

# Implementation of an analytical model accounting for sample inhomogeneities in REFIT

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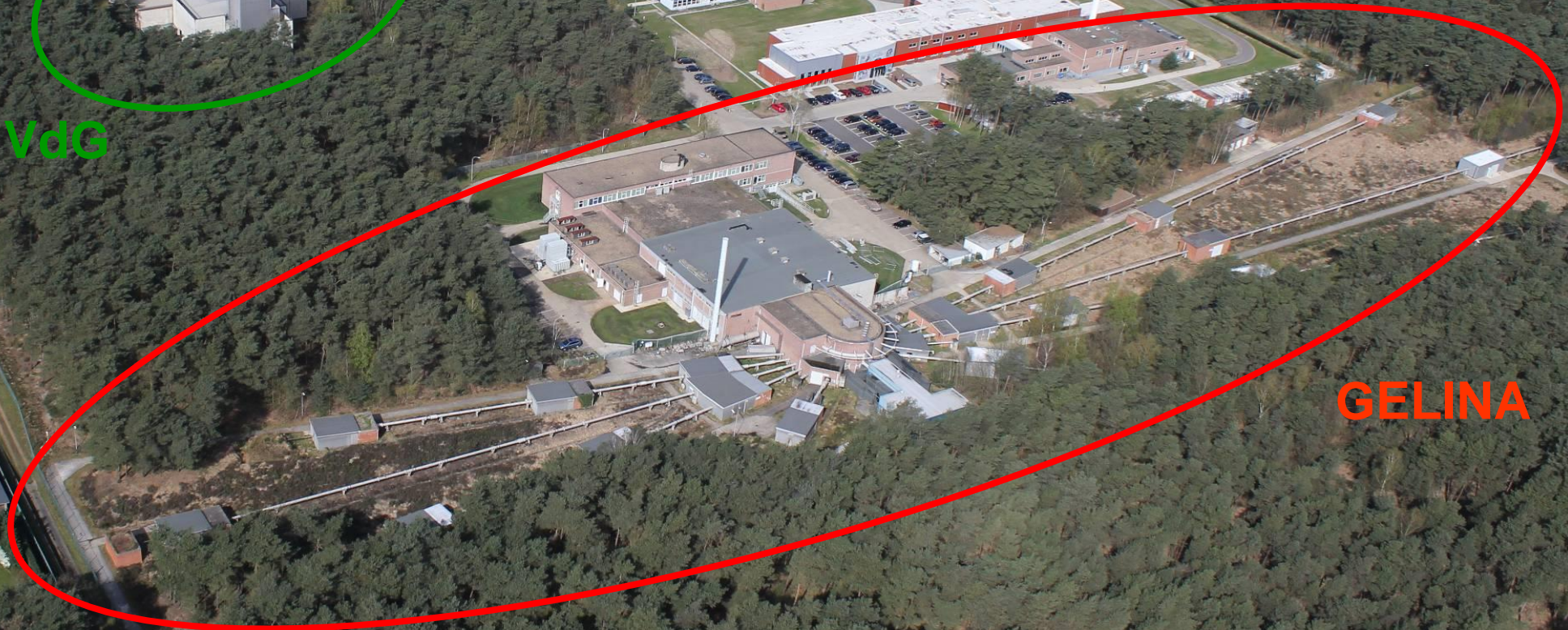
- **Introduction (EC - JRC- IRMM)**
- **Neutron Resonance Transmission and Capture Analysis**
- **Motivation**
- **Initial model validation**
- **Experimental model validation**
- **Summary**



