

# **Status of the general description of fission observables by the GEF code**

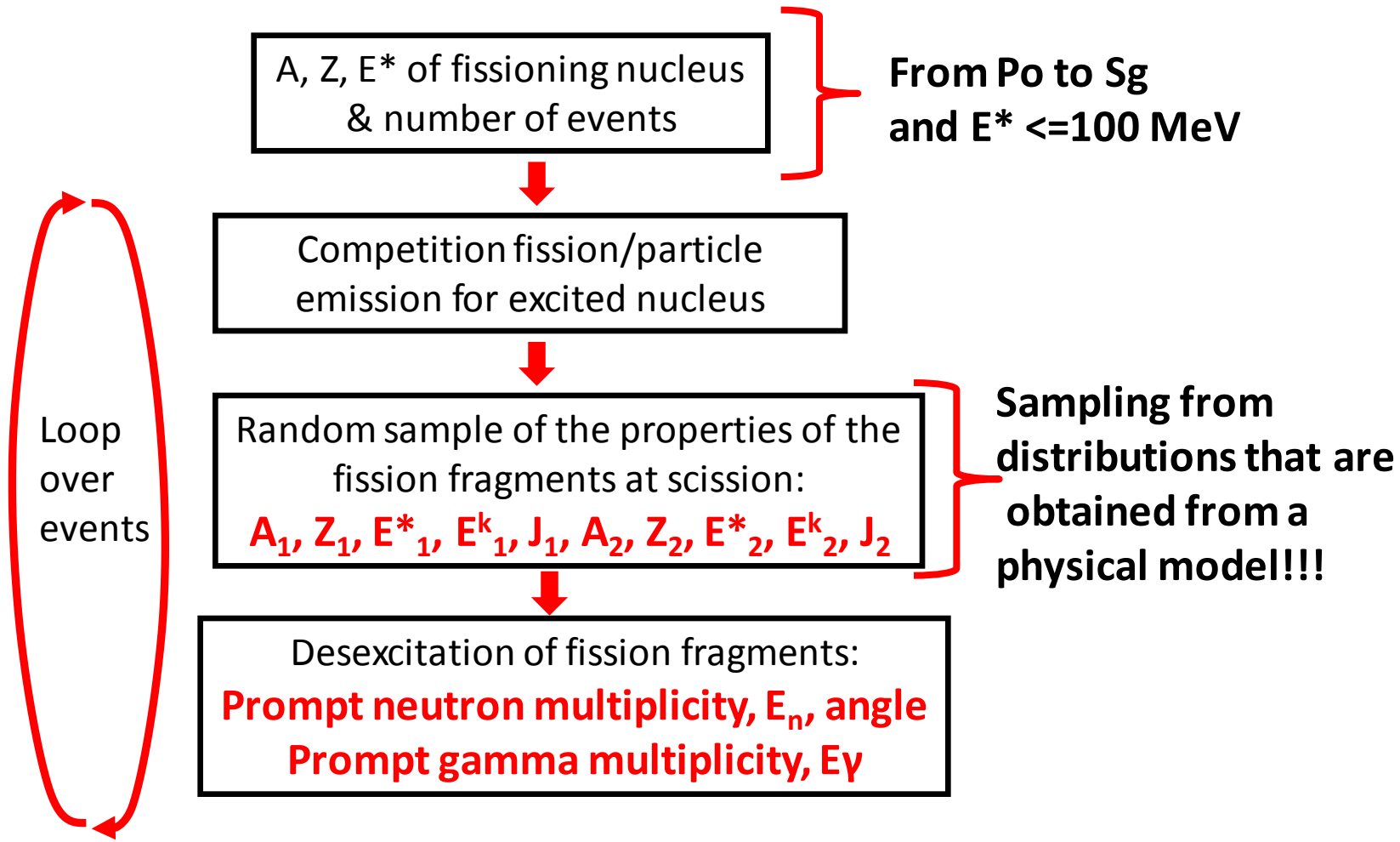
**Beatriz Jurado, Karl-Heinz Schmidt**

**CENBG, Bordeaux, France**

**Supported by EFNUDAT, ERINDA (Short-term visits)**

# The General Fission (GEF) code

## Structure



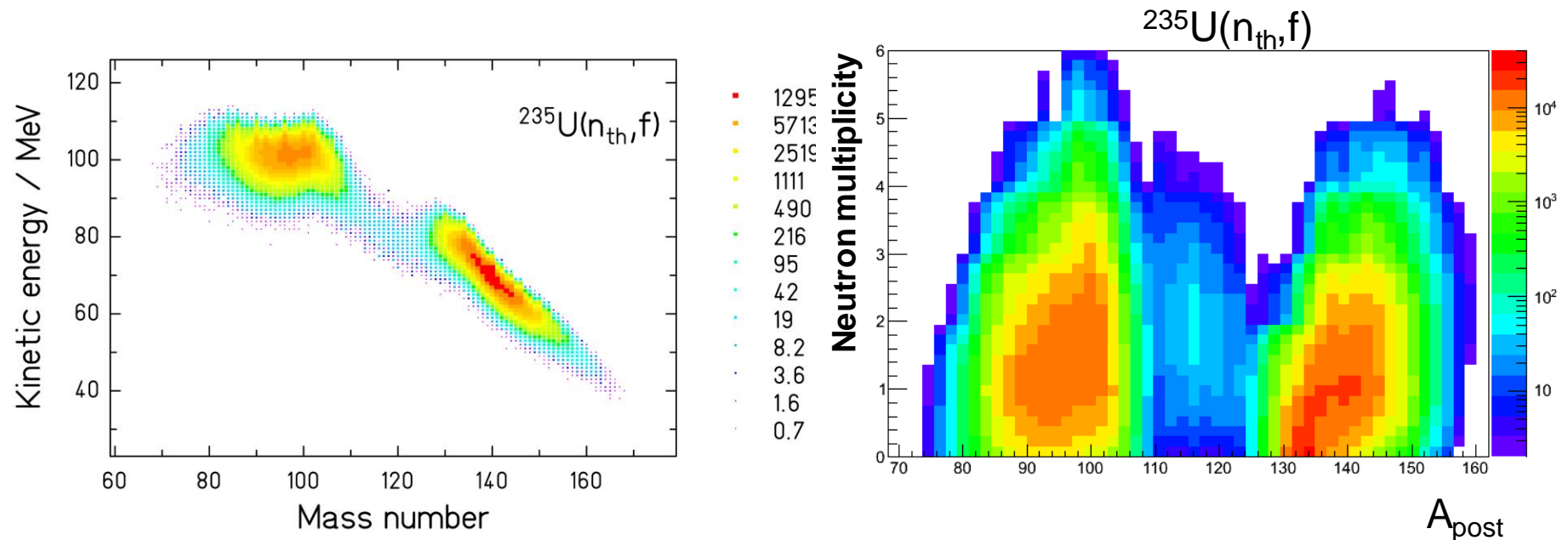
# Output of GEF

Results for essentially all fission observables

List-mode output  
can be used as an  
event generator

Fichier Edition Format Affichage ?											
* Z1	Z2	A1post	A2post	I1pre	I2pre	n1	n2	TKEpre	TKEpost		
* Calculation with nominal model parameters											
38	54	94	139	1.5	7.5	1	2	169.3808	167.3484		
37	55	92	140	3	0	2	2	161.8703	158.89		
35	57	88	146	4.5	8.5	1	1	161.9046	160.3561		
35	57	90	143	0.5	14.5	1	2	158.5806	156.6665		
36	56	90	145	3	4	0	1	177.1083	176.6457		
36	56	92	143	4	11	0	1	174.0105	173.5394		
35	57	88	147	2	16	0	1	169.2923	168.8658		
37	55	93	141	5	8	1	1	172.1861	170.601		
40	52	98	135	3.5	3.5	1	2	178.0875	175.9526		
40	52	102	132	7.5	13.5	1	1	171.735	170.2318		
38	54	97	137	5	5	1	1	180.9156	179.2917		

All possible correlations between quantities



# Ideas behind GEF

**Combination of physical concepts and experimental information**

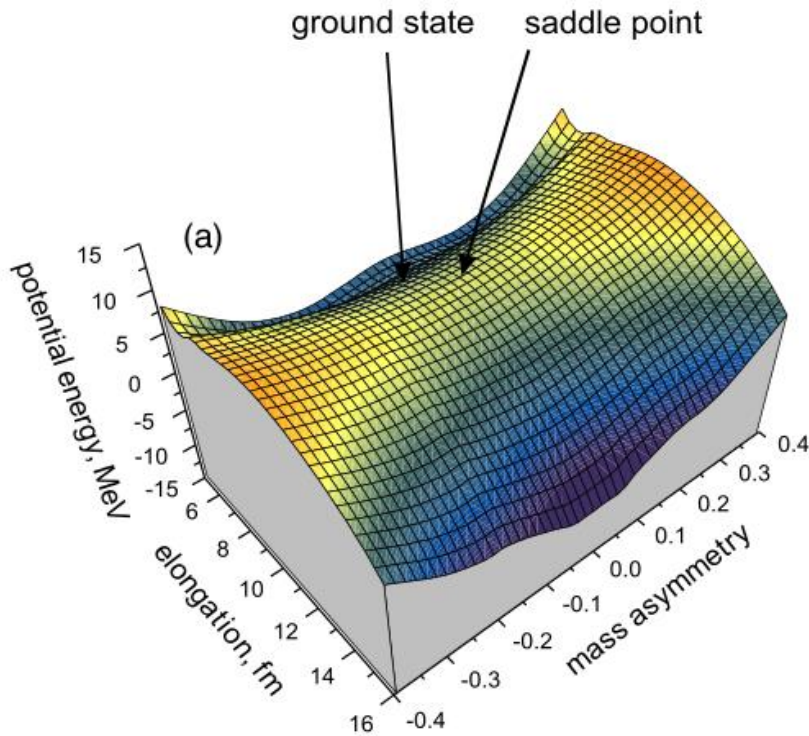
**Two examples:**

**→ Fragment yields**

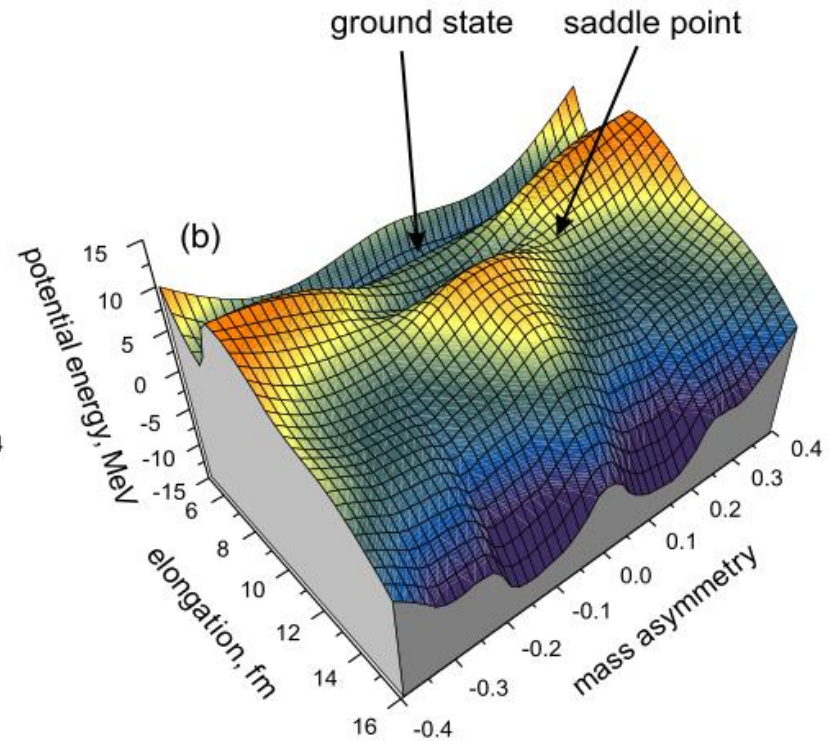
**→ Partition of excitation energy**

# Potential energy landscape

2-dim. calculation by A. Karpov, 2008



liquid-drop potential

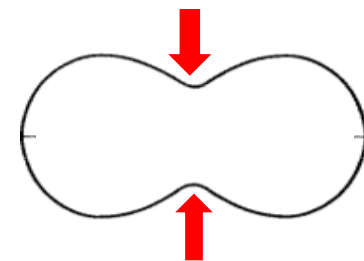


With shell effects

**Shells behind outer saddle:**

**Property of the nascent fragments**

**Quantum-mechanical effect caused by the neck!**



# Use of the separability principle in GEF

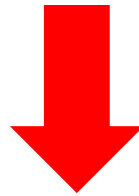
**Separability principle** { Macroscopic potential depends on fissioning nucleus  
Shell effects depend essentially on the fission fragments

## **Stiffness of macroscopic potential**

→ Deduced from experimental yields of symmetric mode for each fissioning nucleus

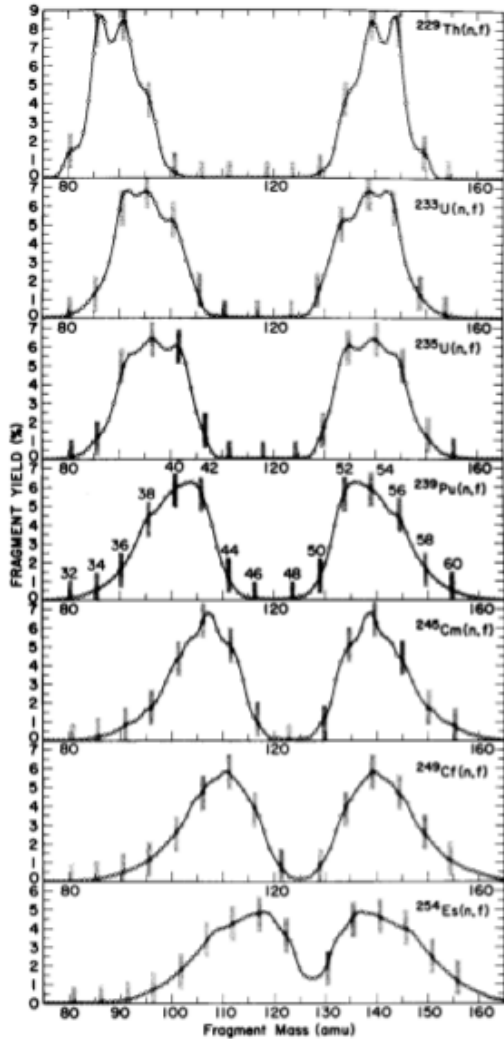
## **Position, strength and curvature of shells**

