Importance of nuclear triaxiality for electromagnetic strength, level density and neutron capture cross sections in heavy nuclei

E. Grosse, A.R. Junghans, and R. Massarczyk
TU Dresden and Helmholtz-Zentrum Dresden-Rossendorf

Radiative capture of fast neutrons

Nuclear shape and dipole strength

Level densities and nuclear shape

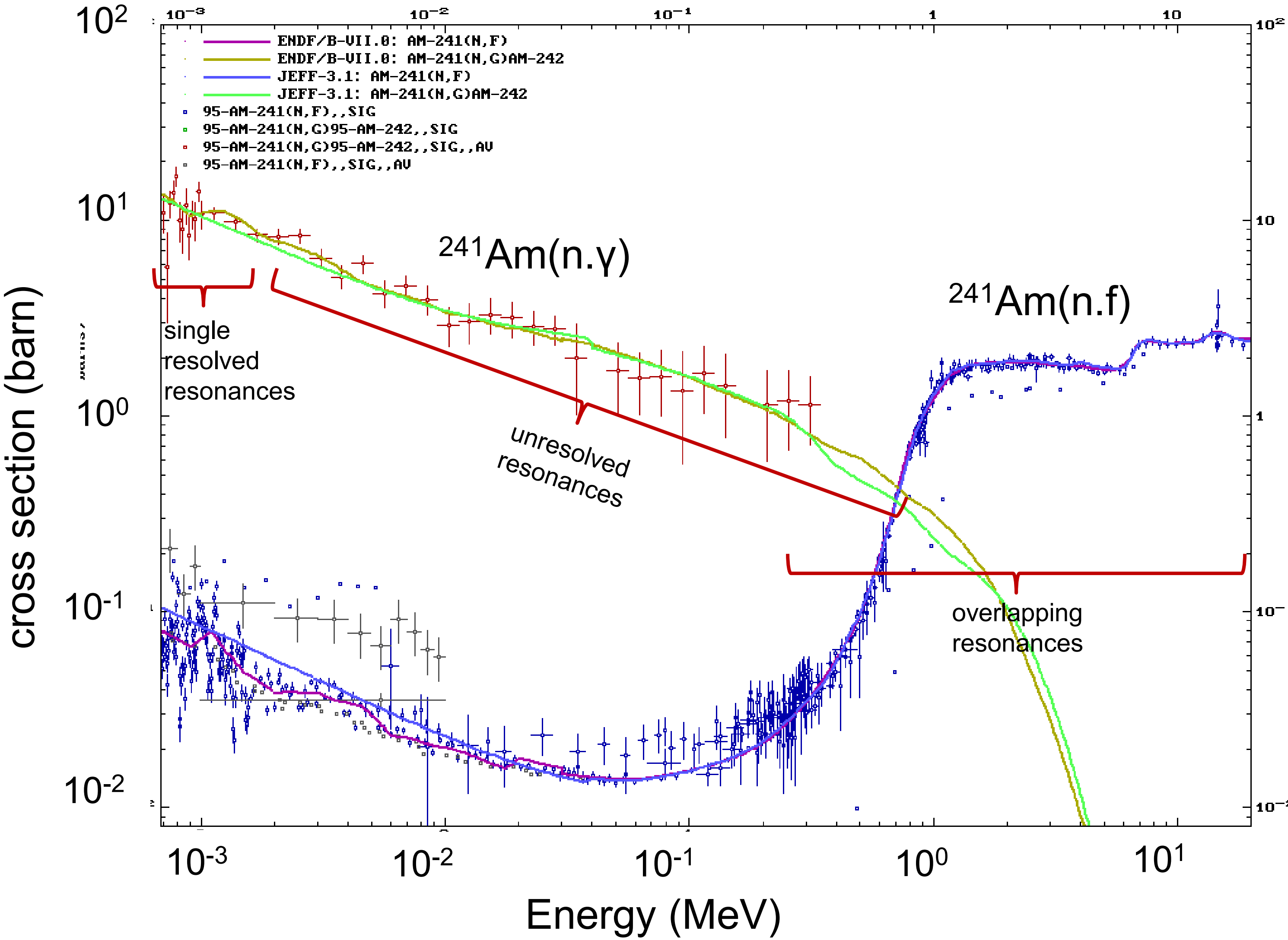
Electric dipole and other radiative strength

Predictions for capture cross sections

Work in progress - 2013
in collaboration with R. Beyer, R. Schwengner, A. Wagner and partners in EFNUDAT and ERINDA

ERINDA
HZDR

*Neutron induced **fission** in competition to **radiative neutron capture***



***better measurements** needed*
*as well as **model calculations***

Salvatores. NEA report No. 6090 (2006)

Radiative capture, *averaged over resonances **R***
*and summed over final states **f** in γ-decay of multipolarity λ, and ℓ_n*

$$\left\langle \sigma_{n,\gamma}(E_{\mathrm{n}}) \right\rangle_R \approx 2\pi^2 \lambdabar_{\mathrm{n}}^2 \sum_{\mathrm{f},\ell,\lambda} (2\ell+1) \left\langle \rho(E_R) \cdot \frac{\Gamma_{\mathrm{n}} \times \Gamma_\gamma(E_R - E_{\mathrm{f}})}{\Gamma_{\mathrm{n}} + \Gamma_\gamma} \right\rangle_R \qquad E_R = S_n + E_n = E_{\mathrm{f}} + E_\gamma \approx S_n$$

$\Gamma_{\mathrm{n}} \succ \Gamma_\gamma$ *formation · decay* $\ell \equiv \ell_{\mathrm{n}};\quad \lambda \equiv \lambda_\gamma;\quad \Delta \equiv \Delta(E_{\mathrm{n}})$

$$\cong 2\pi^2 \lambdabar_{\mathrm{n}}^2 \sum_{\ell,\lambda} (2\ell+1) \left\langle \sum_{\mathrm{m}} C(I_{\mathrm{R}}, I_{\mathrm{f}}, \lambda, n_{\mathrm{f}}) \int_0^{E_R} \rho(E_{\mathrm{f}}, I_{\mathrm{f}}) E_\gamma^{2\lambda+1} f_\lambda(E_\gamma)\, dE_\gamma \right\rangle_R \qquad E_\gamma^{2\lambda+1} f_\lambda(E_\gamma) = \Gamma_\gamma(E_\gamma)\rho(E_R)$$

Level density ρ enters for 0<E_{f}<E_{R},

$\boldsymbol{\rho(E_x,I) = \rho_{int}(E_x,I) \cdot K_{coll}(I,\beta,\gamma)}$

Photon strength f_λ *is assumed to depend on **E_γ** only,*

*and not on **E_x** (**Axel-Brink hypothesis**)*

*Average radiative capture cross section is proportional to **$\rho(E_{\mathrm{f}},\mathrm{I_f})$** and to photon strength **$f_\lambda(E_\gamma)$;***

Both, *are influenced by nuclear symmetry,* i.e.***shape*** (***β*** *and* ***γ***)**.**

For many*, if not nearly all,* ***heavy nuclei*** *only the* ***ℛ -symmetry*** *is formally well established,*

whereas usually ***spherical*** *or* ***axial symmetry*** (*β and γ*) *are assumed* ***ad hoc****.*

Triaxial shapes are of importance for ***radiative neutron capture***

as well as for ***fission,*** *and thus for* ***transmutation*** *physics.*

Feshbach et al., Phys. Rev. 71, 145 (1947)
Hughes et al., Phys. Rev. 75, 1781 (1949)
Bartholomew et al., Adv. in Nucl. Phys. 7 (1973) 229

ERINDA
HZDR

For many properties of heavy nuclei ***triaxiality*** *plays an important role – an issue* ***not*** *contained in many* ***model calculations****.*

Probabely the disregard of ***triaxiality*** *is related to numerical problems (of theorists), resulting from performing the* ***angular momentum projection*** *in three dimensions exactly.*

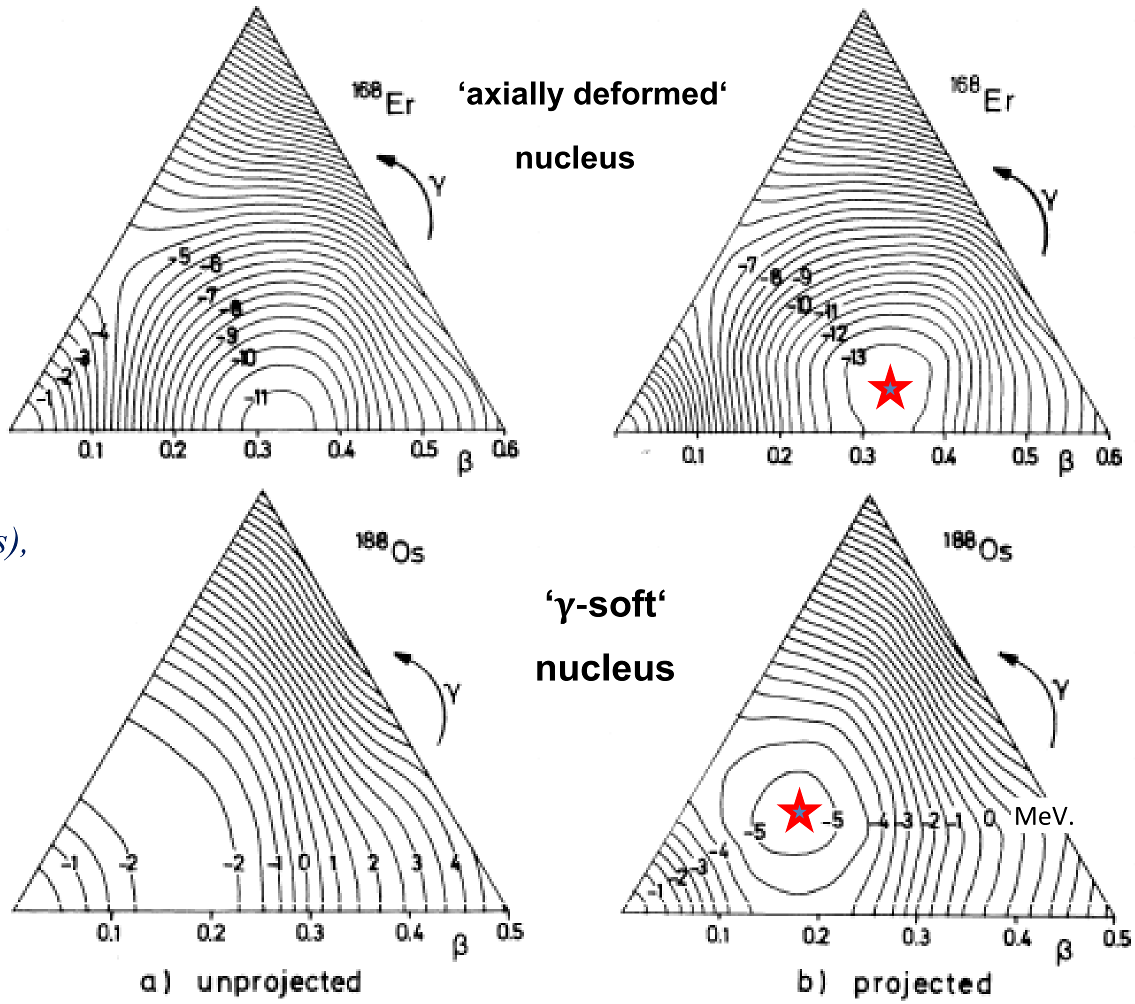
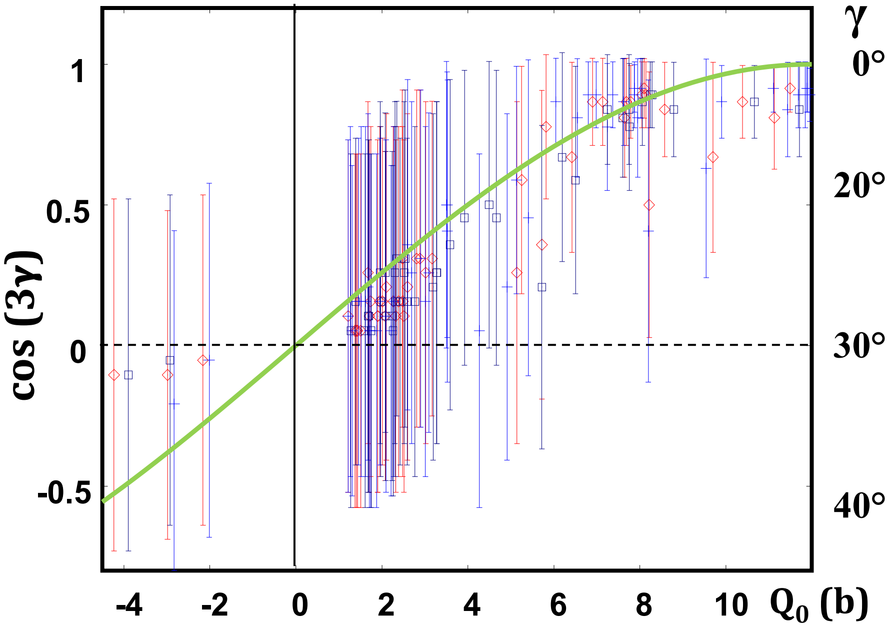


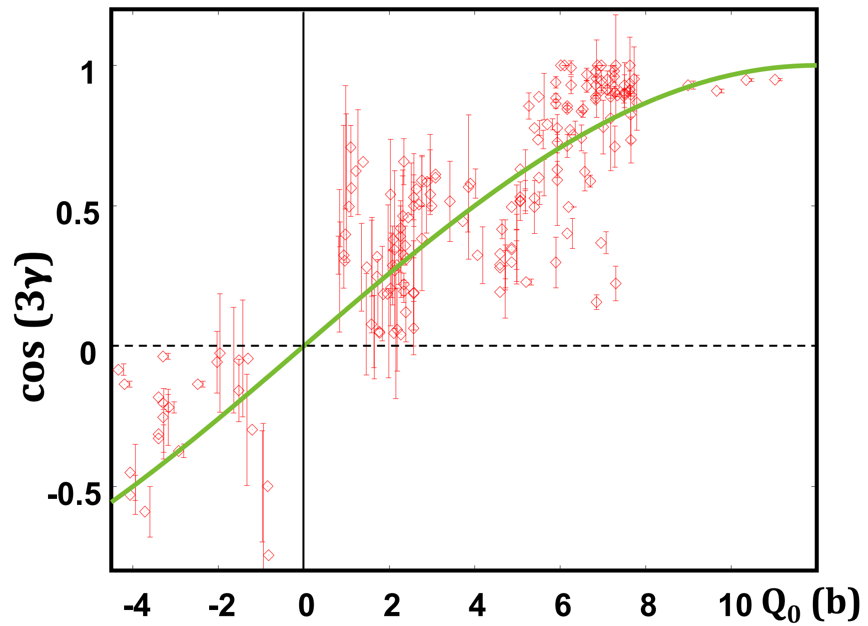
FIG. 1. Energy surface in the β-γ plane for the nuclei ^{168}Er and ^{188}Os (a) without angular momentum projection and (b) with exact three-dimensional angular momentum projection. The units on the equipotential lines are megaelectron-

Hayashi, Hara, Ring , PRL 53 (1984) 337

Triaxial deformation

seen in ***Hartree-Fock-Bogolyubov*** *calculations with Gogny force in accord to* ***Coulex data*** *.*





CHFB calculation *for stable isotopes,*
bars indicate variance.
Calculation also performed for exotic nuclei
=> ***global predictions*** *can be based on them.*

Coulex-data, *rotation invariant analysis,*
with experimental uncertainty bars.
=> Rigid triaxial deformation.

Bertsch et al., PRL 99 (2007) 032502
Delaroche et al., PRC 81 (2010) 014303
(CEA/DAM & UoWash.)

