y , and z become

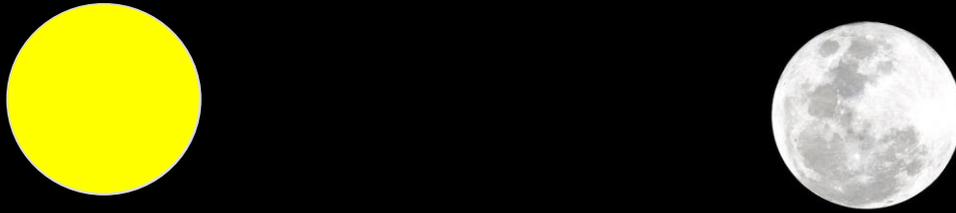
$$x = -\sin l \cos i + \sin i \cos \Omega t$$

$$y = -\sin l \sin i \sin \Omega_M t + \cos i \sin \Omega_M t \cos \Omega t - \sin \Omega t \cos \Omega_M t$$

$$z = -\sin l \sin i \cos \Omega_M t + \cos i \cos \Omega_M t \cos \Omega t + \sin \Omega_M t \sin \Omega t$$

where i is the inclination of the orbit, Ω is frequency of 1 day, and Ω_M is the orbit period.

The sun and moon have very nearly the same angular size, viewed from the Earth:



Each is about 30', half a degree.

The conventional wisdom is that this is a coincidence. I think there is a very good reason. It is tidal.