→ Deduced from experimental yields and shapes of asymmetric modes, essentially the same for all fissioning nuclei



**Description of a large variety of fissioning systems with the same set of parameters!!**

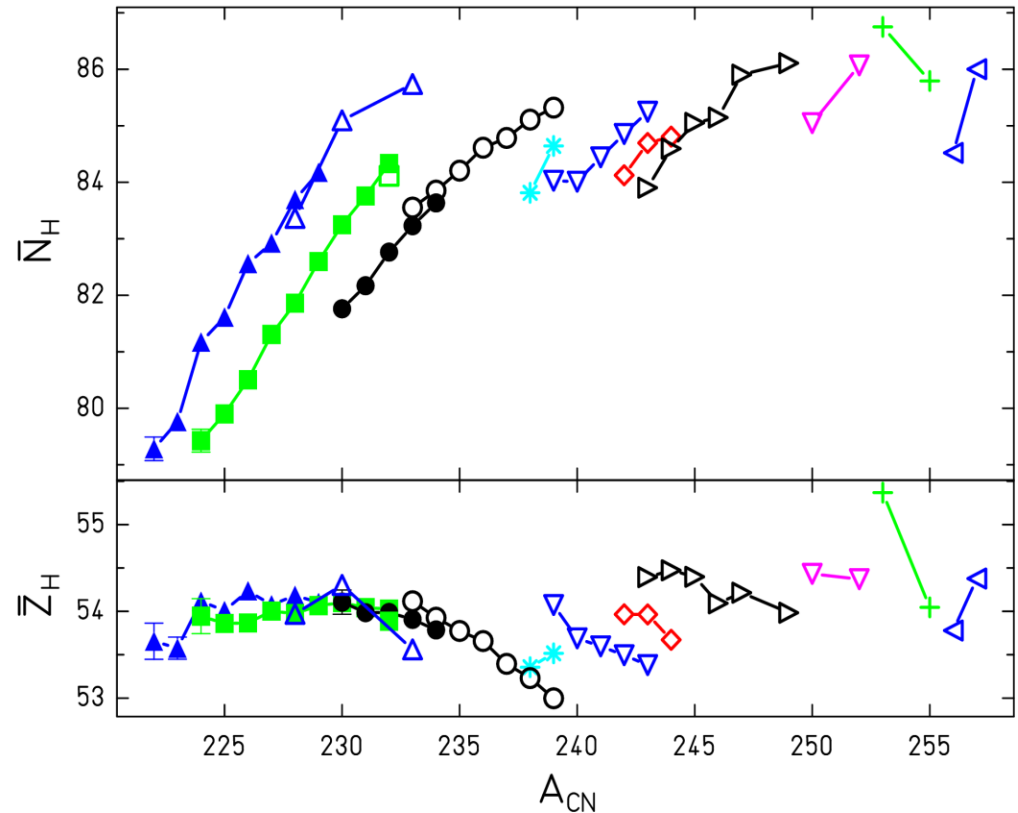
# Empirical information on the main shells



Unik et al., 1973

$\langle A \rangle \approx 140$

Shell effects in neutron number



Böckstiegel et al., NPA 802 (2008) 12

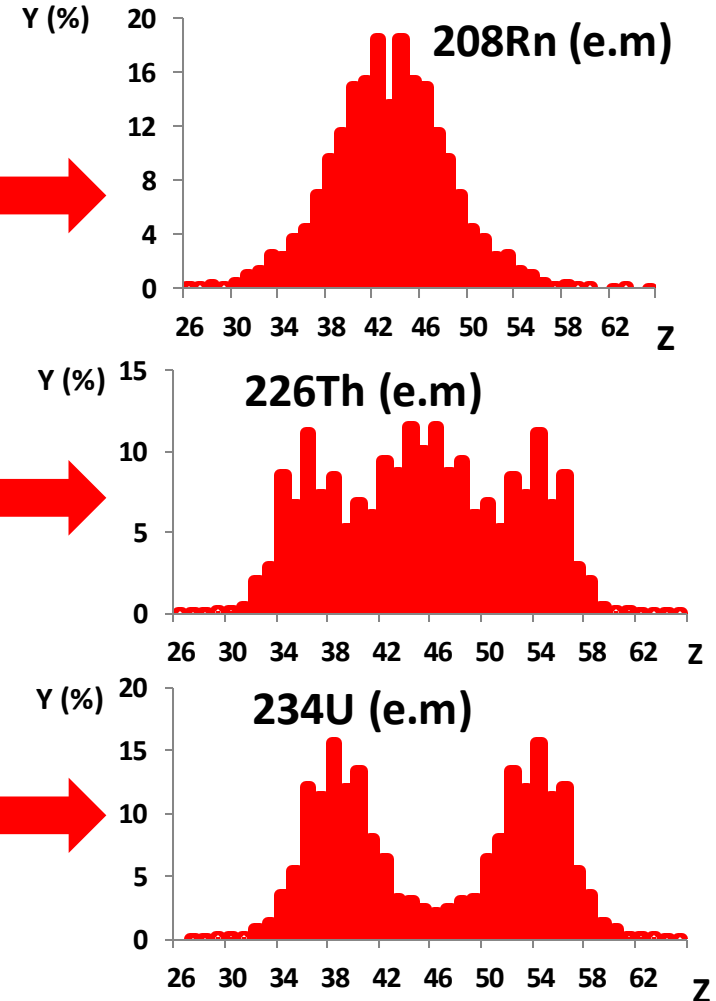
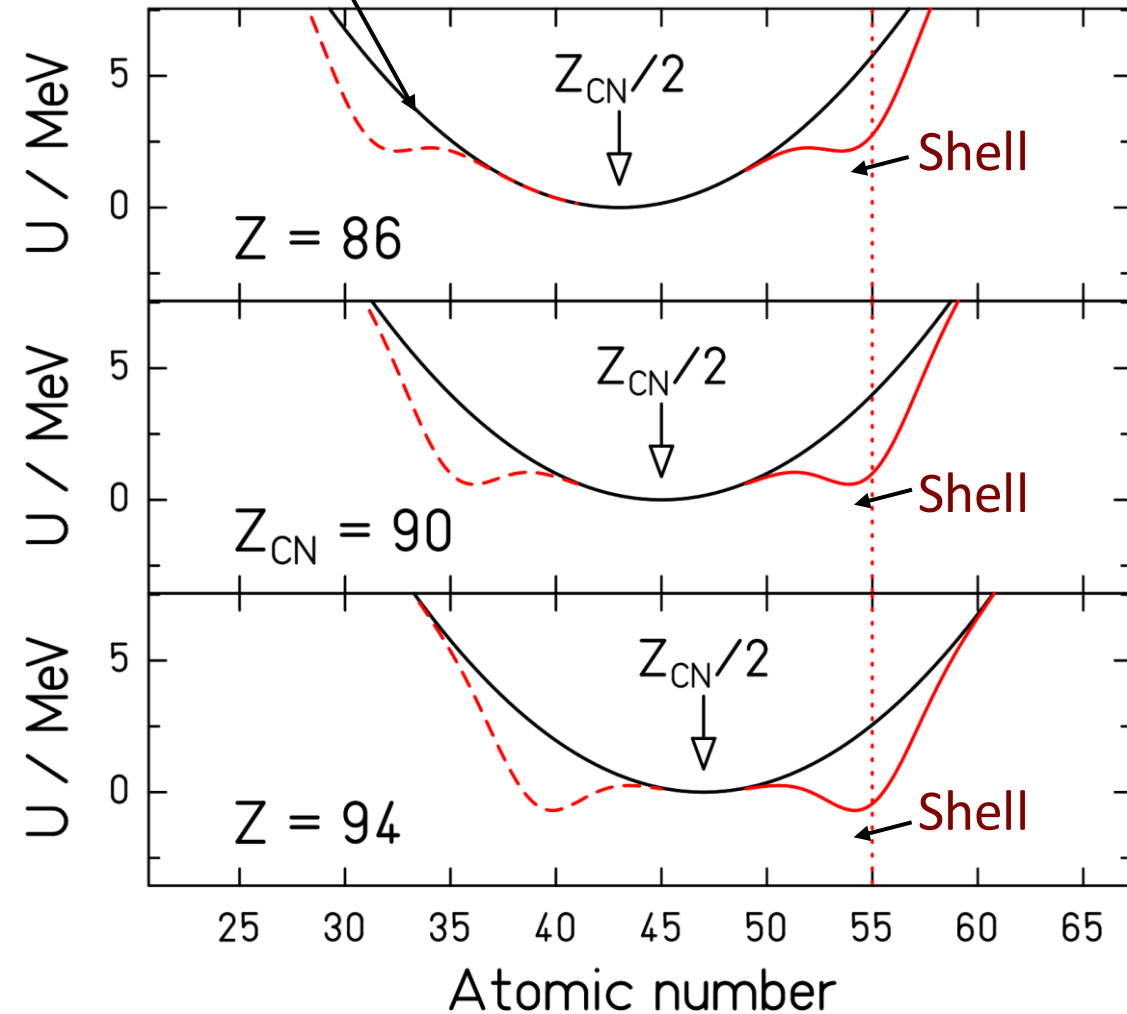
GSI data with long isotopic chains:

**New empirical result:  $\langle Z \rangle \approx 54$**

**Strong variation of  $\langle A \rangle$  !**

# Final potential

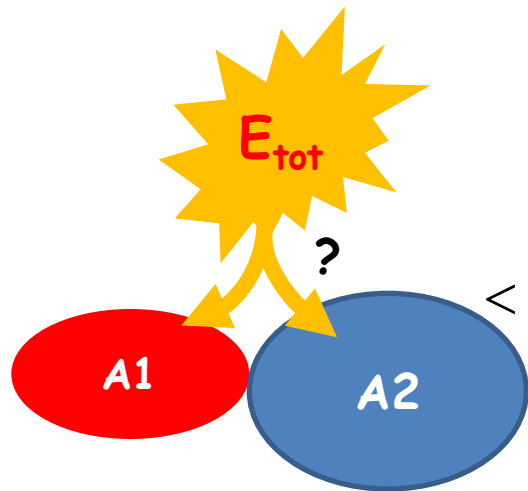
Liquid-drop potential



**Interplay between liquid-drop potential and shells explains observed transition from symmetric to asymmetric fission**

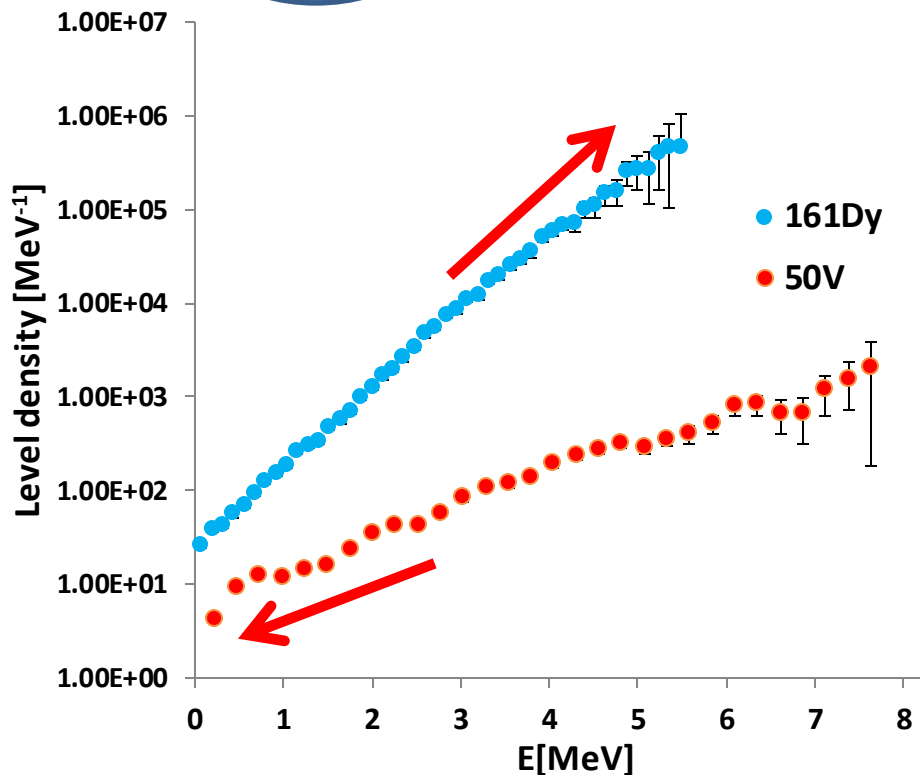


# Partition of excitation energy before scission (Statistical mechanics)



$$\langle E_L \rangle = \frac{\int_0^{E_{tot}} E \rho_L(E) \rho_H(E_{tot} - E) dE}{\int_0^{E_{tot}} \rho_L(E) \rho_H(E_{tot} - E) dE}$$