JRC-IRMM major provider in Europe of ND for nuclear energy applications



VdG

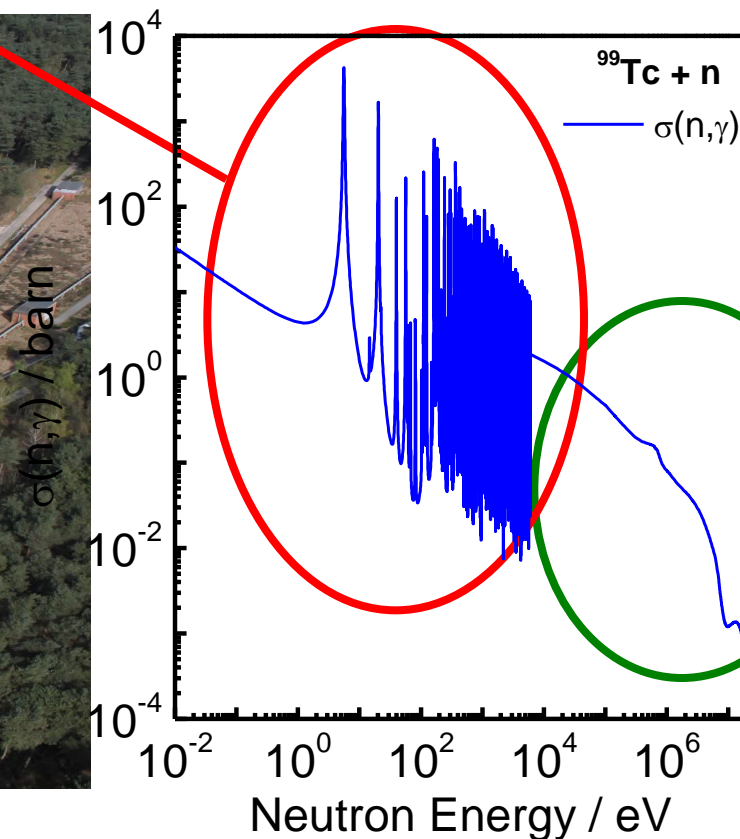


GELINA

## GELINA

## Van de Graaff

$^{99}\text{Tc}(n,\gamma)$  cross section

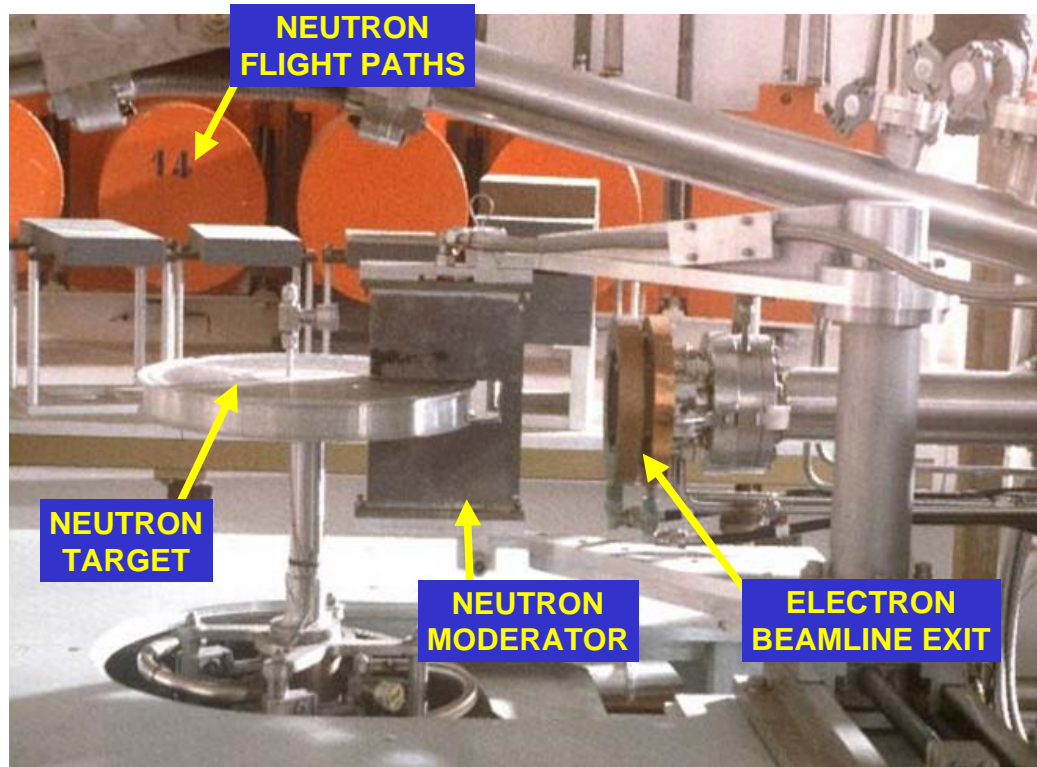


**White neutron source  
+  
Time-of-flight (TOF)**

**Mono-energetic neutrons  
(cp,n) reactions**



- Pulsed white neutron source  
( $10 \text{ meV} < E_n < 20 \text{ MeV}$ )
- High resolution TOF measurements
- Multi-user facility: 10 flight paths
- Flight path lengths: 10 m – 400m
- Measurement stations with special equipment to perform:
  - Total cross section measurements
  - Partial cross section measurements

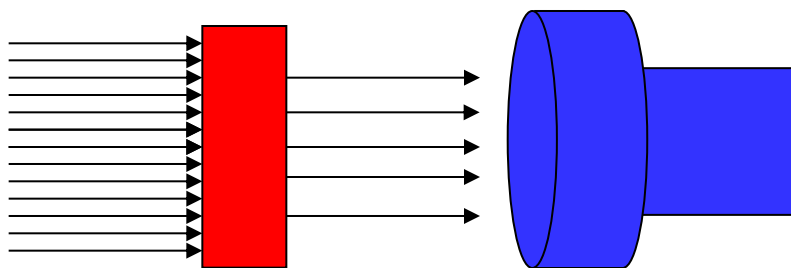


- $e^-$  accelerated to  $E_{e^-, \max} \approx 140 \text{ MeV}$
- Bremsstrahlung in U-target (rotating & cooled with liquid Hg)
- $(\gamma, n)$ ,  $(\gamma, f)$  in U-target
- Low energy neutrons by moderation (water moderator in Be-canning)

## Transmission

$$T \cong e^{-n \sigma_{\text{tot}}}$$

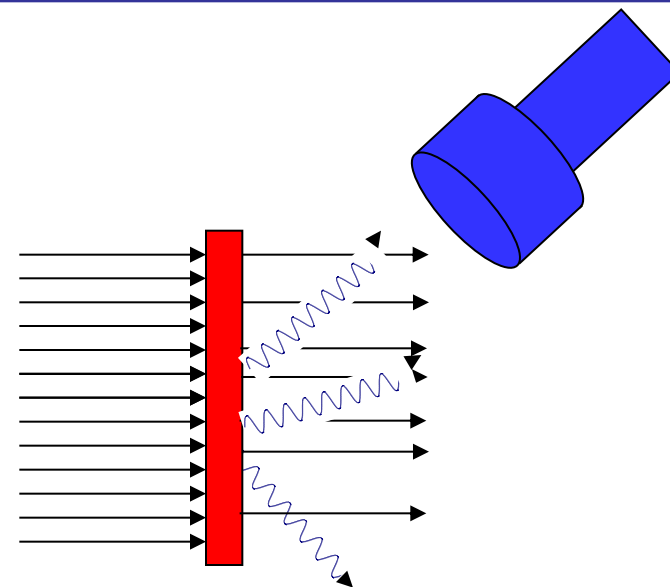
**T** : transmission  
Fraction of the neutron beam traversing  
the sample without any interaction