$$\frac{M_S}{r_S^3} = \frac{M_M}{r_M^3}$$

1. What if the solar and lunar tides were the same? Then...

$$\frac{\rho_S D_S^3}{r_S^3} = \frac{\rho_M D_M^3}{r_M^3}$$

2. Mass is proportional to density times volume. (D is diameter.)

$$\theta_S = \left(\frac{\rho_M}{\rho_S} \right)^{1/3} \theta_M$$

3. The angular size θ is D/r , and must therefore be almost the same for the sun and the moon.

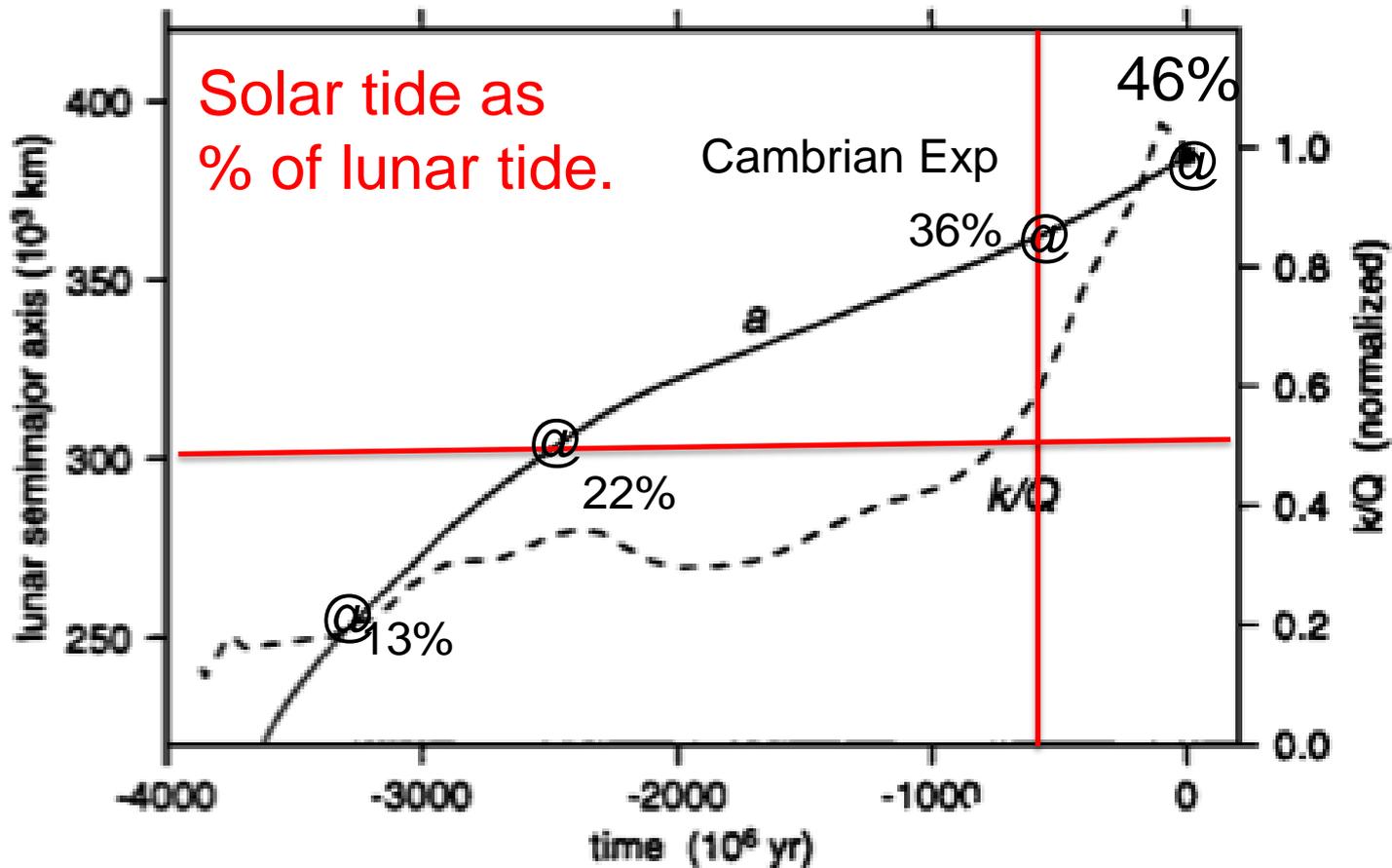
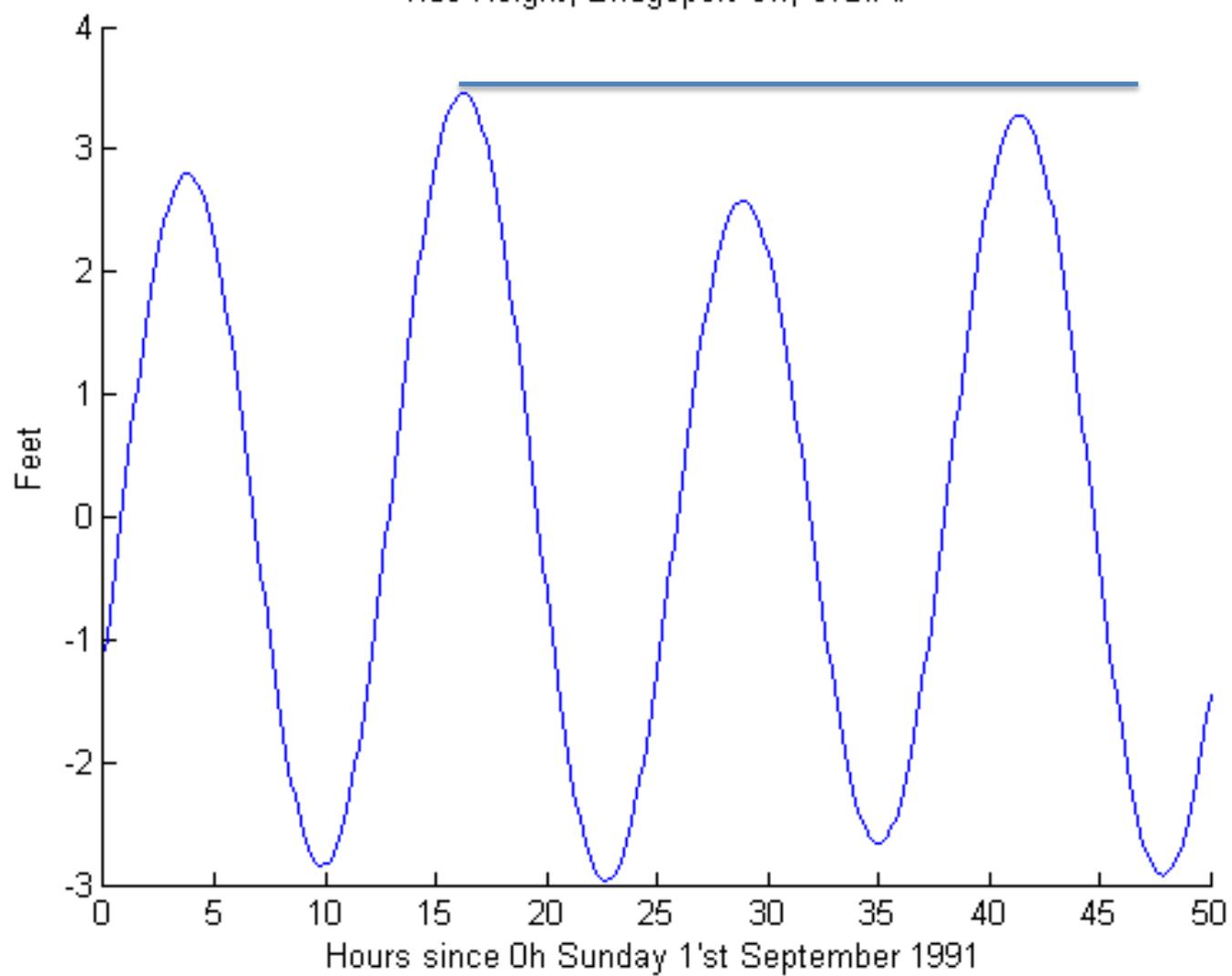