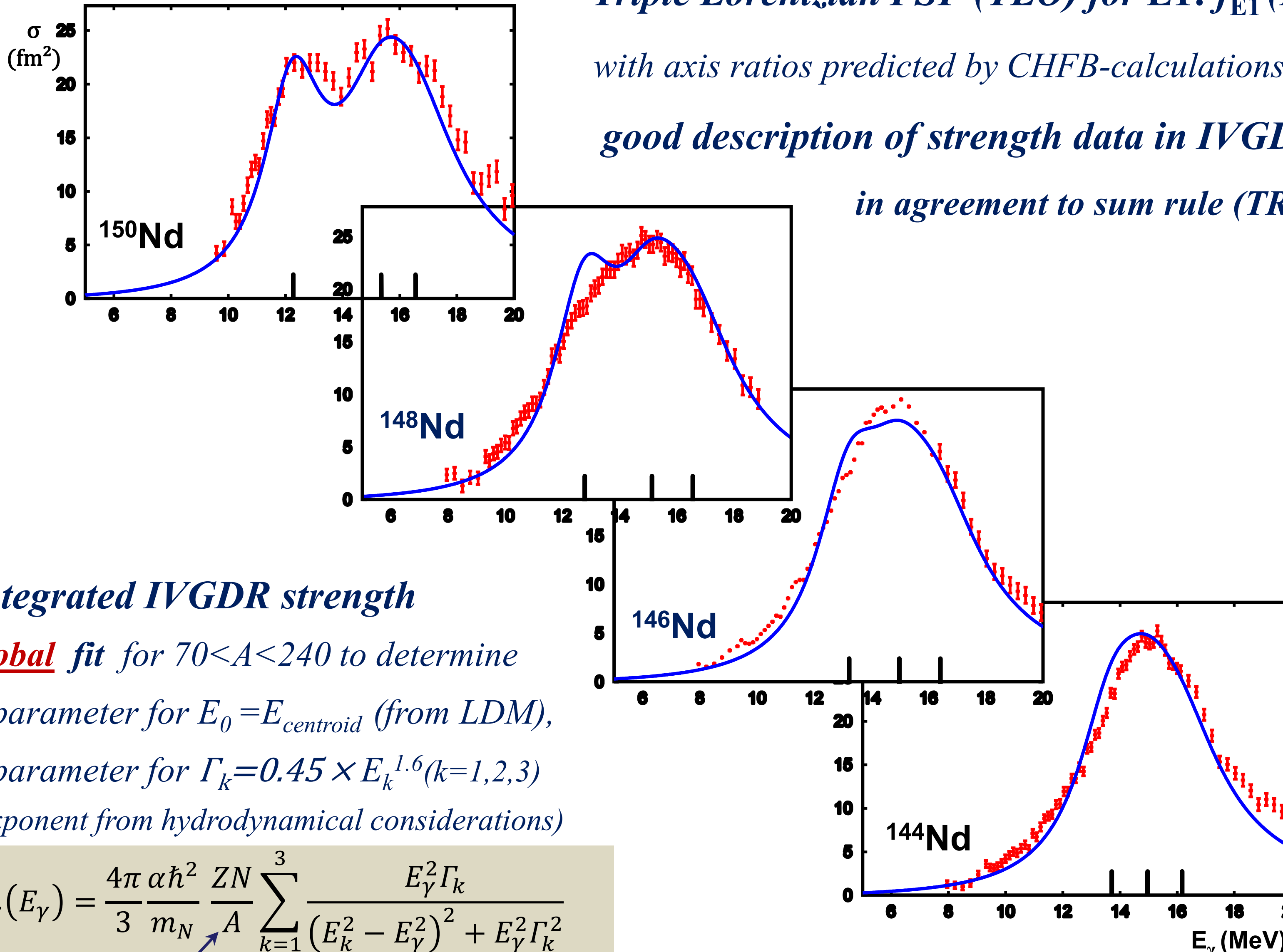
Toh et al., PRC 87 (13) 041304 (**Gamma-sphere**)
Cline, Annu. Rev. Nucl. Part. Sci. **36 (86) 683**
Srebnry, Czosnyka et al., NP A **766** (06) 25
(Rochester-Warsaw collaboration)

Triple Lorentzian PSF (TLO) for E1: $f_{E1}(E_\gamma)$

with axis ratios predicted by CHFB-calculations ▶

good description of strength data in IVGDR,

in agreement to sum rule (TRK).



Integrated IVGDR strength

global fit for 70<A<240 to determine

1 parameter for $E_0 = E_{centroid}$ (from LDM),

1 parameter for $\Gamma_k = 0.45 \times E_k^{1.6}$ (k=1,2,3)

(exponent from hydrodynamical considerations)

$$\sigma_{abs}(E_\gamma) = \frac{4\pi}{3}\frac{\alpha\hbar^2}{m_N}\frac{ZN}{A}\sum_{k=1}^{3}\frac{E_\gamma^2\Gamma_k}{\left(E_k^2 - E_\gamma^2\right)^2 + E_\gamma^2\Gamma_k^2}$$

TRK-sum rule

Junghans et al., PLB 670 (08) 200; Myers et al., PR C 15 (77) 2032
Bush and Alhassid, NPA **531** (91) 27; Delaroche et al., PRC 81 (10) 014303

Carlos et al., NP A 172 (71) 437

Triple Lorentzian PSF (TLO)

also fits **E1**-*data very well for nuclei*

usually considered spherical

(axis ratios from CHFB calculations

and integral from TRK sum rule).

Instantaneous shape sampling

improves fit (CHFB → variance).

$$f_{E1}(E_\gamma) = \frac{4\,\alpha}{3\pi\, g\, m_N c^2} \frac{ZN}{A} \sum_{k=1}^{3} \frac{E_\gamma \Gamma_k}{\left(E_k^2 - E_\gamma^2\right)^2 + E_\gamma^2 \Gamma_k^2}$$

Integrated strength, shape of IVGDR

and agreement to sum rule (TRK)

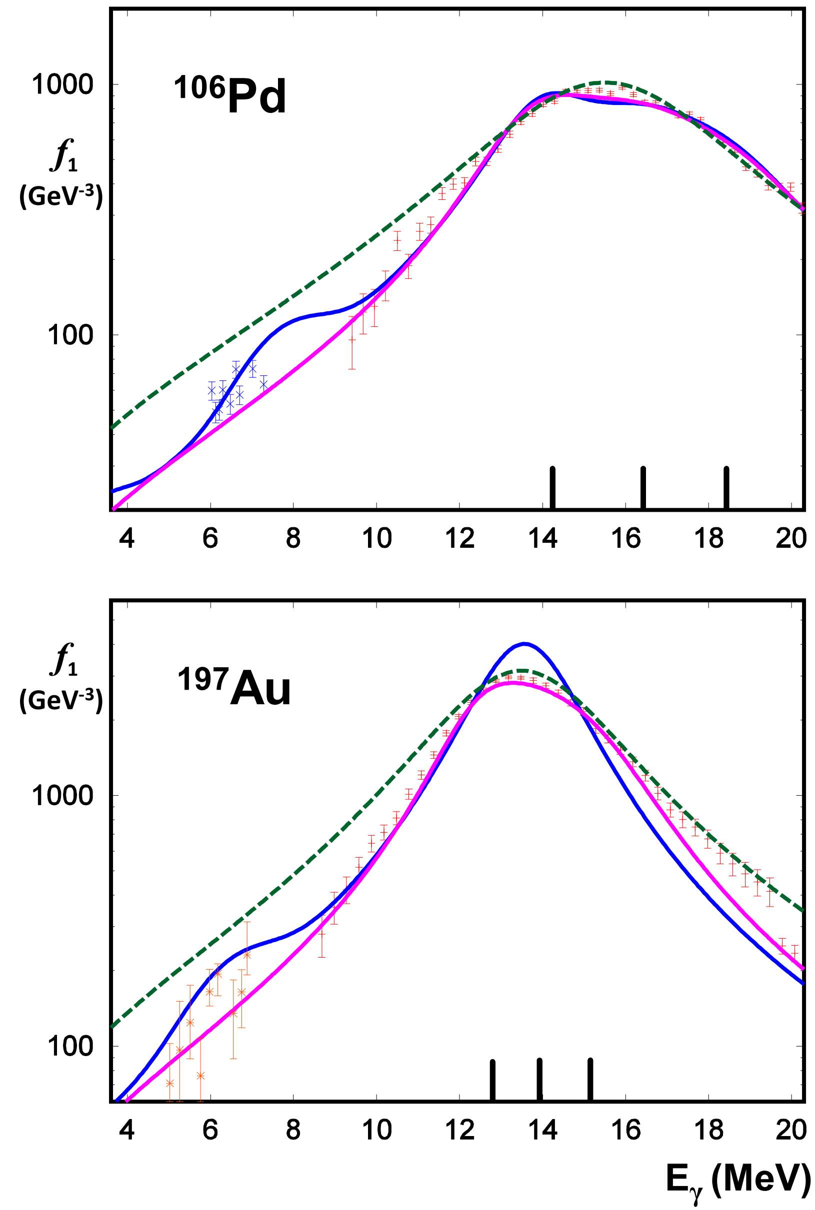
are well predicted.

The tail region is confirmed by ***various*** *data;*

additional components needed to improve fit.

Different tails *as* **RIPL-3** *with* **SLO**,

the standard distibuted by **IAEA**.



Junghans et al., PLB 670 (2008) 200
Pluiko, ADNDT 97 (2011) 567

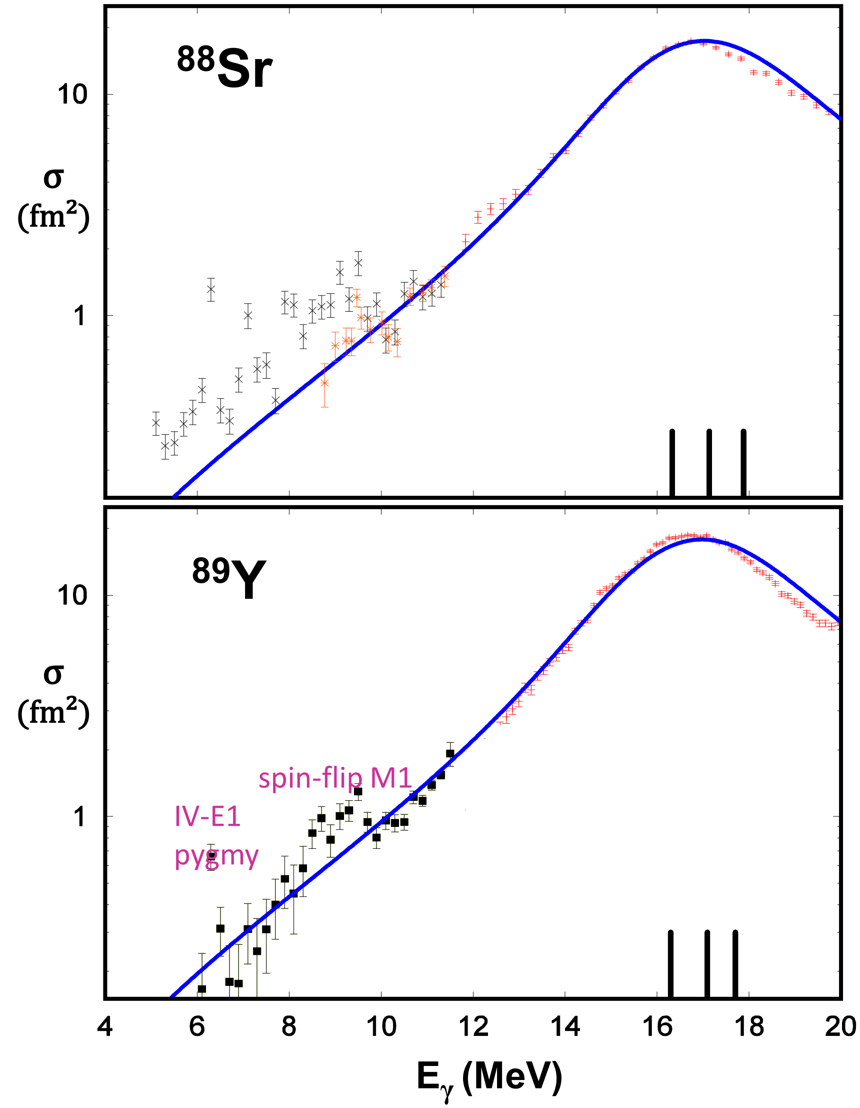
A. Leprêtre et al., NPA 175, 609 (1971) ; J.Kopecky & M.Uhl, Phys. Rev. C 41 (1990) 1941
A. Veyssière et al., NPA159, 561 (1970) ; G.A.Bartholomew et al., Adv. in Nucl. Phys. 7 (1973) 229

*A **triple** **Lorentzian (TLO)** fits* **E1**-
data for odd nuclei (A>70) equally well
with axis ratios from CHFB and
E_k from LDM (even neighbors);
only global fit parameters.

$$\sigma_{abs}(E_\gamma) = g_{eff}(\pi\hbar c)^2 E_\gamma f_\lambda(E_\gamma)$$

$$g_{eff} = \sum_{r=1,m} \frac{2J_r+1}{2J_0+1} = 2\lambda+1 \; for\ all\ J_0$$

$$m = \min(2\lambda+1,\ 2J_0+1)$$



Integrated strength, shape of IVGDR
and agreement to sum rule (TRK)
are very similar for odd nuclei
and even neighbours;
*the tail region is confirmed by **ELBE** data.*
but at variance to **RIPL-3**, *distibuted by* **IAEA**.

For n-capture by e-e target nuclei the photon strength in e-o nuclei is important !

Myers et al., PR C 15 (77) 2032
Bush and Alhassid, NPA **531** (91) 27

S. Datta et al., Phys. Rev. C 8 (73) 1421; R. Schwengner et al., Phys. Rev. C 76 (07) 034321
A. Leprêtre et al., Nucl. Phys. A 175 (71) 609; N. Benouaret et al., Phys. Rev. C 79, 014303 (09)

*The **intrinsic state density** $\rho_{\text{int}}(E_x) = \frac{e^S}{\sqrt{d}}$ is sensitive to **shell, deformation** and **pairing** corrections:*

*Effects **of shells and deformation** are known from masses and liquid drop calculations,*

but damped from δW_o (at the ground state) with increasing t: $\delta W(t) = \delta W_o \cdot \frac{\tau^2 \cosh\tau}{(\sinh\tau)^2} \xrightarrow{t\to\infty} 0$, $\xrightarrow{t\to 0} \delta W_o$; $\tau = \frac{2\pi^2 t}{\hbar\varpi_{sh}} A^{1/3}$

as controlled by the average shell energy $\hbar\varpi_{sh} \cong \hbar^2/m_N A^{1/3} \cong 41/A^{1/3}$ MeV.

***Pairing** causes a condensation at $\boldsymbol{E_{con}}$ and a critical temperature $\boldsymbol{t_c}$:* $\Delta_0 = \frac{12}{\sqrt{A}}$; $E_{con} = \frac{3a\Delta_0^2}{2\pi^2}$; $\frac{t_c}{\Delta_0} = 0.567$.

*with the level density parameter **a** approximately given by* $a \cong \pi^2 A/4\varepsilon_F \cong A/14$.

A large backshift $E_{bs} = E_{con} + n\,\Delta_0$ (n= 0,+1,+2 *for o-o, odd and e-e nuclei*) *is **positive** – also for small A*

*and **reduces*** $\rho_{\text{int}}(E_x) = \frac{e^S}{\sqrt{d}}$.

At energy $E_c = at_c^2 + E_{bs} - \delta W(t_c)$ *a phase transition occurs – in the two phases **S** and $\boldsymbol{E_x}$ are given by:*

*In the **super-fluid phase (SFM)** ($t<t_c$) an interpolation from E_c to E_{gs} is controlled by:*

$$S\,t = F_{SF} S_c\, t_c; \qquad E_x = F_{SF} E_c$$

$$S_c = S(t_c, FG); \qquad F_{SF} = \left(\frac{t}{t_c}\right)^{2.5} \xrightarrow{t\to t_c} 1, \quad \xrightarrow{t\to 0} 0$$

$$d(E_x, SF) = d_c = d(t_c, FG)$$

*In the **Fermi-gas (FGM)** phase ($t>t_c$) one has:*

$$S = 2at - \frac{\delta W(t)}{t} + \frac{\delta W_0}{t}\frac{\tau}{\sinh(\tau)} \xrightarrow{t\to\infty} 2at, \quad \xrightarrow{t\to 0} 0$$

$$E_x = at^2 + E_{bs} - \delta W(t) \xrightarrow{t\to\infty} E_{bs} + at^2, \quad \xrightarrow{t\to 0} E_{bs} - \delta W_0$$

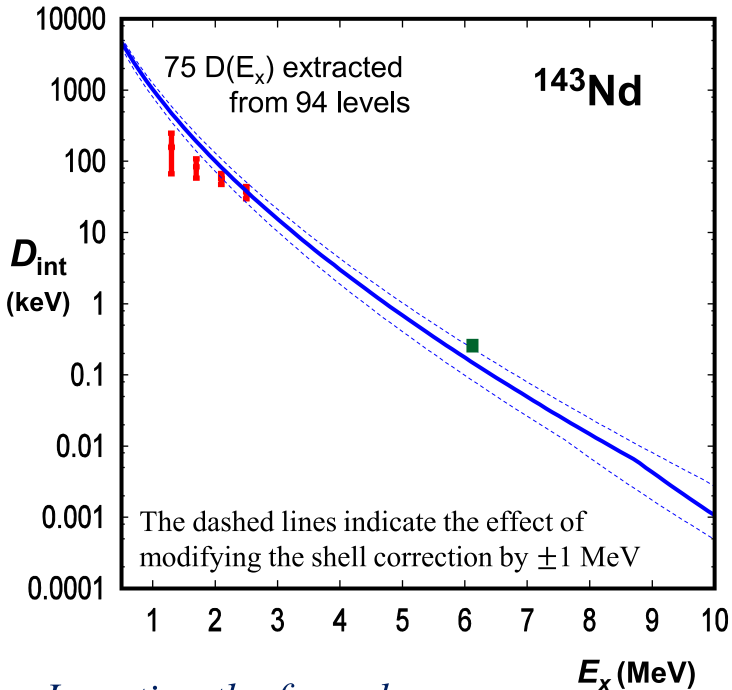
$$d(t, FG) = \frac{18}{\pi} S^3 t^2$$

*The determinant **d** relates to the saddle point method.*

Ignatyuk et al., PRC 47 (93) 1504;
Landau-Lifschitz V, §59; Goriely, NP A 605 (96) 28

Schmidt & Jurado, PRC 83, 014607 (11);
Bohr and Mottelson, vol. I, 2 & 2B (69) & II, 6-523 (75) ; Kataria & Kapoor, PRC 18 (78) 549.