## Capture

$$Y_{\gamma} \approx (1 - e^{-n \sigma_{\text{tot}}}) \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}}$$

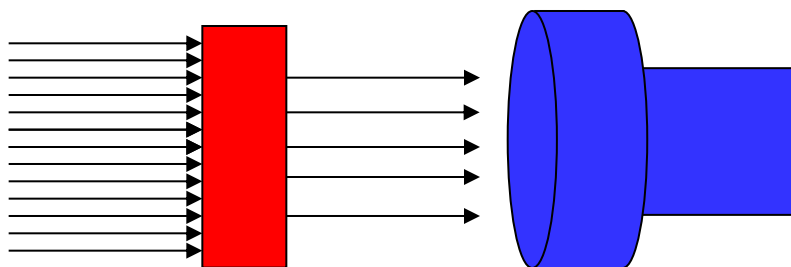
**$Y_{\gamma}$**  : capture yield  
Fraction of the neutron beam creating a  
(n, $\gamma$ ) reaction in the sample



## Transmission

$$T \cong e^{-n \sigma_{\text{tot}}}$$

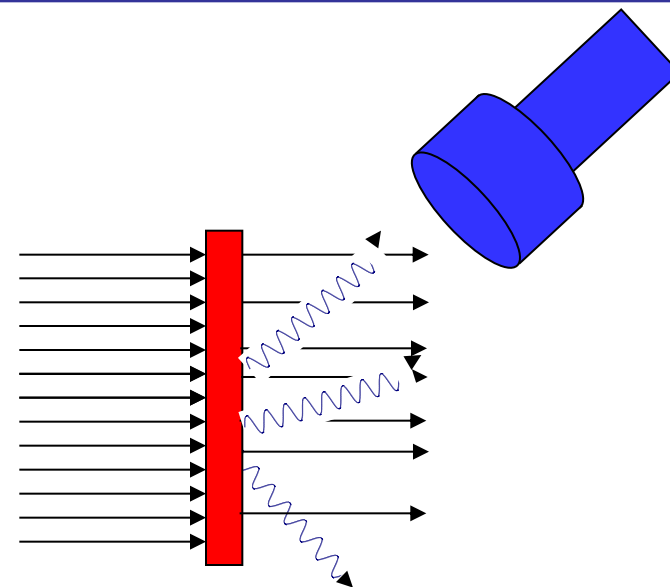
$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}}$$



## Capture

$$Y_{\gamma} \approx (1 - e^{-n \sigma_{\text{tot}}}) \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}}$$

$$Y_{\gamma,\text{exp}} = \frac{C_{\gamma}}{\varepsilon_{\gamma} \phi}$$

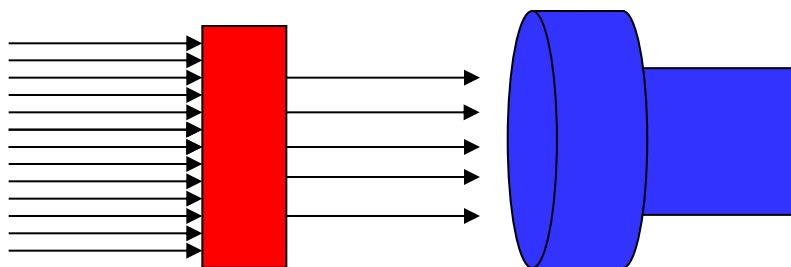




## Transmission

$$T \cong e^{-n \sigma_{\text{tot}}}$$

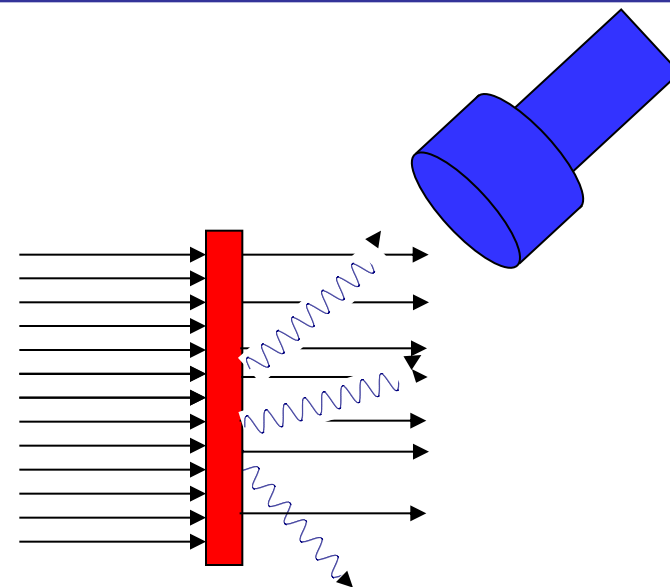
$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}}$$