Only constrain:  
 $E_L + E_H = E_{tot}$



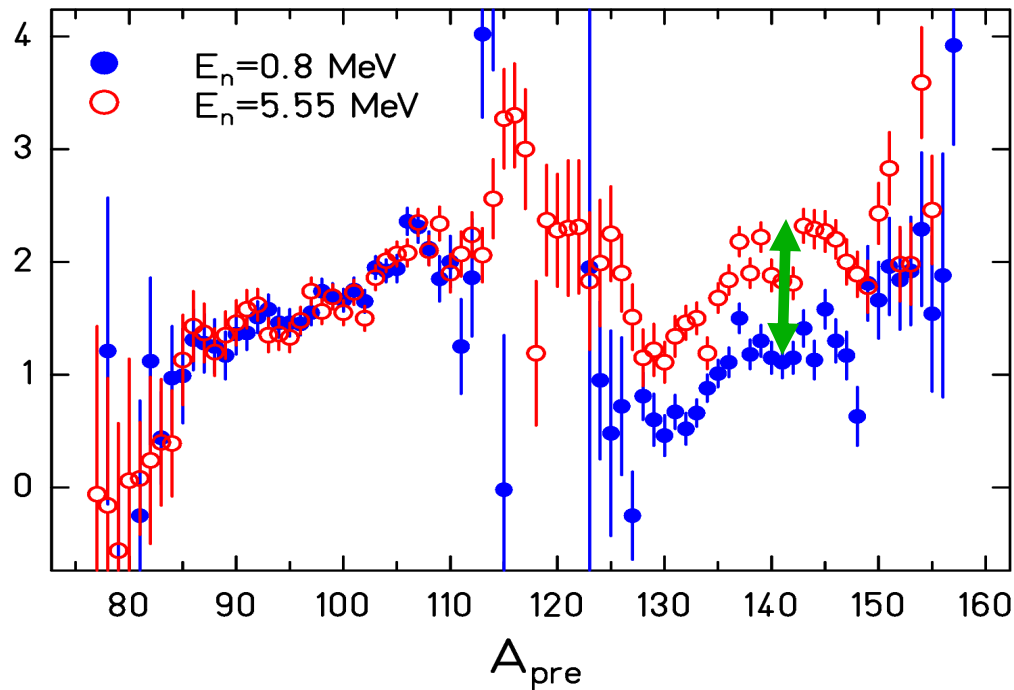
**Highest statistical weight  
for configurations with a cold  
light fragment!**

**Due to the constant-  
temperature behaviour, the  
excitation energy is transferred  
to the heavy fragment →  
Energy Sorting!**

# Neutron yields at different energies

## Signature of energy sorting

### $^{237}\text{Np}(n,f)$



The increase in emitted neutrons corresponds to an increase in  $E^*_{\text{heavy}}$  of  $4.8 \pm 0.2$  MeV.

All the additional  $E^*$  is found in the heavy fragment!

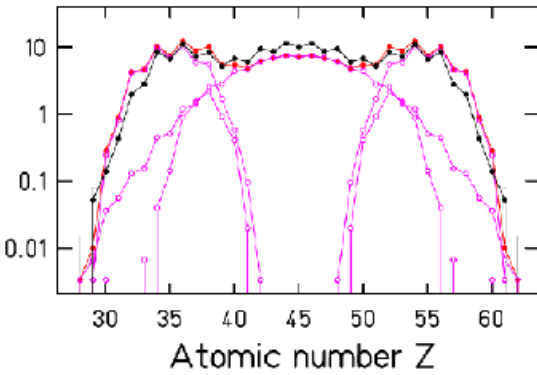
- K.-H. Schmidt, B. Jurado, Phys. Rev. Lett. 104 (2010) 212501
- K.-H. Schmidt, B. Jurado, Phys. Rev. C 83 (2011) 014607
- K.-H. Schmidt, B. Jurado, Phys. Rev. C 83 (2011) 061601 (R)
- K.-H. Schmidt, B. Jurado, submitted to Phys. Rev. Lett (2013)

# **Comparison with experimental data and evaluations**

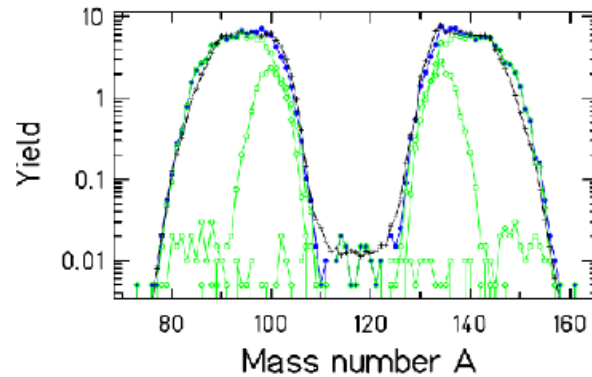
**All the results obtained with a single  
parameter set!**

# Yields

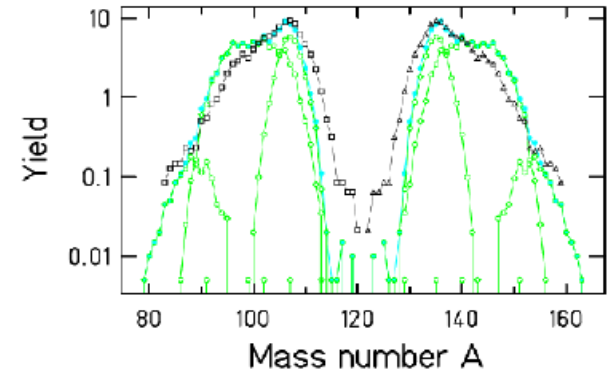
$^{226}\text{Th}(\text{e.m.})$



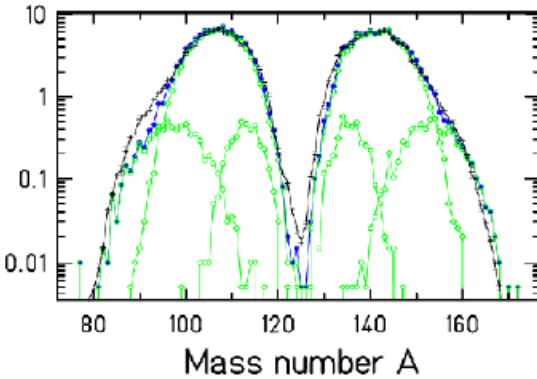
$^{235}\text{U}(\text{n}_{\text{th}}, \text{f})$



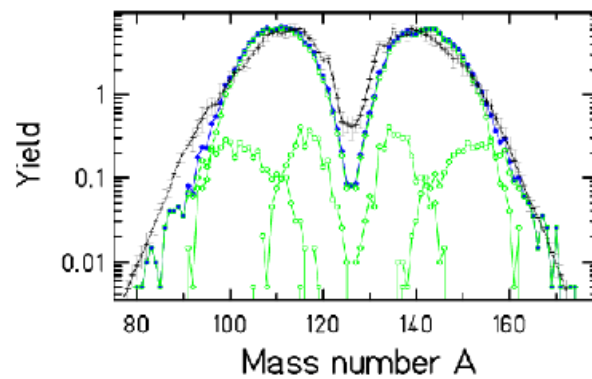
$A_{\text{pre}}, ^{242}\text{Pu}(\text{sf})$