*The **level density** formalism proposed here compares well to s-wave resonances and to bound states – and the temperatures derived from them*



Inverting the formula

$$\frac{1}{D(E,I)} = \rho(E,I) \cong \frac{2I+1}{4} \cdot \rho_{int}(E) = \frac{2I+1}{4D_{int}(E)}$$

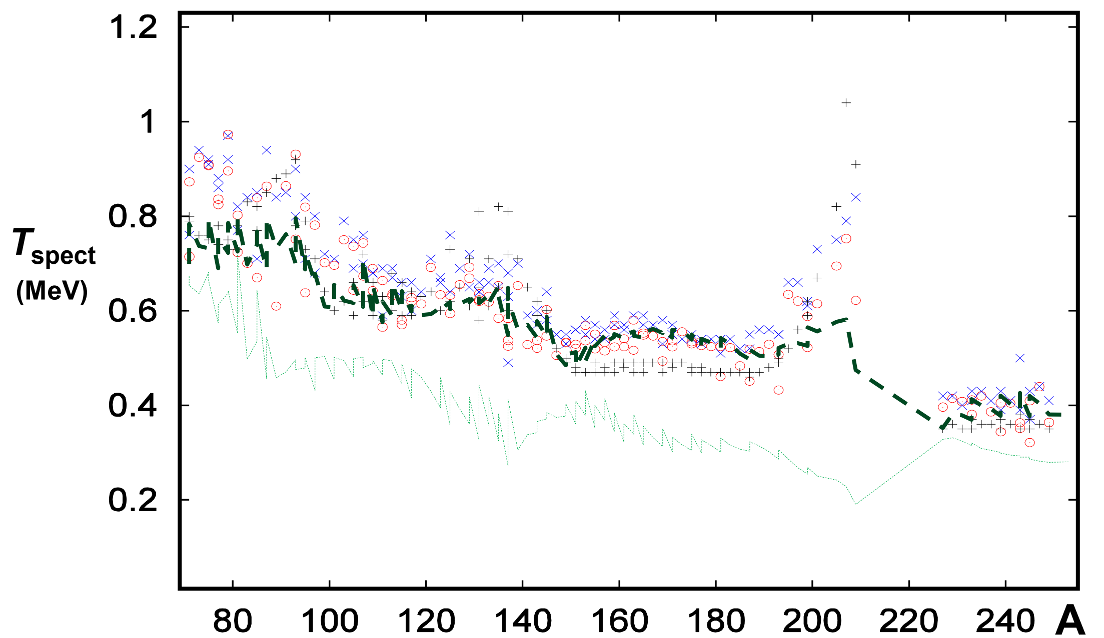
*allows to extract ρ_{int} from the observed average level distances D(E,I) for **all I**;*

when σ is taken from systematics.

D_{int}(bound levels, various I^{π})

D_{int}(S_n, ½$^+$)

Ignatyuk (2009), www-nds.iaea.org/RIPL-3/resonances/resonances0.dat



*The 'spectral' temperatures as derived **locally** from excited levels – with **significant scatter** – by:*

Koning et al., NPA 810 (08) 13 +

v.Egidy et al., PRC 80 (09) 054310 x

Belgya et al., RIPL-2/3 o

*agree reasonably well to the **prediction:** - - - - - - -, **based on triaxiality only and on global parameters*** **(a =.07(A+2A$^{2/3}$) ≅ A/14).**

Belgya, Capote et al.,(2012) www-nds.iaea.org/RIPL-3/levels/levels/

Nuclear shapes *have an important influence on* ***level densites*** *via collective enhancement* $\mathbf{K_{coll}}$:

Collective rotation *induces band for each intrinsic state and levels are pulled down*

Included adiabatically, level densities are considerably larger than the ***state density***:

$$\rho_{\text{int}}(E) = \frac{e^{S(\mathrm{a},t)}}{\sqrt{d(\mathrm{a},t)}}$$

For ***triaxial*** *nuclei this rotational enhancement at low energy is largest.*

$\mathcal{R}$-symmetry is the only constraint; for small I and $E_{rot} \ll E_x$ one has :

$$\rho(E,I) \cong \sum_{\tau=1}^{2I+1} \rho_{\text{int}}(E - E_{rot}(I,\tau)) \xrightarrow{I \text{ small}} \frac{2I+1}{4} \cdot \rho_{\text{int}}(E)$$

$\mathcal{R}$-symmetry

Triaxial nuclei are considered the ***general case –***

and the enhancement in $\boldsymbol{\rho(E,I)}$ *allows a* ***reduction of*** $\boldsymbol{\rho_{\text{int}}(E)}$

as compared to 'conventional' prescriptions:

In ***axially deformed*** *nuclei (no rotation about the symmetry axis) one uses:*

$$\rho(E,I) \xrightarrow{I \text{ small}} \frac{2I+1}{\sqrt{8\pi}\,\sigma} \cdot \rho_{\text{int}}(E).$$

For ***spherical*** *nuclei the level density for a given I is usually obtained from*

$\rho(E,M=I) - \rho(E,M=I+1)$, leading to a reduction of ρ by more than 100:

$$\rho(E,I) \xrightarrow{I \text{ small}} \frac{2I+1}{\sqrt{8\pi}\,\sigma_s^3} \cdot \rho_{\text{int}}(E);$$

Spin dispersion and cut off *related to* ***moments of inertia:***

$\mathfrak{J}_j$ for collective rigid rotationon – and $\mathfrak{J}_s$ for statistical fermionic motion.

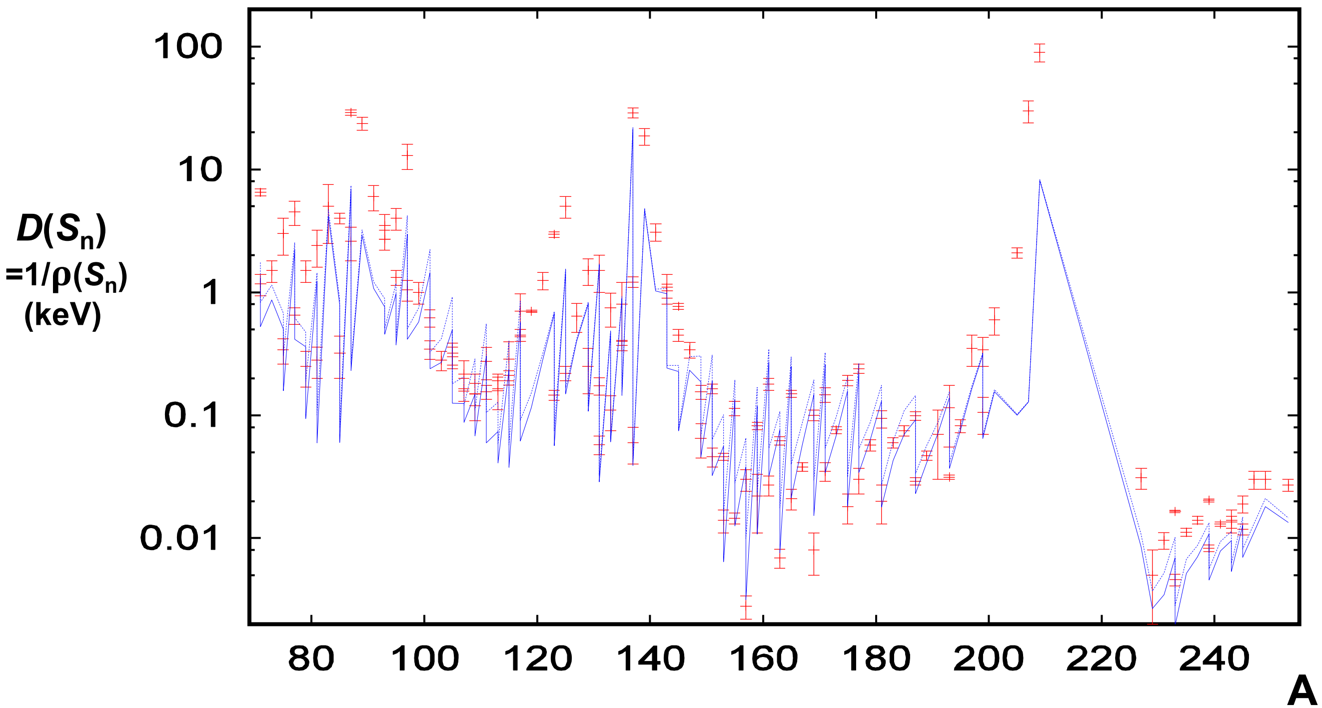
Capture of s-neutrons by e-e nuclei yields $\mathbf{I_R}$**=½⁺**

$$\sigma_s^{\,2} = \frac{\mathfrak{J}_s \cdot T}{\hbar^2}; \quad \mathfrak{J}_j \propto \frac{\mathrm{A}m_n(R_k^2 + R_l^2)}{5}; \quad \mathfrak{J}_s \equiv \mathfrak{J}_3; \quad T = \left(\frac{d\ln(\rho_{\text{int}})}{dE}\right)^{-1}$$

Bjørnholm, Bohr & Mottelson, Rochester conf. on fission (1973) , Bohr & Mottelson, Vol II, 4-63 (1975)

Ericson, Nucl. Phys. 6 (1958) 62.
Bethe, Phys. Rev. **50**, 332 (1936)

In 132 nuclei with **70<A<250** *the* ***mean distance*** *of* ***s-wave resonances*** *at* $\mathbf{S_n}$, I_R=½⁺
is well reproduced assuming ***triaxiality*** **–** *with no sensitivity to β & γ*



assuming:
rigid m.o.inertia,and
spherical shape

axial shape

triaxial shape w/o,
with vibrational enhancement

The only free parameter (level density p.) $\mathbf{a} = \frac{A+3A^{2/3}}{14}$ *is close to the value for nuclear matter* $\mathbf{a} = \frac{\pi^2 A}{4\varepsilon_F} \cong \frac{A}{14}$.

The comparison to the data at (S_{n}, ½⁺) *clearly demonstrates the* ***effect of reduced symmetry***,
i.e. the importance of ***triaxiality***, *whereas the actual values for* $\boldsymbol{\beta}$ *and* $\boldsymbol{\gamma}$ *are unimportant..*

But: absolute values depend significantly on the choice of shell correction $\boldsymbol{\delta W_o}$;
Myers & Swiatecki, corrected for deformation by LDM, was used.

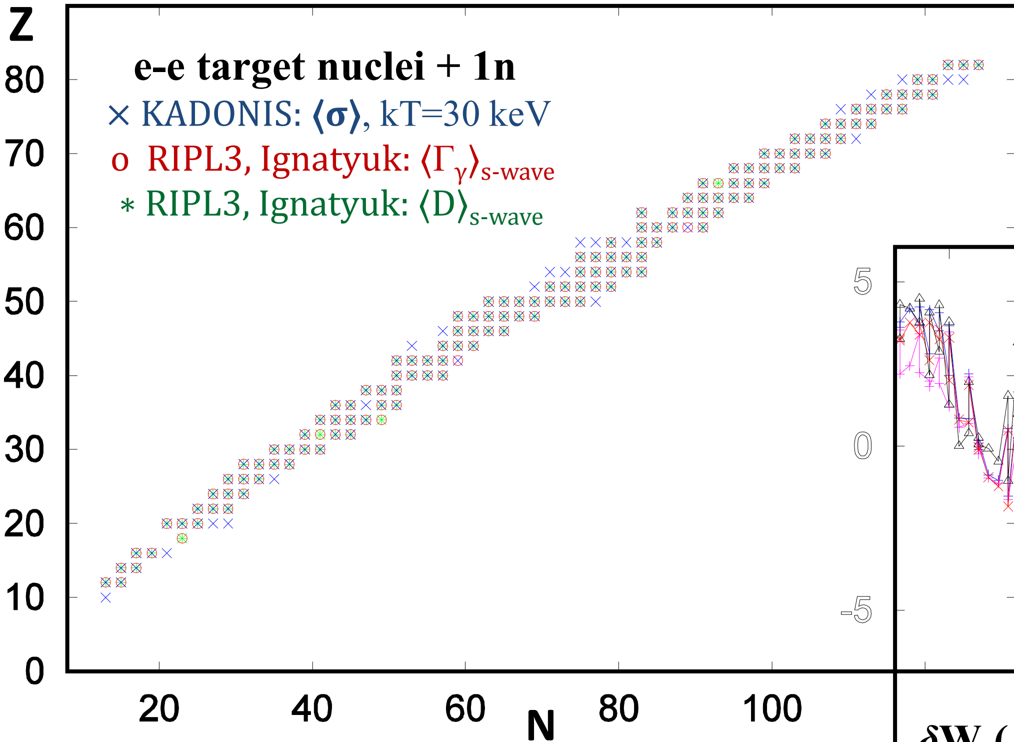
Ignatyuk, nds.iaea.org/RIPL-3/resonances/resonances0.dat

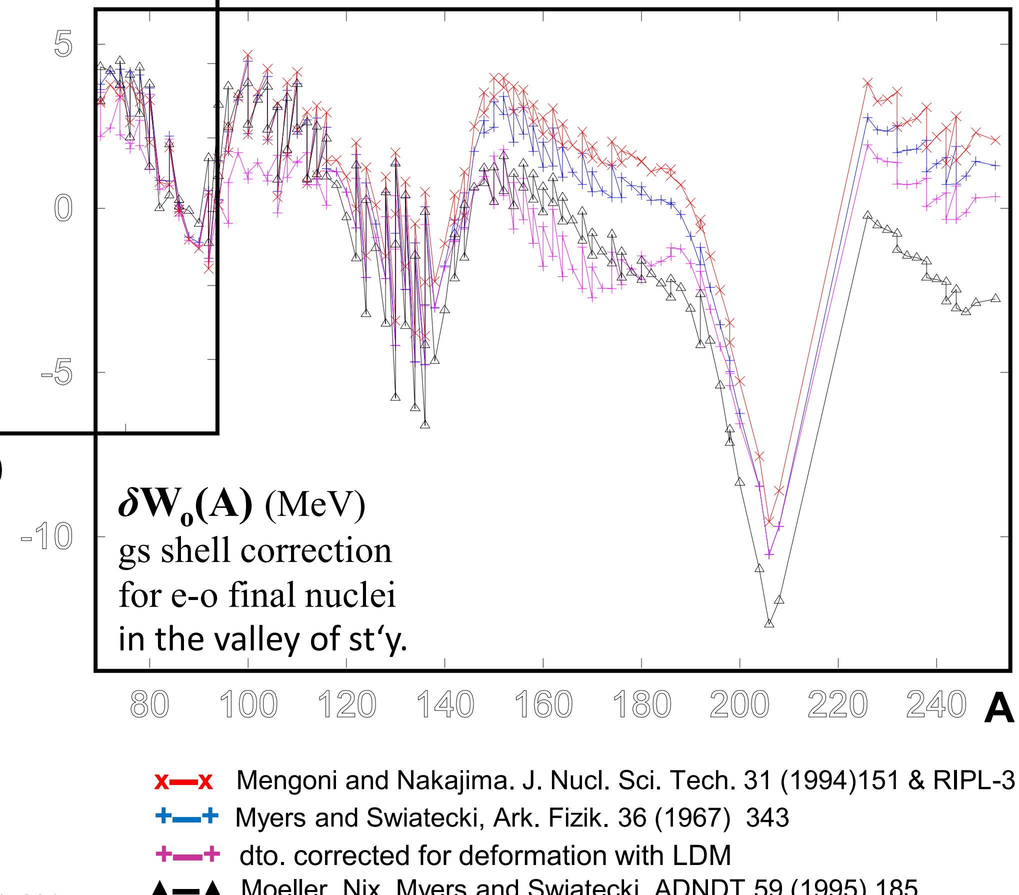
Myers, Swiatecki, Ark. Fizik. 36 (67) 343; nds.iaea.org/RIPL-3/densities/shellcor-ms.dat

Data at S_n for the average level distance ⟨D⟩ and radiative width ⟨ Γ_γ ⟩

are known for more than 125 e-e target nuclei

Liquid drop model masses
and shell correction parameters
are available for many more nuclei,
but significant discrepancies exist
– already in the valley of stability;





The information on shell effects is crucial for level density predictions and for radiative n-capture cross sections.