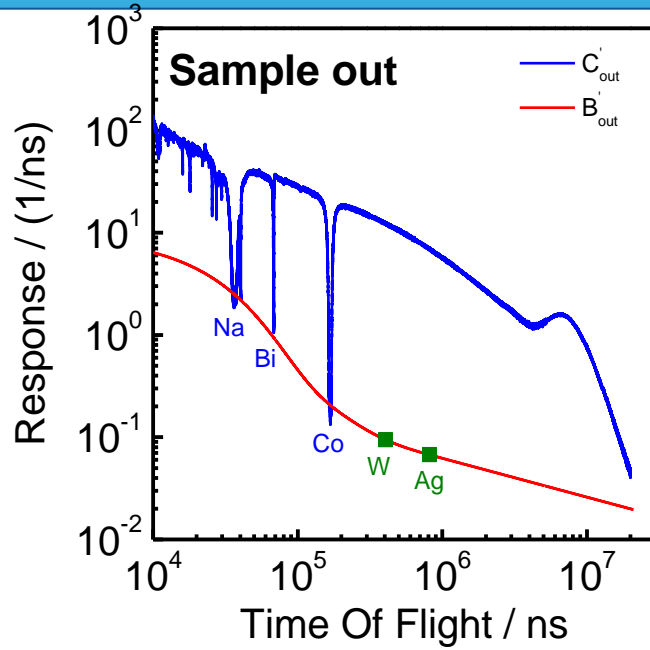


## Capture

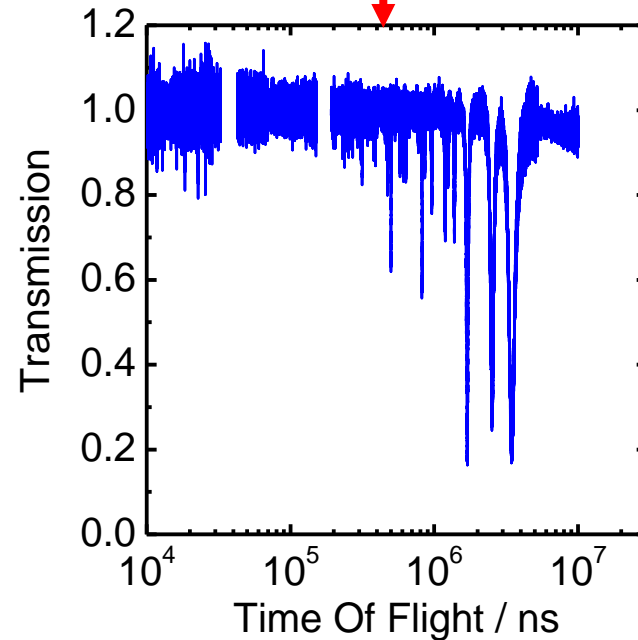
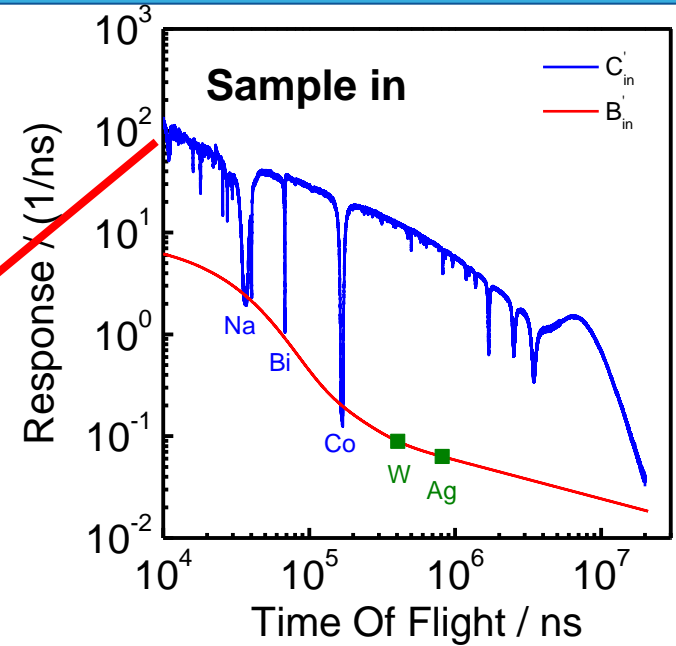
$$Y_{\gamma} \approx (1 - e^{-n \sigma_{\text{tot}}}) \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}}$$