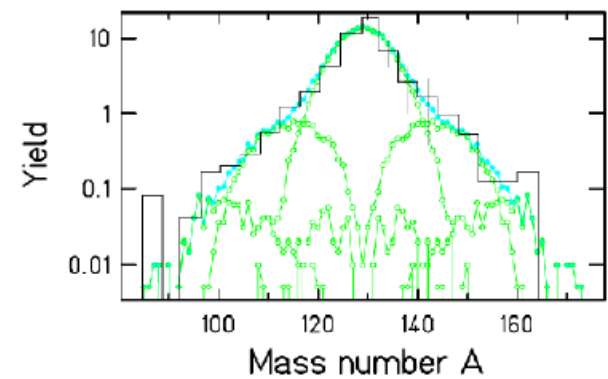
$^{252}\text{Cf}(\text{sf})$



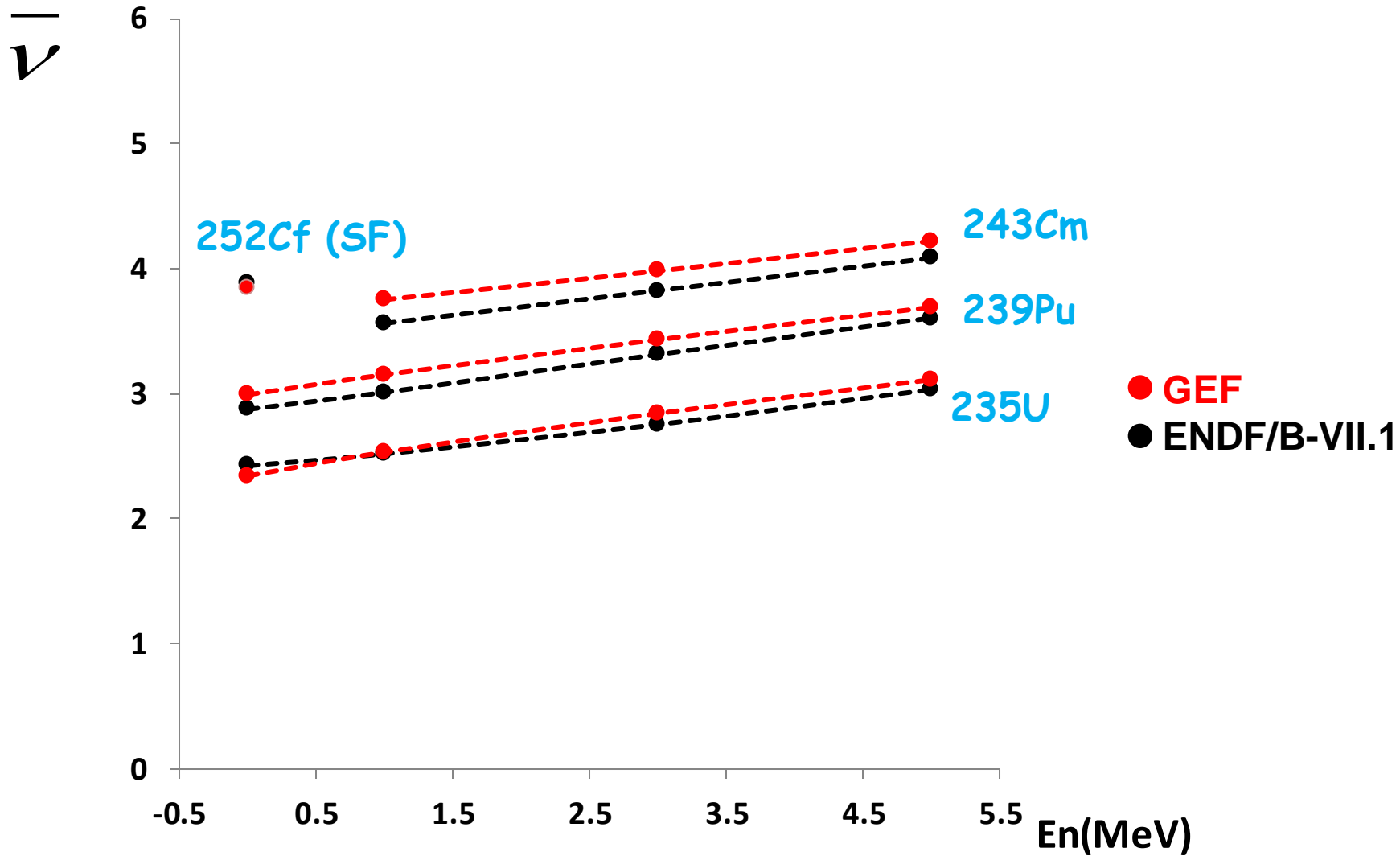
$^{256}\text{Fm}(\text{sf})$



$A_{\text{prov}}, ^{258}\text{Fm}(\text{sf})$



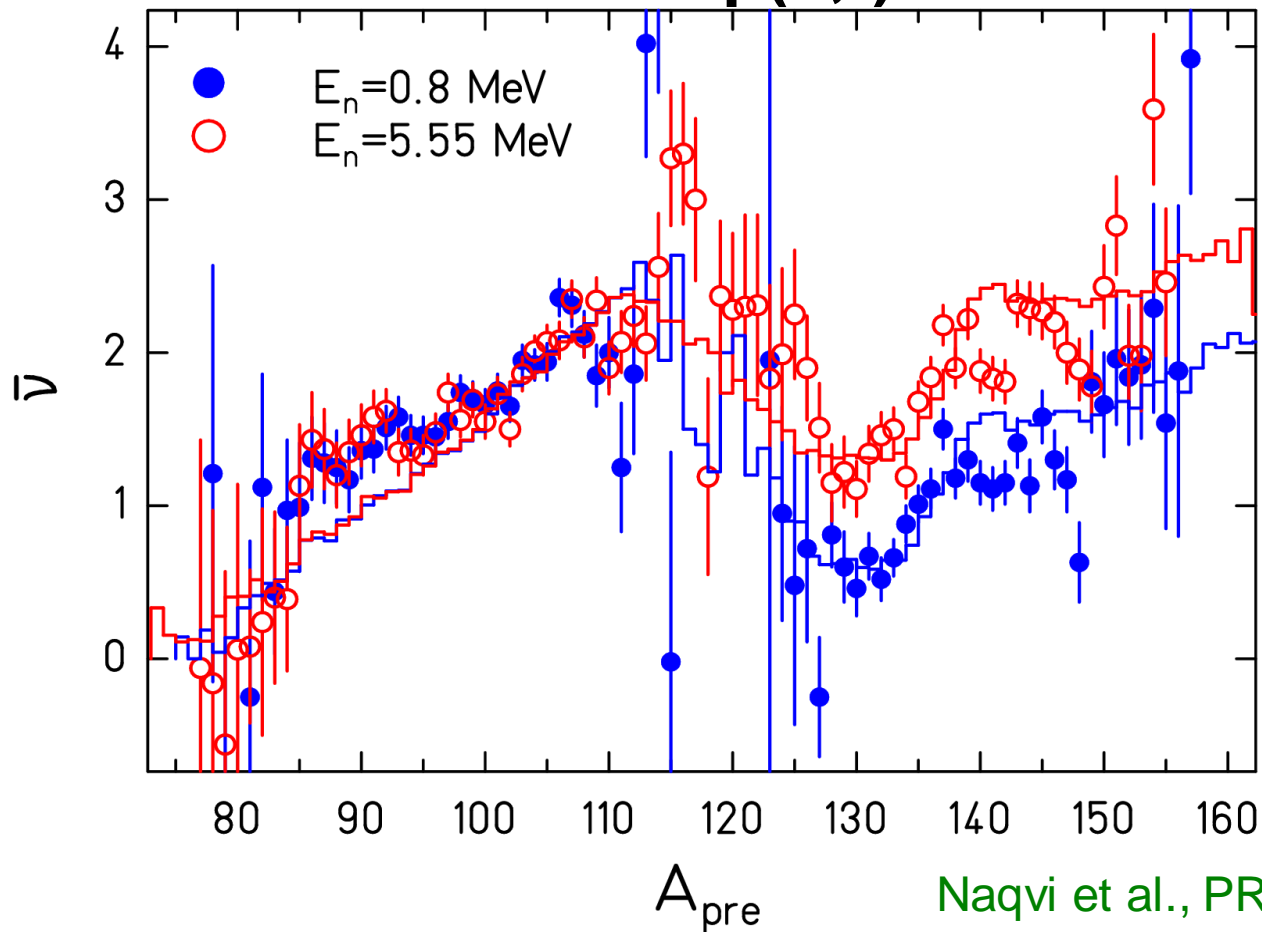
# Average number of prompt neutrons



**Differences <0.2 neutrons for all systems!**

# Prompt neutrons as a function of fragment mass

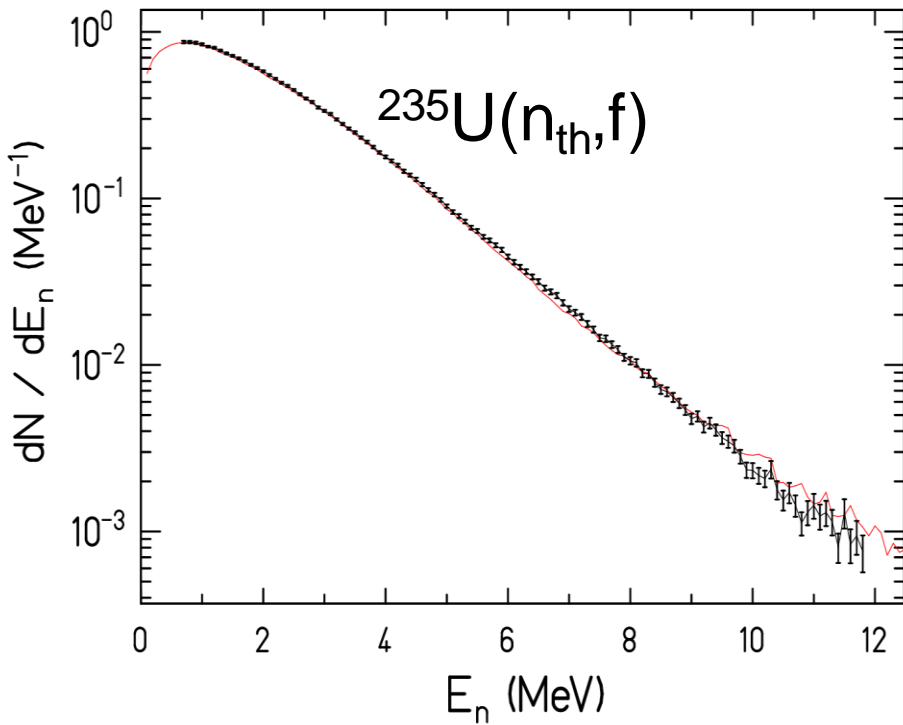
**$^{237}\text{Np}(n,f)$**



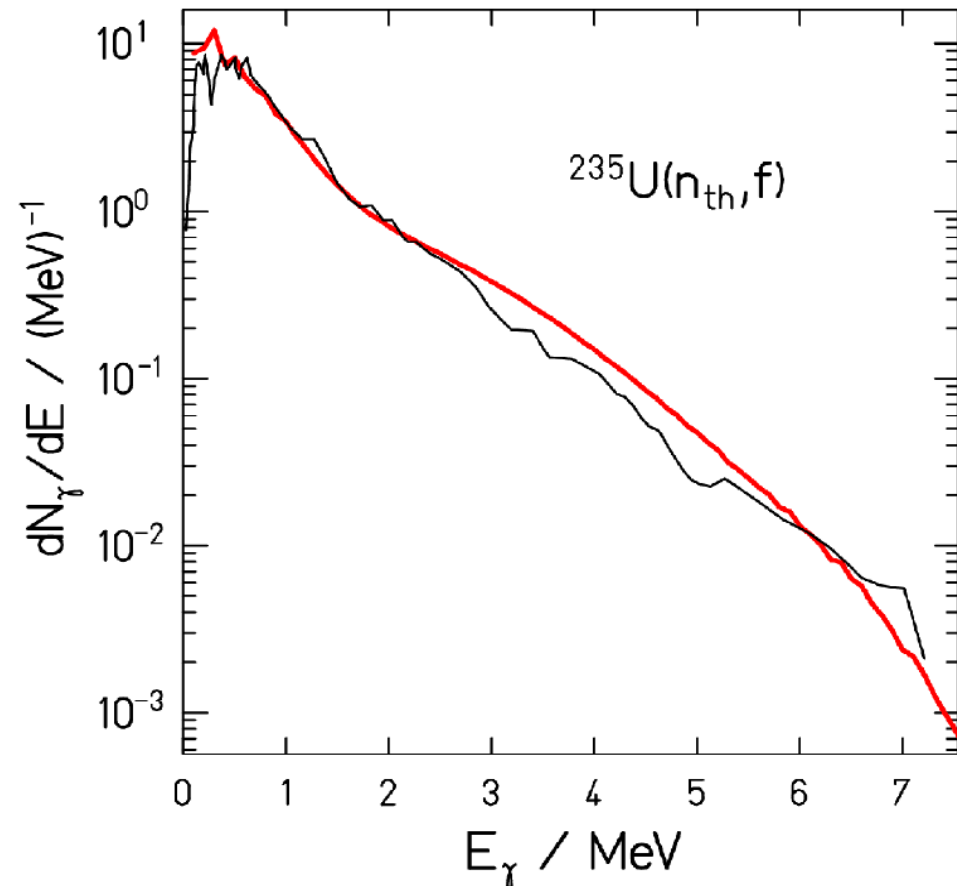
**Good description thanks to energy sorting!**

# Prompt-neutron and gamma spectra

## Neutrons



## Gammas

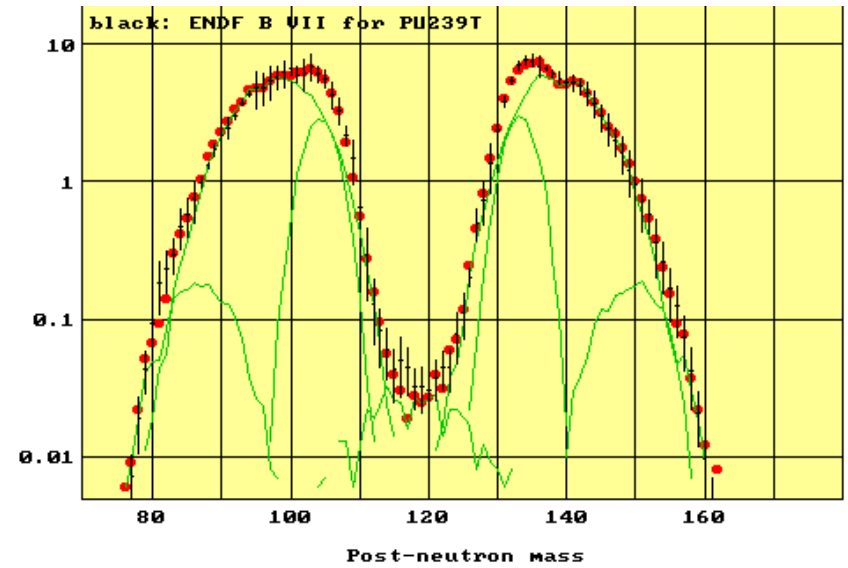
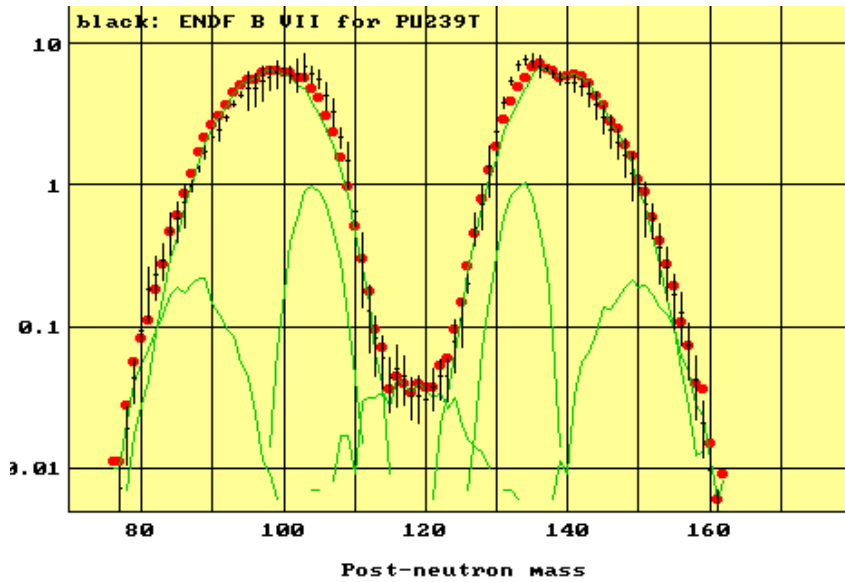


# **GEF: a useful tool for reactor physics**

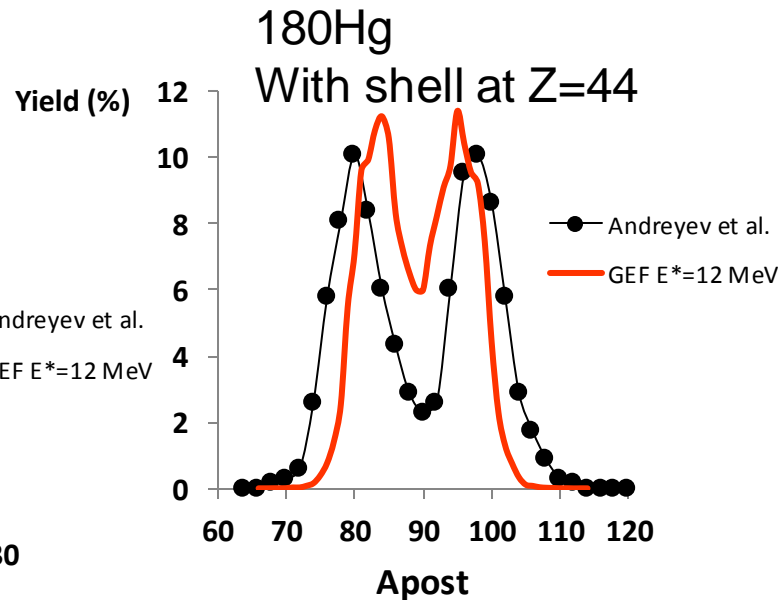
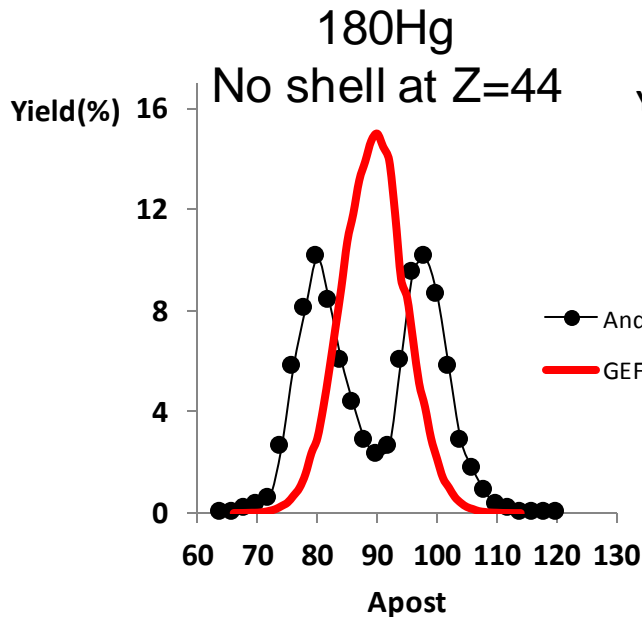
- Independent and cumulative yields in ENDF format (GEFY)**
- Deterministic version of GEF, subroutine (GEFSUB)**
- Error bars for yields from perturbed model parameters (covariance matrix for yields)**
- Production of isomers**



# GEF: a useful tool for fundamental physics



**Shell effect at Z=44 needed to reproduce all the data in a coherent way!**



**This shell is responsible for the asymmetric fission observed for light, neutron-deficient fissioning nuclei!!!**

# Conclusions

- GEF combination of physical concepts from quantum mechanics and statistical mechanics and specific experimental information within a general approach
- GEF gives reliable predictions for essentially all fission observables, also for nuclei where no data exist!
- It is a very useful tool for reactor physics (GEF Yields will be part of next JEFF edition)
- It serves to reveal the sensitivity of fission observables to basic nuclear properties (e.g. shell at  $Z=44$ )

## ...and perspectives

- Inclusion of ternary fission
- Include proton-, electron- and photon-induced fission
- Improve nuclear structure information of the fission fragments
- Perform a quantitative assessment of the deviations between GEF and experimental and evaluated data
- etc...