Compilation of Maxwellian averages allow global test.

R. Capote et al., NDS 110 (2009) 3107
www-nds.iaea.org/RIPL-3/

I. Dillmann et al., PRC 81 (2010) 015801
AIP Conf. Proc. **819**, 123; www.kadonis.org

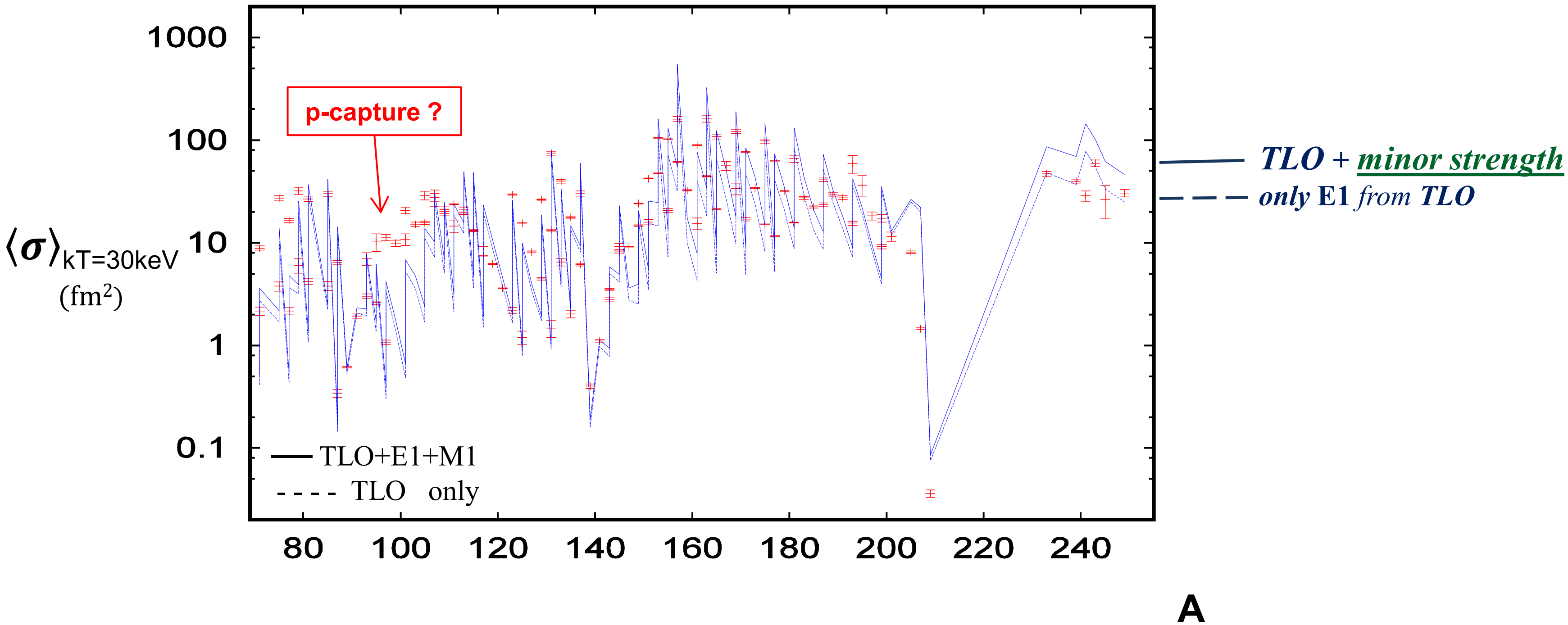
A simultaneous global prediction of

average ***level distances*** *at* S_n *and* ***photon widths*** *for* ***radiative neutron capture*** *(unresolved resonance region)*

allows ***test of*** *the* ***TLO-photon strength*** $f_1(E_\gamma)$ *and the* ***level density parameterization.***

Maxwellian averages ***are a good measure for keV neutrons***

$$\langle\sigma\rangle_{kT} = \frac{2}{\sqrt{\pi}} \frac{\int_0^\infty \sigma(E_n)\, E_n\, e^{-E_n/kT}\, dE_n}{\int_0^\infty E_n\, e^{-E_n/kT}\, dE_n} \qquad \Phi = dN/dE_n \sim \sqrt{E_n} \cdot e^{-E_n/kT}$$



good agreement to Maxwellian averages for >100 nuclei *with predominant s-capture.*

Global predictions are possible, as ⟨σ⟩ depend significantly only on a –

and also on $f_1(E_\gamma)$*, on the nuclear symmetry, and the choice of shell correction* δW_o

Grosse et al., to be published in Physics Procedia (13)

Dillmann et al., PRC81 (10) 015801
AIP Conf. Proc. **819**, 123; www.kadonis.org

Various collective modes *contribute to the* ***photon strength*** *in* ***radiative capture:***

E1: *IVGDR, fit by TLO with* ***sum rule (TRK)*** *and global* ***spreading width*** $\Gamma \propto E_{GDR}^{1.6}$: $\int f dE \approx 12.8\ GeV^{-2}$

isoscalar(IS) E1 *strength in 'pygmy' resonance at* $E_{py} \approx 0.5 \cdot E_{GDR} \approx 6$ *MeV:* $\int f dE \approx 0.1\ GeV^{-2}$

vibration-coupling : $(2^+ \times 3^-)1^-$ @ $E_{sum} \approx 3$ **MeV**; $I \propto B(E2) \cdot B(E3)$: $\int f dE \approx 0.024\ GeV^{-2}$ } *≈3 MeV*

M1: ***orbital*** *(scissors)* ***mode*** @ ≈ **3 MeV** ; $I_{sc} \approx Z^2 \cdot \beta^2$: $\int f dE \approx 0.046\ GeV^{-2}$

isoscalar and isovector components of spin-flip mode @ ≈ 7 MeV: $\int f dE \approx .042\ GeV^{-2}$

'zero pole' originating from a recoupling of nucleon spins within equal configurations: $\int f dE \approx 0.014\ GeV^{-2}$

E2: *quadrupole vibrations @ ≈ 1-2 MeV contribute* $\int f dE < 10^{-2}$, *the GQR @≳ 9 MeV* $\int f dE < 0.2\ GeV^{-2}$.

The parameters of these minor contributions to strength are approximated based on intensive experimental studies at e-beams (Urbana, Bartol, Stuttgart, Darmstadt, Dresden, Duke,..); they determine transition strength $f_\lambda(0 \rightarrow R)$ *from ground to excited states and resonances R.*

Axel-Brink hypothesis *predicts same strength on top of any quasi-particle state* E_x,

causing collectively enhanced decay transitions $f_\lambda(R \rightarrow E_x) = f_\lambda(0 \rightarrow R)$.

Respective structures may appear in CN-reaction spectra (BNL, LASL, Oslo,..)

and they contribute to ***radiative capture*** *of p and n – especially for* $E_\gamma \approx 3$ **MeV**.

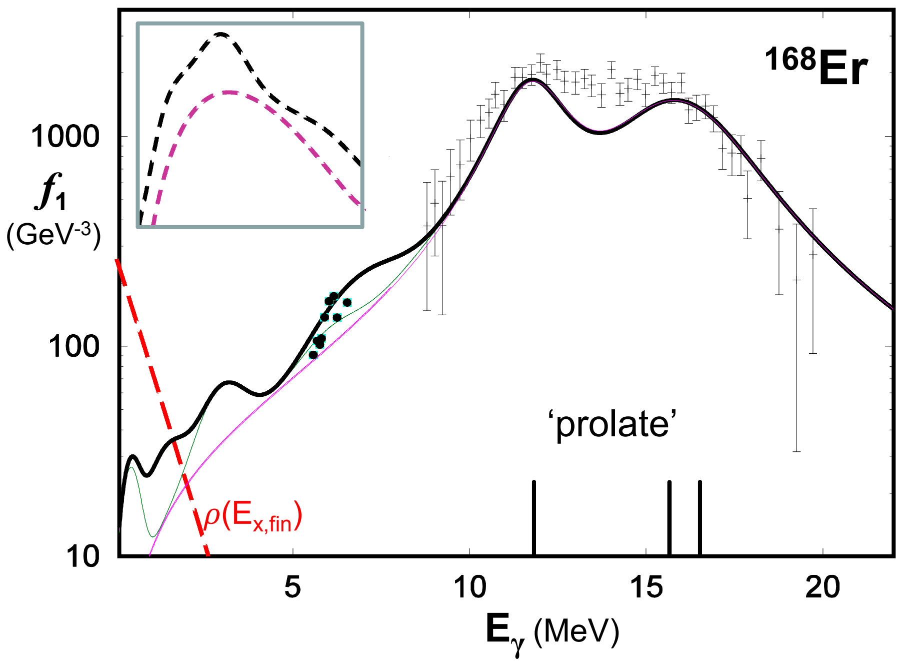
Poelhekken et al., PLB 278 (92) 423
Pysmenetska et al., Phys. Rev. C 73 (2006) 017302
von Garrel et al., Phys. Rev. C 73 (2006) 054315
Kneissl, Nuclear Physics News 16 (2006) 27

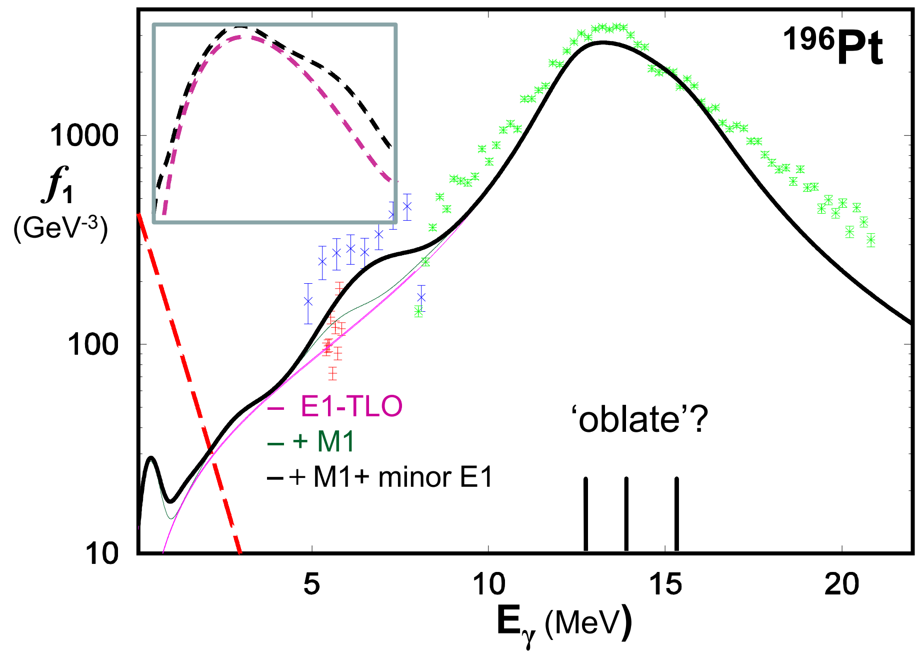
Heyde et al., Rev,Mod.Phys 82 (2010) 2365
Enders et al., Phys. Rev. C 71 (2005) 014306
Richter, Prog. Part. Nucl. Phys. 34 (1995) 261

Overlap *between* ***final level density*** $\rho(E_x)$ *and* ***photon width*** $\Gamma(E_\gamma)$ *peaks at ≈ 3 MeV;*

it determines 1st photon yield and sensitivity of radiative capture cross sections to $\rho(Ex)$ *and* $f(E\gamma)$ *.*

Additional ***'minor'*** *strength near 3-5 MeV (scissors M1, pygmy E1,* $(2^+\otimes3^-)_{1-}$*) leads to some enhancement.*





Triple Lorentzian E1-PSF (TLO) *causes ≲* ***80%*** *of yield;*

minor components non-negligible ► need of new experimental investigations.

Good description of ***dipole strength*** *data in* ***IVGDR*** *and (n,γ)-data* ***in the tail***

using axis ratios from HFB and widths $\Gamma_k \propto E_k^{1.6}$ *.*

Gurevich et al., NPA,351(81) 257

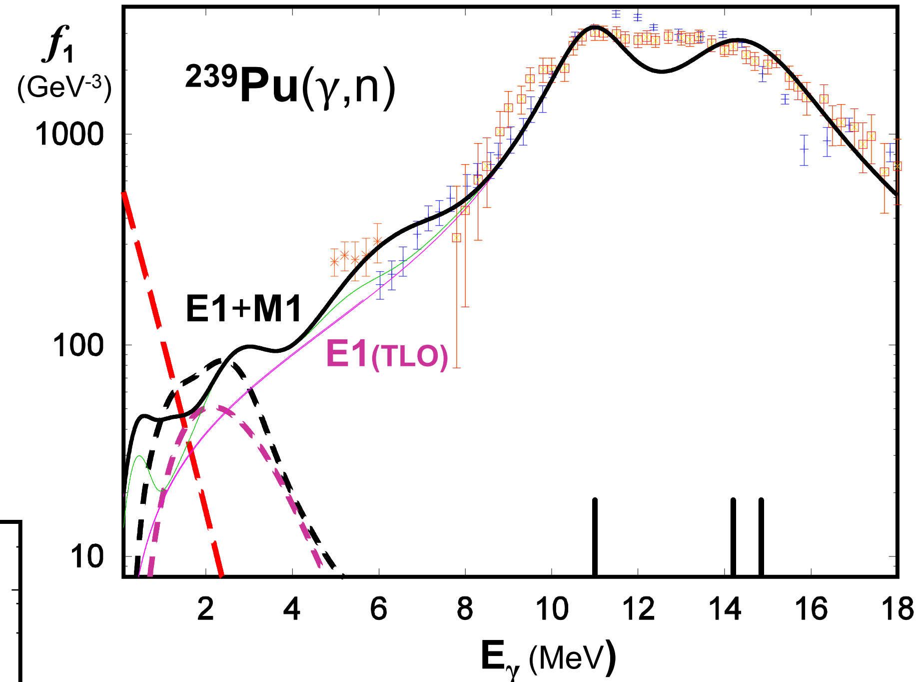
Mughabghab & Dunford, PLB 487(00)155

Massarczyk et al., PRC **87**(13) 044306

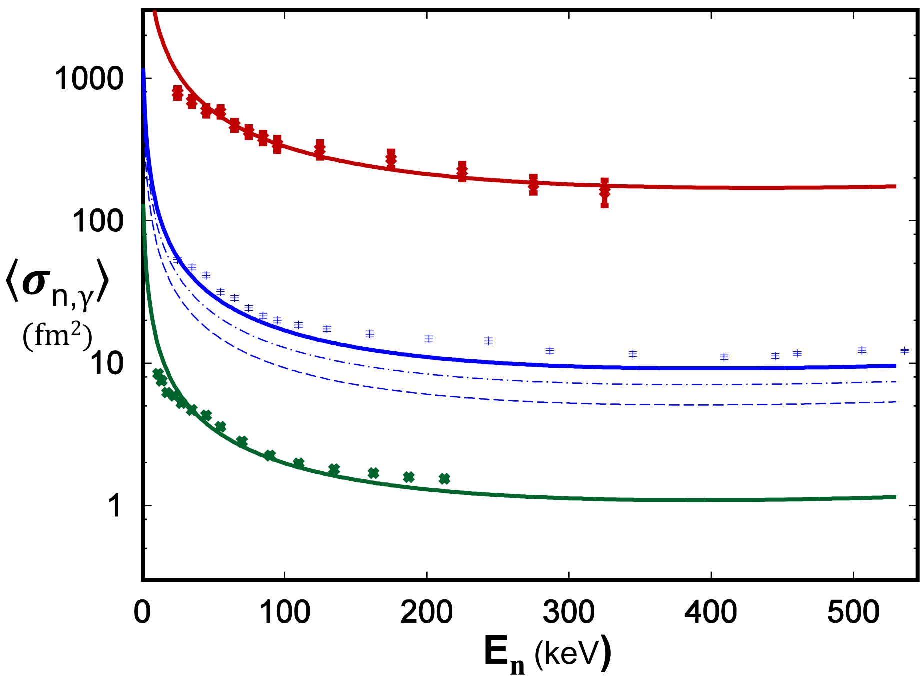
Goryachev & Zalesnyi, Yad. Fiz. **27** (78) 1479

ERINDA HZDR

The ***photon strength*** *and* ***level density parametrizations*** *presented here also work well for* ***actinides:***



Calculation agrees well to data without any new parameter, indicating a possible use for transmutational applications.