$$Y_{\gamma,\text{exp}} = \frac{C_{\gamma}}{\varepsilon_{\gamma} \phi}$$

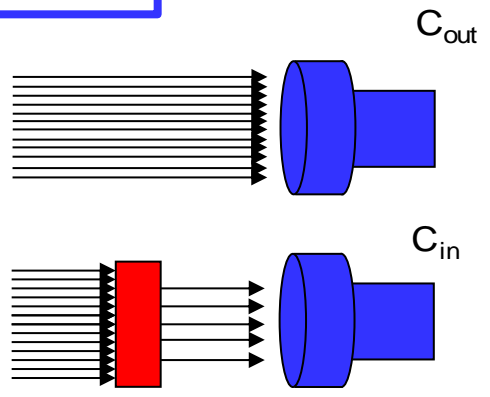




$$T_{exp} = \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}$$

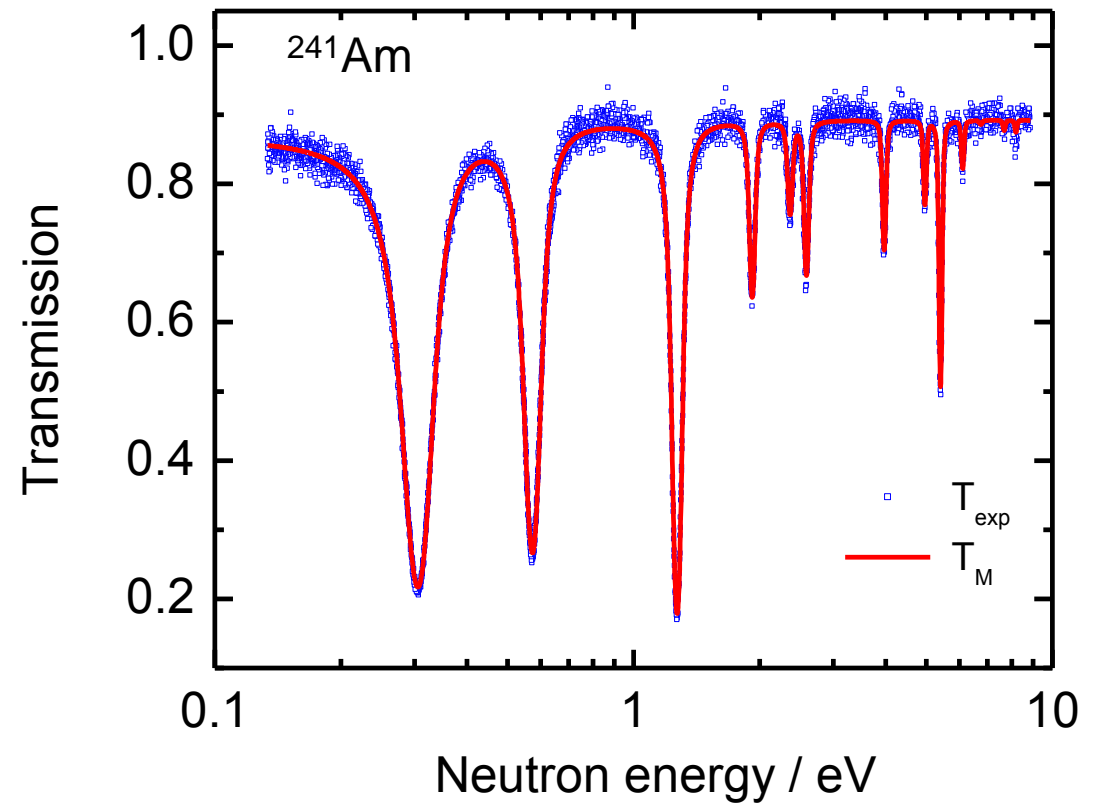


$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}}$$



$$T_M(t_m) = \int R(t_m, E) e^{-n \sigma_{\text{tot}}(E)} dE$$

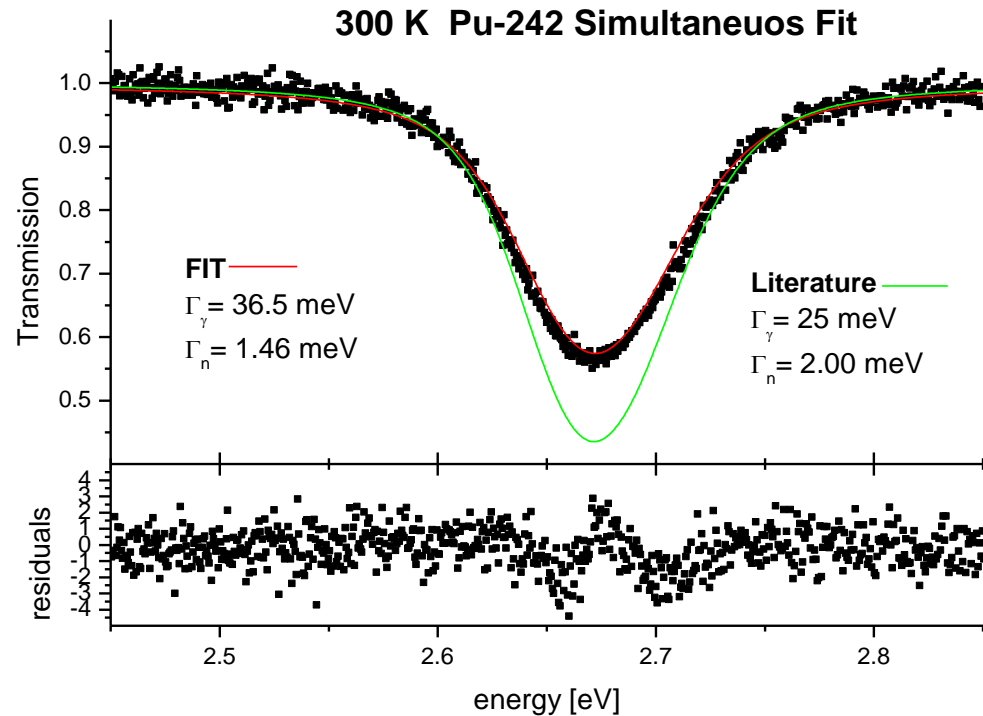
$R(t_m, E)$  response of TOF-spectrometer