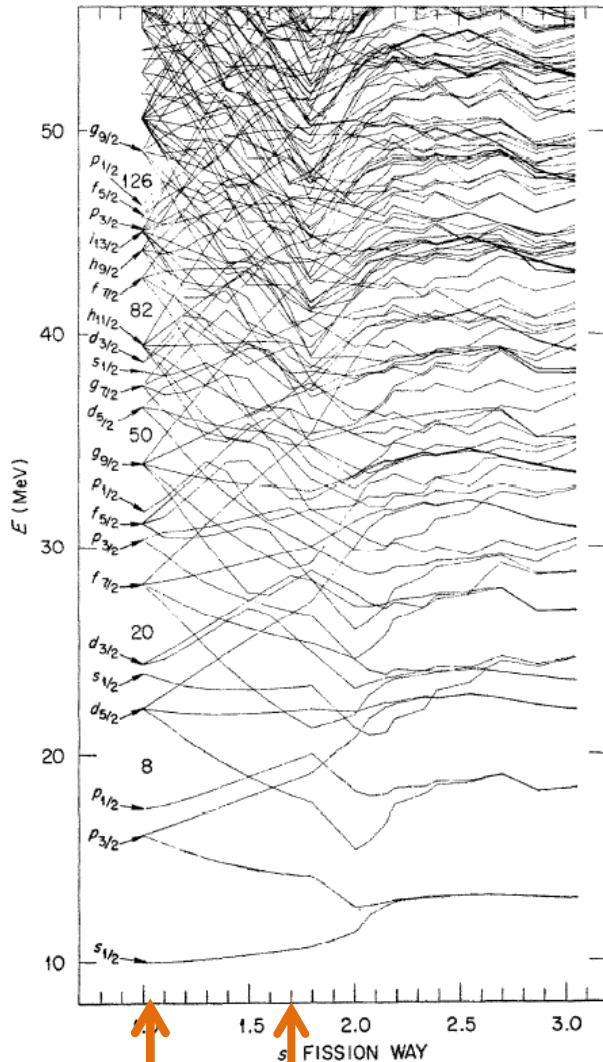
**Download and further information can be found in :**

[www.khs-erzhausen.de](http://www.khs-erzhausen.de) or [www.cenbg.in2p3.fr/GEF](http://www.cenbg.in2p3.fr/GEF)

# Influence of fragment properties on the fission process

Neutron shell-model states of  $^{236}\text{U}$

(U. Mosel, H. W. Schmitt, Nucl. Phys. A 165 (1971) 73)



•Fragments acquire their individual properties near the second barrier

--> Shell-effects

--> Pairing

(H.J. Krappe et al., NPA 690 (2001) 431)

--> Congruence energy

(W.D. Myers et al. NPA 612 (1997) 249)

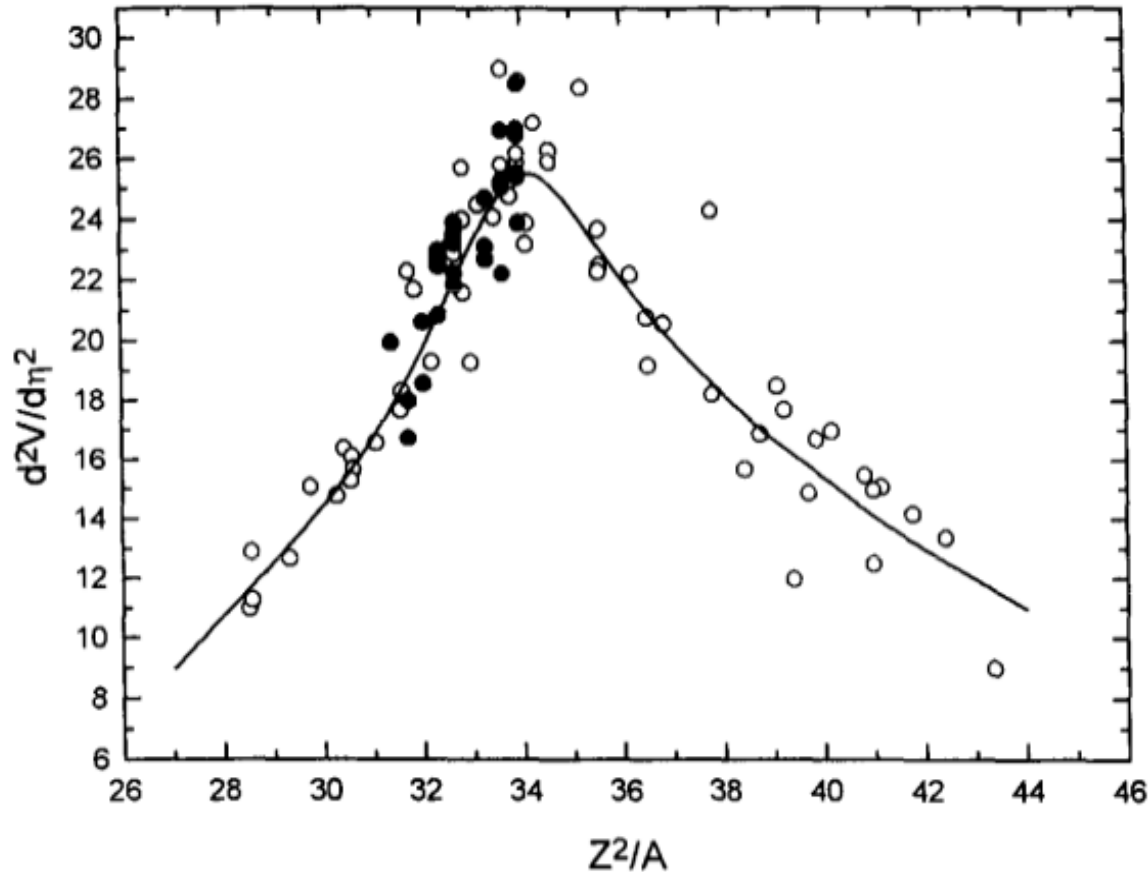
•Level crossing considerably reduced after second barrier

Most of the energy is dissipated near the second barrier

Ground state  
Second barrier

→ → ...  
Scission

# Macroscopic potential



From measured  $\sigma_A$  of symmetric fission mode.

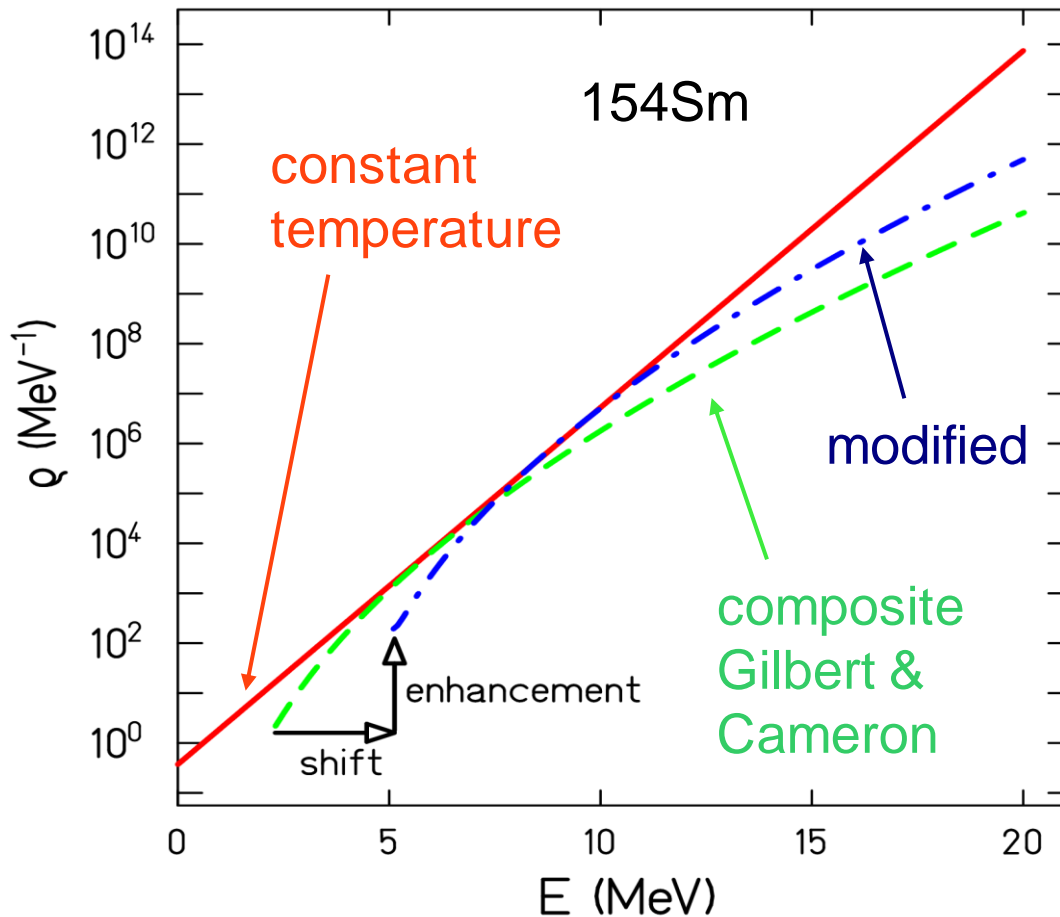
$$\sigma^2 = \frac{\text{temperature}}{\text{stiffness}}$$

(Harmonic oscillator in a heat bath.)

Mulgin et al., NPA 640 (1998) 375

Stiffness vs. mass asymmetry of the macroscopic potential.  
A unique function of fissility.

# Improved level-density description



## Benefit of composite formula:

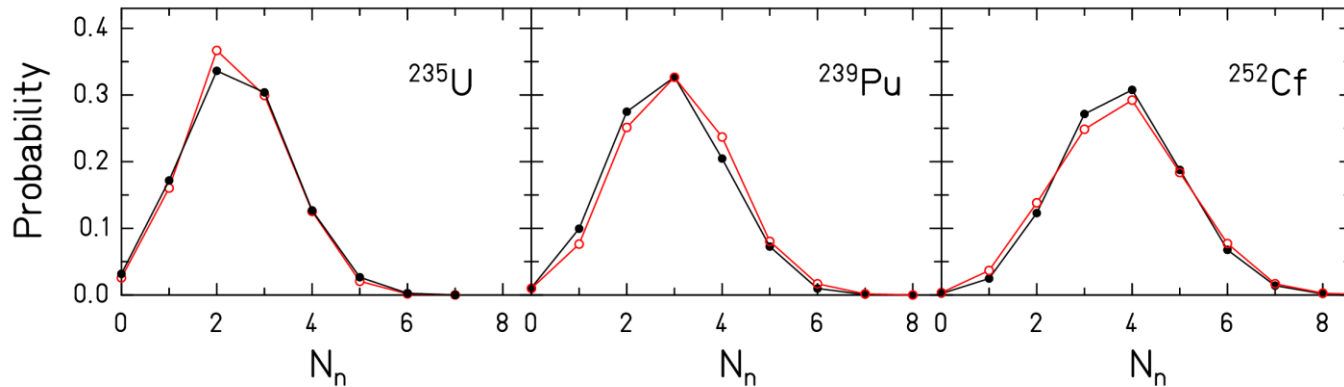
1. Constant temperature reflects large heat capacity due to pairing correlations. Empirical systematics can be used.
2. Fermi gas above the critical pairing energy (Independent-particle)

## Proposed modifications:

1. **Increased shift** parameter required for stability of pairing. (Condensation must enhance binding energy!)
2. **Enhancement factor** reflects contribution of collective excitations.

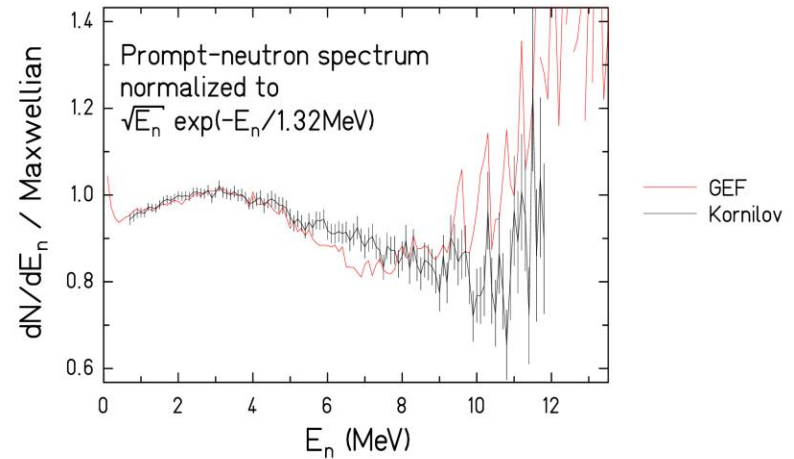
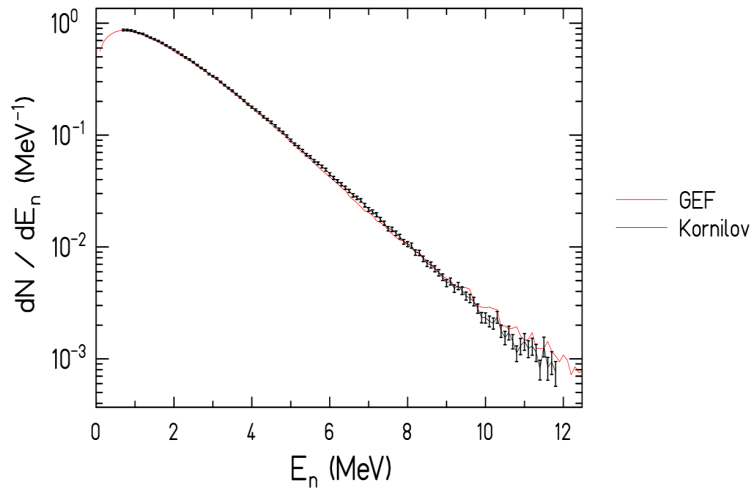


# GEF: Prompt-neutron multiplicity



Fluctuations of the prompt-neutron multiplicities are attributed to shape fluctuations of the nascent fragments

# GEF: Prompt-neutron spectra



$^{235}\text{U}(n_{\text{th}},f)$

Very good reproduction for all measured systems.  
Predictions for other systems without need for any  
experimental information or specific adjustments.