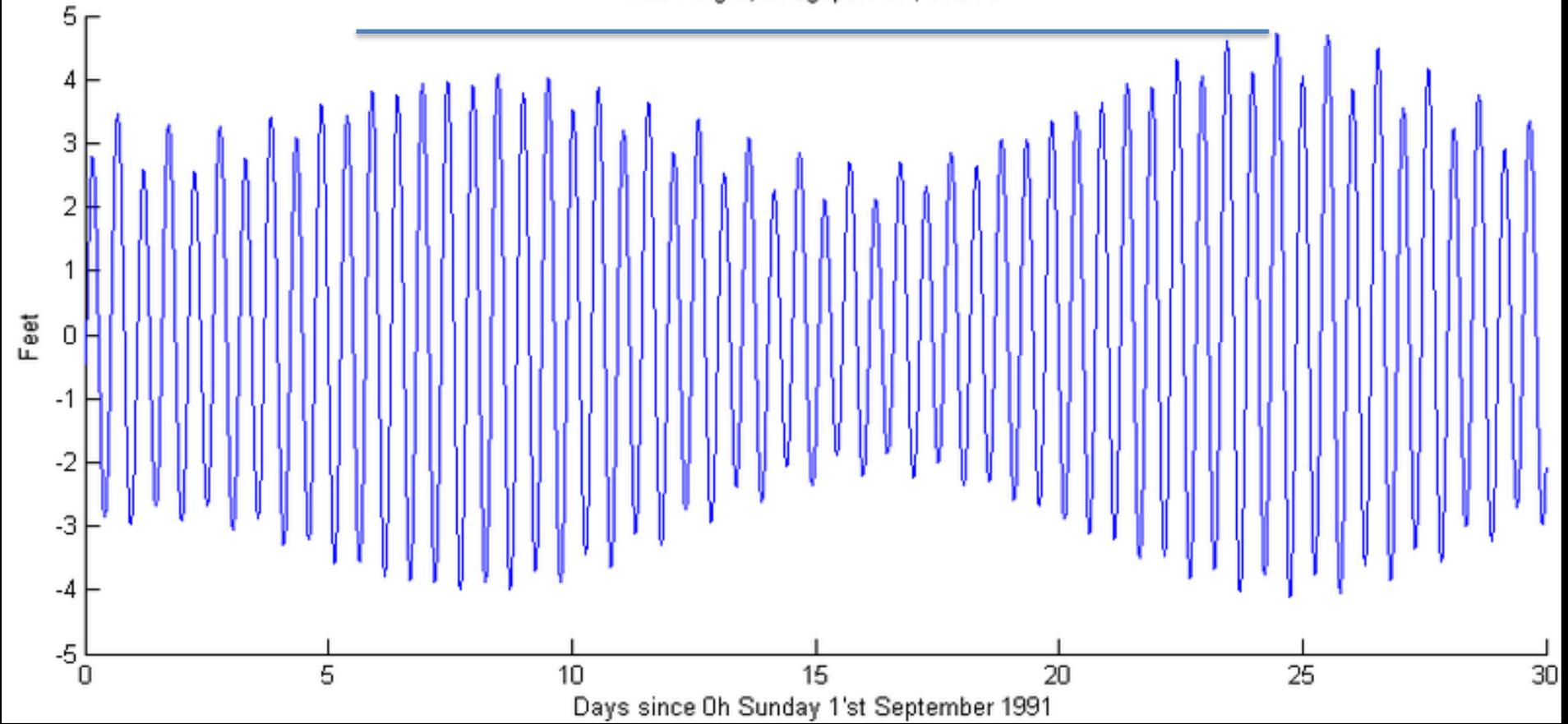
Figure 2. Orbital and tidal evolution. Solid curve is computed variation in orbital semimajor axis a according to *Webb* [1982]. Dashed curve is variation in parameter (k/Q), which has been numerically computed from *Webb's* $a(t)$ via Eqs. (1) and (2).

What if there is some kind of threshold for rapid biological evolution, when the sun's tides are about $1/3$ of the moon's? Perhaps this is related to a lack of commensurability and forming isolated tidal pools...

Tide Height, Bridgeport Cn, U.S.A.



Tide Height, Bridgeport Cn, U.S.A.



Take the Cambrian explosion solar contribution of 36% at face value:

$$\frac{M_S}{r_S^3} = \frac{0.36 M_M}{r_M^3}$$

$$\frac{\rho_S D_S^3}{r_S^3} = \frac{0.36 \rho_M D_M^3}{r_M^3}$$

$$\theta_S = \left(\frac{0.36 \rho_M}{\rho_S} \right)^{1/3} \theta_M$$

with $\rho_M / \rho_S = 3.35 / 1.4 = 2.4$,

$$\theta_S = 0.95 \theta_M$$

From Newton linking terrestrial “falling” with celestial orbits, to Vafa & Strominger’s calculation of black hole entropy, gravity has been a beautifully unifying concept.

Even its simple predictions are nonintuitive:
time scales are independent of the mass.

It predicts, nay demands, an expanding universe.

It explains why stars are hot, and that there would be stars even without nuclear physics.

Perhaps, through gravitational tides, we are here...and the Sun and Moon *look* the same size!

As we have some way to go, there will be more to come.