REFIT, M. Moxon

## Ignoring sample inhomogeneities:

- Underestimation of  $\sigma_{\text{peak}}$
- Overestimation of  $\Gamma$



**Transmission + Capture data**



**sample properties**

**resonance parameters**

## Expected transmission

- **Homogeneous sample:**
- **Heterogeneous sample:**

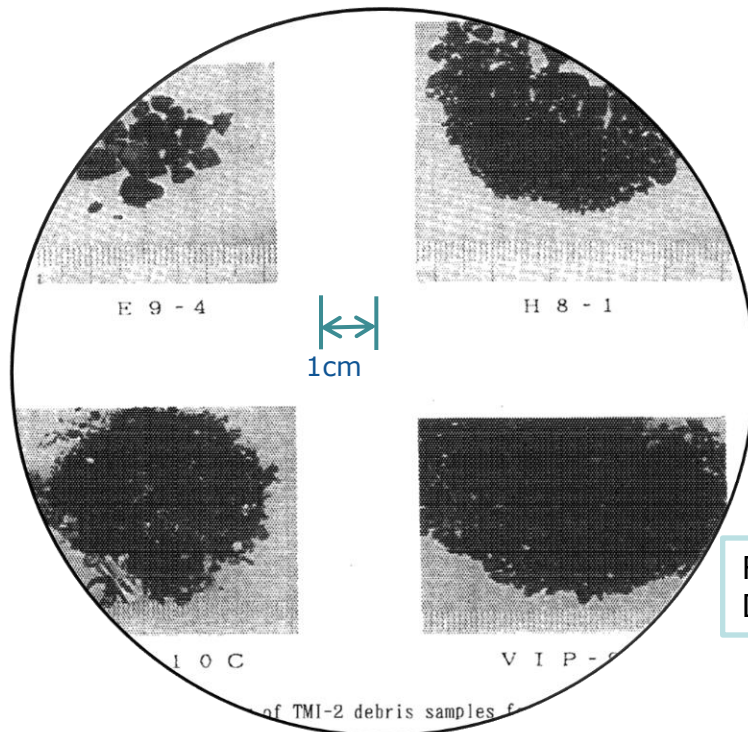
$$T = e^{-n \sigma_{\text{tot}}}$$

$$\bar{T} = \int T(n') p(n') dn' = \int e^{-n' \sigma_{\text{tot}}} p(n') dn'$$

$\neq$

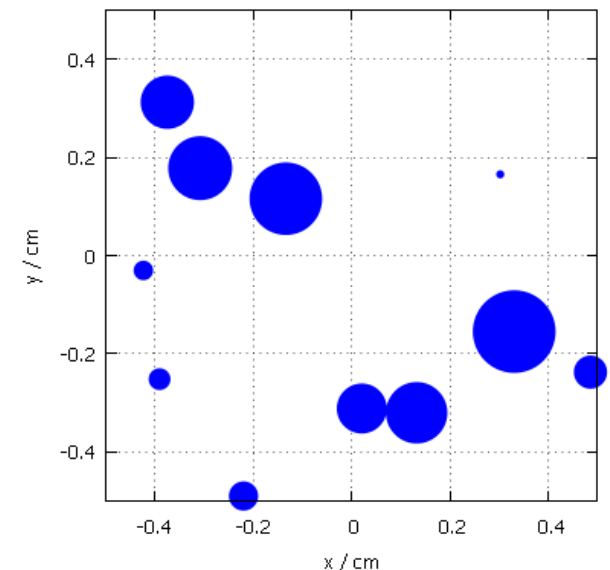
$$T(\bar{n}) = e^{-\bar{n} \sigma_{\text{tot}}} = e^{-\left(\int n' p(n') dn'\right) \sigma_{\text{tot}}}$$

**$p(n')$  : distribution of areal density**



From H. Uetsuka, et al., "Gamma Spectrometry of TMI-2 Debris" (written in Japanese), JAERI-Research 95-084.

- Analytical models to account for inhomogeneities of a powder sample:
  - **Kopeccky et al.** (ND2007)
  - **LP Model** (Levermore, Pomraning et al., J. Math. Phys. 27, 2526, 1986)
  - ...
- Comparison with transmission spectra produced by stochastic calculations (MC simulations)
- LP model performs the best



## ■ Kopeccky et al.:

- **Macroscopic model to describe the variation of the thickness**

Kopeccky et al., ND2007 , Nice , pp. 623 – 626; Schillebeeckx et al., NDS 113 (2012) 3054 - 3100

$$\bar{T} = \left[ \int e^{-\sum_k n'_k \sigma_{tot,k}(E)} p(x) dx \right] (1 - f_h) + f_h$$

$$n'_k = \frac{n_k}{1 - f_h}$$

$$p(x) = \frac{1}{x\sqrt{2\pi s^2}} \exp\left(-\frac{(\ln x + s^2/2)^2}{2s^2}\right)$$

## ■ LP Model:

- **Microscopic model**

Levermore, Pomraning et al., J. Math. Phys. 27, 2526, 1986

$$\bar{T} = \left\{ \frac{r_+ - \tilde{\Sigma}}{r_+ - r_-} \right\} e^{-r_+ R} + \left\{ \frac{\tilde{\Sigma} - r_-}{r_+ - r_-} \right\} e^{-r_- R}$$

$$\tilde{\Sigma} = p_\beta \lambda_\alpha + p_\alpha \lambda_\beta + \lambda_\alpha^{-1} + \lambda_\beta^{-1}$$

$$2r_\pm = \bar{\Sigma} + \tilde{\Sigma} \pm \sqrt{(\bar{\Sigma} - \tilde{\Sigma})^2 + 4\theta}$$