# Initial conditions: angular momentum

- Angular momentum “pumping” of orbital angular momentum by quantum-mechanical uncertainty principle

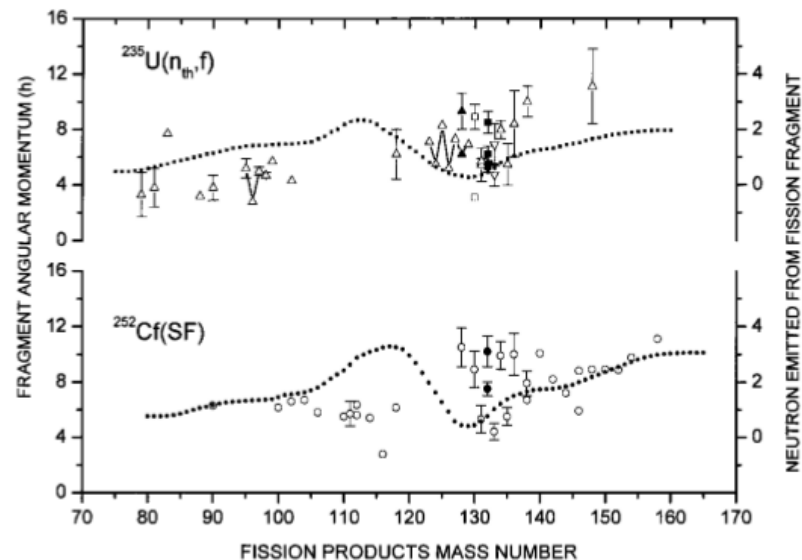
- S. G. Kadmsky, Phys. Atom. Nuclei 70 (2007) 1628

- Enhanced spin by unpaired nucleons

- B. S. Tomar, R. Tripathi, A. Goswami, Pramana 68 (2007) 111

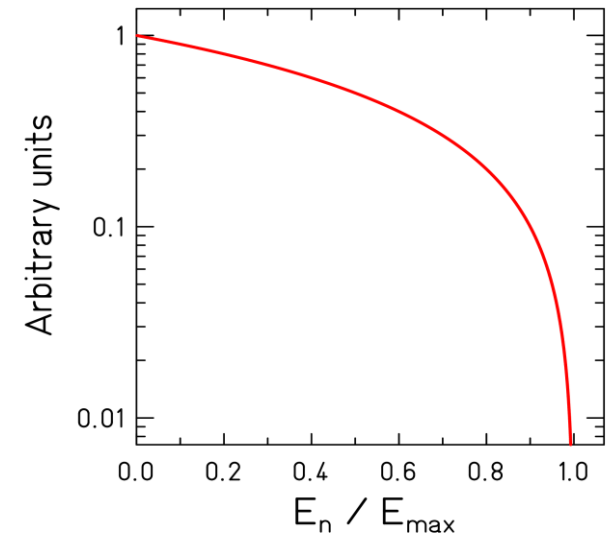
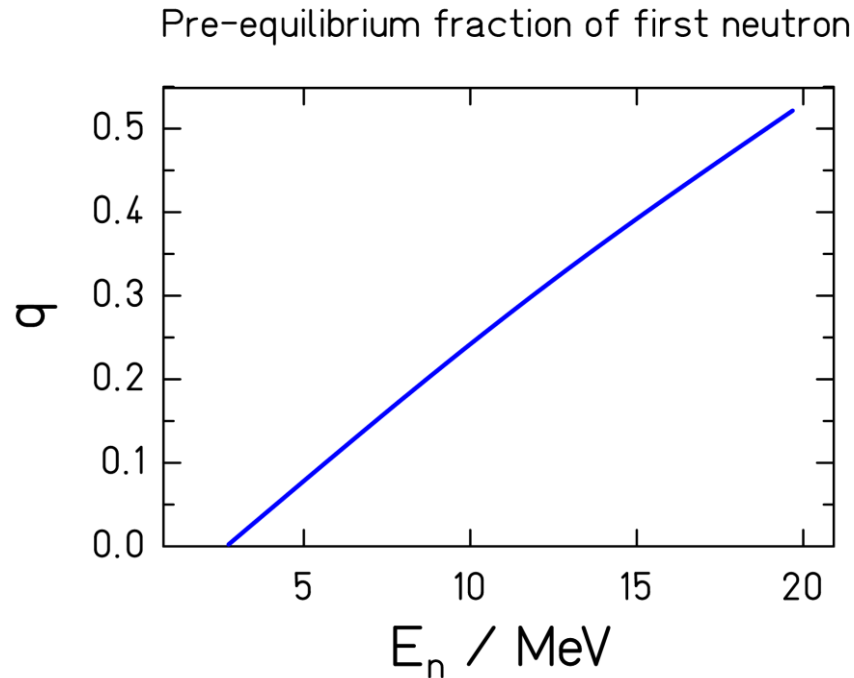
- Empirical scaling

- H. Naik et al, Eur. Phys. J. A 31 (2007) 195



# 1. step: formation of the CN

- Neutron-induced fission:  
Pre-equilibrium emission



Shape of first-step neutron spectrum

# Fission barriers (global description)

- Topographic theorem:

$$B_{max} = B_{ld} - Shell_{GS} \quad [1]$$

- $B_{ld}$  from Thomas-Fermi model [2]

- $Shell_{GS}$  from  $BE_{exp} - BE_{ld}$  (Thomas-F.) [1]

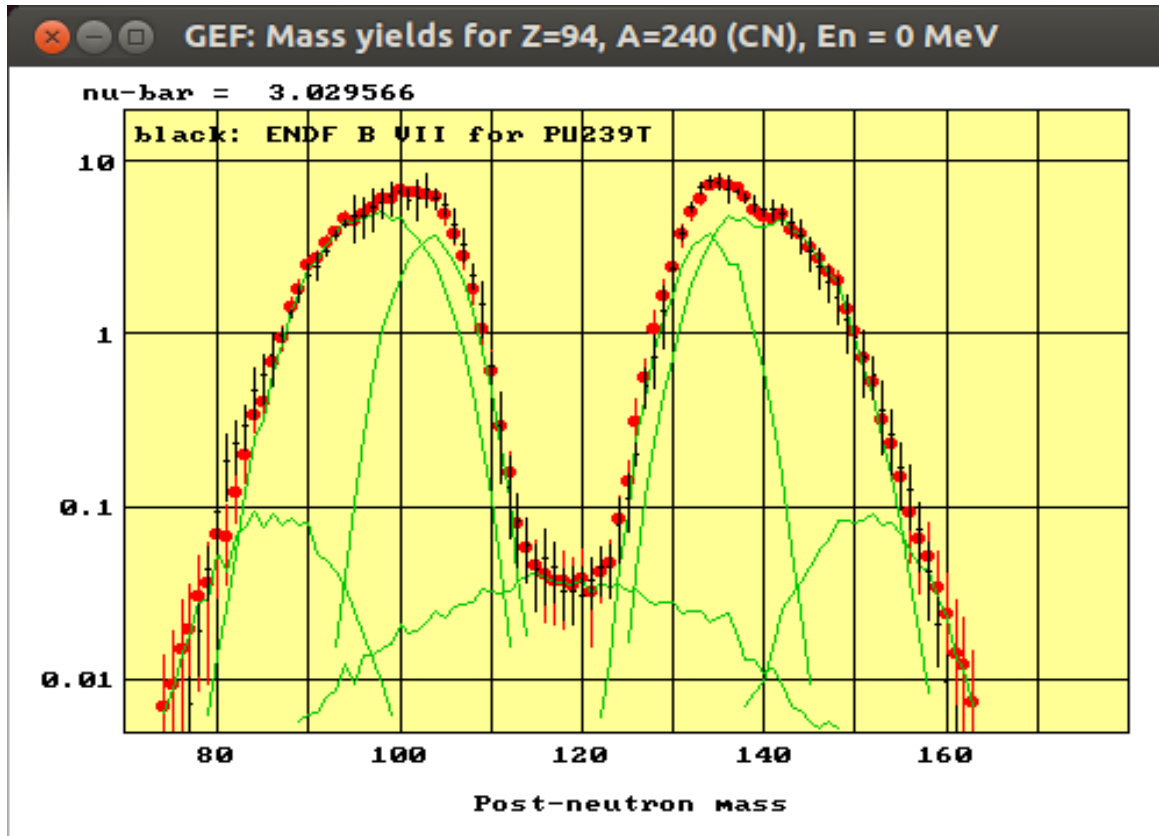
- Correction for systematic deviation of  $B_{ld}$  as  $f(Z)$

- Correction for nuclei with  $B_A \approx B_B$   
(Position of liquid-drop barrier at 2nd minimum)

[1] W. D. Myers, W. J. Swiatecki, Nucl. Phys. A 601 (1996) 141

[2] W. D. Myers, W. J. Swiatecki, Phys. Rev. C 60 (1999) 014606

# Uncertainty estimates from perturbed-parameter calculations



Model parameters fluctuate inside their uncertainty range (deduced from the fit procedure).

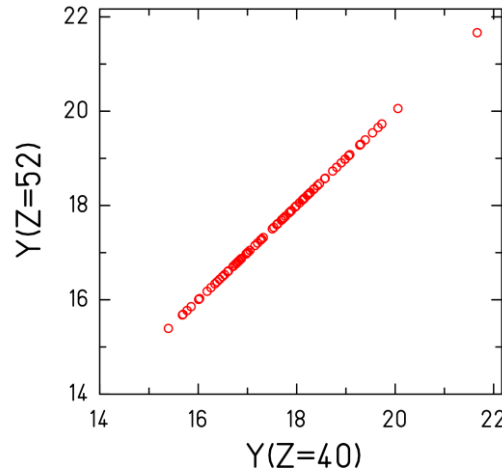
Fluctuations of the results → uncertainties of the fission observables. (Used for GEFY.)

red: Error bars of the model calculation.  
black: Error bars of the evaluated data.

Applicable to any of the fission properties.

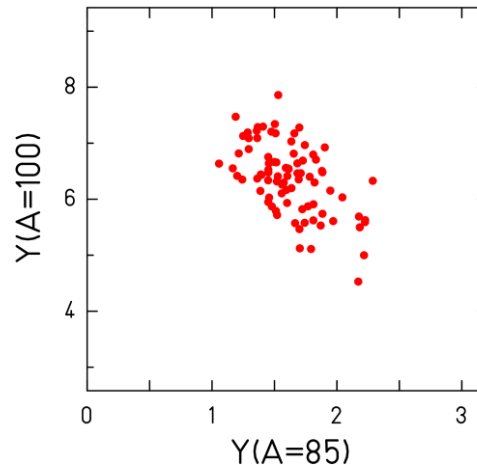
# GEF calculations provide a covariance matrix

- relations required by physics



Two complementary elements (Strictly correlated)

- relations required by the model



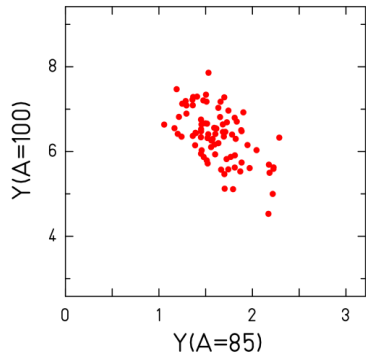
Two masses from different modes. (Slightly anti-correlated)

Result of perturbed GEF calculations for

# Establishing the covariance matrix

The covariance between two variables  $x$  and  $y$  is defined by:

$$\text{cov}(x, y) = \sum_{i=1}^N \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}$$



GEF result:  
Calculation  
with perturbed  
parameter  
values.

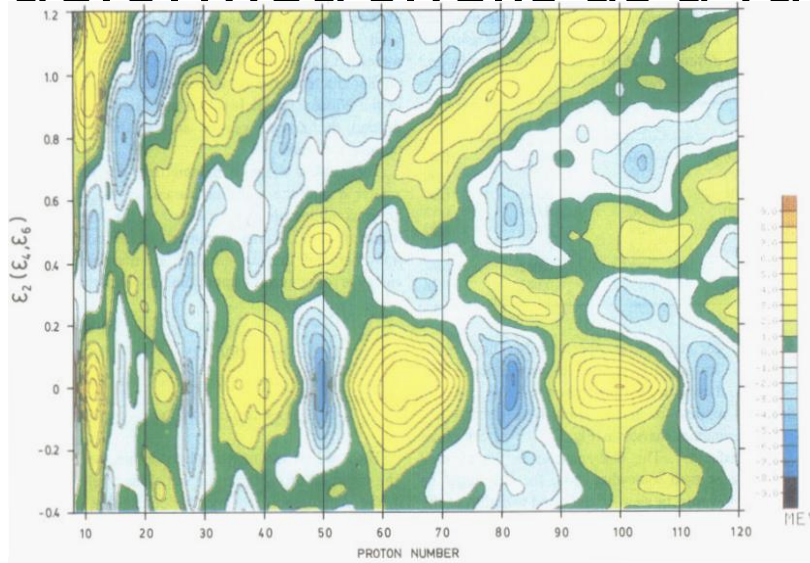
The covariance values between any two yield values  $Y_a$  and  $Y_b$  can be determined from the GEF calculation with perturbed parameters.