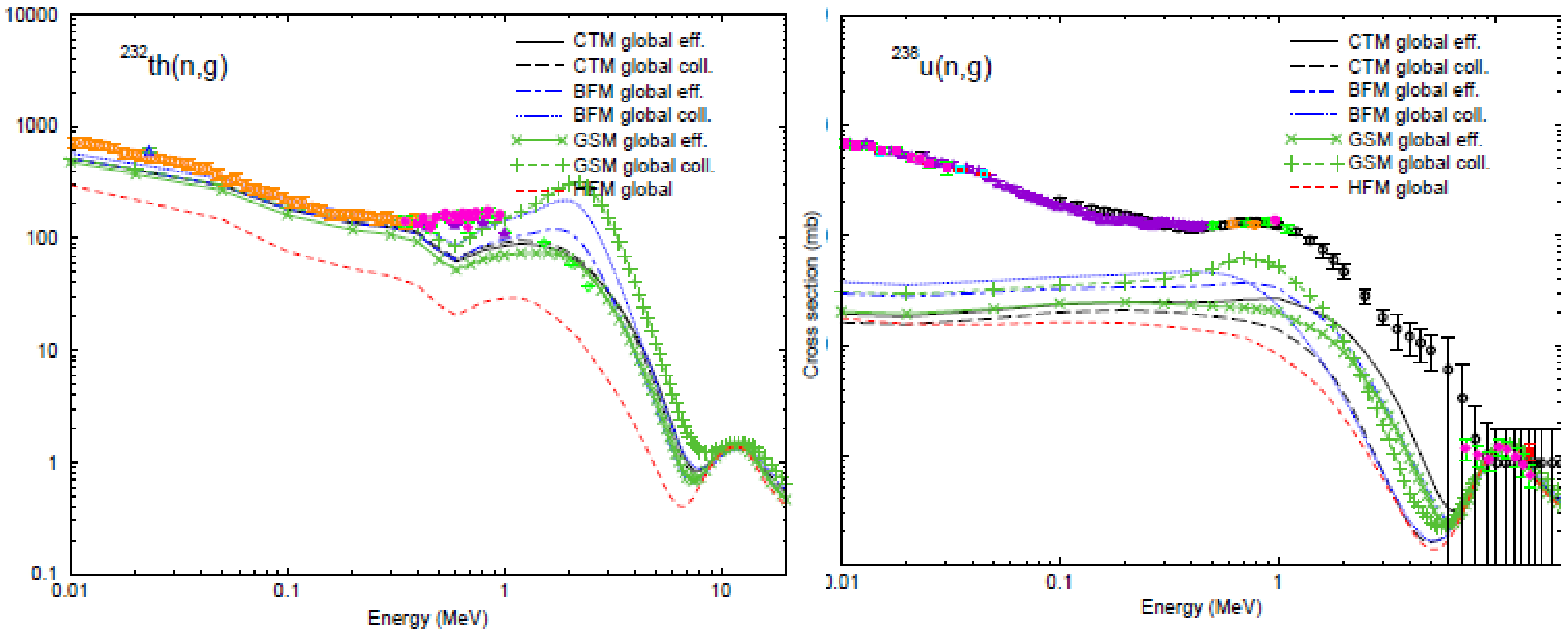


Photon strength other than GDR-tail => isovector E1 has ≳ 30 % influence on radiative capture cross section
(mainly orbital M1).

K. Wisshak et al., NSE,137,183 (2001)
R.R.Spencer,F.Käppeler, Wash. conf. ,2,620 (1975)
L.W.Weston,J.H.Todd, NSE,63,143 (1977)

+ G.M. Gurevich et al., Nucl. Phys. A 351, 257 (1981)
□ B.L. Berman et al., Phys. Rev. C 34 (1986) 2201
x Y. Birenbaum, et al., Phys. Rev. C 36 (1987)1293

*The **photon strength** and **level density parametrizations** used by **TALYS-1.4** do **not** work well for **actinides:***



*Apparently **global parametrizations** for **photon strength** and **level density** as used by us avoid eventual false measurements of photon cross sections in the IVGDR, in its tail and at S_n.*
*For **actinides (transmutation etc.)** an extrapolation to **unstable isotopes** is important.*

http://www.talys.eu/documentation

Conclusions

Transmutation of nuclear waste *needs numerical simulations and these need good input – like* ***global*** *parameterizations derived from basic theory only.*

Ad hoc assumptions about ***spherical or axial symmetry*** *of heavy nuclei simplify (most) theorist's efforts,* ***but*** *experimentalist's observations indicate that nearly all of these* ***nuclei are less symmetric****:*

1. *Multiple Coulomb excitation and other spectroscopic data are well described assuming* ***triaxiality****;*
2. *the strength of the IVGDR agrees to the classical sum-rule (TRK) if a* ***triple splitting*** *is admitted;*
3. *level distances observed at S_n show collective enhancement indicating* ***3 rotational axes****.*

A respective ***combined analysis of*** $\rho(E_x)$ *and* $\langle\sigma(n,\gamma)\rangle$ *needs a small number of* ***global*** *parameters only: A nuclear matter* ***level density parameter*** **a**, ***large backshift*** *($E_{con}+n\Delta_0$)*, ***triaxiality*** *(γ-value unimportant).*

Radiative processes *are dominated by the tail of the electric* ***dipole strength,*** *described for 70<A<240 by a* ***triple Lorentzian (TLO)*** *with 2 free* ***global*** *parameters;* ***no*** *need for a variation of Γ_{GDR} with E_γ. Maxwellian averaged* ***neutron capture*** *data and $\langle\Gamma_\gamma\rangle$ are predicted with no extra free parameters;*

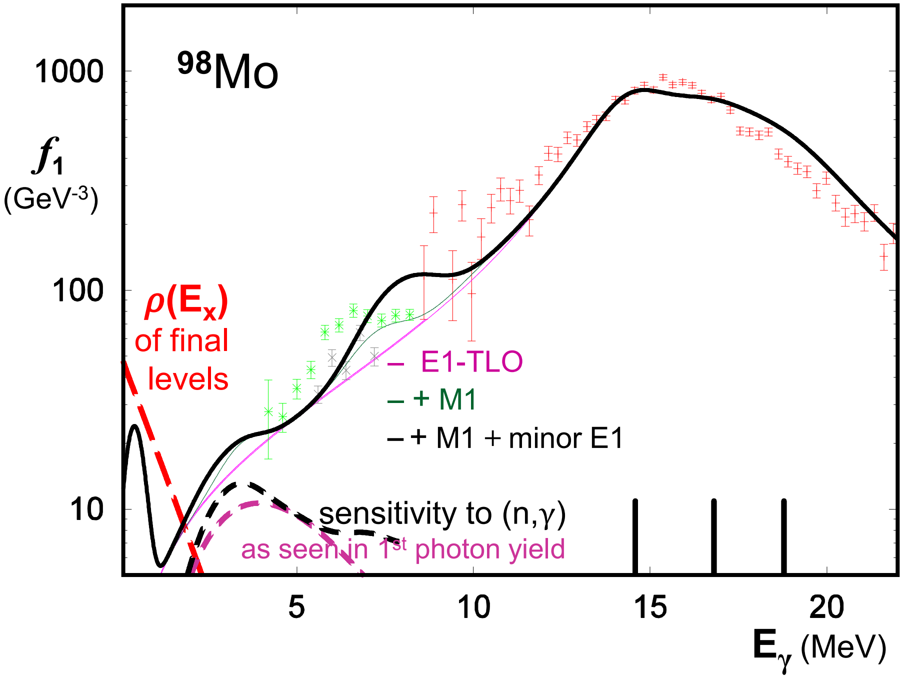
M1 ***orbital*** *strength - important for large β - as well as* **E1($2^+\otimes3^-$)** *need further investigation – studies should also clarify the effect of spin, parity and of shell corrections.*

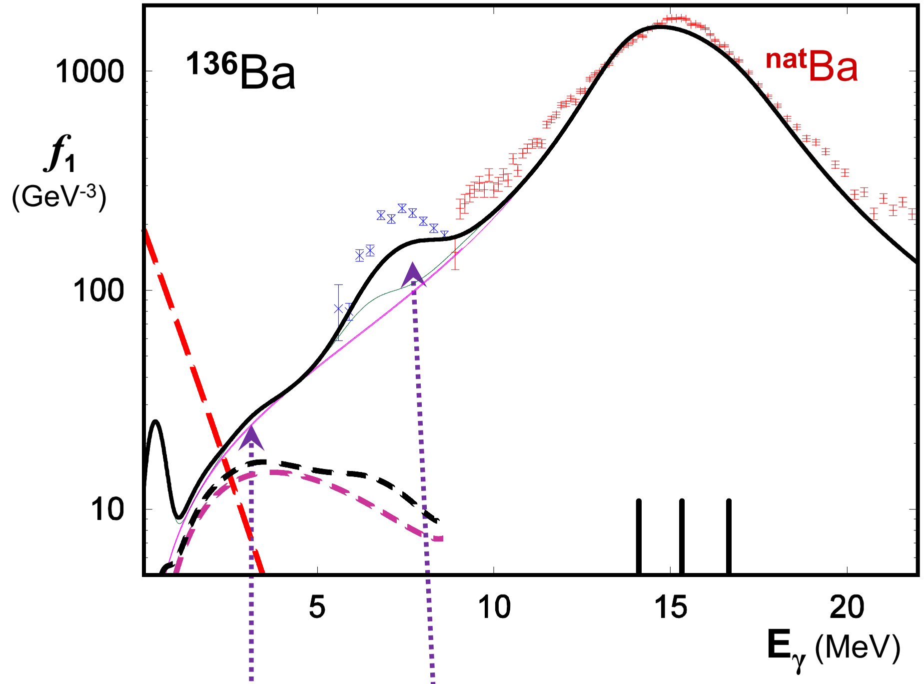
The present analysis shows features clearly at ***variance to TALYS, RIPL-3,*** *..which show remarkable uncertainties and ambiguities, as triaxiality and collective enhancement effects are not well regarded.*

! Most heavy nuclei are <u>triaxial</u> ⇒ $\Gamma_{GDR} \propto E_{GDR}^{\ 1.6}$ & TRK sum rule holds
⇒ ρ collectively enhanced & $E_{bs}=E_{con}+\Delta_o$
Some additional photon strength ⇒ capture data well described globally!

ERINDA
HZDR

ELBE data for ^{98}Mo and ^{136}Ba show significant excess over TLO; lower energy data needed to quantify it.





TLO: triple Lorentzian for IV-E1 covers ≲ 80%

*The influence of **photon strength** on radiative neutron capture results from the overlap of Γ_γ (small at low E_γ as consequence of $E_\gamma^{2\lambda+1}$ and TLO) and level density ρ below S_n (small at low E_x).*

minor E1:** $2^+ \times 3^-$; **pygmy (isoscalar E1)

***M1:** orbital (scissors); isoscalar; isovector*

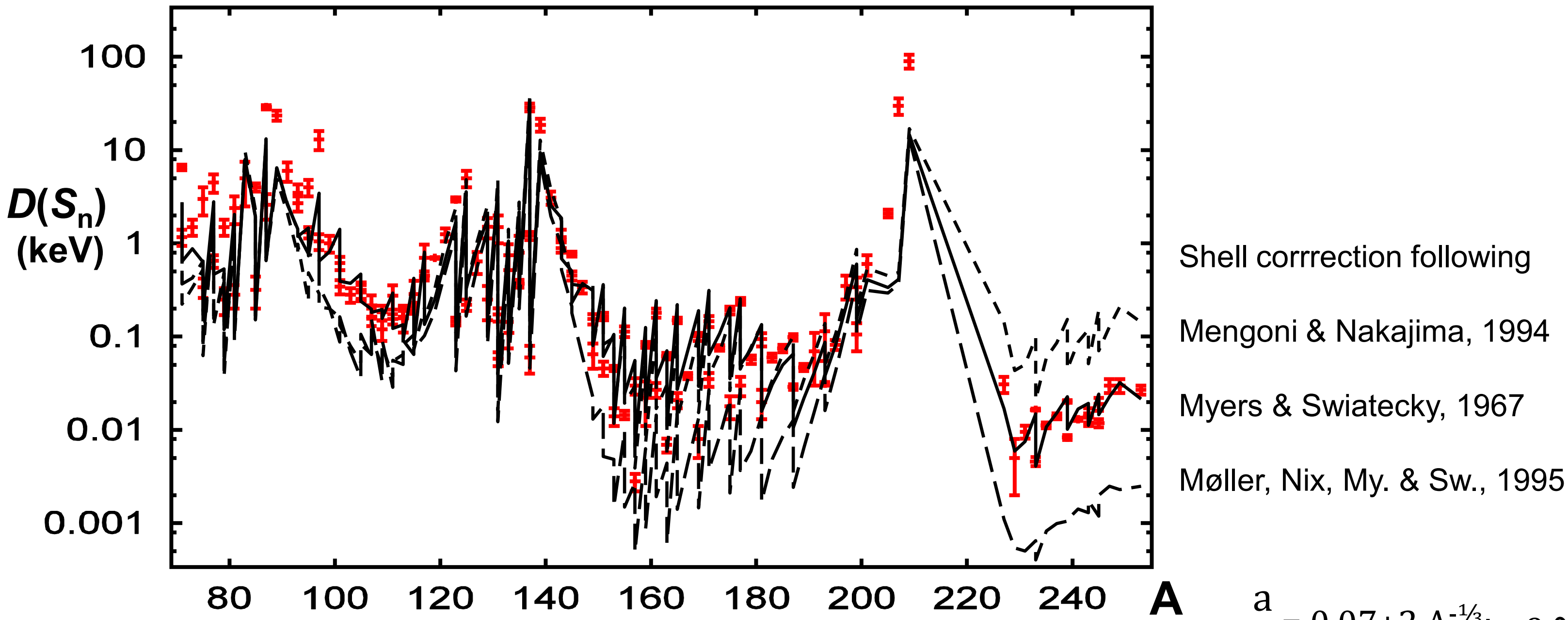
Orbital M1 and vib-coup E1 are especially important for radiative neutron capture,
the photon strength at very low energy
has nearly no influence on primary γ-transitions.

Rusev et al., PRC 79 (09) 061302
Rusev et al., AIP conf.proc.1099 (09)

Massarczyk et al., PRC C 86 (12) 014319
Beil et al., NPA172 (71) 426, NPA227 (74) 427

ERINDA
HZDR

*__Average resonance distance D =1/ρ__ at Sn in comparison to calculations using differently determined shell corrections.*__



$\frac{a}{A} = 0.07+2\ A^{-1/3}; \quad \rho \propto \frac{\sigma_1\ \sigma_2}{\sigma_s\ \sigma_s}$

E1: GDR-TLO + E2⊗E3 + is-pygmy

M1: spin-flip + scissors + zero pole

132 nuclei with ρ measured at S_n,

3000 shape samples (Delaroche et al.),

rotational & vibrational enhancement (Bjørnholm et al.)