$$\theta = (\Sigma^\alpha - \Sigma^\beta)^2 p_\alpha p_\beta$$

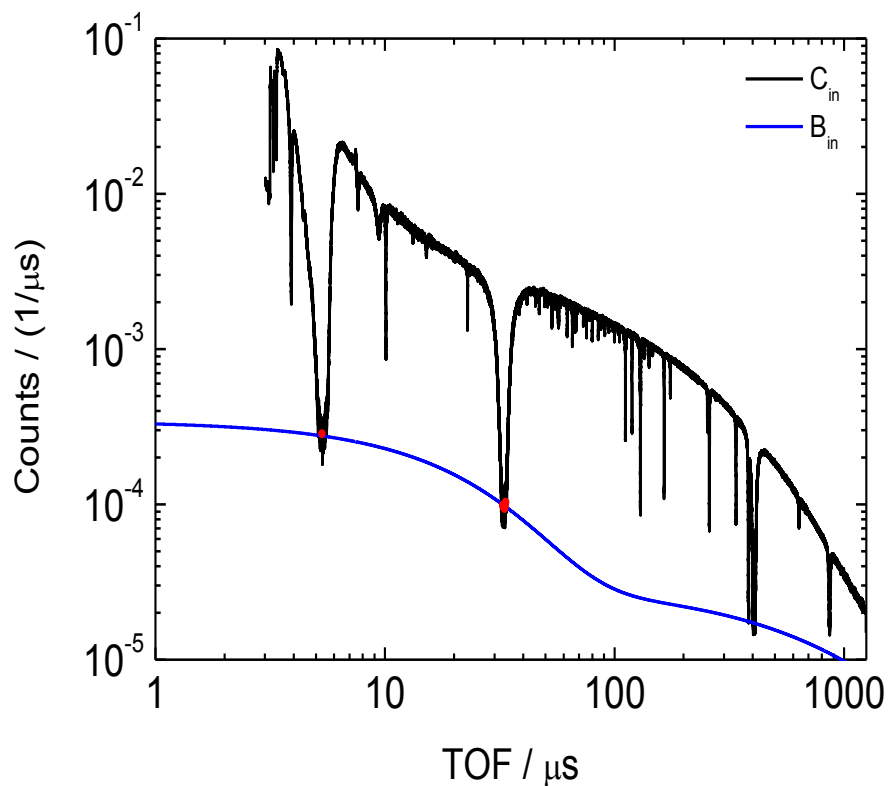


- **Experimental validation of the model**

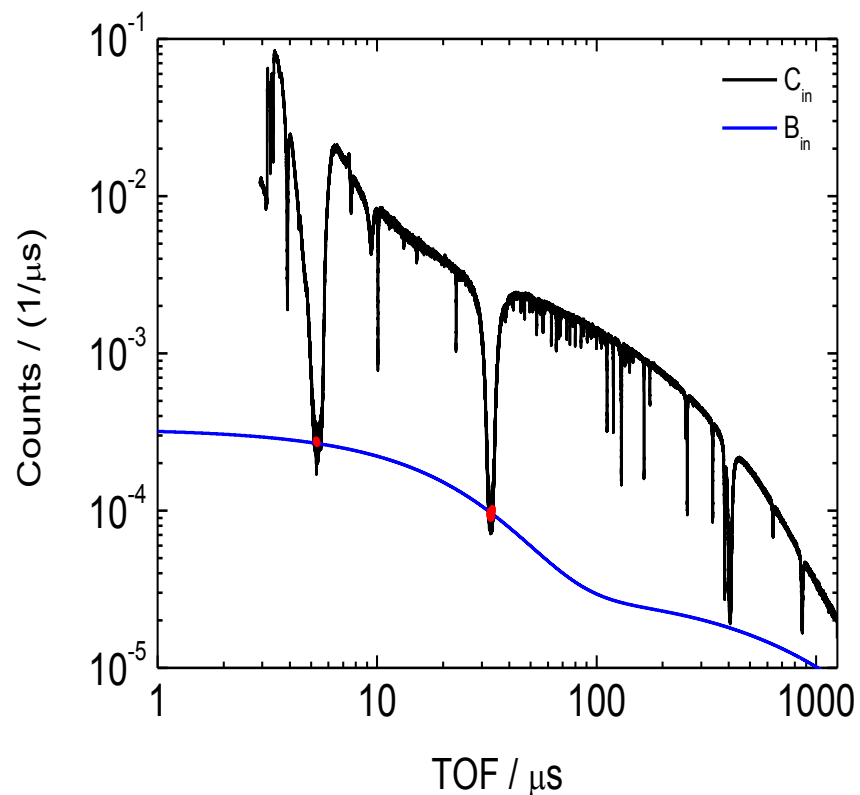
**= based on capture and transmission measurements at  
GELINA:**

- **Cu powder samples with known grain size distribution**
- **W powder samples with known grain size distribution**

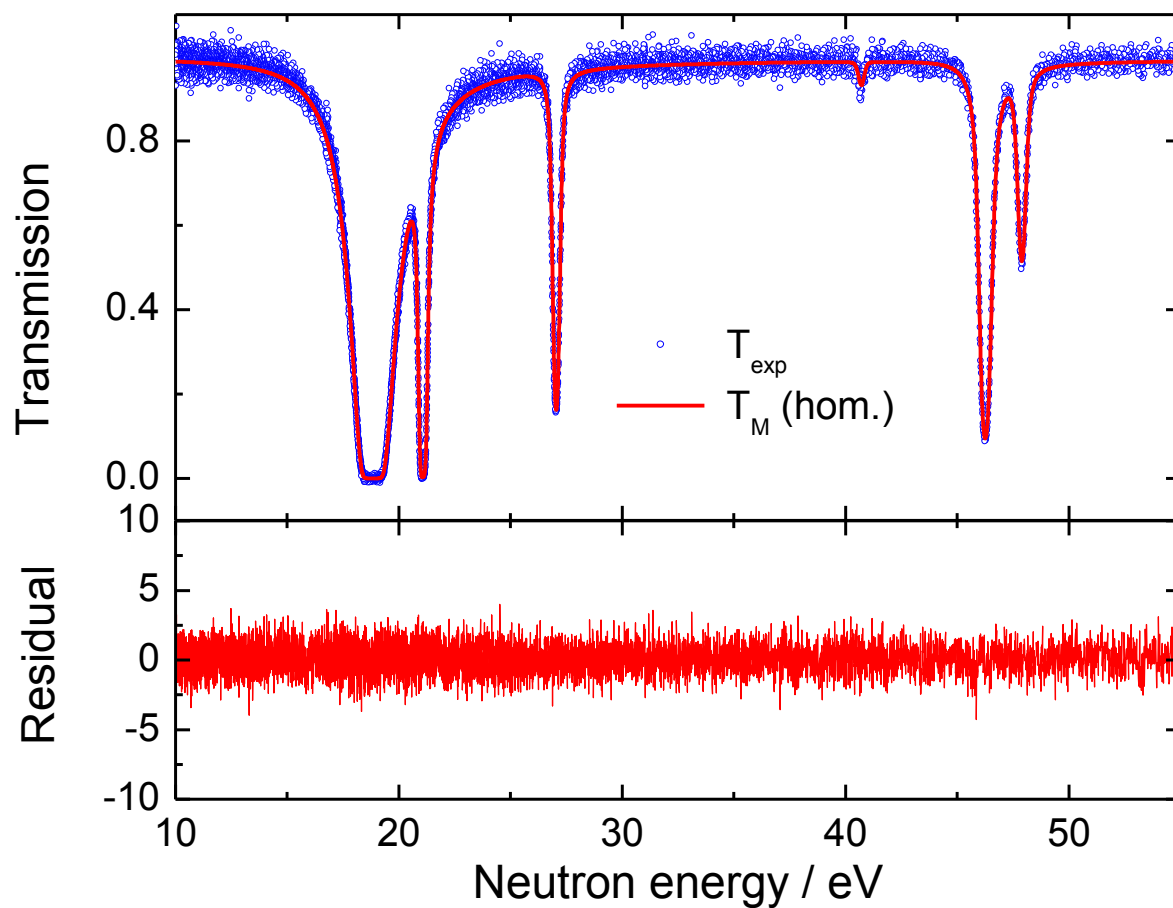
$^{nat}\text{W}$ -metal disc  
 (80 cm diameter, 14 g  $^{nat}\text{W}$ )