All covariance values form the covariance matrix.

The covariance matrix represents the internal logic dependences (trivial ones and model-specific ones) of GEF

# Aspects of statistical mechanics in fission dynamics

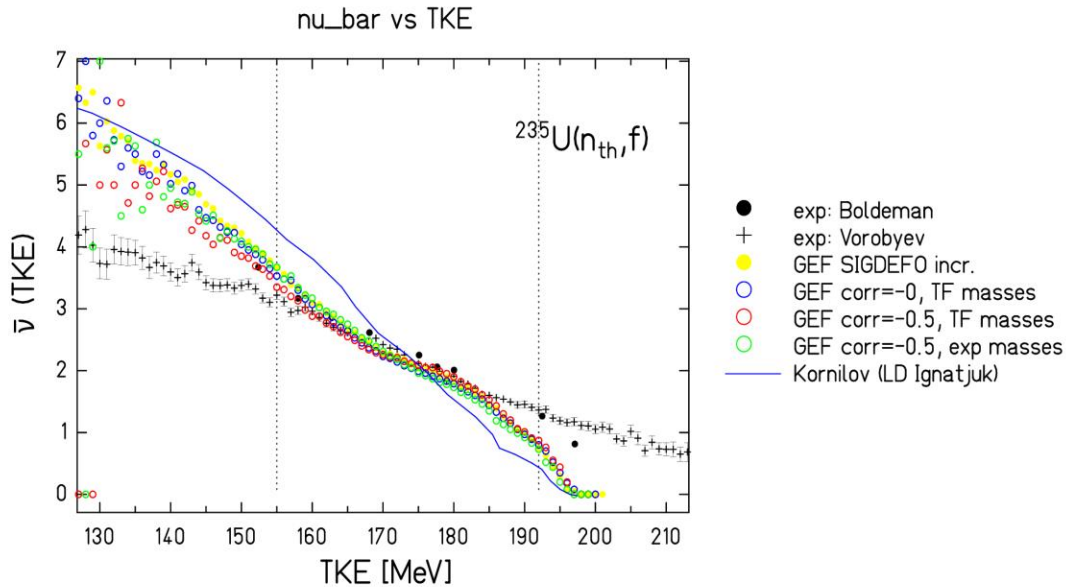
- Deformation energy at scission from deformed shells as a function of  $Z$  (saw-tooth



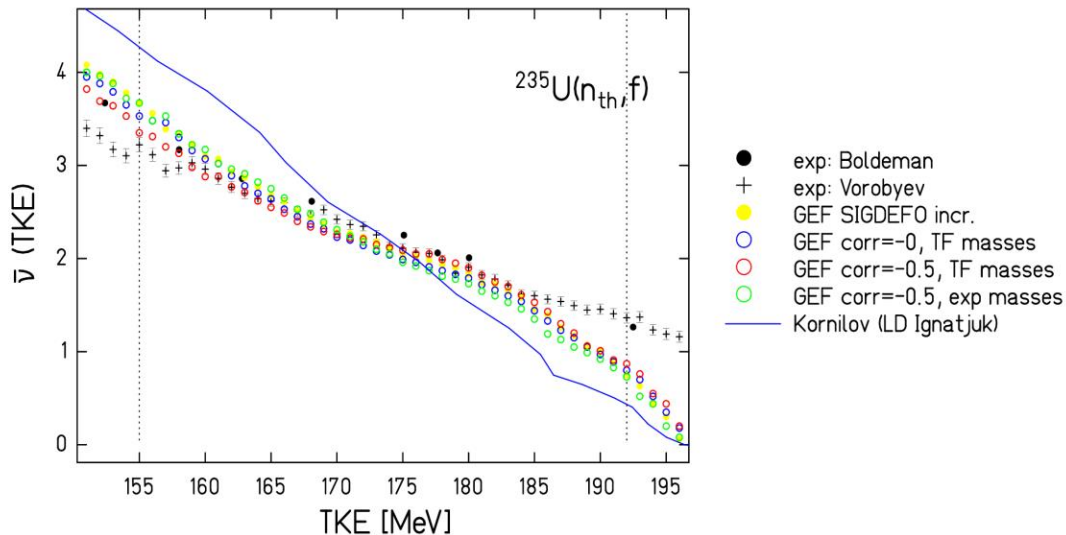
I. Ragnarsson,  
R. K. Sheline,  
Phys. Scr. 29 (1984) 38

- Energy sorting of thermal energy at scission ( $\nu(\Lambda) = f(E^*)$  even-odd effect in  $Z$  yields)

# Other correlations of fission observables



Prompt-neutron yield vs. TKE, quoted as an indication for scission neutrons.



GEF result differs from analytical estimations. GEF considers all (also hidden) correlations.



# Level densities of deformed nuclei:

$a = 0.073 A / \text{MeV} + 0.095 B_s A^{(2/3)} / \text{MeV}$  (A. V. Ignatyuk et al., Sov. J. Nucl. Phys. 21 (1975) 612.)

$B_s = 1 + 2/5 * \alpha_2^2 - \dots$

$\beta = 3/2 * \alpha_2$

$A=96, \beta=0.2$  to  $0.6 \rightarrow$  increase of level density of 13%

$A=140, \beta=0.2$  to  $0.6 \rightarrow$  increase of level density of 14%

$a$  increases by 1%

For  $S_L$  and  $S_2$  there is no net effect because both fragments are deformed.

For  $S_1$  this influence is minor compared to the difference in the slopes of the level densities of the two nascent fragments in asymmetric fission.

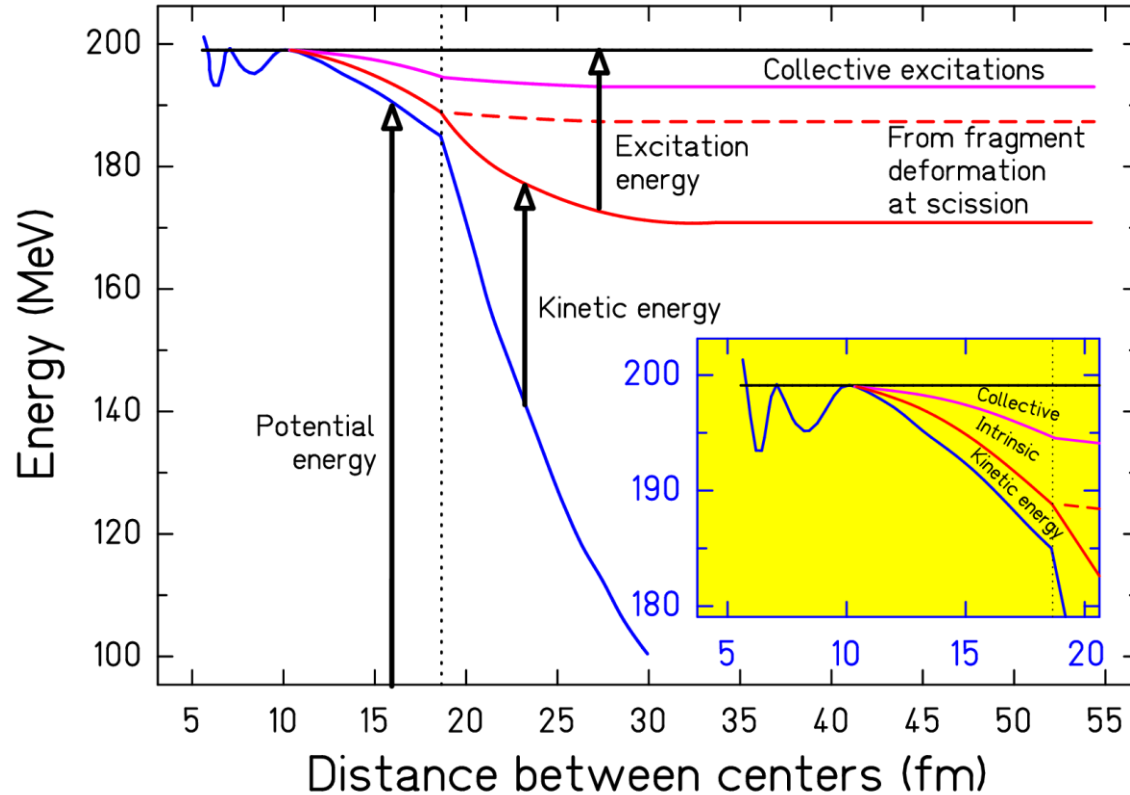
$T$  and  $a$  are closely related [D. Bucurescu, T. von Egidy, Phys. Rev. C 72 (2005) 067304]-->

$T$  decreases only by 1%, --> Negligible compared to the difference in log slopes for asymmetry

Note however that:

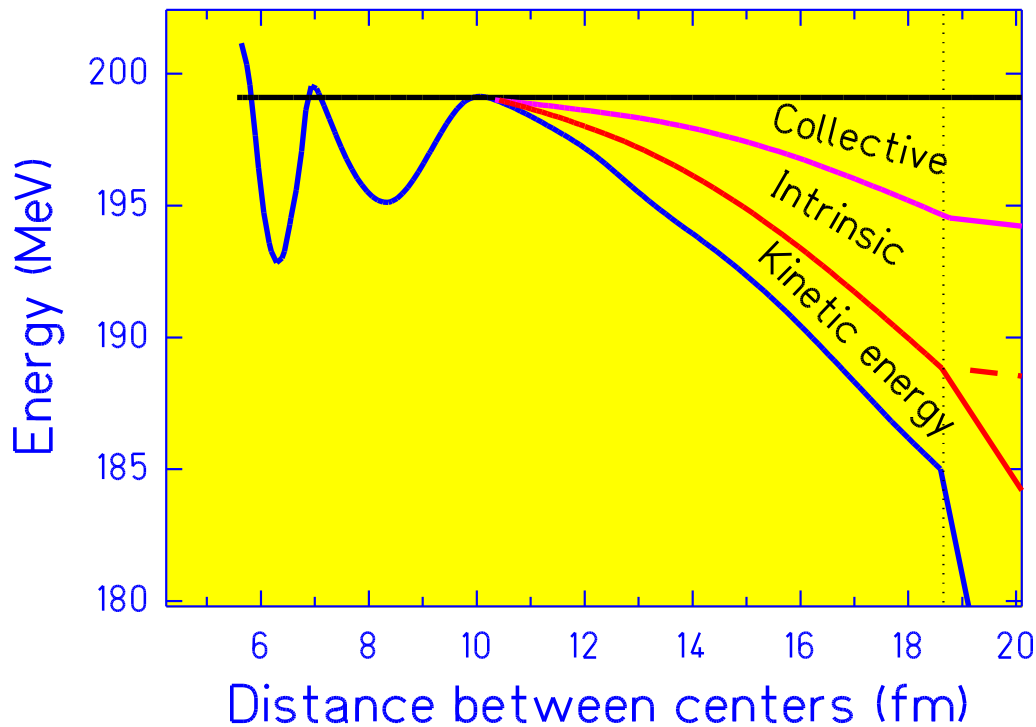
A shell effect of a few MeV causes a modification of the level density by a factor 10!!