Radiative capture, *averaged over resonances and summed over final states in γ-decay*

Average radiative capture cross section *is proportional to* $\boldsymbol{\rho(E_f)}$ *and to photon strength* $\boldsymbol{f_\lambda}$.

$$\langle \sigma_{n,\gamma}(E_n)\rangle_R \approx 2(2\ell+1)\pi^2 \lambda_n^2 \rho(E_{R,\ell})\Delta \left\langle \frac{\Gamma_n \times \Gamma_{R\gamma}}{\Gamma_n + \Gamma_{R\gamma}} \right\rangle_\ell ; \quad \langle \Gamma_n \rangle = \frac{(2\ell+1)\, S_\ell}{(\ell+1)\,\rho(E_{R,\ell})} ; \quad \langle \Gamma_{R\gamma} \rangle = \left\langle \sum_f \bar{\Gamma}_\gamma (R \to f) \right\rangle ; \quad E_R = S_n + E_n = E_f + E_\gamma \approx S_n$$

formation *decay*

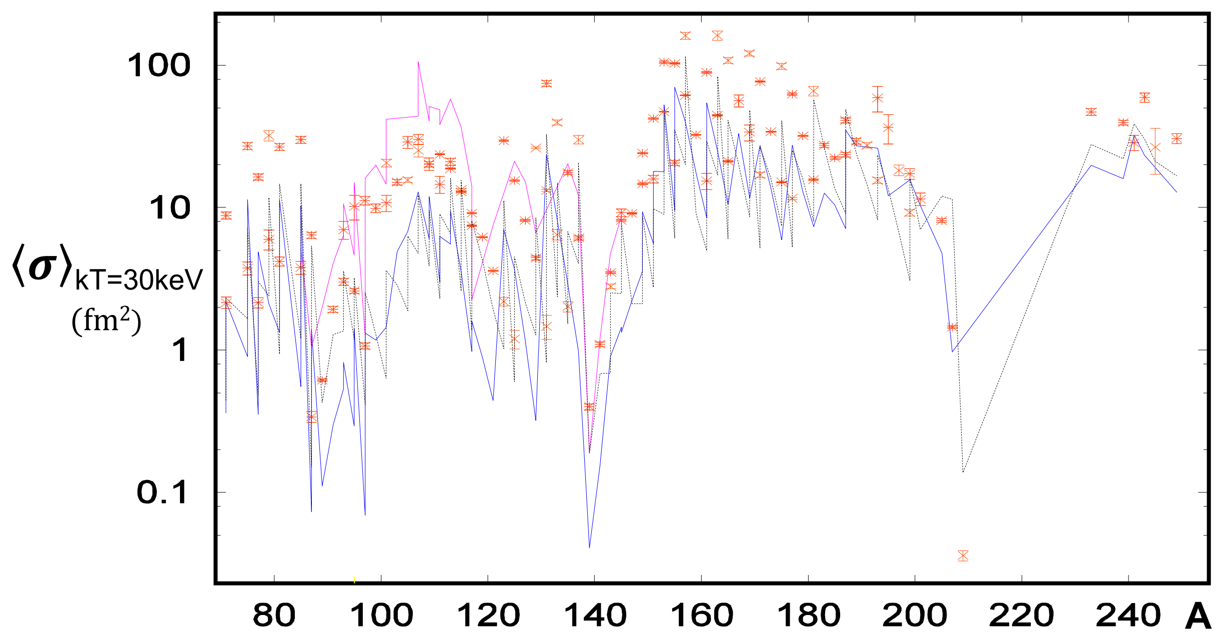
Level density $\boldsymbol{\rho(S_n)}$***, neutron strength S*** *and* $\boldsymbol{\langle \Gamma_\gamma \rangle}$***,*** *are tabulated in RIPL-3 for ℓ=0 &1, and* σ_R *can be calculated as Maxwellian averages, which are a good measure for fast neutrons (overlapping resonance region) with flux:*

$$\Phi = dN/dE_n \sim \sqrt{E_n} \cdot e^{-E_n/kT} \qquad \langle \sigma \rangle_{kT} = \frac{2}{\sqrt{\pi}} \frac{\int_0^\infty \sigma(E_n)\, E_n\, e^{-E_n/kT}\, dE_n}{\int_0^\infty E_n\, e^{-E_n/kT}\, dE_n}$$

$$\left\langle \frac{\Gamma_n \times \Gamma_{R\gamma}}{\Gamma_n + \Gamma_{R\gamma}} \right\rangle \approx 0.87 \frac{\langle \Gamma_n \rangle \times \langle \Gamma_{R\gamma} \rangle}{\langle \Gamma_n \rangle + \langle \Gamma_{R\gamma} \rangle}$$

Porter - Thomas fluctuations

⇒ Moldauer effect



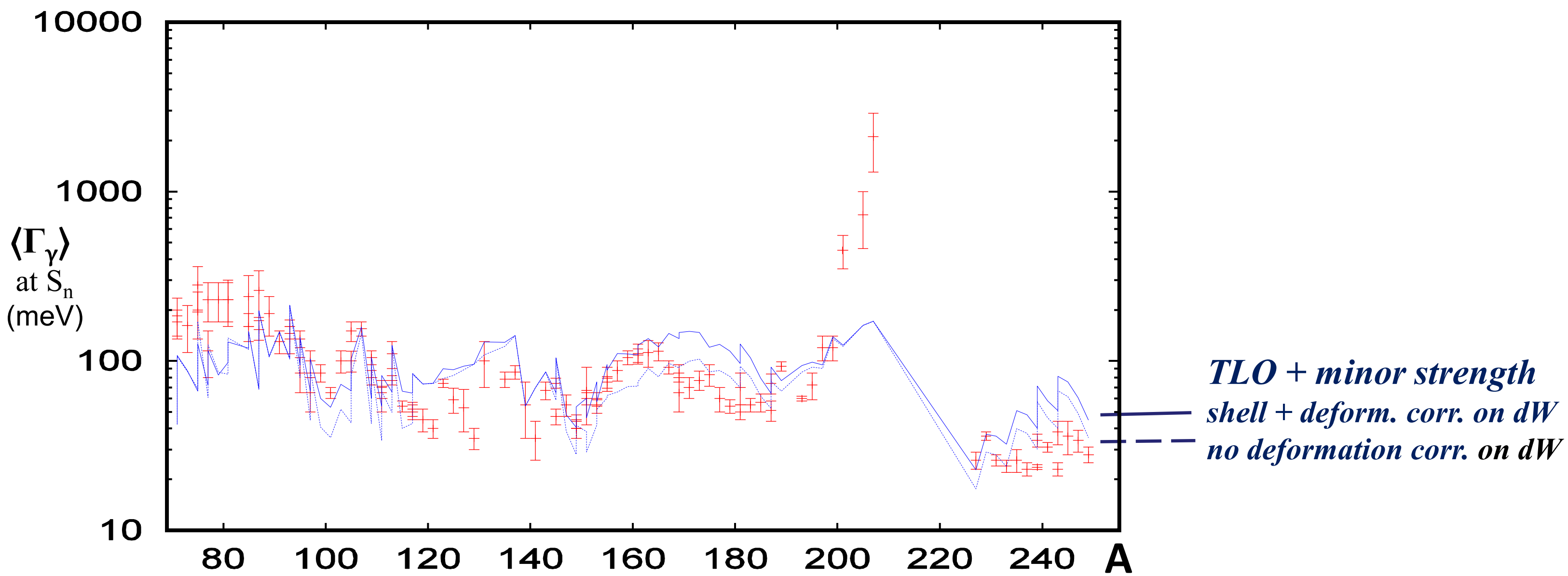
H. Feshbach et al., Phys. Rev. 71, 145 (1947)
S.F. Mughabghab et al., Academic Press NY, (1981)
A. Ignatyuk, Landolt Börnstein, Nuclei and Atoms, 7: Neutron induced reactions, (2009)
R. Capote, A. Ignatyuk et al., RIPL-3 (2008) nds.iaea.org/RIPL-3/resonances/resonances0.dat

***Average radiative width** is nearly independent of* $\boldsymbol{\rho(S_n)}$;

it mainly depends on slope of $\boldsymbol{\rho(E_x)}$ *in the final nucleus below* S_n *, if* $\boldsymbol{f_1(E_\gamma)}$ *is known.*

$$\langle \sigma_R(n,\gamma)\rangle \approx 2(2\ell+1)\pi^2 \bar{\lambda}_n^2 \rho(E_R)\cdot\langle\Gamma_{R\gamma}\rangle = 2(2\ell+1)\pi^2\bar{\lambda}_n^2\cdot C\int_0^{E_R}\rho(E_f)\,E_\gamma^3 f_1(E_\gamma)\,dE_\gamma$$

$$\langle\Gamma_{R\gamma}\rangle = \left\langle \sum_f \bar{\Gamma}_\gamma(R\to f)\right\rangle = C\times\int_0^{E_R}\rho(E_f)\Gamma_\gamma\,dE_\gamma = C\times\int_0^{E_R}\frac{\rho(E_f)}{\rho(E_R)}E_\gamma^3 f_1(E_\gamma)\,dE_\gamma$$



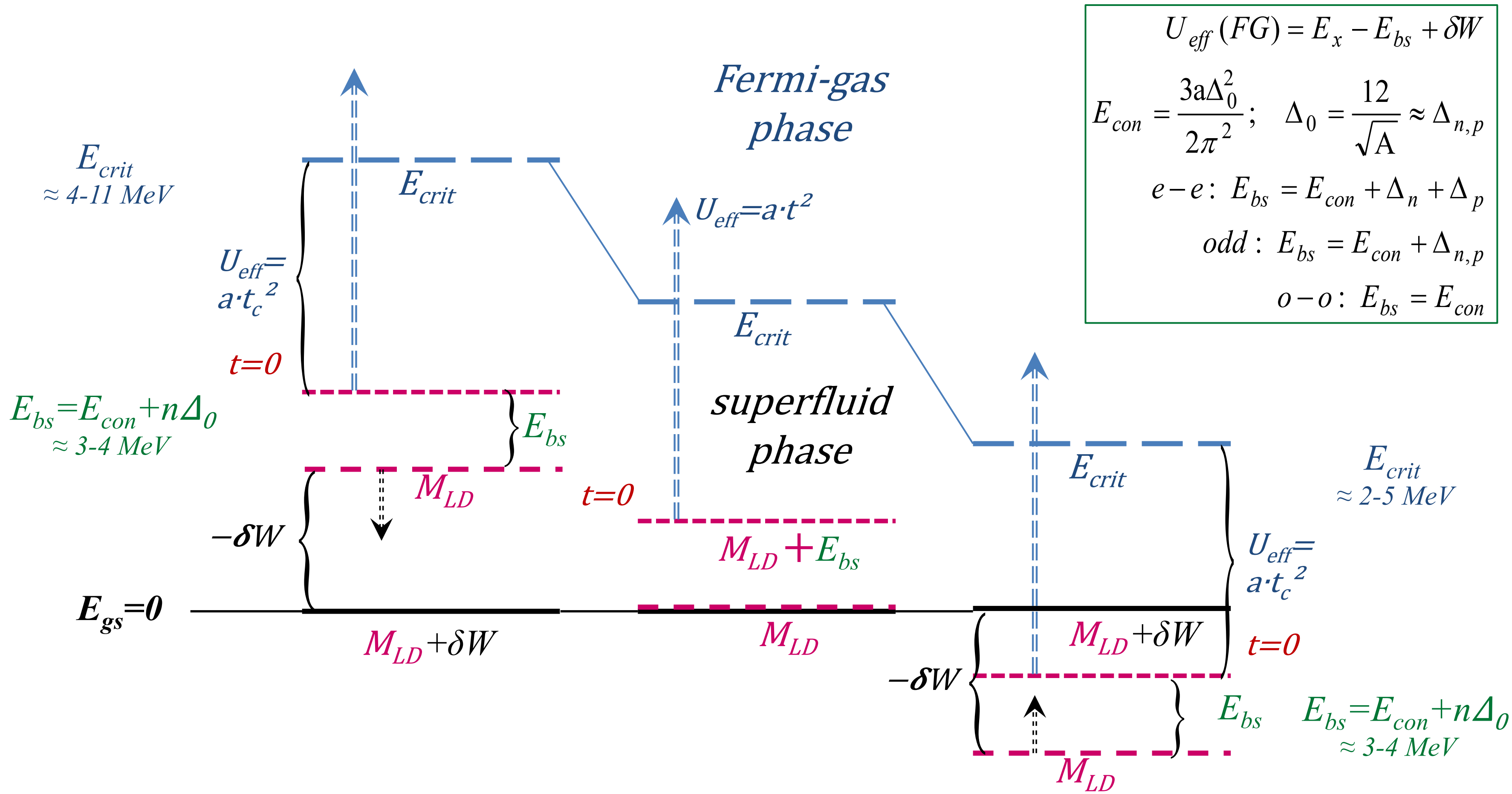
The average photon width can be determined in neutron capture by combining neutron width (i.e. resonant neutron interaction) and branching into photon emission.

*For this analysis **non-resonant** processes and the **neutron strength function** may be important.*

A.Ignatyuk, www-nds.iaea.org/ RIPL-3 (2008)

Schematic energy relations *in the two nuclear phases*

indicating the account for shell effects and pairing



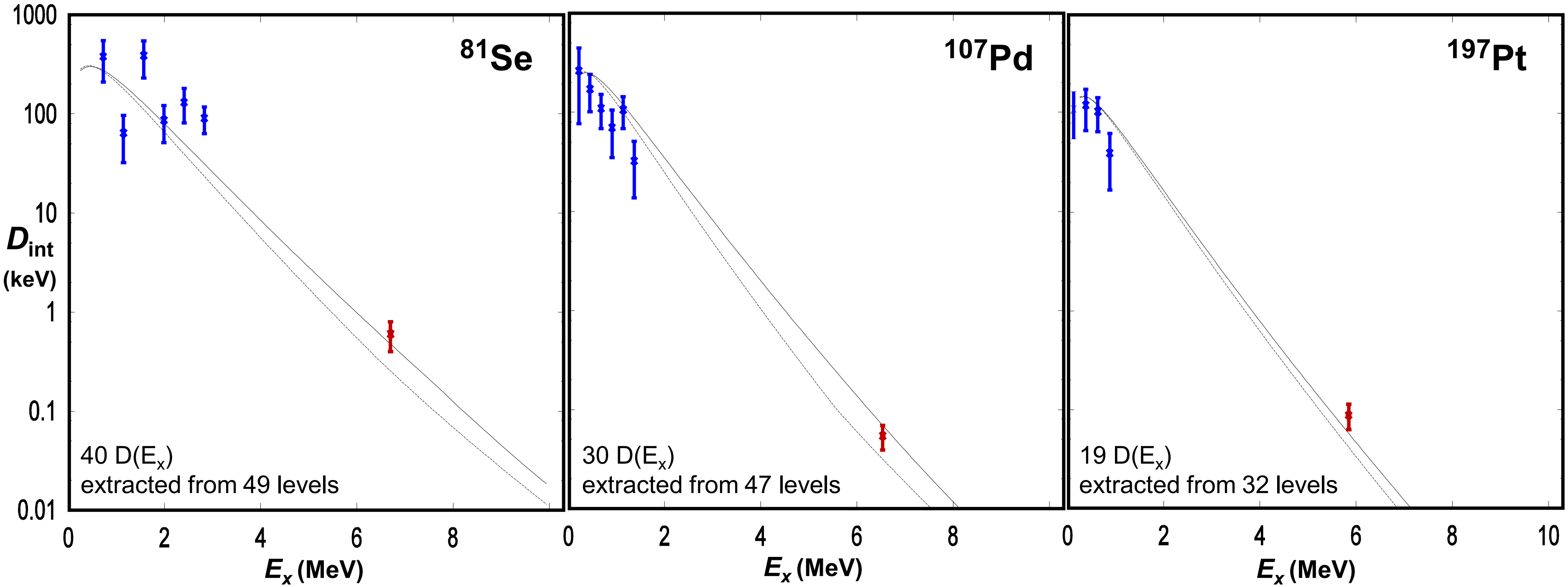
shell correction energy $\quad \delta W_o < 0 \quad\quad \delta W_o = 0 \quad\quad \delta W_o > 0$

$|\delta W| \approx$ *0-11 MeV, damped with increasing t,E:*

in valley of stability $|\delta W_o| < 2$ *MeV for t,E→0*

Bohr and Mottelson, vol. I, 2 (69) & II, 6 (75); Ignatyuk et al., PRC 47 (93) 1504; Svirin, PPN 37 (06) 475; Schmidt & Jurado, PRC 83, 014607 (11)

*The **level density** formalism proposed here compares well to bound states and s-wave resonances*



Inverting the formula $$\frac{1}{D(E,I)} = \rho(E,I) \cong \frac{2I+1}{4} \cdot e^{-\frac{I(I+1)}{2\sigma_s^2}} \cdot \rho_{\text{int}}(E) \xrightarrow{I \text{ small}} \frac{2I+1}{4} \cdot \rho_{\text{int}}(E) = \frac{2I+1}{4D_{\text{int}}(E)}$$

*allows to extract ρ_{int} from the observed average level distances D(E,I) for **all I**;*
when σ is taken from systematics; for low spin I this has a small effect only.