$^{nat}\text{W}$ -powder mixed with  $^{nat}\text{S}$ -powder  
 (80 cm diameter, 14 g  $^{nat}\text{W}$ , 3.5 g  $^{nat}\text{S}$ )

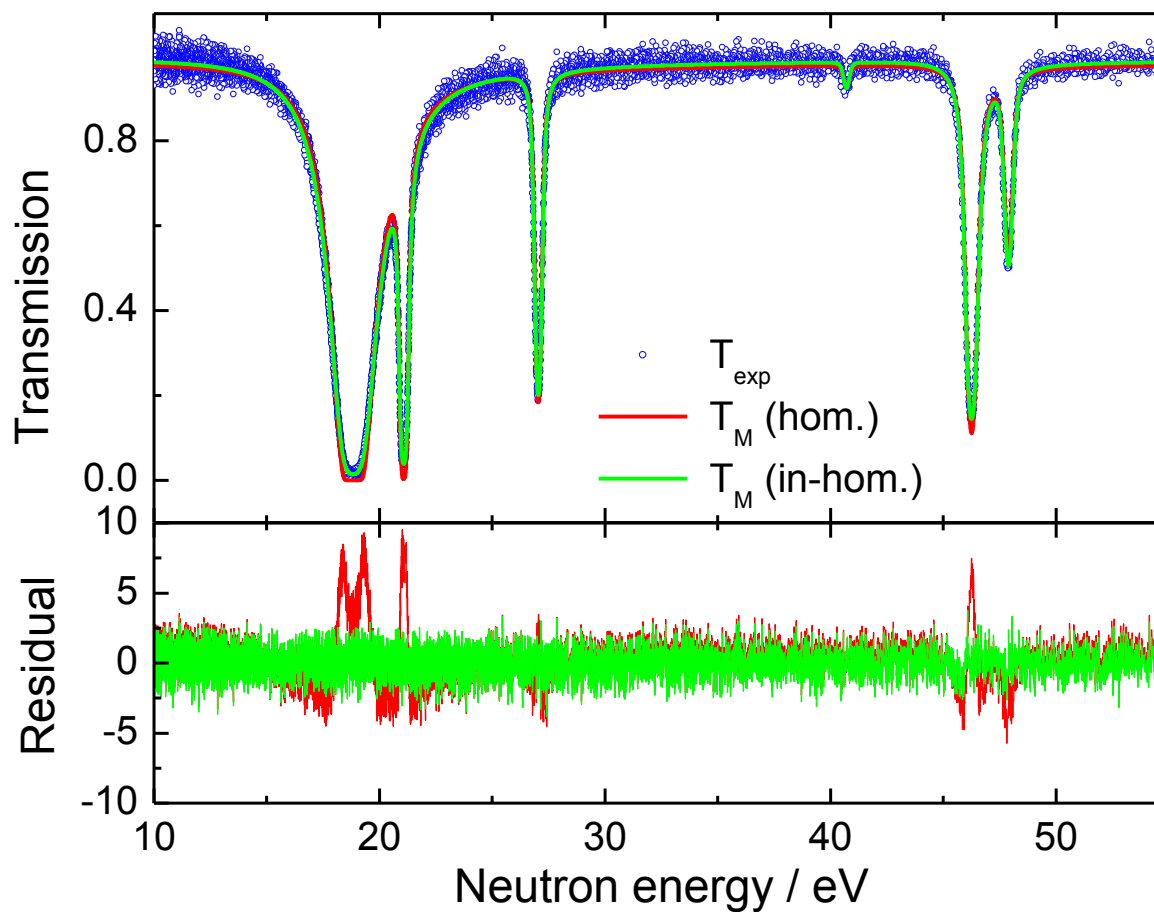


$^{nat}\text{W}$ -metal disc  
(80 cm diameter, 14 g  $^{nat}\text{W}$ )





$^{nat}\text{W}$ -powder mixed with  $^{nat}\text{S}$ -powder  
(80 cm diameter, 14 g  $^{nat}\text{W}$ , 3.5 g  $^{nat}\text{S}$ )



- **Model accounting for inhomogeneities of a powder sample is implemented in REFIT**
- **Initial validation of possible models with MC simulations**
  - LP Model performs the best**
- **Experimental validation of the model with capture and transmission measurements at GELINA**
  - W powder samples with known grain size distribution**

## Contributors:

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