# Initial conditions: Energies



- Energy transformations

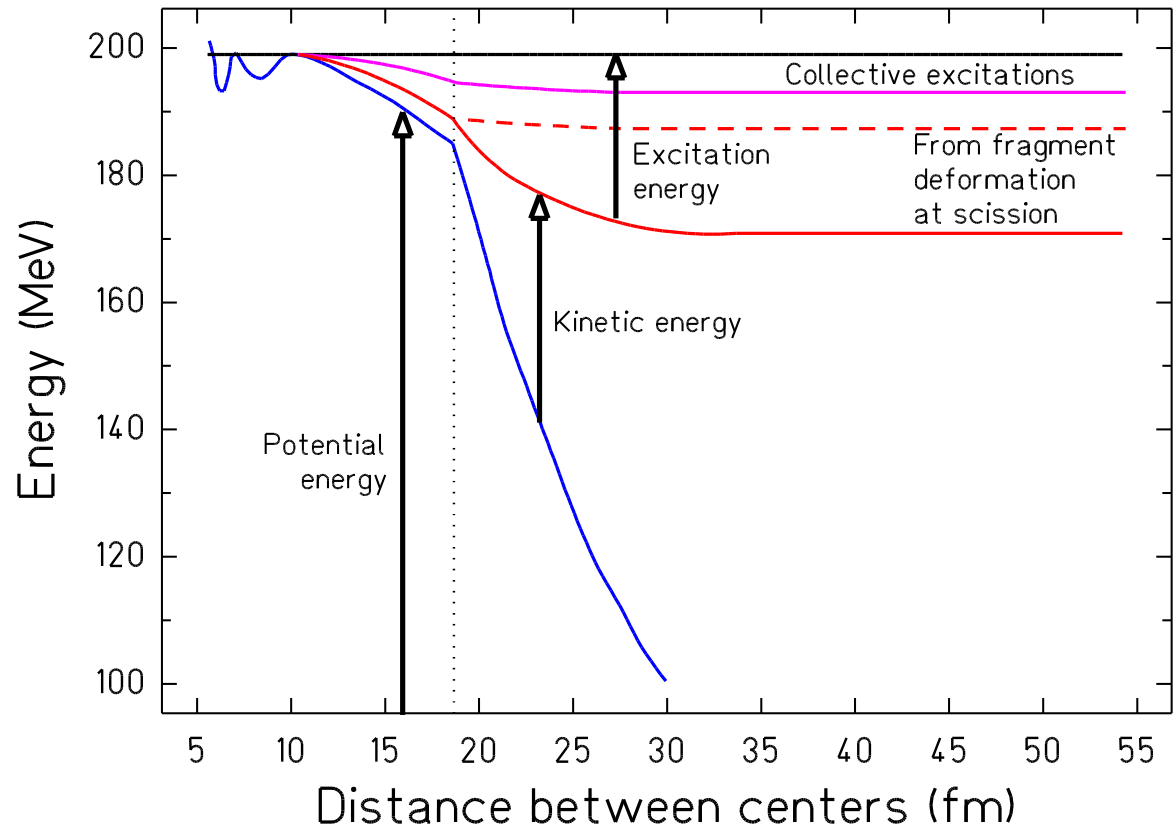
# Energy considerations: situation at scission



Energy at saddle  $E^*_{\text{sad}} = E^*_{\text{CN}} - \text{FB}$   
 Intrinsic  $E^*$ , "heat"  
 (We consider  $E^*_{\text{CN}} < 15 \text{ MeV}$ )  
 Increase in  $E_{\text{beam}}$  mainly goes to  
 intrinsic  $E^*$



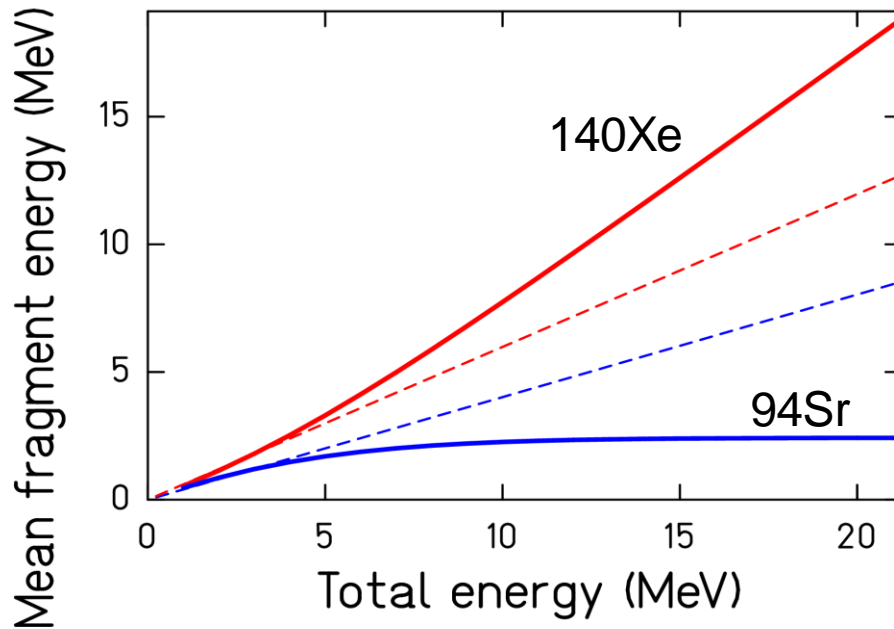
# Energy considerations: fully accelerated fragments



Total  $E^*$   
(TXE)

Total  $E_{kin}$   
(TKE)

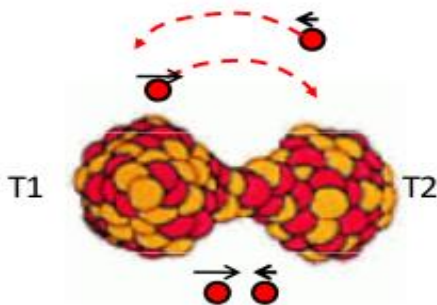
# Initial conditions: Energy sorting



$^{94}\text{Sr}$  and  $^{140}\text{Xe}$

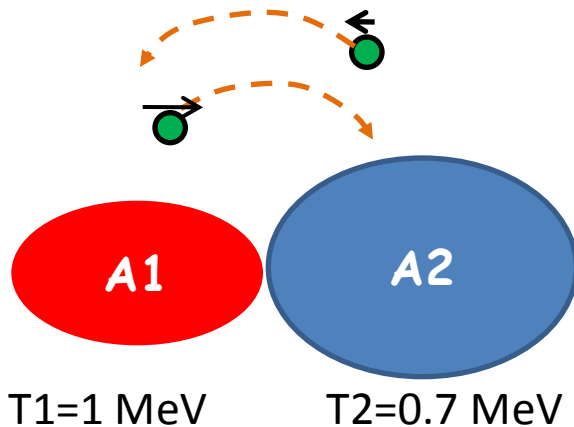
← Fermi gas

K.-H. Schmidt,  
B. Jurado,  
PRC 83 (2011) 061601



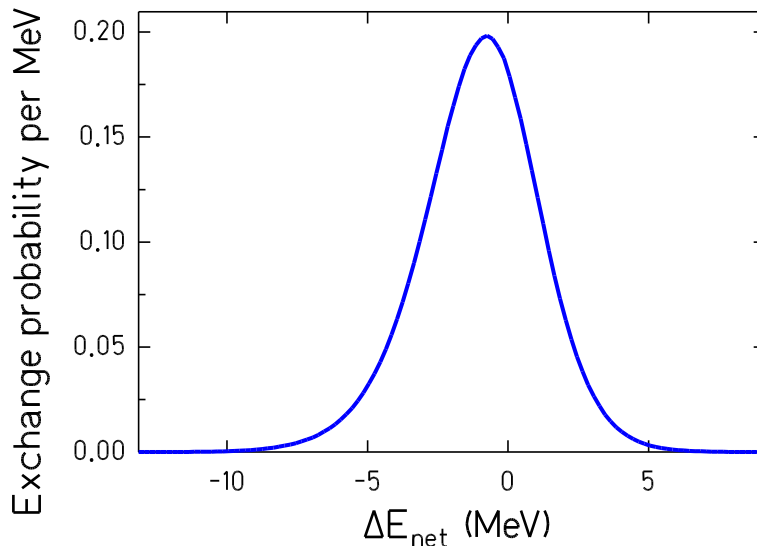
- Sorting of  $E^*$  at scission caused by constant nuclear temperature. (Data from “Oslo group”.)

# Microscopic view of energy transfer



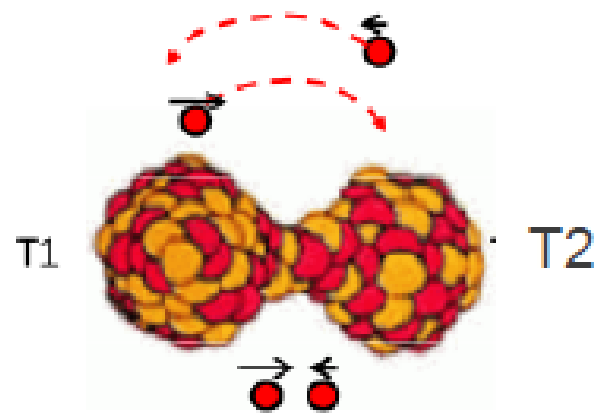
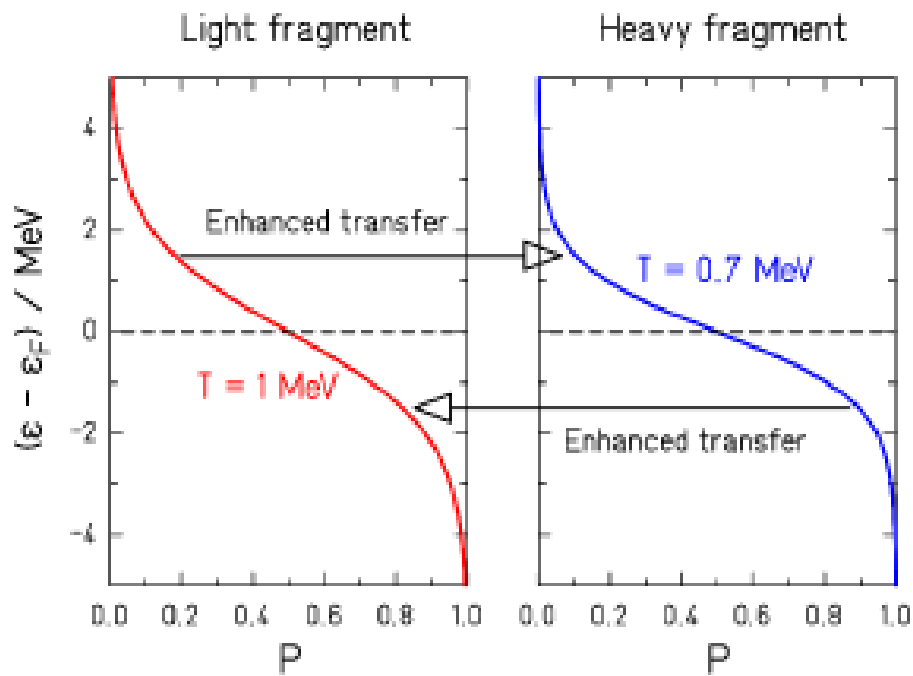
Heat is mainly transferred via nucleon exchange through the neck region

## Simple model calculation



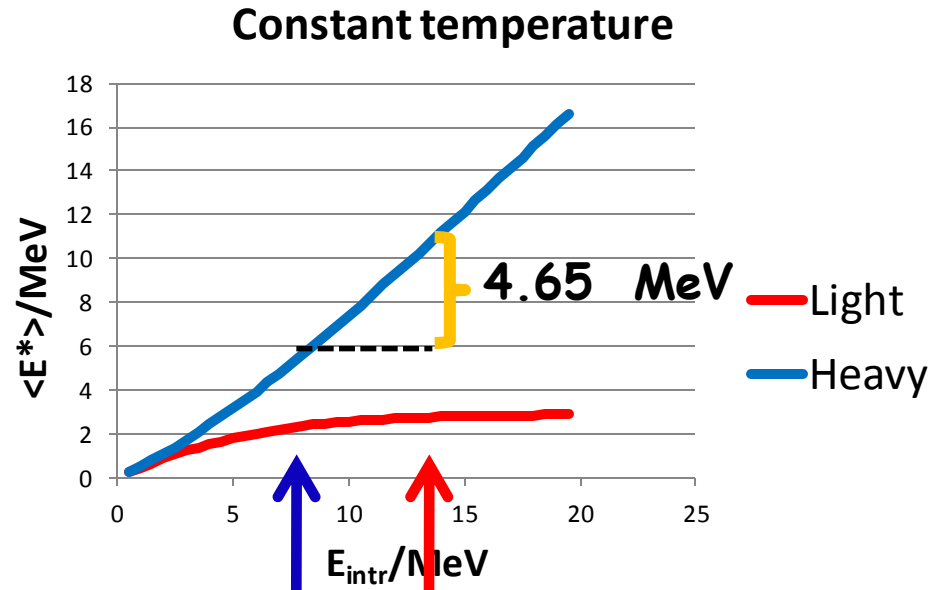
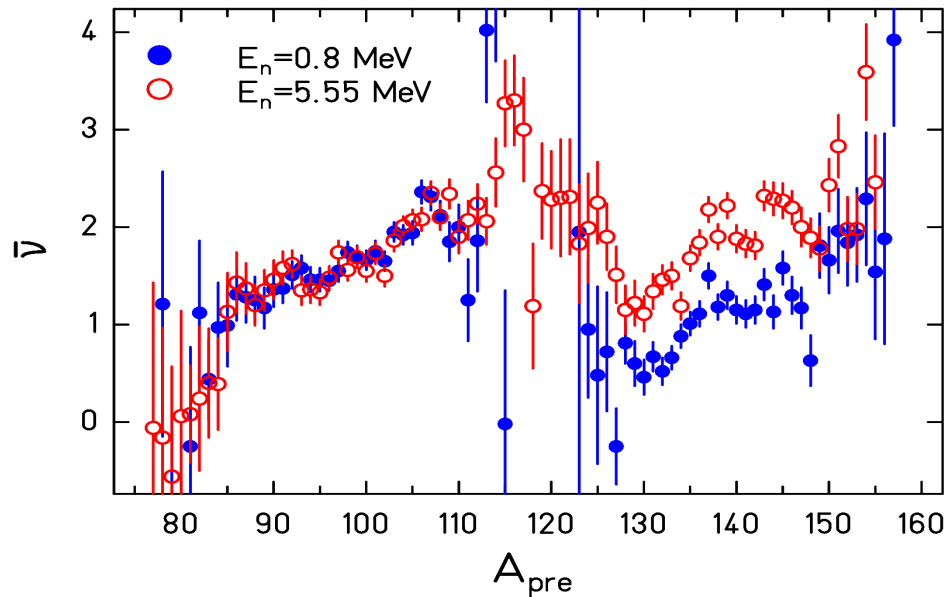
Probability distribution of the change  $\Delta E$  of the excitation energy of the light nucleus after one nucleon exchange.

Energy transfer occurs in large steps of about 0.96 MeV and is subject to strong fluctuations  $\sigma \sim 3$  MeV!!



Heat transport (Randrup NPA 327 (1979) 490)

# $\bar{\nu}$ en fonction de l'énergie : Signature du tri d'énergie d'excitation



L'augmentation du nombre de neutrons correspond  
à une augmentation d' $E^*_{lourd}$  de  $4.8 \pm 0.2$  MeV

→ Peut être seulement expliqué avec le tri d'énergie d'excitation!!!