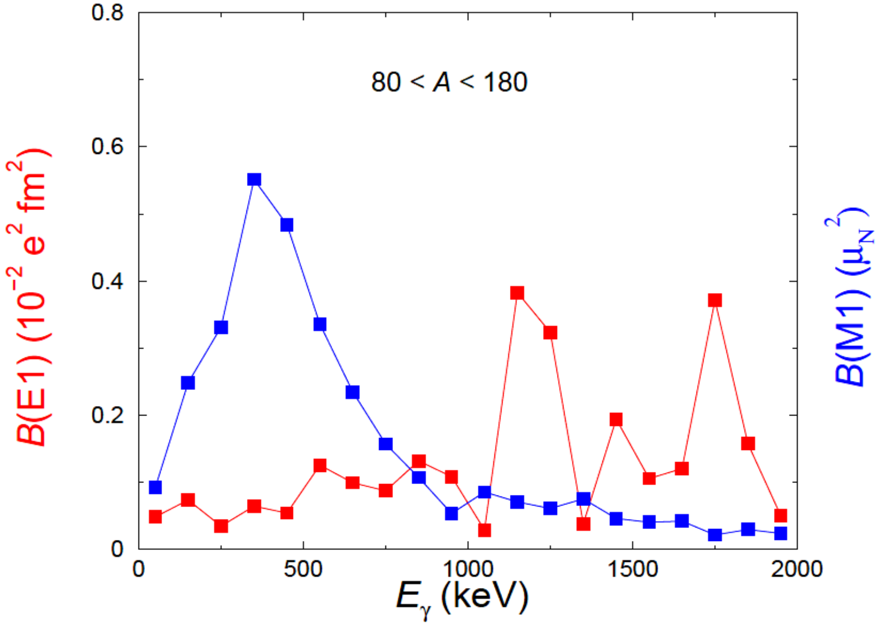
D_{int}(bound levels, various I^{π})
$D_{\text{int}}(S_n, ½^+)$

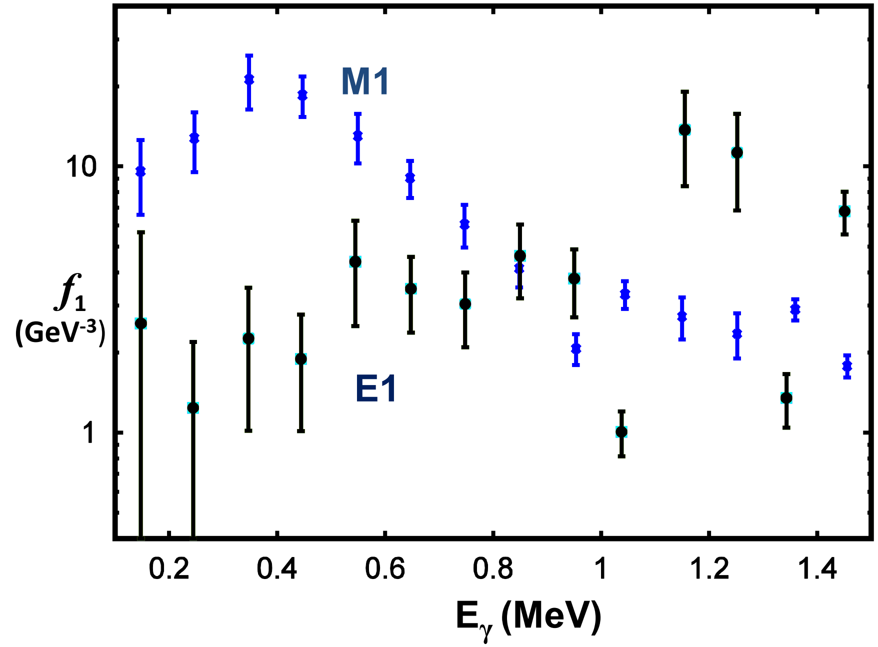
The dashed line indicates the effect of increasing the shell correction by 1 MeV.

Belgya, Capote et al.,(2012) www-nds.iaea.org/RIPL-3/levels/levels/
Ignatyuk (2009), www-nds.iaea.org/RIPL-3/resonances/resonances0.dat

Low energy photon strength

*Information on low energy strength may be obtained from **gamma transition rate averages***
***(ENSDF,** Evaluated Nuclear Structure Data File).*
Application at high excitation because of Axel-Brink hypothesis.





$$f_{E1}(E_\gamma) = \frac{1}{\Delta}\sum_{R\in\Delta}\frac{16\pi\alpha}{27(\hbar c)^2}\mathrm{B}(\mathrm{E1}\uparrow) \approx \frac{349}{\Delta}\sum_{\Delta}\mathrm{B}(\mathrm{E1}\uparrow)$$

$$f_{M1}(E_\gamma) = \frac{1}{\Delta}\sum_{R\in\Delta}\frac{16\pi\alpha}{27(2m_N c^2)^2}\frac{\mathrm{B}(\mathrm{M1}\uparrow)}{\mu_N^2} \approx \frac{3.85}{\Delta}\sum_{\Delta}\frac{\mathrm{B}(\mathrm{M1}\uparrow)}{\mu_N^2}$$

*More data needed on **photon strength** for $\mathbf{E_\gamma}$ in the range 2-5 MeV,*

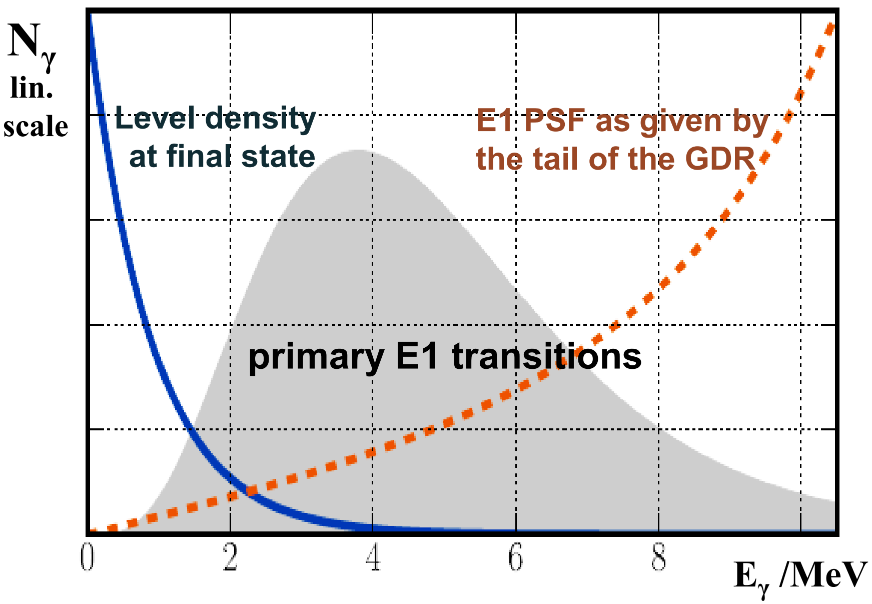
at $\mathbf{E_x} \lesseqqgtr S_n$ this is a real challenge.

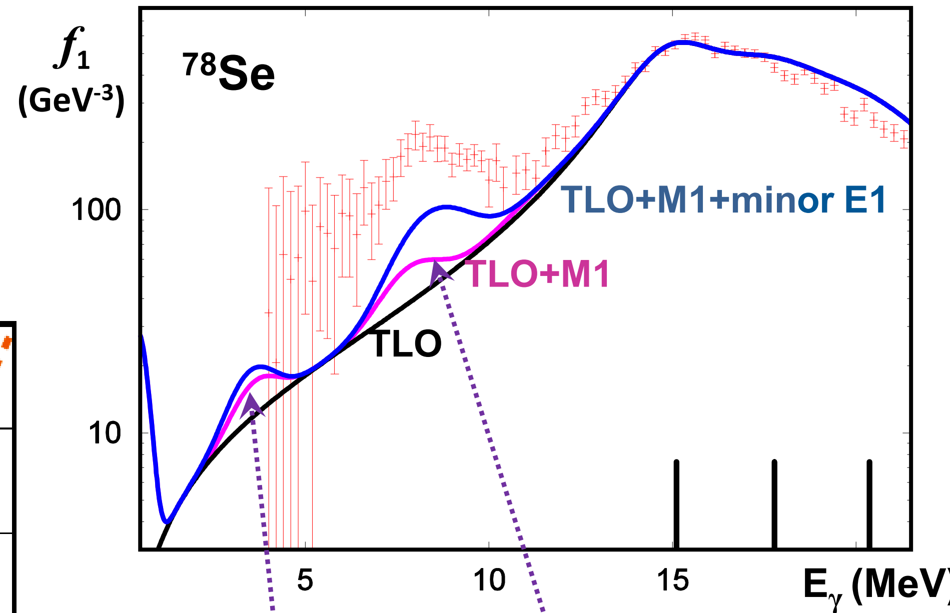
R. Schwengner, (2012)

ELBE data *for ^{78}Se*

show significant excess over TLO;

data for lower energies needed to identify it.





TLO: three Lorentzians for IV-E1

preliminary proposition for:

minor E1: $2^+\times3^-$; pygmy (isoscalar E1)

M1: orbital** (scissors); **isoscalar; isovector

*The influence of **photon strength** on radiative neutron capture results from the overlap of **Γ_γ***

(small at low E_γ as consequence of $E_\gamma^{2\lambda+1}$)

and level density ρ below S_n (small at low E_x).

This relates to Axel-Brink hypothesis.

***Orbital M1** and **vib-coup E1** are important*

for radiative neutron capture,

the photon strength at very low energy

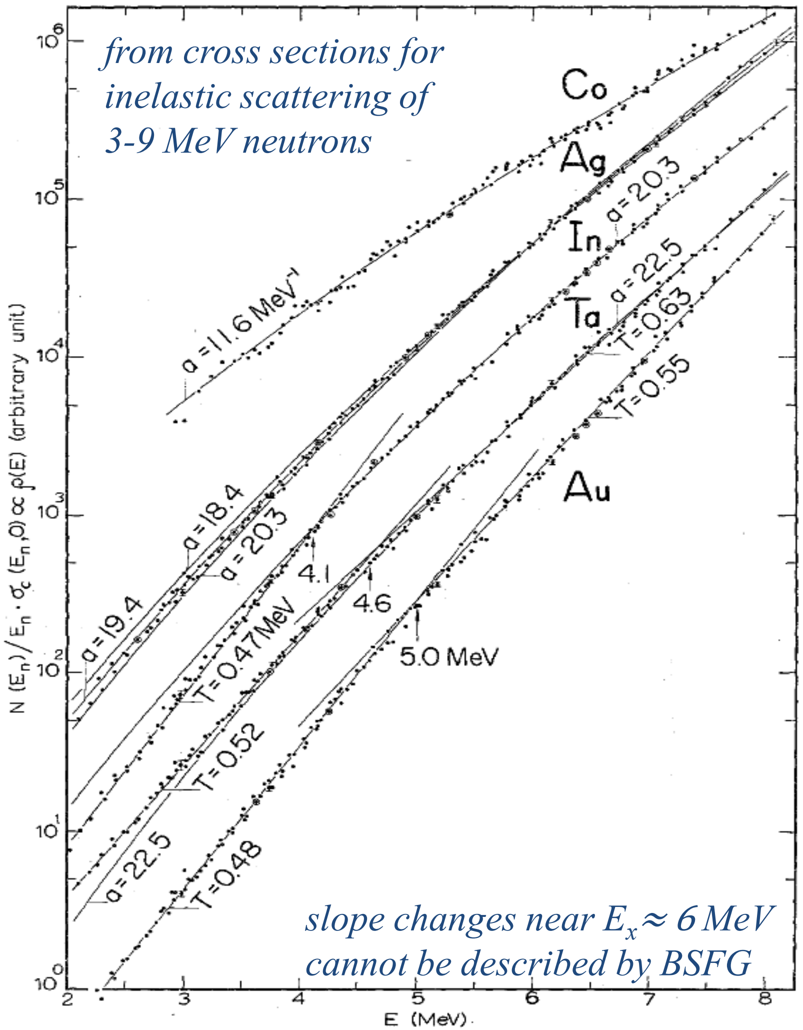
has nearly no influence on primary γ-transitions.

ERINDA

HZDR

Schramm et al., PRC 85 (12) 014311

*The **level density ρ** in heavy nuclei is strongly influenced by the **pairing effect**,*
which lowers the level energies at low excitation such that at a critical temperature t_c
a transition from a paired superfluid phase to a Fermi gas phase is observed;
a phenomenological treatment of the shell, pairing and deformation effects is global.



***Pairing** is accounted for by the critical temperature t_c and a condensation energy E_{cond},*

both related to the pairing gap Δ_o :

$$\Delta_0 = \frac{12}{\sqrt{A}}; \quad \frac{t_c}{\Delta_0} = 0.567; \quad E_{cond} = \frac{3a\Delta_0^2}{2\pi^2}$$

with the level density parameter **a** *approximately given by*

$$\mathbf{a} \cong \pi^2 A/4\varepsilon_F \cong A/14.$$

In the Fermi gas phase the energy is corrected by

a backshift $E_s = E_{cond} + \mathbf{n}\,\Delta_o$

with **n**= **0,+1,+2** *for o-o, odd and e-e nuclei.*

This choice of **n** *makes o-o nuclei the ‘reference‘*

and assures, that E_s is positive also for

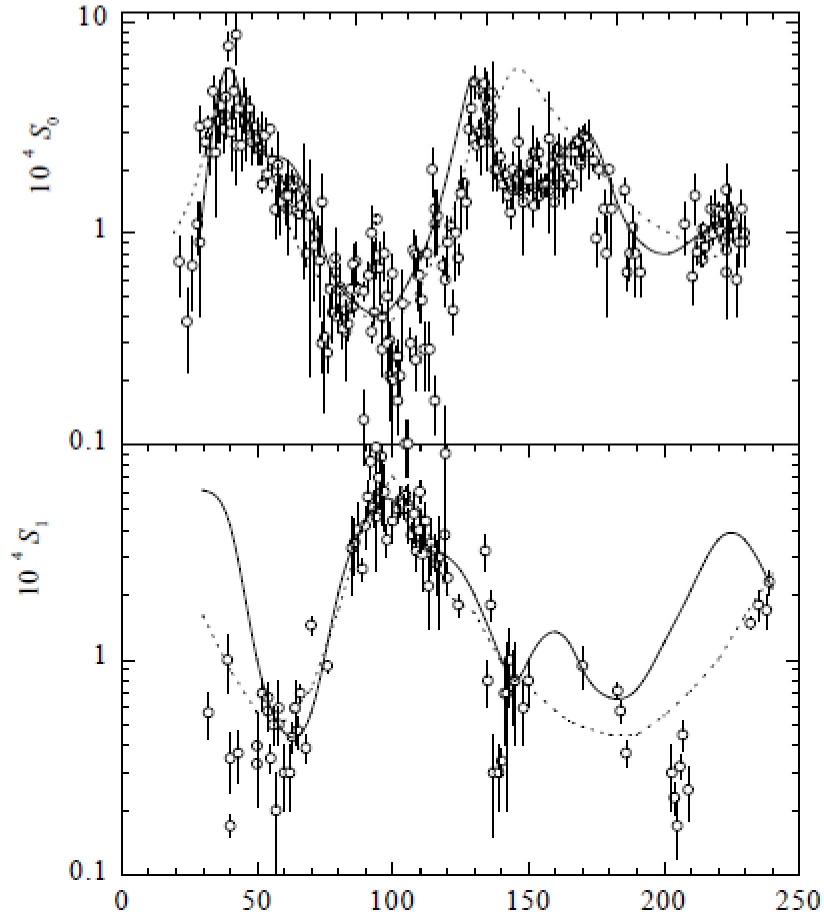
o-o nuclei with small A – at variance to the usual

‘inconsistent‘ *choice* **n**= **-2,-1,0** *for o-o, odd, e-e nuclei.*

For odd nuclei this leads to a reduction of the level density.

Ignatyuk et al., PRC 47 (93) 1504 & IAEA-INDC(CCP)-233/L (85)

Schmidt & Jurado, PRC **83**, 014607 (11)



Strength functions of s- and p-wave neutrons as a function of the mass number.
Results of calculations are shown by dashed curves for the spherical optical model and solid ones for the coupled-channels method.

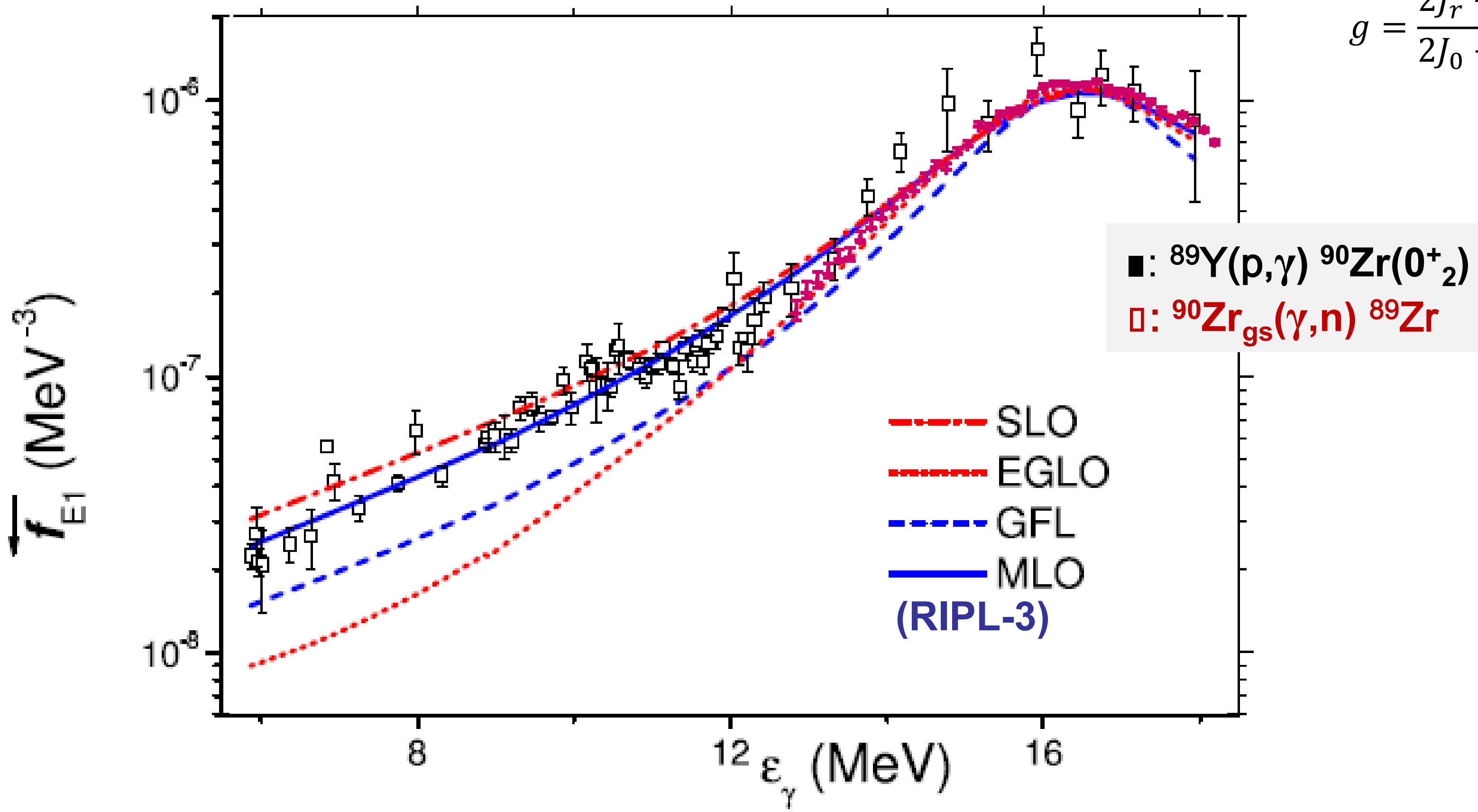
A. Ignatyuk, Landolt Börnstein,
Nuclei and Atoms, 7: Neutron induced reactions,

Photon strength function (PSF): $f_\lambda(E_\gamma)$

*single Lorentzian (SLO) for spherical nucleus ^{90}**Zr**.*

$$\frac{\overline{\Gamma_{lu}(E_\gamma)}}{\overline{E}_\gamma^{2\lambda+1}}\rho_u = f_\lambda(E_\gamma) = \frac{\overline{\sigma_{abs}}(E_\gamma)}{g(\pi\hbar c)^2\,\overline{E}_\gamma^{2\lambda-1}}$$

$$g = \frac{2J_r+1}{2J_0+1}$$



***Axel-Brink hypothesis** confirmed:* *E1-strength on top of* ^{90}Zr (0^+_2) *coincides with GDR on* ^{90}Zr$_{gs}$

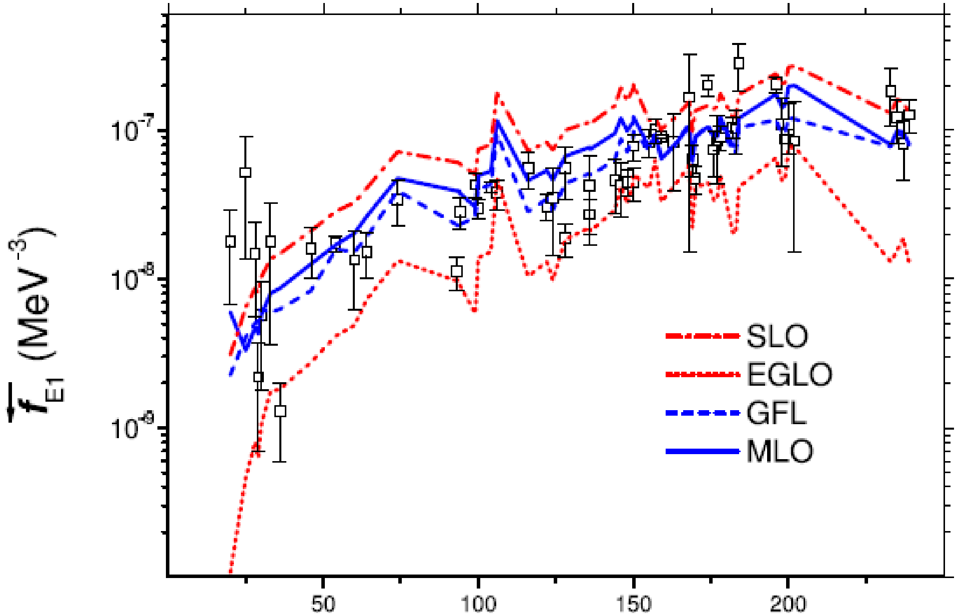
Lorentzians describe data in the IVGDR as well as below 12 MeV (= threshold for ^{90}Zr(γ,n))

R. Capote et al., NDS 110 (2009) 3107
www-nds.iaea.org/RIPL-3/

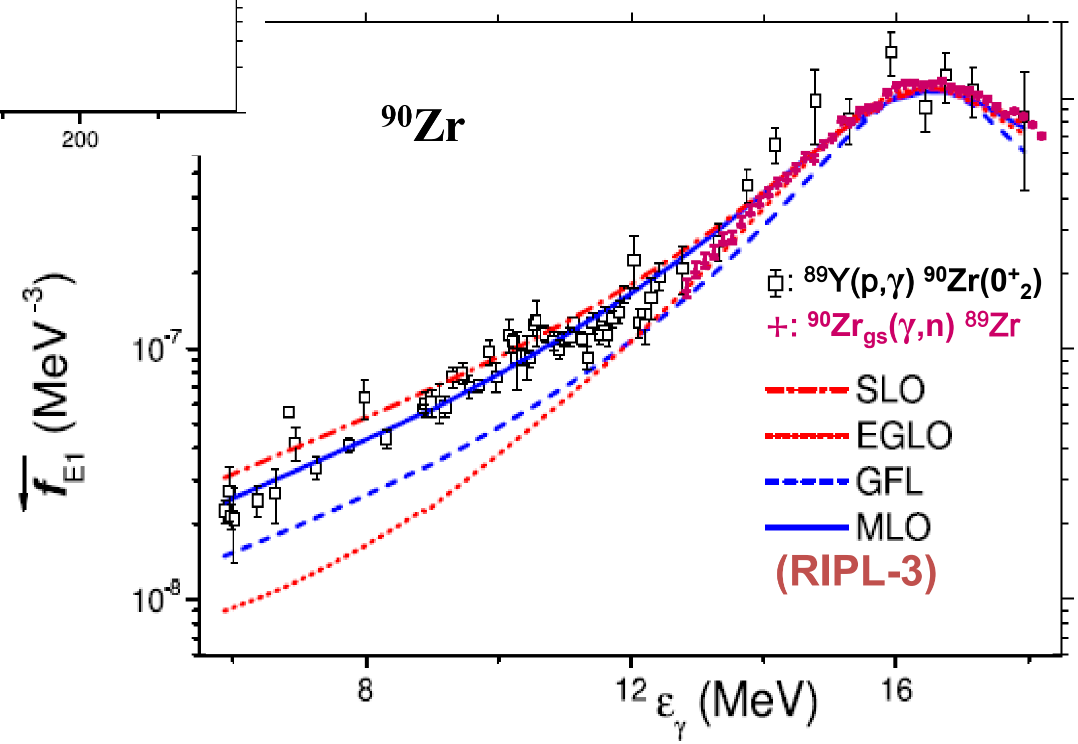
Z. Szeflinski et al., Phys.Lett.B 126 (1983)159
G.A.Bartholomew et al., Adv. in Nucl. Phys. 7(1973) 229

*Electric dipole **strength functions** in **RIPL-3.***

$$\frac{\overline{\Gamma_{lu}(E_\gamma)}}{\overline{E}_\gamma^{2\lambda+1}}\,\rho_u = f_\lambda(E_\gamma) = \frac{\overline{\sigma_{abs}(E_\gamma)}}{g\,(\pi\hbar c)^2\,\overline{E}_\gamma^{2\lambda\text{-}1}}$$



These 4 E1-*strength functions rely on **local** fits to the IVGDR to obtain a width* **Γ** *and an integrated strength* **I** *for each isotope; in the low energy slope* f_{E1} *is proportional to* **I** *and nearly* ∝ *to* **Γ**.



In contrast to this diversity

TLO *is a strength function with **globally** determined parameters:*

I *corresponds to TRK sum rule and*

Γ *stems from fit to >30 GDR-shapes:*

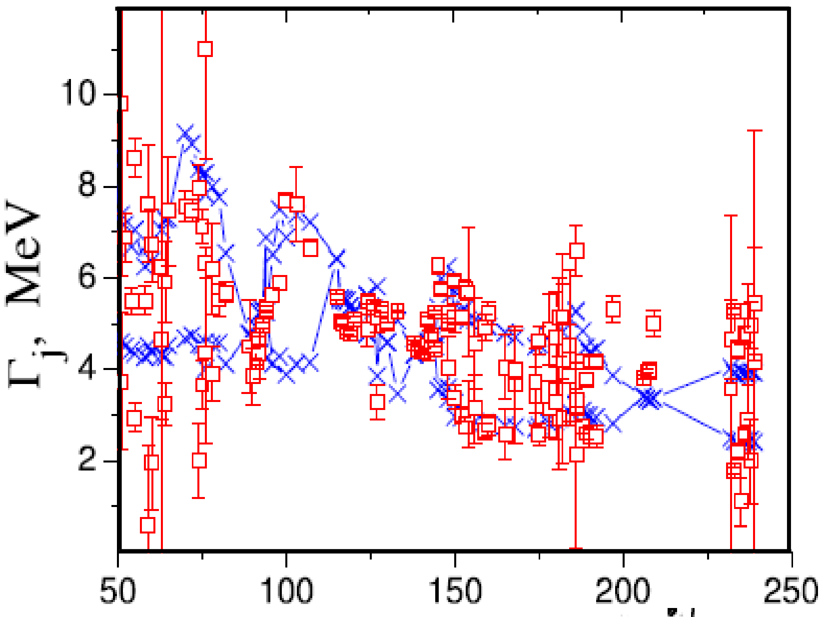
$\Gamma_k = 0.45\ E_k^{1.6}$

http://www-nds.iaea.org/RIPL-3/gamma/gamma-strength-exp.dat

R. Capote et al., NDS 110 (2009) 3107 www-nds.iaea.org/RIPL-3/

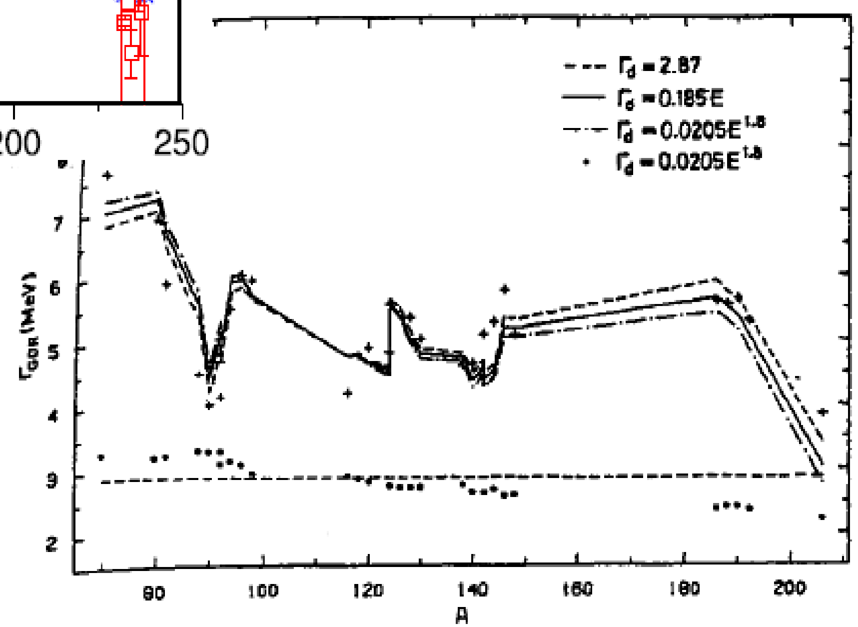
Local GDR fits => *erratically variing* Γ_r

=> ***Difference of TLO to previous parameterizations*** (TALYS, NON-SMOKER)



TALYS, EMPIRE, . . :
$\Leftarrow \oplus \Gamma_{E1}(E_\gamma) \propto E_\gamma^{\,2}$

NON-SMOKER:
$\Gamma_{E1}(E_\gamma) \propto E_\gamma^{\,1/2} \oplus \Longrightarrow$

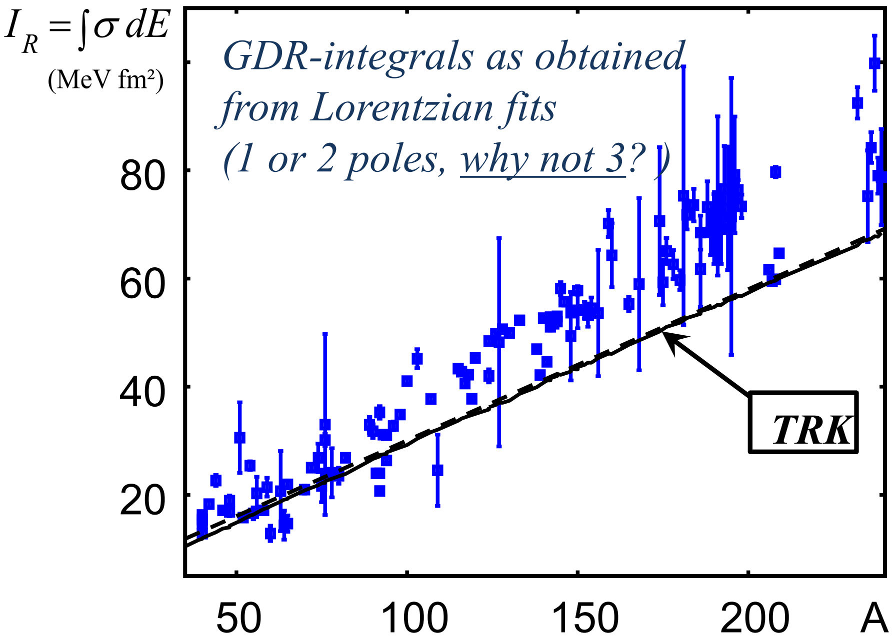


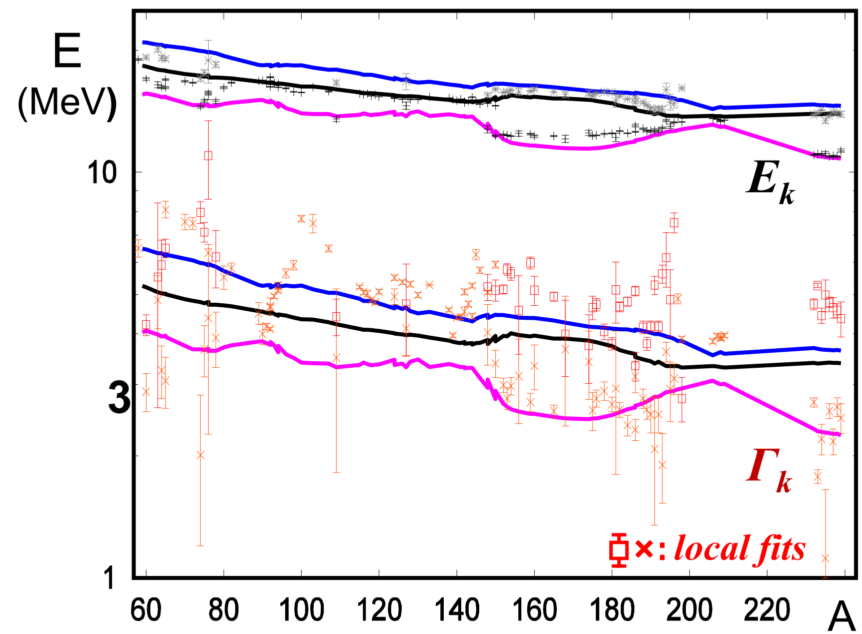
TLO *uses* **TRK,**
global fit *for A>70 and*
independence of Γ_{E1} *on* E_γ

V.A. Pluiko, www-nds.iaea.org/RIPL-3/gamma/gdrparameters-exp.dat;
ADNDT 97 (2011) 567; R. Capote et al., NDS 110 (2009) 3107

Thielemann & Arnould (1983) ; contr. to conf. on
Nucl. Data for Science and Techn., Böckhoff ed.

Local GDR fits => I_r, which often exceed TRK sum & erratically variing Γ_r





sum rule predicts dipole strength I varying only smoothly with A;

individual fits yield large scatter in I & Γ due to neglect of <u>triaxiality.</u>

<u>TLO</u> fixes I_R to TRK sum and E_k, Γ_k to LDM, hydrodynamics and βγ from HFB

$\Gamma_{E1}(E_\gamma) = const(E_\gamma)$

W. Kuhn, Z. Phys. 33 (1925) 408;
F. Reiche and W. Thomas, ibid. 34
M. Gell-Mann et al., PR 95 (1954) 1612

J.-P. Delaroche et al., PRC 81 (10) 014303
B. Bush and Y. Alhassid, NPA **531** (91) 27

V.A. Pluiko, www-nds.iaea.org/RIPL-3/gamma/gdrparameters-exp.dat;
ADNDT 2011; R. Capote et al., NDS 110 (2009) 3107